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Fabien Prieur

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Strategic Fossil Expansion and the Timing of the Energy Transition*

Fabien Prieur†

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Abstract

We develop a dynamic model of exhaustible resource exploitation, with exploration, in which a regulator determines the end date of the fossil regime by trading off industry profits against climate damages. The weight assigned to damages reflects the fossil industry's pre-existing political influence. We compare Nash and Stackelberg interactions between the industry and the regulator. Under Nash behavior, regulation shortens the fossil regime and reduces cumulative emissions relative to the unregulated benchmark. Under Stackelberg leadership, however, a monopoly may increase exploration relative to the Nash outcome in order to delay the transition. Calibrating the model to global oil market data, we obtain that strategic leadership increases reserves by approximately 7% relative to the Nash outcome and delays the transition by about 2-3 years. The analysis thus provides an explanation for sustained upstream fossil fuel investment despite announced net-zero commitments.

Keywords: exploration, energy transition, political influence, Nash vs Stackelberg interaction

JEL classification: D72, C73, Q54

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†CEE-M, Univ Montpellier, CNRS, INRAE, Institut Agro Montpellier. fabien.prieur@umontpellier.fr.

1 Introduction

Climate policy seeks to accelerate the transition away from fossil fuels. Yet upstream investment in fossil fuel exploration remains substantial. Why would firms continue expanding reserves when climate regulation may render a large fraction of those reserves stranded?

Recent evidence highlights this tension. Despite net-zero commitments, global investment in oil and gas exploration and production has remained broadly stable over the past decade and continues to represent a sizable share of total energy investment (International Energy Agency, 2025b). At the same time, climate science indicates that a large proportion of existing fossil fuel reserves must remain unburned to limit warming to 1.5C (Welsby et al., 2021). These observations suggest a disconnect between long-run climate objectives and fossil companies' exploration behavior. This paper proposes a strategic explanation for this pattern.

We develop a dynamic model in which exploration affects not only future extraction profits but also the regulator's decision of when to terminate the fossil regime. The fossil industry first chooses a costly exploration level that determines the initial stock of reserves and subsequently extracts along an optimal depletion path. A regulator determines the end date of the fossil regime by maximizing the difference between total discounted benefits from the exploitation of fossil fuels and total discounted climate damages. The relative weight assigned to damages reflects the inverse of the industry's lobbying power. When the regulator partially internalizes industry profits–due to political influence or broader economic concerns–exploration affects the cost-benefit analysis underlying the timing of the energy transition. Exploration thus becomes a strategic instrument.

We compare three scenarios. In the benchmark case, the industry freely chooses the stopping time of extraction. In a Nash regime, exploration and transition timing are chosen simultaneously, with each player treating the other's decision as given. In a Stackelberg regime, the industry anticipates the regulator's stopping rule when choosing exploration. This distinction is central: under Stackelberg leadership, exploration is selected with full awareness of its impact on the regulator's termination decision. We abstract from continuous policy instruments such as carbon taxes in order to isolate the interaction between exploration and regime termination. In practice, phase-out commitments, moratoria, and net-zero deadlines operate as discrete policy instruments. Our objective is to examine how exploration affects the extensive margin of fossil fuel use through its influence on the timing of transition.

The analysis yields three main results. First, relative to the unregulated benchmark, a regulator who internalizes climate damages shortens the fossil regime and reduces cumulative emissions. However, extraction is front-loaded: emissions are higher at each date within the shorter exploitation period. Second, under Nash interaction, exploration declines relative to the benchmark because a shorter expected regime reduces the marginal value of reserves. Third, under Stackelberg leadership, a monopoly may increase exploration relative to the Nash outcome in order to delay the transition. Strategic expansion arises only when two conditions hold. On the one hand, the regulator's stopping rule must be locally increasing in exploration. In that region, additional reserves raise the opportunity cost of termination sufficiently to induce postponement. On the other, the shadow value of the resource, which captures private marginal return to exploration, must be sufficiently low at the exploration level associated with maximal delay. In that case, exploration generates a wedge between Nash and strategic incentives, leading the industry to expand reserves beyond the Nash level. A numerical analysis calibrated to global oil market data implies that strategic leadership increases reserves by approximately 7% relative to the Nash outcome and delays the transition by about 2-3 years. These magnitudes are economically meaningful and arise precisely when equilibrium exploration lies within the delay-responsive region of the regulator's stopping rule.

The paper contributes to three strands of literature. First, it relates to works on dynamic strategic exploitation of exhaustible resources and endogenous timing (Salant, 1976, Gaudet and Lasserre, 1988, Gerlagh and Liski, 2011). Gaudet and Lasserre (1988) analyse endogenous stopping and exploration decisions in competitive and monopolistic settings. We extend this framework by introducing a regulator who determines the terminal date through an optimal stopping problem that takes climate damage into account. Second, the paper relates to the political economy of climate policy. Pressure groups can exert political influence directly through lobbying activities, such as contributions to politicians' electoral campaigns and informational lobbying of decision-makers, and indirectly through information campaigns intended to persuade public opinion. Yu (2005) examines the competition between industrial and environmental lobbies, explaining why the latter perform better in the latter form of political influence. Other contributions focus on a different strategy employed by industrial and fossil lobbies, which involves disseminating doubt about either the scientific evidence (Bramoullé and Orset, 2018) or the credibility of rival interest groups, typically environmental NGOs (Chiroleu-Assouline and Lyon, 2020). Rather than modelling lobbying explicitly, we treat political influence in a simplified form through the regulator's objective function, demonstrating how real investment decisions can influence policy timing. In terms of the policy implications of lobbying, there is growing evidence that industrial lobbies can successfully prevent the enactment of regulations (see Meng and Rode, 2019 for US climate policy) or reduce the probability of policy enactment (Kang, 2015). Lobbying can also result in less stringent regulations, leading to more allowances being given for free in the context of the EU ETS (Anger et al., 2016, Winkler, 2022). It can also result in lower taxes (Anger et al., 2015, Richter et al., 2009). This paper identifies a novel channel of influence: real investment decisions may shape regulatory timing.

Third, the analysis relates to research on intertemporal distortions in resource extraction, including the 'green paradox' literature (Sinn, 2012). Unlike models where anticipation of future carbon taxes accelerates extraction, this mechanism operates through the endogenous termination of the fossil regime and the strategic manipulation of reserve size.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 derives the optimality conditions governing exploration and termination decisions. Section 4 characterizes the equilibrium outcomes under the different strategic regimes and market structures. Section 5 conducts a numerical analysis. Section 6 concludes.

2 Model

The analysis extends Gaudet and Lasserre (1988)'s model of the exploitation of a non-renewable resource, whose available stock at instant t is denoted by S,[1] by a competitive industry vs a monopoly. The structure of the fossil market has certainly changed over the last thirty years, resulting in a market that is more competitive than before. Yet we keep considering these two extreme market structures as benchmarks for the analysis.

At each instant, type-i industry, $i = c, m$, extracts the resource at rate $q^i \geq 0$, which forms the market supply. Market demand is captured by the invariant inverse demand function, $p(q)$, with $p^{'}(q) < 0$. We assume away extraction costs. So the instantaneous profit is: $B^i(q^i)$, with $B^c(q^c) = pq^c$ and $B^m(q^c) = p(q^m)q^m$. In addition to extraction, the industry has to decide on its exploration effort. Exploration is modeled in a very simple way. There is no uncertainty surrounding this activity. The industry must simply discover or acquire the initial resource stock, S_0, in order to launch exploitation.[2] Exploration involves a cost $E(S_0)$, with $E'(S_0), E''(S_0) \geq 0$. The evolution of the resource is governed

[1] When unnecessary, the time index is omitted.

