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Limit-Pricing and the (Un)Effectiveness of the Carbon Tax

Saraly Andrade de Sa – Julien Daubanes

WP 2014.07

Limit-Pricing and the (Un)Effectiveness of the Carbon Tax*

by

Saraly Andrade de Sá
Institute for Environmental Decisions at ETH Zurich
E-mail address: *saraly.andrade@env.ethz.ch*

and

Julien Daubanes
Center of Economic Research at ETH Zurich
E-mail address: *jdaubanes@ethz.ch*

May, 2014

Short abstract

This paper questions the ability of a carbon tax to reduce oil extraction. Demand for oil is very price inelastic. Facing such demand, an extractive cartel induces the highest price that does not destroy its demand: it tolerates "non-drastic" substitutes but deters substitution possibilities that have the potential to drastically deteriorate its demand. Limit-pricing equilibria of non-renewable resource markets sharply differ from conventional Hotelling outcomes. Oil taxes become neutral. Policies only reduce current oil extraction when they support existing non-drastic substitutes. Since the carbon tax applies to oil and to its current carbon substitutes, it induces higher oil current production.

Long abstract

All existing studies on the design of the optimal carbon tax assume that such instrument can effectively curb current carbon emissions. Yet as this paper argues, the effectiveness of a carbon tax is very limited when limit pricing arises on the oil market.

Demand for energy, for fossil fuels like oil in particular, is notoriously very price inelastic, even in the long run. Facing such demand, an extractive cartel may increase its profits with higher prices, as long as those prices do not destroy its demand. The demand for oil features kinks, each corresponding to the entry price of one competing substitute. Some substitutes may be tolerated by an oil-extracting cartel (e.g. other fuels, including existing biofuels, solar and wind sources of energy...). However, when a substitution possibility has the potential to drastically deteriorate its market share, the cartel maximizes its profits by inducing the "limit price" that deters its entry. Limit-pricing equilibria of non-renewable resource markets sharply differ from the conventional Hotelling outcome; for instance, taxes on the cartel's resource become neutral regardless of their dynamics.

Environmental policies may still reduce current extraction quantities when limit pricing occurs. For that, policies must support the production of existing substitutes, i.e. those not deterred by the cartel's pricing. Unlike it, a carbon tax may increase current oil extraction: while its direct application to the oil (carbon) resource may be neutral, its application to oil's (carbon) substitutes induces higher oil production.

I. Introduction

There are three basic facts about the market for oil and its energy substitutes. First, the demand for energy is very price inelastic; in particular, the long-run price elasticity of the demand for oil is commonly estimated to be lower than one.[1] Second, oil reserves are highly concentrated; the OPEC cartel controls most of them.[2] Third, although the oil resource serves the largest share of the demand for energy, several other energy goods exist that compete with it, as for instance other fuels, various biofuels and flows of renewable energy from alternative sources.

Under standard cost conditions, a monopoly facing a relatively inelastic demand may increase its profits by charging higher prices (reducing quantities supplied). Yet, there is a limit up to which this monopoly can do so: high enough prices warrant the profitability of substitutes that would destroy the monopoly's demand. When substitution possibilities are represented by a perfect substitute that is producible without limit under constant returns ("backstop technology" as coined by Nordhaus, 1973), the monopoly maximizes its profits by inducing the limit price which deters substitution: below, higher prices increase profits; above, profits vanish.

This static limit-pricing theory carries over to the case of an extractive monopoly that exploits a finite stock of resource over time: as long as there is some resource to be exploited, the monopoly's profits are maximum when the limit price is induced at each date. The monopolistically-supplied resource may be oil, and its backstop substitute may be some high-potential future-generation biofuel. Under stationary market conditions, limit pricing means a constant extraction path together with a constant price path until

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[1]E.g. Hausman and Newey (1995), Krichene (2002), Kilian and Murphy (2014). See Krichene (2002) for a review of empirical studies on the issue: most estimates contradict Pindyck's (1979) 1.01 finding. See also Hausman and Newey (1995) and Kilian and Murphy's (2014) discussion on the consistency of their short-run estimate with standard long-run estimates.

[2]According to the US Energy Information Agency (EIA), "OPEC member countries produce about 40 percent of the world's crude oil. [...] OPEC's oil exports represent about 60 percent of the total petroleum traded internationally". Available at http://www.eia.gov/finance/markets/supply-opec.cfm. More importantly, still according to the EIA, 72 percent of proved oil reserves were controlled by OPEC members in 2011 (http://www.eia.gov/).

the resource is entirely depleted. The limit-pricing outcome of non-renewable-resource markets resulting from low demand elasticity has been first noticed by Hoel (1978) and Salant (1979),[3] and further considered by Dasgupta and Heal (1979, Page 343) and by Newbery (1981).

There are two basic limitations to the exercise of market power by an oil cartel like the OPEC. The academic literature inspired by Hotelling (1931) has extensively examined the intertemporal constraint that stocks to be exploited are exhaustible.[4] In contrast, the instantaneous constraint that high prices may trigger the entry of some oil substitutes has remained a recurrent business view.

Besides its empirical foundation, combining the observed low demand elasticity with existing substitution possibilities, the appeal of the limit-pricing theory further relies on its explanatory advantages; unlike conventional models à la Hotelling (1931), the limit-pricing type of equilibria immediately accounts for the well-established stationarity of oil prices and quantities (see Gaudet, 2007).[5] The relevance of these equilibria can also be substantiated on the ground of various accounts by OPEC-related personalities.[6] As

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[3]Hoel and Salant's papers dealt with the limit-pricing phase that may follow the ordinary non-renewable-resource monopoly pricing stage of Stiglitz (1976). When the demand for the resource has a lower-than-one price elasticity, Salant and Hoel rightly anticipated that limit pricing may occur at all dates. On the limit-pricing-phase curiosity when demand is not so inelastic, see also the recent investigation on the effect of backstop subsidies by van der Ploeg and Withagen (2012).

[4]As Pindyck (1987) put it, "potential monopoly in extractive resource markets can be limited by the depletability of reserves".

[5]The constant-price and constant-quantity outcome of limit pricing sharply contrasts with the conventional Hotelling-type interior equilibrium where the monopoly's marginal revenue rises at the profit-discounting rate (Stiglitz, 1976); this is so despite identical stationary conditions. Indeed, following the well-known result that a static monopoly never operates in regions of the demand curve where the price elasticity is less than unity, Stiglitz (1976) and many others assumed away so low elasticity levels. As Stiglitz (1976) put it, this is to avoid that "one can obtain larger profits by reducing [the quantity]"; thus it guarantees that there exists a solution to the monopoly's problem even in absence of a backstop substitute. A less-than-one price elasticity means that the monopoly's before-cost revenue is increasing with price. Hence, restrictions on the demand sensitiveness may also be embedded in the form of the monopoly's gross revenue function; for instance, Lewis, Matthews and Burness (1979) assumed it to be decreasing with price everywhere.

[6]In a famous 1974 interview, Jamshid Amuzegar, then Iran's Minister of the Interior and the Shah's right-hand oil expert, when explaining that OPEC's strategy is to have the oil price following the industrialized countries' inflation, had these revealing words: "The first of our (...) principles is that the price of oil should be equivalent to the cost of alternative sources of energy." (Time Magazine, October 14, 1974, Page 36.). More recently, OPEC Secretary General Abdullah al-Badri commented on oil prices being around US$130: "We are not happy with prices at this level because there will be destruction as far as demand is concerned". He later identified US$100, as being a "comfortable" price

Cairns and Calfucura (2012) concluded from their recent analysis of the opaque OPEC behavior, Saudi Arabia's (and OPEC's) dominant strategy is indeed to "restrain the price to conserve its market in the long-run."

Limit pricing on non-renewable-resource markets has recently gained renewed attention (e.g. the application by van der Ploeg and Withangen, 2012, Page 353). A closely related line of research was also initiated by Gerlagh and Liski (2011, 2012) and followed by Jaakkola (2012). Their models provide the dynamic counterparts in resource markets of strategic entry-prevention equilibria in the spirit of Bain (1956) and Modigliani (1958). Oil-exporting countries strategically interact with oil-consuming nations which may costly switch to alternative sources of energy; exporters maintain low enough prices for such investment strategy to remain dominated. In contrast with that research, when account is taken of the inelasticity of demand, limit pricing arises regardless of strategic interactions; market power by a coordinated demand side is not required for that.[7]

This paper examines the effects of taxes – like the carbon tax – on a non-renewable resource – like oil – when limit pricing arises from the low elasticity of resource demand. The taxation of non-renewable resources is revisited in that context. Much research efforts currently revolve around the effects of carbon taxation and the design of the optimal carbon tax: see the influential works by Metcalf (2008), Sinn (2008), Golosov, Hassler, Krusell and Tsyvinski (2014), van der Ploeg and Withagen (2014), among many others. It is hoped that both the taxation of carbon resources like oil and the support to non-carbon substitutes are effective instruments to curb carbon emissions that are responsible for global warming. Moreover, relatively high tax rates are already applied to oil products in most countries. From existing governmental commitments and in light of current national and international policy discussions on climate change mitigation (see Metcalf, 2010, for the United States), it is to be anticipated that tax rates on carbon energies may further

(http://www.reuters.com/article/2012/05/03/us-opec-supply-idUSBRE8420UY20120503).

