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Exploitation and Recycling of Rare Earths *

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We study the exploitation of recyclable exhaustible resources such as rare earths and Phosphorus. We use a standard Hotelling model of resource exploitation that includes a primary sector and a recycling sector. We show that, when the primary sector is competitive, the price of the recyclable resource increases through time. This result stands in contrast to durable resources, for which the optimal price path is either decreasing or U-shaped (Levhari and Pindyck, 1981). We then show a new reason why the price of an exhaustible resource may decrease: when the primary sector is monopolistic, the primary producer has incentives to delay its production activities in order to delay recycling. As a consequence, the price path of the recyclable resource may be U-shaped. We also show that a technological improvement in the recycling sector increases the price in the short term but decreases it later.

Keywords: Rare earths, Phosphorus, recycling, competition, market power, optimal control
JEL codes: Q31, Q53.

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1 Introduction

Recyclable exhaustible resources, such as rare earths and Phosphorus, are increasingly important industrial inputs. Indeed, rare earth elements are important inputs for the production of many modern technologies, such as cell phones, light bulbs, automobiles, hybrid car batteries and gearboxes, and wind turbines (Chakhmouradian and Wall, 2012). Phosphorus (derived from Phosphate rocks) is essential for soil fertility and has no substitute in agricultural production processes (Cordell et al., 2009). Due to economic development and an increasing world population, demand for rare earths and phosphorus has been growing rapidly and is expected to grow even more in the future (Alonso et al. 2012, Steen 1998).[1]

Historically, the supply of these resources has been highly concentrated. Until 2010, China controlled 95% of the production of rare earths (Chakhmouradian and Wall, 2012), while a handful of countries, including Morocco, China, and the U.S.A, controlled most of the world's Phosphate rock production (IFDC, 2010). However, prospects for the supply of rare earths and Phosphate rocks differ. Since 2010, the supply of rare earths has become less concentrated as China currently possesses less than 40% of rare earth reserves, while the supply of Phosphate rocks has become more concentrated, as 85% of these reserves are currently located in Morocco and Western Sahara.[2]

One strategy to increase the supply of these resources is recycling. The recycling of rare earths is in its infancy (UNEP, 2011), currently focused on scrap materials that contain high amounts of rare earth elements and on high value rare earths, including magnets, lamp phosphors, and nickel hybrid batteries (Golev et al., 2014). European fertilizer firms, such as Ostara and CNP-Technology already produce recycled Phosphorus fertilizers (Hukari and Nätto 2015) and, according to the U.K. Environment Agency (EA, 2012), approximatively 70% of Phosphorus from sewage sludge produced in the U.K. is currently recycled as biosolids to be used as fertilizer.[3]

In order to assess the effect of recycling, it is necessary to understand how the primary sector may react. Since the main input to the production of recycled materials is the stock of scrap, the emergence of recycling activities may affect the dynamics of both the extraction of the exhaustible resource as well as the price of the final goods.

In this paper, we study the impact of a recycling sector in a stylized economic model of exhaustible resource extraction. We develop a Hotelling model of resource extraction in which the consumption good is produced from virgin or recycled materials. Virgin materials are extracted from a finite stock of a virgin resource and recycled materials are derived from the stock of scrap. The stock of recyclable scrap grows with current consumption of the virgin resource at

[1] Alonso et al. (2012) predict that the demand for rare earths will increase by 5 to 9 percent per annum until 2025. According to EFMA (2000) and Steen (1998), the demand for phosphorus may increase by as much as 50 to 100% by 2050 with increased global demand for food.

[2] After the 2008 peak in demand due to the pro-biofuel policy in the U.S. and to the 150% increase in the export tariff on phosphate in China, Phosphate rock reserve estimates for Morocco and Western Sahara increased from 6 to 50 billion tons, while reserve estimates remained below 6 billion and 2 billion tons for China and other countries (U.S.A, South Africa, Jordan, and Russia), respectively (USGS, 2011).

[3] A new German ordinance (AbfKlärV) will make phosphorus recovery obligatory for most sewage sludge. It has been notified to the European Commission and may enter into force in 2018. See http://www.phosphorusplatform.eu/scope-in-print/scope-in-press/1327-german-sludge-p-recycling-ordinance-notified-to-europe

a given recyclability rate. We assume a competitive recycling sector in which production costs decrease with the stock of recyclable scrap. As a consequence, production in the primary sector generates a positive externality that benefits to the recycling sector. To ensure consistency with the various possible (future and present) market structures in the rare earths and Phosphate rocks extraction sectors, we consider two polar cases: competitive and monopolistic extraction.

Our main results are the following. We show that, if the primary sector is competitive, the optimal level of production for firms in the primary sector is such that the price of the resource grows at the discount rate (this is the so-called Hotelling rule) because these firms assume that their production will not increase the stock of scrap. We then show that, if the extraction sector is monopolistic, the dynamics of the price of the resource depend on the recyclability rate. When the recyclability rate is sufficiently low, the price of the resource increases over, which is similar to the case without a recycling sector. When the recyclability rate is sufficiently high, the monopolistic firm has an incentive to postpone extraction. As a consequence, the price of the resource is U-shaped, i.e. the price first decreases and then increases. We also generate insights concerning the impact of technological progress in the recycling sector when the primary sector is monopolistic. We show that an increase in the recyclability rate has first a positive and then a negative effect on the price of the resource. This suggests that technological progress in the recycling sector can be detrimental to consumers in the short-run and beneficial for them in the long run. We also show that the exhaustion date increases with both the initial stock and the recyclability rate of the resource.

The present paper is related to the literature dealing with durable resources. Durable resources differ from other resources (among which, recyclable resources) in that their demands are for quantities of stock in circulation rather than for flows of production. Producers of durable goods use similar production technologies and consumers typically consume durable goods for a certain period of time. Primary and recycled goods, in contrast, are typically produced using two different production processes. As stated in Levhari and Pindyck (1981), demand is a stock relationship for durable resources while it is a flow relationship for recyclable resources. Our results highlight important differences between recyclable and durable resources. We show that the two assumptions lead to quite different results. Indeed, Levhari and Pindyck (1981) show that, in the case of a competitive industry that produces a durable good, the price of the resource first decreases and may increase thereafter. In contrast, we find that the price of the resource is always increasing in the context of a competitive extraction sector.

