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Optimal Timing of Carbon Capture Policies under Learning-by-doing*

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Abstract

Using a standard Hotelling model of resource exploitation, we determine the optimal energy consumption paths from three options: dirty coal, which is non-renewable and carbon-emitting; clean coal, which is also non-renewable but carbon-free thanks to carbon capture and storage (CCS); and solar energy, which is renewable and carbon-free. We assume that the atmospheric carbon stock cannot exceed an exogenously given ceiling. Taking into account learning-by-doing in CCS technology, we show the following results: i) Clean coal exploitation cannot begin before the outset of the carbon constrained phase and must stop strictly before the end of this phase; ii) The energy price path can evolve non-monotonically over time; and iii) When the solar cost is low enough, an unusual energy consumption sequence along which solar energy is interrupted for some time and replaced by clean coal may exist.

Keywords: Climate change; Energy substitution; Carbon Capture and Storage; Learning-by-doing.

JEL classifications: O44, Q31, Q42, Q54, Q55.

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1 Introduction

A large body of evidence has demonstrated the potential of carbon capture and storage (CCS) in mitigating CO_2 emissions, especially from concentrated sources (e.g., the power generation sector). Given that fossil fuels supply over 85% of all primary energy needs, CCS appears as the only technology that can substantially reduce CO_2 emissions while allowing fossil fuels to meet the world's expanding energy demand (Herzog, 2011). According to Hamilton *et al.* (2009), the mitigation cost for the capture and compression of emissions from gas power plants is about \$52/tCO_2. Adding transport and storage costs ranging from \$5 to \$15/tCO_2, a carbon price of about \$60-65/tCO_2 is needed to make these plants competitive. However, these costs exhibit a high variance. Rubin *et al.* (2012) report added capital and operating costs of CCS with respect to non-CCS conventional thermic plants within a range of 60-80% for SPC plants and 30-50% for IGCC plants.[1]

Considering this cost evidence, it is widely acknowledged that the actual deployment of CCS in the medium term depends heavily on the implementation of effective GHG emissions mitigation policies. IEA (2008) reports the need to begin deploying CCS around 2025 and to expand the use of this technology up to a 20% share of carbon emissions abatement by 2050 to meet the 450 ppm stabilization scenario. According to the technico-economic literature (Van den Broek *et al.*, 2009), this would require a carbon price of about \$100/tCO_2 for NGCC plants[2] and \$40/tCO_2 for IGCC plants, based on current state-of-the-art technology.

The uncertainty regarding the effective prospects of world climate change policies surely explains why after an enthusiastic period in the middle of the previous decade, CCS optimism has since been seriously tempered by a lack of effective development of the technology. Carbon capture technology is still in its infancy, with approximately 20 plants in operation (Thronicker and Lange, 2015). Given this context, the potential of learning about CCS technology has attracted a growing interest. The promoters of CCS have long advocated that, although more costly, the technology could benefit from a large potential of cost reduction thanks to learning-by-doing.

Learning-by-doing postulates a progressive reduction of the production costs in the cumulative production capacity. In the engineering literature, the so-called learning rate refers to the percentage cost reduction that could result from doubling the production capacity. Van den Broek *et al.* (2009) reports learning rates for CCS varying between 5% and 18% for capital costs and between 0 and 30% for operating and maintenance costs

[1]SPC: Super critical pulverized coal plants; IGCC: Integrated gasification combined cycle plants.
[2]NGCC: natural gas combined cycle power plants.

regarding the main types of thermic plants. Such a huge variance reflects the complexity of the learning assessment exercise. Cost reduction is typically achieved through more energy efficient plants, better management of other pollution mitigation devices (e.g., SO_2 emission control) and dedicated progresses for CCS operation at the plant level. Moreover, learning about energy efficiency can also benefit non-CCS plants. If such technological spillovers could help with the deployment of CCS, they may also benefit production modes that are both more competitive and more polluting. Finally, all existing cost figures have been computed with respect to the highly efficient installations located in North America or Western Europe. Most coal-fired plants installed today in emerging countries, such as China or India, use cheaper techniques. However, the potential difficulty of applying CCS technologies designed for efficient Western plants to these low cost plants remains an open issue (Sheng Li *et al.*, 2012).

There are four main economic drivers affecting CCS deployment. The first driver is the implementation of environmental policies targeting carbon concentration stabilization objectives throughout the current century. The second is the world's evolving demand for energy. The third driver is the change in the supply of fossil fuels and its resulting impact on the CCS economic potential. Finally, the fourth main economic driver affecting deployment is the competition between CCS and other carbon-free options from renewable sources, such as solar, wind or biomass. Within this context, the pros and cons of CCS can be summarized as follows: on the positive side, CCS allows for the use of fossil fuel, a relatively abundant and cheap energy source, without contributing to global warming. Furthermore, as a clean technological alternative to conventional fossil energy, it increases the social efficiency of the energy production system, thus lowering the social cost of carbon. Conversely, some proposed negative aspects are that the deployment of CCS is highly dependent on the availability of safe, convenient storage facilities and the avoidance of leakage risks to local populations and natural environments. Another concern is that relatively cheap CCS can also delay the development of clean renewable alternatives and/or hinder R&D in other energy production techniques (e.g., hydrogen).

Accounting for all of these issues, the objectives of this paper are two-fold. First, we want to assess the relative strengths of these pros and cons using a stylized economic model that incorporates the previously mentioned drivers. Second, we want to describe the effects of learning-by-doing as it pertains to the optimal timing of CCS use, the dynamics of the relative competitiveness in carbon sequestration compared to other clean energy sources, and its impact on the social cost of carbon pollution under a global atmospheric carbon concentration stabilization objective.

The main drivers affecting the economic potential of CCS are highly interconnected.

In the literature, these linkages are taken into account in two ways. Many authors use integrated assessment models to evaluate the role of CCS (Mc Farland *et al.*, 2003, Edenhofer *et al.*, 2005, Gerlagh and van der Zwaan, 2006, Grimaud *et al.*, 2011). These models underline the high sensitivity of this option concerning critical technological pitfalls and climate policy variables. However, in most cases, CCS is unlikely to become profitable until the second half of the century. Other authors pursue a more analytical route by introducing CCS in theoretical models of climate change. Lafforgue *et al.* ($2008^{a,b}$) illustrates optimal CCS policies by using a model of energy substitution, explicitly taking into account the scarcity of sequestration sites. In a recent contribution, Grimaud and Rouge (2014) examine the CCS deployment problem using an endogenous growth model.

The issue of learning-by-doing has also attracted a great deal of attention concerning the 'induced technical change' debate. Following Goulder and Matthai (2000), it is recognized that learning about clean technologies can have ambiguous effects on the timing of the energy transition toward a carbon-free economy. According to Manne and Richels (2004) or Gerlagh (2006), such ambiguous features remain for CCS; learning should be expected to play a role in reducing the cost of carbon sequestration and thus the opportunity cost of carbon pollution. However, learning-by-doing influences not only the deployment of new techniques but also the production of energy and thus the energy price dynamics. This point is examined by Chakravorty *et al.* (2012) with learning-by-doing in clean renewable energy technologies. They show that learning generally induces price fluctuations over the course of the energy transition from polluting fossil fuels toward clean energy. Our own modeling framework is similar to this work but with a main difference: benefiting from learning-by-doing in CCS requires the burning of fossil fuels and thus forces us to confront their increasing scarcity. These two effects work in opposition of one another: the learning effect, which decreases with cumulative experience, and the scarcity effect, which increases according to the depletion of fossil resources. As the learning effect initially should override the scarcity effect and then becomes dominated by the latter, the average cost of the combined process, which consists of burning coal while abating its polluting by-product, should first decrease and then increase. In contrast, learning about renewable energies continuously decreases their cost, as no depletion is at work here to reverse the effect of the cumulative experience, thus leading to a different range of possible optimal paths.

The findings in the literature may be broadly summarized as follows. CCS needs both relatively cheap access to fossil fuels and a strong public intervention against global warming. Learning-by-doing has an ambiguous effect on the timing of CCS deployment. Learning can induce energy price fluctuations even in the so-called 'Hotelling models'.

Given this context, we pose the following questions: How do the aforementioned drivers influence the timing of CCS implementation? How should the trigger energy price, which makes CCS competitive, be determined? Does learning-by-doing imply an early implementation of CCS? What is the impact of learning on the energy price dynamics? How does CCS, with or without learning, affect the timing and use of other clean renewable alternatives? Existing studies have addressed these questions either through static models, which neglect the properly dynamic aspects of CCS deployment, or by considering each influencing factor individually.

To overcome such limitations, we develop the following modeling framework. Energy needs can be supplied by two primary sources: the first source is non-renewable and carbon-emitting (coal), whereas the second source is renewable and clean (solar energy). Furthermore, two energy generation modes from fossil fuels are available: the first is a conventional technique that releases carbon emissions into the atmosphere, and the second involves using CCS to abate emissions. We assume constant marginal costs of energy delivery by conventional fossil fuel and renewable energy techniques. The CCS option incurs an additional cost, but it also benefits from learning-by-doing. Following a route pioneered by Chakravorty *et al.* (2006), we assume that a given carbon stabilization cap is enforced. Hence, two clean energy options allow for the relaxation of this constraint: solar energy and CCS. The design of the optimal energy consumption path results from a comparison at any point in time between the respective marginal costs of these various options: conventional energy production, CCS or clean renewable energy. The design also depends on the energy demand and the carbon stabilization target.

In this setting, we show the following results. It is never optimal to deploy CCS before the economy is constrained by the carbon stabilization objective. When CCS is used, learning may induce non-monotonic evolutions of the energy price, which stands in contrast with the usual message coming from Hotelling models with a climate constraint and in which the economy simultaneously faces the depletion of fossil fuels and the rise of a pollution stock. When the solar cost is low enough, an unusual sequence of energy consumption can occur in which solar energy consumption is interrupted for some time and replaced by clean coal exploitation. These findings remain valid if, instead of the complete depletion of the non-renewable resource implied by our constant marginal costs assumption, an economic depletion process wherein the amount of the resource ultimately extracted has to be endogenously determined is considered. More interestingly, our results are also robust to an explicit account of the limited carbon storage capacity.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces the optimal program of the social planner and describes the first-order conditions. Section 4 characterizes the optimal paths depending on the severity of the climate constraint and the relative cost of renewable energy. We discuss the results in Section 5. Section 6 explores the robustness of our findings against alternative assumptions. Section 7 concludes the paper.

2 The model

The economy produces energy services from two primary sources: a potentially carbon-emitting, non-renewable resource (coal) and a clean renewable resource (solar). We denote by q the energy services production rate. Consuming q generates an instantaneous gross surplus $u(q)$. The function $u(\cdot)$ is twice continuously differentiable, strictly increasing, $u'(q) > 0$, strictly concave, $u''(q) < 0$, and satisfies the first Inada condition: $\lim_{q\downarrow 0} u'(q) = +\infty$. We denote by $p(q)$ the marginal gross surplus function $u'(q)$ and by $q(p)$ its inverse (i.e., the energy demand function).

Polluting non-renewable resource

Let $X(t)$ be the available stock of coal at time t, measured in energy services equivalent units, X^0 be the initial endowment, with $X(0) \equiv X^0 > 0$, and $x(t)$ be the instantaneous extraction rate, measured in the same units, so that $\dot{X}(t) = -x(t)$. The unit processing cost of coal, denoted by c_x, is assumed to be constant and hence equal to the marginal cost.[3] Burning and consuming coal generates carbon emissions that are proportional to its use. Let ζ be the unitary pollutant content of coal so that, lacking any abatement policy, the pollution flow released into the atmosphere amounts to $\zeta x(t)$. However, the effective flow of carbon emissions might be lowered thanks to the CCS option.

