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Intertemporal Abatement Decisions under Ambiguity Aversion in a Cap and Trade*

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Abstract

We study intertemporal abatement decisions by an ambiguity averse firm covered under a cap and trade. Ambiguity aversion is introduced to account for the prevalence of regulatory uncertainty in existing cap-and-trade schemes. Ambiguity bears on both the future permit price and the firm's demand for permits. Ambiguity aversion drives equilibrium choices away from intertemporal efficiency and induces two effects: a pessimistic distortion of beliefs that overemphasises 'detrimental' outcomes and a shift in the effective discount factor. Permit allocation is non neutral and the firm's intertemporal abatement decisions do not solely depend on expected future permit prices, but also on its own expected future market position. In particular, pessimism leads the expected net short (resp. long) firm to overabate (resp. underabate) early on relative to intertemporal efficiency. We show that there is a general incentive for early overabatement and that it is more pronounced under auctioning that under free allocation.

Keywords: Emissions trading; Regulatory uncertainty; Permit banking; Ambiguity aversion.
JEL Classification codes: D81; D92; Q58.

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1 Introduction

As compared to standard markets a specificity of markets for pollution permits is that the supply of permits is fixed, exogenous and imposed by a regulatory authority. In other words, the scarcity created by the cap on emissions is of a political nature and pollution permits are not natural commodities with intrinsic value.[1] Rather, their value ultimately relates to the credibility of both the regulator in supervising the market and the regulatory scheme itself. Firms' anticipation and perception of the future regulatory stringency will therefore guide their abatement, compliance and clean technology investment strategies through time.

Moreover, carbon markets[2] do not function in a vacuum and are influenced by policies outside of their perimeters. External and uncertain factors such as macroeconomic conditions, the usage of offset credits for compliance and the reach of complementary policies can erode the stringency of the cap. Policy overlap[3] can be fortuitous as in the EU Emissions Trading System (ETS). It can also be explicitly built into an open regulatory system such as the California ETS which, operating in conjunction with a set of complimentary measures, constitutes a safety net ensuring the attainment of the state-wide target. In any case, this translates into significant uncertainty about baseline emission levels which Borenstein et al. (2015) estimate to be «at least as large as uncertainty about the effect of abatement measures».[4]

In most permit markets, permits can be banked for future demonstration of compliance. Under certainty this flexibility allows a least discounted cost solution and abatement is efficiently spread over time. In particular, absence of arbitrage requires that the permit price grows at the interest rate (Hotelling, 1931; Rubin, 1996).[5] In analysing the effects of regulatory interventions under discontinuous compliance, Hasegawa & Salant (2014) find that

«Permit markets may be subject to three kinds of uncertainty: (1) uncertainty about the aggregate demand for permits that will be resolved by an information disclosure at a fixed date in the future; (2) aggregate demand shocks in each period; and (3) regulatory uncertainty.»

[1] A pollution permit (or allowance) is not a property right per se because it can be limited, modified or simply cancelled. Having inherently 'ill-defined property rights' leaves flexibility to the regulator to react to realised shocks, whatever their nature, and adjust the design features of the scheme accordingly.

[2] Carbon markets are our leading example. The terms carbon and pollution will be used interchangeably. Notice that our analysis of the impacts of regulatory risks on firms' market behaviours also applies to other cases of 'insecure' natural resource tradable property rights, see e.g. Grainger & Costello (2014).

[3] Policy overlap is not limited to climate change and energy policies. For instance, Schmalensee & Stavins (2013) underline the impact of railway deregulation on the US SO_2 trading programme.

[4] Borenstein et al. (2015) show that significant baseline uncertainty coupled with little price elasticity is likely to conduce to either very high or very low price levels with high price volatility.

[5] Note that permits are commodities whose storage costs are negligible. The cost of carry price should therefore correspond to the spot price grown at the interest rate.

Only considering points (1-2) and provided that firms are risk neutral, the same rationale applies in the stochastic market equilibrium where intertemporal efficiency obtains in expectations (Schennach, 2000). Point (3), however, is essential in that discretionary regulatory interventions are known to distort intertemporal optimality of agents' decisions due to their anticipation of future regulatory actions (Kydland & Prescott, 1977; Salant & Henderson, 1978). In practice, such regulatory risks, be they upside or downside in nature, have shown to bear on permit prices.[6] In particular, Salant (2016) shows that regulatory uncertainty does weigh on price formation in the EUETS even under the assumption that agents are risk neutral.[7] This suggests that regulatory risks cannot be entirely hedged against. Note that Koch et al. (2016) find that the EUETS is highly responsive to political events and announcements, which gives empirical support to the theory developed by Salant.

Through permit banking, prices reflect expectations about future market developments and may comprise premiums associated with holding permits.[8] For instance, Bredin & Parsons (2016) show that the EUETS changed from initial backwardation to contango in late 2008. That is, in Phase II of the EUETS, futures prices were higher than cost of carry prices with implied premiums of significant sizes. This may suggest that firms were hedging themselves against increasing permit prices. Interestingly, Bredin & Parsons (2016) also note that this term structure reflects a sort of fear that is not consistent with the types of reforms discussed at the EU level.[9] Moreover, according to Hintermann et al. (2016), positive permit prices in oversupplied Phase II indicate banking on the part of firms as they expect a binding emissions constraint in the future and «because of their awareness of regulatory uncertainty».

Faced with regulatory uncertainty, firms lack confidence and/or relevant information to properly assign one probability measure uniquely describing the stochastic nature of their decision problems. This corresponds to a situation characterised by ambiguity. By contrast, risk refers to situations where such probabilities are perfectly known and unique. The paper thus examines intertemporal abatement decisions by a risk neutral ambiguity averse ETS-liable firm

[6] Examples are many. The price rise in early 2016 in the New Zealand ETS is attributable to the announcement that the 2:1 compliance rule should be abolished. Similarly, downward pressure on prices in Chinese pilots results from regulatory uncertainty about the transition to a national market, especially regarding the carry-over provision for pilot permits into the national market. Prices in RGGI increased when the 45% slash in the cap was under discussion, but before it was actually passed and implemented.

[7] Salant (2016) draws from his analysis of the «peso problem» and the gold spot price in the 70's that conflicted with the assumption of rational expectations under risk neutrality (Salant & Henderson, 1978).

[8] Permit prices may thus not be 'right' in that they may not reflect (intertemporal) marginal abatement costs. This wedge can also be sustained by other factors such as transaction costs or market power; see Hintermann et al. (2016) for a review of the empirical literature on the price determinants in the EUETS.

[9] As discussed, changes in the permit supply via the Market Stability Reserve or an adjustment of the annual cap-decreasing factor should shift the term structure as a whole, not just its slope.

to take account of the prevalence of regulatory uncertainty. Ambiguity neutrality constitutes our natural benchmark and corresponds to the situation where the firm's optimal abatement stream is determined by the least discounted expected cost solution (Schennach, 2000).

We consider a firm covered under a two-date cap and trade, i.e. the scheme starts at the beginning of date 1 and terminates at the end of date 2. We assume that the firm is already compliant at date 1 but can still undertake additional abatement and bank permits into date 2 in anticipation of date-2 requirements. At date 1, however, both the date-2 market price and the firm's demand for permits are ex-ante ambiguous and exogenous to the firm. This reflects that regulatory uncertainty (*i*) directly bears on price formation (Salant, 2016); (*ii*) also affects the firm's baseline level of emissions via direct or indirect policy overlaps (Borenstein et al., 2015). We note that regulatory uncertainty could also affect the firm's allocation of permits. However we choose to keep permit allocation as a parameter in the model to be able to measure its influence on intertemporal abatement decisions.[10] Indeed, we will show that neutrality of permit allocation does not hold under ambiguity aversion.[11]

Ambiguity entirely and exogenously resolves between the two dates.[12] We solve the firm's intertemporal cost minimisation programme by backward induction and compare the optimal level of date-1 abatement under ambiguity aversion relative to ambiguity neutrality. We consider a smooth ambiguity model of choice (Klibanoff et al., 2005) in which the firm is confronted with different possible scenarios about the future regulatory framework, i.e. objective probability distributions for the related permit price and demand forecasts, and has subjective beliefs over this set of scenarios.[13] Attitudes towards ambiguity originate in the relaxation of linearity between objective and subjective lotteries. Ambiguity aversion corresponds to the additional aversion (w.r.t. risk aversion) to being unsure about the probabilities of outcomes and conduces the firm to favour abatement streams that reduce the level of ambiguity.

We show that ambiguity aversion drives equilibrium abatement stream choices away from intertemporal efficiency. Before analysing the effects of joint permit price and firm's baseline ambiguities we consider each source of ambiguity in isolation. This will also allow us to separate the two ambiguity aversion induced effects. First, with pure baseline ambiguity

[10]Ultimately the firm's gross effort of abatement (baseline minus allocation) would be impacted by regulatory uncertainty, which we already capture by letting the baseline be ambiguous.

[11]Neutrality of allocation does not hold as soon as one of the assumptions sustaining the market equilibrium solution of Montgomery (1972) and Krupnick et al. (1983) is relaxed. See Hahn & Stavins (2011) for a review.

[12]Learning is perfect and exogenous to the firm because it can readily observe the prevailing market price and its own demand at date 2, and cannot influence the extent of learning by its date-1 actions.

[13]Consider for instance that these objective scenarios are provided by groups of experts, e.g. BNEF, Energy Aspects, ICIS-Tschach, Point Carbon, diverse academic fora or think tanks, etc.

and from the perspective of the risk neutral firm, the cap and trade can be assimilated to a tax regime where the tax is set at the expected permit price. Ambiguity aversion induces an upward (resp. downward) shift in the firm's discount factor when it exhibits Decreasing (resp. Increasing) Absolute Ambiguity Aversion. Early overabatement therefore occurs relative to the benchmark under DAAA, which we define as ambiguity prudence as in Berger (2014) and Gierlinger & Gollier (2017). We also note that the DAAA-induced increase in the discount factor can create a downward pressure on future permit prices.

Second, under pure price ambiguity, ambiguity aversion induces another effect by which the firm pessimistically distorts its subjective beliefs and overweights 'detrimental' scenarios. When the firm expects to be net short (resp. long) it will overemphasise scenarios where high (resp. low) permit prices are relatively more likely. This raises (resp. lowers) the firm's estimate of the future price relative to the benchmark and raises (resp. lowers) its incentive for early abatement accordingly. As compared to the benchmark the ambiguity averse firm does not solely base its present abatement decisions on the expected future permit price but also on its expected future market position. Note that this ultimately hinges upon the allocation of permits which is thus non-neutral. In particular, we identify allocation thresholds below (resp. above) which pessimism leads the firm to overabate (resp. underabate) early on.

Third, under both price and baseline ambiguities, we show that early overabatement occurs when the conditions for early overabatement under pure price ambiguity obtain and, in addition, high-price scenarios coincide with high-baseline scenarios. We then briefly extend the model and consider a continuum of firms identical but for permit allocation where the aggregate ambiguity on firms' baselines endogenously determines the ambiguous permit price. This allows us to refine the threshold condition on permit allocation and we show that there is a general tendency towards early overabatement under a symmetric allocation of permits. This can provide a behavioural explanation for the observed accumulation of unused, banked permits in all existing ETSs in addition to other permit-oversupply sustaining physical factors (Goulder, 2013; Newell et al., 2013; de Perthuis & Trotignon, 2014; Tvinnereim, 2014).[14]

The two ambiguity aversion induced effects can be aligned or countervailing, the direction and magnitude of which depend on both the degree of ambiguity aversion and permit allocation. An increase in ambiguity aversion always increases the magnitude of the pessimistic distortion in the sense of a monotone likelihood ratio deterioration (Gollier, 2011) and we

[14] In parallel, permit prices have declined and keep hovering at low levels or just above price floors when such price support mechanisms exist. This has sparked short-term regulatory interventions (e.g. ex post supply management) as well as structural design reforms (e.g. in the form of price or quantity-based containment permit reserves) in all existing ETSs that is adding to the level of regulatory uncertainty.

show that it can increase that of ambiguity prudence only when ambiguity prudence is not too strong relative to ambiguity aversion. Therefore, a higher degree of ambiguity aversion is not necessarily conducive to a larger adjustment in early abatement (in absolute terms). With a parametrical example we numerically show that early abatement decreases with allocation and that the magnitude of the pessimistic distortion is generally greater than that of the shift in the discount factor. This shows that, under ambiguity aversion, early abatement is higher under auctioning than free allocation and suggests that the distortion away from intertemporal efficiency is greater under a cap and trade than an emissions tax.

The remainder is organised as follows. Section 2 reviews the related literature. Section 3 presents our model and assumptions. Section 4 analyses the effects of ambiguity aversion on intertemporal abatement decisions relative to ambiguity neutrality. In particular, Section 4.1 considers the case of pure firm-level baseline ambiguity and Section 4.2 that of pure permit price ambiguity. The case of joint price and baseline ambiguities is presented in Section 4.3 while Section 4.4 considers the case of market-wide demand ambiguity with endogenous permit price. Finally, Section 5 illustrates our results numerically and Section 6 concludes.

2 Related literature

The paper combines two strands of literature, namely dynamic abatement and investment incentives under environmental policies and decision-making under ambiguity aversion.

Dynamic abatement and investment incentives. The paper first extends Baldursson & von der Fehr (2004) to ambiguity aversion. Similarly, Baldursson & von der Fehr show that risk averse firms that expect to be short (resp. long) on the permit market overinvest (resp. underinvest) in abatement technology relative to risk neutrality.[15] Note, however, that in our setup firms expecting to be net long (resp. short) can still overabate (resp. underabate) when they exhibit DAAA (resp. IAAA). In practice, there is an asymmetry between long and short entities since the former are under no compulsion to sell and can adopt a passive wait-and-see attitude as long as uncertainty is high and experience is being gained (Ellerman et al., 2010).[16] Note also that both cap and trade and an emissions tax deteriorate under

[15] Ben-David et al. (2000) find similar results which are also supported by laboratory experiments (Betz & Gunnthorsdottir, 2009). Note also that the design of the market (e.g. price containment mechanisms) will affect permit price formation and banking decisions (Holt & Shobe, 2016). See Kollenberg & Taschini (2016) for an analysis of the EUETS Market Stability Reserve with risk averse firms.

[16] In early Phase I of the EUETS industrial companies (acknowledged to be long) did not see as significant an effect of the carbon price on their output cost as power companies did (acknowledged to be short).

ambiguity aversion while a tax regime remains intertemporally efficient under risk aversion (Baldursson & von der Fehr, 2004).[17] The paper also extends Chevallier et al. (2011) who examine the impacts of a risk on permit allocation on firms' banking decisions.[18] They find that banking increases consecutive to an increase in risk if, and only if the third derivative of the firm's production function is positive. Relatedly, Colla et al. (2012) show that the presence of speculators with whom risk averse firms can trade permits augments the risk bearing capacity of the market and tends to reduce permit price volatility.

The paper follows the literature on dynamic investment incentives under environmental regulations in that it generally considers exogenous shocks on permit prices and firms' demands, see Requate (2005) for a review. Partial equilibrium models tend to favour tax over cap and trade essentially because in the latter the permit price can comprise a real option value and thus deviates from marginal abatement costs, see e.g. Xepapadeas (2001) with permit price uncertainty and Chao & Wilson (1993) with aggregate demand uncertainty. This literature further distinguishes between irreversible and reversible investments and generally shows that the former tend to decrease with uncertainty (Blyth et al., 2007) while the latter can be used as a hedge and tend to increase with uncertainty (Chen & Tseng, 2011).[19] For instance, Zhao (2003) finds that irreversible investment incentives decrease in the level of abatement cost uncertainty, but more so under a tax than an ETS. Note also the 'partial substitutability' between abatements and low-carbon investments (Slechten, 2013). Finally, Albrizio & Costa (2014) explicitly analyse the effects of policy uncertainty on irreversible and reversible investments by ETS-liable firms in a model where the regulator's preferences are observed (and the associated cap set) only once firms have made their investment decisions.

Decision-making under ambiguity. Following the seminal contribution of Ellsberg (1961) it is now well documented that most individuals treat ambiguity differently than objective risk, i.e. they prefer gambles with known rather than unknown probabilities.[20] There exist alternatives to Subjective Expected Utility (Savage, 1954), see Etner et al. (2012) and Machina & Siniscalchi (2014) for a review. These models of choice differ in their treatments of objective and subjective probabilities and preferences are no longer linear in the probabilities. They can roughly be grouped into three categories. The first category represents

[17] In addition, note that alternatively considering price and quantity regulations allows us to separate the two ambiguity aversion induced effects.

[18] If firms can pool risks banking may be a risk-management tool besides smoothing abatement over time.

[19] In the words of Laffont & Tirole (1996) low-emission investment constitutes a 'bypass' of permit markets.

[20] In particular, Ellsberg (1961) showed that rational decision-makers behaved in ways incompatible with the Savagian axiomatisation, e.g. the sure-thing principle.

non-additive beliefs, i.e. the probability of an outcome depends on its ranking among all possible outcomes (Schmeidler, 1989; Chateauneuf et al., 2007). The second category considers that agents have a set of multiple subjective priors. Gilboa & Schmeidler (1989) provided behavioural foundations for Multiple-priors (or Maximin) Expected Utility (MEU) preferences. Ghirardato et al. (2004) later axiomatised the α-maxmin model of choice which considers a convex combination of maximal and minimal expected utilities over the set of multiple priors.

The third category corresponds to Recursive Expected Utility models. In these models agents have a second-order subjective prior over a set of first-order objective measures and they are EU-maximisers over the two layers of uncertainty (Klibanoff et al., 2005, or KMM). Compared to the other two categories, a KMM model of choice has the advantage of disentangling ambiguity itself (or 'beliefs') from attitudes (or 'tastes') towards ambiguity. It comes with nice comparative statics and tractability properties to which the decision-making under risk machinery readily applies, can be embedded in a dynamic framework (Klibanoff et al., 2009) and nests other models of choice under ambiguity aversion as special cases.[21]

Ambiguity aversion has been applied to a variety of fields in economics, such as finance (Gollier, 2011; Gierlinger & Gollier, 2017), formation of precautionary savings (Berger, 2014), self-insurance and self-protection (Alary et al., 2013; Berger, 2016) or health (Treich, 2010; Berger et al., 2013), and can explain otherwise unaccounted for empirical facts such as the equity premium puzzle (Collard et al., 2016) or the negative correlation between asset prices and returns (Ju & Miao, 2012). Closer to our paper is the emerging theory of the competitive firm à la Sandmo (1971) under ambiguity aversion (Wong, 2015a) and the integration of risk and model uncertainty in Integrated Assessment Models (Millner et al., 2013; Berger et al., 2017). There is also mounting evidence that individuals tend to display ambiguity aversion and especially DAAA, see e.g. Berger & Bosetti (2016) and references therein.

The paper develops a two-period model to analyse what is fundamentally a fully-fledged dynamic problem. This is sufficient to capture the essence of the two ambiguity aversion induced effects and simplifies the problem at hand in two respects. First, considering more than two periods is technically difficult; for instance, Collard et al. (2016) assume CAAA to simplify Euler equations. This means that Collard et al. solely consider pessimism distortions while shifts in levels are abstracted away. Second, another difficulty relates to the incorporation of new information to update beliefs and preferences.[22] This issue is mechanically absent in

[21] When ϕ displays CAAA with $\phi(x) = \frac{e^{-\alpha x}}{-\alpha}$, Klibanoff et al. (2005) show that, under some conditions, the KMM model approaches the MEU criterion when the ambiguity aversion coefficient α tends to infinity.