Our model reveals a new reason why the price of a resource may decrease: a firm with market power in the extraction sector will (strategically) choose to delay extraction in order to reduce the opportunities for recycling. This differs from the case of durable resources (Levhari and Pindyck, 1981), in which the initial price decrease is due to the growing amount of the durable good in circulation and the resulting decrease in demand (the price of the resource increases at some point in time only if the durable good depreciates).

There are other explanations for U-shaped price profiles of exhaustible resources. Pindyck (1978) shows that this may occur when exploration and reserve accumulation are taken into account. In a model with exogenous technical change and endogenous change in grades, Slade (1982) also finds that U-shaped price profiles may occur. These studies do not consider the possibility of recycling.

The environmental advantages of recycling have long been recognized in the economic literature (Smith, 1972; Weinstein and Zeckhauser, 1974; Hoel, 1978).[4] The present paper is related to literature that focuses on the possible adverse social consequences of market power in the extraction sector. A series of papers consider a monopolistic extraction sector facing a competitive recycling sector (Gaskins, 1974; Swan, 1980; Martin, 1982; Suslow, 1986; Hollander and Lasserre, 1988) and show that, despite the presence of a competitive recycling sector, the extraction firm maintains (at least some of) its monopoly rents.[5] However, none of these papers show that market power in the primary sector can result in a U-shaped price profile of the non-renewable resource.

The remainder of the paper is structured as follows. Section 2 introduces the model in which the monopolist of the exhaustible resource faces a competitive recycling sector. Section 3 studies price dynamics in the case of a competitive primary producer sector. Section 4 focuses on the qualitative properties of the optimal path in the case of a monopolistic primary producer. The main conclusions from this analysis are gathered in section 5.

2 The Model

The economy produces a quantity Q of a consumption good. The consumption good can be produced from a non-renewable resource or from recycled materials. For simplicity we assume that the virgin and recycled materials are perfect substitutes. The primary sector faces a competitive sector of recycling firms.

Non-renewable resource and scrap dynamics

Let $X(t) \geq 0$ be the residual stock of virgin resource at time t, X^0 be the initial stock, with $X(0) \equiv X^0 > 0$, and $x(t) \geq 0$ be the extraction rate at time t, so that:

$$\dot{X}(t) = -x(t). \tag{1}$$

The unit cost of extraction of the virgin resource is assumed to be zero.

Let $S(t) \geq 0$ be the stock of (recyclable) scrap at time t, with a zero initial stock, $S(0) = 0$. Let $r(t) \geq 0$ be the quantity of recycled materials marketed at time t, so that the total quantity consumed at time t is then $Q(t) = x(t) + r(t)$. Let $\alpha \in [0, 1]$ be the proportion of the resource that is not recycled and becomes recyclable scrap. The dynamics of the scrap material writes $\dot{S}(t) = \alpha(Q(t) - r(t))$ or,

$$\dot{S}(t) = \alpha x(t), \tag{2}$$

where α represents the recyclability rate of the non-renewable resource.

The recycling sector

[4] André and Cerdá (2006) provide a model that takes into account the interactions of the material composition of output and waste as potentially recyclable products.

[5] Gaudet and Van Long (2003) consider the possibility of imperfect competition in the recycling sector. They show that, when primary and secondary production decisions are made simultaneously, the presence of the recycling sector may increase the market power of the primary producer. Weikard and Seyhan (2009), motivated by the case of Phosphorus, consider a model of competitive resource extraction and the possibility of saturated demand (i.e. taking into account the possibility that soil can become saturated with Phosphorus).

The recycling sector is assumed to be competitive and the marginal cost of recycling to be a decreasing function of the stock of scrap and an increasing function of the quantity of recycled materials, $c(S, r)$.

In equilibrium in the recycling sector, the price of the consumption good must equal the marginal cost of recycling:

$$p(Q(t)) = c(S(t), r(t)). \tag{3}$$

The primary sector

The discounted profits in the primary sector, with discount rate $\delta \geq 0$, are given by:

$$\int_0^{+\infty} e^{-\delta t} px dt, \tag{4}$$

In the following, we will consider two polar cases: the case of a competitive primary sector and the case of a monopolistic primary sector. In the case of a competitive primary sector, resource owners behave as price takers, and they consider the price of the resource to be a function of time, $p \equiv P(t)$. In the case of a monopolistic primary sector, the owner of the resource takes into account how extracted quantities affect the total quantity of material supplied (virgin as well as recycled) and the effect of this supply on the price of the resource, i.e. $p \equiv p(Q(t))$.

3 Competitive primary sector

In this section, we consider the case of a competitive primary sector. In this case, producers take the price, P, as well as the total quantity, Q, as given. They consider the following problem:

$$\underset{\{x(t), t \geq 0\}}{Max} \int_0^{+\infty} e^{-\delta t} P(t) x(t) dt, \tag{5}$$

$$\begin{aligned} s.t. \; \dot{X}(t) &= -x(t), & (6) \\ X(t) &\geq 0, \; x(t) \geq 0. & (7) \end{aligned}$$

The Hamiltonian and the Lagrangian for this optimal control problem are as follows:[6]

$$H = Px + \lambda_X(-x), \tag{8}$$

$$L = H + \mu_X X + \mu_x x, \tag{9}$$

where λ_X is the co-state variable associated with the stock X, and, μ_X, μ_x are the multipliers associated with the non-negativity constraints $X \geq 0$, and $x \geq 0$. The competitive solution is found by solving problem (5) subject to (6) and (7) and then using (3), (2) and $P(t) = p(Q(t))$, $\forall t$, to determine the recycling level and the market clearing price. The Maximum Principle

[6]We drop the time index when there is no possible confusion.