[7]The limit-pricing equilibria considered here may nevertheless be interpreted as Bain-Modigliani or Gerlagh-Liski strategic equilibria once the limit price is appropriately defined as a best response of a Nash game where players' strategies are considered given.

increase and that a more favorable fiscal treatment will be given to their non-carbon substitutes.

Yet, there exists no study of taxation-induced changes in non-renewable-resource equilibrium quantities that consider limit-pricing situations, whether in the literature on non-renewable-resource taxation (e.g. Gaudet and Lasserre, 2013) or in the literature about market power on resource markets. Studies on the specific effect of taxes on resource monopolies are entirely based on Stiglitz's (1976) Hotelling-type interior equilibria; e.g. Bergstrom, Cross and Porter (1981) or Karp and Livernois (1992). As we will see, exclusively relying on this conventional treatment of monopoly power on non-renewable resource markets may lead to wrongly assess the effects of large-scale environmental taxation policies.

We start with the standard setting of Hoel (1978), Salant (1979), Dasgupta and Heal (1979) and Newbery (1981): a finite stock of homogenous resource is exploited by a monopoly that faces a relatively price-inelastic demand; substitution opportunities are represented by the availability of a backstop technology. In that setting, we introduce a specific tax applied to the extracted flow of resource and we examine its effect in the spirit of Gaudet and Lasserre (2013). Unlike Hotelling models where only constant-present-value taxes are neutral (Dasgupta, Heal and Stiglitz, 1981), we show that resource taxes have in general no effect in presence of limit pricing: as long as resource extraction remains profitable, a rise in the resource tax does not affect equilibrium quantities at any date. Hence, the goal of reducing the consumption flow of the oil resource cannot be achieved by directly penalizing its extraction. Additionally, subsidies to the backstop substitute result in more resource being extracted at each date. This is the object of Section 2.

While a backstop substitute is a standard modeling device, it has no actual counterpart. Unlike ordinary goods producible without limit, the production of energy goods usually exhibits decreasing returns to scale because it relies on some scarce primary factors.[8]

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[8]For non-renewable substitutes to oil (other carbon fuels, uranium), scarcity arises from the finiteness of total exploration prospects and/or from the fact that low-cost reserves specifically are limited. Similarly for standard biofuels, as well as for solar and wind energy production, scarcity arises from land

On these grounds, in Section 3, we do away with the often-made constant-returns-to-scale assumption, and consider substitutes to oil that have imperfectly-elastic supplies. Each substitute is characterized by its own entry price, which is its marginal cost at the origin, and has a rising marginal cost function; that is, its production may be profitable while it does not serve the entire market. The resource (residual) demand curve progressively reflects the multiplicity of such substitution possibilities, with kinks and changing demand elasticity at those kinks (Marshall, 1920).

These heterogenous substitutes differ by the elasticity of their supply curve. In contrast with backstop substitutes, the resource-exploitation phase is compatible with some substitutes being on use. The analysis of the limit-pricing equilibrium draws a fundamental distinction between two kinds of substitutes. On the one hand, some substitutes are unable to meet a large fraction of the energy demand because of too rapidly rising marginal costs (we call them "non-drastic"); their low-elasticity supply curves do not eat enough of the resource demand to induce the monopoly to deter their entry. On the other hand, limit pricing is caused by substitutes which, once profitable, are able to serve a relatively large fraction of the energy demand because their costs are not rising so fast (we call them "drastic").[9] Their elastic supply curves cause the demand for the resource to become elastic enough, thus representing a sufficient threat to its market share to deserve deterrence by the resource supplier. In this setting, resource taxes remain neutral. However, subsidies to non-drastic (on-use) substitutes and to drastic (high-potential) substitutes have effects of opposite directions on equilibrium resource quantities: only subsidies to

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limitations. For instance, at the microeconomic level of a wind turbine, returns to scale should be increasing because the turbine involves a fixed set-up cost and almost-constant marginal costs of maintenance; at the macroeconomic level however, the unit cost of wind energy output must be increasing both because of land supply limitations and because the marginal land is of worse quality as far as wind is concerned. See for instance Chakravorty, Magné and Moreaux (2008) and Heal (2009) on land requirements and large-scale substitution of fuel products. Land availability is considered an issue as soon as further use of land causes rents to rise.

[9]Our terminology is directly inspired by the use of "drastic" and "non-drastic" in the economics of innovation. The expression "drastic innovation" was coined by Arrow (1962) and is now widely used in industrial organization (e.g. Gilbert, 2006) and in the new economics of growth (e.g. Aghion and Howitt, 2009): "drastic" innovations make old products obsolete while "non-drastic" ones leave a positive profit to the incumbent. In general, whether innovations are drastic or not affects the outcome of entry-deterrence games.

non-drastic substitutes induce a reduction of the extraction flow.

The carbon tax is applied to the carbon content of the oil resource; according to the above results, like a regular resource tax, the carbon tax has no effect on the equilibrium resource quantity. Yet other energy goods contain carbon, that are substitutes to oil. Those carbon substitutes are currently produced and so are non-drastic substitutes in our analysis. Because the carbon tax is also applied to these substitutes, it turns out to induce a greater equilibrium resource extraction.

Finally in Section 4, with further details in the Appendix, we discuss limit-pricing equilibria in less parsimonious models integrating various aspects of the oil market: the heterogeneity of the resource and the endogeneity of its ultimately-extracted quantities; more complex market structures than the pure monopoly model; uncertainty over future extraction profits; non-stationary technologies. The message delivered by Section 3 on the effects of resource taxes and of subsidies to substitutes on current extraction survives.

II. Effects of environmental taxation with a backstop technology

This section follows the limit-pricing model of Hoel (1978), Salant (1979), Dasgupta and Heal (1979) and Newberry (1981), that assumes a backstop substitute. We study the effects of taxes on a non-renewable resource and of subsidies to the backstop technology.

A. Static limit pricing

Assume that a monopoly produces some energy resource flow q at a constant marginal cost $c > 0$.

The total energy demand is given by the function $\overline{D}(p)$ of its price p; it is continuously differentiable and strictly decreasing. We assume that the price elasticity of the energy demand is lower than one all along the demand curve, that is $\xi_{\overline{D}}(p) \equiv -\overline{D}'(p)p/\overline{D}(p) < 1$.

There is a backstop technology by which a competitive sector can produce a perfect substitute to the resource at a constant positive marginal cost $b > c$. The demand notion that is relevant to the monopoly is the residual demand it faces. Let us denote it with

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$D(p) \leq \overline{D}(p)$. When $p < b$, the production of the substitute is not profitable and thus the residual demand for the resource is the entire energy demand $D(p) = \overline{D}(p)$. When $p > b$, the substitute becomes profitable and more attractive than the resource, whose demand is thus destroyed: $D(p) = 0$. For notational simplicity and without any consequence on our message, we assume, as is standard, that if $p = b$ consumers give priority to the resource: at this price, the monopoly may serve the entire demand $D(b) = \overline{D}(b)$. To sum up, the residual demand $D(p)$ that the monopoly faces is continuous everywhere and continuously differentiable everywhere but at the kink $p = b > 0$, such that its price elasticity $\xi_D(p) = \xi_{\overline{D}}(p)$ is lower than one until the kink at which it jumps to infinity. This demand schedule is illustrated in Figure 1.
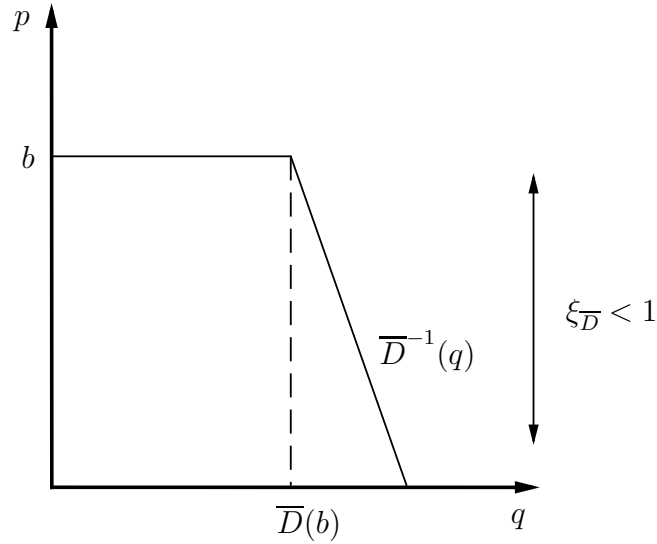
Figure 1: Residual demand for the resource in the presence of a backstop technology

Which production level maximizes the monopoly's profits in that context? If the monopoly supplies an amount q that is lower than the threshold quantity $\overline{D}(b)$, it induces the limit price b and its spot profit is $(b - c)q$, which is strictly increasing in q until $\overline{D}(b)$. With a higher supply $q > \overline{D}(b)$, the monopoly depresses the price below b; then its spot profit as function of the resource quantity becomes $\left(\overline{D}^{-1}(q) - c\right)q$, which is strictly decreasing in q because demand is sufficiently inelastic. Indeed, marginal profit may be

written $p\left(1 - 1/\xi_{\overline{D}}(p)\right) - c$, where $\xi_{\overline{D}}(p) < 1$ implies the term into parentheses to be positive. To sum up, the instantaneous profit is

$$\pi(q) = \begin{cases} (b-c)q, \text{ increasing, for } q \leq \overline{D}(b) \\ \left(\overline{D}^{-1}(q) - c\right)q, \text{ decreasing, for } q > \overline{D}(b) \end{cases}, \tag{1}$$

which is maximized by the supply level $q^m = \overline{D}(b)$ that induces the limit price $p = b$, the maximum price that deters the entry of the backstop.