[22] Note that Klibanoff et al. (2009) are able to retain dynamic consistency by defining preferences recursively, assuming 'rectangularity' of subjective beliefs together with prior-by-prior Bayesian updating, but do

our two-period model. Millner et al. (2013) opt for two polar exogenous learning scenarios: one where ambiguity resolves after the first period, the other with persistent and unchanged ambiguity throughout. Guerdjikova & Sciubba (2015) consider two similar types of learning structures, one where the true scenario is determined in the first period, another where the 'hidden' scenario is a Markov process and cannot never be identified, as in Ju & Miao (2012). Alternatively, Gierlinger & Gollier (2017) and Traeger (2014) use a one-step-ahead formulation consisting of nested sets of identical ambiguity structures.

3 The model

We consider a firm whose production's by-product is atmospheric pollution. The firm is regulated under a cap-and-trade system. To demonstrate compliance the firm can abate emissions and/or buy pollution permits on the market. There are two dates $t = 1, 2$. At date 1, the date-2 permit price τ and the firm's date-2 baseline level of emissions b (or production output) are ambiguous in a sense that will be defined below. Ambiguity vanishes at the beginning of date 2, i.e. the firm's date-2 abatement depends on its date-1 abatement and the price and baseline realisations. We then analyse the firm's optimal date-1 abatement decisions under ambiguity aversion relative to the ambiguity neutral benchmark.

The economic environment. Regulation is effective at both dates and terminates at the end of date 2. As in Chevallier et al. (2011) we assume that date-1 compliance is effective and that all inter-firm permit trading opportunities on the market are exhausted. The firm may still undertake additional date-1 abatement a_1 in the perspective of more stringent date-2 requirements. This frees up a corresponding amount of permits that are banked into date 2.[23] This assumption ensures that the Rubin-Schennach banking condition is always satisfied and assumes corner solutions away (Rubin, 1996; Schennach, 2000). There are two alternative descriptions of this framework where regulation is effective at date 2 only. For instance a_1 may also correspond to (i) investments in abatement technology in anticipation of future regulation; or (ii) 'early reduction permits' handed out to the firm for its early abatements.[24] Given an abatement stream $(a_1; a_2)$ the firm's date-2 level of emissions is

not accommodate the dynamic three-color-urn Ellsberg example in Epstein & Schneider (2003).

[23]In the case of the EUETS presented in the Introduction, date 1 corresponds to Phase II with a non-binding constraint on emissions and date 2 to Phase III and beyond with an expected permit scarcity.

[24]These interpretations are equivalent provided that a given level of abatement or investment cuts down emissions by a corresponding amount, and that date-1 abatement or investment reduces both date-1 and date-2 emissions by the same amount. Notice, abatements and investments are substitutes (Slechten, 2013).

thus $b - a_1 - a_2$. Let ω denote the firm's endowment of permits at date 2. Then a positive (resp. negative) value for $b - a_1 - a_2 - \omega$ denotes a short (resp. long) market position and the amount of permits it has to buy (resp. can sell) on the market at date 2. Abatement cost functions are given by twice continuously differentiable functions C_1 and C_2. Abatement is said to have long-term effect in the sense that C_2 also depends on the level of date-1 abatement, i.e. $C_2 \equiv C_2(a_1, a_2)$. The marginal cost of date-1 abatement is thus $\partial_{a_1}(C_1 + C_2)$. Abatement costs are assumed to be strictly increasing and convex on $[0; \infty)$ with no fixed cost, i.e. $C_1', C_1'' > 0$ with $C_1(0) = C_1'(0) = 0$ and $\partial_{a_2} C_2, \partial^2_{a_2 a_2} C_2 > 0$ with $C_2(\cdot, 0) = 0$. The firm also faces decreasing abatement opportunities (Bréchet & Jouvet, 2008), i.e. $\partial^2_{a_1 a_2} C_2 \geq 0$. This is compensated by a positive learning-by-doing effect (Slechten, 2013) which is captured by assuming that $\partial^2_{a_1 a_1}(C_1 + C_2) \geq \partial^2_{a_1 a_2} C_2$ and $\partial^2_{a_2 a_2} C_2 \geq \partial^2_{a_1 a_2} C_2$. When we want to be able to derive analytical results we will assume that abatement cost functions are equipped with the following quadratic specification where for all $a_1, a_2 \geq 0$

$$C_1(a_1) = c_1 a_1^2/2 \text{ and } C_2(a_1, a_2) = c_2 a_2^2/2 + \gamma a_1 a_2, \tag{1}$$

with $c_1, c_2 > 0$ and $c_2 > \gamma$ for our assumptions on abatement costs to obtain. Assuming a quadratic specification is a usual and mild assumption (Newell & Stavins, 2003).[25] This also allows us to clearly single out the effects of ambiguity aversion on optimal abatement streams as it guarantees intertemporal efficiency under ambiguity neutrality (see Proposition 4.3). Note that $1/c_t$ measures the firm's flexibility in abatement at date t and γ denotes the long-term abatement effect coefficient. For tractability reasons we will sometimes need to assume that there is no long-term effect of abatement, i.e. $\partial_{a_1} C_2 \equiv 0$ or $\gamma = 0$.

The firm's objective under uncertainty. We consider a partial-equilibrium model that focuses solely on the firm's abatement and permit trading decisions. The model ignores both the interactions with the goods' market and the firm's production decisions.[26] Denote by $\zeta_t > 0$ the firm's net profits on the goods' markets at date $t = 1, 2$ that are independent of the firm's volume of emissions. To solve for the firm's optimal abatement stream we proceed in two steps using backward induction. At date 2 the firm observes the couple (τ, b) of realised

[25] This corresponds to a second-order Taylor expansion of abatement cost functions centred around baselines. A linear term would merely translate our results by a constant term and is thus omitted for convenience.

[26] This is a restrictive yet usual assumption, see e.g. Zhao (2003) and Baldursson & von der Fehr (2004). It can be justified if firms produce different goods and/or belong to different sectors. While an interaction between the goods' market and environmental policy undoubtedly exists, its direction and magnitude are uncertain. For instance, Martin et al. (2014) show that the UK carbon tax has reduced both energy use and intensity, but find no evidence of impacts on employment or production. See Requate (1998) and Baldursson & von der Fehr (2012) for a treatment of the interaction between permit trading and the output market.

permit market price and individual level of baseline emissions. For any given level of date-1 abatement $a_1 \geq 0$ the firm maximises its date-2 profit, that is

$$\max_{a_2 \geq 0} \pi_2(a_1, a_2; \tau, b) = \zeta_2 - C_2(a_1, a_2) - \tau(b - a_1 - a_2 - \omega). \tag{2}$$

Date-2 optimality requires that $\partial_{a_2} C_2(a_1, a_2^*) = \tau$, where the optimal date-2 abatement is implicitly defined such that $a_2^* \equiv a_2^*(a_1; \tau)$. With cost specification (1) it comes that

$$a_2^*(a_1; \tau) = (\tau - \gamma a_1)/c_2. \tag{3}$$

At date 1, however, both the date-2 permit price and baseline emissions are uncertain. Let the price risk $\tilde{\tau}$ be described by the objective cumulative distribution G^0 supported on $\mathrm{T} = [\underline{\tau}; \bar{\tau}]$ with $0 < \underline{\tau} < \bar{\tau} < \infty$. Let also the baseline risk $\tilde{b}$ be described by the objective cumulative distribution L^0 with support on $\mathrm{B} = [\underline{b}; \bar{b}]$ with $0 < \underline{b} < \bar{b} < \infty$. These two risks are assumed to be independent, i.e. there is no connection between the prevailing market price and the firm's baseline.[27] This parallels a frequent assumption in the literature on firms' decisions under uncertainty that price and production shocks are independent stochastic variables (Viaene & Zilcha, 1998; Dalal & Alghalith, 2009). We consider that the firm is risk neutral. The firm's date-1 optimal abatement decision thus satisfies

$$\bar{a}_1 \doteq \arg\max_{a_1 \geq 0} \left\langle \pi_1(a_1) + \beta \mathbb{E}_{G^0, L^0} \left\{ \pi_2(a_1, a_2^*(a_1; \tilde{\tau}); \tilde{\tau}, \tilde{b}) \right\} \right\rangle, \tag{4}$$

where $\beta \in [0; 1]$ is the firm's discount factor and $\pi_1(a_1) = \zeta_1 - C_1(a_1)$ is the date-1 profit (note the absence of trade terms). Combining optimality conditions at both dates then yields

$$C_1'(\bar{a}_1) + \beta \mathbb{E}_{G^0} \left\{ \partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau})) \right\} = \beta \langle \tilde{\tau} \rangle = \beta \mathbb{E}_{G^0} \left\{ \partial_{a_2} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau})) \right\}, \tag{5}$$

where $\langle \tilde{\tau} \rangle \doteq \mathbb{E}_{G^0}\{\tilde{\tau}\}$ is the expected permit price. Therefore intertemporal efficiency obtains in expectations since expected marginal abatement costs are equated at both dates. For a price realisation $\tau \in \mathrm{T}$ the abatement stream $(\bar{a}_1; a_2^*(\bar{a}_1; \tau))$ corresponds to the Rubin-Schennach least discounted cost solution (Rubin, 1996; Schennach, 2000). Note that Equation (5) is independent of both the firm's baseline risk and permit allocation ω. With quadratic

[27] The cap stringency is determined by the difference between the aggregate level of baseline emissions and the cap. The permit price should reflect the shadow price associated with this constraint. Independence can be justified if the market for permits is competitive or because «there is a complex interaction between BAU emissions, abatement quantities, and allowance prices» (Hintermann et al., 2016). Section 4.4 considers the case of an endogenous price that solely reflects the cap stringency.

cost specification (1) it comes

$$\bar{a}_1 = \frac{(c_2 - \gamma)\beta \langle \tilde{\tau} \rangle}{c_1 c_2 - \beta\gamma^2}. \tag{6}$$

Due to quadratic abatement cost functions the optimal level of date-1 abatement under uncertainty $\bar{a}_1$ is invariant to any mean-preserving spread in $\tilde{\tau}$, cf. Proposition 4.3. It is also clear from Equation (6) that $\bar{a}_1$ is solely dictated by the discounted expected date-2 permit price and does not depend on the expected market position at date 2.

Introduction of ambiguity. We further let the market price and the firm's baseline be ambiguous. Ambiguity is introduced in the sense of Klibanoff et al. (2005) and the firm is uncertain about G^0 and L^0. More precisely, the firm is confronted with a set of objective probability measures for both $\tilde{\tau}$ and $\tilde{b}$ and is uncertain about which of those truly govern the two risks. For each realisation θ (called θ-scenario) of the random variable $\tilde{\theta}$ we let $G(\cdot;\theta)$ and $L(\cdot;\theta)$ denote the objective probability measures for $\tilde{\tau}_\theta$ and $\tilde{b}_\theta$, the θ-scenario price and baseline risks, respectively. Ambiguity is represented by a second-order subjective probability distribution for $\tilde{\theta}$ denoted F and supported on $\Theta = [\underline{\theta};\bar{\theta}]$. The measure F represents the firm's beliefs about which scenario it feels will materialise. While we consider that G and L are second-order dependent across θ-scenarios we assume for consistency with the uncertain case that G and L are first-order independent given a θ-scenario, i.e. $\mathbb{E}_{G,L}\{\cdot|\theta\} \equiv \mathbb{E}_G\{\cdot|\theta\}\mathbb{E}_L\{\cdot|\theta\}$. An ambiguity neutral firm compounds first and second order lotteries and corresponds to a Savagian expected profit maximiser w.r.t. the compound risk measures $\bar{G} \doteq \mathbb{E}_F\{G(\cdot;\tilde{\theta})\}$ and $\bar{L} \doteq \mathbb{E}_F\{L(\cdot;\tilde{\theta})\}$. We assume that there is no bias in the ambiguity neutral firm's beliefs, i.e. $\bar{G} \equiv G^0$ and $\bar{L} \equiv L^0$. That is, an ambiguity neutral firm is not affected by the introduction of ambiguity nor a shift in the level of ambiguity.

Ambiguity aversion. Attitudes towards ambiguity originate in the relaxation of the reduction of compound first and second order lotteries. The construction of the firm's objective can be decomposed into three steps: first, in any given θ-scenario the firm computes its expected date-2 profits w.r.t. $G(\cdot;\theta)$ and $L(\cdot;\theta)$; second, each θ-scenario first-order expected date-2 profits is transformed by an increasing function ϕ; third, the firm's second-order expected date-2 profits obtain by taking the expectation of the ϕ-transformed first-order expected date-2 profits w.r.t. F. Ambiguity aversion is characterised by a concave ambiguity function ϕ. As defined in Equation (9a) we denote by $\mathcal{V}(a_1;\theta)$ the firm's expected profit at date 2 in scenario $\theta \in \Theta$ when it abates $a_1 \geq 0$ at date 1. Under ambiguity aversion, Jensen's inequality yields

$$\phi^{-1}\Big(\mathbb{E}_F\{\phi(\mathcal{V}(a_1;\tilde{\theta})\}\Big) \leq \mathbb{E}_F\{\mathcal{V}(a_1;\tilde{\theta})\}. \tag{7}$$

The left-hand side of Inequality (7) is the date-2 ϕ-certainty equivalent expected profit while the right-hand side corresponds to ambiguity neutrality (ϕ is linear) since expectations is taken w.r.t. compound probability distributions. In words, the ambiguity averse firm dislikes any mean-preserving spread in the space of second-order expected profits. Finally note that since the firm is taken to be risk neutral the function ϕ actually characterises aversion towards model uncertainty (Marinacci, 2015).[28] Because ambiguity aversion requires stronger aversion towards model uncertainty than towards risk our assumption leads to an overestimation of the effects of ambiguity aversion (Berger & Bosetti, 2016). However this assumption allows us to derive clear analytical results and provide interesting insights.[29]

The firm's objective under ambiguity. Note that date-2 optimality and Equation (3) hold irrespective of both the presence of ambiguity and the firm's attitude towards ambiguity. However the optimal date-1 abatement decision under ambiguity aversion hinges upon the ambiguity level, as perceived from date 1, in conjunction with the degree of ambiguity aversion. We use the recursive smooth ambiguity model of choice of Klibanoff et al. (2009).[30] Since ambiguity is resolved at the beginning of date 2 the firm's programme writes

$$\max_{a_1 \geq 0} \pi_1(a_1) + \beta\phi^{-1}\Big(\mathbb{E}_F\{\phi(\mathcal{V}(a_1;\tilde{\theta}))\}\Big), \tag{8}$$

where the θ-scenario-expected profitability from date-1 abatement $\mathcal{V}(a_1;\tilde{\theta})$ satisfies

$$\mathcal{V}(a_1;\theta) \doteq \mathbb{E}_{G,L}\{\tilde{V}(a_1;\theta)|\theta\}, \tag{9a}$$

$$\text{where, } \tilde{V}(a_1;\theta) \doteq \max_{a_2} \pi_2(a_1, a_2; \tilde{\tau}_\theta, \tilde{b}_\theta). \tag{9b}$$

In the above $\mathbb{E}_F$ denotes the expectation parameter taken w.r.t. F conditional on all relevant information available to the firm at date 1. Similarly $\mathbb{E}_{G,L}\{\cdot|\theta\}$ denotes the expectation

[28]See Guetlein (2016) for comparative static results on risk aversion under smooth ambiguity aversion. Risk neutrality on the part of firms is a standard assumption as they should be able to diversify risk. Firms can still exhibit ambiguity aversion which is a different psychological trait. Note that there is empirical evidence of ambiguity aversion for actuaries (Cabantous, 2007). Moreover, Brunette et al. (2015) show that individuals are less risk averse but more ambiguity averse in a group than alone. If we see firms as groups of individuals making joint decisions this may underpin our assumption of a risk-neutral ambiguity-averse firm.

[29]If the firm was risk averse and maximised the utility of its profits at each date, joint conditions on both the utility and ambiguity functions would emerge to determine the direction of the date-1 abatement adjustment, see e.g. Gierlinger & Gollier (2017), Berger (2016) and Wong (2015a). In particular, the criterion to sign pessimism in Proposition 4.4 would have to be restated. In our case this also renders the firm's programme ill-defined since e.g. the date-1 profit is a decreasing function of abatement.

[30]Note that in Klibanoff et al. (2009) the scenario space Θ is finite. Here we consider its continuous extension with a continuous subjective distribution F. Note also that the KMM axiomatisation is based on acts rather than probability distribution on the outcome spaces T and B.

parameter taken w.r.t. $G(\cdot;\theta)$ and $L(\cdot;\theta)$ conditional on the true scenario being θ. Note that Programme (8) is well-defined provided that ambiguity tolerance $-\phi'/\phi''$ is concave (Gierlinger & Gollier, 2017; Berger, 2016). Since this condition is satisfied for the standard ϕ functions we use for numerical simulations in Section 5 we assume that it holds throughout the paper. It follows from the Envelop Theorem applied to $\tilde{V}$ that for all $a_1 \geq 0$ and $\theta \in \Theta$

$$\tilde{V}_{a_1}(a_1;\theta) = \tilde{\tau}_\theta - \partial_{a_1} C_2(a_1, a_2^*(a_1;\tilde{\tau}_\theta)). \tag{10}$$

With quadratic specification (1) the θ-scenario-expected marginal profitability from date-1 abatement reads

$$\mathcal{V}_{a_1}(a_1;\theta) = ((c_2-\gamma)\bar{\tau}_\theta + \gamma^2 a_1)/c_2, \tag{11}$$

where $\bar{\tau}_\theta \doteq \mathbb{E}_G\left\{\tilde{\tau}_\theta|\theta\right\}$ denotes the expected permit price at date 2 in scenario $\theta \in \Theta$. By construction $\mathcal{V}_{a_1}$ is positive, i.e. the firm always have an incentive to bank at date 1.

4 Optimal abatement decisions under ambiguity

4.1 Tax regime: Cap and trade under firm's baseline ambiguity

First consider that $\partial_\theta G(\cdot;\theta) \equiv 0$. Because the firm is risk neutral this situation can be assimilated to a tax regime where the date-2 proportional tax rate on emissions is $\mu \doteq \mathbb{E}_{\bar{G}}\{\tilde{\tau}\}$.[31] In this case, ω can be interpreted as a tax-threshold liability where the tariff is charged only on the difference between emissions and the threshold (Pezzey & Jotzo, 2013). Note that the θ-scenario-expected marginal profitability from date-1 abatement satisfies $\mathcal{V}_{a_1}(a_1;\theta) = \mu - \partial_{a_1} C_2(a_1, a_2^*(a_1;\mu)) > 0$ where both μ and $\partial_{a_1} C_2$ are deterministic. Hence $\mathcal{V}_{a_1}$ is deterministic and does not depend on the θ-scenario considered.

Ambiguity neutrality. With a linear ϕ the necessary first-order condition for Programme (8) defines the optimal level of date-1 abatement under ambiguity neutrality by

$$-C_1'(\bar{a}_1^\mu) + \beta \mathcal{V}_{a_1}(\bar{a}_1^\mu) = 0. \tag{12}$$

[31] The date-2 tax rate is thus certain and exogenously given. Its optimality, whatever the underlying optimality criterion, is not the focus of the paper. Note that the tax regime is such that the date-1 tax rate is zero. This is without loss of generality and roughly captures that tax rates generally rise over time.

