Intermittent renewable electricity generation with smartgrids

Prudence Dato[*], Tunç Durmaz[†] and Aude Pommeret[‡]

April 16, 2016

Abstract

The paper aims at analysing the efficient mix of investment in intermittent renewable energy, energy storage and central grid electricity provision. The novelty of our two-period model is that when there is no electricity production from the renewable capacity, electricity can either be provided through an energy storage device or purchased from the central grid. In addition, we study the consequences of demand side management, which the design of the model will allow to study in a simple way. First, we consider smart meters. They serve at observing the second period electricity price that is uncertain ex ante. Second, the household is allowed to sell electricity to the central grid. We derive the optimal microgrid capacity in terms of solar panel installation and energy storage devices depending on whether the electric grid is smart, or there are smart meters only or neither of the two. We further study the consequences for peak period electricity consumption and find conditions where having access to smartgrids, or smart meters only, generate adverse outcomes by causing more purchases from the grid, and therefore, more electricity provision from power companies. Moreover, we derive the conditions that favor the installation of smart meters. In particular, we show that for a strictly positive installation cost, it is not worth installing a smart meter unless it allows taking advantage of sufficiently low electricity prices.

Keywords: *Renewables, Microgrid, Smartgrids, Energy Storage, Peak-shaving, Demand response*

1 Introduction

Fighting climate change implies a drastic reduction in polluting fossil fuel use. Such a significant change could only be achieved through an energy transition towards clean and

[*]IREGE, University of Savoie

[†]SEE, City University of Hong Kong

[‡]SEE, City University of Hong Kong and IREGE, University of Savoie.

renewable sources for electricity generation. This paper is about the economic analysis of the energy transition. It focuses in particular on the integration of renewable electricity generation facilities (e.g., solar and wind power), because grid purchases of electricity is expected to displace the use of fossil fuels in buildings, industry and transportation in the near future. In addition, decentralized electricity generation using renewables or microgrids could solve the outage problem arising in congested countries like the US, mainly because of deregulation.[1] They would also enable the developing countries to have a better access to clean energy.

The fact that many renewable sources of energy are inherently intermittent and unpredictable, however, makes their integration challenging. This suggests that one cannot ignore energy storage and smartgrids opportunities when studying microgrid penetration. Therefore, in this paper we also study the optimal renewable energy and microgrid penetration for a household (HH) who can have access to smart devices such as smart meters, batteries and so on.

The literature considering the penetration of renewables in the energy mix consists so far in two rather separate trends. On the one hand, macro-dynamic models à la Hotelling consider renewable energy as an abundant and steady flow available with certainty, they ignore variability and intermittency and focus on the cost issue (see for instance Hoel and Kverndokk, 1997 or Tahvonen, 1997). Another strand of literature studies the design of the electric mix (fossil fuels and renewables) when intermittency is taken into account (see Ambec and Crampes, 2012 and 2015) or when storage takes care of peak electricity (see Crampes and Moreaux, 2010) or of excess nuclear production during periods of low demand (Jackson, 1973). A recent reference survey on the economics of solar electricity (Baker et al., 2013) emphasizes the lack of economic analysis of a decentralized clean energy provision through renewable sources. We fill this gap by considering a two-period setting that accounts for intermittency and storage.

In addition, smart grids and electricity demand management has received a lot of attention recently in particular in the media (see The Economist, 2009 or The Telegraph, 2015a and 2015b). Without smartgrids, the lack of transparency on the distribution side of the system is particularly apparent to consumers. Most people do not know how much electricity they are using until they are presented with a bill. Nor do most people know what proportion of their power is generated by nuclear, coal, gas or some form of renewable energy, or what emissions were produced in the process. Moreover a smart grid will make it easier to co-ordinate the intermittent and dispersed sources of power, from rooftop solar panels or backyard wind-turbines, for example. Electricity at different periods are a priori gross complements in the consumer utility function. Demand-side management policies, such as the development of smart meters, will be modeled as policies aiming at increasing the substitutability between electricity at different period, i.e. at incentivizing agents to consume or store electricity when it is cheap to produce. In this paper we account for two levels of smart grids. The first one is smart meters without which nothing smart, as far as electricity use/generation is concerned, can happen. If

[1] Microgrids are groups of interconnected loads and distributed energy resources that act as single controllable entities and operate autonomously with respect to the grid.

consumers are equipped to receive information on spot prices they become more reactive to peak load pricing and can take better decisions (Borenstein and Holland, 2005, or Joskow and Tirole, 2007). The second level concerns the possibility to sell to the central grid. It requires additional infrastructure and may not become widespread as quickly as smart meters. However in many countries like the UK or Germany for example, it is already possible for the HH to sell excess electricity generation to the grid.

The aim of the paper is to analyse the efficient mix of investment in intermittent renewable, storage and central grid electricity provision. The originality of our two-period model is that electricity is produced when the renewable source of energy (solar) is available, and when it is not, electricity can either be provided through a storage device or by the central grid. In addition, we study the consequences of demand-side management, which the design of the model will allow us to study in a simple way. First, we consider smart meters. They serve at observing the second period electricity price that is uncertain ex ante. Second, we allow the HH to sell electricity to the central grid. Note that depending on electricity prices, the HH may buy from the central grid during the first period, store and sell back to the central during second period. We first derive the optimal microgrid capacity, in terms of solar panel installation and storage devices depending on the available smart grids. Then we study the consequences for grid electricity consumption and peak electricity of the penetration of micro grids and smart grids. For instance, we can find situations where having access to smartgrids will generate an adverse effect by causing more purchase from the grid. Finally we derive the conditions favoring the optimal adoption of smart meters. In particular, we show that for any strictly positive cost it is not worth installing a smart meter for any second period electricity price.

The remainder of this paper is structured as follows. The model is presented in section 2. We analyze the optimal microgrid and storage device under smart grids in section 3. We study the optimal decision for microgrid and storage when smart grids reduce to smart meters and when there is no smart meters in section 4. We analyse in section 5 the consequences for energy consumption and purchases of electricity from the grid of the different assumption made on the smart grids in the previous sections. In section 6 we discuss the relevance of smart meter adoption. Finally, we conclude in section 7.

2 The model

We assume a two-period economy. During the first period, a HH invests K_1 (e.g., solar panels) to generate renewable energy (RE) whose total usage cost for the two periods is rK_1. Once the RE investment is made, it serves to produce K_1 kilowatt-hour (kWh) of electricity in the first period. The RE generation during the second period, however, depends on the state of nature, which have two possible realizations. Let P_s denote the probability that there will be sun in the next period. Conversely, $P_n = 1 - P_s$ denotes the probability that the weather will be cloudy causing no solar power generation. Therefore, with probability P_s (or P_n), RE generation in the second period will be K_1 (or 0) kWh.