B. Intertemporal limit pricing of extraction

Consider now that the resource is non-renewable; it is available in a finite quantity $Q_0 > 0$, that is to be extracted over the continuum set of dates $t \in [0, +\infty)$. The above static reasoning carries over to the case of a non-renewable resource as long as date-t flow of extraction q_t does not exceed the remaining exploitable amount at that date.

In that case, the monopoly's problem becomes intertemporal. Assuming a discount rate $r > 0$, the stream of discounted profits amounts to

$$\int_0^T \pi(q_t)e^{-rt} \, dt, \tag{2}$$

where the function $\pi(q_t)$ is given by the function (1) and where the terminal date T is free. The monopoly chooses the extraction path $(q_t)_{t \geq 0}$ in such a way as to maximize (2) under the exhaustibility constraint

$$\dot{Q}_t = -q_t, \tag{3}$$

where $Q_t \geq 0$ denotes the remaining stock at date t, with a given $Q_0 > 0$.

In such intertemporal problems, the relevant instantaneous objective is the Hamiltonian function. The Hamiltonian does not only consist of the present-value static profit objective $\pi(q_t)$; it is corrected by a linear term that reflects the opportunity cost of extracting the scarce resource. For the problem of maximizing (2) under (3), the Hamiltonian

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writes

$$\mathcal{H}(q_t, Q_t, \lambda_t, t) \equiv \pi(q_t)e^{-rt} - \lambda_t q_t, \tag{4}$$

where $\lambda_t \geq 0$ is the multiplier associated with constraint (3). λ_t must be interpreted as the discounted scarcity value of the resource. By the Maximum Principle, it is constant over time at the producer's optimum: $\lambda_t = \lambda$.[10]

The optimal choice of extraction q_t must maximize the Hamiltonian (4) at all dates of the extraction period. Since both $\pi(q)$ as per (1) and λq are linear in q, the Hamiltonian is maximized by the same constant supply level $q^m = \overline{D}(b)$ as the instantaneous revenue $\pi(q)$ in (1), as long as the discounted marginal revenue $(b-c)e^{-rt}$ remains greater than the scarcity value λ (See Figure 2).
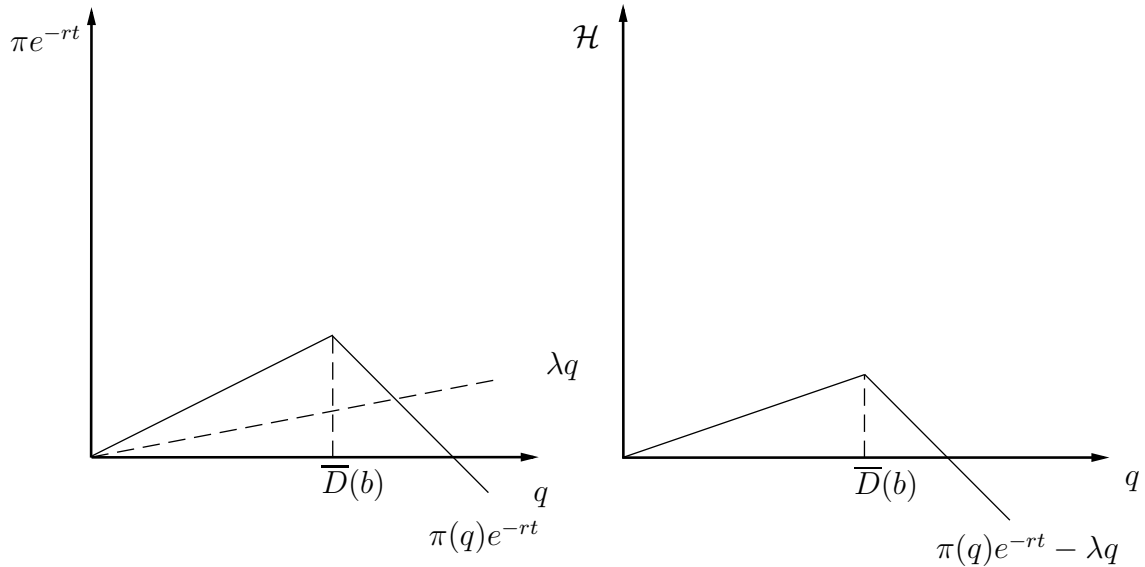
Figure 2: Instantaneous profit and Hamiltonian value

Assume, as an assumption to be contradicted, that λ is nil. Then $(b-c)e^{-rt}$ is always strictly positive, so that extraction is also positive at the constant level $q^m = \overline{D}(b) > 0$. Clearly, this violates the exhaustibility constraint (3) with $Q_t \geq 0$ in finite time.

Therefore $\lambda > 0$. Now contradict that $b - c < \lambda$: in that case, $(b-c)e^{-rt} < \lambda$ for

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[10]The time independence of λ along the optimal producer path is standard in models of Hotellian resources. It arises from the fact that the Hamiltonian does not depend on Q_t because the resource is homogenous. In Section 4, we examine the case of heterogenous resources.

all $t \geq 0$ and there would be no extraction at all; since $b > c$, this would be strictly dominated by some positive extraction. Thus it must be the case that the marginal profit $(b - c)e^{-rt}$ is greater than or equal to λ, but decreasing, over an initial period of time $[0, T^m]$. Because of discounting at rate $r > 0$, this period ends at a terminal date T^m such that $(b - c)e^{-rT^m} = \lambda > 0$. Since $\lambda > 0$, T^m must also be the exhaustion date, that is $T^m = Q_0/q^m = Q_0/\overline{D}(b)$. Combining the last two conditions characterizes λ.

The following proposition summarizes the properties of the limit-pricing equilibrium in absence of taxation policies.

Proposition 1 *In presence of a backstop substitute producible at the constant marginal cost $b > c$,*

1. *The monopoly supplies the constant quantity $q^m = \overline{D}(b) > 0$, and so induces the limit price $p^m = b$ that deters the backstop-substitute production, at all dates t of the extraction period $[0, T^m]$;*

2. *The limit-pricing equilibrium leads to a complete exhaustion of the resource at date $T^m = Q_0/\overline{D}(b)$;*

3. *Along the extraction period, the monopoly's marginal extraction profit is decreasing at rate $r > 0$, despite the constant scarcity component λ; the latter is decreasing in the initial reserve stock Q_0 and increasing in the backstop unit cost b.*

It can easily be verified that deviations from this extraction path would decrease the sum of the monopoly's discounted profits. Two types of deviations are possible. First, consider reallocations of an infinitesimal quantity $\Delta > 0$ of resource from any date t to any date $t' \neq t$ such that $t, t' < T^m$. Reducing extraction by Δ at date t decreases present-value profits by $(b-c)\Delta e^{-rt}$ while increasing extraction at date t' decreases profits as well, since profits are decreasing for quantities exceeding q^m. Second, consider reallocations of an infinitesimal quantity $\Delta > 0$ of resource from any date $t \leq T^m$ to any date $t' > T^m$. Again, reducing extraction by Δ at date t decreases present-value profits by $(b-c)\Delta e^{-rt}$.

On the other hand, increasing extraction at date t', from zero, by Δ, increases present-value profits by $(b - c)\Delta e^{-rt'}$. However, since $r > 0$, the overall effect on the discounted stream of profits remains negative.

To sum up in the context of this section, facing a relatively inelastic resource demand, the monopoly finds it optimal to choose the flat extraction path of level $q^m = \overline{D}(b)$ that induces the highest constant price $p^m = b$ which prevents the backstop's production from taking the entire demand. Unlike conventional treatments of monopoly power on resource markets where the demand elasticity is assumed greater than one, the limit-pricing equilibrium also satisfies the stylized fact that equilibrium pricing occurs on a part of the oil's demand curve where price elasticity is lower than unity.

C. Taxes on the non-renewable resource

Let θ_t be a specific resource tax (or subsidy if negative) applied to the producer resource price p_t at each date $t \geq 0$ to determine the consumer price $p_t + \theta_t$.[11]

The consumer price at which the substitute becomes profitable is b, regardless of the tax. Thus the tax does not affect neither the quantity $q_t = \overline{D}(b)$ below which the backstop enters, nor the consumer price b the market establishes in this case, but the producer price which becomes $b - \theta_t$.

Also, when $q_t \geq \overline{D}(b)$ so that only the resource is produced, the tax-inclusive consumer price is given by the inverse demand $\overline{D}^{-1}(q_t)$, and the price accruing to the producer becomes $\overline{D}^{-1}(q_t) - \theta_t$.

It turns out that the problem of the previous section is only modified to the extent that the instantaneous profit becomes

$$\pi(q_t) = \begin{cases} (b - \theta_t - c)q_t, \text{ increasing, for } q_t \leq \overline{D}(b) \\ \left(\overline{D}^{-1}(q_t) - \theta_t - c\right) q_t, \text{ decreasing, for } q_t > \overline{D}(b) \end{cases} . \tag{5}$$

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[11]This is a consumer tax. As shown for instance by Bergstrom et al. (1981), its effect is formally equivalent to that of a tax falling on the producer.