Clean versus dirty coal energy production

Instead of expressing the CCS process in terms of some captured portion of the potential emission flow, we proceed formally by considering two types of techniques for producing final energy services from coal. The first route is a conventional technique that involves releasing carbon emissions into the atmosphere – we call this technique 'dirty' coal. The second option involves the capture and sequestration of carbon emissions – we call this technique 'clean' coal. Thus, 'clean' and 'dirty' do not refer to different natural qualities of coal, which are assumed to be homogenous for our purpose here, but to different types

[3] This unit cost includes all the costs that must be incurred to supply ready-for-use energy services to the end users (i.e., extraction, industrial processing and transportation costs).

of coal processing for the supply of energy services. Given x_c and x_d as their respective consumption rates, so that $x_c + x_d = x$, the dynamics of X is:

$$\dot{X}(t) = -[x_c(t) + x_d(t)], \quad X(t) \geq 0 \tag{1}$$

$$x_c(t) \geq 0 \quad \text{and} \quad x_d(t) \geq 0. \tag{2}$$

Let $S(t)$ be the cumulative clean coal consumption from time 0 up to time t. Assuming that $S(0) = 0$ for the sake of simplicity, we have:

$$S(t) = \int_0^t x_c(\tau)d\tau \quad \text{and} \quad \dot{S}(t) = x_c(t). \tag{3}$$

CCS cost and learning-by-doing

Producing energy from clean coal is more expensive than from dirty coal because an additional unit CCS cost, denoted by $c_s(S)$, must be incurred. Then, the unit cost of clean fossil energy amounts to $c_x + c_s(S)$. We assume that this additional cost depends on the cumulative clean coal consumption S and that the larger the S, the larger the cumulative experience in CCS operations and the lower the c_s. We thus consider a twice continuously differentiable function $c_s(\cdot)$, such that $c_s'(S) < 0$ and $c_s''(S) > 0$ for $S \in (0, X^0)$, with $\lim_{S \downarrow 0} c_s(S) \equiv \bar{c}_s < \infty$ and $c_s(X^0) \equiv \underline{c}_s > 0$. Note that as the objective of our study is to isolate the pure learning effect, we neglect any possible locking of the learning experience by limited reservoir capacity constraints.[4]

Atmospheric pollution stock and damages

Let $Z(t)$ be the level of the atmospheric carbon concentration at time t and Z^0 be the initial concentration, so that $Z(0) \equiv Z^0 \geq 0$. The pollution stock is assumed to be self-regenerating at a constant and positive proportional rate α. As only dirty coal is carbon-emitting, the dynamics of Z is given by:

$$\dot{Z}(t) = \zeta x_d(t) - \alpha Z(t). \tag{4}$$

Following Chakravorty *et al.* (2006), we assume that so long as Z does not overshoot some critical ceiling $\bar{Z}$, climatic damages will be almost negligible. However, for carbon concentrations larger than $\bar{Z}$, the damage will be immeasurably larger than the sum of the

[4]Due to the scarcity of most accessible storage sites, the average CCS cost should also increase with S possibly up to some upper bound $\bar{S}$ corresponding to the global capacity of the geological carbon sinks. This type of stock effect is examined in Lafforgue *et al.* (2008[a]). In the case of multiple carbon sinks, they state that the different reservoirs must be filled according to the increasing order of their respective costs. We show in Section 6 that all of the qualitative results of the paper remain valid if this scarcity effect is taken into account.

discounted gross surplus generated along any path triggering this overextension.[5] Therefore, the economy must constrain the carbon stock level to remain below this cap:

$$\bar{Z} - Z(t) \geq 0 \quad \text{and} \quad \bar{Z} - Z^0 > 0. \tag{5}$$

An implication of this constraint is that when the ceiling is reached, the maximum quantity of dirty coal that can be consumed is given by the natural self-regeneration capacity of the environment. If $\bar{x}_d$ denotes this quantity, then (4) implies that $\bar{x}_d = \alpha\bar{Z}/\zeta$.

Solar energy production

Solar energy is processed at a constant average cost c_y. We make y^n be the constant natural flow of this renewable energy and $y(t)$ be its consumption rate at time t (in energy services units), with the usual non-negativity constraint:

$$y(t) \geq 0. \tag{6}$$

When solar energy is the only energy source in use, the problem becomes static and the optimal consumption amounts to $\tilde{y}$ such that $u'(\tilde{y}) = c_y$. Assume $\tilde{y} \leq y^n$ so that y^n is large enough to meet the energy demand at the marginal cost c_y. This implies that no rent has ever needed to be charged for the efficient exploitation of this resource. Last, we assume that c_y is larger than c_x to justify the use of coal.

Relaxing the ceiling constraint can be achieved by using either clean coal or solar energy, and the relative competitiveness of these two options depends on their respective costs. That is why we will distinguish between the cases of 'high' and 'low' solar energy costs, with what we mean by 'high' and 'low' being specified in Section 4.

Discounting and welfare

The social discount rate ρ is assumed to be positive and constant. Social welfare is defined as the discounted sum of the net surplus, provided that the cap constraint on the pollution stock is satisfied.

3 The social planner program

At the end user stage, when all of the costs have been covered, the energy services delivered from any source (clean coal, dirty coal or solar) are perfect substitutes, so that $q = x_c + x_d +$

[5] Although a seemingly extreme assumption, it has been shown (Amigues *et al.*, 2011) that in a model where two types of damages are explicitly taken into account (manageable damages expressed as an increasing function of the pollution stock and catastrophic damages in the present form), the main qualitative properties of the optimal policy of the simpler pure ceiling model are preserved.

y. The social planner problem consists of determining the path $\{(x_c(t), x_d(t), y(t)), t \geq 0\}$ that maximizes social welfare:[6]

$$\max_{\{x_c, x_d, y\}} \int_0^\infty \{u(x_c + x_d + y) - c_x[x_c + x_d] - c_s(S)x_c - c_y y\} \, e^{-\rho t} dt \ ,$$

subject to the state equations (1), (3) and (4), and to the inequality constraints (2), (5) and (6). Let λ_S, λ_X and $-\lambda_Z$ be the co-state variables of S, X and Z, respectively[7], and let ν's be the Lagrange multipliers associated with the inequality constraints on the state variables and γ's be those corresponding to the inequality constraints on the control variables. Therefore, the current valued Hamiltonian $\mathcal{H}$ and Lagrangian $\mathcal{L}$ are:

$$\begin{aligned} \mathcal{H} &= u(x_c + x_d + y) - c_x[x_c + x_d] - c_s(S)x_c - c_y y - \lambda_X[x_c + x_d] - \lambda_Z[\zeta x_d - \alpha Z] + \lambda_S x_c \\ \mathcal{L} &= \mathcal{H} + \nu_X X + \nu_Z[\bar{Z} - Z] + \gamma_c x_c + \gamma_d x_d + \gamma_y y \ . \end{aligned}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial x_c} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + c_s(S) - \lambda_S - \gamma_c \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial x_d} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + \zeta\lambda_Z - \gamma_d \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_y - \gamma_y \tag{9}$$

$$\dot{\lambda}_S = \rho\lambda_S - \frac{\partial \mathcal{L}}{\partial S} \quad \Rightarrow \quad \dot{\lambda}_S = \rho\lambda_S + c_s'(S)x_c \tag{10}$$

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{\partial X} \quad \Rightarrow \quad \dot{\lambda}_X = \rho\lambda_X - \nu_X \tag{11}$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \quad \Rightarrow \quad \dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \ , \tag{12}$$

combined to the usual complementary slackness conditions. The transversality condition is given by:

$$\lim_{t \uparrow \infty} e^{-\rho t} \left[\lambda_S(t)S(t) + \lambda_X(t)X(t) + \lambda_Z(t)Z(t)\right] = 0. \tag{13}$$

With a constant marginal extraction cost, the mining rent λ_X grows at the social rate of discount as long as the stock of coal is not exhausted.[8] Thus, letting $\bar{t}_X$ be the time at which exhaustion occurs, we have: $\lambda_X(t) = \lambda_{X0} e^{\rho t}$, $t \leq \bar{t}_X$, with $\lambda_{X0} \equiv \lambda_X(0)$. From (13), we conclude that if coal has a positive initial value $\lambda_{X0} > 0$, then it must be exhausted along the optimal path.

[6] We drop the time index when this causes no confusion.

[7] Using $-\lambda_Z$ as the co-state variable of Z, we can directly interpret $\lambda_Z \geq 0$ as the unitary tax on the pollution emissions generated by dirty coal consumption.

[8] This pure Hotelling-like behavior has been frequently criticized for its lack of realism with respect to the observed trends of fossil resources prices (see Livernois, 2009, for a thorough examination of this problem). Section 6 extends the analysis to extraction costs that increase according to past cumulative extraction, thus making endogenous the amount of coal finally extracted. We show that even if they affect the mining rent dynamics, such cost structures do not alter the main qualitative results of the analysis.

We denote by $\underline{t}_Z$ and $\bar{t}_Z$ the time at which the ceiling constraint begins to bind and the time at which this constraint definitively ceases to be active, respectively. We therefore call the time interval $[\underline{t}_Z, \bar{t}_Z]$ the 'ceiling period'. Before this period, as ν_Z is nil, (12) implies: $\lambda_Z(t) = \lambda_{Z0} e^{(\rho+\alpha)t}$, $t \leq \underline{t}_Z$, with $\lambda_{Z0} \equiv \lambda_Z(0)$. After this period, λ_Z must be nil: $\lambda_Z(t) = 0$, $t \geq \bar{t}_Z$.

For the problem to be meaningful, we assume that the initial coal endowment X^0 is large enough (equivalently that $\bar{Z} - Z^0$ is small enough) to ensure that the ceiling constraint (5) binds along the optimal path. When constrained by the carbon cap, the economy cannot produce dirty coal above the rate $\bar{x}_d$ allowed by natural dilution. Because the reserves of coal are constantly declining, there must exist a finite time such that, even without any cap, the production of dirty coal falls below $\bar{x}_d$. At this time, either coal is exhausted, in which case $\bar{t}_Z = \bar{t}_X$, or not. If coal is not exhausted, then the continuous depletion process implies a constant decline of coal extraction, which means that the carbon cap constraint cannot continue binding until the exhaustion of X^0. Hence, we can conclude that $\bar{t}_Z \leq \bar{t}_X$. During an initial period $[0, \underline{t}_Z)$, the carbon stock Z increases until the carbon cap is reached. Then, it is maintained at its mandated level during the ceiling period $[\underline{t}_Z, \bar{t}_Z)$. Next, either coal is exhausted ($\bar{t}_Z = \bar{t}_X$) and the economy exploits only solar energy, or a last phase $[\bar{t}_Z, \bar{t}_X)$ of coal exploitation begins, during which the remaining coal reserves constantly decline to exhaustion and the carbon constraint is no longer relevant.

Let $\underline{t}_c$ and $\bar{t}_c$ be the dates at which clean coal production begins and ceases, respectively. (10) implies that $\lambda_S(t) = \lambda_{S0} e^{\rho t}$ for $t \leq \underline{t}_c$, with $\lambda_{S0} \equiv \lambda_S(0)$. Last, we denote by t_y the time at which solar energy production begins.

A common cost component of the two types of coal is $c_x + \lambda_X$, the processing cost augmented by the mining rent. We denote by p^F (F for tax-free and abatement cost-free) this common component:

$$p^F(t) = c_x + \lambda_{X0} e^{\rho t} \;\Rightarrow\; \dot{p}^F(t) = \rho \lambda_{X0} e^{\rho t} > 0. \tag{14}$$

Alternatively, p^F may be interpreted as the price of extracted coal before processing and thus before the possible release of carbon emissions. In addition to this common component, the full marginal cost of dirty coal, denoted by c_m^d, must also include the imputed social marginal cost of pollution: $c_m^d = p^F + \zeta \lambda_Z$. On the other hand, the full marginal cost of clean coal, denoted by c_m^c, must include the marginal abatement cost reduced by the marginal value of learning in clean coal processing (i.e., the learning rent): $c_m^c = p^F + c_s(S) - \lambda_S$.

4 Qualitative properties of the optimal paths

Independent of the relative competitiveness of clean coal and solar energy, the optimal paths share several common features. We describe these general properties in the next subsection. The following subsections study the implications of the relative competitiveness of the two carbon-free energy options. With respect to solar energy, clean coal production benefits from a comparative advantage thanks to learning-by-doing, but it requires the actual use of coal whose scarcity increases. The balance sheet of these pro and con arguments varies over time in complex ways, leading to the rich set of possible scenarios we are going to describe.

To focus on this relative competitiveness issue, we restrict the analysis to cases where clean coal has to be exploited along the optimal path. We first show that the production of carbon-free energy, either clean coal or solar, should not begin before the ceiling period. Moreover, clean coal and solar energy production cannot be simultaneously conducted. Next, we successively consider the cases of high and low solar energy costs. In both cases, the energy price may increase non-monotonically over time if the effect of learning-by-doing is strong enough. Given a high solar cost, the CCS option may be delayed until after the beginning of the ceiling period. Moreover, given a low solar cost, it may be optimal to consume solar energy during disconnected time intervals.