[2] So this is a one-shot decision. There is no room for further exploration during the fossil phase that would translate into increases in the stock of resource.

by: $\dot{S} = -q$.

We will envision three scenarios, $j = o, n, s$, that differ in terms of the identity of the decision-maker who chooses the date, T, when the fossil regime comes to an end. The first is Gaudet and Lasserre (1988)'s scenario in which the fossil industry optimally chooses the stopping time. We refer to the authors' paper for a presentation of the decision problem.[3] It will be used as a point for comparison with our two original scenarios in which the timing decision is in the hands of the regulator.

In scenarios $j = n, s$, industry i's problem can be written as:

$$\max_{\{q^{ij}(t)\}, S_0^{ij}} \int_0^{T^{ij}} B^i(q^{ij}(t))e^{-rt}dt - E(S_0^{ij}) \tag{1}$$

s.t. $\dot{S}^{ij}(t) = -q^{ij}(t)$, and $S^{ij}(T^{ij}) = 0$, with r the discount rate.

The regulator cares not only about the industry's profitability but also about the climate damage associated with the consumption of fossil fuels. In the absence of pollution decay, the concentration of greenhouse gases (GHG), P, evolves according to: $\dot{P} = -\dot{S} \Leftrightarrow P = P_0 + S_0 - S$, with P_0 given. In turn, the damage is captured by $D(P)$, with $D'(P), D''(P) > 0$ for $P > 0$.

The regulator's objective may incorporate industry profits for several reasons, including employment concerns, fiscal dependence on fossil revenues, or political influence. Pressure groups can influence the decision-making process through various strategies including direct lobbying, via contributions to political campaigns, informational lobbying, and indirect political influence. Rather than modeling lobbying expenditures explicitly, we treat political influence as embedded in the regulator's objective function, which yields the following

[3] In this case, there is no regulator, and T becomes an additional decision variable for the industry, that is associated with a specific transversality condition.

decision problem[4]

$$\max_{T^{ij}} \int_0^{T^{ij}} [B^i(q^{ij}(t)) - \omega D(P^{ij}(t))]e^{-rt}dt$$

s.t. $P^{ij}(t) = P_0 + S_0^{ij} - S^{ij}(t)$, P_0, $\{q^{ij}(t)\}_{t=0}^{T^{ij}}$ and $\{S^{ij}(t)\}_{t=0}^{T^{ij}}$ given. The parameter $\omega > 0$ denotes the relative weight assigned to climate damages. A lower value of ω corresponds to stronger lobbying power by the fossil industry. This parameter, together with the initial stock of GHG, P_0, are expected to play a key role in shaping the regulator decision.

In this situation, an interaction naturally arises between the industry and the regulator. In the second scenario, decision-makers are considered to take the decision of the other as given, when forming their own decision. This refers to a Nash type of strategic interaction according to which decisions S_0 and T are taken simultaneously. In the third and main one, the industry is a strategic leader in the Stackelberg game.[5] We assume perfect foresight: when the industry chooses its exploration strategy, it perfectly anticipate the regulator's reaction. This is precisely through this channel that the industry will be able to influence the public policy, i.e., the timing of the energy transition.

We abstract from continuous instruments such as carbon taxes in order to isolate the strategic interaction between exploration and regime termination. Introducing an optimal tax would not eliminate the timing problem: in many policy environments, phase-out commitments or moratoria operate as discrete instruments. Our objective is to study how exploration affects the extensive margin of fossil fuel use. We do not account either of the industry's investments in fossil infrastructure, or any other type investments made in anticipation to what comes after the energy transition (for instance, investments in renewable energies). We also abstract from additional features such as uncertainty, stochastic shocks,

[4] This reduced-form approach allows us to focus on how real investment decisions interact with regulatory timing, independently of the specific micro-foundations of political influence.

[5] So, we adopt the opposite perspective of the standard second best approach, which assigns the position of strategic leader to the regulator. Considering the second best would provide us with another reference scenario. But it seems a bit at odds with the existence of lobbying activities. And, it would make the analysis unnecessarily complicated.

or continuation payoffs. This explains why we impose $S^{ij}(T^{ij}) = 0$ as a condition in the industry problem. There is no reason to leave some resource unexploited in this context.

Including these additional ingredients would make the analysis richer. But this is beyond the scope of the current analysis. Our ambition is to build the simplest setting possible that allows us to conduct a tractable analysis and to draw neat conclusions regarding the comparison between the three different scenarios presented above: the optimal transition one (industry chooses T), the Nash interaction one, and the strategic exploration one (Stackelberg interaction). The first two will be used mostly to investigate the features of the last one, and to understand under which conditions a fossil industry with political influence may want to maintain exploration at a high level to delay the transition.

The key distinction between the Nash and Stackelberg scenarios lies in the way the industry adapts to the regulator decision. Under Nash interaction, the industry takes T^{in} as given, while when acting as a strategic leader, it accounts for its capacity to influence the regulator's optimal stopping problem. The latter situation is modeled as in Long et al. (2017),[6] and consists in defining the stopping time as a function of the state of the system at the beginning of the planning period, that is as a function of the level of exploration, S_0^{is}: $T^{is}(S_0^{is})$.

In the next Section, we highlight the optimality conditions associated with these decisions and interpret them.

[6]The authors develop a two-player differential game with both continuous and timing strategies. They introduce the new concept of piecewise closed-loop Nash equilibrium, that extends the concept of Markov perfect equilibrium to the resolution of games with timing strategies.

3 Optimality conditions for exploration and timing strategies

We characterize the equilibrium by examining the necessary optimality conditions associated with exploration and termination decisions. For the sake of the analysis, we will focus on interior solutions, especially when it comes to the stopping problem.

Let $\mathcal{H}^i(t) = B^i(q^i(t)) - \lambda^i(t)q^i(t)$ be the (current value) Hamiltonian, with $\lambda^i(t)$, the shadow price of the resource. In all scenarios, the industry's intertemporal extraction problem leads to the following set of necessary optimality conditions, for an interior solution:

$$\begin{cases} B^{i\prime}(q^{ij}) = \lambda^{ij} \\ \dot{\lambda}^{ij} = r\lambda^{ij} \\ \dot{S}^{ij} = -q^{ij} \end{cases} \tag{2}$$

The optimality conditions underlying the exploration and timing decisions, for scenarios, $j = o, n, s$, and market structures, $i = c, m$, are presented in Proposition 1 below, with $\mathcal{H}^{i*}(t)$ the maximized Hamiltonian (see the Appendix A.1).

Proposition 1. *Optimal transition:*

$$\begin{aligned} &\lambda_0^{io} = E'(S_0^{io}), \\ &\mathcal{H}^{i*}(T^{io}) = 0. \end{aligned} \tag{3}$$

Nash interaction:

$$\begin{aligned} &\lambda_0^{in} = E'(S_0^{in}), \\ &B^i(q^{in}(T^{in})) = \omega D(P_0 + S_0^{in}). \end{aligned} \tag{4}$$

Strategic exploration:

$$\begin{aligned} &\lambda_0^{is} + \mathcal{H}^{i*}(T^{is})e^{-rT^{is}}T^{i\prime}(S_0^{is}) = E'(S_0^{is}), \\ &B^i(q^{is}(T^{is})) = \omega D(P_0 + S_0^{is}). \end{aligned} \tag{5}$$

Denote by $V^{ij}(S_0^{ij}, T^{ij})$ the maximized present value of type-i industry's benefits in scenario j, i.e., the value obtained once substituting the optimal extraction policy, the one that solves (2), in the objective function. The first two scenarios share the first optimality condition related to the choice of S_0^{ij}. This condition sets the marginal benefit of exploration, captured by λ_0^{ij}, which is the marginal value of a variation in the initial stock, $\lambda_0^{ij} = \frac{\partial V^{ij}(S_0^{ij}, T^{ij})}{\partial S_0^{ij}}$, to the marginal cost. The key difference between the Nash and Stackelberg regimes then lies in the additional term

$$\mathcal{H}^{i*}(T^{is})e^{-rT^i}T^{is\prime}(S_0^{is}) \gtreqless 0.$$

This strategic term reflects the fact that exploration alters the regulator's stopping rule. If an increase in reserves delays termination, the industry internalizes this benefit when choosing exploration. The present value gain, for the industry, of changing this date, $T^{is\prime}(S_0^{is})$, is $\mathcal{H}^{i*}(T^{is})e^{-rT^{is}}$, where the Hamiltonian represents the marginal value of varying the horizon time T^{ij}, $\mathcal{H}^{i*}(T^{is}) = \frac{\partial V^{ij}(S_0^{ij}, T^{ij})}{\partial T^{ij}}$.