The modification amounts to integrating the tax θ_t to the marginal cost c. Assume that the adjusted cost $c + \theta_t$ still satisfies the condition $b > c + \theta_t$ at all dates $t \in [0, T^m]$, such that the marginal profit remains positive for some production. In that case, the analysis of the previous section carries over, unchanged. Thus the path of resource taxes is completely neutral to the monopolist's extraction. Indeed, the quantity that the monopoly needs to supply so as to deter the backstop production remains $\overline{D}(b)$, regardless of whether the resource is taxed or not; the monopoly chooses the same limit-pricing quantity as in absence of tax. Meanwhile, its revenues are reduced by the tax burden $\theta_t \overline{D}(b)$ at all dates of the extraction phase.

With a very high tax $\theta_t \geq b - c$ at some date $t \in [0, T^m]$ however, (5) shows that the marginal profit becomes negative for all quantities. In that case, the monopoly is better-off with a zero extraction: high taxes that expropriate the entire profit cause supply interruptions.

The following proposition summarizes the effect of resource taxation in a limit-pricing equilibrium.

Proposition 2 *As long as they warrant non-negative extraction profits, resource taxes leave the resource limit-pricing-equilibrium extraction unchanged.*

Neutral resource taxes exist in standard Hotelling models. Dasgupta et al. (1981) showed that specific resource taxes that grow at the rate at which profits are discounted are neutral to the extraction of a competitive sector; such taxes leave unaffected the intertemporal no-arbitrage condition that prevailed in any Hotelling competitive equilibrium. Karp and Livernois (1992) showed that this neutrality result also applies when extraction is monopolistic.[12] Under competition as well as in a monopoly, extreme taxes eat the entire Hotelling rent and do not warrant any extraction.

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[12]In Hotelling equilibria, whether under competition or monopoly, there exists a family of optimal resource tax/subsidy paths. This family is indexed by a tax component Ke^{rt}, where K is some scalar, that is constant in present value. As Karp and Livernois (1992) put it: "If the amount Ke^{rt} is added to [the optimal unit tax], the monopolist will still want to extract at the efficient rate, provided that the dynamics rationality constraint is satisfied (...)."

The neutrality result of Proposition 2 is reminiscent of the result that resource taxes may be neutral in Hotelling equilibria when they grow at the rate of discount. The novelty lies in the fact that resource taxation in limit-pricing equilibria is of a strong form in the sense that it does not require taxes to obey any particular dynamics.

D.   Subsidies to the backstop substitute

Alternatively, let γ_t be a specific subsidy to the backstop substitute, applied to the backstop's producer price, which is also its marginal cost b. Thus, the problem in absence of taxation is only modified to the extent that the price of the backstop substitute b should be replaced by the consumer net-of-subsidy price $b - \gamma_t$.

Unlike a resource tax, a backstop subsidy γ_t always affects the limit-pricing equilibrium. In the sequel, let us assume away the case of subsidies that would make the substitute available to consumers for free: we assume $\gamma_t < b$. When the substitute price is reduced to $b - \gamma_t$, the resource supply that deters its production rises to $\overline{D}(b - \gamma_t) > \overline{D}(b)$; with a substitute subsidy, the monopoly must supply more so as to deter its entry.

Also, low resource quantities $q_t < \overline{D}(b - \gamma_t)$ that warrant the production of the substitute, no longer induce the market price b, but the lower price $b - \gamma_t$. Thus, for $q_t < \overline{D}(b - \gamma_t)$, the marginal extraction profit of the monopolist becomes $b - \gamma_t - c$.

With backstop substitute subsidies, the instantaneous extraction profit becomes

$$
\pi(q_t) = \begin{cases} (b - \gamma_t - c)q_t, \text{ increasing, for } q_t \leq \overline{D}(b - \gamma_t) \\ \left(\overline{D}^{-1}(q_t) - c \right) q_t, \text{ decreasing, for } q_t > \overline{D}(b - \gamma_t) \end{cases}. \tag{6}
$$

First consider the case of extreme subsidies $\gamma_t \geq b - c$. In that case, the marginal extraction profit is always negative. Indeed, $b - \gamma_t < c$, so that extraction is not warranted. When $\gamma_t < b - c$, the extraction marginal profit is positive for low quantities $q_t \leq \overline{D}(b - \gamma_t)$. In that case, the analysis applies as in absence of subsidies, and the monopoly chooses the limit-pricing supply $q_t = \overline{D}(b - \gamma_t)$ that deters the backstop production. This modification of the limit-pricing equilibrium is illustrated in Figure 3.

The following proposition summarizes the effect of subsidies to a backstop substitute.

Proposition 3 *As long as they warrant non-negative extraction profits, subsidies to the backstop substitute increase the resource limit-pricing-equilibrium current extraction.*

Extreme subsidies that make the extraction profit vanish, cause resource supply interruptions.
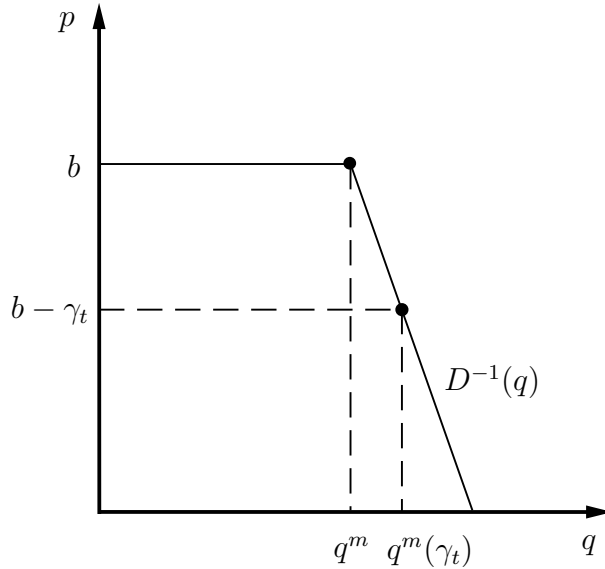
Figure 3: Limit-pricing equilibrium and the effect of a backstop subsidy

If taxation policies aim at reducing the current oil extraction quantity, the model of this section yields a quite pessimistic message. Not only are resource taxes strongly neutral, but subsidizing the backstop substitute induces the monopoly to increase its supply.

III.   Several decreasing-returns-to-scale substitutes

The parsimonious model of Section 2 relies on the existence of a backstop technology. Although a simple and meaningful modeling device, the assumption that a substitute to oil is producible without limit under conditions of constant returns to scale has been questioned in the Introduction.

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Whether in conventional Hotelling-type equilibria or in the limit-pricing equilibrium of Section 2, such backstop technology is never used before the exhaustion date, after which it becomes the exclusive source of energy. Empirical evidence shows that substitutes to oil are currently traded and consumed on energy markets, such as other regular fuels, biofuels, and alternative energies. Yet, each substitute remains far from meeting a large fraction of the energy demand.

In this section, we do away with the single-substitute assumption and allow for the possibility that some substitutes may be used along the resource extraction phase. As a matter of fact, the existing substitutes to oil may differ according to the substitution opportunities they offer. Limit-pricing is not incompatible with some substitutes being deterred while others are produced during the resource exploitation phase.

A. The model

The elasticity of the residual demand is often interpreted as the extent of substitution opportunities (e.g. Lewis et al., 1979). Marshall (1920) argued that, ordinarily, demand curves should be expected to have the property that the price elasticity is increasing with price.

In this section, we assume that there are two substitutes whose entries sequentially kink the resource demand and increase its elasticity; it will be clear shortly that the analysis easily accommodates more than two substitutes. As in Section 2, substitutes are perfect and produced competitively. However, their production now exhibits decreasing returns to scale: they can only offer limited substitution possibilities.[13]

As argued in the Introduction, the supply of energy goods is subject to limitations. These limitations may arise because of the scarcity of some factors.[14] Whether this scarcity is static (e.g. land, as in the case of biofuels, and wind and solar energies) or dynamic (e.g. finite exploitable reserves, as in the case of other fossil fuels), higher

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[13]Similarly, one may consider substitutability to be partial because substitutes only replace oil for some uses (Hoel, 1984).

[14]See especially Footnote 8.

instantaneous prices always warrant a higher instantaneous supply, yet at some greater marginal costs.[15] Thus for simplicity, we assume that the production of substitutes is static and the only good we explicitly treat as non renewable is the resource supplied by the monopoly.

One substitute will be called "drastic" and will turn out to play the same role as the backstop substitute of Section 2: for prices greater than its entry price $p^d > c$, the drastic substitute offers high substitution opportunities that will induce the resource monopoly to deter its production. The other substitute will be called "non drastic". For prices greater than its entry price $p^n > c$, the non-drastic substitute offers relatively low substitution possibilities; unlike the drastic substitute, the resource monopoly's optimal pricing will not imply the non-drastic substitute to be deterred. We assume

$$c < p^n < p^d, \tag{7}$$

so as to exclude the uninteresting case where the non-drastic substitute is deterred at the same time as the drastic one.[16] Thus the non-drastic substitute will be produced along the resource exploitation limit-pricing phase. We now examine the 3 sections of the residual resource demand curve, which is represented in Figure 3.