In the first period, energy can be stored and transferred to the second period. Storing energy is costly due to the loss of energy during the restoration process. Denoting the amount of energy stored in the first period by S_1, the available amount of energy that can be consumed in the second period will then be ϕS_1. Here, $\phi < 1$ is the round-trip efficiency parameter.

In addition to storing energy to deal with the intermittent energy generation from the renewable resource, we assume that the micro-grid is connected to a central grid. Consider the following probability tree diagram, which illustrates the state dependent cost of purchases from the electric grid.

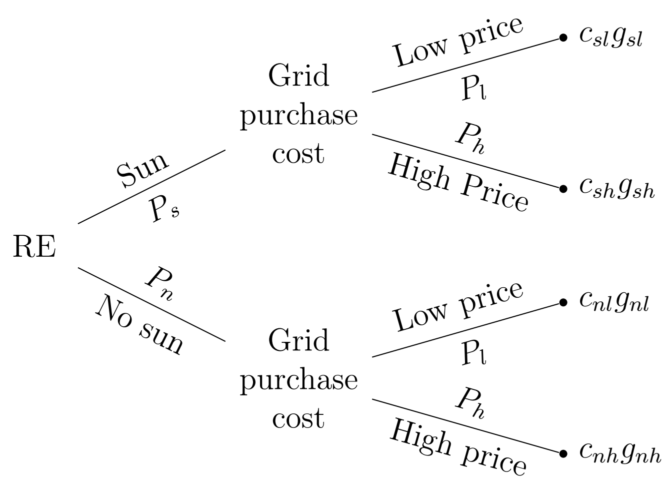

Figure 1: *Central grid purchase costs*

In the diagram, P_l denotes the probability of low price on the grid, while $P_h = 1 - P_l$ is the probability of a high price. In the first period, the unit cost of electricity on the grid is c_1. In the second period, however, the price on the grid will depend on the state. When there is sun and the price on the grid is low, the amount of expenditure made to purchase electricity will be $c_{sl}g_{sl}$, with g_{sl} the quantity of electricity and c_{sl} the unit cost. Similarly, when there is sun and the price on the grid is high, the total cost of purchasing electricity from the grid will be $c_{sh}g_sh$. The remaining entries on the diagram can be interpreted in a similar fashion.

At each period the HH has an instantaneous gross surplus over energy consumption. For $j = s, n$ and $i = l, h$ let $u(K_1 + g_1 - S_1)$ and $u(\mathbb{1}_s(j)K_1 + \phi S_1 + g_{ji})$, where

$$\mathbb{1}_s(j) = \begin{cases} 1, & \text{if } j = s \\ 0, & \text{otherwise,} \end{cases}$$

denote these surpluses in the first and second periods, respectively. It is assumed that $u' > 0$ and $u'' < 0$ where u' and u'' are the first- and second-order derivatives of the surplus function, respectively.

3 Microgrid connected to the main electrical grid with smart grids

In this section, we consider the optimal decisions in terms of solar panel and energy storage investments, and purchases from and sales to the main electric grid of a HH. To do this we consider that the HH is equipped smart meters that connects the home to the smart grid for two-way exchanges of information and energy. In light of Figure 1 the benevolent planner solves the following programme:

$$\max_{\{K_1,S_1,g_1,g_{ij}\}} u\left(K_1+g_1-S_1\right)-c_1g_1+\sum_j\sum_i P_jP_i\left[u\left(\mathbb{1}_s(j)K_1+\phi S_1+g_{ji}\right)-c_{ji}g_{ji}\right]-rK_1$$
$$\text{s.t.}\quad K_1\le\overline{K}, S_1\ge 0, K_1\ge 0 \text{ and } S_1\le\overline{S},$$

where $j=s,n$ and $i=l,h$. The Lagrangian function reads as

$$\mathcal{L}(\cdot)=u\left(K_1+g_1-S_1\right)-c_1g_1+\sum_j\sum_i P_jP_i\left[u\left(\mathbb{1}_s(j)K_1+\phi S_1+g_{ji}\right)-c_{ji}g_{ji}\right]-rK_1$$
$$+\nu_1(\overline{K}-K_1)+\nu_2S_1+\nu_3K_1+\nu_4(\overline{S}-S_1). \tag{1}$$

We denote the optimal HH decisions in the presence of the smartgrids by the 'g' superscript. The first order conditions with respective to K_1^g, S_1^g, g_1^g and g_{ji}^g then yield

$$u'\left(K_1^g+g_1^g-S_1^g\right)+P_s\sum_i P_iu'\left(K_1^g+\phi S_1^g+g_{si}^g\right)-r=\nu_1-\nu_3, \tag{2a}$$
$$\phi\sum_j\sum_i P_jP_iu'\left(\mathbb{1}_s(j)K_1^g+\phi S_1^g+g_{ji}^g\right)-u'\left(K_1^g+g_1^g-S_1^g\right)=\nu_4-\nu_2, \tag{2b}$$
$$u'\left(K_1^g+g_1^g-S_1^g\right)=c_1, \tag{2c}$$
$$u'\left(\mathbb{1}_s(j)K_1^g+\phi S_1^g+g_{ji}^g\right)=c_{ji}, \tag{2d}$$

respectively. Plugging the first order necessary conditions for g_1^g and g_{ji}^g in Eqs. (2a) and (2b) gives

$$c_1+P_s\sum_i P_ic_{si}-r=\nu_1-\nu_3,$$
$$\phi\sum_j\sum_i P_jP_ic_{ji}-c_1=\nu_4-\nu_2.$$

The FOCs drop to the primitives of the model, that is, the prices. Different cases emerge depending on the relative cost of the solar panel installation to the cost of purchasing electricity from the central grid on the one hand, and the relative cost of storage (in terms of loss during the restoration process) to the price on the grid on the other hand.

Here, we focus on the case where solar panels and storage are relatively cheap.[2] Thus,

$$
\begin{aligned}
& c_1 + P_s \sum_i P_i c_{si} - r > 0, \\
& \phi \sum_j \sum_i P_j P_i c_{ji} - c_1 > 0.
\end{aligned} \tag{3}
$$

In such a case and consistent with the intuition, we have corner solutions since it is optimal to install solar panels and store energy as much as possible: $K_1^g = \overline{K}$ and $S_1^g = \overline{S}$. A similar analysis will then give

$$
\begin{aligned}
& g_1^g > 0 \quad \text{if} \quad c_1 < u'(\overline{K} - \overline{S}), \\
& g_1^g \leq 0 \quad \text{otherwise.}
\end{aligned}
$$

Furthermore, the way the grid will be used in the second period will depend on the following conditions:

$$
\begin{aligned}
& g_{ji}^g > 0 \quad \text{if} \quad c_{ji} < u'(\mathbb{1}_s(j)\overline{K} + \overline{S}), \\
& g_{ji}^g \leq 0 \quad \text{otherwise.}
\end{aligned}
$$

If electricity is relatively cheap (resp. expensive) at the period and the state that are considered, it will be purchase (resp. sold).