requires that the following conditions hold:

$$\frac{\partial L}{\partial x} = P - \lambda_X + \mu_x = 0, \tag{10}$$

$$\dot{\lambda}_X = \delta\lambda_X - \frac{\partial L}{\partial X} = \delta\lambda_X - \mu_X, \tag{11}$$

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{12}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0, \tag{13}$$

and one transversality constraint is given by:

$$\lim_{t \to +\infty} e^{-\delta t} \lambda_X(t) X(t) = 0, \tag{14}$$

When both extraction and residual stock levels, $x(t)$ and $X(t)$, are strictly positive, we have $\mu_x = 0$ and $\mu_X = 0$. Substituting these respective values into (10) and (11) yields:

$$P - \lambda_X = 0, \tag{15}$$

$$\dot{\lambda}_X = \delta\lambda_X \tag{16}$$

Differentiating (15) with respect to time gives:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \frac{\dot{P}}{P} \tag{17}$$

From (16), we have:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \delta \tag{18}$$

The combination of (17) and (18) yields:

$$\frac{\dot{P}}{P} = \delta \tag{19}$$

Equation (19) is known as Hotelling's rule: the price of the resource grows at the discount rate. Since $\delta > 0$, the price of the resource increases over time. This result reveals a major difference between recyclable goods and durable goods. The price of a durable exhaustible resource decreases with the amount of the durable good in circulation. It is then either always decreasing or U-shaped when the resource extraction sector is competitive (Levhari and Pindyck, 1981).

4 Monopolistic primary sector

In this section, we consider the case of a monopolistic primary sector. We derive several qualitative properties regarding the optimal time path of virgin resource extraction, the equilibrium recycling quantity, and the price of the consumption good. We then provide comparative statics in order to highlight the role of recycling technology in the dynamics of the price.

For simplicity, we assume in the rest of the paper that the inverse demand for the consumption good and the cost of recycling are linear,

$$p\left(Q\left(t\right)\right)=1-Q\left(t\right)\ \text{ and } c\left(S\left(t\right),r\left(t\right)\right)=1-b-\beta\left(S\left(t\right)-r\left(t\right)\right), \tag{20}$$

with $\beta>0$ and $b\in(0,1)$. Parameter b is a measure of the added value of recycled material compared to scrap.

Solving the recycling sector equilibrium condition (3), we characterize the equilibrium quantity of recycled material at time t as follows:

$$r\left(t\right)=\frac{b+\beta S\left(t\right)-x\left(t\right)}{1+\beta}. \tag{21}$$

Thus, the quantity of recycled material at time t increases with the quantity of scrap and decreases with the quantity of extracted resource. This result is quite intuitive. Since recycling relies on scrap, the higher the stock of scrap, the larger the recycling firms' production. Recycling at time t decreases with the quantity of virgin product sold at time t because recycled and virgin products are strategic substitutes. In the following, we assume that the right hand side of (21) is nonnegative.

The current value Hamiltonian H and Lagrangian L are defined as follows:

$$H=p\left(Q\right)x+\lambda_X\left(-x\right)+\lambda_S\left(\alpha x\right), \tag{22}$$

$$L=H+\mu_X X+\mu_S S+\mu_x x, \tag{23}$$

where λ_X and λ_S are the co-state variables associated with the stocks X and S, and μ_X,μ_S,μ_x are the multipliers associated with the non-negativity constraints $X\geq 0$, $S\geq 0$, and $x\geq 0$. The total quantity writes $Q=x+r=\left[b+\beta\left(S+x\right)\right]/\left(1+\beta\right)$.

The necessary conditions include:

$$\frac{\partial L}{\partial x}=\frac{\beta}{1+\beta}p'\left(Q\right)x+p\left(Q\right)-\lambda_X+\alpha\lambda_S+\mu_x=0, \tag{24}$$

$$\dot{\lambda}_X=\delta\lambda_X-\frac{\partial L}{\partial X}=\delta\lambda_X-\mu_X, \tag{25}$$

$$\dot{\lambda}_S=\delta\lambda_S-\frac{\partial L}{\partial S}=\delta\lambda_S-\mu_S-\frac{\beta}{1+\beta}p'\left(Q\right)x, \tag{26}$$

$$x\geq 0,\mu_x\geq 0,\mu_x x=0, \tag{27}$$

$$X\geq 0,\mu_X\geq 0,\mu_X X=0, \tag{28}$$

$$S\geq 0,\mu_S\geq 0,\mu_S S=0, \tag{29}$$

and two transversality conditions:

$$\lim_{t\to+\infty}e^{-\delta t}\lambda_X\left(t\right)X\left(t\right)=0, \tag{30}$$

$$\lim_{t\to+\infty}e^{-\delta t}\lambda_S\left(t\right)S\left(t\right)=0, \tag{31}$$

and $S\left(0\right)=0$ and $X^0>0$ are given.

In the rest of the paper we focus on solutions in which the exhaustion date of the virgin resource, T^*, is finite, that is $T^* < +\infty$.[7]

4.1 Optimal extraction path, recycling path and price dynamics

Full resolution of the monopolist's programme yields the extraction and recycling paths as well as the dynamics of the price of the consumption good. We first show that the optimal extraction and the recycling paths are monotonic.

Proposition 1 [Optimal Path]: *The optimal extraction path is such that the quantity of extracted material decreases while the quantity of recycled material increases over time:* $\dot{x}^*(t) \leq 0$ *and* $\dot{r}^*(t) \geq 0$.

All proofs can be found in the appendix.

Proposition 1 states that the optimal level of extraction decreases through time. This result is in line with the standard Hotelling model. Indeed, the extracting firm discounts time, choosing to extract more of the resource today and less tomorrow. The quantity of marketed recycled material, in contrast, increases over time. The intuition of these results is as follows. The stock of scrap increases over time, which reduces the unit cost of recycling. This, in turn, provides incentives for recycling firms to increase their production. At the same time, the quantity of extracted material decreases, causing the market price of the resource and the level of recycling to increase.

The price dynamics of the consumption good is established in the following proposition for the case in which scrap material has a low level of recyclability.

