Welfare division in the commons

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Preliminary version

Abstract

Humans own equal rights on natural resources that are available for free on earth such as water, fisheries or clean air. The extraction of those resources is often regulated with property rights such as tradable quotas or emissions permits. I examine the welfare distribution induced by regulated extraction of open-access resources when users differ on access. A welfare distribution is accepted when every group of users obtains at least what it would get under free-access. The "no-transfer" solution prescribes to share the resources efficiently without any transfer - or net trade - among users. It is the most egalitarian welfare distribution that is accepted when people enjoy same benefit from resource use. With heterogeneous benefits, it is the only solution that is accepted when the resource is supplied sequentially across time. The results highlight the limits of market-based solutions for managing natural resources.

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1 Introduction

Economic development relies mostly on the exploitation of natural resources available on earth such as water, fisheries, biodiversity, minerals, fossil fuels, land or forests. Our production and consumption activities impact environmental resources such as clean air or clean water. Since at least Hardin (1968), it has been recognized that the uncontrolled extraction of scarce natural resources is inefficient. It leads to over-extraction across time and space, miss-allocation among users and even depletion. Human must coordinate their use of resource and pollution activities to avoid the so-called "tragedy of the commons".

Economists often recommend the assignment of property rights as a solution to this tragedy. Setting resource quotas and pollution emission permits limits extraction. The resource extracted is optimally allocated to the users by allowing them to trade those rights in a competitive market. However, introducing property rights on resources that are free is not neutral. It affects the division of the welfare from resource extraction. Some resource users might oppose regulated solutions because they turn out to be worse off than under free access. They also might perceive the welfare distribution induced by property rights as unfair.

For instance, global air pollution can be optimally controlled with tradable emission allowances. Yet the allocation of allowances among polluting firms impacts their profit. Some of them might enjoy windfall profits from owning and selling allowances while others incur costs of buying permits and controlling pollution. Similarly, water used for irrigation can be better managed through tradable water rights or quotas. Yet some farmers might experience a loss of welfare compared to unregulated extraction. Others might become far more wealthy by selling their rights to municipalities or industries rather than irrigating their own land. Setting water markets might exacerbate inequality and, therefore, tension among farmers.

The paper aims to investigate the acceptance of regulated solutions to the use of open-access resources. To do that, I analyze the fairness properties of welfare distributions induced by regulated extraction of open-access common-pool resources. The main criteria for acceptance is that resource users are better-off with regulated extraction than under open-access. They should be better off not only individually but also collectively. Group of users can improve their welfare by coordinating the management of resources they fully control under free-access. This defines a *free-access* lower bound on welfare: the higher welfare that a group of users can achieve by dividing optimally the amount of resource it can rely on. I examine welfare distributions that satisfy the free-access lower bound for *all* groups of users. A regulation is accepted if the induced welfare distribution satisfies the free-access lower bounds.

To address this problem, I consider a general framework in which users might differ on their access on resources. Users are connected to one or several pools of resources. The resources extracted are perfectly substitutable for all users connected to them. We thus refer to a single "resource" available in several "pools". The ability to extract some pools and not others can be due to geographical proximity or to technological capability. For instance, the pools can be several water reservoir connected to farmers's land via rivers and canals. The resource pools might be reserves of several sources of energy (e.g. oil, coal, natural gas or biomass). Users differ on their capacity to extract these reserves (e.g. if they control the offshore versus land drilling technology) or to transform each source of energy into electricity (e.g. if they run a coal or natural gas power plant).

For air pollution and emission permits, the pool could be the maximal among of air pollution - such as sulfur oxides or particulate matters - that the air quality standard can allow at one location. Users are then firms running production plants (e.g. thermal power utilities) in different locations.

Another feature of the model is that users enjoy increasing benefit from extracting and consuming the resource up to some satiated level with decreasing return. The resource being somehow essential to users, the marginal benefit is very high when close to zero resource. It is decreasing with resource consumption, thereby capturing diminishing returns in resource use.

A resource sharing problem is defined by a set of users - with increasing and concave benefit from resource use - connected to some resource pools. I examine regulations that are accepted in *all* potential resource sharing problem.