Combining optimality conditions at both dates then yields

$$C_1'(\bar{a}_1^\mu) + \beta\partial_{a_1} C_2(\bar{a}_1^\mu, a_2^*(\bar{a}_1^\mu, \mu)) = \beta\mu = \beta\partial_{a_2} C_2(\bar{a}_1^\mu, a_2^*(\bar{a}_1^\mu; \mu)). \tag{13}$$

The (aggregate) marginal date-1 abatement cost is equated to the marginal date-2 abatement cost, i.e. intertemporal efficiency obtains. With quadratic specification (1) the optimal abatement stream is $(\bar{a}_1^\mu; a_2^*(\bar{a}_1^\mu; \mu))$, where $\bar{a}_1^\mu$ obtains from Equation (6) with $\mu = \langle\tilde{\tau}\rangle$. With no long-term dependency (i.e. $\gamma = 0$), a_2^* is independent of a_1 and the firm's overall level of abatement under ambiguity neutrality is $\bar{a}_1^\mu + a_2^*(\mu) = \beta\mu/c$ where $1/c = 1/c_1 + 1/(\beta c_2)$ is the firm's aggregate flexibility in abatement over the two dates. The overall abatement volume is efficiently apportioned between the two dates

$$\bar{a}_1^\mu = \frac{c}{c_1}\Big(\frac{\beta\mu}{c}\Big) \text{ and } a_2^*(\mu) = \frac{c}{\beta c_2}\Big(\frac{\beta\mu}{c}\Big), \tag{14}$$

that is in proportion to each date abatement flexibility. Notice, the ambiguity neutral benchmark corresponds to a decision under risk, here for a risk neutral firm. Baldursson & von der Fehr (2004) show that intertemporal efficiency continues to hold in a tax regime under risk aversion. As exposed below, however, this does not carry over to ambiguity aversion.

Ambiguity Aversion. With a concave ϕ the necessary first-order condition for Programme (8) defines the optimal level of date-1 abatement under ambiguity neutrality by

$$-C_1'(\hat{a}_1^\mu) + \beta\mathcal{A}(\hat{a}_1^\mu)\mathcal{V}_{a_1}(\hat{a}_1^\mu) = 0, \tag{15}$$

where the shift in levels $\mathcal{A}$ is a function defined by

$$\mathcal{A}(a_1) \doteq \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1;\tilde{\theta}))\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(a_1;\tilde{\theta}))\})}. \tag{16}$$

Proposition 4.1 characterises the impact of ambiguity aversion on the firm's optimal date-1 abatement decision relative to ambiguity neutrality.

Proposition 4.1. *Ambiguity aversion is conducive to higher (resp. lower) date-1 abatement than under ambiguity neutrality if, and only if, the firm displays Decreasing (resp. Increasing) Absolute Ambiguity Aversion. Moreover, under Constant Absolute Ambiguity Aversion, the introduction of ambiguity aversion does not affect the firm's date-1 abatement decision.*

Proof. Relegated to Appendix A.1. □

Except when the firm displays CAAA the tax regime is not intertemporally efficient under ambiguity aversion. This suggests that the relative merits of an emissions tax *vs.* emissions trading highlighted by Baldursson & von der Fehr (2004) under risk aversion would tend to fade away under ambiguity aversion. Moreover, Proposition 4.1 is in line with the literature on the formation of precautionary saving under ambiguity aversion, e.g. Osaki & Schlesinger (2014) and Gierlinger & Gollier (2017). Because the firm overabates at date 1 relative ambiguity neutrality i.f.f. it exhibits DAAA, we follow Berger (2014) and Gierlinger & Gollier (2017) in assimilating DAAA with prudence towards ambiguity.[32] With this definition,

Corollary 4.2. *The firm overabates at date 1 relative to ambiguity neutrality, i.e. forms precautionary date-1 abatement, if, and only if, it displays prudence towards ambiguity.*

Comparing the optimality conditions under ambiguity neutrality (Equation (12)) and ambiguity aversion (Equation (15)) wee see that ambiguity aversion operates a shift in the firm's 'effective' discount factor from β to $\beta\mathcal{A}$. A value higher than unity for function $\mathcal{A}$ indicates ambiguity prudence and the discount factor is shifted up (resp. down) when the firm exhibits DAAA (resp. IAAA). In words, ambiguity prudence puts relatively more weight on date-2 profits than under ambiguity neutrality − lowering impatience, as it were − which leads to date-1 overabatement. Another interpretation is that DAAA intensifies the importance of any date-2 profit risk and can thus be assimilated to a «preference for an earlier resolution of uncertainty», see e.g. Theorem 4 in Strzalecki (2013).

4.2 Cap and trade under pure permit price ambiguity

Now let $\partial_\theta L(\cdot;\theta) \equiv 0$. This corresponds to a cap-and-trade regime under pure price ambiguity, i.e. ambiguity is extrinsic to the firm and transmitted via the permit price only. Without loss of generality further let the firm's baseline b be certain for convenience.

[32] We note that this definition is presently unsettled. For instance, Baillon (2017) defines ambiguity prudence by the less demanding condition that ϕ''' be positive (DAAA $\Rightarrow \phi''' > 0$). This definition parallels that of risk prudence under Expected Utility and can be defined in terms of lotteries. However, $\phi''' > 0$ is not sufficient to guarantee the formation of precautionary banking with the KMM certainty equivalent representation theorem we use. For instance, adopting an approach similar to Kimball (1990), Osaki & Schlesinger (2014) show that only under DAAA is the ambiguity precautionary premium bigger than the ambiguity premium. DAAA is thus the 'natural' definition for ambiguity prudence in our analysis. Note that Berger (2016) underlines the similarity between the KMM and Kreps-Porteus/Selden recursive formulations: just like DARA is required for precautionary saving under risk aversion in the K-P/S models, is DAAA required for precautionary saving under ambiguity aversion in the KMM framework. It could be argued that DAAA should be a standard assumption as it parallels the widely accepted DARA property for risk attitudes. Finally note that there is empirical evidence for DAAA (Berger & Bosetti, 2016).

Ambiguity neutrality. With a linear ϕ the necessary first-order condition for Programme (8) defines the optimal level of date-1 abatement under ambiguity neutrality by

$$-C_1'(\bar{a}_1) + \beta \mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\} = 0. \tag{17}$$

Since the ambiguity neutral firm compounds lotteries and we assume its beliefs are unbiased, Equation (17) coincides with the first-order condition for Programme (4) under uncertainty. Intertemporal efficiency hence obtains in expectations, see Equation (5). As is standard the effects of uncertainty on optimal decisions relate to the third derivative of the profit (utility) function (Leland, 1968; Kimball, 1990). In our case in particular,

Proposition 4.3. *Assume time separability, i.e. $\partial_{a_1} C_2 \equiv 0$. Then, in the face of an increase in uncertainty in the sense of a mean-preserving spread (Rothschild & Stiglitz, 1971), the ambiguity neutral firm overabates at date 1 if, and only if, $C_2''' > 0$.*

Proof. Relegated to Appendix A.2 □

Therefore, date-1 overabatement under ambiguity neutrality is conditional on the positivity of the third derivative of the abatement cost function. Note that Chevallier et al. (2011) obtain a similar result with a risk on future allocation of permits. Note again from Equation (6) that $\bar{a}_1$ does not depend on the expected market position at date 2 and that, with the quadratic costs (1), it is invariant to any mean-preserving spread in $\tilde{\tau}$ also.

Ambiguity aversion. With a concave ϕ the necessary first-order condition for Programme (8) defines the optimal level of date-1 abatement under ambiguity aversion by

$$-C_1'(\hat{a}_1) + \beta \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\})} = 0. \tag{18}$$

Normalising and decomposing the fraction in Equation (18) into two terms yields

$$-C_1'(\hat{a}_1) + \beta \mathcal{A}(\hat{a}_1)\mathbb{E}_F\{\mathcal{D}(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\} = 0, \tag{19}$$

where function $\mathcal{A}$ is defined in Equation (16) and $\mathcal{D}$ is a distortion function satisfying, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$\mathcal{D}(a_1;\theta) \doteq \frac{\phi'\left(\mathcal{V}(a_1;\theta)\right)}{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1;\tilde{\theta}))\}}. \tag{20}$$

In addition to the shift in levels $\mathcal{A}$, there is another ambiguity aversion induced effect via $\mathcal{D}$ which distorts the second-order subjective prior F. By concavity of ϕ the distortion function

$\mathcal{D}$ overweights those θ-scenarios with low-$\mathcal{V}$ values.[33] This can be interpreted as pessimism in the sense of a monotone likelihood ratio (MLR) deterioration (Gollier, 2011; Gierlinger & Gollier, 2017). In particular, the pessimistically distorted second-order subjective measure H is such that, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$H(a_1;\theta) \doteq \int_{\underline{\theta}}^{\theta} \mathcal{D}(a_1;X)\mathrm{d}F(X) = \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1;\tilde{X}))|\tilde{X} \leq \theta\}}{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1;\tilde{\theta}))\}}F(\theta), \tag{21}$$

with $H(\cdot;\underline{\theta}) = 0$, $H(\cdot;\bar{\theta}) = 1$ and $\partial_\theta H(\cdot;\theta) > 0$. By concavity of the objective function,

$$\hat{a}_1 \geq \bar{a}_1 \;\Leftrightarrow\; \mathcal{A}(\bar{a}_1)\mathbb{E}_H\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\} \geq \mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\}. \tag{22}$$

Controlling for the shift in levels $\mathcal{A}$, introducing ambiguity in the ambiguity averse firm's decision is identical to a shift in the ambiguity neutral firm's subjective beliefs from F to H, where H overemphasises low-profit θ-scenarios relative to F. Intuitively, this will be conducive to date-1 overabatement provided that these 'low-profit' scenarios have 'high' marginal profitabilities from date-1 abatement.

Proposition 4.4. *Under CAAA, pessimism raises date-1 abatement relative to ambiguity neutrality if, and only if,* $(\mathcal{V}(\bar{a}_1;\theta))_\theta$ *and* $(\mathcal{V}_{a_1}(\bar{a}_1;\theta))_\theta$ *are anticomonotone. Otherwise the two ambiguity aversion induced effects can be aligned or countervailing. When* ϕ *exhibits DAAA (resp. IAAA) ambiguity aversion is conducive to higher (resp. lower) date-1 abatement than under ambiguity neutrality only if anticomonotonicity (resp. comonotonicity) holds.*

Proof. Relegated to Appendix A.3. □

In words, anticomonotonicity requires that low-$\mathcal{V}$ scenarios coincide with high-$\mathcal{V}_{a_1}$ scenarios. Controlling for the shift in levels $\mathcal{A}$, this ensures that the firm overabates at date 1 relative to ambiguity neutrality. Note that similar (anti)comonotonicity criteria obtain with other representation theorems to sign the effects of ambiguity aversion, see Appendix C for MEU and α-maxmin preferences. The underlying relation between (anti)comonotonicity and pessimism is further illustrated in Examples 4.5 and 4.6.

Example 4.5. *Let* $\Theta = \{\theta_1, \theta_2\}$, $F = (q, \theta_1; 1-q, \theta_2)$ *with* $0 \leq q \leq 1$ *and* ϕ *exhibit CAAA* ($\mathcal{A} \equiv 1$). *Assume that* $\mathcal{V}(\cdot;\theta_2) \geq \mathcal{V}(\cdot;\theta_1)$. *Pessimism thus overweights scenario* θ_1 *relative*

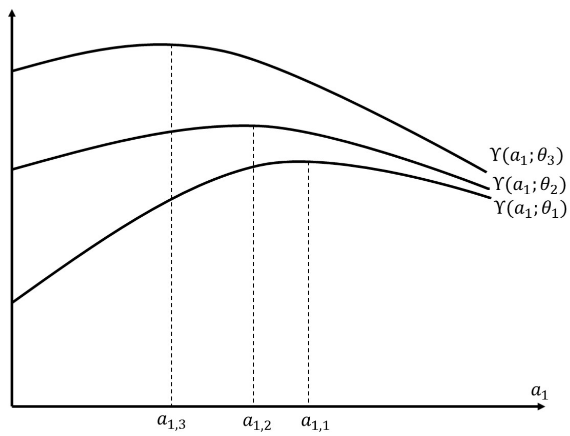
[33] The distortion function $\mathcal{D}$ is a Radon-Nikodym derivative. It is akin to the martingale distortion in robust control theory (Hansen & Sargent, 2001; Strzalecki, 2011).

to θ_2, i.e. $H = (\hat{q}, \theta_1; 1-\hat{q}, \theta_2)$ with $q \leq \hat{q} \leq 1$.[34] *Then, under ambiguity neutrality, date-1 abatement with the subjective prior H ($\bar{a}_{1,H}$) is higher than with F ($\bar{a}_{1,F}$) i.f.f.*

$$\hat{q}\mathcal{V}_{a_1}(\bar{a}_{1,F};\theta_1) + (1-\hat{q})\mathcal{V}_{a_1}(\bar{a}_{1,F},\theta_2) \geq q\mathcal{V}_{a_1}(\bar{a}_{1,F};\theta_1) + (1-q)\mathcal{V}_{a_1}(\bar{a}_{1,F},\theta_2),$$

which is true i.f.f. anticomonotonicity holds, i.e. $\mathcal{V}_{a_1}(\cdot;\theta_1) \geq \mathcal{V}_{a_1}(\cdot;\theta_2)$.

Figure 1: The effect of pessimism under anticomonotonicity (Example 4.6)



Note: $\mathcal{V}(\cdot;\theta)$ and $\mathcal{V}_{a_1}(\cdot;\theta)$ (and thus $\Upsilon(\cdot;\theta)$ and $\Upsilon_{a_1}(\cdot;\theta)$) are anticomonotonic w.r.t. θ-scenarios; $a_{1,i}$ denotes the optimal level of date-1 abatement for a risk neutral firm solely considering scenario θ_i.

Example 4.6. *Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$ and $\Upsilon(a_1;\theta_i)$ denote the net intertemporal expected revenue from date-1 abatement $a_1 \geq 0$ in scenario θ_i, i.e. $\Upsilon(a_1;\theta_i) = \pi_1(a_1) + \beta\mathcal{V}(a_1;\theta_i)$. Let Θ be ordered such that $\Upsilon(\cdot;\theta_i)$ is increasing in i. Assume that anticomonotonicity holds, i.e. $\Upsilon_{a_1}(\cdot;\theta_i)$ is decreasing in i. This is depicted in Figure 1 where $a_{1,i}$ is the optimal date-1 abatement in scenario θ_i. Anticomonotonicity implies that $a_{1,i}$ is decreasing with i and that the higher date-1 abatement the narrower the spread in $\Upsilon(\cdot;\theta)$ across θ-scenarios.*

The ambiguity averse firm dislikes any mean-preserving spread in the space of conditional second-order expected profit. Accordingly, pessimism adjusts date-1 abatement in the direction of a reduced spread in $\mathcal{V}(\cdot;\theta)$ across θ-scenarios. We note that anticomonotonicity is quite demanding a condition, which requires that $\mathcal{V}(\cdot;\theta)$ do not cross between θ-scenarios to clearly sign the covariance in Proof A.3, and could be relaxed somewhat.[35] It might be

[34]Note that $\hat{q} = 1$ with MEU preferences. This illustrates that a KMM model of choice converges to MEU in the limiting case of infinite ambiguity aversion (Klibanoff et al., 2005).

[35]We could not get there analytically but this is illustrated with numerical simulations in section 5. Note that Berger et al. (2017) transform the anticomonotonicity criterion into a 'convergence effect' between scenarios. They are able to do so because they use a binary structure, i.e. a good and a bad state, and ambiguity bears solely on the chances that these two states occur.

sufficient that the discrepancy in $\mathcal{V}(\cdot;\theta)$ across θ-scenarios diminishes with date-1 abatement in some rough sense for pessimism to raise it relative to ambiguity neutrality.

To further account for the shift in levels $\mathcal{A}$, momentarily assume for clarity that there is no long-term effect of abatement, i.e. $\partial_{a_1} C_2 \equiv 0$. In this case, Condition (22) rewrites

$$\hat{a}_1 \geq \bar{a}_1 \;\Leftrightarrow\; \mathcal{A}(\bar{a}_1)\left(\langle\tilde{\tau}\rangle + \mathcal{P}(\bar{a}_1)\right) \geq \langle\tilde{\tau}\rangle\,, \tag{23}$$

where $\mathcal{P}$ can be interpreted as a pessimism-only price distortion function satisfying, for all $a_1 \geq 0$,

$$\mathcal{P}(a_1) \doteq \frac{\mathrm{Cov}_\theta\{\phi'(\mathcal{V}(a_1;\tilde{\theta}));\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}}{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1;\tilde{\theta}))\}}. \tag{24}$$

Note that anticomonotonicity is equivalent to a non-negative $\mathcal{P}$. In other words, the ambiguity averse firm adjusts date-1 abatement upwards (resp. downwards) when its pessimistically-distorted estimate of the date-2 permit price is higher (resp. lower) than under ambiguity neutrality. It directly follows from Equation (23) that

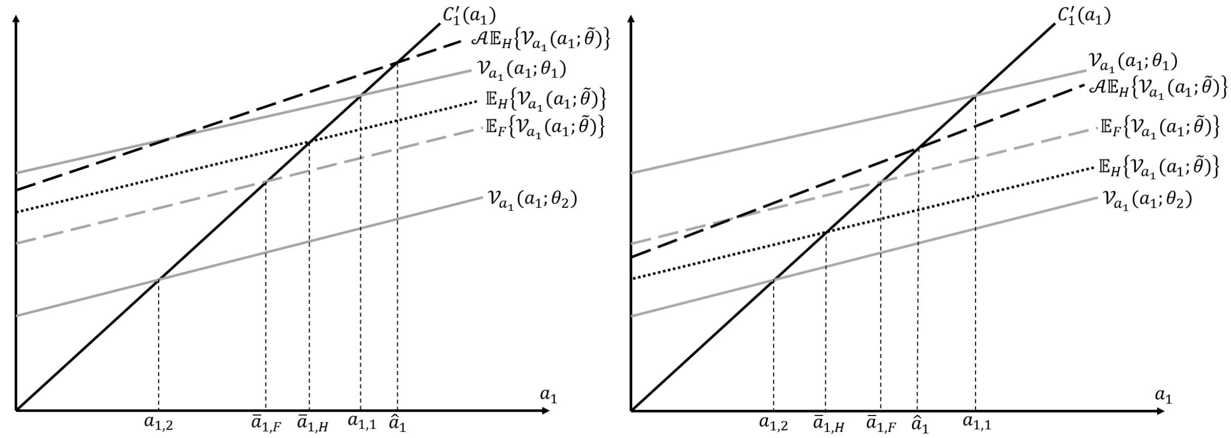
Proposition 4.7. *Let $\partial_{a_1} C_2 \equiv 0$. Then, the following equivalence conditions obtain*
(i) When ϕ displays CAAA, $\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq 0$;
(ii) When ϕ displays DAAA, $\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq (1-\mathcal{A}(\bar{a}_1))\langle\tilde{\tau}\rangle/\mathcal{A}(\bar{a}_1) < 0$;
(iii) When ϕ displays IAAA, $\hat{a}_1 \leq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \leq (1-\mathcal{A}(\bar{a}_1))\langle\tilde{\tau}\rangle/\mathcal{A}(\bar{a}_1) > 0$.