4.1 General properties of the optimal paths

4.1.1 Intertemporal tradeoffs induced by learning-by-doing

Learning from cumulative experience implies that the more clean coal is used, the lower its future cost will be. This suggests that λ_S, which captures this positive externality, must be at least temporarily positive. However, this positive externality is present at some time t only if clean coal continues to be produced after t. Once clean coal exploitation ceases, the externality disappears.

Proposition 1 *The co-state variable associated with cumulative clean coal production is positive until the end of clean coal exploitation and nil thereafter:* $\lambda_S(t) > 0$ *for* $t < \bar{t}_c$ *and* $\lambda_S(t) = 0$ *for* $t \geq \bar{t}_c$.

Proof: Solving the differential equation (10) using (13) yields:

$$\lambda_S(t) = -\int_t^{\infty} c_s'(S) x_c e^{-\rho(\tau - t)} d\tau = -\int_t^{\infty} \dot{c}_s e^{-\rho(\tau - t)} d\tau. \quad (15)$$

The learning rent at time t identifies with the cumulative discounted sum of all future marginal cost cuts allowed by a slight accumulation of experience, which is expressed as $dS = x_c$. Clearly, λ_S is strictly positive as long as $\int_t^\infty x_c d\tau > 0$ (i.e., as long as $t < \bar{t}_c$) and nil thereafter. ■

If λ_S measures the value of increased experience using the clean coal option (i.e., when $\underline{t}_c < t < \bar{t}_c$), it has a slightly different interpretation prior to the beginning of the active clean coal production period (i.e., when $t < \underline{t}_c$). In this case, (15) is equivalent to:

$$\lambda_S(t) = -e^{-\rho(\underline{t}_c - t)} \int_{\underline{t}_c}^{\bar{t}_c} c_s'(S) x_c e^{-\rho(\tau - \underline{t}_c)} d\tau \equiv e^{-\rho(\underline{t}_c - t)} K_c \ , \quad t \leq \underline{t}_c \ , \tag{16}$$

where K_c stands as the total cumulative marginal value of the experience acquisition plan that spans the entire duration of the clean coal exploitation period. Prior to the beginning of this plan, λ_S simply measures the discounted marginal value of experience evaluated from time t.[9]

Equation (7) states the optimal learning arbitrage equation. To interpret this relation, let $\pi_m^c \equiv p - p^F - c_s(S)$ be the subsidy-free unit profit from clean coal exploitation. Thus, (7) implies that $\pi_m^c = -\lambda_S < 0$. Benefiting from learning requires negative current returns until $\bar{t}_c$ when $\pi_m^c = 0$, that is, as long as clean coal is exploited. Taking (15) into account, (7) is equivalent to:

$$\pi_m^c(t) = \int_t^{\bar{t}_c} c'(S(\tau)) x_c(\tau) e^{-\rho(\tau - t)} d\tau.$$

On the one hand, increasing by dx_c the use of clean coal at time t induces an additional loss of $\pi_m^c(t) dx_c$. On the other hand, it allows for an additional cost reduction $c'(S(\tau)) x_c(\tau) e^{-\rho(\tau - t)} dx_c$ in present value at all later times $\tau > t$. Following the optimal learning path, the cumulative sum of these additional cost reductions must exactly balance the initial additional loss.

The fact that the economy decides to use clean coal despite its negative returns results from the competition between clean and dirty coals. Exploiting more dirty coal has a net opportunity cost amounting to $\zeta \lambda_Z$. To avoid paying this cost, the energy industry may prefer to bear the additional cost of clean coal energy generation $c_s(S)$. Lacking learning opportunities, (7) and (8) show that the industry is indifferent between the two coal exploitation regimes when $\zeta \lambda_Z = c_s(S)$. With learning possibilities, the same indifference arbitrage condition states that $\zeta \lambda_Z = c_s(S) - \lambda_S < c_s(S)$. Note that if both types of coal are used, then learning, by reducing the opportunity cost of the clean coal option below its additional cost, also reduces the opportunity cost of dirty coal pollution.

[9] Although the value of experience is clearly initially nil if clean coal is not exploited, this does not imply that its marginal value λ_S should be nil, as shown in (16).

Equation (7) may also be interpreted as a modified Hotelling rule. Assume that the energy industry faces an exogenously given and time-decreasing additional cost $c_s(t)$. Then, the standard Hotelling rule prescribes that the clean coal exploitation rate should equal the marginal net surplus in present value at all times during actual clean coal production. In the present model, the marginal net surplus identifies with the unit net surplus, thus resulting in the following form of the Hotelling rule:

$$\{p(t) - c_x - c_s(t)\} e^{-\rho t} = \mu_0, \tag{17}$$

where μ_0 measures the marginal present value of the clean coal exploitation plan. However, what captures the relation (7) is not a consequence of the declining trend of the future costs of clean coal production but the possibility opened by learning of manipulating this trend to increase the value of the clean coal exploitation plan. To clarify this point, consider a clean coal exploitation plan satisfying the Hotelling rule (17) but in a learning context where $c_s(t)$ would depend on previous clean coal production decisions. Assume that the economy tries to improve industry profits above this reference plan at some time t through an increase dx_c in clean coal use over a short time interval $[t, t + dt)$. At the end of the interval, such an alteration would generate an increase of the experience level amounting to $dS = dx_c dt$, though at the price of an accelerated depletion of fossil reserves $dX = -dx_c dt$. Next, during a second time interval $[t+dt, t+h)$, the economy decides to keep its experience gain dS; that is, it reverts to the reference accumulation plan $\dot{S}(t) = x_c(t)$. As $x_c(t)$ is not modified, $p(t) - c_x$ is not modified either. However, the experience gain allows for an increase in the unit net surplus during the time interval $[t + dt, t + h)$, which amounts to:

$$\int_{t+dt}^{t+h} [c_s(S) - c_s(S + dS)]\, x_c e^{-\rho(\tau-(t+dt))} d\tau \approx -dx_c dt \int_{t+dt}^{t+h} c_s'(S) x_c e^{-\rho(\tau-(t+dt))} d\tau \ .$$

During the final time interval $[t + h, t + h + dt)$, the economy reverts to the reference path by reducing the production level of clean coal by an amount dx_c, thus reducing the experience level to its reference level by the end of the interval. Note that this adjustment also restores the coal resource stock to its reference level after $t+h+dt$. The Hotelling rule (17) being satisfied, the change has no net welfare effect between the first and the third time interval. Thus, the overall balance of the adjustment amounts to the net surplus gain from a higher experience level during the second time interval, which is clearly positive, and contradicts the optimality of the Hotelling rule (17).

At time t, the additional net surplus generated by the small increase in clean coal use, $dx_c(t) > 0$, includes not only the variation of the current surplus, $[p(t)-c_x-c_s(S(t))]dx_c(t)$, but also the sum of the future clean cost reductions generated by this additional experience level, the value of which amounts to $\int_t^{\bar{t}_c} c_s'(S) x_c e^{-\rho(\tau-t)} d\tau dx_c(t)$ at time t. Equating over time the discounted value of this complete marginal net surplus at each date of clean coal exploitation, we have the following modified Hotelling rule:

$$\left\{p(t) - c_x - c_s(S(t)) - \int_t^{\bar{t}_c} c'(S)x_c e^{-\rho(\tau - t)} d\tau\right\} e^{-\rho t} = \lambda_{X0} \ . \tag{18}$$

Comparing (17) to (18), it appears that $\mu_0 < \lambda_{X0}$ for a given energy price path $\{p(t), t \geq 0\}$ and for any cost path $\{c_s(t), t \geq 0\}$, even for optimal ones.[10] Conversely, for a given marginal value of the clean coal exploitation plan, the energy price is lowered by an explicit account of the experience acquired from the production of clean coal energy. Benefiting from the learning opportunities requires a larger use of clean coal with respect to a passive behavior, taking as given the declining trend of production costs. This does increase the scarcity of the resource, which is measured by the coal scarcity rent λ_X, and induces a higher supply of clean coal, thus leading to a lower energy price.

4.1.2 Optimal timing of the clean options

We now describe the optimal timing of clean coal and solar energy. Proposition 2 shows that neither of these two carbon-free options should be used before the ceiling period and that the clean coal exploitation phase must be strictly closed before the end of this period. However, as we shall see later (cf. sections 4.2 and 4.3), it may or may not be optimal to immediately begin clean coal exploitation once the ceiling has been reached.

Proposition 2 *(i) Neither clean coal nor solar energy must be used before the beginning of the ceiling period:* $\underline{t}_c \geq \underline{t}_Z$ *and* $t_y \geq \underline{t}_Z$*. (ii) Clean coal exploitation must be closed before the end of this period:* $\bar{t}_c < \bar{t}_Z$*.*

Proof: See Appendix A.1.

From Proposition 2, we conclude that if it is optimal to use clean coal, its exploitation must occur during the ceiling period: $[\underline{t}_c, \bar{t}_c] \subseteq [\underline{t}_Z, \bar{t}_Z)$. The intuition behind this result is quite clear. If the economy is never constrained by a cap, the comparative advantage of dirty coal with respect to its clean alternatives prevents the use of clean coal or solar energy before the depletion of fossil reserves. Thus, the problem only has interest when the cap constraint eventually binds. The equations (8), (12) and (14) show that the energy price must rise before the beginning of the ceiling period. Independent of the learning process, the full marginal cost of the first unit of clean coal is $p^F + \bar{c}_s - \lambda_S$; thus, the clean coal option cannot be introduced before the energy price has at least reached this cost

[10] To understand why rule (17) is false even if $c_s(t) = c_s(S(t))$, note that the experience accumulation condition $S(t) = \int_0^t x_c d\tau$, which is equivalent to $\dot{S}(t) = x_c(t)$, must be taken into account – a condition that is neglected in (17).

level. However, if that occurs before the beginning of the ceiling period, then clean coal exploitation could satisfy all of the energy demand at this price and further in time at even lower prices, thanks to learning. Thus, dirty coal production should be abandoned so that the economy would never again accumulate carbon and hence would never be constrained by the cap, which presents a contradiction.

As noted before, dirty coal use is more profitable than clean coal use when the economy is no longer constrained by the carbon cap. Consequently, the clean coal option should be abandoned at most when the carbon cap is no longer binding. The fact that clean coal use should end before the end of the ceiling period results from a slightly more subtle argument. We shall see in Proposition 5 that λ_Z decreases during the ceiling phase, which improves the relative competitiveness of dirty coal. As λ_Z drops to zero when the ceiling period ends and as $c_s(S)$ remains above $\underline{c}_s$, dirty coal becomes more competitive than clean coal before the end of the cap period.

We turn now to the competition between clean coal and solar energy. The following proposition states that these two clean options are mutually exclusive.

Proposition 3 *Clean coal and solar energy may never be simultaneously exploited.*

Proof: See Appendix A.2.

This result is because the full marginal cost of clean coal cannot be constant over any non-degenerate time interval. Thus, it cannot be equal to the marginal cost of solar energy, except at some particular time. In other words, the two clean options encounter the Herfindahl logic of the least cost resource exploitation priority principle.[11] The non-constancy of the full marginal cost of clean coal can be easily explained. As stated at the end of Section 3, this cost c_m^c is defined as the sum of two components: the common cost of coal use, p^F, and the specific component of clean coal use, $c_s(S) - \lambda_S$. From (14) and (10) respectively, we find that $\dot{p}^F = \rho\lambda_X > 0$ and $d\left(c_s(S) - \lambda_S\right)/dt = c_s'(S)x_c - \dot{\lambda}_S = -\rho\lambda_S < 0$. Therefore, p^F and $c_s(S) - \lambda_S$ move in opposite directions, thus introducing the possibility of time fluctuations in c_m^c before $\bar{t}_c$. A time differentiation yields $\dot{c}_m^c = \rho(\lambda_X - \lambda_S)$. Subsequently, depending on the relative magnitudes of the resource scarcity rent and the learning rent, the marginal opportunity cost of clean coal use may either increase or decrease. The following subsections will more closely study the implications of this potentially non-monotonous behavior.

[11] See Herfindahl (1967). Note that the Herfindall principle holds so long as the consumption flows are not constrained, as pointed out by Amigues *et al.* (1998) and Holland (2003) but also so long as the pollution content of coal is homogenous (see Chakravorty *et al.*, 2008 for a thorough analysis of the heterogeneous polluting resources case).