Suppose first that the benefits from resource use are identical. Users differ solely on their access to the resource. In this case, an egalitarian division of welfare seems quite natural. People having equal rights as well as same needs and satisfaction from resource use, they should be treated equally.[1] Property rights on resources can be defined to equalize welfare resource extraction and after trade. Unfortunately, the egalitarian solution generally fails to satisfy the free-access lower bounds. Some users who have access to more resource than others would oppose it because they lose compared to free-access extraction. To reconcile an egalitarianism with the free-access lower bounds, I rely on Dutta and Ray (1989)'s concept of egalitarian solution under participation constraints. Among the welfare distributions that satisfy the free-access lower bounds (the participation constraints in Dutta and Ray, 1989), it is the one that is not Lorenz dominated by any other welfare distribution. Surprisingly, the free-access lower bounds forbid any redistribution of welfare. It is shown that the egalitarian solution under participation constraints is the so-called *no-transfers* solution. It requires to share the resource efficiently *without* any transfer of welfare among users. It assigns to each user exactly her or his benefit from consuming the efficient allocation of resource. Although users have all same benefit function, due to unequal access to resource pools, some users consume more than others under the efficient resource allocation. Nevertheless, those inequality cannot be mitigated: any transfer of welfare from a wealthy user to a poorest one - i.e. consuming less resource - violates the free-access lower bounds.

With heterogenous benefits from resource use, the no-transfers solution does satisfy the free-access lower bounds. Yet other welfare distributions do in general with a notable exception: when all users can consume their satiated level. In this case, the free-access lower bound of every user (or group of users) coincides with the welfare assigned by the no-transfers solution. Therefore any user would get strictly less than her or his free-access lower bound if it transfers part of her or his welfare to another user.

The uniqueness of the no-transfers solution when consumption is satiated has interesting implications. Suppose that the resource division problem is repeated across time. Typically, some amount of resource is shared, through regulated extraction, before new resource is supplied and shared and so forth. Now users would accept regulations if they do not lose compared to the free-access extraction of, not only the current amount of resource, but also any future supply. The free-access lower bounds can thus extended to sequential resource supply: any group of agents should enjoy at least the welfare of free-access extraction of the current amount of resource and

[1] Unequal treatment might be motivated by unequal access. Yet, if users are not held responsible for their access to resource pools, they should not be rewarded for wider access or penalized for limited access.

also of any future supply. It turns out that the no-transfers solution is the only one that satisfies the free-access lower bounds with sequential supply. It is unique because future resource supply could allow users to consume their satiated level in which case only the no-transfer solution does satisfy the free-access lower bounds.

The results have implications for the design of regulations on natural resources and environmental pollution. It questions the assignment of tradable rights as a solution to the tragedy of the commons. When people enjoy the same benefit from resource use, net trades of those rights exacerbates inequality and/or reduces the welfare of some of the users compared to free-access. In particular, the equal division of resource with trade would benefit to users who have access to more resource pools because they sell their rights on the more valued pools at a higher price. When people differ on benefit from resource use, the sequence of net trade of resource rights can make some users worse-off compared to free-access. Those users would likely oppose such market-based solutions to the tragedy of the commons. In this sense, our theoretical analysis is consistent with the popular opposition of tradable rights on free resources such as water or clean air.

The paper is related to cooperative game theory and axiomatic analysis of resource division (Thomson, 2008). The free-access lower bounds correspond to the core bounds of a cooperative game induced by common-pool resource extraction. It is a cooperative game with externalities: the welfare that a coalition of players can guarantee to itself depends on the behavior of other players (Bloch, 1996). The free-access lower bounds assume the worst that can potentially happen for the coalition: all other players extract the resource they are connected to up to be satiated. As a consequence, the lower bounds on welfare are quite low. Nevertheless, they constraint quite a lot the core solutions in our framework.

Our model shares some features with the river sharing problem introduced by Ambec and Sprumont (2002) and then extended by Ambec and Ehlers (2008) with satiated benefits. However the spatial structure and timing differ. In the river sharing problem, users have sequential access to a single resource, water. Therefore the most upstream users can extract all resource they have access to under free-access. In contrast, here, users have unequal access to several pools of resources. Moreover, all those who are connected to the same pool have symmetric and simultaneous access. They therefore can rely only on the resource pools the fully control or what is remaining from the shared pools.[2] In a recent papers, Bochet, Ilkilic and Moulin (2013) investigate the allocation of a divisible good located under in different pools under unequal access with a similar graph structure than here. They also focus egalitarian solutions. However, they deal with non-transferable utility: only one good is assigned, no trade or compensations are allowed. The focus is on the allocation of the good, while I am interested the distribution of welfare. The paper complements the literature on common-pool resource extraction under asymmetric information (e.g. Montero, 2008). It investigates the efficiency properties of regulation mechanisms while I am interested in the welfare impact of regulated extraction.