In all scenarios, the second condition governs the choice of T. In the optimal case, the choice is made by the industry based on the standard transversality condition that states that the optimal stopping time occurs when the value of expanding the fossil regime, as captured by the Hamiltonian, becomes worthless. In the two other cases, the stopping time is chosen by the regulator that puts an end to the fossil regime when the net benefit of staying in this regime, that is defined as the weighted difference between instantaneous profits and climate damages, becomes nil.

Observing that $B^{c\prime}(q^{cj}) = p$, we get, for the competitive industry, that $\mathcal{H}^{cj*}(t) = 0$, for all t and all j. It means that for this particular industry and market structure, optimality conditions for the Nash and strategic exploration scenarios coincide. There is no incentive to strategically influence the timing of the energy transition in this case.

The next Section digs deeper into the analysis, and the comparison between solutions (S_0^{ij}, T^{ij}) to the above systems.

4 Outcomes in the different scenarios, for the different industries

Our aim is to characterize and to compare the solutions of the different scenarios, for given market structure. This should enable us to find the conditions under which there will be *too much* exploration by the industry despite – or actually because of – the perspective of the energy transition. For that purpose, we need to know more about the features of the solution, which makes it convenient to use a specification of the model. In fact, we use a linear specification of the inverse demand, $p(q) = a - bq$, where a is the choke price, that is enough to solve the intertemporal extraction problem explicitly.

Next, observing that systems (3)-(5) share exactly one optimality condition, the resolution strategy consists in proceeding to a comparison by pairs of solutions (S_0^{ij}, T^{ij}), starting with the optimal transition and Nash scenarios. Using a quadratic specification of the cost of exploration, $E(S_0) = \varepsilon \frac{S_0^2}{2}$, the following results can be established (the proofs are displayed in the Appendices A.2.1 and A.4).

Proposition 2. *In the Nash scenario, the regulator puts an end to the fossil regime sooner than in the optimal transition scenario. In turn, type-i industry, subject to the climate regulation, chooses a lower level of exploration compared to what would be chosen with a free stopping time:*

$$T^{in} < T^{io} \text{ and } S_0^{in} < S_0^{io} \text{ for } i = c, m$$

The regulator's choice of the stopping time is not only dictated by market conditions. Its cost-benefit analysis logically leads to a sooner transition compared to what would be optimal for the industry.[7] This outcome reflects the corrective role of regulation in internalizing the climate externality. As to the industry, the initial shadow value of the resource is increasing in this date. A shorter exploitation period then reduces the marginal benefit of exploration and results in a lower initial stock. We therefore obtain that cumulative GHG emissions will be lower under the regulation than in its absence. However because the initial, or constant value, shadow value is lower in the Nash scenario, extraction rates are higher at every instant of the shorter exploitation phase. Consequently, the resulting climate damage is felt sooner under the regulation. This timing effect pushes in the opposite direction of the size effect (cumulative emissions diminish), and shares similarities with the forces that drive the so-called *green paradox* (Sinn, 2012).

We already observed that the outcomes for the Nash and Stackelberg scenarios are the same for the competitive industry. For the monopoly, we obtain (see the Appendix A.2.2):

Proposition 3. *The regulator's stopping time, $T^m(S_0)$, is non-monotone in the exploration level if*

$$\lim_{S_0 \to 0} G^{m\prime}(S_0) + 2br > 0. \tag{6}$$

More precisely, there exists a unique $\tilde{S}_0^m \in (0, \bar{S}_0)$ such that $T^{m\prime}(S_0) \geq 0 \Leftrightarrow S_0 \leq \tilde{S}_0^m$.

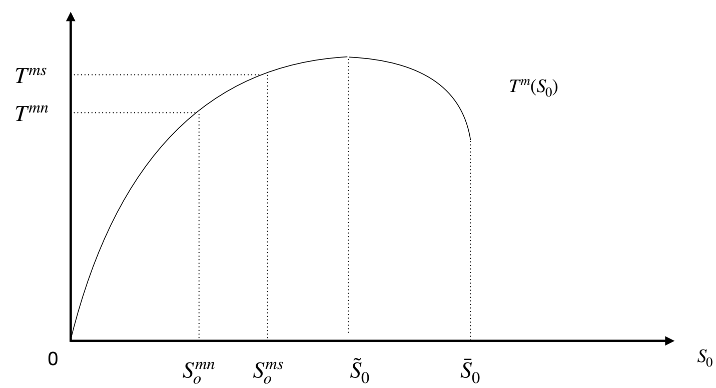
The upper bound, $\bar{S}_0$, on the level of exploration is a necessary condition for the regulator's decision problem to exhibit an interior solution, $T^m > 0$, in the Nash and Stackelberg scenarios. As to condition (6), it also imposes an upper bound, but on the initial GHG concentration. If it is met, then there is room for the regulator to adapt its strategy in response to any variation in the level of exploration.[8] In this case, the features of the stopping time $T^m(S_0)$, that can be interpreted as a reaction function in the strategic scenario only, indicate that for sufficiently low levels of exploration, the regulator responds to an increase

[7] In fact, in the absence of regulation, the industry would opt for the stopping time associated with the highest possible level of exploration.

[8] With a sufficiently large initial stock of pollution, but not too large for an interior solution to exist, the stopping time would always be decreasing in S_0.

in exploration by postponing the transition because the increase in the industry's profits outweighs the increase in the climate damage in its trade-off. For high enough S_0, the opposite reaction is required. This results in an inverted U-shaped stopping time (Figure 1), where the critical level $\tilde{S}_0^m$ represents the delay-maximizing level of fossil reserve.

Figure 1: Optimal stopping time as a function of the exploration level



Denote the initial shadow value by $\lambda_0^m = \lambda_0^m(T, S_0)$. Given the above information, we can show that:

Proposition 4. ***1. Existence:*** *A necessary and sufficient condition for the existence of a unique interior solution to the monopoly's exploration problem in the Nash scenario is:*

$$\lim_{S_0 \to \bar{S}_0} \lambda_0^m(T^m(S_0), S_0) < \lim_{S_0 \to \bar{S}_0} E'(S_0). \tag{7}$$

This condition is also sufficient for the existence of an interior solution in the Stackelberg case.

2. Effect of strategic leadership: *Suppose condition (6) is met. Condition*

$$\lambda_0^m(T^m(\tilde{S}_0^m), \tilde{S}_0^m) < E'(\tilde{S}_0^m), \tag{8}$$

is necessary and sufficient to obtain the following ranking between Nash and Stackelberg exploration levels: $S_0^{ms} < S_0^{mn} < \tilde{S}_0^m$.