Proposition 2 [Price Dynamics]: *The price of the final good is never decreasing if the recyclability rate is sufficiently low. Formally, there exists* $\widehat{\alpha} > 0$ *such that* $\partial p^*/\partial t \geq 0$ *for all* t *if* $\alpha \leq \widehat{\alpha}$.

Proposition 2 states that the standard result of an increasing resource price holds if the level of recyclability is low. In this case, the existence of the recycling sector has a limited impact on the optimal choice of the monopolistic firm. We show in the following Proposition that this result no longer holds when the level of recyclability is sufficiently large.

Proposition 3 [Non Monotonic Price]: *If the recyclability rate is sufficiently large, then the price of the final good first decreases and then increases. Formally, if* $\alpha > \widehat{\alpha}$ *there exists* $0 < \widehat{t} < T^*$ *such that* $\partial p^*/\partial t < 0$ *if* $t \in [0, \widehat{t})$ *and* $\partial p^*/\partial t \geq 0$ *if* $t \in [\widehat{t}, T^*]$.

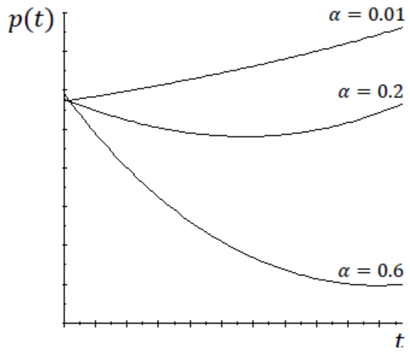
Proposition 3 states that, when the recyclability rate is sufficiently large, then the optimal price path is U-shaped. The intuition behind this result lies in the fact that the monopolistic firm has an incentive to postpone extraction of the virgin resource. In delaying extraction, the monopolist delays the increase in the stock of scrap, which in turn delays the reduction of recycling costs. This results in a delayed increase in the quantity of marketed recycled materials.

[7] A sufficient condition is $1 - \frac{\beta \alpha X^0}{1-b+\beta} > 0$. This condition can be derived from equation (67) in the proof of Proposition 1.

Propositions 2 and 3 can be illustrated using the following numerical example:

Numerical Example A: Let $X^0 = 1$, $\beta = 1$ and $\delta = 0.02$. Figure 1 shows the price path over time for each value of α.

Figure 1: The price path is either increasing or U-shaped



4.2 The Role of Recyclability

In this section, we focus on the influence of the recyclability rate on the exhaustion date, the extraction rate, and the price of the consumption good.

We first show that the exhaustion date increases with the availability of the virgin and recycled materials.

Proposition 4 [Exhaustion date and Recyclability]: *The optimal exhaustion date increases with both the initial stock and the recyclability rate of the resource. Formally:*

$$\frac{\partial T^*}{\partial X^0} > 0 \text{ and } \frac{\partial T^*}{\partial \alpha} > 0$$

Proposition 4 states that the date of exhaustion of the resource is delayed with the initial stock, which is intuitive. It also states that the exhaustion date increases with the recyclability rate of the resource. This is due to the fact that a higher recyclability rate provides stronger incentives for the monopolist to delay extraction of the virgin resource.

In the next proposition, we focus on the effect of an increase in the recyclability rate on the optimal extraction and recycling paths.

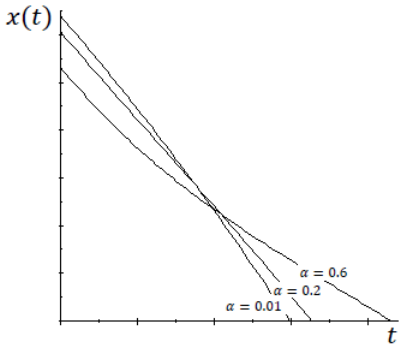
Proposition 5 [Extraction and Recyclability]: *Early extraction decreases while later extraction increases with recyclability. Formally, there exists a date* $0 < \tilde{t} < T^*$ *such that*

$$\frac{\partial x^*}{\partial \alpha} < 0 \text{ for } t \in [0, \tilde{t}) \text{ and } \frac{\partial x^*}{\partial \alpha} \geq 0 \text{ for } t \in \left[\tilde{t}, T^*\right).$$

Proposition 5 shows that when the level of recyclability of the resource increases, extraction is delayed. Thus, extraction first decreases and then increases as the recyclability rate increases. The intuition of this result is as follows. When the virgin resource is not exhausted $(X > 0)$, the dynamics of the shadow price of the virgin resource follows Hotelling's rule. This (shadow) price grows at a rate equal to the discount rate, $\dot{\lambda}_X/\lambda_X = \delta > 0$. This means that the extracting firm has incentives to extract the resource early. However, unlike in a standard Hotelling model, the extracting firm in our model also faces the recycling sector. When there is a stock of scrap ($S > 0$), the dynamics of the shadow price of the scrap stock is driven by the following differential equation: $\dot{\lambda}_S = \delta\lambda_S + \frac{\beta}{1+\beta}x$. If there is no extraction ($x = 0$), then the Hotelling rule holds for the stock of scrap, $\dot{\lambda}_S/\lambda_S = \delta > 0$, and the owner of the virgin resource has an incentive to delay extraction in order to maintain a small stock of scrap. If the level of extraction is positive, $\frac{\beta}{1+\beta}x > 0$, there is a tendency for the shadow price of scrap to increase. This reinforces the owner's incentives to delay extraction. Thus, the higher the recyclability rate, the stronger the incentives to postpone extraction. Since the resource is exhausted in finite time and the initial stock is fixed, extraction will increase with the recyclability rate at some point in time. This result can be illustrated in the following numerical example.

Numerical Example B: Let $X^0 = 1$, $\beta = 1$ and $\delta = 0.02$. Figure 2 depicts the optimal extraction path for different values of α.

Figure 2: Optimal extraction path for various levels of the recyclability rate



We can now derive comparative static results regarding the effect of an increase in the recyclability rate on the dynamics of the consumption good price.