The rest of the paper proceeds as follow. Section 2 introduces the model and describes the efficient allocation of resource. Welfare distributions are analyzed in Section 3. Section 3.1 defines the free-access lower bounds. Section 3.2 describes the no-transfers solution. It shows that it

[2] Ambec (2008) analyzes a single resource shared by several users under symmetric access and similar preferences than the present paper. However, the focus is on the Walrasian allocation with equal endowment which characterized with fairness principles.

satisfies the free-access lower bounds. Section 3.3 considers the case of homogenous benefits. It shows that the no-transfers solution is the egalitarian solution under the free-access lower bounds. Section 3.4 extends the model to heterogeneous benefits and sequential supply of resource. It characterizes the no-transfers solution as the only one that satisfies the free-access lower bounds with sequential supply of resource. Section 4 concludes the paper.

2 The resource sharing problem

2.1 The Model

A set of agents $N = \{1, ..., n\}$ are sharing a homogenous good called "resource" located in different places called "pools". Each agent $i \in N$ enjoys the same benefit $b_i(x_i)$ from consuming x_i units of the resource. We assume b_i increasing and concave up to a satiated consumption level $\hat{x}_i$ (which may be infinite): $b'_i(x_i) > 0$ for every $x_i \leq \hat{x}_i$, $b'_i(\hat{x}_i) = 0$, $b'_i(x_i) \leq 0$ for every $x_i > \hat{x}_i$ and $b''_i(x_i) < 0$ for every $x_i \leq \hat{x}_i$. We normalize the benefit of zero consumption to zero: $b_i(0) = 0$. We further assume that $b'_i(0)$ high enough (e.g. $b'_i(0) = +\infty$) so that assigning no resource to one agent is never efficient. Let $M = \{1, ..., m\}$ be the set of sources. Let e_j denote the amount of resources available at pool j for every $j \in M$ with $e_j > 0$. Each agents i has access to some subset $S_i \subseteq M$ of the sources. Symmetrically, each pool j can supply a subset $R_j \subseteq N$ of agents.

A resource sharing problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e}) \equiv (N, M, \mathcal{S}, \boldsymbol{b}, \boldsymbol{e})$ is defined by a set of agents $N = \{1, ..., n\}$, a set of pools $M = \{1, ..., m\}$, the sets of pools $\mathcal{S} = \{S_i\}_{i \in N}$ that are connected to each user, the benefit functions $\boldsymbol{b} = (b_1, ..., b_n)$ and the amount of resource $\boldsymbol{e} = (e_1, ..., e_m)$ available in each pool. We denote a problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ because we will vary $(\boldsymbol{b}, \boldsymbol{e})$ later on.[3]

An example of resource sharing problem with three agents and three pools is represented in Figure 1.

[3]Note that the model encompasses the extreme cases of equal access to all sources $S_i = M$ for every $i \in N$, as well as exclusive access to pools $S_i \cap S_j = \emptyset$ for every $i, j \in N$.

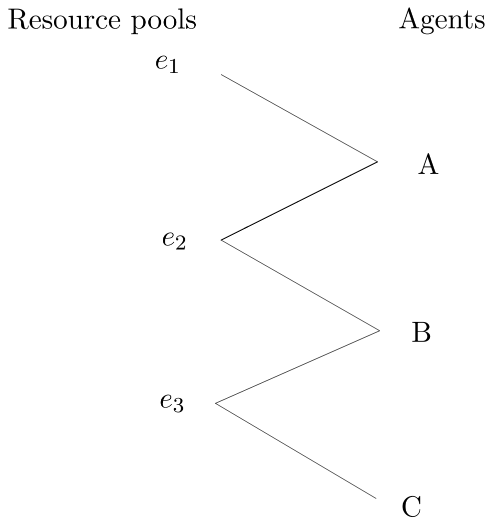

Figure 1: Example of resource sharing problem with unequal access

Agent 1 has exclusive access to pool 1, he or she shares pool 2 with agent 2. Agents 2 and 3 share pool 3. Formally, $S_1 = \{1, 2\}$, $S_2 = \{2, 3\}$ and $S_3 = \{3\}$.

An allocation of resource is a matrix $\boldsymbol{X} = [x_{ji}]_{j \in M, i \in N}$ where $x_{ji} \geq 0$ denotes i's extraction from pool j for every $i \in N$ and $j \in M$. The resource allocation $\boldsymbol{X}$ is feasible if it satisfies the following resource constraints for $j = 1, ..., m$:

$$\sum_{i \in R_j} x_{ji} \leq e_j, \tag{1}$$

A feasible resource consumption plan is a vector of (resource) consumptions $\boldsymbol{x} = (x_i)_{i \in N}$ that can be achieved with a feasible resource allocation $\boldsymbol{X}$ with $x_i = \sum_{j \in S_i} x_{ji}$ for every $i \in N$. Agent i enjoys a benefit $b_i(x_i)$ from consuming x_i for $i = 1, ..., n$. The total benefit from consuming $\boldsymbol{x}$ is thus $\sum_{i \in N} b_i(x_i)$. An efficient consumption plan $\boldsymbol{x}^*$ maximizes the total welfare.