ı) For all prices $p \leq p^n$, no substitute is competing with the resource at all.[17] The residual demand the monopoly is facing is the entire demand $D(p) = \overline{D}(p)$. Since the residual demand $D(p)$ will turn out to be monotonic, the range of prices $p \leq p^n$ that deter both substitutes is induced by sufficiently high monopoly extraction

$$q \geq q^n, \text{ with } q^n \equiv \overline{D}(p^n), \tag{8}$$

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[15]In the case of a non-renewable substitute, supply is still characterized by the equalization of price with marginal costs, while marginal costs are adjusted to comprise the opportunity costs of extraction. See Sweeney (1993, Pages 775-776) for the interpretation of the instantaneous supply of a non-renewable resource.

[16]In principle, there may be substitutes, drastic or non-drastic, with entry prices exceeding the equilibrium limit price, that are not produced over the limit-pricing extraction phase.

[17]In this section as in Section 2, the assumption that substitutes' production is nil at their entry price is made for notational simplicity.
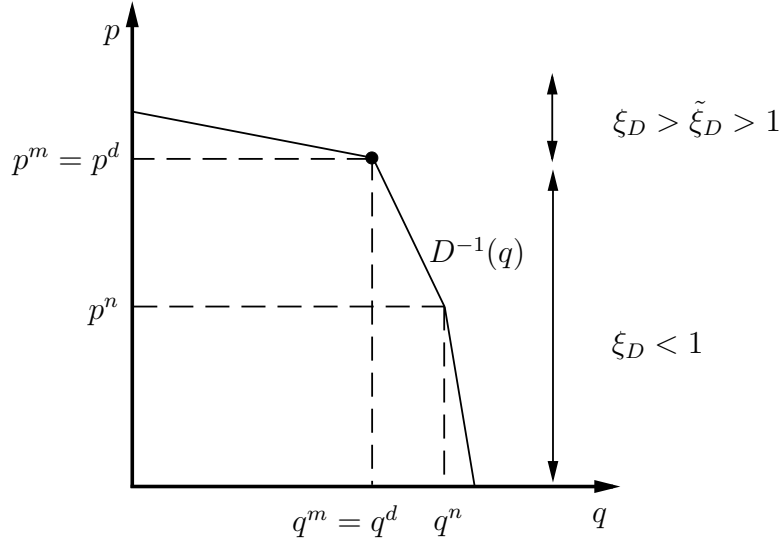
16

Figure 4: Residual demand and limit-pricing equilibrium with drastic and non-drastic substitutes

over which

$$\pi(q) = \left(D^{-1}(q) - c\right) q \text{ is decreasing} \tag{9}$$

since

$$\xi_D(q) = \xi_{\overline{D}}(q) < 1. \tag{10}$$

$\imath\imath$) For prices $p^n < p \leq p^d$, only the non-drastic substitute is competing with the resource, as the price exceeds its entry price p^n, which is its marginal cost at the origin: $p^n \equiv C^{n\prime}(0) > 0$. Unlike the backstop of Section 2, the non-drastic substitute is unable to meet a large fraction of the resource demand without exhibiting substantial cost increase. Thus the marginal cost $C^{n\prime}(x^n)$ of producing a quantity x^n of non-drastic substitute is differentiable, strictly increasing and the non-drastic-substitute supply function $S^n(p) \equiv C^{n\prime -1}(p)$ is continuous, with $S^n(p) > 0$ if and only if $p > p^n$. We assume that its elasticity of supply $\xi_{S^n}(x^n) = C^{n\prime}(x^n)/(C^{n\prime\prime}(x^n)x^n)$ is low in the sense that the elasticity of the residual demand $D(p) = \overline{D}(p) - S^n(p)$ satisfies

$$\xi_D(q) = \frac{e}{q}\xi_{\overline{D}}(e) + \frac{x^n}{q}\xi_{S^n}(x^n) < 1, \tag{11}$$

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where $e = q + x^n$ is the total energy supply. The range of prices $p^n < p \leq p^d$ over which only the non-drastic substitute is produced is induced by monopoly's supply

$$q^d \leq q < q^n, \text{ with } q^d \equiv \overline{D}(p^d) - S^n(p^d); \tag{12}$$

over this range, it follows from (11) that

$$\pi(q) = \left(D^{-1}(q) - c\right) q \text{ is decreasing.} \tag{13}$$

iii) For prices $p > p^d$, the two substitutes are competing with the resource because the price also exceeds the entry price p^d, which is the drastic substitute's marginal cost at the origin: $p^d \equiv C^{d\prime}(0)$. The marginal cost of producing a quantity x^d of drastic substitute $C^{d\prime}(x^d)$ is differentiable and strictly rising because of decreasing returns; yet relatively slowly, which reflects the capacity of the drastic substitute to meet a substantial part of the demand without implying a large increase in marginal cost. Accordingly, we assume that the drastic-substitute supply function $S^d(p)$ has a sufficiently high elasticity $\xi_{S^d}(x^d) = C^{d\prime}(x^d)/\left(C^{d\prime\prime}(x^d)x^d\right)$ to ensure, as will be verified shortly, that

$$\pi(q) = \left(D^{-1}(q) - c\right) q \text{ is increasing} \tag{14}$$

over the range of monopoly's supplies

$$q < q^d, \text{ where } q^d \equiv \overline{D}(p^d) - S^n(p^d), \tag{15}$$

that induce the production of the drastic substitute. When the two substitutes are produced, the residual demand faced by the monopoly is $D(p) = \overline{D}(p) - S^n(p) - S^d(p)$, and the profit in (14) is increasing strictly if $\xi_{S^d}(x^d)$ is high enough for the residual demand elasticity

$$\xi_D(q) = \frac{e}{q}\xi_{\overline{D}}(e) + \frac{x^n}{q}\xi_{S^n}(x^n) + \frac{x^d}{q}\xi_{S^d}(x^d) \tag{16}$$

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to satisfy

$$\xi_D(q) > \tilde{\xi}_D \equiv \frac{p^d}{p^d - c} > 1. \tag{17}$$

Indeed in the context of this section, $\pi(q) = \left(D^{-1}(q) - c\right)q$ and the marginal revenue may be written $\pi'(q) = D^{-1}(q)\left(1 - 1/\xi_D(q)\right) - c$, where $\xi_D(q) \equiv -D^{-1}(q)/\left(D^{-1\prime}(q)q\right)$. Thus $\pi'(q) > 0$ if and only if the demand elasticity is sufficiently high over $q < q^d$; precisely, if and only if $\xi_D(q) > D^{-1}(q)/\left(D^{-1}(q) - c\right)$. Since the reciprocal of the Lerner index (right-hand side) is increasing in q, its supremum is $\sup_{q \leq q^d} D^{-1}(q)/\left(D^{-1}(q) - c\right) = D^{-1}(q^d)/\left(D^{-1}(q^d) - c\right)$, which is also $\tilde{\xi}_D$ in (17). Therefore, the first inequality in (17) ensures that $\pi(q)$ is increasing over $q < q^d$.[18]

To sum up, the instantaneous profit is continuous and such that

$$\pi(q) = \begin{cases} \left(D^{-1}(q) - c\right)q, & \text{increasing, for } q < q^d \\ \left(D^{-1}(q) - c\right)q, & \text{decreasing, for } q^d \leq q < q^n \\ \left(\overline{D}^{-1}(q) - c\right)q, & \text{decreasing, for } q \geq q^n \end{cases} \tag{18}$$

and is thus maximized by the supply level

$$q^m = q^d \equiv \overline{D}(p^d) - S^n(p^d). \tag{19}$$

The dynamic analysis of Section 2 applies as before once the backstop marginal cost b has been replaced by the entry price p^d, and once q^m is given by (19). Thus, the following proposition summarizes the properties of the limit-pricing equilibrium in the context of this section.

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[18]Conventional monopoly pricing with non-kinked demand induces a point of the demand curve where demand elasticity is equal to the reciprocal of the Lerner index: $\xi_D(q) = D^{-1}(q)/\left(D^{-1}(q) - c\right)$. Thus assumption (17) rules out the existence of such equilibria.

Proposition 4 *In presence of a drastic and a non-drastic substitutes, with entry prices $c < p^n < p^d$,*

1. *The monopoly supplies the constant quantity $q^m = q^d = \overline{D}(p^d) - S^n(p^d)$ as per (19), and so induces the limit price $p^m = p^d$ that deters the drastic-substitute production, at all dates t preceding the exhaustion date $T^m = Q_0/\left(\overline{D}(p^d) - S^n(p^d)\right)$;*

2. *All along the extraction period $[0, T^m]$, the non-drastic substitute is produced in the constant quantity $S^n(p^d) > 0$.*

In absence of taxation policies, the limit-pricing equilibrium is thus depicted as in Figure 4. As far as taxation policies are concerned, the distinction between drastic, deterred substitutes, and in use, non-drastic substitutes, will turn out to be fundamental.