Proposition 6 [Price and Recyclability]: *The price first increases and then decreases with the recyclability rate . Formally, there exists a date* $0 < t' < T^*$ *such that*

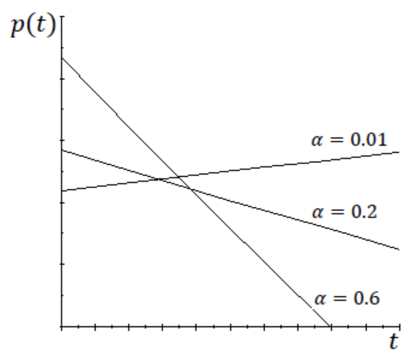
$$\frac{\partial p^*}{\partial \alpha} > 0 \text{ for } t \in [0, t') \text{ and } \frac{\partial p^*}{\partial \alpha} \leq 0 \text{ for } t \in \left[t', T^*\right).$$

A consequence of Proposition 6 is that a higher recyclability rate is not always beneficial to consumers. In the short-run ($t < t'$), consumers must pay a higher price for the consumption good. However, in the long run ($t' < t$), consumers pay a lower price for the consumption good (thanks to the increased supply). The intuition underlying this result is as follows. The price of the final good negatively depends on the marketed quantity of virgin and recycled materials. When the recyclability rate increases, the stock of scrap tends to increase, which leads to an increase in recycling. However, because the stock of scrap increases with extraction by a factor $\alpha < 1$, the increase in recycling cannot compensate for the short-run decrease in extraction, and the marketed quantity of the consumption good decreases. This explains why the price of the final good increases in the short run. Approaching the exhaustion date, an increase in the recyclability rate increases both the stock of scrap and the level of extraction. This explains why the price of the final good decreases in this period. Gaskins (1974) also finds that the price of the resource increases in the short run in the presence of a recycling sector. However, in contrast with the present analysis, he focuses on the steady state and finds that the steady state price of the resource is close to the competitive price.

This result can be illustrated in the following numerical example.

Numerical Example C: Let $X^0 = 1$, $\beta = 1$ and $\delta = 0.02$. Figure 3 depicts the optimal price path for different values of α.

Figure 3: Comparative statics: The price first increases and then decreases with α



5 Conclusion

Recycling rare earths and Phosphorus appears to be a promising strategy to increase the supply of these important exhaustible resources. Although the production of the these resources is concentrated in a few countries, reserve estimates suggest that the supply of rare earths will become more competitive while the supply of Phosphorus will become less competitive in the future.

Motivated by these two examples, we built a model of resource extraction in which the primary sector faces a recycling sector. We have shown that, when the primary sector is competitive, the price of the recyclable resource increases through time. This result stands in contrast to durable resources, for which the optimal price path is either decreasing or U-shaped (Levhari and Pindyck, 1981). We have also shown that, when the primary sector is monopolistic, the price of the recyclable resource may be U-shaped when the recyclability rate is sufficiently large. This occurs because the primary producer has incentives to delay the extraction of the resource in order to delay recycling. We have also shown that the price of the resource first increases and then decreases as the recyclability rate increases. Our results suggest that market power in the primary sector may lead to phases in which the price of the resource decreases. They also suggest that consumers may oppose progress in recycling technologies if they sufficiently weight the negative short term effect at the expense of the positive long term effect.

Appendix

Proof of Proposition 1: Using the equilibrium recycling condition (21) and substituting, we have $p(Q) = \frac{\beta}{1+\beta}(a - x - S)$, where $a = (1 - b + \beta)/\beta$. The maximization problem then formally writes:

$$\underset{\{x\}}{Max} \int_{0}^{+\infty} \frac{\beta e^{-\delta t}}{1+\beta}(a - x - S)\, x dt, \tag{32}$$

subject to (21), (1), (2), $X, S, x \geq 0$, X^0 and S^0 given.

Thus, for the new problem, the necessary conditions include:

$$\frac{\partial L}{\partial x} = a - 2x - S - \lambda_X + \alpha \lambda_S + \mu_x = 0, \tag{33}$$

$$\dot{\lambda}_X = \delta \lambda_X - \frac{\partial L}{\partial X} = \delta \lambda_X - \mu_X, \tag{34}$$

$$\dot{\lambda}_S = \delta \lambda_S - \frac{\partial L}{\partial S} = \delta \lambda_S - \mu_S + x, \tag{35}$$

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{36}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0, \tag{37}$$

$$S \geq 0, \mu_S \geq 0, \mu_S S = 0, \tag{38}$$

and two transversality constraints:

$$\lim_{t \to +\infty} e^{-\delta t} \lambda_X(t) X(t) = 0, \tag{39}$$

$$\lim_{t \to +\infty} e^{-\delta t} \lambda_S(t) S(t) = 0. \tag{40}$$

Let us assume that the solution is such that $x(t) > 0$ and $X(t) > 0$ over $[0, T)$ and $x(t) = X(t) = 0$ for $t \geq T$. We also assume that $S(t) > 0$ for all $t > 0$.

First consider the first phase in which $t \in [0, T)$. Since $x(t) > 0$, $X(t) > 0$ and $S(t) > 0$, using (36), (37), and (38), we have $\mu_x = \mu_X = \mu_S = 0$. Then (34) writes

$$\dot{\lambda}_X = \delta \lambda_X, \tag{41}$$

and then

$$\lambda_X = c_1 e^{\delta t}, \tag{42}$$

where c_1 is a constant to be determined later.