A regulation distributes the total welfare through transfers. Let us denote by t_i net transfer received by agent i (it is a payment if negative). Transfers that distribute the total welfare must be budget-balanced: $\sum_{i \in N} t_i = 0$. We denote by u_i the utility of agent i where

$$u_i = b(x_i) + t_i$$

Given the consumption plan $\boldsymbol{x}$, a welfare distribution can be equally defined by budget-balanced transfers $\boldsymbol{t} = (x_i)_{i \in N}$ or utility levels $\boldsymbol{u} = (u_i)_{i \in N}$.

A solution to the problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ is an allocation $(\boldsymbol{x}, \boldsymbol{t})$ which specifies a feasible consumption plan $\boldsymbol{x}$ and budget-balanced transfers $\boldsymbol{t}$. An example of allocation is the Walrasian allocation from the assignment of property rights on the resource to users. Consumption and transfers are determined by trades and market prices. Formally, an allocation of property rights is a matrix $\boldsymbol{W} = [w_{ji}]_{j\in M, i\in N}$ where $w_{ji} \geq 0$ denotes i's endowment of resource from pool j for every $i \in N$ and $j \in M$. Given endowments $\boldsymbol{W}$ and equilibrium prices p_j for the resource in pool $j = 1, ..., M$, users choose how much they trade and consume from each pool. Under the assumption of price-taker users, the first fundamental theorem of welfare applies and, therefore, the equilibrium resource allocation is unique and efficient. Transfers are defined by the value of net trades. Hence, an arbitrary assignment of property rights $\boldsymbol{W}$ leads to a unique solution $(\boldsymbol{x}^*, \boldsymbol{t}^e)$ where $\boldsymbol{x}^*$ is the efficient consumption plan and $\boldsymbol{t}^e$ is defined by $t_i^e = \sum_{j\in M} p_j(e_{ji} - x_{ji}^e)$ for $i = 1, ..., n$ where p_j are the equilibrium prices for $j = 1, ..., m$.

In what follow, I first describe the efficient resource allocations $\boldsymbol{X}^*$ which determines the unique efficient resource consumption scheme $\boldsymbol{x}^*$, before examining transfers $\boldsymbol{t}$ or, equivalently, welfare distributions $\boldsymbol{u}$.

2.2 Efficiency

An efficient resource consumption plan $\boldsymbol{x}^*$ is defined by a feasible resource allocation matrix $\boldsymbol{X}^*$ that maximizes the sum of benefits subject to the feasibility constraints:

$$\begin{array}{ll} \max_{\boldsymbol{X}} \sum_{i\in N} b_i \left(\sum_{j\in M} x_{ji}\right) \text{s.t.} & \\ x_{ji} = 0 & \forall (j,i) \in M\backslash S_i \times N \\ x_{ji} \geq 0 & \forall (j,i) \in S_i \times N \\ \sum_{i\in R_j} x_{ji} \leq e_j & \forall\, j \in M \end{array} \tag{2}$$

The first set of constraints assigns zero resources to agents from pools they are not connected to. The second one makes sure that extraction is non-negative. The third ones limit extraction to resource availability at each pool.

The program (2) has multiple solutions. However, it leads to a unique resource consumption allocation $\boldsymbol{x}^* = (x_1^*, ..., x_n^*)$. Denoting μ_j and λ_{ji} the Langrangian multipliers associated to the resource constraint of pool j and the non-negativity constraint for agent i's allocation of pool j for any $(j, i) \in S_i \times N$, we obtain the following first-order conditions for every $(j, i) \in S_i \times N$:

$$b_i'(x_i^*) = \mu_j - \lambda_{ji}.$$

For each user $i \in N$, the first-order condition equalizes her marginal benefit to the shadow value of the resource she has access to, which is given by the resource and non-negative (or connectedness) constraint. The above first-order conditions have several implications.