4 Microgrid connected to the main electrical grid but with no sale to the grid

In this section, we first consider the optimal decisions in terms of solar panel investment, energy storage and purchases from the grid of a HH equipped only with a smart meter. In this regard, the HH can observe the prices on the electric grid but cannot sell electricity to it. In the following subsection, we will study the case with no smart meters.

In the benevolent planner programme, there are two additional positivity constraints on purchases from the grid; that is, ($g_1^m \geq 0$ and $g_{ji}^m \geq 0$). The Lagrangian function reads as

$$
\begin{aligned}
\mathcal{L}(\cdot) = {} & u\left(K_1 + g_1 - S_1\right) - c_1 g_1 + \sum_j \sum_i P_j P_i \left[u\left(\mathbb{1}_s(j) K_1 + \phi S_1 + g_{ji}\right) - c_{ji} g_{ji}\right] - r K_1 \\
& + \nu_1(\overline{K} - K_1) + \nu_2 S_1 + \nu_3 K_1 + \nu_4(\overline{S} - S_1) + \nu_5 g_1 + \sum_j \sum_i \nu_{ij} g_{ij}.
\end{aligned} \tag{4}
$$

[2]We are convinced that this will be the case in a not-too-distant future. When solar panels and energy storage devices are sufficiently expensive such that they are not utilized, then our analysis can be deemed as less useful. Nevertheless, it is certainly possible to analyse the other cases and allow our study to be more exhaustive.

Let the 'm' superscript denote the optimal value for the decision variables for the case where the HHs only have access to smart meters. The FOCs with respect to K_1, S_1, g_1 and g_{ji} yield

$$u'\left(K_1^m + g_1^m - S_1^m\right) + P_s \sum_i P_i u'\left(K_1^m + \phi S_1^m + g_{si}^m\right) - r = \nu_1 - \nu_3, \tag{5a}$$

$$\phi \sum_j \sum_i P_j P_i u'\left(\mathbb{1}_s(j) K_1^m + \phi S_1^m + g_{ji}^m\right) - u'\left(K_1^m + g_1^m - S_1^m\right) = \nu_4 - \nu_2, \tag{5b}$$

$$u'\left(K_1^m + g_1^m - S_1^m\right) - c_1 = -\nu_5, \tag{5c}$$

$$u'\left(\mathbb{1}_s(j) K_1^m + \phi S_1^m + g_{ji}^m\right) - c_{ji} = -\nu_{ji}, \tag{5d}$$

respectively. Plugging the FOCs for g_1^m and g_{ji}^m in Eqs. (5a) and (5b) gives

$$c_1 + P_s \sum_i P_i c_{si} - r = \nu_1 - \nu_3 + \nu_5 + P_s \sum_i P_i \nu_{si}, \tag{6a}$$

$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \nu_4 - \nu_2 - \nu_5 + \phi \sum_j \sum_i P_j P_i \nu_{ji}. \tag{6b}$$

As in the previous section, we consider the case where Eq. (3) holds. That is,

$$c_1 + P_s \sum_i P_i c_{si} - r > 0,$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 > 0.$$

In light of these equations, several scenarios can emerge. As an example it is possible to face a scenario where it is optimal to use all the storage capacity and yet install no solar panels. One can also consider a case in which it is optimal to fully use the total capacity for solar panels but store no energy. It is also possible to think of a scenario in which the solar panel investment and energy storage decisions take interior values. Figure 2 illustrates various cases for investment and grid purchase decisions by considering different electricity prices on the grid in the first period. Without attempting to calibrate the model, the parameter values we use are $r = 0.05$; $\phi = 0.49$; $P_s = 2/3$; $P_l = 1/2$; $c_1 = 0.1$; $c_{sl} = 0.05$; $c_{sh} = 0.25$; $c_{nl} = 0.1$; $c_{nh} = 0.3$, $\max(\overline{K}) = 4 and \max(\overline{S}) = 4$.

When the price on the grid in the first period is sufficiently low, the figure shows that it will be optimal to store energy at the capacity by purchasing electricity only from the grid. In this case, there will be no investments for solar energy. When the storage capacity is sufficiently high, it is seen that there will be no purchases of electricity from the grid in the second period. It can be seen that for higher prices for the price on the grid in the first period, we start encountering new scenarios.

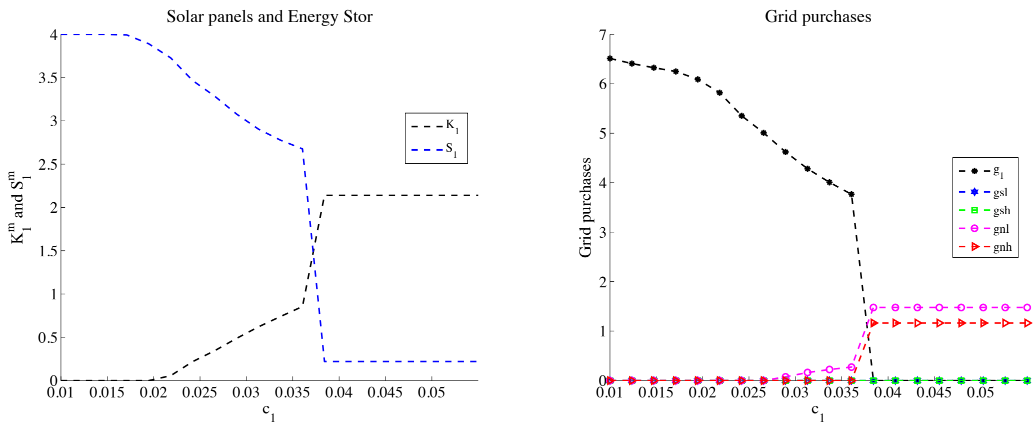

Figure 2: *A case for interior solution*

4.1 Interior solution

We first focus on the case with interior solutions, that is, $\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu_5 = 0$. This implies that $g^m_{sh} = g^m_{nh} = 0$.[3] From an analytical point of view, removing access to the smart grid is equivalent to replacing two constraints, namely $K_1 \leq \overline{K}$ $S_1 \leq \overline{S}$, by two constraints on the grid purchases in the second period.