Proposition 4.7 characterises the conditions about the relative strengths and directions of the pessimistic price distortion $\mathcal{P}$ and the shift in levels $\mathcal{A}$ in determining the direction of the date-1 abatement adjustment under ambiguity aversion. Note that these two effects can be aligned or countervailing. For an instance of the latter case, let ϕ display DAAA and assume that $\mathcal{P}(\bar{a}_1) \in \left[\frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)}\langle\tilde{\tau}\rangle\,;0\right]$. Anticomonotonicity does not hold and pessimism only would lead to date-1 underabatement. However the upward shift in the firm's discount factor is large enough for precautionary date-1 abatement to form overall.

Figure 2 graphically depicts the joint effects of pessimism and ambiguity prudence where for clarity we let $H(\cdot;\theta)$ and $\mathcal{A}$ be constant functions of date-1 abatement (see Appendix D when they are allowed to vary). Note that Figure 2 separates the pessimism effect ($\bar{a}_1 = \bar{a}_{1,F} \to \bar{a}_{1,H}$) from the ambiguity prudence effect ($\bar{a}_{1,H} \to \hat{a}_1$) in terms of date-1 abatement adjustment. Pessimism operates a vertical translation of the F-averaged expected marginal profitability from date-1 abatement within the $\mathcal{V}_{a_1}(\cdot;\theta_2)$-$\mathcal{V}_{a_1}(\cdot;\theta_1)$ band in the direction of the lower $\mathcal{V}$-value scenario. Ambiguity prudence then increases the slope of the H-averaged expected marginal profitability from date-1 abatement.

Figure 2: Joint effects of ambiguity prudence and pessimism

(a) Aligned effects (b) Countervailing effects



Note: $\Theta = \{\theta_1, \theta_2\}$, $\mathcal{V}_{a_1}(\cdot;\theta_1) \geq \mathcal{V}_{a_1}(\cdot;\theta_2)$ and $F = (.5, \theta_1; .5, \theta_2)$. Fig. 2a: anticomonotonicity holds so that H overweights θ_1 relative to θ_2 as compared to F, and the two effects are aligned. Fig. 2b: comonotonicity holds so that H overweights θ_2 relative to θ_1 as compared to F, and the two effects are countervailing. In this case, ambiguity prudence dominates pessimism in terms of adjustment magnitude ($\hat{a}_1 - \bar{a}_{1,H} > |\bar{a}_{1,H} - \bar{a}_1|$).

While Propositions 4.4 and 4.7 are intuitively appealing the anticomonotonicity criterion lacks some concreteness. Proposition 4.8 provides more tangible conditions under which this criterion holds and characterises how it translates with the quadratic specification (1).

Proposition 4.8. *Let ϕ exhibit CAAA. Then, the ambiguity averse firm overabates at date 1 relative to ambiguity neutrality if, and only if*

(i) it expects to be in a net short position at date 2 under the abatement stream $(\bar{a}_1; a_2^*(\bar{a}_1; \tau_\theta^*))$ in all θ-scenarios where $\tau_\theta^* \doteq \int_{\mathrm{T}} x\partial_\theta G(x;\theta)\mathrm{d}x \Big/ \int_{\mathrm{T}} \partial_\theta G(x;\theta)\mathrm{d}x$;

(ii) for a given date-2 permit allocation ω, it abates too little at date 1 under ambiguity neutrality $\bar{a}_1 \leq \min_{\theta\in\Theta} \langle a_{1,\theta} \doteq b - \omega - a_2^*(\bar{a}_1; \tau_\theta^*)\rangle$, or reciprocally,

(iii) its date-2 allocation is relatively small $\omega \leq \omega^* \doteq \min_{\theta\in\Theta} \langle \omega_\theta^* \doteq b - \bar{a}_1 - a_2^*(\bar{a}_1; \tau_\theta^*)\rangle$.

Proof. Relegated to Appendix A.4. □

Proposition 4.8 shows that pessimism can alternatively conduce to overabatement or under-abatement at date 1 relative to ambiguity neutrality and that this depends on the expected market position at date 2. This ultimately relates to the firm's endowment of permits which is thus non neutral under ambiguity aversion. By contrast, the optimal level of date-1 abatement under ambiguity neutrality is solely driven by the $\bar{G}$-expected permit price. While we can intuitively appreciate that the level of date-1 abatement under ambiguity aversion should be decreasing with permit allocation, Appendix A.8 shows that no clear results of comparative statics obtain. Section 5 will confirm this in a numerical example.

Note that date-1 overabatement occurs only in those 'unfavourable' situations where the firm expects to be a net buyer of permits under abatement streams $(\bar{a}_1; a_2^*(\bar{a}_1; \tau_\theta^*))$ in all θ-scenarios. In these situations pessimism overweights those θ-scenarios in which high permit prices are relatively more likely. In turn, this inflates the firm's estimate of the future permit price and thus leads to overabatement. Put otherwise, evaluated at $a_1 = \bar{a}_1$, the marginal benefit of date-1 abatement (a lowering of the likelihood of effectively being net short and of the volume of permit purchases) outweighs the marginal cost of date-1 abatement for sure. Symmetrically, in those 'favourable' situations where the firm expects to be net long in all θ-scenarios pessimism will overemphasise low-price θ-scenarios, which leads to a smaller price estimation than in the benchmark and in turn to underabatement. Otherwise, as soon as the firm is net long in some θ-scenarios and net short in others we cannot conclude a priori.[36]

Anticomonotonicity translates into threshold criteria on initial conditions, i.e. $\bar{a}_1$ or ω. Similarly, Berger (2016) obtains threshold conditions in translating anticomonotonicity in the case of self-insurance and self-protection under ambiguity aversion, in the specific case where ambiguity is concentrated on on state.[37] Note that pessimism acts in line with a 'two-sided' precautionary principle. If date-2 permit allocation is sufficiently high (resp. low) for the firm to expect to be net short (resp. long) in all θ-scenarios $\omega < \omega^*$ (resp. $\omega > \omega^*$) then the pessimistic firm will overabate (resp. underabate).

Corollary 4.9. *The ambiguity averse and prudent firm overabates at date 1 relative to ambiguity neutrality only if the conditions (i-iii) in Proposition 4.8 hold.*

Now further consider that the firm displays prudence towards ambiguity. Whatever the effect of pessimism ambiguity prudence will always push towards an increase in date-1 abatement. In particular, this will rob ambiguity aversion of its symmetric pessimism-only effects on date-1 abatement adjustment. When the firm expects to be net short in all θ-scenarios pessimism and ambiguity prudence reinforce one another and overabatement is amplified relative to sole pessimism. Otherwise, when the effect of pessimism is unclear or opposite to that of ambiguity prudence it is not clear a priori which of the two effects will dominate.[38]

[36] Again, this suggests that anticomonotonicity might actually be too strong a criterion to sign pessimism.

[37] There is also an noticeable parallel between permit banking and both self-insurance and self-protection: banking is costly, but (*i*) reduces the likelihood of being in a net short position at date 2 (role of self-protection); (*ii*) for a given date-2 net position, it increases date-2 profits by either increasing sales or reducing purchases of permits (role of self-insurance).

[38] In section 5, a parametrical example will show that under DAAA ,underabatement occurs when ω is

Ambiguity prudence is therefore in line with a 'one-sided' precautionary principle whereby a sufficiently high allocation ($\omega > \omega^*$) does not guarantee that there will be underabatement while overabatement always occurs when allocation is low enough ($\omega < \omega^*$).

Finally, Proposition 4.8 highlights that clear comparative statics results under ambiguity aversion are hard to come by. For instance, signing pessimism in general is a difficult exercise which requires imposing restrictive threshold conditions.[39] While the mechanics behind pessimism and ambiguity prudence are intuitive, how these concretely transpose is not straightforward. First, these two effects can be aligned or countervailing. Second, both H and $\mathcal{A}$ function values are endogenous to the optimisation programme which ultimately hinges upon initial conditions. Third, our results depend on both the underlying modelling assumptions and the abatement cost functional forms we use.

Increase in ambiguity aversion. In the sense of Klibanoff et al. (2005) firm 2 is more ambiguity averse than firm 1 if firm 2's ambiguity function ϕ_2 writes as an increasing, concave transformation of that of firm 1, ϕ_1. Denote by $\mathcal{A}_i$ and $\mathcal{D}_i$ firm i's ambiguity prudence coefficient and distortion function. If we let $\hat{a}_i$ denote firm i's optimal level of date-1 abatement under ambiguity aversion, then, by concavity of the objective function,

$$\hat{a}_2 \geq \hat{a}_1 \Leftrightarrow \mathcal{A}_2(\hat{a}_1)\mathbb{E}_F\{\mathcal{D}_2(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\} \geq \mathcal{A}_1(\hat{a}_1)\mathbb{E}_F\{\mathcal{D}_1(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\}. \quad (25)$$

Proposition 4.10 separates out two effects consecutive to an increase in ambiguity aversion.

Proposition 4.10. *Consider two ambiguity averse and prudent firms 1 and 2 and assume that there exists a function ψ such that $\phi_2 = \psi \circ \phi_1$ with $\psi' > 0$ and $\psi'' \leq 0$. Then,*
(i) (Gollier, 2011) firm 2 is more pessimistic than firm 1 in the sense of a MLR deterioration;
(ii) assuming that ψ is almost quadratic, i.e. $\psi''' \simeq 0$, a necessary condition for a larger upward shift in levels for firm 2 is that firm 1's ambiguity prudence is not too strong relative to ambiguity aversion, i.e. $-\phi_1''/\phi_1' \leq -\phi_1'''/\phi_1'' \leq -3\phi_1''/\phi_1'$.

Proof. Relegated to Appendix A.5. □

First, point (i) states that an increase in ambiguity aversion is equivalent to an increase in pessimism, i.e. a relatively more concave ϕ_2 places relatively more weight on those low-profit

high enough. This is so because the shift in levels $\mathcal{A}$ has almost no impact relative to pessimism around ω^*.

[39] The characterisation of the cut-off allocation volume ω^* will be refined in section 4.4. Appendix F shows that when price ambiguity is binary the conditions to sign pessimism are milder though not unequivocal as in Snow (2011), Alary et al. (2013), Wong (2015a) and Berger (2016).

scenarios than ϕ_1. In particular, assuming CAAA on the part of both firms, an increase in ambiguity aversion is always conducive to a larger adjustment date-1 abatement (in absolute terms).[40] Second, point (*ii*) only provides a necessary condition regarding the direction of the shift in levels because it is difficult to characterise when $\mathcal{A}_2$ is uniformly larger than $\mathcal{A}_1$. Moreover, ψ must be equipped with an additional property and we impose the simplest one, namely $\psi''' = 0$. In words, point (*ii*) states that when the ambiguity prudence effect for firm 1 is already relatively strong, increasing ambiguity aversion might not further increase the upward shift in levels (that of firm 2). Note that Guerdjikova & Sciubba (2015) also find a cut-off condition on the strength of ambiguity prudence in a market survival context.[41] This result motivates further work on higher-order ambiguity prudence (Baillon, 2017).

4.3 Cap and trade under price and firm's baseline ambiguities

This section considers the case where the two first-order independent ambiguities on the firm's baseline and the market permit price are simultaneously present. Note that the baseline ambiguity can be interpreted as a multiplicative background risk. Let us state

Proposition 4.11. *Let ϕ exhibit CAAA. Then, the ambiguity averse firm overabates at date 1 relative to ambiguity neutrality if, and only if its date-2 permit allocation is relatively small $\omega \leq \min_{\theta\in\Theta} \left\langle \bar{b}_\theta - \bar{a}_1 - a_2^*(\bar{a}_1; \tau_\theta^*) \right\rangle$ and $\mathrm{Cov}_\theta\{G, L\} \geq 0$.*

Proof. Relegated to Appendix A.6. □

The first difference with Proposition 4.8 relates to the definition of the allocation threshold which now comprises $\bar{b}_\theta \doteq \mathbb{E}_L\{\tilde{b}_\theta|\theta\}$ the θ-scenario expected baseline. The second difference is the additional covariance criterion. It states that θ must rank $G(\cdot;\theta)$ and $L(\cdot;\theta)$ in the same order in the sense of first-order stochastic dominance. In words, pessimism triggers overabatement when allocation is low enough, i.e. the firm expects to be net short at date 2, and those θ-scenarios where high prices are more likely coincide with those θ-scenarios where high firm-level demand for permits is more likely. Symmetrically, pessimism triggers underabatement when allocation is high enough, i.e. the firm expects to be net long at date

[40] A similar result for precautionary saving formation is in Proposition 3 of Osaki & Schlesinger (2014).

[41] Consider a market populated by both ambiguity neutral (i.e. SEU-maximisers) and ambiguity averse individuals. The latter tend to disappear with time because they form 'wrong beliefs' as compared to SEU-maximisers. Guerdjikova & Sciubba (2015) show that only those ambiguity-averse agents displaying strong ambiguity prudence relative to ambiguity aversion will survive in the market, $-\phi'''/\phi'' > -2\phi''/\phi'$.

2, and high-price scenarios coincide with low firm-level demand scenarios. When neither of the above holds it is difficult to determine a clear-cut condition to sign pessimism for sure.[42]

4.4 Cap and trade under pure market-wide baseline ambiguity

In this section we consider that the market-wide ambiguity on firms' baselines is the sole determining factor of the permit price ambiguity which endogenously emerges from the market. We consider a continuum $\mathcal{S}$ of infinitesimally small and competitive firms indexed by s. The mass of firms is S. All firms have the same abatement technology (C_1, C_2), subjective beliefs F and ambiguity functions ϕ.[43] Therefore, firms are identical but for their initial allocation $\omega(s)$ which is the key determinant of the date-1 abatement adjustment under ambiguity aversion. Firms are subject to individual baseline ambiguity. To be able to derive clear analytical results we let abatement cost function be time separable and the θ-scenario firm-level baseline uncertainty $\tilde{b}_\theta(s)$ be equipped with a specific structure such that for all $\theta \in \Theta$ and $s \in \mathcal{S}$, $\tilde{b}_\theta(s) = \bar{b}_\theta + \tilde{\epsilon}_\theta(s)$.[44] That is, individual baselines comprise a first term $\bar{b}_\theta$ common to all firms but specific to any given θ-scenario, and an idiosyncratic term $\tilde{\epsilon}_\theta(s)$ such that for all $\theta \in \Theta$, $\left(\tilde{\epsilon}_\theta(s)\right)_{s\in\mathcal{S}}$ are i.i.d. with $\mathbb{E}_L\left\{\tilde{\epsilon}_\theta(s)|\theta\right\} = 0$ and finite variance. Now fix a θ-scenario. By the Law of Large Numbers for a continuum of i.i.d. variables the θ-scenario aggregate level of baseline emissions level is deterministic and given by

$$\int_{\mathcal{S}} \tilde{b}_\theta(s)\mathrm{d}s = \int_{\mathcal{S}} \bar{b}_\theta \mathrm{d}s + \int_{\mathcal{S}} \tilde{\epsilon}_\theta(s)\mathrm{d}s = S\bar{b}_\theta. \tag{26}$$

Fix also an aggregate emission ceiling Ω and permit allocation plan $\left(\omega(s)\right)_{s\in\mathcal{S}}$. Date-2 cost-efficiency requires that all firms abate up to the realised market price τ whatever their baseline realisations, i.e. $\forall s \in \mathcal{S}$, $C_2'\left(a_2^*(s)\right) = \tau$. Note that all firms abate by the same amount $a_2^* \equiv a_2^*(s)$, $\forall s \in \mathcal{S}$. Date-2 market closure in the considered θ-scenario yields

$$\int_{\mathcal{S}} \Big(\tilde{b}_\theta(s) - a_1(s) - a_2(\theta) - \omega(s)\Big)\mathrm{d}s = 0 \;\Rightarrow\; a_2^*(A_1; \bar{b}_\theta) = \bar{b}_\theta - \frac{A_1 + \Omega}{S}, \tag{27}$$

[42] Appendix E provides numerical simulations with joint market price and firm's demand ambiguities.

[43] It is difficult to define the market equilibrium when firms have heterogeneous attitudes towards ambiguity and subjective beliefs, see e.g. Danan et al. (2016). In Appendix B we consider the case of a permit market populated by a mix of (equally) ambiguity averse and neutral firms.

[44] With long-term dependency our results carry over if we suppose symmetric allocation of permits. This ensures that all firms abate the same at both dates. However no trade occurs in equilibrium since firms are identical along all relevant dimensions.

where A_1 is the aggregate date-1 abatement volume carried into date 2. The resulting θ-scenario permit price is thus $\tau_\theta = C_2'(a_2^*(A_1; \bar{b}_\theta)) > 0$ and deterministic (given the θ-scenario). Noting that individual date-1 abatement decisions have no influence on the date-2 permit price, i.e. $\partial_{a_1}\tau_\theta = 0$, it follows that for all $a_1 \geq 0$ and $\theta \in \Theta$, $\mathcal{V}_{a_1}(a_1; \theta) = \tau_\theta$. Denote by $\Psi(s;\theta) \doteq \bar{a}_1 + a_2^*(\bar{A}_1; \bar{b}_\theta) + \omega(s) - \bar{b}_\theta$ firm s' expected net position on the market in scenario θ under ambiguity neutrality. Proposition 4.12 refines the cut-off condition for the formation of precautionary date-1 abatement under ambiguity aversion.

Proposition 4.12. *Let $\partial_{a_1} C_2 \equiv 0$ and firms display prudence towards ambiguity. Then, firm $s \in \mathcal{S}$ forms precautionary date-1 abatement only if $\min_{\theta\in\Theta} \Psi(s;\theta) < \frac{C_2'(a_2^*(\bar{A}_1;\bar{b}_\theta))}{C_2''(a_2^*(\bar{A}_1;\bar{b}_\theta))}$. This is always the case under symmetric allocation of permits.*

Proof. Relegated to Appendix A.7. □

First, given that firms are identical, symmetric allocation of permits would correspond to grandfathering. In this case pessimism and ambiguity prudence are aligned and lead to date-1 overabatement. This result is suggestive of a general tendency towards precautionary permit banking which can contribute to the observed formation of permit surpluses in existing ETSs. Second, note that the anticomonotonicity criterion is somewhat laxer than under pure price ambiguity since net long positions under abatement streams $(\bar{a}_1; a_2^*(\bar{A}_1; \bar{b}_\theta))$ can be sufficient to conduce to date-1 overabatement (provided that these positions are not too big).

5 Numerical illustration

For clarity we ignore long-term effects of abatement ($\gamma = 0$) and assume that the firm has the same abatement technology at both dates which we normalise to unity, i.e. $c_1 = c_2 = 1$ and $\beta = 1$. When the firm exhibits CAAA (resp. DAAA) we take $\phi(x) = \frac{e^{-\alpha x}}{-\alpha}$ (resp. $\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}$) where $\alpha > 0$ (resp. $\alpha > 1$) is the coefficient of absolute ambiguity aversion. If $\hat{a}_1^\alpha$ denotes the optimal date-1 abatement when the degree of ambiguity aversion is α then it solves the implicit equation

$$\hat{a}_1^\alpha = \mathcal{A}(\hat{a}_1^\alpha)\left(\langle\tilde{\tau}\rangle + \mathcal{P}(\hat{a}_1^\alpha)\right). \tag{28}$$

By extension, let $\hat{a}_1^\infty$ denote the optimal date-1 abatement with the MEU representation theorem and $\hat{a}_1^0 = \bar{a}_1$ under CAAA. Similarly $\hat{a}_1^1 = \bar{a}_1$ under DAAA.