Consider two agents that are connected to the same pool. First, if they both extract some resource from this pool, they should enjoy the same marginal benefit. Technically speaking, if two agents l and h consume resource from the same pool j, it means that $x_{jl}^* > 0$ and $x_{jh}^* > 0$. Therefore the resource constraint associated to pool j is binding, while the non-negativity constraints are not for both users. Thus $\mu_j > 0$ and $\lambda_{jl} = \lambda_{jh} = 0$. The first-order conditions implies $b_l'(x_l^*) = b_h'(x_h^*) = \mu_j$. Remarkably, if the two users have same benefit function, they consume the same amount of resource: if $b_l = b_h$ then $x_l^* = x_h^*$.

Second, if one of the two users connected to the same pool extract from it but not the other, then the later enjoy a lower marginal benefit than the former. If say users l extracts from j but not h, it means that h has its non-negativity constraint binding for pool j but not l: $\lambda_{jh} > 0$ and $\lambda_{jl} = 0$. The first-order conditions imply $b'_l(x^*_l) > b'_h(x^*_h)$. In case of same benefit functions $b_l = b_h$, it means that users l consumes less that h: $x^*_l < x^*_h$. Intuitively, if l is consuming from the shared pool j but not h, it is because h has access to other pools which are more 'abundant" in the sense that they have lower shadow values. Therefore h should be assigned resources from those abundant pools instead of pool j which is left for l and other users more constrained that h.

A third useful implication of the first-order condition allows to compare the shadow value of resource pools. Two pools that are used by a same user should have same shadow value. Formally, if user i consumes from two pools j and l, that is if $x^*_{ij} > 0$ and $x^*_{il} > 0$ meaning that $\lambda_{ji} = \lambda_{li} = 0$, then $\mu_j = \mu_l$.

Using the above properties, we can rank resource pools according to their shadow values and link those pools to the marginal benefit of their users. Formally, the efficient consumption plan $\boldsymbol{x}^*$ solution to (2) defines a partition $\{M_k\}_{k=1}^K$ of the set of pools M. Each subset M_k is the set of resource pools with same shadow value p_k for $k = 1, ..., K$ with $p_1 > .... > p_K$ where $p_k = \mu_j$ such that all pools $j, l \in M_k$ have same shadow values $\mu_j = \mu_l = p_k$ for $k = 1, ..., K$. Similarly, we can rank users according to their marginal benefit. The the efficient consumption plan $\boldsymbol{x}^*$ defines a partition $\{N_k\}_{k=1}^K$ of the set of users N. Each subset N_k is the set of users with same marginal benefit $b'_i(x^*_i) = b'_l(x^*_l)$ for every $i, l \in N_k$ for $k = 1, ..., K$. Marginal benefits increase moving from N_k to N_{k+1}. All users which belong to the same set N_k are extracting from resource pools with same shadow value or price equals to their marginal benefit: $b'_i(x^*_i) = b'_l(x^*_l) = p_k$ for every $i, l \in N_k$ for $k = 1, ..., K$. Thus users from N_k are extracting resource from pools in M_k with price p_k for $k = 1, ..., K$. Some users in N_k might have access to resource pool with higher prices M_1 to M_{k-1} but are not using them in the efficient solution.[4]

In the case of homogeneous benefits $b_i = b_j$ for every $i, j \in N$, the ranking of marginal benefits translates into resource consumption levels. It means that users in N_1 consume the same amount of resource say x^1, strictly less than agents in N_2 who consumes x^2 and so on for $N_3, ..., N_K$. The efficient consumption plan defines a partition of $\{N_k\}_{k=1}^K$ of the set of agents N and consumptions $\{x^k\}_{k=1}^K$ with $x^1 < .... < x^K$ where every agent in N_k consumes x^k for $k = 1, ..., K$.

In the example from Figure 1, the efficient allocations depend on the amount of resources in each resource pool. The three figures below represent three distinct solutions for 15 units of resource to be shared but located differently when benefits are identical $b_1 = b_2 = b_3 = b$. We assume that $\hat{x}$ is high enough (e.g. equals to 15).

[4]Note that if the resource is sufficiently abundant in some pools, the marginal benefit of agents in N_K is nil. They consume their satiated level $x^* = \hat{x}_i$ for every $i \in N_K$. Moreover, the price of resource at the pools belonging to M_K is nil: $p_K = 0$.