Conditions (7) and (8) are boundary conditions. Since the latter is stronger than the former, let us interpret (8) only, which is the critical one to obtain our main result.[9] Formally, this condition states that the initial marginal value of reserves evaluated at $\tilde{S}_0^m$ must lie below marginal exploration cost. To understand under which circumstances it is more likely to hold, we conduct comparative statics with respect to the key parameter ω, the relative weight placed on climate damage in the regulator's objective. We get that the optimal stopping time $T^m(S_0)$ is decreasing in ω. Ceteris paribus, the larger the weight of the climate damage in the regulator's objective, the sooner the transition. This is because a larger ω tends to increase the cost of extending the fossil regime at each date, while leaving the benefit unchanged. We expect that the delay-maximizing level of reserve decreases in ω as well. This is the case if the initial shadow value is sufficiently low: $\lambda_0^m(T^m(\tilde{S}_0^m), \tilde{S}_0^m) < \frac{a}{2}$ (see the Appendix A.3).

Now suppose that ω is sufficiently low,[10] which results in a pretty high $T^m(S_0)$, for any S_0, and a pretty large $\tilde{S}_0^m$. In this situation, climate policy is rather lax, as the regulator is responsive to industry interests. Furthermore, the industry has significant leeway to influence the regulator's decision. The conditions are therefore ripe for a strategic increase in fossil fuel reserves. Indeed, the potential gain from delaying the transition is substantial, as is the incentive to increase reserve levels compared to the scenario where the effect of exploration choice on the transition date is ignored. In any case, the transition occurs strictly later under Stackelberg interaction than under Nash interaction. But here this is explained by the fact that strategic exploration exceeds the Nash level.

Strategic expansion thus requires two conditions. First, the regulator's stopping rule must be locally increasing in exploration, so that additional reserves can delay the transi-

[9]Condition (7) ensures that exploration is not pushed to the maximal level $\bar{S}_0$, beyond which the regulator's stopping problem would cease to admit an interior solution. It guarantees that the marginal value of reserves eventually falls below marginal exploration cost.

[10]But not too low otherwise the regulator's stopping time is very close to the industry's optimal one and it is not worth manipulating reserves.

tion. Second, exploration must not already be privately optimal at the reserve level that maximizes delay. When both conditions hold, a wedge emerges between the private value of reserves and their strategic value in influencing policy timing. The monopoly then expands exploration beyond the Nash level in order to internalize the delay effect.

In order to illustrate our findings and to check whether they are relevant in the real world situation, we next proceed to a numerical analysis based on plausible calibration of the model.

5 Quantitative analysis

The calibration of the model requires two modifications. First we introduce a parameter, κ, that converts units of fossil consumption into GHG emissions. This modifies the law of motion of P that becomes $\dot{P} = \kappa q$, and results in the following relationship between the stock of resource and the stock of pollution: $P = P_0 + \kappa(S_0 - S)$ for all t. Second we extend the damage function to account for the social cost of carbon (SCC). Damages will also be defined in terms of the difference between current and preindustrial GHG concentration, P_{PI}. Using a quadratic specification of this function, this is done by incorporating a parameter δ such that: $D(P) = \frac{\delta}{2}(P(t) - P_{PI})^2$.

The oil demand block is calibrated to recent global market conditions. Baseline consumption is set to $q_0 = 104.4$ mb/d, consistent with the International Energy Agency's projection of world oil demand for 2026 (International Energy Agency, 2025a). The benchmark price is $p_0 = 69$ USD per barrel, corresponding to the U.S. Energy Information Administration's estimate of the 2025 annual average Brent crude oil price (U.S. Energy Information Administration, 2026). We adopt a medium-run price elasticity of demand $\eta_d = -0.45$, consistent with long-run empirical estimates reported by Cooper (2003). Proven world crude oil reserves are set to 1,567 billion barrels based on the OPEC Annual Statistical Bulletin (Organization of the Petroleum Exporting Countries, 2025). Atmospheric CO_2 concentration is initialized at $P_0 = 426.5$ ppm, reflecting recent global measurements reported by National Oceanic and Atmospheric Administration (2024), and the pre-industrial benchmark is set to $P_{\mathrm{PI}} = 278$ ppm following the IPCC Sixth Assessment Report (Intergovernmental Panel on Climate Change, 2021). The climate damage function is calibrated to match a social cost of carbon of 190 USD per ton of CO_2, corresponding to the U.S. Environmental Protection Agency's central estimate under a 2% discount rate (U.S. Environmental Protection Agency, 2023). We adopt a baseline discount rate of $r = 5\%$ in the model and adjust the implied marginal damage accordingly. The conversion 1 ppm ≈ 7.81 $GtCO_2$ follows standard carbon-cycle accounting reported in Intergovernmental Panel on Climate Change (2021). The parameter κ converts cumulative oil extraction into an increase in atmospheric CO_2 concentration. One barrel of crude oil releases approximately 0.43 tons of CO_2 when combusted (U.S. Environmental Protection Agency, 2024). Hence, one gigabarrel (Gb) of oil generates about 0.43 $GtCO_2$. Using again the standard atmospheric conversion above, this implies that extracting one Gb of oil increases atmospheric concentration by $\kappa = 0.43/7.81 \approx 0.055$ ppm. As to the political weight parameter, we set $\omega = 0.4$, , implying that the regulator internalizes 40% of climate damages. This value ensures an interior stopping time while capturing partial political influence of the fossil sector. Table 1 summarizes the set of parameter values.

Third because we conduct a partial equilibrium analysis and costs (climate damage) and benefits (industry's profits) are not of the same scale, we need to proceed to some pseudo-normalization, or rescaling, of the damage function for the calibration. Without it, there would be no interior solution to the government's stopping problem, unless it was for a very tiny ω.

Table 1: Baseline calibration

Parameter	Value	Description
Demand (inverse demand $p(q) = a - bq$)		
q_0	104.4 mb/d	Baseline world oil demand
p_0	69 USD/bbl	Baseline Brent price
η_d	-0.45	Medium-run price elasticity of demand
b	1.4687 \$/ (mb/d)	Slope of inverse demand
a	222.3333 \$/bbl	Intercept of inverse demand
Reserves		
S_0^{data}	1567 Gb	World proven crude oil reserves (calibration target)
Climate / damages		
P_0	426.5 ppm	Initial atmospheric CO_2 concentration
P_{PI}	278 ppm	Pre-industrial CO_2 benchmark
r	0.05	Discount rate
ω	0.4	Weight on damages in regulator objective
SCC	190 USD/tCO_2	Social cost of carbon (external benchmark)
δ	5.0×10^8 \$/ppm^2/yr	Damage curvature in $D(P) = \frac{\delta}{2}(P - P_{\text{PI}})^2$
κ	0.0553 ppm/Gb	CO_2 concentration increase per Gb oil consumed

Let $B_0 = p_0 q_0$ and $D_0 = D(P_0)$ respectively be the initial profit and climate damage, we redefine the damage function, at any date, as:

$$\tilde{D}(P) = \frac{B_0}{D_0} D(P).$$

Then the regulator's objective, in the numerical analysis, can be written as

$$W^{num} = B_0 \int_0^T e^{-rt} \left[\frac{B(q(t))}{B_0} - \omega \frac{D(P(t))}{D_0} \right] dt,$$

and the regulator's optimal stopping rule becomes:

$$B(q(T)) = \omega \frac{B_0 D(P_0 + \kappa S_0)}{D_0}.$$

Admittedly this necessary simplification comes at the cost of neutralizing the influence of the damage parameter δ.