The economic intuition as follows. If it is not possible to sell to the grid when there is sun and a high price on the electric grid, then there is no incentive to buy an infinite amount of solar panels. On the other hand, when there is no sun, the electricity price is high and it is not technically possible to sell to the grid, then there is no incentive to have an infinite amount of storage capacity. Also note that it must always be true that

$$g^m_{nl} > g^m_{nh} > g^m_{sh} \text{ and } g^m_{nl} > g^m_{sl} > g^m_{sh}.$$

The optimal levels of S_1 and K_1 can be calculated as follows. Using interior solutions, Eqs. ($6a$) and ($6b$) read as

$$c_1 + P_s \sum_i P_i c_{si} - r = P_s P_h \nu_{sh}, \tag{7a}$$

$$\sum_j \sum_i P_j P_i c_{ji} - \frac{c_1}{\phi} = P_h \sum_j P_j \nu_{jh}. \tag{7b}$$

By replacing Eq. ($5d$) with ν_{ji} in Eqs. (7a) and (7b), the optimal levels of S_1 and K_1 can be calculated from the following system of equations:

$$c_1 + P_s(P_l c_{sl} + P_h u'(K^m_1 + \phi S^m_1)) = r, \tag{8a}$$

$$P_n(P_l c_{nl} + P_h u'(\phi S^m_1)) + P_s(P_l c_{sl} + P_h u'(K^m_1 + \phi S^m_1)) = c_1/\phi. \tag{8b}$$

[3]This is due to the fact that assuming $g_{sh} = 0$ only will lead to infinitely many solutions for S^m_1 and K^m_1.

The interpretation is as follows. Eq. (8a) shows that the marginal cost of solar panel should equal, first, the avoided marginal cost of buying from the grid in the first period, second, avoided marginal cost of buying from the grid when there is sun and the price on the grid is low, and third, the marginal benefit of consuming energy generated by the HH, that is, $u'(K_1^m + \phi S_1^m)$, when there is sun and the price is high. On the other hand, Eq. (8b) indicates that the marginal cost of storage, c_1/ϕ, that is, the opportunity cost of forgone consumption in period 1 adjusted for the storage loss, should equal the expected avoided marginal cost of buying from the grid plus the expected marginal benefit of consuming energy generated by the HH.

The optimal levels for the number of solar panels and energy storage will be calculated from the following equations:

$$u'(\phi S_1^m) = \frac{c_1/\phi - P_n P_l c_{nl} + c_1 - r}{P_n P_h}, \tag{9a}$$

$$u'(K_1^m + \phi S_1^m) = \frac{r - c_1 - P_s P_l c_{sl}}{P_s P_h}. \tag{9b}$$

for $g_{jl}^m > 0$.

4.2 Solar power constrained

When the solar power is constrained by the available physical capacity, and therefore, $K_1^m = \overline{K}$, the following conditions for the multipliers,

$$\nu_2 = \nu_3 = \nu_4 = \nu_5 = 0, \text{ and } \nu_1 > 0,$$

allow us to write (cf. Eqs. (6a) and (6a))

$$c_1 + P_s \sum_i P_i c_{si} - r = \nu_1 + P_s \sum_i P_i \nu_{si} > 0,$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \phi \sum_j \sum_i P_j P_i \nu_{ji} > 0.$$

A necessary condition for an interior solution is $g_{sh}^m = 0$. This is because, on the margin, the benefit from storing energy at the capacity will be lower than its cost in the first period, which would have been otherwise had the HH sold to the grid.[4] One way to circumvent this problem is to pick a lower level of energy storage and avoid consuming from the grid in the state when there is sun and the price is high.

[4]The optimal solution dictates $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$ in the smart grid case so that some electricity can optimally be sold in both periods.

4.3 Storage constrained

When energy storage is constrained by the available capacity for the device, and thus, $S_1^m = \overline{S}$, we have the following conditions for the multipliers:

$$\nu_1 = \nu_2 = \nu_3 = \nu_5 = 0, \text{ and } \nu_4 > 0.$$

Eqs. (6a) and (6a) then allow us to write

$$c_1 + P_s \sum_i P_i c_{si} - r = P_s \sum_i P_i \nu_{si} > 0,$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \nu_4 + \phi \sum_j \sum_i P_j P_i \nu_{ji} > 0.$$

Similar to the previous subsection, a necessary condition for an interior solution is $g_{sh}^m = 0$. Otherwise, using a higher number of solar panels or consuming from the grid when there is sun and a high price on the electric grid will lead to a lower expected marginal return from solar power generation.

4.4 Solar power and storage constrained

When the installation of both the solar power and energy storage is constrained by the available physical capacity, and therefore, $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$, we have the following conditions for the multipliers:

$$\nu_2 = \nu_3 = \nu_5 = 0, \nu_1 > 0 \text{ and } \nu_4 > 0.$$

This leads us to

$$c_1 + P_s \sum_i P_i c_{si} - r = \nu_1 + P_s \sum_i P_i \nu_{si}, \tag{10a}$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \nu_4 + \phi \sum_j \sum_i P_j P_i \nu_{ji}. \tag{10b}$$

As a result, there is no restriction as to the use of the grid in the second period.

4.5 Microgrid problem in the absence of smart meters

In this section, we consider the optimal decisions of a HH who is not equipped with a smart meter device. Therefore, the HH can neither observe the second period central grid electricity price nor sell to the grid. Consider a price tariff such that c_1 and c_2 are the prices of electricity on the grid in the first and second period, respectively. The HH solves the same program as in Section 4.1 for special values of the parameters such that $P_i = 1$ and $c_{ji} = c_2$. Therefore, for an interior solution for K_1, S_1 and g_1 and assuming that

$$c_1 + P_s c_2 - r > 0 \text{ and}$$
$$\phi c_2 - c_1 > 0$$

holds, we must have $g^o_s = g^o_n = 0$. We denote the optimal decisions for the HHs who do not have access to any smart device by the'o' superscript. Accordingly, no electricity will be purchased from the grid in the second period. S^o_1 and K^o_1 can be calculated from the following system of equations:

$$c_1 + P_s u'(K^o_1 + \phi S^o_1) = r, \tag{11a}$$

$$P_s u'(K^o_1 + \phi S^o_1) + P_n u'(\phi S^o_1) = c_1/\phi. \tag{11b}$$

As this is a special case of Section 4.1, the interpretation of the equations will be similar to that of Eqs. (8a) and (8b). Therefore, we refer the reader to p.9.

The optimal values for solar panels and energy storage device can be calculated from Eqs. (11a) and (11b):

$$u'(\phi S^o_1) = \frac{c_1/\phi + c_1 - r}{P_n}, \tag{12a}$$

$$u'(K^o_1 + \phi S^o_1) = \frac{r - c_1}{P_s}. \tag{12b}$$

Next proposition summarizes the results obtained in this section.

Proposition 1.

- *If the HH is equipped with smart meters only then*
  1. *if neither solar panel nor storage device are capacity constrained, there exist interior solutions for solar panel investment, storage, electricity purchase in first period, and there is no electricity purchase in second period if the price is high.*
  2. *if only solar panel (resp. storage device) is capacity constrained, there exits interior solutions for storage (resp. solar panel investment) and electricity purchase in the first period, and there is no electricity purchase in the second period if the price is high and there is sun.*
- *If the HH is not equipped with a smart meter and there are no capacity constraints, then there exist interior solutions for solar panel investment, storage, electricity purchase in first periods and there is no electricity purchase during the second period.*

5 Electricity consumption and grid activity

In this section we discuss the implications of the smart grid and smart meters for electricity consumption and the grid activity. Following the same parametric conditions that satisfy Eq. (3), we first do a comparison between the case with a smart grid and the case with smart meters but without the smart grid. This is then followed by a comparison between the cases with smart meters in the absence of a smart grid and the case without smart meters.