4.1.3 Dynamics of prices and quantities

Prior to the beginning of the ceiling phase, the economy produces only dirty coal. As the energy price continuously increases during the initial phase, the exploitation of dirty coal should decline. Once the cap $\bar{Z}$ is reached, dirty coal use is constant and equal to $\bar{x}_d$. At the end of the ceiling period, either coal is exhausted or not and then dirty coal use declines once again until exhaustion.

When considering the dynamics of clean coal use, it should be expected that fluctuations of the clean coal marginal cost will translate into non-monotonous evolutions of its exploitation rate. To confirm this point, note that when the clean coal option is in use, its total marginal cost must be equal to the energy price: $p = c_m^c$. This implies that $\dot{p} = \dot{c}_m^c$ and thus that $\dot{x}_c = \dot{c}_m^c / u''(q)$. During any time phase of increasing cost ($\dot{c}_m^c > 0$), the energy price should increase and thus the clean coal energy production rate must decrease, whereas the reverse happens during any phase of declining cost ($\dot{c}_m^c < 0$). Hence, $\dot{x}_c$ may be either positive or negative. However, we can show that x_c, and thus also p, can follow only two types of paths.

Note that because λ_S tends toward 0 at the end of the clean coal exploitation period (cf. Proposition 1) and is necessarily continuous in this type of deterministic model, there must exist some terminal sub-interval $(\bar{t}_c - \Delta, \bar{t}_c)$, $0 < \Delta \leq \bar{t}_c - \underline{t}_c$, during which x_c decreases and the energy price p increases. Considering now the entire clean coal exploitation phase, Proposition 4 states that the sign of $\dot{x}_c$ may change at most only once.

Proposition 4 *During a phase of simultaneous exploitation of clean and dirty coal, the energy price either increases monotonically or first decreases and then increases. Similarly, clean coal production either decreases monotonically or first increases and then decreases.*

Proof: See Appendix A.3.

Equation (10) shows that the growth rate of the learning rent is lower than the discount rate (i.e., the growth rate of the coal scarcity rent). If $\lambda_X - \lambda_S > 0$ at the beginning of the clean coal exploitation phase, then $\dot{\lambda}_X/\lambda_X > \dot{\lambda}_S/\lambda_S$ implies that the rent gap increases and thus that the energy price increases while clean coal exploitation decreases all throughout the phase. Note that the operational margin $p - c_x - c_s(S)$, which is the gross margin before provision for the resource depletion and the experience accumulation, is positive and permanently increasing until the end of the clean coal production period. In the opposite situation, the rent gap is initially negative and declines in absolute value until its change in sign from negative to positive. The energy price follows a parallel motion, whereas the clean coal exploitation rate follows an inverted U-shaped pattern. In this case, the operational margin is initially negative before turning to positive when approaching the end of the clean coal exploitation phase.

4.1.4 Dynamics of shadow values

We now describe the dynamics of the shadow values of learning and pollution during the active clean coal production phase. Proposition 5 implies that the learning rent λ_S either constantly decreases or is inverted U-shaped, insofar as it first increases before decreasing at the end of the clean coal exploitation period. Regardless of the type of path that is followed by the energy price during the ceiling period, there is only one type of path for the shadow cost λ_Z of pollution during this period.

Proposition 5 *(i) When clean coal production exhibits an inverted U-shaped pattern and x_c reaches its maximum level at time $t'_c \in (\underline{t}_c, \bar{t}_c)$, then the learning rent decreases during the declining phase of x_c: $\dot{\lambda}_S(t) < 0$, $t \in (t'_c, \bar{t}_c)$. (ii) The shadow marginal cost of pollution decreases during the ceiling period: $\dot{\lambda}_Z(t) < 0$, $t \in (\underline{t}_Z, \bar{t}_Z)$.*

Proof: See Appendix A.4.

The final common characteristics shared by all possible optimal paths concerns their respective behaviors prior to the ceiling period. From Proposition 2, we know that this phase must occur before the beginning of the clean coal exploitation phase: $[0, \underline{t}_Z) \subseteq [0, \underline{t}_c)$. During this initial phase, the full marginal cost of the clean coal option amounts to $c_m^c = c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0})e^{\rho t}$, which may either increase or decrease depending on whether the initial shadow marginal cost of coal λ_{X0} is larger or smaller than the initial shadow marginal value λ_{S0} of the cumulative experience in abatement.

Let $\bar{p} \equiv u'(\bar{x}_d)$. From the previous discussion, it may be concluded that the energy price cannot be higher than $\bar{p}$ when the carbon cap constraint binds. If $c_y > \bar{p}$, then solar energy is not competitive during the ceiling phase. However, after the ceiling phase, the economy faces a conventional resource depletion problem in which solar energy stands as a backstop resource. It is well known that solar energy cannot be produced prior to coal exhaustion in such a situation. We call the 'high' solar cost case the configuration $c_y > \bar{p}$, wherein solar energy is only exploited after the complete depletion of coal. In the reverse case, a configuration we label the 'low' solar cost case $\bar{p} > c_y$, solar energy is introduced during the ceiling phase and is simultaneously used alongside coal until coal is completely depleted.

4.2 The high solar cost case: $c_y > \bar{p}$

Given the characteristics of the clean coal exploitation phase, three main types of optimal scenarios can be identified. These may be revealed by building a phase portrait in the (S, p) plane. Within the time interval $[\underline{t}_c, \bar{t}_c]$, $p = u'(\bar{x}_d + x_c)$ is a decreasing function of x_c. This implicitly defines the production of clean coal during the interval, which is denoted by $x_c^d(p)$, as a decreasing function of p such that $x_c^d(p) = 0$ for $p = \bar{p}$ and $x_c^d(p) > 0$ for $p < \bar{p}$. The time differentiation of (7) while using (10) yields: $p = c_x + c_s(S) + (\lambda_X - \lambda_S) = c_x + c_s(S) + \dot{p}/\rho$. The optimal trajectory $\{(S(t), p(t)), t \in [\underline{t}_c, \bar{t}_c)\}$ is thus a solution of the following autonomous differential system:

$$\dot{S}(t) = x_c^d(p(t)) \tag{19}$$

$$\dot{p}(t) = \rho\left[p(t) - c_x - c_s(S(t))\right]. \tag{20}$$

Equation (19) is only defined for $p \leq \bar{p}$, and the locus $\dot{S} = 0$ corresponds to the horizontal line $p = \bar{p}$ in the phase plane (S, p). On the other hand, the locus $\dot{p} = 0$ defines the curve $p_p(S) \equiv c_x + c_s(S)$, with $p_p(0) = c_x + \bar{c}_s$, $p_p'(S) = c_s'(S) < 0$ and $\lim_{S \uparrow X^0} p_p(S) = c_x + \underline{c}_s$. It can be proved that the system (19)-(20) has a unique solution when using the particular values $S(\underline{t}_c) = 0$ and $p(\underline{t}_c) \equiv p_0^c$. An algorithmic argument able to identify the optimal level of p_0^c as a function of the model fundamentals is presented in the companion technical appendix.

Two cases must be considered: (i) The strong environmental constraint case, $\bar{p} > c_x + \bar{c}_s$. In this case, $\bar{Z}$ is sufficiently low for $\bar{p} = u'(\alpha\bar{Z}/\zeta)$ to be higher than the upper bound over the unit exploitation cost of clean coal. (ii) The weak environmental constraint case, $\bar{p} < c_x + \bar{c}_s$. The phase portrait of S and p for these two cases is illustrated in Figure 1.

[Figure 1 here]

The strong environmental constraint case: $\bar{p} > c_x + \bar{c}_s$

In all scenarios, clean coal exploitation requires that $p_0^c < \bar{p}$. This implies that clean coal exploitation must begin precisely at the time the pollution cap is reached: $\underline{t}_c = \underline{t}_Z$. The top panel of Figure 1 depicts the possible trajectories of S and p in this case. Two types of optimal scenarios, labeled Scenarios 1 and 2, are possible depending on whether p_0^c is larger or smaller than $c_x + \bar{c}_s$. Figure 2 illustrates the optimal price and consumption paths in Scenario 1 where $c_x + \bar{c}_s < p_0^c$.[12] The optimal sequence of energy use is the following:

[12] The common reading of Figures 2-5 is the following. The top panel draws the optimal price path (bold line). The successive fragments of this trajectory are determined by: i) the full marginal cost of

i) only dirty coal until the carbon cap is reached; ii) both dirty and clean coal during the initial part of the ceiling period; iii) only dirty coal during the remainder of the ceiling period; iv) only dirty coal during the post-ceiling phase; and v) an infinite duration phase of solar energy use.

[Figure 2 here]

Observe that during the clean coal exploitation phase, its full marginal cost constantly increases; thus, the price should also increase, whereas the use of clean coal should decrease. Furthermore, the continuity in price at time $\underline{t}_Z$ implies that the use of dirty coal jumps down when the carbon cap is attained, whereas the clean coal exploitation rate is set at a strictly positive level, which compensates for this jump.

Figure 3 illustrates the optimal paths in Scenario 2 where $p_0^c < c_x + \bar{c}_s < \bar{p}$. As $p_0^c < \bar{p}$, the dirty coal exploitation rate collapses at time $\underline{t}_Z$, whereas the production of clean coal jumps up by an equivalent amount, as in Scenario 1. The sequence of energy use is also the same as in Scenario 1. However, in Scenario 2, the energy price first falls before increasing up to $\bar{p}$ by the end of the clean coal exploitation phase. Simultaneously, clean coal production first increases before dropping to zero. Such an unusual increase in coal consumption once bound by the pollution cap results from the high learning potential of the CCS technology. The experience acquisition process induces an initial decrease of c_m^c, the full marginal cost of the clean coal option, and thus results in an initial decrease of the energy price.

[Figure 3 here]

When facing a strong environmental constraint (i.e., a low carbon concentration stabilization objective), the economy should begin to produce clean coal as soon as possible, irrespective of learning possibilities. Under constant marginal extraction cost, 'as soon as possible' means immediately upon the event of the ceiling constraint beginning to bind. The clean coal option allows for the consumption of more fossil fuel than would be authorized by natural dilution only. However, lacking learning opportunities, the depletion process of fossil fuels implies a constant reduction in the use of clean coal until the economy relies upon the maximum possible rate of dirty coal consumption. Learning possibilities do not change this feature, although they may allow for an initial phase of expanding use

dirty coal during the pre-ceiling phase ($c_m^d = p^F + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$), during the ceiling phase ($c_m^d = \bar{p}$) and during the post-ceiling phase ($c_m^d = p^F$); ii) The full marginal cost of clean coal before and during its use ($c_m^c = p^F + \bar{c}_S - \lambda_{S0}e^{\rho t}$ and $c_m^c = p^F + c_s(S) - \lambda_S$, respectively). The bottom panel depicts the composition of the energy portfolio, and the bold line is the optimal trajectory of q.

of clean coal before the depletion process takes over the learning process and reverses the trend toward a constant decline of clean coal consumption.

Adopting a weaker carbon concentration objective slightly modifies the previous rationale. First, lacking learning opportunities, clean coal production is not competitive, and the economy should prefer to stick only to dirty coal production at its constrained level $\bar{x}_d$. Second, it is not necessarily optimal to begin producing clean coal as soon as the carbon cap is attained, even considering available learning possibilities. We examine this case now.

The weak environmental constraint case: $\bar{p} < c_x + \bar{c}_s$

The phase portrait for this case is illustrated in the lower panel of Figure 1. If $p_0^c < \bar{p}$, then the optimal clean coal exploitation path corresponds to Scenario 2. If $p_0^c = \bar{p}$, then it is characterized by Scenario 3. In this last situation, Figure 4 shows that the optimal sequence of phases is modified as follows. Clean coal production begins to be competitive only after the outset of the ceiling period. The phase of joint exploitation of both types of coal still occurs during the ceiling period, but it is now flanked by two phases of exclusively dirty coal use: $\underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$. Once again, this sequence results from the fluctuating behavior of the full marginal cost of clean coal, as illustrated in Figure 4. Contrary to the previous cases (c.f. Figure 2), the exploitation of clean coal now begins smoothly, $\lim_{t \downarrow \underline{t}_c} x_c(t) = 0$, and there is no longer an abrupt drop in dirty coal use at time $\underline{t}_c$.