Before we move to the results, note that exploration costs are assumed quadratic, $E(S_0) = \frac{\varepsilon S_0^2}{2}$, and the curvature parameter is calibrated to match observed proven reserves (1,567 Gb) under the equilibrium concepts considered in the numerical analysis. This leads to the calibrated values displayed in Table 2. Although the exploration cost parameter ε is not directly observable, the implied marginal exploration cost $E'(S_0) = \varepsilon S_0$ can be expressed in dollars per barrel. In our baseline calibration, this corresponds to values on the order of a few dollars per barrel at the observed reserve level. This magnitude is broadly consistent with estimates of finding and development costs for conventional oil resources reported by international agencies and industry sources (e.g.U.S. Energy Information Administration, 2022, International Energy Agency, 2023), which typically place the lower end of marginal supply costs in the range of a few dollars per barrel.[11]

Results are exposed in Table 2.[12] Under the baseline calibration, the delay-maximizing reserve level is $\tilde{S}_0^m \simeq 1872.52$ Gb. When exploration costs are calibrated so that the Stackelberg outcome matches observed proven reserves (1,567 Gb), the implied Nash monopoly reserve level falls to approximately 1,469 Gb, while the strategic leader expands reserves by about 7% relative to Nash. This expansion delays the energy transition by roughly 2.7 years, which represents around 4–5% of the regulated transition horizon. Hence, the main effect of strategic exploration appears less in very large reserve changes than in a significant postponement of the transition date. The unregulated optimal-transition benchmark yields substantially higher reserve levels and termination dates, indicating that regulation shortens the fossil regime, while strategic leadership partially offsets this effect by increasing exploration within the delay-responsive region of the regulator's stopping rule.

The benchmark calibration sets the political-weight parameter to $\omega = 0.4$. The qualita-

[11] We emphasize, however, that ε should be interpreted as a reduced-form parameter capturing the global convexity of exploration costs rather than a direct measure of firm-level drilling or development expenditure.

[12] The numerical analysis has been done using Maple. The code is available upon request.

Table 2: Key outcomes under alternative calibrations of exploration costs: baseline case

	Stackelberg-calibrated ε	**Nash-calibrated ε**
Exploration-cost calibration		
Target moment	$S_0^{ms} = S_0^{data}$	$S_0^{mn} = S_0^{data}$
ε (in \$/Gb^2)	3.0027×10^6	2.1815×10^6
Exploration levels (Gb)		
$\tilde{S}_0^m$ (delay-maximizing)	1872.52	1872.52
S_0^{mn} (Nash monopoly)	1469.09	1567.00
S_0^{ms} (Stackelberg monopoly)	1567.00	1679.52
S_0^{mo} (optimal transition / unregulated)	1583.73	1714.05
Termination dates (years)		
$T^{mn} \equiv T_m(S_0^{mn})$	61.27	63.93
$T^{ms} \equiv T_m(S_0^{ms})$	63.93	66.75
T^{mo} (optimal transition / unregulated)	76.90	81.71
Strategic distortions (Stackelberg vs Nash)		
$\Delta S_0^{SN} \equiv S_0^{ms} - S_0^{mn}$ (Gb)	97.91	112.52
$\Delta S_0^{SN}/S_0^{mn}$ (%)	6.66	7.18
$\Delta T^{SN} \equiv T^{ms} - T^{mn}$ (years)	2.66	2.82
Gap to unregulated benchmark (optimal vs Nash)		
$\Delta S_0^{ON} \equiv S_0^{mo} - S_0^{mn}$ (Gb)	114.64	147.05
$\Delta S_0^{ON}/S_0^{mn}$ (%)	7.80	9.38
$\Delta T^{ON} \equiv T^{mo} - T^{mn}$ (years)	15.62	17.78

tive results are robust to different values of ω, the key parameter. However, the calibration ceases to be well-defined for sufficiently large values of ω, as the observed reserve level exceeds the maximal feasible reserve level implied by the regulated problem, $\bar{S}_0$. Yet, we can still perform the robustness analysis for lower values of ω (see Table 3). Overall we observe that the effects are qualitatively the same but they become really modest when the weight attached to the climate damage is low. When ω increases, the regulator places greater weight on climate damages, leading to earlier termination dates and lower equilibrium exploration levels. However, the wedge between Nash and Stackelberg outcomes increases. In other words, when the weight attached to the environment gets bigger, the incentive to manipulate reserves by the Stackelberg leader gets stronger, which results in a longer delay.

Table 3: Robustness analysis (ε calibrated on Stackelberg reserves)

	$\omega = 0.2$	$\omega = 0.3$
ε (in \$/Gb^2)	3.13×10^6	3.12×10^6
$\tilde{S}_0^m$ (Gb)	3780,90	2585.28
Nash eq. (S_0^{mn}, T^{mn})	$(1542.46, 70.90)$	$(1524.38, 66.89)$
Stackelberg eq. (S_0^{ms}, T^{ms})	$(1567, 71.40)$	$(1567, 68.26)$
Optimal sol. (S_0^{mo}, T^{mo})	$(1566.93, 76.28)$	$(1567.79, 76.31)$
ΔS_0^{SN} (Gb)	25.54	42.62
ΔT^{SN} (year)	0.50	1.37

6 Conclusion

This paper analyzes how exploration decisions interact with the timing of the energy transition in a second-best environment characterized by political influence. When a regulator determines the termination date of the fossil regime, exploration and policy timing become strategically interdependent.

Under Nash interaction, regulation shortens the fossil regime and reduces cumulative emissions. However, when the industry acts as a strategic leader, exploration may increase relative to the Nash outcome in order to delay the transition. This occurs when the regulator's stopping rule is non-monotonic in exploration—a condition more likely under strong lobbying power and moderate initial atmospheric carbon concentration.

The analysis suggests that sustained exploration need not reflect myopia or disbelief in climate commitments. Instead, exploration may serve as a strategic instrument that increases the economic cost of termination and shapes regulatory timing. The results underscore the importance of institutional design: when regulators partially internalize industry profits, real investment decisions may operate as vehicles of political influence.

The numerical analysis indicates that the strategic distortion is moderate in magnitude but persistent across plausible parameter values. While regulation substantially shortens the fossil regime relative to the unregulated benchmark, strategic leadership offsets part of this effect by expanding reserves within the delay-responsive region of the regulator's stopping rule. The results suggest that even relatively small strategic adjustments in reserve accumulation can have meaningful implications for the timing of the energy transition.

Future research could extend the framework by incorporating uncertainty, endogenous lobbying expenditures, renewable investment, continuous exploration. Empirically, the mechanism suggests that exploration should remain elevated in jurisdictions where regulators place significant weight on industry profitability–such as regions with strong fiscal dependence on fossil revenues or substantial political influence by producers. Testing this prediction represents an important avenue for future work.

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A Appendix

A.1 Optimality conditions

Conditions for the optimal transition scenario are standard, and can be found for instance in Gaudet and Lasserre (1988). As to the new conditions for the strategic scenarios, the second one is obtained by differentiating the regulator's objective function with respect to T^{ij}, taking the sequences $\{q^{ij}(t)\}$ and $\{S^{ij}(t)\}$ as given, and observing that $P^{is}(T) = P_0 + S_0^{is}$.

The first one comes from the following optimization: The strategic industry, when choosing S_0, maximizes the value function, $V^i(S_0, T^{is})$ net of the exploration cost, while taking into account the reaction function of the regulator, $T^{is}(S_0)$:

$$\max_{S_0} V^i(S_0, T^{is}(S_0)) - E(S_0)$$

The optimal exploration level, S_0^{is}, then satisfies:

$$\frac{\partial V^i(S_0^{is}, T^{is})}{\partial S_0} + \frac{\partial V^i(S_0^{is}, T^{is})}{\partial T^{is}} T^{is\prime}(S_0^{is}) - E'(S_0^{is}) = 0,$$

with partial derivatives of the value function satisfying:

$$\frac{\partial V^i(S_0, T^{is})}{\partial S_0} = \lambda_0^{ij} \text{ and } \frac{\partial V^i(S_0, T^{is})}{\partial T^{is}} = e^{-rT^{is}} \mathcal{H}^{i*}(T^{is})$$

and $\mathcal{H}^{i*}(T^{is})$ the maximized current value Hamiltonian (obtained by substitution $q^{is}(t)$ and $\lambda^{is}(t)$ with the solutions to the extraction problem, detailed below, in the Hamiltonian).