5.1 Smart grid vs smart meter

5.1.1 Interior solution

Recall that the interior solution without the smart grid constitutes

$$\nu_1 = \nu_3 = \nu_3 = \nu_4 = \nu_5 = 0.$$

and the superscripts 'g' and 'm' denote the optimal decisions in the smart grid and the smart meter only cases, respectively.

From Eqs. (5c) and (2c) we have

$$g_1^m - g_1^g = (\overline{K} - K_1^m) - (\overline{S} - S_1^m).$$

Furthermore, in the second period, using Eqs. (5d) and (2d), we get

$$g_{ji}^m - g_{ji}^g \geq \mathbb{1}_s(j)(\overline{K} - K_1^m) + \phi(\overline{S} - S_1^m) \geq 0$$

with the first inequality from the left being strict at least for g_{sh}^m and g_{nh}^m.[5]

These two equations allow us to deduce that

$$\begin{aligned}(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) >& (\overline{K} - K_1^m) - (\overline{S} - S_1^m) \\ &+ \sum_j \sum_i P_j P_i \left[\mathbb{1}_s(j)(\overline{K} - K_1^m) + \phi(\overline{S} - S_1^m)\right]\end{aligned} \tag{13}$$

As K_1^m and S_1^m are optimal, the above inequality can be rewritten as:

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > (1 + P_s)(\overline{K} - K_1^m) - (1 - \phi)(\overline{S} - S_1^m) \tag{14}$$

A sufficient condition for buying less from the grid with smart grid is therefore[6]

$$\begin{gathered}(1 + P_s)(\overline{K} - K_1^m) \geq (1 - \phi)(\overline{S} - S_1^m) \\ (\text{or } (1 + P_s)\overline{K} - (1 - \phi)\overline{S} \geq (1 + P_s)K_1^m - (1 - \phi)S_1^m)\end{gathered} \tag{15}$$

Consider the two periods. When the additional electricity that is expected to be generated by the solar panels with smart grids exceeds that of the additional energy lost by the storage devices with smart grids (that is, $(1 - \phi)(\overline{S} - S_1^m)$), there will be less purchase from the grid in the smart grid case.

When the net amount of electricity generated in the smart grid case (that is, $(1 + P_s)\overline{K} - (1 - \phi)\overline{S}$) is higher than it is for the case with smart meters only, then the HH

[5]Otherwise, $g_{sh}^m - g_{sh}^g = 0$, which requires that $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$. From Eqs. (9a) and (9b), one can see that the chance for the two equalities to hold simultaneously (or even individually) is extremely small, and therefore, negligible.

[6]Considering that $\nu_5 > 0$ and $g_1^m = 0$, Eq. (14) and the sufficient condition given by Eq. (15) will still be valid.

will demand a lower amount of electricity, and therefore, purchase less electricity from the grid. On the other hand, if the net amount of electricity generated in the smart grid case is lower, that is, when Eq. (15) is not satisfied, the result is ambiguous. This is mainly related to the fact that in the smart grid case, it is always optimal to store at the capacity when Eq. (3) holds. Yet, when $\overline{K}$ is sufficiently small, the necessary amount of energy that will be stored will be obtained from the grid. Even if there will be a lower amount of grid purchases in the second period, the first period purchase of electricity can be sufficiently high to cause a higher expected amount of purchase from the grid in the smart grid case.

Figure 3 illustrates the differences between the grid purchases for cases with smart grid and with smart meter only. While the first graph from right demonstrates the total purchases from the grid, that is, $g_1^m - g_1^g + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$, the two figures from left demonstrate the grid purchases in the first and second periods, that is, $g_1^m - g_1^g$ and $\sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^*)$, respectively. We are only interested with the qualitative pattern. Therefore, we do not attempt to calibrate the model. The parameter values we use are $r = 0.05; \phi = 0.49; P_s = 2/3; P_l = 1/2; c_1 = 0.03; c_{sl} = 0.02; c_{sh} = 0.3; c_{nl} = .02; c_{nh} = 0.3, \min(\overline{K}) = 1.97, \max(\overline{K}) = 3.97, \min(\overline{S}) = 0.17, \max(\overline{S}) = 2.17$. ($K_1^m = 1.97$ and $S_1^m = .17$.)

In particular, if the accessible solar panel capacity is low (e.g., $\overline{K} = 1.97$) and the accessible energy storage capacity is rather large (e.g., $\overline{S} = 2.17$), having access to smart grid will generate an adverse effect by causing more purchase from the grid. Figure 3 indicates that the difference between grid purchases for the cases with and without smart grid is the highest when the solar and storage capacities are low and high, respectively. Also, higher level of solar panels and stored energy lead to lower amount of expected purchase from the grid.

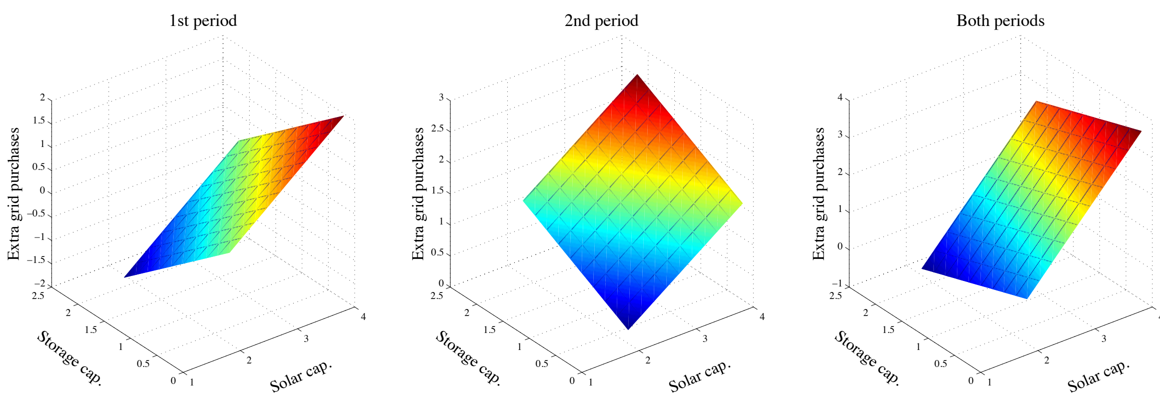

Figure 3: *Difference in purchases from the grid.*

5.1.2 Solar power constrained

When the solar power is constrained by the available physical capacity, and therefore, $K_1^m = \overline{K}$, we have the following conditions for the multipliers:

$$\nu_2 = \nu_3 = \nu_4 = \nu_5 = 0, \text{ and } \nu_1 > 0.$$

Looking at the first period, an interior solution for g_1 implies

$$u'(\bar{K} + g_1^m - S_1^m) = u'(\bar{K} + g_1^g - \bar{S}). \tag{16}$$

As the marginal utility is decreasing with consumption, that is, $u'' < 0$, $g_1^m < g_1^g$. A higher level of energy storage, accordingly, will lead to a higher level of grid purchase in the case with the smart grid in the first period:

$$g_1^m - g_1^g = -(\overline{S} - S_1^m) < 0.$$

In the second period, the expected difference between grid purchases in the smart meter and smart grid cases is

$$\sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) \geq \phi(\bar{S} - S_1^m) > 0. \tag{17}$$

This indicates that the expected purchase in the smart grid case will be higher in the second period.