We take a discrete scenario space $\Theta = [\![-\bar{\theta}; \bar{\theta}]\!]$ and assume that F is uniform over Θ. For all scenario $\theta \in \Theta$, $G(\cdot;\theta)$ is uniform over $\mathrm{T}_\theta = [\underline{\tau} + \theta; \bar{\tau} + \theta]$ where $0 < \bar{\theta} < \underline{\tau}$ and $\bar{\tau} > \underline{\tau} > 0$.

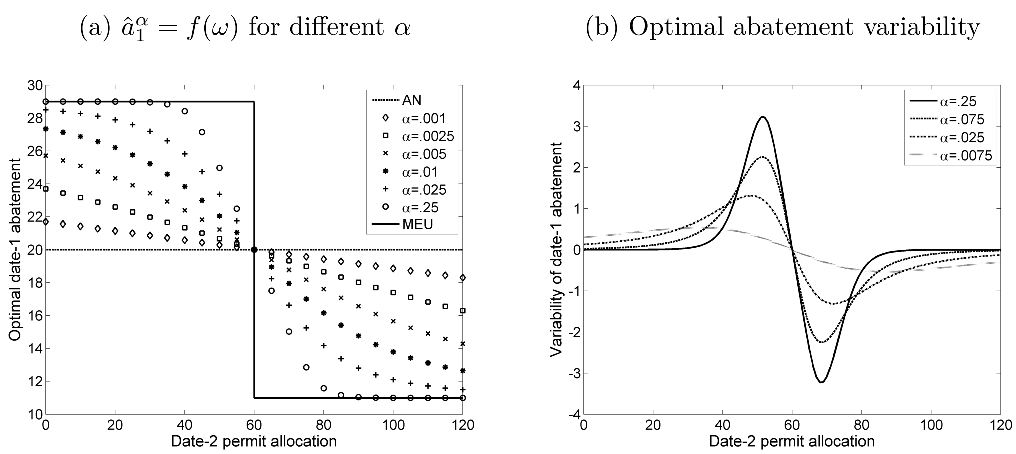
Similarly $L(\cdot;\theta)$ is uniform over $\mathrm{B}_\theta = [\underline{b}+\theta; \bar{b}+\theta]$ where $0 < \bar{\theta} < \underline{b}$ and $\bar{b} > \underline{b} > 0$. The parameters are set such that $\underline{\tau} = 10$, $\bar{\tau} = 30$, $\underline{b} = 50$, $\bar{b} = 150$ and $\bar{\theta} = 9$. Date-2 permit allocation is such that $\omega \in [0; 120]$. By construction, for all $\theta \in \Theta$, $\bar{\tau}_\theta = \langle\tilde{\tau}\rangle + \theta$ and $\bar{b}_\theta = \langle\tilde{b}\rangle + \theta$ where $\langle\tilde{\tau}\rangle = (\bar{\tau}+\underline{\tau})/2 = 20$ and $\langle\tilde{b}\rangle = (\bar{b}+\underline{b})/2 = 100$. Note that $\mathcal{V}_{a_1}(a_1;\theta) = \bar{\tau}_\theta = \langle\tilde{\tau}\rangle + \theta$, i.e. the θ-scenario expected marginal profitability from date-1 abatement is constant. Below we consider cap-and-trade regimes under pure price ambiguity (see Appendix E for joint market price and firm's demand ambiguities). In this case, anticomonotonicity holds provided that, for all $\theta \in \Theta$,

$$\partial_\theta \mathcal{V}(a_1;\theta) \leq 0 \;\Leftrightarrow\; \omega \leq \langle\tilde{b}\rangle - a_1 - \langle\tilde{\tau}\rangle - \theta. \tag{29}$$

Evaluated at $a_1 = \bar{a}_1 = \langle\tilde{\tau}\rangle$, anticomonotonicity holds i.f.f. $\omega \leq \omega^* = \langle\tilde{b}\rangle - 2\langle\tilde{\tau}\rangle - \bar{\theta} = 51$. Symmetrically, comonotonicity at $a_1 = \bar{a}_1$ holds i.f.f. $\omega \geq \langle\tilde{b}\rangle - 2\langle\tilde{\tau}\rangle + \bar{\theta} = 69$.

Cap-and-trade regime under CAAA. Equation (28) simplifies to $\hat{a}_1^\alpha = \langle\tilde{\tau}\rangle + \mathcal{P}(\hat{a}_1^\alpha)$. Figure 3a depicts the variations of $\hat{a}_1^\alpha$ w.r.t. α and ω. Since the codomain of the pessimistic

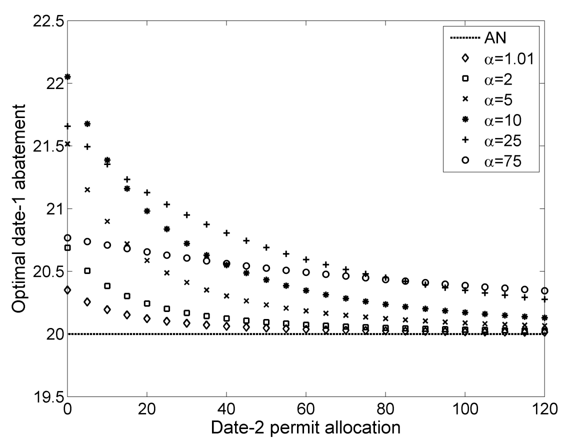
Figure 3: Cap-and-trade regime under CAAA



price distortion $\mathcal{P}$ is bounded to $[-\bar{\theta}, \bar{\theta}]$, $\hat{a}_1^\alpha$ is confined within the range $[\langle\tilde{\tau}\rangle - \bar{\theta}; \langle\tilde{\tau}\rangle + \bar{\theta}]$. The dotted line represents the optimal date-1 abatement under ambiguity neutrality $\hat{a}_1^0$ and is independent of the allocation. The solid line characterises the optimal date-1 abatement level with the MEU representation theorem $\hat{a}_1^\infty$. It is a step function of the allocation: if $\omega < \bar{\omega} = 60$, $\hat{a}_1^\infty = \langle\tilde{\tau}\rangle + \bar{\theta}$; otherwise, $\hat{a}_1^\infty = \langle\tilde{\tau}\rangle - \bar{\theta}$. Other curves depict $\hat{a}_1^\alpha$ for various ambiguity aversion degrees α. First note that the KMM representation describes the continuum between the two polar cases of ambiguity neutrality and MEU. In particular, $\hat{a}_1^\alpha$ unambiguously decreases with ω with a clear threshold $\bar{\omega} = 60$ below (resp. above) which overabatement (resp. underabatement) occurs for all ambiguity aversion degrees. It is noteworthy that this condition is laxer than anticomonotonicity since $\omega^* < \bar{\omega}$.[45] Second, for any given permit allocation the magnitude of the date-1 abatement adjustment $|\hat{a}_1^\alpha - \bar{a}_1|$ increases with α. For instance when $\omega < \bar{\omega}$, $\hat{a}_1^\alpha$-lines are ordered by increasing α and never cross each other, i.e. an increase in ambiguity aversion always leads to higher date-1 abatement.[46] Note also that the bigger α, the more sensitive the variations in $\hat{a}_1^\alpha$ w.r.t. ω around $\bar{\omega}$. In particular, for $\alpha = .25$, $\hat{a}_1^\alpha$ has already converged to its upper (resp. lower) limit when ω reaches 30 (resp. 90). Figure 3b depicts the variability of the date-1 abatement adjustment w.r.t. ω for various ambiguity aversion degrees.[47] The bigger α, the quicker $\hat{a}_1^\alpha$ reacts to ω in a smaller $\bar{\omega}$-centred range. For lower α, the incentive to adjust date-1 abatement is smaller and more evenly spread over the allocation range.

Tax regime under DAAA. Equation (28) simplifies to $\hat{a}_1^\alpha = \mathcal{A}(\hat{a}_1^\alpha)\langle\tilde{\tau}\rangle$ and $\mathcal{V}_{a_1}(a_1;\theta) = \langle\tilde{\tau}\rangle$. Figure 4 depicts the variations of $\hat{a}_1^\alpha$ w.r.t. α and ω and isolates the effects of the shift in levels

Figure 4: Tax regime under DAAA



$\mathcal{A}$. For all $\alpha > 1$, $\hat{a}_1^\alpha$ unambiguously decreases with ω and is always above $\hat{a}_1^1$. That is, $\mathcal{A}$ is a decreasing function of allocation and has steeper variations for smaller ω. Note that for a standard tax regime, i.e. $\omega = 0$, higher ambiguity aversion degrees do not guarantee higher date-1 abatement levels. In particular, there exists a threshold $\bar{\alpha}$ such that $\hat{a}_1^\alpha$ increases (resp. decreases) with α provided that α is below (resp. above) $\bar{\alpha}$. Numerically we find

[45]Again, this suggests that anticomonotonicity might be too strong a requirement to sign pessimism. From the simulations we can infer that $\bar{\omega} \doteq \mathbb{E}_F\{\omega_\theta^*\} = \langle\tilde{b}\rangle - 2\langle\tau\rangle$. That is, ambiguity aversion raises date-1 abatement relative to ambiguity neutrality i.f.f. anticomonotonicity holds in expectations over Θ w.r.t. F.

[46]Under CAAA, only point (i) in Proposition 4.10 holds. The effects of an increase in α are thus clear.

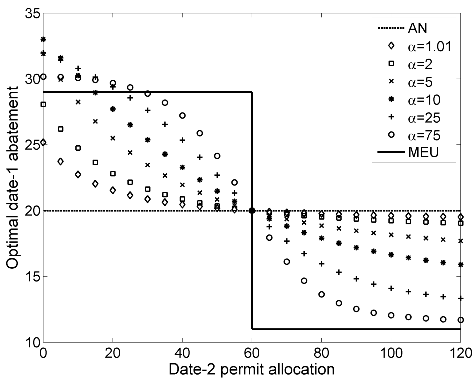
[47]Figure 3b plots $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha)$ as a function of ω. From Equation (23) and injecting the first-order condition for $\hat{a}_1^\alpha$, overabatement occurs i.f.f. $\hat{a}_1^\alpha - \bar{a}_1 + \mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha) > 0$. That is, $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha)$ can be interpreted as a proxy of the incentive to increase $\hat{a}_1^\alpha$ relative to $\bar{a}_1$.

$\bar{\alpha} \simeq 11.5$. For ω high enough, however, note that $\hat{a}_1^\alpha$ is ranked by increasing ambiguity aversion degrees. Note also that the ratio $\hat{a}_1^\alpha/\bar{a}_1 > 1$ is relatively smaller than for a cap and trade under CAAA. This suggests that the magnitude of the shift in levels $\mathcal{A}$ is relatively smaller than the pessimistic distortion $\mathcal{P}$.

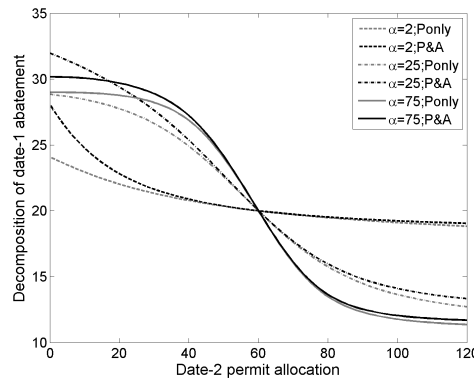
Cap-and-trade regime under DAAA. This case combines the joint effects of $\mathcal{A}$ and $\mathcal{P}$ and $\hat{a}_1^\alpha$ solves Equation (28). Figure 5 depicts the variations of $\hat{a}_1^\alpha$ w.r.t. α and ω. Figure

Figure 5: Cap-and-trade regime under DAAA

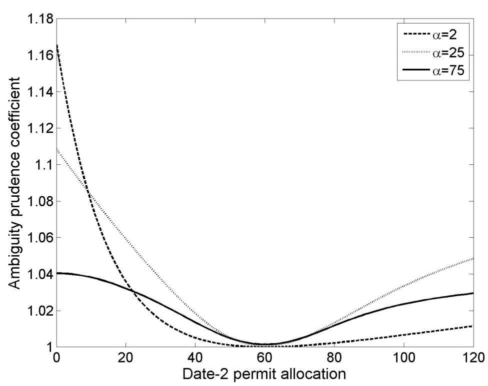
(a) $\hat{a}_1^\alpha = f(\omega)$ for different α



(b) Decomposition of $\mathcal{P}$ and $\mathcal{A}$ effects



(c) Magnitude of the $\mathcal{A}$ effect



5a is similar to Figure 3a save for small disruptions due to the upward shift in level $\mathcal{A}$. It is noteworthy that this upward shift is asymmetric w.r.t. allocation. When $\omega > \bar{\omega}$, $\hat{a}_1^\alpha$ is pushed up towards the $\bar{a}_1$-line, though without breaching it, and the lower limit $\langle\tau\rangle - \bar{\theta}$ is never reached. When $\omega < \bar{\omega}$, date-1 abatement is further adjusted upwards. For relatively low allocation levels the upper limit $\langle\tau\rangle + \bar{\theta}$ can be exceeded. As in the tax regime, for ω low enough, higher ambiguity aversion degrees do not guarantee higher date-1 abatements. More precisely, as Figure 5c shows, the magnitude of the $\mathcal{A}$-adjustment is more pronounced for low α when ω is small. Note that the upward shift is relatively smaller when ω is big enough. Note also that within the $[40; 80]$ band, the $\mathcal{A}$-adjustment is very small and ordered by increasing α. That $\hat{a}_1^\alpha$-lines may cross each other when ω is low enough substantiates Proposition 4.10, i.e. the shift in levels $\mathcal{A}$ may disrupt the $\mathcal{P}$-adjustment. By contrast, no such crossings exist when $\omega > \bar{\omega}$, i.e. there is an asymmetry in the $\mathcal{A}$-adjustment. Relative adjustments in abatement attributable to $\mathcal{A}$ and $\mathcal{P}$ are illustrated in Figure 5b. It is clear that the $\mathcal{A}$-adjustment is more pronounced for lower than bigger ω and that it is almost nil within the $[40; 80]$ band. Except for low allocation levels, this further suggests that pessimism is the main determinant of the date-1 abatement adjustment.

6 Conclusion

Emissions Trading Systems are prone to regulatory uncertainty. In this paper we introduce ambiguity aversion on the part of an ETS-liable firm to account for the prevalence of regulatory uncertainty and analyse the impacts of ambiguity aversion on intertemporal abatement decisions relative to the case of ambiguity neutrality. We show that ambiguity aversion drives the equilibrium abatement stream choices away from intertemporal efficiency. In particular, ambiguity aversion induces two effects, namely a pessimistic distortion of the firm's beliefs and a shift in the firm's discount factor. These two effects can be aligned or countervailing, the direction and magnitude of which depend on the firm's degree of ambiguity aversion and the expected future market position. It is noteworthy that permit allocation is non-neutral. In particular, sole pessimism leads the expected net short (resp. long) firm to overabate (resp. underabate) early on relative to intertemporal efficiency because it overemphasises high-price scenarios. We also show that, under certain conditions, ambiguity aversion creates a general incentive to overabate early on which is more pronounced under auctioning than under free allocation. This can provide a behavioural explanation for the formation of permit surpluses in existing carbon markets that is adding to the other sustaining physical factors described in the Introduction.

In Appendix B we consider the impacts of introducing forwards contracts. Forwards trading can mitigate some of the effects associated with pessimism, but only under the assumption that forwards are fairly priced will pessimism completely vanish. Note, however, that the shift in the firm's discount factor always persists. In sum, this suggests that the introduction of forward contracts is unlikely to (i) restore intertemporal efficiency; (ii) render a cap and trade and an emissions tax equivalent under ambiguity aversion. In Appendix B we also show that if permit allocation is sufficiently asymmetrical across firms, then the equilibrium volume of trade between ambiguity averse firms will be reduced relative to the benchmark. This can contribute to what Ellerman (2000) calls «autarkic compliance» in nascent ETSs in which traded volumes are thin and covered entities tend to cling on to their permit endowments.

Finally, we also show in Appendix B that having a mix of ambiguity neutral and averse firms in the market brings the market equilibrium further away from intertemporal efficiency as compared to the case of a market solely populated by ambiguity averse firms. This indicates one way to extend the paper, that is, to properly define a market equilibrium where agents have heterogeneous beliefs and attitudes towards ambiguity. As underlined in Section 4.4, however, aggregating different beliefs and tastes towards ambiguity is challenging, see e.g. Danan et al. (2016). Another way to build on this paper is to endogenise output decisions. That is, the firm accounts for the induced future output price change in deciding on its present abatement. This is an interesting alley for future research since Baldursson & von der Fehr (2012) find that, considering two types of firms (clean and dirty), accounting for production decisions alters and sometime reverts their 2004 results.[48]

[48] Endogenising output decisions mitigates (resp. exacerbates) risk exposure for dirty firms (resp. clean or highly-allocated dirty firms). With a small allocation risk averse clean and dirty firms alike (both on average and at the margin) reduce investment relative to the risk neutral benchmark.

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Appendices & Supplemental Material

A Collected proofs and analytical derivations

A.1 Proof of Proposition 4.1

By concavity of the objective function, $\hat{a}_1^\mu \geq \bar{a}_1^\mu$ i.f.f. $-C_1'(\bar{a}_1^\mu) + \beta\mathcal{A}(\bar{a}_1^\mu)\mathcal{V}_{a_1}(\bar{a}_1^\mu) \geq 0$, which is equivalent to $\mathcal{A}(\bar{a}_1^\mu) \geq 1$. The proof follows if we prove the following claim:

$$\text{DAAA(resp. IAAA,CAAA)} \Leftrightarrow \mathbb{E}\phi'(\cdot) \geq \text{(resp. } \leq, =)\phi' \circ \phi^{-1}(\mathbb{E}\phi(\cdot)) \Leftrightarrow \mathcal{A} \geq \text{(resp. } \leq, =)1.$$

Let ϕ be thrice differentiable. An agent is said to display Decreasing Absolute Ambiguity Aversion (DAAA) i.f.f. its Arrow-Pratt coefficient of absolute ambiguity aversion $-\phi''/\phi'$ is non-increasing. This is the case when $-\phi'''\phi' + \phi''^2 \leq 0$ or, upon rearranging, when $-\phi'''/\phi'' \geq -\phi''/\phi'$. This is equivalent to $-\phi'$ being more concave than ϕ, i.e. absolute prudence w.r.t. ambiguity exceeds absolute ambiguity aversion. In terms of certainty equivalent this translates into $\phi^{-1}(\mathbb{E}\phi(\cdot)) \geq (-\phi')^{-1}(-\mathbb{E}\phi'(\cdot))$. Applying $-\phi'$ on both sides proves the claim. See also Osaki & Schlesinger (2014) and Guerdjikova & Sciubba (2015) for a proof based on the concepts of ambiguity premium and ambiguity precautionary premium extending similar notions under risk (Pratt, 1964; Kimball, 1990).