B. Taxes on the non-renewable resource

The same way as in Section 2, a unit consumer tax θ_t leaves unchanged the consumer price p^d at which the drastic substitute enters, and thus the extraction quantity q^d given by (15) that induces its entry. It also leaves the entry price p^n unchanged. Thus the tax does only modify (18) to the extent that, for any extraction quantity q_t, the price accruing to the producer is the inverse demand $D^{-1}(q_t)$, reduced by the tax θ_t. Once the elasticity threshold in (17) has been adjusted to $\widetilde{\xi}_D \equiv p^d/\left(p^d - \widetilde{\theta} - c\right)$, with $\widetilde{\theta} = \sup\limits_{t \in [0,T^m]} \theta_t$, the instantaneous profit function becomes

$$\pi(q) = \begin{cases} \left(D^{-1}(q) - \theta_t - c\right)q, \text{ increasing, for } q < q^d \\ \left(D^{-1}(q) - \theta_t - c\right)q, \text{ decreasing, for } q^d \leq q < q^n \\ \left(\overline{D}^{-1}(q) - \theta_t - c\right)q, \text{ decreasing, for } q \geq q^n \end{cases} \quad (20)$$

just as if the extraction cost c was augmented by the levy θ_t.

Thus, to the extent that the tax is not expropriating the entirety of the instantaneous extraction profit, it does not affect the monopoly limit-pricing path of Proposition 4, and the strong neutrality result of such taxes and subsidies holds as per Proposition 2.

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When the rent $p^d - c$ accruing to the producer in absence of tax in the limit-pricing equilibrium of Proposition 4 falls short of the tax levy, things are also like in Section 2: the tax leads to an interruption of the monopoly's extraction.

C. Subsidies to the drastic substitute

Subsidies to the drastic substitute in this section have the same effect as subsidies to the backstop substitute in Section 2.

With a subsidy γ_t^d to the consumption of the drastic substitute, the resource price at which the substitute becomes profitable becomes $p^d - \gamma_t^d$. Therefore, the extraction supply that deters the substitute's entry is increased to

$$q_t^d \equiv \overline{D}(p^d - \gamma_t^d) - S^n(p^d - \gamma_t^d), \tag{21}$$

instead of (15).

Also, whenever the production of the drastic substitute is warranted, as when $q_t < q_t^d$, its supply is increased, with no consequence on the limit-pricing equilibrium. In that case, a resource price p_t implies a producer price for the substitute $p_t + \gamma_t^d$ so that the subsidy reduces the residual demand faced by the monopoly to $D(p_t) = \overline{D}(p_t) - S^n(p_t) - S^d(p_t + \gamma_t^d)$. Hence, any resource supply $q_t < q_t^d$, induces the resource price $D^{-1}(q_t) = \left(\overline{D} - S^n - S^d \right)^{-1}(q_t)$, lower than in absence of subsidy.

The monopoly's revenue is only modified in these two respects, with the exact same consequence as in Section 2 for backstop subsidies: the equilibrium limit-pricing extraction $q_t^m = q_t^d$ is increased as per (21), yet of course to the extent that the subsidy warrants positive extraction.

Proposition 5 *As long as they warrant non-negative extraction profits, subsidies to a drastic substitute increase the resource limit-pricing-equilibrium current extraction.*

Extreme subsidies that make the extraction profit vanish, cause resource supply interruptions.

D. Subsidies to non-drastic substitutes

In the limit-pricing equilibrium of Proposition 4, the production of drastic (like backstop) substitutes is deterred by the monopoly. Currently used substitutes must all be non-drastic. This section shows that, in a limit-pricing context, the effect of subsidies to existing substitutes greatly differ from the effects earlier identified.

With a subsidy γ_t^n to the consumption of the non-drastic substitute, the resource price at which its production is profitable becomes $p^n - \gamma_t^n$. Thus the extraction level below which the substitute enters is reduced to

$$q_t^n \equiv \overline{D}(p^n - \gamma_t^n) \tag{22}$$

instead of (8).

Also, for all resource prices $p_t > p^n - \gamma_t^n$ – equivalently all extraction levels $q_t < \overline{D}(p^n - \gamma_t^n)$ – that warrant the production of the non-drastic substitute, its supply expressed as a function of the resource price is augmented to $S^n(p_t + \gamma_t^n)$. Accordingly, the residual demand is reduced to $D(p_t) = \overline{D}(p_t) - S^n(p_t + \gamma_t^n) - S^d(p_t)$ by the subsidy.

In particular, at the entry price p^d of the drastic substitute, the subsidy γ_t^n lowers the residual demand faced by the monopoly. Thus the extraction to be supplied so as to deter the entry of the drastic substitute is lower: (15) becomes

$$q_t^d \equiv \overline{D}(p^d) - S^n(p^d + \gamma_t^n), \tag{23}$$

lower than in absence of subsidy.

Thus (18) rewrites

$$\pi(q) = \begin{cases} (D^{-1}(q) - c)\, q, & \text{increasing, for } q < q_t^d \\ (D^{-1}(q) - c)\, q, & \text{decreasing, for } q_t^d \leq q < q_t^n \\ \left(\overline{D}^{-1}(q) - c\right) q, & \text{decreasing, for } q \geq q_t^n \end{cases} \tag{24}$$

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where threshold quantities q_t^n and q_t^d are lowered by the subsidy as per (22) and (23).

Thus the dynamic analysis of Section 2 applies, and a limit-pricing equilibrium realizes, in which the monopoly supplies less, so as to induce the unchanged limit price p^d: $q^m(\gamma_t^n) = q_t^d$, decreasing in γ_t^n. Figure 5 depicts the change in the residual demand faced by the monopoly as a consequence of the subsidy to the non-drastic substitute, and the resulting reduction in the limit-pricing quantity.
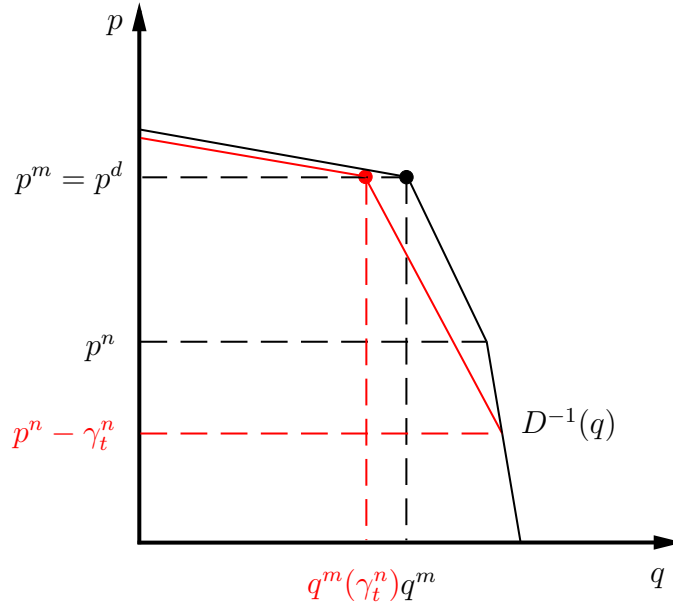
Figure 5: Limit-pricing equilibrium and the effect of a subsidy to a non-drastic substitute

The message of the following proposition sharply contrasts with those of Section 2 and of Proposition 4.

Proposition 6 *Subsidies to a non-drastic substitute,*

1. *Increase the substitute limit-pricing-equilibrium current production;*

2. *Reduce the resource limit-pricing-equilibrium current extraction.*

Like other instruments, extreme subsidies to a non-drastic substitute that make the extraction profit vanish, cause resource supply interruptions.

The carbon tax is applied to the carbon content of energy goods. Thus the carbon tax is formally equivalent to several taxes, each applied to a carbon-energy good, to an extent that reflects its unit carbon content.

In particular, the carbon tax comprises a tax on the oil resource as earlier examined. The result of Proposition 2, also valid in the context of this section, indicates that such tax, if not imposed at an extreme rate, has no effect. Indeed, it does not modify the entry price of the drastic substitute, and thus not the extraction quantity the monopoly is supplying to induce a lower price.

Energy goods that are substitutes to oil also contain carbon (e.g. gas, coal). Those carbon substitutes are currently produced and so are non-drastic substitutes in our analysis. Thus the carbon tax simultaneously act as a tax on a non-drastic substitute. The analysis of this section establishes that such tax (a negative subsidy) reduces the supply of the substitute and increases the supply of the resource. Indeed, with a lower supply of non-drastic substitute, the monopoly is supplying more resource so as to induce a price lower than the drastic substitute's entry price.

Since the carbon tax combines a resource tax with a tax on the non-drastic substitute, its effect immediately results from Propositions 2 and 5, as summarized in the following corollary.

Corollary 1 *The combination of a resource tax that warrants non-negative extraction profits, with a tax on a non-drastic substitute,*

    1. Decreases the substitute limit-pricing-equilibrium current production;

    2. Increases the resource limit-pricing-equilibrium current extraction.

IV. Discussion: ultimately extracted quantities, extraction costs, stationarity and market structures

This paper points at the empirical relevance of limit-pricing equilibria for the oil market and shows that the effects of environmental taxation instruments in such context differ from most conventional studies. In particular, taxes applied to flows of resources, if not prohibitive, are neutral irrespective of their time dynamics. As far as subsidies to oil substitutes are concerned, it is fundamental to make a distinction between two sorts of substitutes. On the one hand, the limit pricing is caused by the entry threat of drastic substitution possibilities. Like backstops, subsidies to drastic substitutes induce equilibrium extraction quantities to increase. On the other hand, limit pricing is compatible with other substitutes to oil being used, as is currently the case. Unlike backstops or drastic substitutes, subsidies to any currently in-use, non-drastic substitutes do reduce current extraction quantities by an amount that depends on their respective elasticity of supply. While we have restricted attention to a single non-drastic substitute for simplicity, extension to several such substitutes is immediate. Since the effect of subsidies depends on the supply elasticity of the substitute, the objective of reducing carbon-resource extraction quantities in a cost-efficient manner may imply selecting non-carbon substitutes on the grounds of their supply elasticity; an issue that is beyond the scope of the present work.