Summing up the two inequalities, the difference between the expected total purchase of electricity in the smart meter and smart grid cases can be expressed as

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > -(1-\phi)(\overline{S} - S_1^m), \tag{18}$$

As the RHS of Eq. (18) is negative, the inequality given by Eq. (18) is no more a sufficient condition to buy less from the grid with smart grid. The intuition is as follows: as the parametric condition dictates that it is optimal to store at the maximum capacity, the grid purchases in the first period can be sufficiently high to cause a higher level of purchase from the grid in the smart grid case.

Figure 4 shows the differences between the grid purchases for cases with and with the smart meter when $K_1^m = \overline{K}$. While the first two graphs from left demonstrate the purchases from the grid in the first and the second period, respectively, the last figure demonstrates the expected sum of the grid purchases in the two periods. Namely, the three figures from left to right demonstrate $g_1^m - g_1^g$, $\sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$ and $g_1^m - g_1^g + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$, respectively. We are only interested with the qualitative pattern, and thus, we do not attempt to calibrate the model. The parameter values that we employ are $r = 0.05; \phi = 0.49; P_s = 2/3; P_l = 1/2; c_1 = 0.030; c_{sl} = 0.03; c_{sh} = 0.3; c_{nl} = 0.03; c_{nh} = 0.3, \overline{K} = 0.3, \min(\overline{S}) = 2.6, \max(\overline{S}) = 4.6.$ ($S_1^m = 2.6$.)

In line with our intuition above, the last figure show that lower values of energy storage capacity, low storage capacity will allow for a higher grid activity with smart

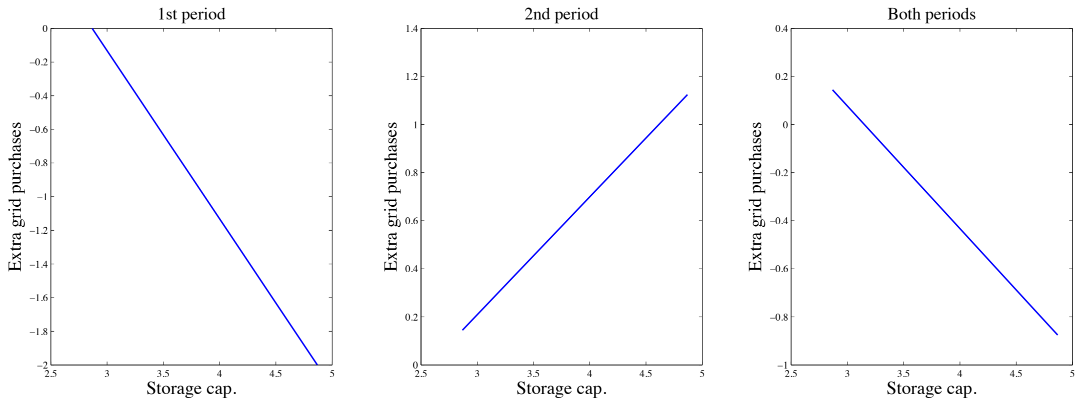

Figure 4: *Difference in purchases from the grid* ($K_1^m = \overline{K}$).

meter only. Nevertheless, with higher storage capacities, which allow for larger amounts of energy to be stored in the first period (see Fig. 4a), the total amount of energy purchased from the grid increases. This happens even if the grid purchases are lower in the second period with the smart grids.

5.1.3 Storage constrained

When the solar power is constrained by the available physical capacity, and therefore, $S_1^m = \overline{S}$, we have the following conditions for the multipliers:

$$\nu_1 = \nu_2 = \nu_3 = \nu_5 = 0, \text{ and } \nu_4 > 0.$$

From Eq. (14), the difference between the purchase of electricity in the smart meter only and smartgrid cases can be expressed as

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > (1 + P_s)(\overline{K} - K_1^m), \tag{19}$$

As $\overline{K} > K_1^m$, the LHS is strictly positive. Therefore, the average level of grid purchases will be higher in the smart meter only case.

5.1.4 Solar power and storage constrained

Recall that we have the following conditions for the multipliers when the installation of both the solar power and energy storage is constrained by the available physical capacity, and therefore, that is, $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$:

$$\nu_2 = \nu_3 = \nu_5 = 0, \nu_1 > 0 \text{ and } \nu_4 > 0.$$

The difference between the purchase of electricity in the smart meter only and smartgrid cases can now be expressed as

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > 0, \tag{20}$$

Therefore, the average level of grid purchases will be higher in the smart meter only case.

5.2 Smart meter vs no smart meter

In this subsection, we compare electricity purchases from the central grid with and without smart meters.

Proposition 2. *For* $P_n > \max(P_n^1, P_n^2)$*, where*

$$P_n^1 \equiv \frac{c_1/\phi - r + c_1}{c_{nl}},$$
$$P_n^2 \equiv \frac{c_{sl} - r + c_1}{c_{sl}},$$

1. *There is more purchase from the grid with smart meters.*
2. *In particular,* $g_1^o < g_1^m$ *and* $g_{ji}^m \geq 0$ *while* $g_j^o = 0$*.*

Proof. Given $u'' < 0$ and using Eqs. (9a) and Eqs. (12a), $S_1^m > S_1^o$ if and only if

$$P_n > \frac{c_1/\phi - r + c_1}{c_{nl}}.$$

Furthermore, using Eqs. (9b) and (12b), $K_1^o + \phi S_1^o > K_1^m + \phi S_1^m$ if and only if

$$P_n > \frac{c_{sl} - r + c_1}{c_{sl}}.$$

Consequently,

$$g_1^m - g_1^o = K_1^o - K_1^m + S_1^m - S_1^o$$

completes the proof. □

In the reverse case, that is, if $P_n < \min(P_n^1, P_n^2)$, then $g_1^o > g_1^m$. Thus, more electricity will be purchased from the grid without smart meters. In the second period, the consumption from the grid for the no smart meter case, $\sum_j P_j(g_j^o)$, is zero. On the other hand, as $g_{jh}^m = 0$, consumption from the grid with the smart meter case reads as $P_l \sum_j P_j(g_{jh}^m) \geq 0$. As a result, for a sufficiently low level of expected purchases from the grid in the smart meter case, the expected grid purchase in the no smart meter case will be higher. Conversely, when the expected purchase from the grid in the second period is sufficiently high, then, in expected terms, the total purchase of electricity will be higher with the smart meter case.