[Figure 4 here]

Conclusion for the high solar cost case

In this case, solar energy is never competitive before the exhaustion of coal. The optimal timing and use scale of the clean coal option thus only depends on its relative competitiveness with respect to dirty coal. Proposition 2 states that clean coal is never competitive until the economy is constrained by the cap, irrespective of the importance of future learning opportunities. For clean coal to be competitive, its opportunity cost c_m^c must be lower than the upper bound $\bar{p}$. Lacking learning, $c_m^c = p^F + \bar{c}_s$ constantly increases, implying that if clean coal is not competitive at the beginning of the ceiling phase, then it will never be competitive. With learning possibilities, CCS is introduced at the onset of the ceiling phase if $c_m^c(\underline{t}_Z) < \bar{p}$, as in the no-learning case. However, the potential learning rate may be so high that, although CCS is not competitive when the carbon cap begins to bind (i.e., if $c_m^c(\underline{t}_Z) > \bar{p}$), it may become competitive, at least temporarily, during the ceiling period. As we discuss now, a low solar cost level implies that clean coal will have to compete not only with dirty coal but also with solar energy, which results in other optimal scenarios of energy consumption.

4.3 The low solar cost scenarios: $c_y < \bar{p}$

As $c_y < \bar{p}$, then, during the ceiling period, dirty coal is necessarily used alongside either solar energy or clean coal. The phase portrait of the dynamics of p and S in this case (not depicted) is very similar to the one for the high solar cost case, with $\bar{p}$ simply being replaced by c_y. Thus, the discussion may be organized by distinguishing the moderately cheap solar cost case, when $c_y > c_x + \bar{c}_s$, from the very cheap solar case, when $c_y < c_x + \bar{c}_s$. The optimal paths correspond to either Scenario 1 or 2 in the first case and to either Scenario 2 or 3 in the second case.

Moderately cheap solar energy: $c_y > c_x + \bar{c}_s$

As $p_0^c < c_y$ is a necessary condition for the use of clean coal in this cost configuration, the phase of joint exploitation of the two types of coal must begin when the ceiling is attained and before the introduction of the solar option. Two possibilities must be considered next.

First, if $c_x + \bar{c}_s < p_0^c < c_y$, then the optimal path corresponds to Scenario 1. The energy price increases all throughout the clean coal exploitation phase. As in the high solar cost case, dirty coal production drops at the beginning of the clean coal production phase – a fall that is compensated for by a parallel upward jump in the production of clean coal. This phase ends when the energy price is equal to c_y and solar energy becomes competitive. Then, according to Proposition 4, the clean coal option is definitively replaced by the solar option: $p(\bar{t}_c) = c_y$ and $\bar{t}_c = t_y$. The production of solar energy substitutes for the production of clean coal while remaining at the ceiling level up to the time at which $p^F = c_y$. Finally, dirty coal exploitation is closed when reserves are exhausted, and thereafter solar energy supplies all energy needs. Note that in this case, the end of the ceiling period coincides with the time at which coal is exhausted: $\bar{t}_Z = \bar{t}_X$.

Second, if $p_0^c < c_x + \bar{c}_s < c_y$, then the optimal path follows Scenario 2. Clean coal production begins at a strictly positive level when the carbon cap is reached, implying a drop in the dirty coal production rate, as in the preceding Scenario 1. However, now clean coal production rises during the initial time phase before declining to zero. The energy price first declines and then rises up to the cost level of solar energy. When this level is attained, clean coal exploitation is shut down and replaced by solar energy exploitation in conjunction with dirty coal exploitation. Thus, $\bar{t}_c = t_y$ as in Scenario 1, and the exploitation of dirty coal at the ceiling continues until the depletion of the coal reserves

$(\bar{t}_Z = \bar{t}_X)$. The economy moves last to a pure solar energy exploitation regime. As they are very similar to those in the high solar cost case, the associated price and consumption paths of Scenarios 1 and 2 for the moderately cheap solar cost case are not depicted here.

With a moderately low cost, solar energy becomes competitive with respect to dirty coal during the ceiling period, though not with respect to clean coal, at least at the beginning. However, the same rationale as in the high solar cost case applies here. Lacking learning opportunities, the coal depletion process will progressively reduce the comparative advantage of clean coal until it is replaced by solar energy. The same holds with learning-by-doing, the only difference being the possibility of an initial time phase at the ceiling that is characterized by an expanding use of clean coal before resource scarcity is able to overrun the declining clean coal cost trend induced by learning. If solar energy is very cheap, then learning becomes necessary for the clean coal option to remain competitive. However, it also becomes possible that solar energy has an initial comparative advantage with respect to clean coal at the beginning of the ceiling phase, thus delaying the use of clean coal.

Very cheap solar energy: $c_y < c_x + \bar{c}_s$

If $p_0^c < c_y$, then the economy follows the optimal path corresponding to Scenario 2. However, solar energy may be cheap enough to become competitive with respect to dirty coal prior to the production of clean coal. Figure 5 illustrates this situation. In such a scenario, solar energy is introduced at the beginning of the ceiling period, $t_y = \underline{t}_Z$, and clean coal exploitation starts only afterward, so that $t_y < \underline{t}_c < \bar{t}_c < \bar{t}_Z$. During the time interval $[\underline{t}_c, \bar{t}_c]$ of joint exploitation of both types of coal, the exploitation of solar energy is interrupted because the energy price is lower than the trigger price c_y. At $t = \bar{t}_c$, solar energy restores its competitiveness with respect to clean coal and replaces it in the energy mix. Subsequently, dirty coal and solar energy are simultaneously used again until time $t = \bar{t}_Z$ when $p^F(t) = c_y$ and the coal stock is exhausted. Observe that clean coal use does not start smoothly at $\underline{t}_c$, as in the high solar cost case. Because solar energy is used before $\underline{t}_c$ and also after $\bar{t}_c$, clean coal use is abruptly initiated at the level of solar energy production at the beginning of the clean coal exploitation phase and is reduced from this level to zero by the end of the phase.

[Figure 5 here]

Conclusion for the low solar cost case

The energy price can never be higher than c_y, the marginal cost of solar energy. If the marginal cost of clean coal is lower than c_y at the time the ceiling is reached, then the CCS option dominates the solar option, at least initially. As learning is unable to eternally counteract the continuous rise of the full marginal cost of clean coal, its exploitation ends once this cost becomes higher than c_y. In this scenario, learning ability only opens a brief window for clean coal generation before the introduction of solar energy during the ceiling phase. If the marginal cost of clean coal is higher than c_y at the time the ceiling is reached, then clean coal is irrelevant with respect to the cheaper solar alternative. However, the continuous decline in the opportunity cost of the clean coal option allows for its eventual use at the expense of the solar option. In such scenarios, solar energy is initially used at the beginning of the ceiling phase in combination with dirty coal while waiting for the clean coal option to become competitive. It appears once again when the competitive advantage of clean coal has been sufficiently reduced due to the increasing scarcity of fossil fuels.

5 Discussion

Our work reveals the potential complexity of the phasing of clean alternatives to polluting fossil fuels, even within a highly stylized framework. The usual time ordering through the least cost, first used principle may be blurred due to learning-by-doing effects. Models in the Hotelling vein usually predict a steady rise of the energy price and an ever-contracting energy supply. Learning abilities may affect the validity of these conclusions, insofar as the supply of clean energy may possibly increase, at least temporarily, which would lead to a decrease in the energy price.

To discuss more in depth the implications of learning-by-doing on CCS use, it is convenient to briefly describe the CCS deployment problem without learning effects. Assume a constant CCS unit cost $\bar{c}_s$. For clean coal to be competitive, the operational cost $c_x + \bar{c}_s$ must be lower than either $\bar{p}$, in the high solar cost case, or c_y, in the low solar cost case. If this competitiveness condition is satisfied, then clean coal is introduced when the carbon cap is reached, and its production stops before the end of the ceiling phase. The energy price constantly increases during the clean coal exploitation phase, whereas the clean coal production rate declines.

In the companion technical appendix, we draw a sensitivity analysis in the absence of learning opportunities. We show that: (i) A lower coal scarcity (i.e., a higher coal initial endowment X^0) decreases the resource rent, increases the shadow cost of carbon, drops the energy price path and thus the trigger price at which CCS becomes competitive, and favors an earlier start of CCS use at a higher rate and for a longer time period; (ii) A worse initial state of the environment (i.e., a lower $\bar{Z} - Z^0$) has the same qualitative effect as a lower resource scarcity level, except that it triggers an initial increase of the energy price path; and (iii) A more expensive CCS technology (i.e., a higher $\bar{c}_s$) decreases the resource rent, increases the shadow cost of carbon, initially increases the energy price path, and delays the introduction of CCS, implying that clean coal is produced at a lower rate over a shorter time period.

These findings are in line with the literature using numerical assessment models. Lacking learning opportunities, two factors favor the deployment of CCS: cheap access to fossil fuels and a stringent atmospheric carbon concentration target. However, note that an early introduction of CCS may be indifferently triggered under a low energy price regime, as in the case of an abundant non-renewable resource, or under a high energy price regime, resulting, for example, from a high shadow cost of carbon pollution. These opposite outcomes suggest the existence of a critical decreasing relationship between coal scarcity and the severity of the carbon cap, which results in the same level of the competitiveness trigger price justifying the use of the CCS option. They also imply that CCS is relevant only for sufficiently high initial endowments pairs (X^0, Z^0).

Another important remark concerns the influence of CCS on the timing of renewable energy use. Without CCS, a larger coal endowment or a worse initial state of the environment delays the transition toward solar energy in the high solar cost case and advances it in the low solar cost case. With CCS but without learning effects, the introduction of solar energy is always delayed. In the high solar cost configuration, this result is simply due to the slowdown in the extraction pace, which is induced by a more severe carbon pollution problem, and CCS plays no role in this. In the low solar cost case, a larger availability of coal and/or a more stringent carbon cap improves the competitiveness of CCS at the expense of solar energy and then delays the exploitation of the later.

The consequences of a higher CCS cost offer a good illustration of the influence of CCS technology characteristics on energy use and the price dynamics. The loss of CCS competitiveness resulting from a higher cost is distributed across both the crude energy input price, p^F, by lowering it, and the end-use energy price, p, by increasing it. It is worth noting that this impact materializes not only during the clean coal exploitation phase, as should be intuitively expected, but also before this phase (i.e., prior to the ceiling period). In terms of timing, CCS introduction is delayed and the clean coal exploitation period is shortened. In the low solar cost case, solar energy use also starts earlier. The first feature results from the lesser competitiveness of clean coal with respect to dirty coal, and the second results from its lesser competitiveness with respect to solar energy.

Describing the effects of larger initial coal endowments in the case of learning possibilities is a more intricate task. In Scenario 3, wherein clean coal is used after the beginning of the ceiling period, we show the following results (cf. companion technical appendix). A larger coal initial endowment, or an equivalently worse initial state of the environment, binds the ceiling constraint earlier. Larger coal reserves induce a pivotal effect on the energy price, initially decreasing the price path and only increasing it when the atmospheric carbon stock nears the cap. In contrast, a worse initial state of the environment increases the entire energy price trajectory. However, these effects are not specific to the present model and would appear even without the CCS option.

More interestingly, a larger coal stock or a worse initial state of the environment induces a forward shift of the entire clean coal exploitation phase. Not only does a more severe carbon problem have no reason to favor an early beginning of CCS in such scenarios, but it should also be a motive for delaying its introduction. The reason for this counter-intuitive behavior is the stopping condition in the learning process. Throughout the learning plan, the marginal gain of learning decreases, whereas the opportunity cost of burning more coal to learn more about CCS technology increases. Thus, the learning process is submitted to a stopping rule according to which learning has to be resumed when the marginal gain from an increased experience is matched by the marginal opportunity cost of the coal use it implies. However, this stopping rule only depends upon the characteristics of the learning curve and the value of the remaining coal reserves for future use after the end of the clean coal exploitation phase. Thus, a higher initial coal stock simply delays the time at which the stopping rule applies. As the length of the clean coal phase is fixed in Scenario 3, the result will be a translation farther in time of the same clean coal exploitation plan and thus of the same learning plan. In this scenario, the marginal value of learning in present value at time 0 is thus reduced by the larger coal reserves or the worse initial state of the environment.