A.2 Proof of Proposition 4.3

For a probability measure G^i, define the function O^i by

$$0 = -C_1'(\bar{a}_1^i) + \beta\mathbb{E}_{G^i}C_2'(a_2^*(\tilde{\tau})) \equiv O^i(\bar{a}_1^i), \tag{A.1}$$

where $\bar{a}_1^i$ is the date-1 optimal abatement when the price risk is distributed according to G^i and a_2^* does not depend on a_1 since we assume time separability. Let the measure G^j be a mean-preserving spread of G^i, i.e. an increase in risk relative to G^i in the sense of Rothschild & Stiglitz (1971). Concavity of the firm's objective function then yields

$$\bar{a}_1^j \geq \bar{a}_1^i \Leftrightarrow O^j(\bar{a}_1^i) \geq O^i(\bar{a}_1^i) = 0 \Leftrightarrow \mathbb{E}_{G^j}C_2'(a_2^*(\tilde{\tau})) \geq \mathbb{E}_{G^i}C_2'(a_2^*(\tilde{\tau})). \tag{A.2}$$

By Jensen's inequality, the last inequality in Equation (A.2) holds true i.f.f. C_2' is convex.

A.3 Proof of Proposition 4.4

By concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ is equivalent to

$$\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1;\tilde{\theta}))\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\} \geq \phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\bar{a}_1;\tilde{\theta}))\})\mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\}. \tag{A.3}$$

With ϕ displays DAAA, a sufficient condition for Inequality (A.3) to hold that

$$\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1;\tilde{\theta}))\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\} \geq \mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1;\tilde{\theta}))\}\mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\}. \tag{A.4}$$

This is exactly $\mathrm{Cov}_\theta\{\phi'(\mathcal{V}(\bar{a}_1;\tilde{\theta}));\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\} \geq 0$. Noting that ϕ' is non-increasing concludes. The above argument holds with equality (resp. reverses) when ϕ is CAAA (resp. IAAA).

A.4 Proof of Proposition 4.8

The proof consists in signing $\mathrm{Cov}_\theta\{\mathcal{V}(\bar{a}_1;\tilde{\theta});\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\}$ and identifying when it is non-positive. With quadratic cost specification (1), for all $\theta \in \Theta$, $\mathcal{V}_{a_1}(\bar{a}_1;\theta)$, $\bar{a}_1$, and a_2^* are given in Equations (11), (6) and (3), respectively. Differentiating $\mathcal{V}_{a_1}(\bar{a}_1;\theta)$ w.r.t. θ and then integrating by parts yields

$$\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) = \frac{c_2-\gamma}{c_2}\int_{\mathrm{T}} x\partial_\theta g(x;\theta)\mathrm{d}x = \frac{\gamma - c_2}{c_2}\int_{\mathrm{T}} G_\theta(x;\theta)\mathrm{d}x, \tag{A.5}$$

where $G_\theta(\cdot;\theta) \doteq \partial_\theta G(\cdot;\theta)$. Similarly, by the Envelop Theorem and differentiation w.r.t. θ,

$$\begin{aligned}
\partial_\theta \mathcal{V}(\bar{a}_1;\theta) &= -\int_{\mathrm{T}} C_2(\bar{a}_1, a_2^*(\bar{a}_1;x)) + x\left(b - \bar{a}_1 - a_2^*(\bar{a}_1;x) - \omega\right)\partial_\theta g(x;\theta)\mathrm{d}x \\
&= -\int_{\mathrm{T}} x\Big(b - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{2c_2}\Big) - \frac{\gamma^2\bar{a}_1^2}{2c_2}\partial_\theta g(x;\theta)\mathrm{d}x \\
&= \int_{\mathrm{T}} \Big(b - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{c_2}\Big)G_\theta(x;\theta)\mathrm{d}x,
\end{aligned} \tag{A.6}$$

where the third equality obtains by integration by parts. For all $x \in \mathrm{T}$, let $k : x \mapsto b - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{c_2}$. We assume that $\underline{\tau} < c_2(b - \omega - \bar{a}_1) < \bar{\tau}$. Notice that k changes sign over T and by continuity there exists $\tau_0 \in \mathrm{T}$ such that $k(\tau_0) = 0$, i.e. $\tau_0 = c_2(b-\omega) - (c_2-\gamma)\bar{a}_1$. For all $\theta \in \Theta$, let us define

$$\Gamma(\tau_0;\theta) \doteq \frac{1}{c_2}\int_{\mathrm{T}} (\tau_0 - x)\, G_\theta(x;\theta)\mathrm{d}x. \tag{A.7}$$

Differentiating w.r.t. τ_0 yields $\Gamma'_\theta(\tau_0) = \frac{1}{c_2}\int_{\mathrm{T}} G_\theta(x;\theta)\mathrm{d}x$. When $G_\theta > 0$, $\partial_\theta \Gamma(\cdot;\theta) > 0$ so that $\Gamma(\underline{\tau};\theta) < 0$ and $\Gamma(\bar{\tau};\theta) > 0$. Symmetrically, when $G_\theta < 0$, $\partial_\theta \Gamma(\cdot;\theta) < 0$ so that $\Gamma(\underline{\tau};\theta) > 0$ and $\Gamma(\bar{\tau};\theta) < 0$. In both cases $\forall\theta \in \Theta$, by continuity of $\Gamma(\cdot;\theta)$ there exists a duple $(\tau^*_\theta; a_{1,\theta})$ such that $\tau^*_\theta = c_2(b-\omega) - (c_2-\gamma)a_{1,\theta}$ defined by $\Gamma(\tau^*_\theta;\theta) = 0$. By definition,

$$\int_{\mathrm{T}} (\tau^*_\theta - x)\, G_\theta(x;\theta)\mathrm{d}x = 0 \;\Rightarrow\; a_{1,\theta} = \frac{c_2}{c_2-\gamma}\left(b - \omega - \frac{\int_{\mathrm{T}} xG_\theta(x;\theta)\mathrm{d}x}{c_2\int_{\mathrm{T}} G_\theta(x;\theta)\mathrm{d}x}\right). \tag{A.8}$$

For a given ω, $a_{1,\theta}$ corresponds to the required date-1 abatement effort in scenario θ when the permit price prevailing at date 2 is $\tau^*_\theta = \mathbb{E}_{G_\theta}\{\tilde{X}\}/\mathbb{E}_{G_\theta}\{\mathbf{1}\}$, i.e. when date-2 abatement is $a^*_2(\bar{a}_1;\tau^*_\theta)$. Two cases then arise depending on the monotonicity of G w.r.t. θ.

1. $G_\theta > 0$: $\forall\theta\in\Theta$, $\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) < 0$ and $\partial_\theta \mathcal{V}(\bar{a}_1;\theta) > 0$ i.f.f. $\frac{c_2}{2}\Big(b-\omega-\Big(1-\frac{\gamma}{c_2}\Big)\bar{a}_1\Big) > \tau^*_\theta$, that is i.f.f. $\bar{a}_1 < a_{1,\theta}$;
2. $G_\theta < 0$: $\forall\theta\in\Theta$, $\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) > 0$ and $\partial_\theta \mathcal{V}(\bar{a}_1;\theta) < 0$ i.f.f. $\frac{c_2}{2}\Big(b-\omega-\Big(1-\frac{\gamma}{c_2}\Big)\bar{a}_1\Big) > \tau^*_\theta$, that is i.f.f. $\bar{a}_1 < a_{1,\theta}$.

In both cases, $\hat{a}_1 > \bar{a}_1$ i.f.f. $\bar{a}_1 < a_{1,\theta}$ $\forall\theta\in\Theta$, that is i.f.f. $\bar{a}_1 < \min_{\theta\in\Theta} a_{1,\theta}$, which proves (ii). Points (i) and (iii) follow straightforwardly.

A.5 Proof of Proposition 4.10

Assume that $\mathcal{V}(\cdot;\tilde{\theta})$ and $\mathcal{V}_{a_1}(\cdot;\tilde{\theta})$ are anticomonotone, i.e. both firms form precautionary date-1 abatement. For all θ in Θ it holds that

$$\frac{\mathcal{D}_2(\hat{a}_1;\theta)}{\mathcal{D}_1(\hat{a}_1;\theta)} = \psi'\circ\phi_1(\mathcal{V}(\hat{a}_1;\theta))\frac{\mathbb{E}_F\{\phi_1'(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\}}{\mathbb{E}_F\{\phi_2'(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\}} \propto \psi'\circ\phi_1(\mathcal{V}(\hat{a}_1;\theta)). \tag{A.9}$$

W.l.o.g. let $\mathcal{V}(\hat{a}_1;\theta)$ be non-decreasing in θ. By definition $\psi'\circ\phi_1(\mathcal{V}(\hat{a}_1;\theta))$ and thus $\frac{\mathcal{D}_2}{\mathcal{D}_1}$ are non-increasing in θ. That is, firm 2 displays a stronger pessimism than firm 1 in the sense that it overemphasises low-$\mathcal{V}$ scenarios even further. Since we assume anticomonotonicity $\mathcal{V}_{a_1}(\hat{a}_1;\theta)$ is non-increasing in θ. It thus holds that

$$\mathbb{E}_F\{\mathcal{D}_2(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\} \geq \mathbb{E}_F\{\mathcal{D}_1(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\}. \tag{A.10}$$

Comparing Equations (25) and (A.10), it is always true that $\hat{a}_2 \geq \hat{a}_1$ provided that $\mathcal{A}_2(\hat{a}_1) \geq \mathcal{A}_1(\hat{a}_1)$. However it is not easy to say when it holds that $\mathcal{A}_2 \geq \mathcal{A}_1$. We note that a necessary condition for this to hold is that firm 2's coefficient of absolute ambiguity prudence is higher than that of firm 1, i.e. $-\phi_2'''/\phi_2'' \geq -\phi_1'''/\phi_1''$. Assuming $\psi''' = 0$ then yields

$$\phi_2'' = (\psi'' \circ \phi_1)\,\phi_1'^2 + (\psi' \circ \phi_1)\,\phi_1'', \text{ and } \phi_2''' = 3\,(\psi'' \circ \phi_1)\,\phi_1'\phi_1'' + (\psi' \circ \phi_1)\,\phi_1'''. \tag{A.11}$$

Noting that $-\phi_2'''/\phi_2'' \geq -\phi_1'''/\phi_1''$ rewrites $-\phi_1'''/\phi_1'' \leq -3\phi_1''/\phi_1'$ concludes.

A.6 Proof of Proposition 4.11

With both price and baseline ambiguities the θ-scenario expected profitability from date-1 abatement writes

$$\mathcal{V}(a_1;\theta) = \iint_{\mathrm{B,T}} \Big(\zeta_2 - C_2(a_1, a_2^*(a_1;x)) - x(y - a_1 - a_2^*(a_1;x) - \omega)\Big) g(x;\theta) l(y;\theta)\mathrm{d}x\mathrm{d}y. \tag{A.12}$$

Assume abatement costs have quadratic specification (1). Because G and L are first-order independent, differentiating Equation (A.12) w.r.t. θ, integrating by parts and evaluating at $a_1 = \bar{a}_1$ gives

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) = \int_{\mathrm{T}} \Big(\bar{b}_\theta - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{c_2}\Big) G_\theta(x;\theta)\mathrm{d}x + \bar{\tau}_\theta \int_{\mathrm{B}} L_\theta(y;\theta)\mathrm{d}y, \tag{A.13}$$

where $\bar{b}_\theta \doteq \mathbb{E}_L\{\tilde{b}_\theta|\theta\}$ and $L_\theta(\cdot;\theta) \doteq \partial_\theta L(\cdot;\theta)$. Note that $\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta)$ is given by Equation (A.5). By the same token as in Proof A.4 anticomonotonicity holds when $G_\theta > 0$ (resp. $G_\theta < 0$) provided that the allocation threshold condition is satisfied and $L_\theta > 0$ (resp. $L_\theta < 0$).

A.7 Proof of Proposition 4.12

Assume quadratic abatement costs as in Equation (1). All ambiguity neutral firms abate by the same amount at date 1 $\bar{a}_1 = \beta \langle \tau_\theta \rangle / c_1$ where

$$\langle \tau_\theta \rangle \doteq \mathbb{E}_F\{\tau_\theta\} = c_2 \mathbb{E}_F\{a_2^*(\bar{A}_1;\bar{b}_\theta)\} = c_2\Big(\langle \tilde{b} \rangle - (\bar{A}_1 + \Omega)/S\Big). \tag{A.14}$$

Noting that $\bar{A}_1 = S\bar{a}_1$ then gives

$$\bar{a}_1 = \frac{c}{c_1}(\langle \tilde{b} \rangle - \Omega/S) \text{ and } a_2^*(\bar{A}_1;\bar{b}_\theta) = \bar{b}_\theta - \frac{c\langle \tilde{b} \rangle}{c_1} - \frac{c\Omega}{\beta c_2 S}. \tag{A.15}$$

Note that the aggregate emission constraint is satisfied in every θ-scenario

$$\int_{\mathcal{S}} \Big(\tilde{b}_\theta(s) - \bar{a}_1 - \bar{a}_2^*(\bar{A}_1; \bar{b}_\theta)\Big)\mathrm{d}s = \Omega. \tag{A.16}$$

Note also that a positive permit price in each θ-scenario requires that, when $A_1 = \bar{A}_1$,

$$\Omega(c_1 - c) > S\Big(c_1 \max_{\theta\in\Theta} \bar{b}_\theta - c\langle\tilde{b}\rangle\Big), \tag{A.17}$$

which we assume is the case. Let us now sign $\mathrm{Cov}_F\{\mathcal{V}(\bar{a}_1; \tilde{\theta}); \mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\}$. We have

$$\mathcal{V}(\bar{a}_1; \theta) = \zeta_2 - C_2(a_2^*(\bar{A}_1; \bar{b}_\theta)) - \tau_\theta(\bar{b}_\theta - \bar{a}_1 - a_2^*(\bar{A}_1; \bar{b}_\theta) - \omega(s)) \tag{A.18a}$$

$$\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1; \theta) = \partial_\theta \tau_\theta = C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))\partial_\theta a_2^*(\bar{A}_1; \bar{b}_\theta) = C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))\partial_\theta \bar{b}_\theta, \tag{A.18b}$$

$$\partial_\theta \mathcal{V}(\bar{a}_1; \theta) = \Big(C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))\Psi(s; \theta) - C_2'(a_2^*(\bar{A}_1; \bar{b}_\theta))\Big)\, \partial_\theta \bar{b}_\theta, \tag{A.18c}$$

since $\partial_\theta \bar{A}_1 = \partial_\theta \bar{a}_1 = 0$ (they are decided ex ante) and where $\Psi(s; \theta) \doteq \bar{a}_1 + a_2^*(\bar{A}_1; \bar{b}_\theta) + \omega(s) - \bar{b}_\theta$ is firm s' expected net position on the market in scenario θ under ambiguity neutrality. Anticomonotonicity holds provided that for all $\theta \in \Theta$, $\Psi(s; \theta) < \frac{C_2'(a_2^*(\bar{A}_1; \bar{b}_\theta))}{C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))}$. Note that this allows a net long (positive) market position which was not the case under pure price ambiguity. Plugging in Equation (A.15) gives $\Psi(s; \theta) = \omega(s) - \frac{\Omega}{S}$ which is nil for a symmetric allocation plan. Therefore, when allocation is symmetric anticomonotonicity holds unconditionally. Assume for simplicity that the ratio of abatement technology between the two dates is unitary, i.e. $c_1 = \beta c_2$. Then,

$$\Psi(s; \theta) < \frac{C_2'(a_2^*(\bar{A}_1; \bar{b}_\theta))}{C_2''(a_2^*(\bar{A}_1; \bar{b}_\theta))} \;\Leftrightarrow\; \omega(s) < \min_{\theta\in\Theta} \Big\langle \omega_\theta \doteq (\Omega/S + 2\bar{b}_\theta - \langle\tilde{b}\rangle)/2 \Big\rangle, \tag{A.19}$$

Noting from Equation (A.17) that $\omega_\theta > \Omega/S$ for all $\theta \in \Theta$ concludes.

A.8 Comparative statics w.r.t. permit allocation

With ϕ CAAA and no long-term effect of abatement Equation (18) rewrites

$$-\, C_1'(\hat{a}_1) + \beta \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1; \tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta})\}}{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1; \tilde{\theta}))\}} = 0. \tag{A.20}$$

Taking the total differential of Equation (A.20) yields

$$\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}\omega} = \frac{\beta\Phi(\hat{a}_1)}{C_1''(\hat{a}_1) - \beta\Phi(\hat{a}_1)}, \tag{A.21}$$

where, since $\mathcal{V}_\omega = \mathcal{V}_{a_1} = \bar{\tau}_\theta$, and omitting arguments so as to avoid cluttering,

$$\Phi(a_1) = \frac{\mathbb{E}_F\{\mathcal{V}_{a_1}^2\phi''(\mathcal{V})\}\mathbb{E}_F\{\phi'(\mathcal{V})\} - \mathbb{E}_F\{\mathcal{V}_{a_1}\phi'(\mathcal{V})\}\mathbb{E}_F\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\}}{\mathbb{E}_F\{\phi'(\mathcal{V})\}^2}. \tag{A.22}$$

In particular, note that $\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}\omega} \in \,]-1;0[$ i.f.f. $\Phi(\hat{a}_1) < 0$. We can show that

$$\begin{aligned} \Phi(\hat{a}_1) &\propto \mathrm{Cov}\{\mathcal{V}_{a_1};\mathcal{V}_{a_1}\phi''(\mathcal{V})\}\mathbb{E}_F\{\phi'(\mathcal{V})\} - \mathrm{Cov}\{\mathcal{V}_{a_1};\phi'(\mathcal{V})\}\mathrm{E}_F\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\} \\ &\propto \mathcal{P}(\hat{a}_1) - \mathcal{P}_2(\hat{a}_1) = \frac{\mathrm{Cov}\{\mathcal{V}_{a_1};\phi'(\mathcal{V})\}}{\mathbb{E}_F\{\phi'(\mathcal{V})\}} - \frac{\mathrm{Cov}\{\mathcal{V}_{a_1};\mathcal{V}_{a_1}\phi''(\mathcal{V})\}}{\mathbb{E}_F\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\}}, \end{aligned} \tag{A.23}$$

where $\mathcal{P}$ is the pessimism-only price distortion and $\mathcal{P}_2$ can be interpreted as a second-order pessimism-only price distortion. These two distortions have positive values when anticomonotonicity holds in which case $\Phi(\hat{a}_1) \leq 0$ i.f.f. $\mathcal{P}_2(\hat{a}_1) \geq \mathcal{P}(\hat{a}_1)$. It is difficult to determine the variations of $\hat{a}_1$ w.r.t. ω because it is hard to sign $\mathcal{P}_2(\hat{a}_1) - \mathcal{P}(\hat{a}_1)$ in general. In line with intuition numerical simulations in Section 5 show that the level of optimal date-1 abatement unambiguously decreases with permit allocation, with intensities depending on the degree of ambiguity aversion and the initial allocation volume itself. This would suggest that $\mathcal{P}_2$ is larger than $\mathcal{P}$. Again, this calls for studying higher orders for ambiguity prudence.