When the probability of having no sun, and therefore, no solar power in the second period, is sufficiently high, and it is not optimal to consume from the grid when the price is high, the HH will find it optimal to store more by consuming a higher amount in the first period. Furthermore, the fact that electricity can be purchased from the grid when the price is low and when there are smart meters also adds to grid purchases. Conversely, when the probability of having sun is sufficiently high, the purchases from the grid in the first period will be lower with smart meters. This is mainly due to the fact that, first, the possibility of no sun is sufficiently low, and second, the HH may need to purchase from the grid only when the price on the grid is low.

Figure 5 illustrates an example for this case. Without attempting to calibrate the model, the parameter values that we employ are $r = 0.5; \phi = 0.59; P_l = 1/2; c_1 = 0.275; c_{sl} = 0.32; c_{sh} = 0.7; c_{nl} = 0.55; c_{nh} = 0.7, \max(\overline{K}) = 6, \max(\overline{S}) = 5$. These parameter values allow us to get interior solutions for the number of solar panels, storage and grid purchase in the first period. In running the simulations, we use 50 linearly equally spaced points between 0.3875 and 0.4541 for P_n. Furthermore, while $\max(P_n^1, P_n^2) = 0.4384$, $\min(P_n^1, P_n^2) = 0.2969$. In line with Proposition 2, when $P_n > \max(P_n^1, P_n^2)$ the expected total grid purchases will be higher in the smart meter case. Even though $P_n < \min(P_n^1, P_n^2)$ is a sufficient condition for a higher level of grid purchases in the no smart meter case, it can be seen on the figure that a higher level of grid purchases will be expected with the use of the smart meter if $P_n > 0.43$. Reading in-

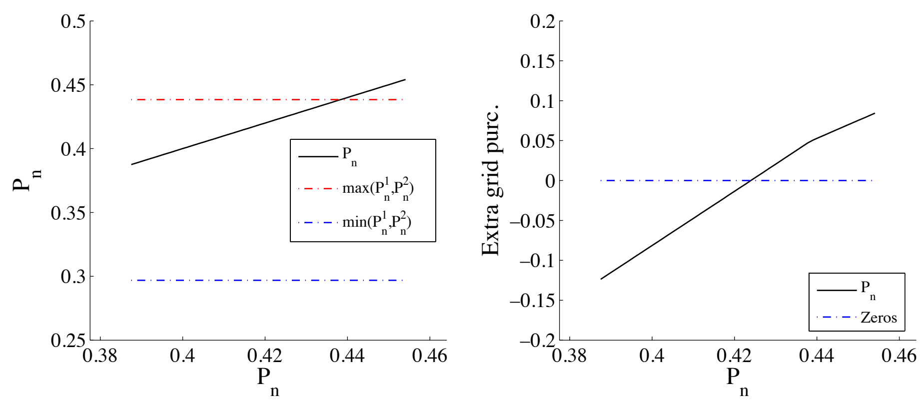

Figure 5

dication: 'Extra grid purc.' corresponds to the expected electricity purchase with smart meter in excess compared to the electricity grid purchase without a smart grid.

6 When to install smart meters?

In this section, we analyse the conditions under which it is relevant to install a smart meter, for a given strictly positive cost, r^m, of this device. The maximum value function for the no smart meter case when the solution is interior can be written as:

$$V^o = u\left(K_1^o + g_1^o - S_1^o\right) - c_1 g_1 + \sum_j P_j \left[u\left(\mathbb{1}_s(j)K^o + \phi S_1^o + g_j^o\right) - c_2 g_j^o\right] - rK_1^o.$$

For the case with smart meters only, recall that the benevolent planner solves the following program:

$$\begin{aligned} V^m = \max_{\{K_1,S_1,g_1,g_{ji}\}} \quad & u\left(K_1 + g_1 - S_1\right) - c_1 g_1 \\ & + \sum_j \sum_i P_j P_i \left[u\left(\mathbb{1}_s(j) K_1 + \phi S_1 + g_{ji}\right) - c_{ji} g_{ji}\right] - r K_1 \\ \text{s.t.} \quad & \overline{K} \geq K_1 \geq 0,\ \overline{S} \geq S_1 \geq 0, g_1 \geq 0 \text{ and } g_{ji} \geq 0, \end{aligned}$$

where $j = s, n$ and $i = l, h$.

We have shown in section 3 that for an interior solution case, that is, $\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu_5 = 0$, and

$$c_1 + P_s \sum_i P_i c_{si} - r > 0,$$

$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 > 0,$$

we must have $g^m_{sh} = g^m_{nh} = 0$. In the case when the price is high, this indicates that there will be no grid purchases in the second period irrespective of whether there are smart meters or not. This suggests that the HH may not benefit from the installation of the smart meter when the price on the grid takes high values.[7] Nevertheless, the installation of the smart meter will be beneficial when the expected avoided cost (or saving) from its use is sufficiently high. This urges us to investigate the behavior of the maximum value function (V^m) with respect to the low price on the grid.

The maximum value function can then be written as

$$\begin{aligned} V^m = & u\left(K^m_1 + g^m_1 - S^m_1\right) - c_1 g^m_1 + P_l \sum_j P_j \left[u\left(\mathbb{1}_s(j) K^m_1 + \phi S^m_1 + g^m_{jl}\right) - c_{ji} g^m_{jl}\right] \\ & + P_h \sum_j P_j u\left(\mathbb{1}_s(j) K^m_1 + \phi S^m_1\right) - r K^m_1 \end{aligned}$$

The following proposition characterizes the smart meter installation decision. For simplicity let $c_{jl} = c_l$.[8]. We note r^m is the usage (or investment) cost of the smart meter.