This corresponds to the first optimality condition for the strategic scenario. The Nash scenario combines the first condition of the optimal scenario with the second of the strategic scenario. This completes the proof of Proposition 1.

A.2 Monopoly

With a linear inverse demand, system (2) can be solved to obtain:

$$\begin{aligned} q^m(t) &= \tfrac{1}{2b}(a - \lambda_0^m e^{rt}) \\ S^m(t) &= S_0^m - \tfrac{1}{2b}(at - \tfrac{1}{r}\lambda_0^m(e^{rt} - 1)) \end{aligned}$$

The expression of the initial shadow value, whose current value is $\lambda^m(t) = \lambda_0^m e^{rt}$, is obtained by solving the condition $S^m(T) = 0$:

$$\lambda_0^m = \frac{r(aT - 2bS_0)}{e^{rT} - 1} \equiv \lambda_0^m(T, S_0), \tag{9}$$

which is assumed to be non-negative, with

$$\begin{aligned} \tfrac{\partial \lambda_0^m}{\partial S_0} &= -\tfrac{2br}{e^{rT}-1} < 0, \\ \tfrac{\partial \lambda_0^m}{\partial T} &= \tfrac{r}{e^{rT}-1}(a - \lambda_0^m e^{rT}) \geq 0, \end{aligned}$$

because $q^m(t) \geq 0$ is equivalent to $\lambda^m(t) = \lambda_0^m e^{rt} \leq a$ for all t.

Note that the specification of the inverse demand allows us to reformulate the second optimality condition derived in the Appendix A.1. For the monopoly, the Hamiltonian is positive for positive extraction rates, $\mathcal{H}^{mj}(t) = b(q^{mj}(t))^2$. So, in the optimal transition scenario, the second optimality condition imposes $q^{mo}(T^{mo}) = 0$, which is equivalent to $\lambda^{mo}(T) = a$.

In the Nash and strategic scenarios, this condition

$$(a - bq^m(T))q^m(T) = \omega D(P_0 + S_0),$$

is a second order polynomial in $q^m(T)$, whose unique admissible solution (featuring $q^m(T) <$

$\frac{a}{2b}$),[13] is:

$$q^m(T) = \frac{a}{2b}(1 - \sqrt(\Delta(S_0))) \Leftrightarrow \lambda^m(T) = a\sqrt(\Delta(S_0)),$$

with $\Delta(S_0) = 1 - \frac{4b\omega D(P_0+S_0)}{a^2} > 0 \leftrightarrow S_0 < \bar{S}_0$. The upper bound imposed on the level of exploration is a necessary condition for the regulator's decision problem to exhibit an interior solution, $T^{ij} > 0$, in the Nash and strategic scenarios. Indeed for all $S_0 > \bar{S}_0$, we obtain $B^i(q^{ij}(T^{ij})) < \omega D(P_0 + S_0)$ for $j = n, s$, and all admissible $q^{ij}(T^{ij})$, so for all $T^{ij} > 0$.[14] For an interesting problem, we focus hereafter on the interval of variation $(0, \bar{S}_0)$ for the exploration strategy.

Let us denote $G^m(S_0) = a\sqrt(\Delta(S_0))$. With a linear inverse demand, the pairs (S_0^{mo}, T^{mo}), (S_0^{mn}, T^{mn}), (S_0^{ms}, T^{ms}) respectively solve:

Optimal transition:

$$\begin{aligned} \lambda_0^{mo} &= E'(S_0^{mo}) \\ \lambda^{mo}(T^{mo}) &= a \end{aligned} \tag{10}$$

VS Nash interaction

$$\begin{aligned} \lambda_0^{mn} &= E'(S_0^{mn}) \\ \lambda^{mn}(T^{mn}) &= G^m(S_0^{mn}) \end{aligned} \tag{11}$$

VS strategic exploration:

$$\begin{aligned} \lambda_0^{ms} + \mathcal{H}^{m*}(T^{ms})e^{-rT^{ms}}T^{is\prime}(S_0^{ms}) &= E'(S_0^{ms}) \\ \lambda^{ms}(T^{ms}) &= G^m(S_0^{ms}) \end{aligned} \tag{12}$$

[13] Note that $\frac{a}{2b}$ is the rate of extraction that maximizes the instantaneous benefit. This sets an upper bound on the extraction rate because it is not optimal for the industry to exceed it.

[14] in the Appendix A.2.2, we show that the extraction rate, evaluated at the ending time, is monotone decreasing in T^{ij}.

A.2.1 Optimal vs Nash

For those two scenarios, the optimality conditions in (10) & (11) can be manipulated simultaneously, i.e., there is no timing. Making use of the definition of λ_0^m, the first condition of these two systems defines the optimal level of exploration as a function of the horizon of the exploitation phase:

$$S_0^m(T) = \frac{raT}{\epsilon(e^{rT}-1)+2br} \text{ for } j = o, n. \tag{13}$$

with

$$\lim_{T\to 0} S_0^m(T) = \lim_{T\to\infty} S_0^m(T) = 0$$

and

$$S_0^{m\prime}(T) = \frac{ra}{(\epsilon(e^{rT}-1)+2br)^2}(\epsilon(e^{rT}-1)+2br-\epsilon rTe^{rT})$$

Then, $S_0^{m\prime}(T) \geq 0 \Leftrightarrow \epsilon(e^{rT}-1)+2br \geq \epsilon rTe^{rT}$ with

$$\lim_{T\to 0} \epsilon(e^{rT}-1)+2br > 0 = \lim_{T\to 0} \epsilon rTe^{rT} \text{ and } \lim_{T\to\infty} \frac{\epsilon rTe^{rT}}{\epsilon(e^{rT}-1)+2br} = \infty.$$

So there exists a unique $\tilde{T}^m \in (0,\infty)$ such that $S_0^{m\prime}(T) \geq 0 \Leftrightarrow T \leq \tilde{T}^m$.

Now, these scenarios are associated with different conditions, w.r.t. the choice of T. Actually these conditions share the same LHS but their RHS differ. More precisely, for any S_0 we have

$$G^m(S_0) < a \Leftrightarrow \Delta(S_0) < 1$$

Let us denote the common LHS as $f^m(T) = \epsilon S_0^m(T)e^{rT}$. The derivative:

$$f^{m\prime}(T) = \frac{\epsilon rae^{rT}}{(\epsilon(e^{rT}-1)+2br)^2}(\epsilon(e^{rT}-(1+rT))+2br(1+rT)) > 0$$

In addition, $f^m(\tilde{T}^m) = a$. So the optimal stopping time of the industry is precisely

$T^{mo} = \tilde{T}^m$, that is associated with the highest level of exploration, $S_0^{mo} = S_0^m(\tilde{T}^m)$.

As to the Nash scenario, because the RHS, $G^m(S_0^m(T))$, is inverted-U shaped with its minimum at $T = \tilde{T}^m$ and always lower than a, we can conclude that there exists a unique T^{mn} satisfying the second optimality condition, that is such that $T^{mn} < T^{mo}$. By the definition of $S_0^m(T)$, this in turn is equivalent to $S_0^{mn} < S_0^{mo}$. Finally, note that for the Nash solution to be well-defined, it is sufficient that $S_0^m(\tilde{T}^m) \leq \bar{S}_0$.

A.2.2 Nash vs Stackelberg

In the strategic scenario, the industry moves first. The problem is solved backward starting with the regulator's decision to choose T for given S_0 (second condition in 12). Then we turn to the industry's problem of choosing S_0 given the regulator's reaction (first condition in 12). This dictates how conditions should be manipulated. For consistency/comparison, the Nash scenario, that displays the same second optimality condition, will also be (re)analyzed according to this specific logic.