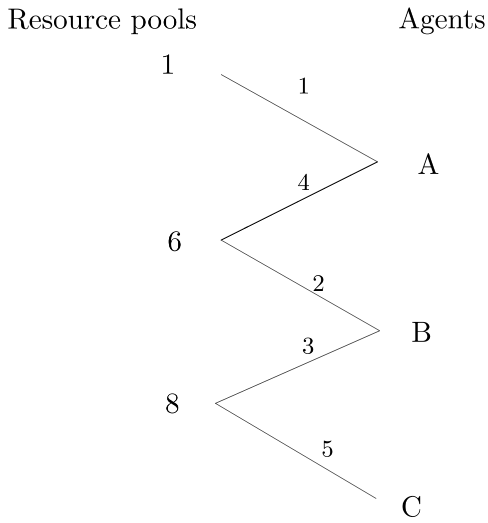

Figure 2: Example with same consumption

In Figure 2, equal sharing of the total among of resource $e_1 + e_2 + e_3 = 15$ can be achieved with the above extraction levels in each link. Each user obtains 5 units of the resource. The decomposition of N is with only one set: $N_1 = N$. Total welfare is $3b(5)$.

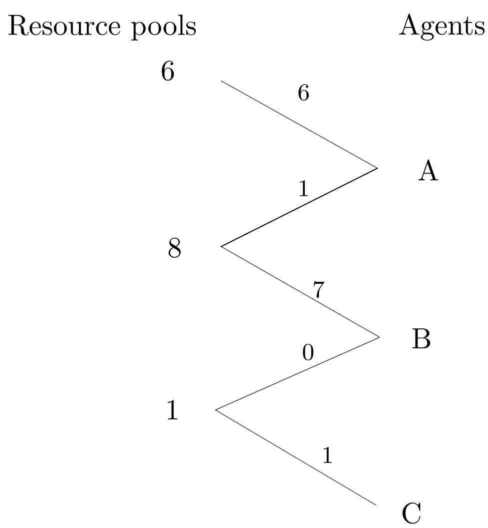

Figure 3: Example with two consumption levels

As in Figure 2, 15 units of resource has to be divided. However, in Figure 3, not enough resource is available in pool 3 to supply agent 3 with 5 units. As a consequence, agent 3 consumes all resource available in pool 3, i.e. $x_3^* = 1$, whereas agents 1 and 2 share the remaining 14 units, each of them getting 7 units: $x_1^* = x_2^* = 7$. The partition of N is now $N_1 = \{3\}$ and $N_2 = \{1, 2\}$. Total welfare is $b(1) + 2b(7) < 3b(5)$: the limited access constraints are binding.

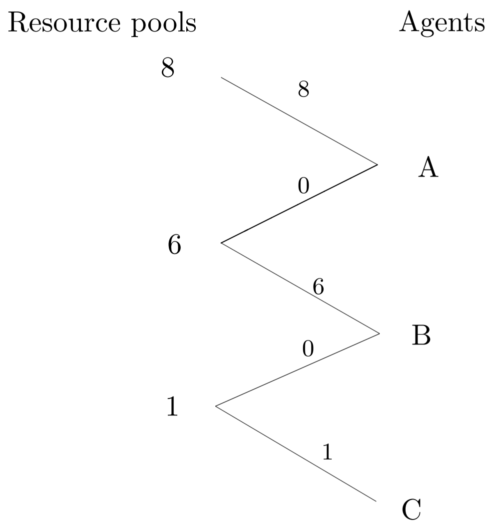

Figure 4 : Example with three consumption levels

In Figure 4, it is not anymore possible to assign 7 units of the resource to agent 2 because only 6 units are available at pool 2. So pool 2 is assigned exclusively by agent 2 while agent 1 exhaust the 8 units from the pool he or she controls. The efficient consumption allocation is $x_1^* = 8$, $x_2^* = 6$ $x_3^* = 1$. The set of agents N is decomposed into three subsets $N_1 = \{3\}$, $N_2 = \{2\}$ and $N_3 = \{3\}$. Total welfare $b(1) + b(6) + b(8)$ is even lower than in Figures 2 and 3 due to limited access.

Before moving to the study of welfare distributions, we establish formally a property of the efficient consumption plan $\boldsymbol{x}^*$. It makes use of the partition $\{N_k\}_{k=1,...K}$ and $\{M_k\}_{k=1,...,K}$ of the set of users N and pools M defines by $\boldsymbol{x}^*$. It requires new notation. For any $T \in N$, let us denote by $\xi(T) = \{j \in M | R_j \subseteq T\}$ the set of pools fully controlled by coalition T. The following Lemma states that agents in coalition $\cup_{l=k}^K N_l$ are extracting only from pools they fully control for $k = 1, ..., K$.

Lemma 1 $\cup_{l=k}^K M_l = \xi(\cup_{l=k}^K N_l)$ *for* $k = 1, ..., K$.