Finally, let us turn to the policy implications of a CCS deployment with learning-by-doing abilities. The implementation of the optimal policy requires both a pollution tax on residual carbon emissions and a clean coal subsidy. The time profile of the pollution tax rate $\zeta\lambda_Z$ has the well-known, inverted-U profile obtained in most global warming studies, in accordance with the research pioneered by Ulph and Ulph (1994) and Tahvonen (1997). This time profile is also the standard profile of the models with ceiling constraints that were pioneered by Chakravorty *et al.* (2006) and the time profile of the mixed model of Amigues *et al.* (2011), in which both small and catastrophic instances of damage are taken into account. In a deterministic context, the unitary clean coal subsidy is abruptly set to the optimal level λ_S when clean coal exploitation begins. Subsequently, this subsidy may follow two possible evolutions. Either it constantly decreases until the end of the clean coal exploitation phase (Scenario 1), or it first increases before decreasing when the economy comes close to exhausting its profitable learning opportunities (Scenarios 2 and 3).

6 Extensions

The present analysis can be extended in several directions. Here, we focus on two of them. First, we introduce stock effects in coal extraction costs. Second, we describe the consequences of a limited carbon storage capacity.

One may wonder if coal exploitation may ultimately end not because of a physical shortage of fuels but due to more economic reasons. To accommodate this idea, assume that the unit extraction cost c_x is not constant but rather is a decreasing function of the remaining reserves.[13] Let $c_x(X)$ be this function, assumed to be twice continuously differentiable and such that $c_x'(X) < 0$, $c_x''(X) > 0$, $c_x(X^0) = \underline{c}_x > 0$ and $c_x(0^+) = +\infty$. The last assumption insures that the total coal endowment X^0 will never be entirely mined, so that some fraction of the coal reserves forever remain underground.

In this new framework, the dynamics of the mining rent is given by $\dot{\lambda}_X = \rho\lambda_X + c_x'(X)x$. The dynamics of $p^F = c_x(X) + \lambda_X$ is thus defined as $\dot{p}^F = -c'(X)x + \dot{\lambda}_X = \rho\lambda_X > 0$. Then, this common cost component increases over time, as in the original model (cf. equation (14)). To compute the final grade of the mined coal, denoted by $\tilde{X}$, remark first that at the optimal coal mining closure time $\bar{t}_X$, $\lambda_X(\bar{t}_X) = 0$. In the high solar cost case, $c_y > \bar{p}$ implies that the economy is no longer constrained by the carbon cap at $\bar{t}_X$. Therefore, price continuity requires that $c_y = p^F(\bar{t}_X) = c_x(X(\bar{t}_X))$, which determines in a unique way $\tilde{X} = X(\bar{t}_X)$. The same applies to the low solar cost case, in which the price remains equal to c_y until the end of coal exploitation. Note that the final grade $\tilde{X}$ depends only on the cost characteristics of coal extraction and solar energy production.

As p^F is still increasing through time and as the total amount of extracted coal is determined only by the features of the last pure solar use phase, the results of Propositions 1-5 remain valid under an economic depletion process induced by increasing extraction costs. The entire set of scenarios identified in Section 4 also remains valid. Finally, note that we could also consider in the same way an increasing cost function $c_x(X)$ to reflect a dominating learning effect in coal extraction activities. As with a decreasing function, because the dynamics of p^F is not modified, our previous findings still hold.

[13]The underlying assumption is that coal deposits have different extraction costs and are exploited by increasing order of their respective costs (cf. Herfindahl, 1967).

Now assume that CCS is constrained by a limited storage capacity $\bar{W}$. Let $W(t)$ be the cumulative carbon stock stored in available deposits at time t: $\dot{W}(t) = \zeta x_c(t)$ until the available storage sites have been filled, which formally occurs when the capacity constraint $\bar{W} - W(t) \geq 0$ binds. Denoting by λ_W the scarcity rent of storage sites, we have $\lambda_W > 0$ if the capacity constraint eventually binds. Thus, the first-order condition (7), with respect to clean coal production, is modified as follows:

$$u'(x_c + x_d + y) = c_x + \lambda_X + c_s(S) - \lambda_S + \zeta\lambda_W - \gamma_c. \tag{7'}$$

Conditions (8)-(12) are unchanged, whereas the transversality condition (13) becomes:

$$\lim_{t\uparrow\infty} e^{-\rho t}\left[\lambda_S(t)S(t) + \lambda_X(t)X(t) + \lambda_Z(t)Z(t) + \lambda_W(t)W(t)\right] = 0. \tag{13'}$$

Let ν_W denote the Lagrange multiplier associated with the storage capacity constraint. The motion of λ_W obeys:

$$\dot{\lambda}_W = \rho\lambda_W - \nu_W. \tag{24}$$

Let $\bar{t}_W$ be the time at which the capacity constraint binds. Then, clean coal exploitation must stop at this time: $\bar{t}_c = \bar{t}_W$. Since $\nu_W(t) = 0$ for $t \leq \bar{t}_c$, we conclude from (24) that $\lambda_W(t) = \lambda_{W0}e^{\rho t}$ for $t \leq \bar{t}_c$, with $\lambda_{W0} \equiv \lambda_W(0)$. During the active clean coal exploitation phase, (7') is then equivalent to:

$$p(t) = c_x + (\lambda_{X0} + \zeta\lambda_{W0})e^{\rho t} + c_s(S(t)) - \lambda_S(t), \quad t \in [\underline{t}_c, \bar{t}_c). \tag{25}$$

Using the CCS option now requires two non-renewable resources that are strict complements: coal and sequestration capacity. The full marginal opportunity cost of clean coal exploitation as given by the R.H.S. of (25) is still the sum of two components. The first component $c_x + \lambda_X + \zeta\lambda_W$ permanently increases over time, whereas the second component $c_s(S) - \lambda_S$ decreases, as in the original model.

The only point to check is that CCS is not deployed prior to the ceiling period. However, with limited sequestration capacities, the argument presented in Proposition 2 is even stronger. To see this point, assume that the economy decides to sequester some polluting emissions z at time $t < \underline{t}_Z$. The other option would be to not sequester z at time t, release it into the atmosphere, and counterbalance what remains of z in the atmosphere by an increase of the carbon capture at time $\underline{t}_Z$. What remains of z at time $\underline{t}_Z$ thus amounts to $z^{-\alpha(\underline{t}_Z - t)} < z$ thanks to natural dilution. Hence, the sequestration effort is lower, thus allowing savings on the sequestration capacity input. This shows that the qualitative properties of the optimal paths as described in the original model are preserved. Of course, limited storage capacities lower the competitiveness of CCS with respect to either dirty coal or solar energy production. However, provided that clean coal is actually exploited despite its lower profitability, the entire set of optimal scenarios of the original model remains valid in this context.

7 Conclusion

Carbon capture and storage appears as a promising method for reconciling an extended use of fossil fuels while mitigating carbon emissions. However, this option faces various pitfalls. First, CCS requires suitable sites able to store carbon dioxide with low leakage risks. Second, CO_2 recovery and compression from thermic energy production plants incurs significant additional costs either for new installations or for existing ones. In view of these constraints, the lack of a present strong policy action against climate change surely explains why the CCS technology remains in its infancy, having only a few plants in operation throughout the world. Nevertheless, promoters of CCS have long claimed that while incurring significant additional costs today, the technology could provide a potentially large amount of cost reduction due to learning-by-doing.

We have studied these issues through a stylized dynamic model. The deployment of CCS under learning possibilities should affect not only the timing of carbon accumulation or the phasing of other carbon-free energy generation techniques but also the trend of energy prices. We have shown that with sufficient learning abilities, CCS use may induce a non-monotonous evolution of the energy price. This opens the possibility of both very complex timing scenarios for the exploitation of CCS and the transition from fossil fuels toward renewable energy sources. We first described these possible scenarios in a simplified context, not taking into account the scarcity of storage sites or the economic conditions determining the final depletion of fossil fuels. We next showed that our results still hold when these features are explicitly considered.

Our work can be extended in several directions. In the context of the induced technical change debate, CCS deployment requires specific subsidies to efficiently monitor the learning-by-doing process of this technology. The same applies to other carbon-free technical options, such as solar energy or biomass production, suggesting that directed technical change monitoring different potential climate change mitigation options should be worth specific study. Such a framework appears also interesting for examining the issue of technological spillovers between CCS and efficiency enhancing energy generation techniques, a point frequently emphasized in the engineering literature. Our modeling setting also neglects the proper investment constraints on CCS development. Our findings suggest that CCS should be used as a sort of interim option allowing for the temporary reduction of the mitigation constraint on carbon emissions while waiting for other carbon-free energy sources to become predominant in the energy mix.

Finally, the implementation of efficient learning trajectories typically requires a combination of policy instruments which evolve over time in possibly complex views. This is particularly striking in the case of CCS. The potentially adverse effects of the subsidization of green energy have recently attracted a great deal of attention under the heading of the 'green paradox' dilemma. The possibility of such effects in the context of CCS development remains an open issue, which requires more specific study.

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Appendix

A.1 Proof of Proposition 2

From an integrating by parts of equation (15), we get the following alternative expression of λ_S which will be useful to prove Propositions 2 and 3:

$$\lambda_S(t) = \left[c_s(S(t)) - c_s(S(\bar{t}_c))e^{-\rho(\bar{t}_c - t)}\right] - \rho \int_t^{\bar{t}_c} c_s(S(\tau))e^{-\rho(\tau - t)}d\tau \ . \tag{A.1}$$

The claim $\underline{t}_c \geq \underline{t}_Z$

Assume that clean coal is exploited prior to the ceiling period: $\underline{t}_c < \underline{t}_Z$. Then: (i) either only clean coal is used during the time interval $(\underline{t}_c, \underline{t}_Z)$; (ii) or there exists a sub-interval $(t'_c, t'_Z) \subset (\underline{t}_c, \underline{t}_Z)$ during which both clean and dirty coals are simultaneously exploited; (iii) or there exists a sub-interval $(t''_c, t''_Z) \subset (\underline{t}_c, \underline{t}_Z)$ during which clean coal and solar energy are simultaneously exploited.

(i) From $Z(\underline{t}_c) < \bar{Z}$ and $\dot{Z}(t) = -\alpha Z(t) < 0$, for $t \in (\underline{t}_c, \underline{t}_Z)$, we conclude that $Z(\underline{t}_Z) < \bar{Z}$, which contradicts the ceiling attainment condition.

(ii) For $t \in (t'_c, t'_Z)$, equating the full marginal costs of both types of coal implies: $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_s(S) - \lambda_S(t)$. Replacing $c_s(S) - \lambda_S$ by its expression (A.1), this last equation yields:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_s(S(\bar{t}_c))e^{-\rho(\bar{t}_c - t)} + \rho \int_t^{\bar{t}_c} c_s(S(\tau))e^{-\rho(\tau - t)}d\tau \ .$$

Differentiating with respect to time and taking (A.1) into account again, we obtain:

$$\begin{aligned} 0 \ &< \ \zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t} \\ &= \ \rho c_s(S(\bar{t}_c))e^{-\rho(\bar{t}_c - t)} - \rho c_s(S(t)) + \rho^2 \int_t^{\bar{t}_c} c_s(S(\tau))e^{-\rho(\tau - t)}d\tau \ = \ -\rho\lambda_S(t) \ < \ 0, \end{aligned}$$

which is a contradiction.

(iii) We show in Proposition 3 that clean coal and solar energy may never be simultaneously exploited.

The claim $t_y \geq \underline{t}_Z$

Assume that solar energy is used prior to the ceiling period: $t_y < \underline{t}_Z$. Then, over the time interval $(t_y, \underline{t}_Z)$, only solar energy must be used as $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} > c_y$ and, from the first claim, clean coal may not be exploited. Hence, Z decreases during the interval. This implies that $Z(\underline{t}_Z) < \bar{Z}$, which contradicts the ceiling attainment condition.

The claim $\bar{t}_c < \bar{t}_Z$

Assume that at time $t \geq \bar{t}_Z$, both types of coal are used. Equating their full marginal costs and taking into account that $\lambda_Z = 0$ for all $t \geq \bar{t}_Z$, we get: $p^F(t) = p^F(t) + c_s(S(t)) - \lambda_S(t)$. Using (A.1) and simplifying, it comes: $c_s(S(\bar{t}_c))e^{-\rho\bar{t}_c} = -\rho\int_t^{\bar{t}_c} c_s(S(\tau))e^{-\rho\tau}d\tau < 0$, which is a contradiction.