Conditions (33), and (35) write

$$a - 2x - S - c_1 e^{\delta t} + \alpha \lambda_S = 0, \tag{43}$$

and,

$$\dot{\lambda}_S = \delta \lambda_S + x, \tag{44}$$

Differentiating (43) with respect to time, we find

$$-2\dot{x} - \dot{S} - \delta c_1 e^{\delta t} + \alpha \dot{\lambda}_S = 0. \tag{45}$$

Using (43) and (45), we find

$$-2\dot{x} - \dot{S} - \delta c_1 e^{\delta t} - \delta \left(a - 2x - S - c_1 e^{\delta t} \right) + \alpha \left(\dot{\lambda}_S - \delta \lambda_S \right) = 0, \tag{46}$$

or,

$$-2\dot{x} - \dot{S} + \delta S + (\alpha + 2\delta)\, x - \delta a = 0, \tag{47}$$

Differentiating (2) with respect to time, we obtain

$$\ddot{S} = \alpha \dot{x}. \tag{48}$$

Substituting (48) and (2) into (47), and rearranging, we have

$$\ddot{S} - \delta \dot{S} - \frac{\alpha \delta}{2} S + \frac{\alpha \delta}{2} a = 0. \tag{49}$$

Solving for the stock of scrap S we find

$$S = c_2 e^{\gamma^+ t} + c_3 e^{\gamma^- t} + a, \tag{50}$$

where $\gamma^+ = \frac{\delta + \sqrt{\delta(2\alpha+\delta)}}{2}$ and $\gamma^- = \frac{\delta - \sqrt{\delta(2\alpha+\delta)}}{2}$.

Differentiating (50) with respect to time, we obtain

$$\dot{S} = \gamma^+ c_2 e^{\gamma^+ t} + \gamma^- c_3 e^{\gamma^- t}. \tag{51}$$

Using (2), we have

$$x = \frac{\gamma^+}{\alpha} c_2 e^{\gamma^+ t} + \frac{\gamma^-}{\alpha} c_3 e^{\gamma^- t}. \tag{52}$$

Substituting (52) into (44), we obtain

$$\dot{\lambda}_S - \delta \lambda_S = \frac{\gamma^+}{\alpha} c_2 e^{\gamma^+ t} + \frac{\gamma^-}{\alpha} c_3 e^{\gamma^- t}. \tag{53}$$

Solving for the shadow price of the stock of scrap λ_S we find

$$\lambda_S = D e^{\delta t} + \frac{\gamma^+}{\alpha (\gamma^+ - \delta)} c_2 e^{\gamma^+ t} - \frac{\gamma^-}{\alpha (\delta - \gamma^-)} c_3 e^{\gamma^- t}. \tag{54}$$

Using $X^0 - X(t) = \int_0^t x dt$ and integrating (52) between 0 and t, we find

$$X^0 - X(t) = \frac{1}{\alpha} \left(c_2 \left(e^{\gamma^+ t} - 1 \right) + c_3 \left(e^{\gamma^- t} - 1 \right) \right). \tag{55}$$

Now consider the second phase in which $t \geq T$. We have $x(t) = 0 = X(t)$ and $S(t) > 0$.

Using (38), we have $\mu_S = 0$. Condition (35) writes

$$\dot{\lambda}_S = \delta \lambda_S, \tag{56}$$

and then

$$\lambda_S = c_5 e^{\delta t}, \tag{57}$$

where c_5 is a constant to be determined later.

Notice that $\dot{S} = \alpha x = 0$, and then

$$S = c_4, \tag{58}$$

where c_4 is a constant to be determined later.

Using (58) and (57), transversality constraint (40) becomes

$$c_4 c_5 = 0. \tag{59}$$

Assume $c_5 \neq 0$. Then, using (58) at $t = T$, we have $S(T) = c_4 = 0$. Combining (50) and (55) and taking $t = T$, we have $\alpha X^0 = S(T)$. Hence, we must have $X^0 = 0$, which is false. We conclude that $c_5 = 0$. Thus, for $t \geq T$, we have

$$\lambda_S = 0. \tag{60}$$

In order to solve for c_1, c_2, c_3, c_4, D and T, we focus on solutions such that λ_S is continuous. Using (54) and (60) at $t = T$, we obtain

$$De^{\delta T} + \frac{\gamma^+}{\alpha(\gamma^+ - \delta)} c_2 e^{\gamma^+ T} - \frac{\gamma^-}{\alpha(\delta - \gamma^-)} c_3 e^{\gamma^- T} = 0. \tag{61}$$

Using $x(T) = 0$ and (52), we have

$$\gamma^+ c_2 e^{\gamma^+ T} + \gamma^- c_3 e^{\gamma^- T} = 0. \tag{62}$$

Using $X(T) = 0$ and (55), we have

$$\alpha X^0 = c_2 \left(e^{\gamma^+ T} - 1\right) + c_3 \left(e^{\gamma^- T} - 1\right). \tag{63}$$

Using (50) at $t = 0$, we have

$$S^0 = c_2 + c_3 + a. \tag{64}$$

Using (43) and (50) at $t = T$, we obtain

$$c_2 e^{\gamma^+ T} + c_3 e^{\gamma^- T} + c_1 e^{\delta T} = 0. \tag{65}$$

Using (50) and (58) at $t = T$, we have

$$c_4 = c_2 e^{\gamma^+ T} + c_3 e^{\gamma^- T} + a. \tag{66}$$

Solving for c_1, c_2, c_3, c_4, D and T from conditions (61)-(66), we obtain

$$\begin{aligned} c_1 &= \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} \left(a - S^0\right), \\ c_2 &= -\frac{\gamma^- e^{\gamma^- T}}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} \left(S^0 - a\right), \\ c_3 &= \frac{\gamma^+ e^{\gamma^+ T}}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} \left(S^0 - a\right), \\ c_4 &= \left(S^0 - a\right) \frac{\left(\gamma^+ - \gamma^-\right) e^{\delta T}}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} + a, \\ D &= \frac{a - S^0}{\alpha} \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}}, \end{aligned}$$

and the exhaustion date T^* is implicitly characterized by :

$$\alpha X^0 = \left(a - S^0\right) \left(1 - \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} e^{\delta T^*}\right). \tag{67}$$

We conclude that the optimal extraction path is, for $t \in [0, T]$:

$$x^*(t) = \frac{\left(a - S^0\right) \delta}{2} \left(\frac{e^{\gamma^+ T^*} e^{\gamma^- t} - e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}\right), \tag{68}$$

the stock of scrap is, for $t \in [0, T]$,

$$S^*(t) = \left(a - S^0\right) \left(1 - \frac{\gamma^+ e^{\gamma^+ T^*} e^{\gamma^- t} - \gamma^- e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}\right), \tag{69}$$

and the market price, for $t \in [0, T]$,

$$p^*(t) = S^0 + \left(a - S^0\right) \left(\frac{\gamma^+ - \gamma^-}{2}\right) \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}. \tag{70}$$