The existing literature has mentioned limit-pricing equilibria of non-renewable-resource monopolies in the simple Hotelling model of Section 2 where substitution possibilities are represented with a backstop technology. Section 3 extends Section 2 by refining the description of substitution possibilities: doing so proves to deliver a sharply different message on the incidence of taxation policies. Yet, our results have been obtained in a relatively parsimonious model.

On the one hand, one may question whether limit-pricing equilibria survive more complex setups. To take that into account, we extend the setting of Section 3 in the Appendix. As long as the equilibrium exhibits limit pricing on the part of the monopoly,

our results on taxation-induced equilibrium changes on current extraction levels survive qualitatively.

On the other hand, traditional Hotelling models with a fixed stock of homogenous resource assume the ultimately extracted quantity as given. The analysis of Sections 2 and 3 has focused on policy-induced changes on current extraction levels. The treatment of the Appendix allows to consider the effect of taxation policies on the cumulative quantity ultimately extracted; an aspect considered very important from an environmental perspective.

A. Reserves' heterogeneity, reserves' development and the ultimately extracted quantity

As a matter of fact, reserves of oil are heterogenous. One standard way to take that into account is to assume that spot extraction costs not only depend on current extraction flows, but also on the total amount of reserves still to be extracted, as when resources are exploited in order of their respective costs. This approach has been initiated by Hotelling (1931), consolidated by Gordon (1967) and recently used for instance by van der Ploeg and Withagen (2012, 2014) in works on the carbon tax.

For an unchanged ultimately extracted quantity, stock effects introduce incentives to extract the resource more slowly (Dasgupta and Heal, 1979), i.e. to save present extraction so as to extend the exploitation period. Appendix A extends the limit-pricing model of Sections 2 and 3 to the Hotelling-Gordon cost representation just described. This extension makes the limit-pricing model comparable with the conventional non-renewable-resource monopoly models of Bergstrom et al. (1981) and Karp and Livernois (1992); it turns out that the limit-pricing outcome survives the introduction of stock effects.

Also with stock effects, the ultimately extracted quantity may become endogenous because extraction can stop before the complete depletion of available reserves: this happens when the benefit derived from the last units to be extracted falls short of too high extraction costs. Thus in the stock-effect model, reserves may become economical or uneconomical as a result of a policy. Yet classical papers on the taxation of resource mo-

nopolies (Bergstrom et al., 1981; Karp and Livernois, 1992) assumed away the possibility that policies might affect the cumulative amount of exploited reserves.

Appendix A considers the case where some available reserves are left unexploited in the limit-pricing equilibrium. Appendix B further shows how a resource tax that is neutral on extraction levels along the exploitation phase increases the amount of abandoned reserves and thus reduces the ultimately extracted quantity.

Another reason why resource taxes may affect the ultimately exploited resource is that it may discourage exploration and development efforts by which reserves become exploitable. In Appendix C, we borrow the approach of Gaudet and Lasserre (1988), also used in for instance by Fischer and Laxminarayan (2005) or Daubanes and Lasserre (2012). In these models, the marginal cost of developing an amount of exploitable reserves is rising, as when resource units are developed in order of their respective development costs; reserves are established so as to equate the marginal development cost to the implicit value of marginal reserves. This extension does not modify the limit-pricing outcome and the tax effect on ultimately developed and exploited quantities mentioned above.

B.   Risk on future profits

Section 2 and Appendix A identify a strictly positive rate of profit discounting as an essential condition to the realization of limit-pricing equilibria. In absence of uncertainty, profits should be discounted at the rate of returns of available assets, e.g. the rate of interest. Future revenues from the exploitation of fossil fuels may be subject to various forms of uncertainty (e.g. Long, 1975). When the risk exists that profits may be destroyed from some future date on, it formally amounts to a premium added to the rate of discount. The profit-discounting rate must thus be interpreted more broadly as reflecting such risks.

C.   Non-stationary conditions

Technologies may not be stationary because of technical progress in the substitutes' sectors. In our treatment, the limit price is the minimum marginal cost of producing a

drastic substitute whose entry would significantly deteriorate monopoly's profits. Assuming that technical change causes the entry price of the drastic substitution possibilities to diminish over time may interestingly account for decreasing resource prices and increasing extraction quantities.

D. Interpretation of the limit price

The limit price may also be interpreted more broadly than the entry price of a drastic (or backstop) substitute. In Gerlagh and Liski (2011), the falling limit price induced by strategic oil suppliers is the price level beyond which irreversible investments on the part of consumers are anticipated to destroy the value of oil. Gerlagh-Liski equilibrium results from strategic interactions which are absent here. But one may borrow from their analysis a reinterpretation of the limit price as the price level beyond which a sufficiently drastic threat to the monopoly's profits would be carried out.

E. Market structures

Finally, the oil extraction sector is not a monopoly or an equivalent perfectly coordinated cartel serving the entire demand. When coordination is not perfect, several suppliers have oligopoly power. For an oligopoly, the elasticity of the residual demand that each individual supplier faces is increasing with the number of such suppliers. Similarly, as remarked by Salant (1976), the residual demand a cartel faces in presence of a competitive fringe is more elastic when the fringe has a larger market share. Equilibria with a lower-than-unity residual-demand elasticity are more likely when the supply sector is highly coordinated. The limit-pricing theory is more relevant when production sectors are more concentrated.

APPENDIX

A Occurrence of the limit-pricing equilibrium in a more complex setup

As far as energy demand is concerned, assumptions are the same as in Section 3. In particular, the total demand $\overline{D}(p)$ is non increasing and has a lower-than-unity price elasticity everywhere. The demand notion that is relevant to the monopoly is the residual demand for the resource $D(p)$. The total demand $\overline{D}(p)$ and the residual demand $D(p)$ differ because there may exist substitutes to the resource. The purpose of this appendix is to study the occurrence of limit-pricing equilibria in a more complex setup, and to revisit resource taxation in that context. Thus it is sufficient to assume that there is one drastic substitute only, with entry price $p^d > 0$ and supply function $S^d(p)$, strictly positive and strictly increasing if and only if $p > p^d$.

Unlike Sections 2 and 3, and following Karp and Livernois (1992), the total cost of extracting a given quantity of oil q_t at date $t \geq 0$ is assumed to take the form $C(q_t, Q_t) \geq 0$, where $Q_t \geq 0$ is the remaining stock to be exploited at this date: $Q_t = Q_0 - \int_0^t q_s \, ds$. This cost function is twice differentiable in its two arguments, increasing in q_t, decreasing in Q_t and such that $\partial^2 C(q_t, Q_t)/\partial q_t \partial Q_t \leq 0$ and that $C(0, Q_t) = 0$ for any $Q_t \geq 0$. We do not restrict $C(q_t, Q_t)$ to be convex here.[19]

In that context, the monopoly's spot profit is

$$\pi(q_t, Q_t) = D^{-1}(q_t)q_t - C(q_t, Q_t), \tag{25}$$

where $D^{-1}(q_t)$ is the inverse demand for oil. A property of the profit in (25) is that $\pi(0, Q_t) = 0$ for any $Q_t \geq 0$. Indeed $C(0, Q_t)$ is nil by assumption and $D^{-1}(q_t)$ is bounded above because $\overline{D}(p)$ is non increasing and $S^d(p)$ is strictly increasing.

Wherever it exists, marginal profit writes

$$\frac{\partial \pi(q_t, Q_t)}{\partial q_t} = D^{-1}(q_t)\left(1 - \frac{1}{\xi_D(q_t)}\right) - \frac{\partial C(q_t, Q_t)}{\partial q_t}, \tag{26}$$

where $\xi_D(q_t) \equiv -D^{-1}(q_t)/\left(D^{-1\prime}(q_t)q_t\right)$ is the price elasticity of the residual demand. $\xi_D(q_t)$ is discontinuous at the level $q^d = D(p^d)$, because of the drastic substitute's entry: for quantities $q_t > q^d$, $\xi_D(q_t)$ is lower than unity, while for $q_t < q^d$, it is assumed greater than the threshold $\widetilde{\xi}_D > 1$. In presence of non-linear extraction costs that depend on the

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[19]For simplicity, we assume away fixed extraction costs: $C(0, Q_t) = 0$ for any $Q_t \geq 0$. As a matter of fact, extraction may require set-up costs (see Hartwick, Kemp and Long, 1986; Neher, 1999). The present analysis solely allows for diminishing marginal extraction costs, that do not reflect well diseconomies of scale arising from fixed costs. Provided it is not prohibitively high, the analysis should survive the integration of a fixed component to the extraction profit derived along the exploitation phase.

remaining extraction, we adapt Section 3's definition of the $\widetilde{\xi}_D$ as the following supremum:

$$\widetilde{\xi}_D \equiv \sup_{q \leq q^d,\, Q \leq Q_0} \frac{D^{-1}(q)}{D^{-1}(q) - \frac{\partial C(q,Q)}{\partial q}} > 1; \tag{27}$$

the adjustment is sufficient to ensure that, regardless of the remaining reserves $Q_t \in [0, Q_0]$, marginal profit with respect to q_t is positive below q^d, as can be easily verified in (26), and negative above.