B Additional considerations and extensions

This appendix first extends our model by allowing for trades of forward contracts. It then analyses the impacts of (i) ambiguity aversion on the equilibrium volume of permit trade; (ii) having a mix of ambiguity averse and neutral firms on the market for permits.

Forward trading. It is natural to investigate to which extent can the introduction of a forwards market diminish the effects of ambiguity aversion and restore intertemporal efficiency. In practice, ETS-liable firms liable have recourse to forward contracts for hedging purposes, e.g. power companies in the EUETS. We consider that firms now have the possibility permits in a forward market at date 1. Let a_f and p_f denote the volume of permits contracted in the forward market and the forward price, respectively. Note that this does not change the optimal abatement decision at date-2. In particular, the firm's recursive programme now writes

$$\max_{a_1 \geq 0, a_f} \left\langle \zeta_1 - C_1(a_1) - p_f a_f + \beta \phi^{-1}\Big(\mathbb{E}_F\{\phi(\mathcal{V}(a_1, a_f; \tilde{\theta}))\}\Big)\right\rangle, \tag{B.1}$$

where, $\forall \theta \in \Theta$, $\mathcal{V}(a_1, a_f; \theta) = \mathbb{E}_G\left\{\zeta_2 - C_2(a_1, a_2^*(a_1; \tilde{\tau}_\theta)) - \tilde{\tau}_\theta(b - a_1 - a_f - a_2^*(a_1; \tilde{\tau}_\theta) - \omega)|\theta\right\}$. The two necessary first-order conditions for $\hat{a}_1$ and $\hat{a}_f$ are given by

$$-C_1'(\hat{a}_1) + \beta \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1, \hat{a}_f; \tilde{\theta})\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\})} = 0, \tag{B.2a}$$

$$\text{and } -p_f + \beta \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \tilde{\theta})\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\})} = 0. \tag{B.2b}$$

By the Envelop, $\mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \theta) = \bar{\tau}_\theta \geq \mathcal{V}_{a_1}(\hat{a}_1, \hat{a}_f; \theta) = \bar{\tau}_\theta - \mathbb{E}_G\left\{\partial_{a_1} C_2(\hat{a}_1, a_2^*(\hat{a}_1; \tilde{\tau}_\theta))|\theta\right\} > 0$, where $\bar{\tau}_\theta = \mathbb{E}_G\left\{\tilde{\tau}_\theta|\theta\right\}$. Thus, absent long-term effect of abatement, intertemporal efficiency in expectations is restored since it holds $\beta\langle\tilde{\tau}\rangle = C_1'(\bar{a}_1) = p_f = C_1'(\hat{a}_1)$ provided that $p_f \in \mathrm{T}$ is predetermined but irrespective of how p_f is priced. Otherwise, present long-term effect of abatement, combining Equations (B.2a) and (B.2b) gives

$$-C_1'(\hat{a}_1) - \beta \mathcal{A}(\hat{a}_1, \hat{a}_f)\mathbb{E}_F\{\mathcal{D}(\hat{a}_1, \hat{a}_f; \tilde{\theta})\mathbb{E}_G\{\partial_{a_1} C_2(\hat{a}_1, a_2^*(\hat{a}_1; \tilde{\tau}_\theta))|\theta\}\} + p_f = 0. \tag{B.3}$$

Assume forward contracts are fairly priced, i.e. the forward price is unbiased $p_f \equiv \beta\langle\tilde{\tau}\rangle$. For any $a_1 \geq 0$, the optimal forward volume $a_f^*(a_1)$ solves $\langle\tilde{\tau}\rangle = \mathcal{A}(a_1, a_f^*)\mathbb{E}_F\{\mathcal{D}(a_1, a_f^*; \tilde{\theta})\tau_\theta\}$. Therefore, $\hat{a}_1 \geq \bar{a}_1$ i.f.f.

$$\mathbb{E}_{\bar{G}}\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau}))\} \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1))\mathbb{E}_F\{\mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \tilde{\theta})\mathbb{E}_G\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau}_\theta))|\theta\}\}. \tag{B.4}$$

With quadratic cost specification (1), Inequality (B.4) is equivalent to

$$\langle\tilde{\tau}\rangle + \gamma(\mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) - 1)\bar{a}_1 \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1))\mathbb{E}_F\{\mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \tilde{\theta})\tau_\theta\}, \tag{B.5}$$

which, under the fair price assumption, is equivalent to $\mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) \geq 1$. In summary,

Proposition B.1. *Consecutive to the introduction of forward contracts,*
(i) absent long-term effect of abatement, intertemporal efficiency in expectations is restored irrespective of how forward contracts are priced;
(ii) present long-term effect of abatement and assuming that forward contracts are fairly priced, intertemporal efficiency in expectations obtains only under CAAA. In particular, under DAAA (resp. IAAA), date-1 overabatement (resp. underabatement) persists.

Absent long-term effect of abatement, the optimal level of date-1 abatement level does not depend on the underlying ambiguity level nor on the firm's attitude towards ambiguity. This is in line with recent extensions of the separation theorem under smooth ambiguity aversion (Wong, 2015b, 2016; Osaki et al., 2016).[49] Present long-term effect of abatement, the introduction of a fairly-priced market for forward contracts only corrects for pessimism but not for the shift in levels $\mathcal{A}$.[50] As far as date-1 abatement decisions are concerned a cap-and-trade regime with fairly-priced forward contracts is hence akin to a tax regime. When forwards are not priced fairly, however, pessimism will remain.

Equilibrium volume of trade. We investigate the impact of ambiguity aversion on the part of firms on the overall volume of trade. Assume CAAA for clarity. Then, when firm s (resp. l) is allocated less (resp. more) than $\min_{\theta\in\Theta}\omega^*_\theta$ (resp. $\max_{\theta\in\Theta}\omega^*_\theta$) it expects to be net short (resp. long) in all θ-scenarios under the abatement stream $(\bar{a}_1; a^*_2(\bar{a}_1;\tau^*_\theta))$. That is, $\hat{a}_1(s) \geq \bar{a}_1 \geq \hat{a}_1(l)$. At date 2, all firms equate their date-2 marginal abatement costs $\partial_{a_2}C_2(a_1;a^*_2)$ to the observed allowance price τ. With quadratic cost specification (1), total abatements for the three types of firms rank such that

$$a^*_2(\hat{a}_1(s);\tau) + \hat{a}_1(s) = \Big(\tau + (c_2-\gamma)\hat{a}_1(s)\Big)/c_2 \geq a^*_2(\bar{a}_1;\tau) + \bar{a}_1 \geq a^*_2(\hat{a}_1(l);\tau) + \hat{a}_1(l). \quad \text{(B.6)}$$

Since the net buying (resp. selling) firm s (resp. l) abates more (resp. less) and buys (resp. sells) less permitss on the market than under ambiguity neutrality, the following holds

Proposition B.2. *Let permits be non-symmetrically distributed such that at least some firms are endowed with $\omega \notin [\min_{\theta\in\Theta}\omega^*_\theta; \max_{\theta\in\Theta}\omega^*_\theta]$. Then, the equilibrium volume of trade is lower when firms are ambiguity averse than when they are ambiguity neutral.*

Similarly, Baldursson & von der Fehr (2004) find that risk aversion reduces the equilibrium volume of trade relative risk neutrality. Ambiguity and risk aversions may thus contribute to what Ellerman (2000) calls «autarkic compliance» in nascent ETSs, i.e. traded volumes are thin (e.g. presently in the SKETS or the Chinese pilots). Because covered entities are waiting for increased price discovery and due to high regulatory uncertainty they tend to

[49]In the presence of pure price ambiguity for a risk-averse ambiguity-averse competitive firm, see Wong (2015b). In the presence of price ambiguity and additive background risk for a risk-neutral and ambiguity-averse competitive firm, see Osaki et al. (2016). In the presence of price ambiguity and additive or multiplicative background risk for a risk-averse ambiguity-averse competitive firm, see Wong (2016).

[50]This contrasts with Wong (2015b), Wong (2016) and Osaki et al. (2016) in that they use the static KMM formulation, hence without the shift in levels $\mathcal{A}$.

hold on to their quota allocation so that trades are scarce. For instance, the volume of trades (both in spot EUAs and futures) increased steadily over Phase I of the EUETS as uncertainty gradually vanished, see e.g. Chapter 5 in Ellerman et al. (2010).

Different tastes for ambiguity. Consider a permit market populated by both ambiguity averse and neutral firms where $\varepsilon \in [0;1]$ denotes the share of ambiguity averse firms. Assume $\partial_{a_1} C_2 \equiv 0$. For any $\varepsilon \in (0;1)$ denote by $\hat{a}_1^\varepsilon$ and $\bar{a}_1^\varepsilon$ the optimal date-1 abatement levels for the ambiguity averse and neutral firms, respectively. Suppose also that ambiguity averse firms are allocated $\omega \leq \min_{\theta\in\Theta} \omega_\theta^*$ so that, in a market that contains either only ambiguity averse or ambiguity neutral firms, optimal date-1 abatement levels satisfy $\hat{a}_1^{\varepsilon=1} = \hat{a}_1 \geq \bar{a}_1^{\varepsilon=0} = \bar{a}_1$ and $\hat{A}_1 = S\hat{a}_1 \geq \bar{A}_1 = S\bar{a}_1$. For any mix ε, assume that market closure at date 2 gives the θ-scenario permit price by $\tau_\theta^\varepsilon = C_2'\Big(\bar{b}_\theta - (\varepsilon\hat{A}_1 + (1-\varepsilon)\bar{A}_1 + \Omega)/S\Big)$.[51] Denoting by $\bar{\tau}_\theta$ and $\hat{\tau}_\theta$ the θ-scenario permit price when $\varepsilon = 0$ and $\varepsilon = 1$, respectively, we have $\hat{\tau}_\theta \leq \tau_\theta^\varepsilon \leq \bar{\tau}_\theta$. Symmetrically, when ambiguity averse firms receive a large allocation $\omega \geq \max_{\theta\in\Theta} \omega_\theta^*$, $\hat{a}_1 \leq \bar{a}_1$, we have $\bar{\tau}_\theta \leq \tau_\theta^\varepsilon \leq \hat{\tau}_\theta$. By comparing the necessary first-order conditions for $\bar{a}_1$ and $\bar{a}_1^\varepsilon$ on the one hand, and for $\hat{a}_1$ and $\hat{a}_1^\varepsilon$ on the other hand, the following holds

Proposition B.3. *Let $\varepsilon \in (0;1)$ denote the share of ambiguity averse firms. Then,*
(i) when they are allocated $\omega < \min_{\theta\in\Theta} \omega_\theta^*$, $\bar{a}_1^\varepsilon < \bar{a}_1 < \hat{a}_1 < \hat{a}_1^\varepsilon$;
(ii) when they are allocated $\omega > \max_{\theta\in\Theta} \omega_\theta^*$, $\bar{a}_1^\varepsilon > \bar{a}_1 > \hat{a}_1 > \hat{a}_1^\varepsilon$.

This shows that having a mix of ambiguity averse and neutral firms in the market where ambiguity averse firms are endowed with a relatively high or low number of permits brings the market further away from intertemporal efficiency. In particular, note that this also alters abatement decisions of ambiguity neutral firms.

C MEU preferences & anticomonotonicity

The anticomonotonicity criterion is robust in the sense that it obtains with other models of choice under ambiguity. This appendix considers the α-maxmin representation theorem (Gilboa & Schmeidler, 1989; Ghirardato et al., 2004). We stick to our interpretation of Θ as the set of possible objective probability distributions. The firm thus grants a weight $\alpha \in [0;1]$ to the worst θ-scenario in Θ and the complementary weight to the best θ-scenario.

[51] This is a conservative assumption. As will be clear from Proposition B.3, defining τ_θ^ε with $\hat{A}_1^\varepsilon$ and $\bar{A}_1^\varepsilon$ instead of $\hat{A}_1$ and $\bar{A}_1$ would further amplify the deviation.

Proposition C.1. *Let the firm exhibit MEU preferences. The ambiguity averse firm over-abates at date 1 relative to SEU preferences if, and only if, the sequences* $(\mathcal{V}(\bar{a}_1;\theta))_\theta$ *and* $(\mathcal{V}_{a_1}(\bar{a}_1;\theta))_\theta$ *are anticomonotone, where* $\bar{a}_1$ *denotes the optimal date 1-abatement under SEU.*

Proof. For the purpose of the proof, let Θ be a discrete finite set of cardinality $k = |\Theta|$ and ordered such that $\theta_1 \leq \cdots \leq \theta_k$. Let $(q_i)_{i=1,\ldots,k}$ be the subjective prior where q_i denotes the firm's subjective probability that the θ_i-scenario will materialise and $\sum_i q_i = 1$. W.l.o.g. let the sequence $(\mathcal{V}(\bar{a}_1;\theta_i))_i$ be non-decreasing in i. We have

$$\bar{a}_1 \doteq \arg\max_{a_1 \geq 0} \Big\langle \Upsilon_{\mathrm{SEU}}(a_1) \doteq \pi_1(a_1) + \beta \sum_{i=1}^{k} q_i \mathcal{V}(a_1;\theta_i) \Big\rangle. \tag{C.1}$$

The α-maxmin objective function reads

$$\begin{aligned} \Upsilon_\alpha(a_1) &\doteq \pi_1(a_1) + \beta\Big(\alpha \min_{\theta\in\Theta} \mathcal{V}(a_1;\theta) + (1-\alpha)\max_{\theta\in\Theta}\mathcal{V}(a_1;\theta)\Big) \\ &= \pi_1(a_1) + \beta\left(\alpha\mathcal{V}(a_1;\theta_1) + (1-\alpha)\mathcal{V}(a_1;\theta_k)\right), \end{aligned} \tag{C.2}$$

and let $\hat{a}_1^\alpha$ be the unique maximiser of Υ_α. By concavity of Υ_α,

$$\hat{a}_1^\alpha \geq \bar{a}_1 \;\Leftrightarrow\; \alpha\mathcal{V}_{a_1}(\bar{a}_1;\theta_1) + (1-\alpha)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k) \geq \sum_{i=1}^{k} q_i \mathcal{V}_{a_1}(\bar{a}_1;\theta_i). \tag{C.3}$$

By virtue of ambiguity aversion it holds $\Upsilon_\alpha \leq \Upsilon_{\mathrm{SEU}}$. That is, for all $a_1 \geq 0$,

$$\alpha\mathcal{V}(a_1;\theta_1) + (1-\alpha)\mathcal{V}(a_1;\theta_k) \leq \sum_{i=1}^{k} q_i\mathcal{V}(a_1;\theta_i). \tag{C.4}$$

Rearranging Equation (C.4) gives

$$\begin{aligned} (\alpha - q_1)\mathcal{V}(a_1;\theta_1) &\leq \sum_{i=2}^{k-1} q_i\mathcal{V}(a_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}(a_1;\theta_k) \\ &\leq \Big(\alpha + \sum_{i=2}^{k} q_i - 1\Big)\mathcal{V}(a_1;\theta_k) = (\alpha - q_1)\mathcal{V}(a_1;\theta_k) \end{aligned} \tag{C.5}$$

since $(\mathcal{V}(a_1;\theta_i))_i$ is non-decreasing in i and $\sum_i q_i = 1$. Since $\mathcal{V}(\cdot;\theta_k) \geq \mathcal{V}(\cdot;\theta_1) > 0$, note that $\alpha \geq q_1$ is a sufficient condition for $\Upsilon_\alpha \leq \Upsilon_{\mathrm{SEU}}$ to hold. Then,

$$\hat{a}_1^\alpha \geq \bar{a}_1 \;\Leftrightarrow\; (\alpha - q_1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_1) \geq \sum_{i=2}^{k-1} q_i\mathcal{V}_{a_1}(\bar{a}_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k). \tag{C.6}$$

Finally note that it is sufficient for Inequality (C.6) this to hold that $(\mathcal{V}_{a_1}(\bar{a}_1;\theta_i))_i$ be non-increasing in i since this would guarantee that

$$\sum_{i=2}^{k-1} q_i \mathcal{V}_{a_1}(\bar{a}_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k) \geq \Big(\alpha + \sum_{i=2}^{k} q_i - 1\Big)\mathcal{V}_{a_1}(\bar{a}_1;\theta_2) \\ = (\alpha - q_1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_2). \tag{C.7}$$

This concludes the proof. □

W.l.o.g. fix $\alpha = 1$, i.e. MEU collapses to Wald's minimax criterion. An increase in the level of ambiguity correspond to an increase in the cardinality of Θ, say from $|\Theta|$ to $|\Theta'|$. Note that this also corresponds to an increase in the degree of ambiguity aversion since

$$\min_{\theta\in\Theta'} \Big\langle \max_{a_1\geq 0} \mathcal{V}(a_1;\theta)\Big\rangle \leq \min_{\theta\in\Theta} \Big\langle \max_{a_1\geq 0} \mathcal{V}(a_1;\theta)\Big\rangle \;\Leftrightarrow\; |\Theta'| \geq |\Theta|. \tag{C.8}$$

That is, 'beliefs' and 'tastes' are not disentangled (this is attributable to the min operator). By linearity of the objective function, Proposition C.1 also applies to the ϵ-contamination model of choice (Eichberger & Kelsey, 1999) which corresponds to a convex combination between a SEU criterion with a confidence degree or weight $\epsilon \in [0;1]$ and Wald's criterion with weight $1-\epsilon$. See also Gierlinger & Gollier (2017) for a treatment of multiplier preferences from robust control theory (Hansen & Sargent, 2001; Strzalecki, 2011).