Since $\gamma^+ > 0 > \gamma^-$, the extraction level $x^*(t)$ characterized in (68) decreases through time, while the stock of scrap increases through time, $\dot{S}^*(t) = \alpha x^*(t) \geq 0$. Recycling is given by

$$r^*(t) = \frac{b}{\beta} + a \left(1 - \frac{\left(\gamma^+ + \frac{\delta}{2\beta}\right) e^{\gamma^+ T^*} e^{\gamma^- t} - \left(\gamma^- + \frac{\delta}{2\beta}\right) e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}\right), \tag{71}$$

and increases through time.□

Proof of Proposition 2: From (70), we know that the price of the consumption good is

$$p^*(t, \alpha) = \frac{a}{2} \sqrt{\delta\left(2\alpha + \delta\right)} \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}, \tag{72}$$

The sign of the derivative with respect to time is given by

$$\frac{\partial p^*}{\partial t} \propto \gamma^- e^{\gamma^+ T^*} e^{\gamma^- t} + \gamma^+ e^{\gamma^- T^*} e^{\gamma^+ t}, \tag{73}$$

which is positive if and only if

$$t \geq T^* + \frac{1}{\gamma^+ - \gamma^-} \ln\left(1 - \frac{\delta}{\gamma^+}\right). \tag{74}$$

Hence, $\frac{\partial p^*}{\partial t} \geq 0$ for all $t \in [0, T]$ if and only if

$$T^* \leq \frac{1}{\gamma^+ - \gamma^-} \ln\left(\frac{\gamma^+}{\gamma^+ - \delta}\right). \tag{75}$$

We know from Proposition 3 that the left hand side of condition (75) is increasing with α. The derivative of the right hand side with respect to γ^+ is

$$\frac{-1}{(2\gamma^+ - \delta)^2}\left(2\ln\frac{\gamma^+}{\gamma^+ - \delta} + \frac{\delta\left(2\gamma^+ - \delta\right)}{\gamma^+\left(\gamma^+ - \delta\right)}\right) < 0. \tag{76}$$

It is decreasing with α. When α goes to 0, γ^+ goes to δ and the right hand side in (75) goes to $+\infty$. A first order approximation of (67) at $\alpha = 0$ leads to $\left(\frac{X^0}{a}\delta + 1 - \delta T^*\right)e^{\delta T^*} \simeq 1$. The solution of this equation is $T^* < +\infty$ because the left hand side is $\frac{X^0}{a}\delta + 1 > 1$ at $T^* = 0$. The solution of the equation increases up to $T^* = \frac{X^0}{a}$ and then decreases and goes to $-\infty$ when $T^* \to +\infty$. This concludes the proof.□

Proof of Proposition 3: The result directly follows from the proof of Proposition 2.□

Proof of Proposition 4: The optimal exhaustion date is implicitly characterized by (67), which can be rewritten as:

$$f\left(\gamma^+, T^*, \alpha, X^0, \delta\right) \equiv 1 - \frac{2\gamma^+ - \delta}{\gamma^+ e^{\gamma^+ T^*} + \left(\gamma^+ - \delta\right)e^{\left(\delta - \gamma^+\right)T^*}}e^{\delta T^*} - \frac{\alpha X^0}{a} = 0, \tag{77}$$

where $\gamma^+ = \left(\delta + \sqrt{\delta\left(2\alpha + \delta\right)}\right)/2$. Its derivative with respect to T^* is given by

$$\frac{\partial f}{\partial T^*} = \frac{\gamma^+\left(\gamma^+ - \delta\right)\left(2\gamma^+ - \delta\right)e^{\delta T^*}}{\left(\gamma^+ e^{\gamma^+ T^*} + \left(\gamma^+ - \delta\right)e^{\left(\delta - \gamma^+\right)T^*}\right)^2}\left(e^{\gamma^+ T^*} - e^{-\left(\gamma^+ - \delta\right)T^*}\right). \tag{78}$$

Since $\gamma^+ \geq \delta$, we have

$$\frac{\partial f}{\partial T^*} > 0. \tag{79}$$

The derivative of f with respect to γ^+ is given by

$$\frac{\partial f}{\partial \gamma^+} = -\frac{\delta\left(e^{\gamma^+ T^*} - e^{-\left(\gamma^+ - \delta\right)T^*}\right) + \left(\gamma^+ e^{\gamma^+ T^*} - \left(\gamma^+ - \delta\right)e^{-\left(\gamma^+ - \delta\right)T^*}\right)\left(\gamma^+ - \delta\right)T^*}{\left(\gamma^+ - \delta\right)^2\left(\frac{\gamma^+}{\gamma^+ - \delta}e^{\gamma^+ T^*} + e^{-\left(\gamma^+ - \delta\right)T^*}\right)^2}e^{\delta T^*}. \tag{80}$$

Since $\gamma^+ > \delta$, we have

$$\frac{\partial f}{\partial \gamma^+} < 0. \tag{81}$$

The derivative of f with respect to α is

$$\frac{\partial f}{\partial \alpha} = -X^0/a < 0 \tag{82}$$

Using (77) and the implicit function theorem, we have:

$$\frac{\partial T^*}{\partial X^0} = -\frac{\partial f/\partial X^0}{\partial f/\partial T} = \frac{\alpha/a}{\partial f/\partial T} > 0. \tag{83}$$

Using the implicit function theorem again, and $\partial\gamma^+/\partial\alpha > 0$, (81), (79) and (82), the derivative of the exhaustion date with respect to the recyclability rate is such that:

$$\frac{\partial T^*}{\partial \alpha} = -\frac{(\partial f/\partial\gamma^+)(\partial\gamma^+/\partial\alpha) + \partial f/\partial\alpha}{\partial f/\partial T} > 0. \tag{84}$$

□

Proof of Proposition 5: The proof proceeds in three steps. We first show that the growth rate of extraction is decreasing through time. Second, we show that the growth rate is increasing with the recyclability rate. We then combine these properties in order to prove the result.