A.2 Proof of Proposition 3

Assume that clean coal and solar energy are simultaneously used during some time interval. Then, their full marginal costs must be equal: $c_y = c_x + \lambda_{X0}e^{\rho t} + c_s(S(t)) - \lambda_S(t)$. Replacing λ_S by its expression (A.1) results in:

$$c_y - c_x = \lambda_{X0}e^{\rho t} + c_s(S(\bar{t}_c))e^{-\rho(\bar{t}_c - t)} + \rho\int_t^{\bar{t}_c} c_s(S(\tau))e^{-\rho(\tau - t)}d\tau \ . \qquad \text{(A.2)}$$

From a differentiation with respect to time, we obtain:

$$0 = \rho\left\{\lambda_{X0}e^{\rho t} + c_s(S(\bar{t}_c))e^{-\rho(\bar{t}_c - t)} - c_s(S(t)) + \rho\int_t^{\bar{t}_c} c_s(S(\tau))e^{-\rho(\tau - t)}d\tau\right\} \ .$$

Using (A.2), it comes: $0 = \rho(c_y - c_x) - \rho c_s(S(t))$. Time differentiating again finally yields: $0 = -\rho c_s'(S(t))x_c(t) > 0$, which is a contradiction.

A.3 Proof of Proposition 4

Assume that $\dot{x}_c(t) = 0$ at some time $t \in [\underline{t}_c, \bar{t}_c)$ and define t_0 as the first time at which the equality holds over the interval $[\underline{t}_c, \bar{t}_c)$. Differentiating (7) with respect to time and using (10) results in: $u''(x_c(t) + \bar{x}_d)\dot{x}_c(t) = \rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)]$. Define $\phi(t) \equiv \lambda_{X0}e^{\rho t} - \lambda_S(t)$. The concavity of $u(\cdot)$ implies that:

$$\dot{x}_c(t) > / = / < 0 \ \Leftrightarrow \ \phi(t) < / = / > 0 \ .$$

Time differentiating ϕ and using (10) again implies: $\dot{\phi}(t) = \rho\phi(t) - c_s'(S)x_c(t)$. Integrating over $[t_0, t]$, with $t_0 < t \leq \bar{t}_c$, and taking into account that $\phi(t_0) = 0$, we obtain:

$$\phi(t) = -e^{\rho t}\int_{t_0}^{t} c_s'(S(\tau))x_c(\tau)e^{-\rho\tau}d\tau > 0, \quad \forall t \in (t_0, \bar{t}_c] \ .$$

We conclude that $\dot{x}_c(t) < 0$ for $t \in (t_0, \bar{t}_c]$. Hence, if the sign of $\dot{x}_c$ changes, then this change occurs only once and from positive to negative.

A.4 Proof of Proposition 5

The claim $\dot{\lambda}_S(t) < 0,\ t \in (t'_c, \bar{t}_c)$

Consider any pair of successive dates t_i and t_{i+1} such that $t'_c \leq t_i < t_{i+1} < \bar{t}_c$, at which $\dot{\lambda}_S$ alternates in sign and between which the sign of $\dot{\lambda}_S$ does not change. We show that $\lambda_S(t_i)$ is a local maximum while $\lambda_S(t_{i+1})$ is a local minimum. As $\lim_{t\uparrow\bar{t}_c} \lambda_S(t) = 0$, there must exist a third date t_{i+2} at which the sign of $\dot{\lambda}_S$ alternates again and reaches a local maximum, which contradicts the above statement from which $\lambda_S(t_{i+1})$ should be a maximum and $\lambda_S(t_{i+2})$ should be a minimum.

To show this statement, note that from (10), as $\dot{\lambda}_S(t_h) = 0$, $h = i, i+1$, then $\rho\lambda_S(t_h) = -c'_s(S(t_h))x_c(t_h)$. As $S(t_{i+1}) > S(t_i)$ and $c''_s > 0$, then $-c'_s(S(t_i)) > -c'_s(S(t_{i+1}))$, whereas since $\dot{x}_c(t) < 0$ for $t \in (t_i, t_{i+1})$, then $x_c(t_i) > x_c(t_{i+1})$. This implies that $\lambda_S(t_i) > \lambda_S(t_{i+1})$. Then, $\lambda_S(t_i)$ is a local maximum whereas $\lambda_S(t_{i+1})$ is a local minimum.

Let t_j and t_{j+1}, $t_j < t_{j+1} < \bar{t}_c$, be the two last successive dates at which λ_S reaches a maximum and drops to a minimum before $\bar{t}_c$, respectively. As $\lim_{t\uparrow\bar{t}_c} \lambda_S(t) = 0$, there must exist a third date $t_{j+2} \in (t_{j+1}, \bar{t}_c)$ at which λ_S attains a local maximum, so that $\lambda_S(t_{j+1}) < \lambda_S(t_{j+2})$, which is in contradiction with the above result.

The claim $\dot{\lambda}_Z(t) < 0,\ t \in (\underline{t}_Z, \bar{t}_Z)$

Let us review the various possibilities that can occur during the ceiling period.

(i) Both types of coal are exploited. As clean coal is exploited, then $p = p^F + c_s(S) - \lambda_S$. Differentiating with respect to time and using (10) implies:

$$\dot{p}(t) = \dot{p}^F(t) + c'_s(S(t))x_c(t) - \dot{\lambda}_S(t) = \dot{p}^F(t) - \rho\lambda_S(t)\ . \tag{A.3}$$

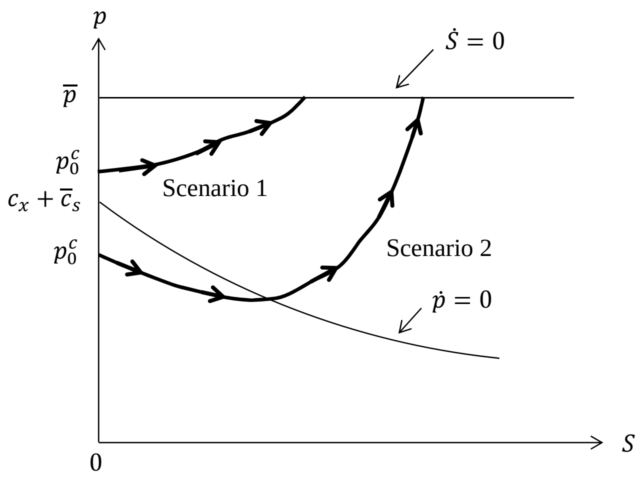
As dirty coal is simultaneously exploited, then we must also have $p = p^F + \zeta\lambda_Z$. Time differentiating and taking (A.3) into account, we get:

$$\dot{p}(t) = \dot{p}^F(t) + \zeta\dot{\lambda}_Z(t) \ = \ \dot{p}^F(t) - \rho\lambda_S(t) \ \Rightarrow\ \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) < 0\ .$$

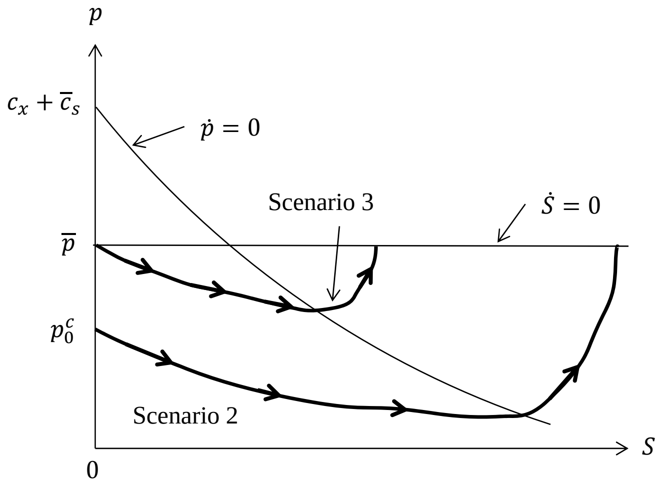
(ii) Both dirty coal and solar energy are exploited. Equating their full marginal costs implies: $p = c_y = p^F + \zeta\lambda_Z$. Time differentiating and using (14), we get:

$$0 = \dot{p}^F(t) + \zeta\dot{\lambda}_Z(t) = \rho\lambda_{X0}e^{\rho t} + \zeta\dot{\lambda}_Z(t) \ \Leftrightarrow\ \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_{X0}e^{\rho t} < 0\ .$$

(iii) Only dirty coal is exploited. In this case, we must have: $p = u'(\bar{x}_d) = p^F + \zeta\lambda_Z$. Hence, from (14), $\dot{\lambda}_Z(t) = -\frac{1}{\zeta}\dot{p}^F(t) = -\frac{\rho}{\zeta}\lambda_{X0}e^{\rho t} < 0$. Finally, note that although $\dot{\lambda}_Z$ is not necessarily continuous, λ_Z is always continuous.



The strong environmental constraint case: $c_x + \overline{c}_s < \overline{p}$



The weak environmental constraint case: $c_x + \overline{c}_s > \overline{p}$

Figure 1: Phase portrait in the (S, p) plane during the active clean coal exploitation phase

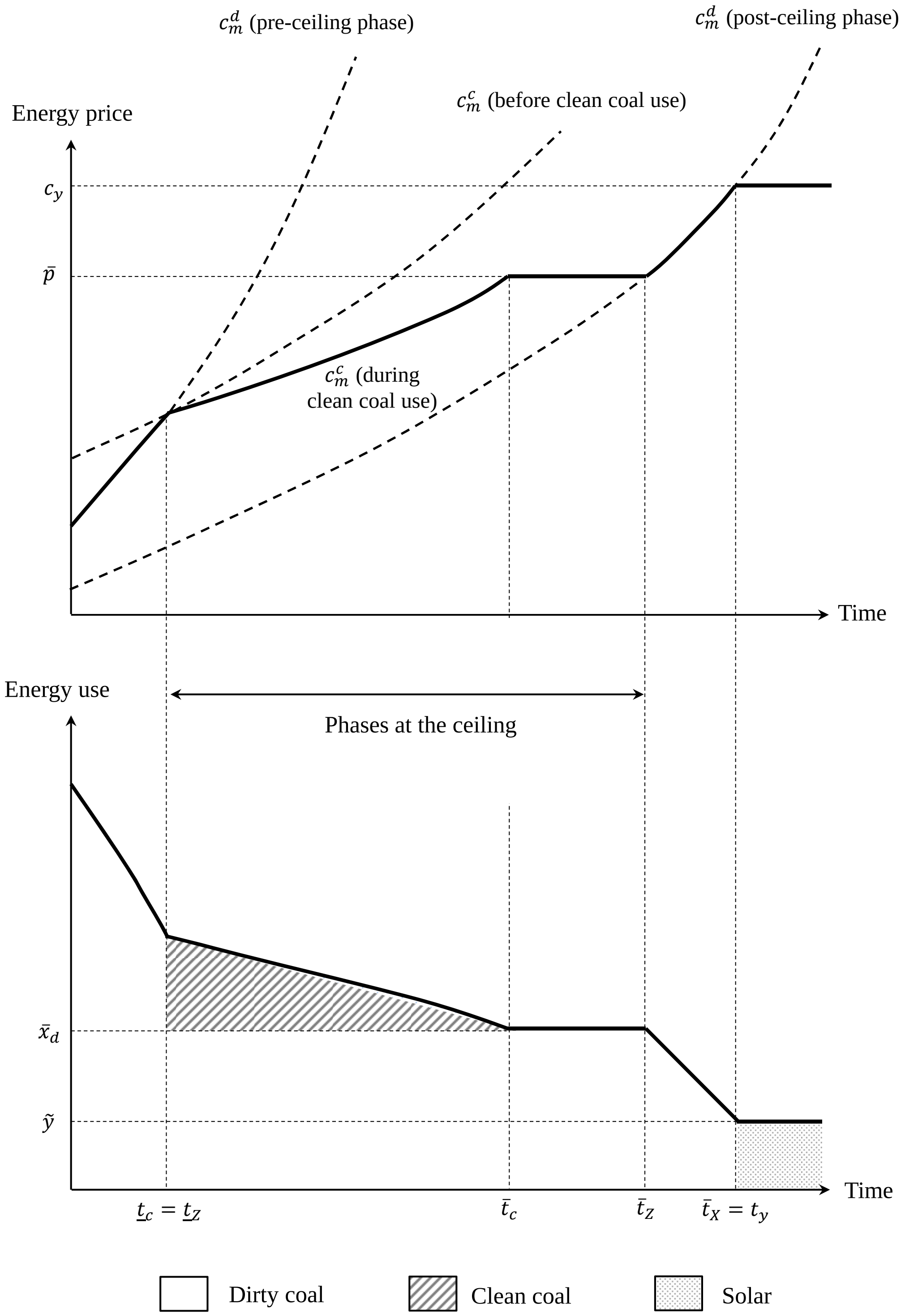


Figure 2: Optimal price (top panel) and quantity (lower panel) paths with high solar costs and a strong environmental constraint: Scenario 1