D The two ambiguity aversion induced effects

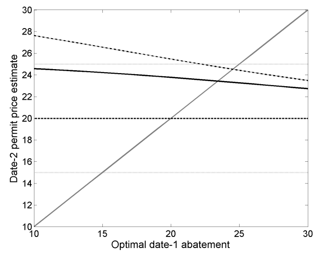
With numerical simulations this appendix further illustrates the decomposition of the two ambiguity aversion induced effects provided in Figure 2 when H and $\mathcal{A}$ are allowed to vary with a_1. There are only two scenarios $\Theta = \{\theta_1 = +5, \theta_2 = -5\}$ with equal probability under the subjective prior $F = (q_1, \theta_1; q_2, \theta_2)$, i.e. $q_1 = q_2 = .5$. We assume $\partial_{a_1} C_2 \equiv 0$ so that for all $\theta \in \Theta$ and $a_1 \geq 0$, $\mathcal{V}_{a_1}(a_1;\theta) = \langle\tilde{\tau}\rangle + \theta$ where the F-expected price is $\langle\tilde{\tau}\rangle = 20$. This means that $\mathcal{V}_{a_1}$-lines will be flat while they were upward-sloping in Figure 2. The pessimistically-distorted prior $H = (\hat{q}_1, \theta_1; \hat{q}_2, \theta_2)$ and shift in levels $\mathcal{A}$ satisfy, for all $a_1 \geq 0$,

$$\hat{q}_i(a_1) = \frac{q_i \phi'\left(\mathcal{V}(a_1;\theta_i)\right)}{q_1\phi'\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi'\left(\mathcal{V}(a_1;\theta_2)\right)} \text{ for } i = \{1,2\}, \tag{D.1a}$$

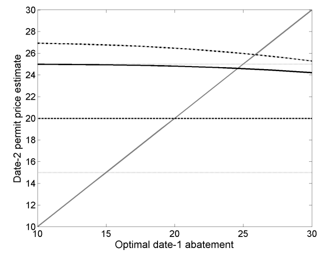
$$\text{and } \mathcal{A}(a_1) = \frac{q_1\phi'\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi'\left(\mathcal{V}(a_1;\theta_2)\right)}{\phi' \circ \phi^{-1}\left(q_1\phi\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi\left(\mathcal{V}(a_1;\theta_2)\right)\right)}. \tag{D.1b}$$

Figure 6: Separation of pessimism and ambiguity prudence

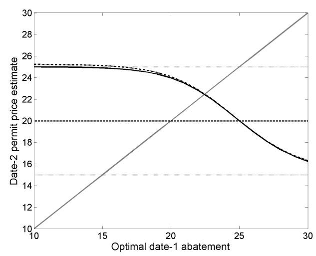
(a) $\alpha = 5$ and $\omega = 20$



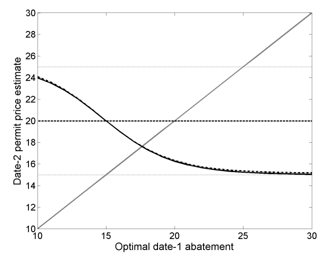
(b) $\alpha = 10$ and $\omega = 20$



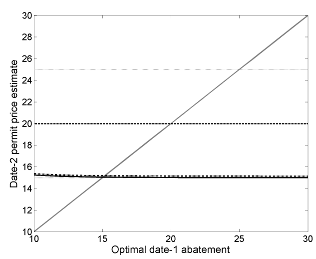
(c) $\alpha = 75$ and $\omega = 55$



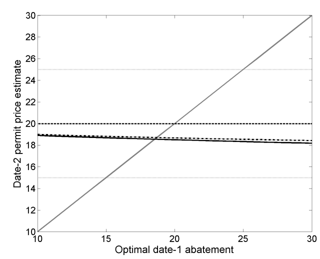
(d) $\alpha = 75$ and $\omega = 65$



(e) $\alpha = 75$ and $\omega = 80$



(f) $\alpha = 5$ and $\omega = 90$



Note: The upward-sloping grey solid line is C_1'. The two flat grey dotted lines are $\mathcal{V}_{a_1}(a_1;\theta_i)$. The flat dark dashed line is $\mathbb{E}_F\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$. The curved black solid line is $\mathbb{E}_H\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$. The curved black dotted line is $\mathcal{A}(a_1)\mathbb{E}_H\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$. The intersection between C_1' and $\mathcal{A}(a_1)\mathbb{E}_H\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$ gives $\hat{a}_1$.

The necessary-first order condition for $\hat{a}_1$ in Equation (19) rewrites

$$-C_1'(\hat{a}_1) + \beta\mathcal{A}(\hat{a}_1)\Big(\langle\tilde{\tau}\rangle + \hat{q}_1(\hat{a}_1)\theta_1 + \hat{q}_2(\hat{a}_1)\theta_2\Big) = 0, \tag{D.2}$$

and is graphically depicted in Figure 6 for different combinations of α and ω. In this numerical example Figure 6 illustrates that the bulk of the variation in date-1 abatement level under ambiguity aversion relative to ambiguity neutrality is driven by pessimism. However note that the relative effects of ambiguity prudence can be relatively significant especially when α is low (Figs. 6a and 6b). Figures 6c and 6d highlight the high sensibility of $\hat{a}_1$ around the threshold $\bar{\omega} = 60$ for relatively high α. Figures 6e and 6f indicate that the pessimistic prior distortion is more pronounced when α is high. Finally, Figures 6b and 6e underline that when ω is outside of the $[40-80]$ band and α is relatively high pessimism redistributes almost all the weight to the worst scenario, i.e. θ_1 (resp. θ_2) when ω is small (resp. high).

E Joint market price and firm's demand ambiguities

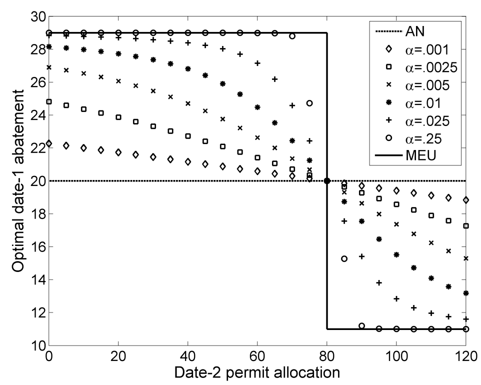
As in Section 5 consider that $\mathrm{T}_\theta = [\underline{\tau}+\theta; \bar{\tau}+\theta]$ and $\mathrm{B}_\theta = [\underline{b}+\theta; \bar{b}+\theta]$, i.e. high-price scenarios coincide with high-demand scenarios. In this case it holds that $\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) = \langle\tilde{\tau}\rangle + \theta$ and

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) \leq 0 \Leftrightarrow \omega \leq \langle\tilde{b}\rangle - \bar{a}_1 + \theta. \tag{E.1}$$

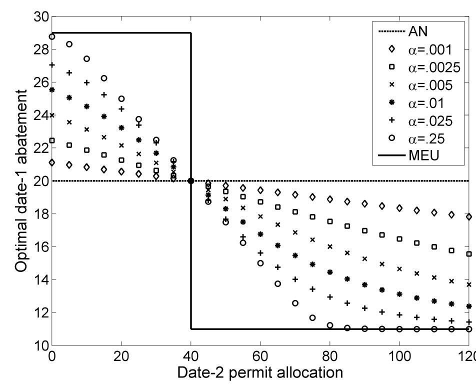
Anticomonotonicity (resp. comonotonicity) thus holds for sure if $\omega \leq 71$ (resp. $\omega \geq 89$). In expectations over Θ, anticomonotonicity (resp. comonotonicity) holds i.f.f. $\omega \leq$ (resp. $\geq$) 80. The situation is depicted in Figure 7a. Now consider that $\mathrm{T}_\theta = [\underline{\tau}+\theta; \bar{\tau}+\theta]$ and $\mathrm{B}_\theta =$

Figure 7: Cap-and-trade regime under CAAA with joint price and demand ambiguities

(a) $\hat{a}_1^\alpha = f(\omega)$ with $\mathrm{Cov}_\theta\{G, L\} > 0$



(b) $\hat{a}_1^\alpha = f(\omega)$ with $\mathrm{Cov}_\theta\{G, L\} < 0$



$[\underline{b}-\theta; \bar{b}-\theta]$, i.e. high-price scenarios coincide with low-demand scenarios. In this case it holds that $\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) = \langle\tilde{\tau}\rangle + \theta$ and

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) \leq 0 \Leftrightarrow \omega \leq \langle\tilde{b}\rangle - 2\langle\tilde{\tau}\rangle - \bar{a}_1 - 3\theta. \tag{E.2}$$

Anticomonotonicity (resp. comonotonicity) thus holds for sure if $\omega \leq 13$ (resp. $\omega \geq 67$). In expectations over Θ, anticomonotonicity (resp. comonotonicity) holds i.f.f. $\omega \leq$ (resp. $\geq$) 40. The situation is depicted in Figure 7b. Comparing Figures 7a and 7b, we see that date-1 overabatement occurs for a wider allocation range as the allocation threshold is higher (resp. lower) when $\mathrm{Cov}_\theta\{G, L\} >$ (resp. $<$) 0 as compared to pure price ambiguity (Figure 3a). Note also that the variability of the adjustment in date-1 abatement increases (resp. decreases) around the threshold when $\mathrm{Cov}_\theta\{G, L\} >$ (resp. $<$) 0.

F Cap and Trade under binary price ambiguity

This appendix considers the case of binary price ambiguity, i.e. in all θ-scenarios $\tilde{\tau}_\theta$ either takes the value $\underline{\tau} > 0$ with probability $p(\theta) \in [0;1]$ or $\bar{\tau}$ with complementary probability and $\Delta\tau = \bar{\tau} - \underline{\tau} > 0$. Let the underlying objective price lottery be $(p, \underline{\tau}; 1-p, \bar{\tau})$. The no-ambiguity bias requires that $p = \mathbb{E}_F\{p(\tilde{\theta})\}$ and thus $\langle\tilde{\tau}\rangle = p\underline{\tau} + (1-p)\bar{\tau}$. W.l.o.g. assume for clarity that abatement cost functions are time separable. Let $\Upsilon(\cdot;\theta)$ denote the θ-scenario expected net intertemporal revenue from date-1 abatement. For all $a_1 \geq 0$ and $\theta \in \Theta$,

$$\begin{aligned}\Upsilon(a_1;\theta) &\doteq \pi_1(a_1) + \beta\mathcal{V}(a_1;\theta) \\ &= \zeta - C_1(a_1) - \beta p(\theta)(C_2(a_2^*(\underline{\tau})) + \underline{\tau}(b - a_1 - a_2^*(\underline{\tau}) - \omega)) \\ &\quad - \beta(1-p(\theta))(C_2(a_2^*(\bar{\tau})) + \bar{\tau}(b - a_1 - a_2^*(\bar{\tau}) - \omega)),\end{aligned} \tag{F.1}$$

where $\zeta = \zeta_1 + \beta\zeta_2$. With quadratic cost specification (1) Equation (F.1) rewrites

$$\Upsilon(a_1;\theta) = \zeta - C_1(a_1) + \beta\Big(p(\theta)\Delta\tau(b - a_1 - \omega - \langle\tau\rangle/c_2) - \bar{\tau}\Big(b - a_1 - \omega - \bar{\tau}/(2c_2)\Big)\Big), \tag{F.2}$$

where $\langle\tau\rangle \doteq (\bar{\tau}+\underline{\tau})/2$ denotes the date-2 average price when $p = .5$. Differentiating Equation (F.2) and evaluating it at $a_1 = \bar{a}_1 = \beta\langle\tilde{\tau}\rangle/c_1$ gives

$$\Upsilon_{a_1}(\bar{a}_1;\theta) = -C_1'(\bar{a}_1) + \beta\mathcal{V}_{a_1}(\bar{a}_1;\theta) = -C_1'(\bar{a}_1) + \beta(\bar{\tau} - p(\theta)\Delta\tau), \tag{F.3}$$

which is decreasing in θ i.f.f. $p(\theta)$ is increasing in θ. By optimality under ambiguity neutrality $\Upsilon_{a_1}(\bar{a}_1;\theta) = 0$ when $p(\theta) = p$. It follows that $\Upsilon_{a_1}(\bar{a}_1;\theta)$ changes sign from positive to negative at $p(\theta) = p$. Intuitively we see from Equation (F.2) that when the firm expects to be net short under the abatement stream $(\bar{a}_1; \langle\tau\rangle/c_2)$, $\Upsilon(\bar{a}_1;\theta)$ is relatively high (resp. low) when $p(\theta)$ is relatively large (resp. small). Therefore, for those θ-scenarios such that $p(\theta) < p$ where $\Upsilon(\bar{a}_1;\theta)$ is relatively low and $\Upsilon_{a_1}(\bar{a}_1;\theta) > 0$, an increase in a_1 will increase $\Upsilon(a_1;\theta)$. Conversely, for those θ-scenarios such that $p(\theta) > p$ where $\Upsilon(\bar{a}_1;\theta)$ is relatively high and $\Upsilon_{a_1}(\bar{a}_1;\theta) < 0$, an increase in a_1 will decrease $\Upsilon(a_1;\theta)$. In these two cases the spread in expected profits across θ-scenarios is reduced. More formally,

Proposition F.1. *Let ϕ exhibit CAAA. Under binary price ambiguity, quadratic and time separable abatement cost functions, the prevalence of ambiguity aversion raises date-1 abatement relative to ambiguity neutrality if, and only if,*
(i) *the objective probability associated with the low-price scenario p is above the threshold* $\bar{p} = (\beta c_2 \bar{\tau} + c_1 \langle \tau \rangle - c_1 c_2 (b - \omega)) / (\beta c_2 \Delta\tau) \in [0; 1]$*; or equivalently,*
(ii) *the firm expects to be net buyer of permits under the abatement stream* $(\bar{a}_1; \bar{a}_2)$ *where* $\bar{a}_1 = \beta\langle\tilde{\tau}\rangle/c_1$ *and* $\bar{a}_2 = \langle\tau\rangle/c_2$*; or equivalently,*
(iii) *the firm's allocation* ω *is below the threshold* $\bar{\omega} = b - \bar{a}_1 - \bar{a}_2$*.*

Proof. By differentiation w.r.t. θ we have, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$\partial_\theta \mathcal{V}(\bar{a}_1; \theta) = p'(\theta)\Delta\tau(b - \omega - \beta\langle\tilde{\tau}\rangle/c_1 - \langle\tau\rangle/c_2), \tag{F.4a}$$

$$\text{and } \partial_\theta \mathcal{V}_{a_1}(\bar{a}_1; \theta) = -p'(\theta)\Delta\tau. \tag{F.4b}$$

Therefore, anticomonotonicity holds i.f.f. $b - \omega - \beta\langle\tilde{\tau}\rangle/c_1 - \langle\tau\rangle/c_2 > 0$, i.e. the firm is net short of permits when it abates $(\bar{a}_1; \bar{a}_2)$. Note that by definition, $\langle\tilde{\tau}\rangle = \bar{\tau} - p\Delta\tau$, which is decreasing with p. Anticomonotonicity thus holds i.f.f.

$$2\beta c_2 \left(\bar{\tau} - p\Delta\tau\right) + c_1 \left(\bar{\tau} + \underline{\tau}\right) < 2c_1 c_2 \left(b - \omega\right), \tag{F.5}$$

that is, i.f.f. $p > \bar{p}$. For $\bar{p}$ to be admissible we need that $\beta\underline{\tau} \leq c\left(b - \omega\right) \leq \beta\bar{\tau}$. To see why it makes sense to have such a price range, note that when the permit price is $c\left(b - \omega\right)$ the firm's gross abatement effort $b - \omega$ is optimally apportioned between the two dates. □

Initial allocation continues to dictate the direction of the date-1 abatement adjustment. This contrasts with results in Snow (2011), Alary et al. (2013), Wong (2015a) and Berger (2016) where the effect of pessimism is clear under a binary ambiguity structure. However the condition for anticomonotonicity to hold is milder than in Proposition 4.8. The ambiguity averse firm must expect to be net short under the sole the abatement stream $(\bar{a}_1; \bar{a}_2)$ − not across all θ-scenarios − for it to overabate. Note that this is akin to a situation where the ambiguity averse firm has no idea about the future price at all and thus considers the equiprobable price scenario. By contrast the ambiguity neutral firm is not affected by ambiguity.

A novel insight from Proposition F.1 is Condition (i). An explicit $\bar{p}$-threshold allows us to characterise the effects of an increase in the ambiguity level, here proxied by the price range $\Delta\tau$, for given degree of ambiguity aversion. To do so we determine the infinitesimal shift δp in $\bar{p}$ consecutive to an infinitesimal increase $\delta\tau > 0$ in $\Delta\tau$. For an upward shift in $\Delta\tau$, i.e. $\bar{\tau}$

increases by $\delta\tau$ with $\underline{\tau}$ fixed, $\bar{p}$ reacts such that

$$2\beta c_2\left(\delta\tau - \bar{p}\delta\tau - \bar{\tau}\delta p - \delta p\delta\tau + \underline{\tau}\delta p\right) + c_1\delta\tau = 0, \text{ i.e. } R^{\uparrow} = \frac{\delta p}{\delta\tau} = \frac{2\beta c_2(1-\bar{p}) + c_1}{2\beta c_2\Delta\tau} > 0, \quad \text{(F.6)}$$

where $\delta p\delta\tau \simeq 0$ in the first order and $R^{\uparrow}$ denotes the rate of increase in $\bar{p}$ consecutive to an increase in $\bar{\tau}$ by $\delta\tau$. For a downward shift in $\Delta\tau$, i.e. $\underline{\tau}$ decreases by $\delta\tau$ with $\bar{\tau}$ fixed,

$$2\beta c_2\left(\bar{p}\delta\tau + \bar{\tau}\delta p + \delta p\delta\tau - \underline{\tau}\delta p\right) + c_1\delta\tau = 0, \text{ i.e. } R^{\downarrow} = -\frac{\delta p}{\delta\tau} = \frac{2\beta c_2\bar{p} + c_1}{2\beta c_2\Delta\tau} > 0, \quad \text{(F.7)}$$

where $\delta p\delta\tau \simeq 0$ again and $R^{\downarrow}$ denotes the rate of decrease in $\bar{p}$ consecutive to a decrease in $\underline{\tau}$ by $\delta\tau$, in absolute terms. It follows that

$$R^{\uparrow} - R^{\downarrow} = (1-2\bar{p})/\Delta\tau > 0 \iff \bar{p} < 1/2. \quad \text{(F.8)}$$

Consider a symmetric price range increase from $\Delta\tau$ to $\Delta\tau + 2\delta\tau$ which preserves $\langle\tau\rangle$ as well as the ambiguity neutral firm's price estimate $\langle\tilde{\tau}\rangle$. The ambiguity averse firm's price estimate, however, shifts from $\langle\tilde{\tau}\rangle_{\Delta\tau}$ to $\langle\tilde{\tau}\rangle_{\Delta\tau+2\delta\tau}$ where

$$\langle\tilde{\tau}\rangle_{\Delta\tau+2\delta\tau} = \langle\tilde{\tau}\rangle_{\Delta\tau} + \delta\tau(1-2p) \geq \langle\tilde{\tau}\rangle_{\Delta\tau} \iff p \leq 1/2 \iff \langle\tilde{\tau}\rangle_{\Delta\tau} \leq \langle\tau\rangle. \quad \text{(F.9)}$$

An increase in the range of price ambiguity hence always brings the ambiguity averse firm's price estimate (resp. $\bar{p}$) closer to $\langle\tau\rangle$ (resp. 1/2). This can be likened to a precautionary principle. More precisely under CAAA,

1. When $\bar{p} > 1/2$, the ambiguity averse firm overabates at date 1 i.f.f. $p \geq \bar{p} > 1/2$, i.e. i.f.f. $\langle\tilde{\tau}\rangle \leq \langle\tau\rangle$. That is, ambiguity aversion raises date-1 abatement when the ambiguity neutral firm foresees a price below $\langle\tau\rangle$ and does not abate enough relative to the $\langle\tau\rangle$-price scenario. Both $\bar{p}$ and $\langle\tilde{\tau}\rangle$ decrease consecutive to a symmetric increase in $\Delta\tau$, which makes the anticomonotonicity criterion relatively laxer.
2. When $\bar{p} < 1/2$, note that the ambiguity averse firm overabates i.f.f. $p \in [\bar{p}; 1/2]$, i.e. even though $\langle\tilde{\tau}\rangle > \langle\tau\rangle$ and the ambiguity neutral firm already abates more at date 1 than under the $\langle\tau\rangle$-price scenario. Both $\bar{p}$ and $\langle\tilde{\tau}\rangle$ increase consecutive to a symmetric increase in $\Delta\tau$, which makes the anticomonotonicity criterion relatively more restrictive.

In other words, when the condition for pessimism to raise date-1 abatement relative to ambiguity neutrality is relatively demanding (resp. lax), an increase in the ambiguity range makes it laxer (resp. more demanding), which is in line with a precautionary principle.