The intertemporal problem of the monopoly in the context of this appendix is to maximize

$$\max_{(q_t)_{t \geq 0}} \int_0^{T^m} \left(D^{-1}(q_t)q_t - C(q_t, Q_t) \right) e^{-rt}\, dt \tag{28}$$

subject to

$$\dot{Q}_t = -q_t, \tag{29}$$

and

$$Q_{T^m} \geq 0, \tag{30}$$

where the terminal date $T^m \geq 0$ is free and defined in such a way that $q_t > 0$ for all $t \leq T^m$ and $q_t = 0$ for all $t > T^m$.

Denoting by λ_t the co-state variable associated with (29), the Hamiltonian function of Problem (28)-(30) writes

$$\mathcal{H}(q_t, Q_t, \lambda_t, t) = \pi(q_t, Q_t)e^{-rt} - \lambda_t q_t. \tag{31}$$

As in Section 2 (see Figure 2), the Hamiltonian takes a null value for $q_t = 0$. As will be verified later, it must be the case that the discounted profit $\pi(q_t, Q_t)e^{-rt}$ does not fall short of the corrective term $\lambda_t q_t$ along the exploitation period, so that the Hamiltonian is maximum at the same limit-pricing level as the extraction profit:

$$q_t^m = q^m = q^d, \ \forall t \in [0, T^m]. \tag{32}$$

By the Maximum Principle, it must also be that $\dot{\lambda}_t = -\partial \mathcal{H}(.)/\partial Q_t$, that is

$$\dot{\lambda}_t = \frac{\partial C(q_t, Q_t)}{\partial Q_t} e^{-rt} < 0, \ \forall t \in [0, T^m]. \tag{33}$$

Consider now the terminal date T^m. On the one hand, the transversality condition associated with the fact that T^m is free tells that the Hamiltonian must take a null value at that date:

$$\left(D^{-1}(q^m)q^m - C(q^m, Q_{T^m}) \right) e^{-rT^m} = \lambda_{T^m} q^m. \tag{34}$$

On the other hand, the transversality condition associated with the constraint (30) that

reserves at T^m must be non negative implies

$$\lambda_{T^m} Q_{T^m} = 0. \tag{35}$$

There are two possibilities as far as the ultimately remaining reserves $Q_{T^m} \geq 0$ are concerned. Either $Q_{T^m} = 0$ as initial reserves Q_0 are completely exhausted. Then, $T^m = Q_0/q^m$, as in Section 2. Or $Q_{T^m} > 0$ because the extraction of the most expensive units is not economical. By (35), leaving some reserves unexploited after T^m is only compatible with a zero implicit value of those reserves: $\lambda_{T^m} = 0$. Following (34), that means that the amount of abandoned reserves must be such that extraction profits are zero at the terminal date; Q_{T^m} is uniquely determined by

$$D^{-1}(q^m)q^m - C(q^m, Q_{T^m}) = 0. \tag{36}$$

In that case, the terminal date is $T^m = (Q_0 - Q_{T^m})/q^m$.

Having determined the reserves Q_{T^m} ultimately left unexploited and the terminal date T^m of the exploitation phase, let us now verify that exploitation is indeed warranted at all dates prior to T^m. Totally differentiating the maximized Hamiltonian $\mathcal{H}(q^m, Q_t, \lambda_t, t)$ given by (31) with respect to time at any date t of the exploitation period $[0, T^m]$ yields $d\mathcal{H}(.)/dt = -r(D^{-1}(q^m)q^m - C(q^m, Q_{T^m}))e^{-rt} - \partial C(q^m, Q_t)/\partial Q \, \dot{Q}_t e^{-rt} - \dot{\lambda}_t q^m$, where $\dot{Q}_t = -q^m$ by (29) so that the last two terms cancel out by (33). Hence,

$$\frac{d\mathcal{H}(.)}{dt} = -r\left(D^{-1}(q^m)q^m - C(q^m, Q_{T^m})\right)e^{-rt},$$

which is strictly negative since $r > 0$. A strictly positive discount rate ensures that the Hamiltonian is strictly decreasing all along the exploitation phase from date 0 to date T^m, when the Hamiltonian is zero by (34), and thus

$$\mathcal{H}(q^m, Q_t, \lambda_t, t) > 0, \ \forall t \in [0, T^m),$$

which verifies that active exploitation is warranted over that period.

Hence limit pricing arises as an equilibrium in this more general model, as a result of a lower-than-one price elasticity of demand, of the presence of a drastic substitute, and of a strictly positive profit-discounting rate.

B   Effect of resource taxation

When there is a unit tax θ_t applied to the consumption of the resource, as in Sections 2 and 3, the instantaneous extraction profit is reduced to

$$\pi(q_t, Q_t) = D^{-1}(q_t)q_t - \theta_t q_t - C(q_t, Q_t). \tag{37}$$

31

The entry of the drastic substitute causes the elasticity of residual demand to be greater than the $\widetilde{\xi}_D$ threshold for quantities $q < q^d$, where like Sections 2 and 3, the presence of the tax requires the following adjustment in (27):

$$\widetilde{\xi}_D \equiv \sup_{q \leq q^d, \ Q \leq Q_0, \ (\theta_t)_{t \in [0, T^m]}} \frac{D^{-1}(q)}{D^{-1}(q) - \theta_t - \frac{\partial C(q,Q)}{\partial q}} > 1;$$

this adjustment ensures that the instantaneous profit is maximum at level $q^m = q^d$.

All along the extraction phase, the analysis follows unchanged where the Hamiltonian (31) integrates (37). That is, provided too high a tax does not cause supply interruptions, it has no effect on the extraction behavior of the monopoly as it does not affect the entry price of the drastic substitute, thus neither the minimum supply $q^m = q^d$ that deters its competition.

Yet in the setting of this appendix, the tax may affect the quantity Q_{T^m} ultimately extracted. Consider for simplicity that the tax is not causing any supply interruption along the extraction phase $[0, T^m]$. With the Hamiltonian adjusted as above, the transversality condition (34) characterizing the last extraction date becomes

$$\left(D^{-1}(q^m)q^m - \theta_{T^m}q^m - C(q^m, Q_{T^m})\right) e^{-rT^m} = \lambda_{T^m}q^m. \tag{38}$$

Thus following the same steps as the zero-tax analysis of Appendix A, either reserves are entirely exhausted and $Q_{T^m} = 0$, or an amount of reserves characterized by

$$D^{-1}(q^m)q^m - \theta_{T^m}q^m - C(q^m, Q_{T^m}) = 0, \tag{39}$$

is abandoned. The comparison between (39) and its no-tax counterpart (36) immediately shows that more reserves are left unexploited – less are ultimately extracted – with a tax $\theta_{T^m} > 0$.

C Costly exploration and development efforts

Finally, consider that reserves $Q_0 - Q_{T^m}$ to be exploited arise from costly exploration and development efforts. Following Gaudet and Lasserre (1988), assume that the production of the stock of reserves to be exploited occurs at date 0 and is subject to decreasing returns because, as exploration prospects are finite, it must be more and more difficult to produce new reserves. When the production of reserves to be exploited is costly, it cannot be optimum to produce more than what is to be exploited. Formally, the cost of producing $Q_0 - Q_{T^m}$ is given by the increasing and strictly convex function $E(Q_0 - Q_{T^m})$. Let us further assume that $E'(0) = 0$ so as to avoid the uninteresting situation where those costs induce the monopoly to produce no reserves at all.

The problem of the monopoly becomes

$$\max_{(q_t)_{t\geq 0}} \int_0^{T^m} \left(D^{-1}(q_t)q_t - C(q_t, Q_t)\right) e^{-rt} \, dt - E(Q_0 - Q_{T^m}) \tag{40}$$

subject to (29) and (30). The integration of reserves production into the monopoly's problem has no consequence on the analysis of the limit-pricing exploitation phase, but on the transversality condition associated with constraint (30).

Specifically, instead of (35), the following must hold. Q_0 may be entirely developed and completely exhausted as before and $Q_{T^m} = 0$ if development and extraction costs make it economical. Such is compatible with the marginal reserve production cost being lower than the implicit value of marginal reserves: $E'(Q_0) \leq \lambda_{T^m}$. Yet when reserves are not completely developed and extracted, $Q_{T^m} > 0$, and the implicit value of marginal reserves λ_{T^m}, instead of being equalized to zero as in absence of reserve production cost, is equalized to the the marginal cost $E'(Q_0 - Q_{T^m})$.

It turns out that the condition characterizing the reserves left undeveloped and unexploited (36) becomes

$$\left(D^{-1}(q^m)q^m - C(q^m, Q_{T^m})\right) e^{-rT^m} = \lambda_{T^m} q^m = E'(Q_0 - Q_{T^m}), \tag{41}$$

where $T^m = (Q_0 - Q_{T^m})/q^m$.

This extension immediately accommodates the case of a tax as previously considered in Appendix B. In that case, the left-hand side of (41) integrates the tax component $-\theta_{T^m} q^m$ of (39), which leads to the same conclusion that the tax increases the amount left undeveloped and unexploited and so reduces the ultimately extracted quantity.

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