We shall deal first with the second condition:

$$\lambda_0^m(S_0, T)e^{rT} = G^m(S_0). \tag{14}$$

$\lambda_0^m(S_0, T)$ is defined for $T \geq \frac{2bS_0}{a} (\Leftrightarrow \lambda_0 \geq 0)$. It represents an increasing function of T, with $\lambda_0(S_0, \frac{2bS_0}{a})e^{r\frac{2bS_0}{a}} = 0$ and $\lim_{T\to\infty} \lambda_0(S_0, T)e^{rT} = \infty$. So it admits a unique solution, $T^m(S_0)$, defined implicitly only.

Total differentiation of (14) yields:

$$T^{m\prime}(S_0) = -\frac{\frac{\partial \lambda_0^m}{\partial S_0^m} e^{rT^m(S_0)} - G^{m\prime}(S_0)}{(\frac{\partial \lambda_0^m}{\partial T^m} + r\lambda_0^m(S_0, T^m(S_0)))e^{rT^m(S_0)}} \tag{15}$$

with $G^{m\prime}(S_0) = a\frac{\Delta'(S_0)}{2\sqrt{(\Delta(S_0))}} < 0$, $G^{m\prime\prime}(S_0) = \frac{a}{\Delta(S_0)}(\Delta''(S_0)\sqrt{(}\Delta(S_0) - (\Delta'(S_0))^2) < 0$.

We obtain: $T^{m\prime}(S_0) \geq 0 \Leftrightarrow \frac{\partial \lambda_0^m}{\partial S_0^m} e^{rT^m(S_0)} \leq G^{m\prime}(S_0) \Leftrightarrow \frac{2br}{e^{rT^m(S_0)}-1} e^{rT^m(S_0)} \geq -G^{m\prime}(S_0)$. Let the LHS be denoted by $h^m(S_0)$. Both functions take positive values. The function in the RHS is increasing in S_0, with $\lim_{S_0 \to \bar{S}_0} -G^{m\prime}(S_0) = \infty$. As to $h^m(S_0)$, it satisfies

$$h^{m\prime}(S_0) = -\frac{2br^2 e^{rT^m(S_0)}}{(e^{rT^m(S_0)} - 1)^2} T^{m\prime}(S_0)$$

and $\lim_{S_0 \to \bar{S}_0} = h^m(S_0) \in (0, \infty)$. Therefore, imposing $\lim_{S_0 \to 0} = h^m(S_0) > \lim_{S_0 \to 0} -G^{m\prime}(S_0)$, or even the stronger conditions on the parameters

$$\lim_{S_0 \to 0} G^{m\prime}(S_0) + 2br > 0,$$

is sufficient to conclude that there exist an odd number of intersections between $h^m(S_0)$ and $-G^{m\prime}(S_0)$. This condition imposes an upper bound on P_0 because: $-\lim_{S_0 \to 0} G^{m\prime}(S_0) = \frac{2b\omega D(P_0)}{a\sqrt(\Delta(0))}$, with derivative w.r.t. P_0 equal to $\frac{2b\omega}{a\Delta(0)}(D''(p_0)\sqrt(\Delta(0)) + \frac{2b\omega(D'(P_0)^2)}{a^2\sqrt(\Delta(0))}) > 0$.

Next we can demonstrate that the number of intersection is exactly one. Working by contradiction, assume that there exist $n = 3$ points of intersection, $\tilde{S}_0^1 < \tilde{S}_0^2 < \tilde{S}_0^3$. These points also correspond to extrema of the function $h^m(S_0)$. So they should satisfy: $h^{m\prime}(\tilde{S}_0^k) = 0$ and $h^m(\tilde{S}_0^k) = -G^{m\prime}(\tilde{S}_0^k)$ for $k = 1, 2, 3$. It must then be that $\lim_{S_0 \to \tilde{S}_0^{1+}} h^{m\prime}(\tilde{S}_0) < 0$ and $\lim_{S_0 \to \tilde{S}_0^{2-}} h^{m\prime}(\tilde{S}_0) > 0$. This in turn implies that there must exist a $\hat{S}_0 \in (\tilde{S}_0^1, \tilde{S}_0^2)$ such that $h^{m\prime}(\hat{S}_0) = 0$ and $h^m(\hat{S}_0) < -G^{m\prime}(\hat{S}_0)$. Such a $\hat{S}_0$ would be an extremum of function $h^m(S_0)$, but not an intersection between $h^m(S_0)$ and $G^{m\prime}(S_0)$. So, the contradiction.

In sum, there exists a unique $\tilde{S}_0^m$ such that $h(\tilde{S}_0^m) = -G'(\tilde{S}_0^m) \Leftrightarrow T^{m\prime}(\tilde{S}_0^m) = 0$. The optimal stopping time – or the reaction function – of the regulator, $T^m(S_0)$, is a bell-shaped function of the the exploration decision of the industry. This completes the proof of Proposition 3

The second step of the proof consists in substituting $T^m(S_0)$ in the industry's optimality condition and search for a solution. We conduct the analysis for the Nash scenario with the aim of deriving conclusions for the Stackelberg scenario.

Define $F^m(S_0)$ as the LHS of the second condition: $F^m(S_0) = \lambda_0^m(S_0, T^m(S_0))$. We need to find the solution(s) to:

$$F^m(S_0) = E'(S_0)$$

given that $E'(0) = 0$ and $E''(S_0) > 0$ (no need to use the specification here), and for $S_0 \in (0, \bar{S}_0)$.

The features of $F^m(S_0)$ are as follows:

$$F^{m\prime}(S_0) = \frac{\partial \lambda_0^m}{\partial S_0} + \frac{\partial \lambda_0^m}{\partial T} T^{m\prime}(S_0)$$

which is equivalent to:

$$F^{m\prime}(S_0) = \frac{1}{(\frac{\partial \lambda_0^m}{\partial T} + r\lambda_0^m(S_0, T))e^{rT^m(S_0)}} \left(\frac{\partial \lambda_0^m}{\partial S_0} \frac{\partial \lambda_0^m}{\partial T} (e^{rT^m(S_0)} - 1) + r\lambda_0^m \frac{\partial \lambda_0^m}{\partial S_0} + \frac{\partial \lambda_0^m}{\partial T} G^{m\prime}(S_0) \right) < 0$$

after some manipulations and given the expressions of these derivatives. Given $F^m(S_0) \geq 0$ for all $S_0 \geq 0$, a necessary and sufficient condition for the existence of a unique S_0^{mn}, solution to the equation above, is:

$$\lim_{S_0 \to \bar{S}_0} F^m(S_0) < \lim_{S_0 \to \bar{S}_0} E'(S_0). \tag{16}$$

This together with the transition date, $T^{mn} = T^m(S_0^{mn})$, fully characterizes the solution to the Nash scenario, already studied in the Appendix A.2.1.

The corresponding condition, in the strategic scenario, can be written as:

$$H^m(S_0) = E'(S_0)$$

with,

$$H^m(S_0) = F^m(S_0) + \mathcal{H}^m(T^m(S_0))e^{-rT^m(S_0)}T^{m\prime}(S_0).$$

In order to conduct the existence analysis, we first exploit the qualitative properties of this extra (second) term to locate $H^m(S_0)$ with respect to $F^m(S_0)$. Indeed, from the behavior of $T^m(S_0)$, we know that $H^m(S_0) \geq F^m(S_0)$ if and only if $S_0 \leq \tilde{S}_0^m$. Then, there are two possible cases.