Proposition 3. *There exists a price on the electric grid $\hat{c}(r^m)$ such that $V^m - r^m \geq V^o$ if and only if $c_l \leq \hat{c}(r^m)$, with $\frac{\partial \hat{c}(r^m)}{\partial r^m} < 0$.*

Proof. In a first step, we prove that there exits some level $\tilde{c}$ such that for $c_l \leq \hat{c}_{nl}$ we have $V^o = V^m$. It is easy to see using Eq. (5d) that for a high enough c_{ji} the problem exactly reduces to that without smart meter, therefore $V = V_m$. In a second step, the

[7]Focusing on the cases where $K_1 = \overline{K}$ and $S_1 = \overline{S}$, the reasoning will remain the same unless $g_{jl} = 0$.
[8]The reasoning is unaltered for the general case.

relationship between c_l and V^m can be shown as

$$\begin{aligned}\frac{\partial V^m}{\partial c_l} &= -P_l \sum_j P_j g^m_{jl} < 0, \\ \frac{\partial^2 V^m}{\partial c_l^2} &= -P_l \sum_j P_j \frac{\partial g^m_{jl}}{\partial c_l} > 0,\end{aligned} \tag{21}$$

where

$$\begin{aligned}\frac{\partial g^m_{nl}}{\partial c_l} &= \frac{1}{u''(\phi S^m_1 + g^m_{nl})} - \phi \frac{\partial S^m_1}{\partial c_l} < 0, \\ \frac{\partial g^m_{sl}}{\partial c_l} &= \frac{1}{u''(K^m_1 + \phi S^m_1 + g^m_{sl})} - \left(\frac{\partial K^m_1}{\partial c_l} + \phi \frac{\partial S^m_1}{\partial c_l} \right) < 0, \\ \frac{\partial S^m_1}{\partial c_l} &= -\frac{P_l}{2P_h} \frac{1}{u''(\phi S^m_1)} > 0, \\ \frac{\partial K^m_1}{\partial c_l} + \phi \frac{\partial S^m_1}{\partial c_l} &= -\frac{P_l}{2P_h} \frac{1}{u''(K^m_1 + \phi S^m_1)} > 0.\end{aligned}$$

For c_l sufficiently high, g^m_{sl} and g^m_{nl} will be zero. Let $\hat{c}_{sl}$ and $\hat{c}_{nl}$ ($\hat{c}_{nl} > \hat{c}_{sl}$) be the prices that lead to $g^m_{sl} = 0$ and $g^m_{nl} = 0$, respectively. Therefore,

$$\left. \frac{\partial V^m}{\partial c_l} \right|_{c_l \geq \hat{c}_{nl}} = -\rho g^m_{jl} = 0,$$

and $V^m(c_l)|_{c_l \geq \hat{c}_{nl}} = V^m(\hat{c}_{nl})$.

Consequently, $V^m(\hat{c}_{nl}) - r^m \geq V^o(c_2)$, where c_2 is given and constant, is not possible, and there does not exit any $r^m > 0$ such that it will always be optimal to invest in smart meters. Yet, due to $\partial V^m / \partial c_l < 0$, $\partial^2 V^m / \partial c_l^2 > 0$ and the Inada condition $\lim_{x \to 0} u'(x) = +\infty$, $V^m(c_l) - r^m$ will cross $V^o(c_2)$ once, when $V^m(\hat{c}(r^m)) - r^m = V^o(c_2)$. ($\hat{c}(r^m) < \hat{c}_{nl}$.) It is straightforward to show that $\frac{\partial \hat{c}(r^m)}{\partial r^m} < 0$. This implies that if $c_l < \hat{c}(r^m)$, $V^m(c_l) - r^m > V^o(c_2)$ and it will be optimal to have the smart meter installed. □

One of the take away from this section is that the smart meter will become beneficial to the HH once it can allow taking advantage of a sufficiently low electricity price. If second period low price is not sufficiently low, the installation of a smart meter may not be worth it. Figure 6 illustrates this case.

7 Conclusion

Climate change, congested central grids in developed countries and lack of access to electricity in developing countries are problems that could be mitigated through the penetration of the electricity market of renewables like wind and solar together with microgrids. The intermittent nature of renewables coupled with the non-reactivity of

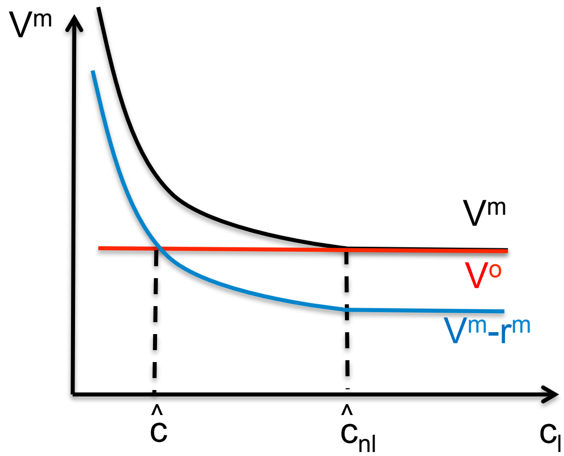

Figure 6

consumers to short term fluctuations in electricity provision suggest the implementation of new technologies.The first one would concern storage devices to reduce the the burden of intermittency. The second one would alleviate both storage and consumers non-reactivity issues and would consist into smartgrids. The later can take the form of smart meters or of infrastructures allowing to sell back to the central grid the excess electricity generated in the microgrid. In this paper we study the implications of such technologies for the electricity purchase from the central grid. We show that it is not always the case that by accessing to smartgrids, electricity from the central grid and therefore GHG emissions will be reduced. In addition, smart meters are only desirable if they allow to exploit sufficiently large fluctuations in demand provision (that show up in the electricity prices in our model).

More could be done within our framework. We could appraise the suitability of microgrid in case of a constrained access to the central grid or a risk of black-out as it is the case both in developed countries like the US or developing countries like India. In addition, we could explore cases where solar panels or storage devices are so expensive that they may be suitable with the smartgrids but not without them. Finally our results could serve as a basis to design environmental policies such as subsidies either for microgrids or smartgrids.

8 References

Ambec S. and C. Crampes (2012), Electricity provision with intermittent sources of energy, Resource and Energy Economics 34 (2012) 319-336

Ambec S. and C. Crampes (2015), Environmental Policy with intermittent sources of energy, Mimeo.

Baker E., M. Fowlie, D. Lemoine and S. S. Reynolds, The Economics of Solar Electricity, Annual Review of Resource Economics, 5:387-426, 2013.

Borenstein S. and Holland S., 2005, On the Efficiency of Competitive Electricity Markets with Time-Invariant Retail Prices, Rand Journal of Economics 36, Autumn.

Crampes C. and Moreaux M., 2010, Pumped Storage and Cost Saving, Energy Economics, 32: 325-333.

Jackson R., 1973, Peak Load Pricing Model of an Electric Utility Using Pumped storage, Water Resource Research 9(3) 556-562.

Joskow P. and and Tirole J., 2007, Reliability and Competitive Electricity Markets, Rand Journal of Economics 38(1), 60-84.

Hoel M. and S. Kverndokk, Depletion of Fossil Fuels and the Impacts of Global Warming, Resource and Energy Economics, 18: 115-136, 1996.

O. Tahvonen, Fossil fuels, stock externalities, and backstop technology, Canadian Journal of Economics, 30: 855-874, 1997.

The Economist, 2009, Building the Smart Grid, june 4th, 2009.

The Telegraph 2015a, Smart meters: will you pay more for peak electricity?, May 24th 2015.

The Telegraph 2015b, My solar power smart meter shaves Â£250 off energy bills, August 28th 2015.