Proof: First observe that efficiency implies $\xi(\cup_{l=k}^K N_l) \subset \cup_{l=k}^K M_l$. Otherwise some pools that belong to $\xi(\cup_{l=k}^K N_l)$ will be not extracted, because agents outside $\cup_{l=k}^K N_l$ do not have access to them (by definition), which contradicts that $\boldsymbol{x}^*$ is efficient. Second we show that $\xi(\cup_{l=k}^K N_l) \supset \cup_{l=k}^K M_l$. Suppose not. Then there exists $j \in \cup_{l=k}^K M_l$ but $j \notin \xi(\cup_{l=k}^K N_l)$. Since $j \notin \xi(\cup_{l=k}^K T_l)$, there exists an agent i outside $\cup_{l=k}^K N_l$ who has access to pool j: $\exists i \in N \setminus \cup_{l=k}^K N_l$ such that $j \in S_i$. Pick an agent $h \in \cup_{l=k}^K N_l$ who extracts some resource from j: $x_{jh}^* > 0$. Since $i \in N \setminus \cup_{l=k}^K N_l$ and $h \in \cup_{l=k}^K N_l$, then $b_i'(x_i^*) > b_h'(x_h^*)$. Moreover, $\exists \epsilon > 0$ such that $x_{jh}^* - \epsilon > 0$ and $b_i'(x_i^* + \epsilon) \geq b_h'(x_h^* - \epsilon)$. Consider the following feasible resource consumption plan: $x_i' = x_i' + \epsilon$, $x_h' = x_h^* - \epsilon$, $x_j' = x_j^*$ for every $j \neq i, h$. It changes welfare by $(b_i(x_i^* + \epsilon) - b_i(x_i^*)) - (b_h(x_h^*) - b_h(x_h^* - \epsilon)) > (b_i'(x_i^* + \epsilon) - b_h'(x_h^* - \epsilon)\epsilon \geq 0$ where the first inequality is due to the concavity of b_i and b_h. This contradicts that $\boldsymbol{x}^*$ is efficient. ■

3 The free-access lower bounds

3.1 Definition

Users agree to implement a specific policy to coordinate extraction if it is their self-interest to do so. They must be better-off with the welfare distribution implemented by the policy than without. We need to compute the welfare that a user or a group of users can achieve on its own under free-access. Let us consider an arbitrary coalition of users T. They can surely rely on the resource pools they fully control. What's about the pool of resources shared with users outside T? Let us for instance consider the example represented in Figure 2 with a satiated consumption level $\hat{x} = 5$ for all agents. Agent A can at least consume the 1 unit of resource from resource pool 1 she controls. How much does she can expect from the 6 units she shares with B? At least 1 unit because agent B will never extract more than its satiated level of 5 units. More than that because B is also connected to a resource pool of 8 units shared with agent C. Since C will not extract more than 5 out of the 8 units, 3 units are left to B so that B would not extract more than 3 on the pool shared with A. Thus A can expect at least 3 units from the pool shared with B.

More generally, coalition T would object any welfare distribution that assign less that what it could achieve while leaving enough to outsiders up to be satiated. That is letting the outsiders exhaust the resource pools they fully control and obtain their residual demand on the pool shared with the members of coalition T.

Formally, for an arbitrary coalition T, let us denote by $T^c \equiv N \backslash T$ the set of users outside N. A feasible satiated allocation for T^c - the outsiders of coalition T- is a matrix $\boldsymbol{X}_{T^c}$ such that:[5]

$$
\begin{aligned}
x_{ji} \geq 0 \quad & \forall (j,i) \in S_{T^c} \times T^c \\
x_{ji} = 0 \quad & \forall (j,i) \in M \backslash S_i \times T^c
\end{aligned} \tag{3}
$$

$$\sum_{i \in R_j \cap T^c} x_{ji} = e_j \qquad \forall j \in \xi(T^c) \tag{4}$$

$$\sum_{i \in R_j \cap T^c} x_{ji} \leq e_j \qquad \forall j \in S_T \cap S_{T^c} \tag{5}$$

$$\sum_{j \in S_i} x_{ji} \leq \hat{x}_i \qquad \forall\, i \in T^c \tag{6}$$

Let us denote by $\Omega(T^c)$ the set of all feasible satiated allocations for T^c. The highest welfare that coalition T can achieve with the feasible satiated allocation $\boldsymbol{X}^c_{T^c}$ solves the following maximization program:

$$
\begin{array}{lll}
v(T, \boldsymbol{X}^c_{T^c}) \equiv & \max_{\boldsymbol{X}_T} \sum_{i \in T} b_i \left(\sum_{j \in M} x_{ji} \right) & \text{s.t.} \\
& x_{ji} = 0 & \forall (j,i) \in M \backslash S_i \times T \\
& x_{ji} \geq 0 & \forall (j,i) \in S_i \times T \\
& \sum_{i \in T \cap R_j} x_{ji} \leq e_j - \sum_{i \in R_j \backslash T} x^c_{ji} & \forall\, j \in S_T
\end{array} \tag{7}
$$