Differentiating (68), we can write the growth rate of extraction:

$$\tau\left(\gamma^+, T^*\right) \equiv \frac{\dot{x}^*(t)}{x^*(t)} = -\frac{(\gamma^+ - \delta)\, e^{\left(2\gamma^+ - \delta\right)(T^* - t)} + \gamma^+}{e^{(2\gamma^+ - \delta)(T^* - t)} - 1}. \tag{85}$$

The derivative of the growth rate with respect to T^* is

$$\frac{\partial\tau}{\partial T^*} = \frac{\gamma^+ + \delta + (2\gamma^+ - \delta)\gamma^+}{\left(e^{(2\gamma^+ - \delta)(T^* - t)} - 1\right)^2}\delta e^{\left(2\gamma^+ - \delta\right)(T^* - t)} > 0. \tag{86}$$

The derivative of the growth rate with respect to γ^+ is

$$\frac{\partial\tau}{\partial\gamma^+} = \delta\frac{G(t)}{\left(e^{(2\gamma^+ - \delta)(T^* - t)} - 1\right)^2},$$

where $G(t) = 2(T^* - t)(2\gamma^+ - \delta)\, e^{\left(2\gamma^+ - \delta\right)(T^* - t)} + 1 - e^{2\left(2\gamma^+ - \delta\right)(T^* - t)}$. Notice that $G'(t) = -2(T^* - t)(2\gamma^+ - \delta)^2 e^{\left(2\gamma^+ - \delta\right)(T^* - t)} < 0$ and $G(T^*) = 0$. Hence $G(t) > 0$ and then

$$\frac{\partial\tau}{\partial\gamma^+} > 0. \tag{87}$$

We know from Proposition 4 that T^* increases with α and we also know that γ^+ increases with α. Using (86) and (87), we conclude that

$$\frac{d\tau}{d\alpha} > 0. \tag{88}$$

In other words, we have

$$\frac{\partial^2 \ln(x)}{\partial t \partial\alpha} > 0. \tag{89}$$

Hence $\ln x$ has the single crossing property with respect to time and the recyclability rate.

According to Proposition 4, the exhaustion date increases with recyclability, $\frac{\partial T^*}{\partial\alpha} > 0$. Hence recyclability necessarily increases extraction when time approaches the exhaustion date. Since the initial stock does not depend on the recyclability rate, recyclability necessarily decreases extraction at some point in time. Thanks to the single-crossing property, there exists a date $0 < \tilde{t} < T^*$ such that $\frac{\partial x^*}{\partial \alpha} < 0 \Longleftrightarrow t < \tilde{t}$.□

Proof of Proposition 6:

The stock of scrap can be rewritten as follows:

$$S^*(t) = F(\gamma^+, T^*) = a\left(1 - \frac{\gamma^+ e^{\gamma^+(T^*-t)} + (\gamma^+ - \delta) e^{-(\gamma^+-\delta)(T^*-t)}}{\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{-(\gamma^+-\delta)T^*}} e^{\delta t}\right).$$

The derivative of this function with respect to T^* is:

$$\frac{\partial F}{\partial T^*} = -a\frac{\gamma^+(\gamma^+ - \delta)(2\gamma^+ - \delta)\left[e^{\delta T - \gamma t} - e^{\delta T + (\gamma^+-\delta)t}\right]}{\left(\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{-(\gamma^+-\delta)T^*}\right)^2} e^{\delta t} \geq 0, \tag{90}$$

and its derivative with respect to γ^+ is

$$\frac{\partial F}{\partial \gamma^+} = -a\frac{-t(\gamma^+)^2 e^{\gamma^+(2T^*-t)} + \left[(2T^* - t)(\gamma^+)^2 - \delta\right] e^{\delta T - \gamma^+ t} - \left[2T^*(\gamma^+)^2 - \delta\right] e^{\delta T + (\gamma^+-\delta)t}}{\left(\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{-(\gamma^+-\delta)T^*}\right)^2 e^{-\delta t}} \geq 0. \tag{91}$$

Since T^* and γ^+ both increase with α, using (90) and (91) we conclude that S^* increases when α increases.

Now consider the growth rate of the price. Using (70), it can be written as follows:

$$\frac{\dot{p}^*}{p^*} = \frac{\gamma^+ - (\gamma^+ - \delta) e^{(2\gamma^+-\delta)(T^*-t)}}{1 + e^{(2\gamma^+-\delta)(T^*-t)}} \equiv H(\gamma^+, T^*). \tag{92}$$

Its derivative with respect to γ^+ is given by:

$$\frac{\partial H}{\partial \gamma^+} = \frac{1 - e^{2(2\gamma^+-\delta)(T^*-t)} - 2(2\gamma^+ - \delta)(T^* - t) e^{(2\gamma^+-\delta)(T^*-t)}}{\left(1 + e^{(2\gamma^+-\delta)(T^*-t)}\right)^2} \leq 0. \tag{93}$$

Since H is also decreasing with T^* and both T^* and γ^+ increase with α, we conclude that

$$\frac{\partial^2 \ln p^*}{\partial t \partial \alpha} < 0. \tag{94}$$

This means that $\ln p^*$ has the single-crossing property with respect to t and α. We know that

$$\begin{aligned} p^*(T^*) &= 1 - S^*(T) - x^*(T) \\ &= 1 - \alpha X^0. \end{aligned}$$

Then $p^*(T^*)$ decreases with α. Moreover, we have

$$\begin{aligned} p^*(0) &= 1 - S^*(0) - x^*(0) \\ &= 1 - x^*(0). \end{aligned} \tag{95}$$

Using Proposition 5, we know that x^* decreases with α at $t = 0$. Hence p^* increases with α at $t = 0$. Using the single-crossing property (94), we conclude that there exists $t' \in (0, T^*)$ such that $\partial p^*/\partial \alpha > 0 \iff t' \in [0, t')$.$\square$

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