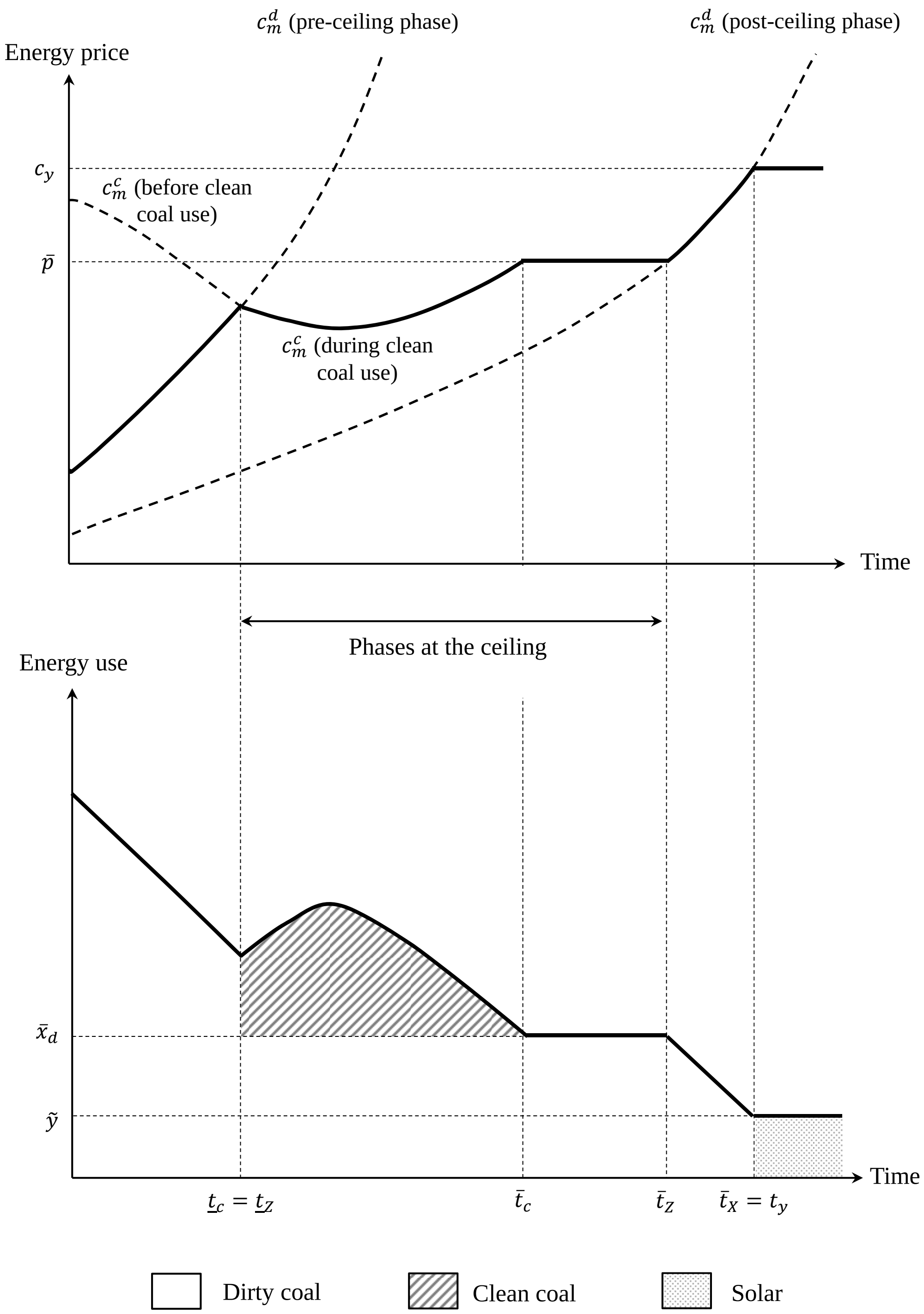


Figure 3: Optimal price (top panel) and quantity (lower panel) paths with high solar costs and a strong environmental constraint: Scenario 2

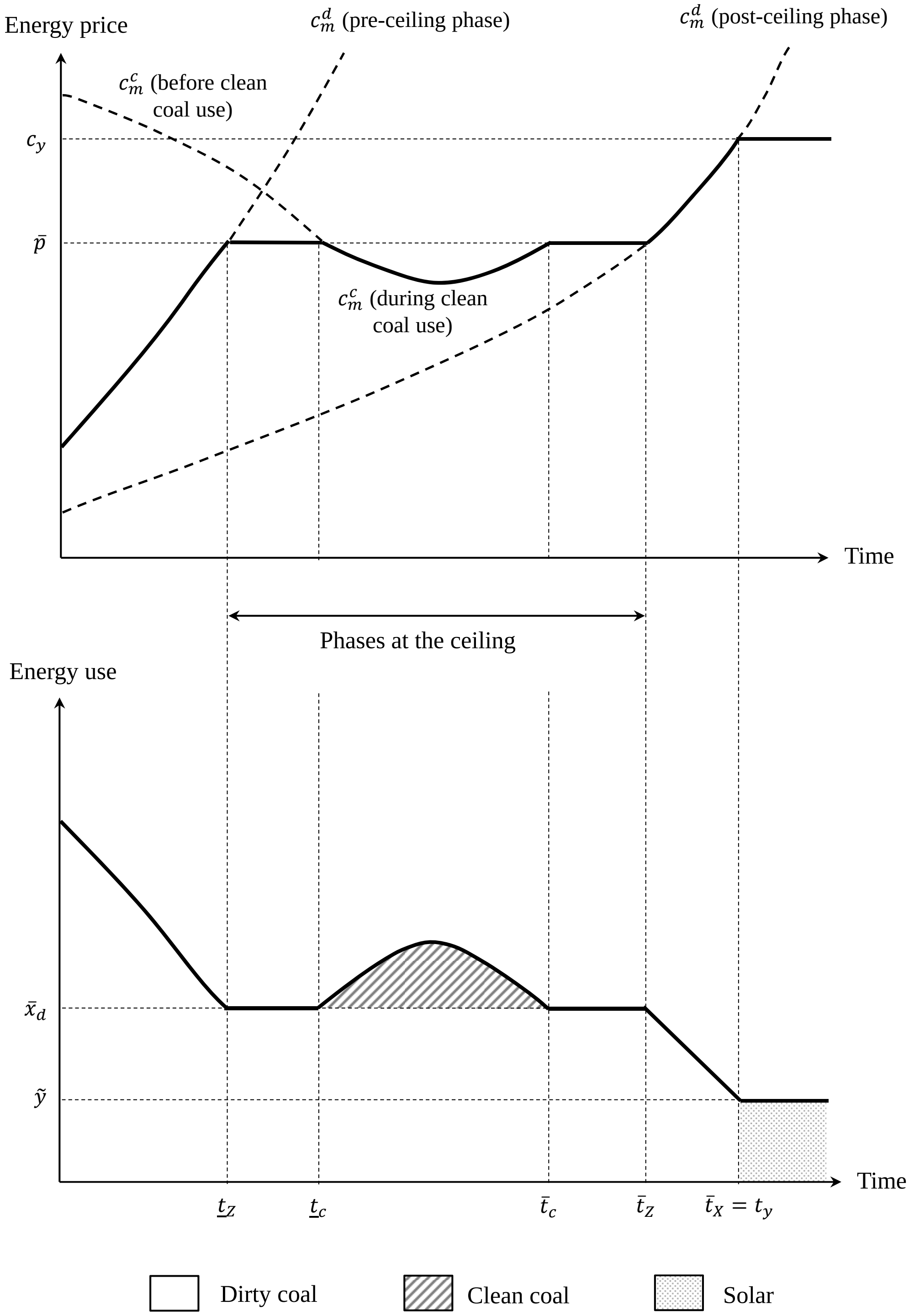


Figure 4: Optimal price (top panel) and quantity (lower panel) paths with high solar costs and a weak environmental constraint: Scenario 3

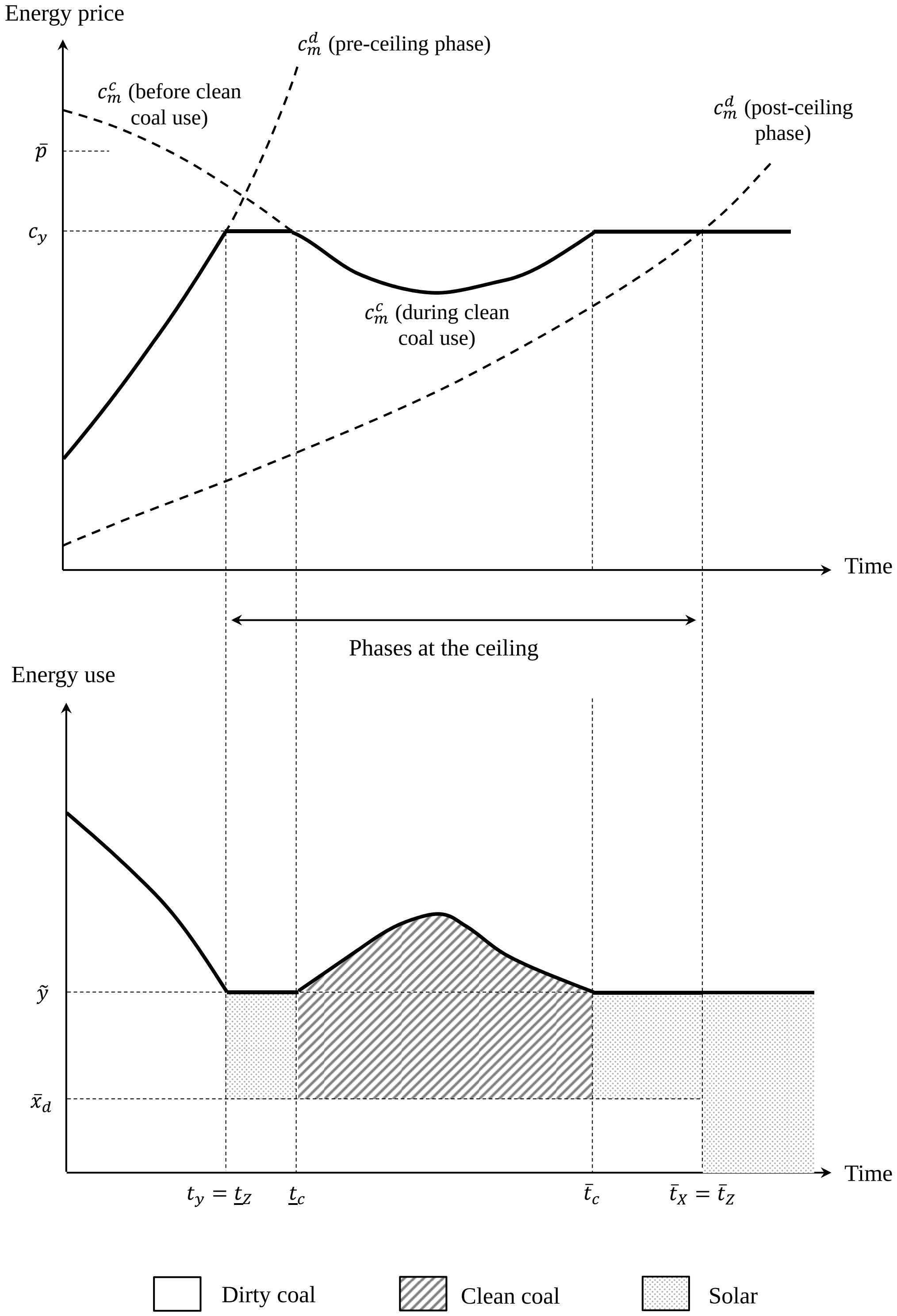


Figure 5: Optimal price (top panel) and quantity (lower panel) paths with a very cheap solar energy: Scenario 3