If the condition $F^m(\tilde{S}_0^m) < E'(\tilde{S}_0^m)$ holds (it is stronger than 16), then we can conclude that S_0^{mn} is lower than $\tilde{S}_0^m$, and $S_0^{ms} \in (S_0^{mn}, \tilde{S}_0^m)$. Else, we obtain: S_0^{mn} is greater than $\tilde{S}_0^m$, and $S_0^{ms} \in (\tilde{S}_0^m, S_0^{mn})$. This completes the proof of Proposition 4.

A.3 Comparative statics

We can conduct some comparative statics on $T^m(S_0)$. Let us define X as the vector of parameters: $X = (a, b, r, \omega, P_0)$ and x an element of this vector. Then, let us make the dependence of the functions in the optimality condition defining T^m over (some elements of) X explicit: $\lambda_0^m = \lambda_0(S_0, T; X)$ and $G^m(S_0) = G(S_0; X)$. Total differentiation of this condition yields:

$$\frac{\partial T^m}{\partial x} = -\frac{\frac{\partial \lambda_0}{\partial x} e^{rT} - \frac{\partial G}{\partial x}}{(\frac{\partial \lambda_0}{\partial T} + r\lambda_0)e^{rT}} \text{ for } x \neq r$$

Direct comparison of the terms at the numerator gives: $\frac{\partial T^m}{\partial \omega}, \frac{\partial T^m}{\partial P_0} < 0$, because $\frac{\partial \lambda_0}{\partial x} = 0$ and $\frac{\partial G}{\partial x} < 0$ for $x = \omega, P_0$. We also have $\frac{\partial T^m}{\partial r} > 0$, whereas the effect of a change in a and b is unclear (these parameters affect both the cost and benefit of extending the fossil phase).

Let us now analyze how $\tilde{S}_0^m$ responds to changes in ω. Denote $h^m(S_0)$ as $h(S_0; \omega)$. $\tilde{S}_0^m$ solves

$$h(\tilde{S}_0^m; \omega) + G(\tilde{S}_0^m; \omega) = 0.$$

Differentiating this expression we get

$$\frac{\partial \tilde{S}_0^m}{\partial \omega} = -\frac{\frac{\partial h}{\partial \omega} + \frac{\partial G}{\partial \omega}}{G^{m\prime\prime}(\tilde{S}_0^m)}$$

The opposite of the denominator is positive. As to the numerator,

$$\frac{\partial h}{\partial \omega}+\frac{\partial G}{\partial \omega}=\frac{a}{2\sqrt{(}\Delta(\tilde{S}_0^m)}\left(-\frac{2br}{e^{rT^m(\tilde{S}_0^m)-1}}\frac{\partial \Delta(\tilde{S}_0^m)}{\partial \omega}\left[\frac{r}{(e^{rT^m(\tilde{S}_0^m)}-1)(\frac{\partial \lambda_0}{\partial T}+r\lambda_0)}-\frac{1}{\lambda_0}\right]+\frac{\partial \Delta'(\tilde{S}_0^m)}{\partial \omega}\right),$$

it is negative if

$$\lambda_0 < \frac{a}{2},$$

because $\frac{\partial \Delta(\tilde{S}_0^m)}{\partial \omega} = -\frac{4bD(P_0+\tilde{S}_0^m)}{a^2} < 0$ and $\frac{\partial \Delta'(\tilde{S}_0^m)}{\partial \omega} = -\frac{4bD'(P_0+\tilde{S}_0^m)}{a^2} < 0$.

A.4 Competitive industry

Again, system (2) can be solved to obtain:

$$\begin{aligned}
q^c(t) &= \tfrac{1}{b}(a-\lambda_0^c e^{rt}) \\
S^c(t) &= S_0^c - \tfrac{1}{b}(at - \tfrac{1}{r}\lambda_0^c(e^{rt}-1)) \\
\lambda_0^c &= \tfrac{r(aT-bS_0)}{e^{rT}-1} \equiv \lambda_0^c(S_0,T)
\end{aligned}$$

with

$$\begin{aligned}
\tfrac{\partial \lambda_0^c}{\partial S_0} &= -\tfrac{br}{e^{rT}-1} < 0, \\
\tfrac{\partial \lambda_0^c}{\partial T} &= \tfrac{r}{e^{rT}-1}(a-\lambda_0^c e^{rT}) \geq 0,
\end{aligned}$$

For the competitive industry, in the optimal transition scenario, we know that the Hamiltonian is zero at all instant, and the extraction stops when λ_0^c sets at the choke price level, a. In the Nash and strategic scenarios, condition

$$(a-bq^c(T))q^c(T) = D(P_0+S_0),$$

is a second order polynomial in $q^c(T)$, with unique admissible solution:

$$q^c(T) = \frac{a}{2b}(1-\sqrt{(}\Delta(S_0))) \Leftrightarrow \lambda^c(T) = \frac{a}{2}(1+\sqrt{(}\Delta(S_0))) \equiv G^c(S_0),$$

with $\Delta(S_0)$ defined in Appendix A.2. Remind that for a competitive industry, the Nash and Stackelberg solutions coincide. The analysis of solutions in the optimal and Nash scenarios follows the same steps as in the Appendix A.2.1. Herebelow, we summarize the main findings. From the definition of λ_0^c, the first optimality condition defines the optimal level of exploration as a function of T:

$$S_0^c(T) = \frac{raT}{\epsilon(e^{rT}-1)+br}$$

with $\lim_{T\to 0} S_0^c(T) = \lim_{T\to\infty} S_0^c(T) = 0$ and $S_0^{c\prime}(T) = \frac{ra}{(\epsilon(e^{rT}-1)+br)^2}(\epsilon(e^{rT}-1)+br-\epsilon rTe^{rT})$.

Then, $S_0^{c\prime}(T) \geq 0 \Leftrightarrow \epsilon(e^{rT}-1)+br \geq \epsilon rTe^{rT}$ with

$$\lim_{T\to 0} \epsilon(e^{rT}-1)+br > 0 = \lim_{T\to 0} \epsilon rTe^{rT} \text{ and } \lim_{T\to\infty} \frac{\epsilon rTe^{rT}}{\epsilon(e^{rT}-1)+br} = \infty.$$

So there exists a unique $\tilde{T}^c \in (0,\infty)$ such that $S_0^{c\prime}(T) \geq 0 \Leftrightarrow T \leq \tilde{T}^c$. Conditions governing the choice of T share the same LHS but their RHS differ. More precisely, for any S_0 we have $G^c(S_0) = \frac{a}{2}(1+\sqrt{(}\Delta(S_0)) < a$. Let us denote the common LHS as $f^c(T) = \epsilon S_0^c(T)e^{rT}$, with derivative:$f^{c\prime}(T) = \frac{\epsilon rae^{rT}}{(\epsilon(e^{rT}-1)+br)^2}(\epsilon(e^{rT}-(1+rT))+br(1+rT)) > 0$. Because $f^c(\tilde{T}^c) = a$, the optimal stopping time of the industry is $T^{co} = \tilde{T}^c$, which results in the exploration level $S_0^{co} = S_0^c(\tilde{T}^c)$. In the Nash scenario, $G^c(S_0^c(T))$ is inverted-U shaped with its minimum at $T = \tilde{T}^c$ and always lower than a. So there exists a unique solution T^{cn}, with $T^{cn} < T^{co}$. This in turn yields the ranking: $S_0^{cn} < S_0^{co}$. This completes the proof of Proposition 2. Note that the Nash solution is well-defined if $S_0^c(\tilde{T}^c) \leq \bar{S}_0$.

Finally note that because $S_0^c(T) > S_0^m(T)$,$f^c(T) > f^m(T)$ for T given. So, $T^{co} < T^{mo}$. This is equivalent to $S_0^c(T^{co}) > S_0^m(T^{mo})$ because $S_0^i(T^{io}) = \frac{a}{\epsilon e^{rT^{io}}}$ for $i = c, m$.