The free-access lower bound of coalition T is the highest welfare that it can achieve over all

[5] We assume that no member of T^c would get more that its assign welfare it the pool fully controlled by T^c are exhausted. This is without loss of generally since, under free disposal, we can always redefine the amount of resource in the controlled pools to assign the satiated consumption level.

feasible satiated allocations for outsiders:

$$v(T) \equiv \max_{\boldsymbol{X}_T \in \Omega(T^c)} v(T, \boldsymbol{X}_{T^c}) \tag{8}$$

The free-access lower bounds are quite low as users have very pessimistic expectations about the amount of resource left by others. For any shared pool, they expect that outsider will be able to extract what they need up to be satiated. Users in T expect to be able to extract resource from the pools they fully control: for those pools $R_j \backslash T = \emptyset$ and, therefore, the right-hand side in last constraint (resource availability) in (8) is simply e_j. However, for the contested pools, they expect the users outside T to extract each resource up to be satiated.

Free-Access Lower bounds: A solution $(\boldsymbol{x}, \boldsymbol{t})$ satisfies the *free-access lower bounds* if, for all coalitions $T \subseteq N$,

$$\sum_{i \in T} (b_i(x_i) + t_i) \geq v(T).$$

The free-access lower bounds uniquely determine the consumption plan for a given resource sharing problem. It is easy to show that the free-access lower bound for the "grand coalition" N forces the resource consumption scheme to be efficient. Therefore, the free-access lower bound for coalition N implies:

$$\sum_{i \in N} (b_i(x_i) + t_i) \geq v(N) = \sum_{i \in N} b_i(x_i^*).$$

The above inequality is binding by definition of $\boldsymbol{x}^*$. Hence the consumption plan must be $\boldsymbol{x}^*$.

3.2 The no-transfer solution

The "no-transfer" solution allocates the resource efficiently among users without any transfer. Denoted $(\boldsymbol{x}^*, \boldsymbol{0})$, it assigns to every user her benefit with the efficient consumption plan $u_i^0 \equiv b_i(x_i^*)$ for $i = 1, ..., n$. It turns out that, in any resource sharing problem, the no-transfers solution satisfies the free-access lower bounds.

Proposition 1 *The no-transfers welfare distribution satisfies the free-access lower bounds. It is the only one if* $\boldsymbol{x}^* = \hat{\boldsymbol{x}}$.

Proof: First, I show that the no-transfers solution assigns $\sum_{i \in T} b_i(x_i^*)$ to members of T. It can be found by solving the following program:

$$\begin{array}{ll} \max_{\boldsymbol{X}_T} \sum_{i \in T} b_i \left(\sum_{j \in M} x_{ji} \right) \text{s.t.} & \\ x_{ji} = 0 & \forall j \in M \backslash S_i, \ \forall\, i \in T \\ x_{ji} \geq 0 & \forall j \in S_i, \ \forall i \in T \\ \sum_{i \in T \cap R_j} x_{ji} \leq e_j - \sum_{i \in R_j \backslash T} x_{ji}^* & \forall\, j \in S_T \end{array} \tag{9}$$

Programs (9) and (7) have same objective and control variables. They differ only on the last set of constraints. Since $\sum_{j \in S_{T^c}} \sum_{i \in T^c} x_{ji}^* \leq \sum_{j \in S_{T^c}} \sum_{i \in T^c} x_{ji}^c$, there exists a feasible satiated allocation

$\boldsymbol{X}^c_{T^c}$ such that $\sum_{i\in R_j\setminus T} x^*_{ji} \leq \sum_{i\in R_j\setminus T} x^c_{ji}$ for every $j \in S_T$. The last constraint is thus more stringent in (7) than in (9), which implies that the objective of the program is not lower:

$$\sum_{i\in T} b_i(x^*_i) \geq v(T).$$

Uniqueness is shown by observing that if $\boldsymbol{x}^* = \hat{\boldsymbol{x}}$, the stand-alone welfare of any single agent is her or his peak benefit: $v(i) = b_i(\hat{x}_i)$ for $i = 1, ..., n$. It is due to the fact that the residual demand of the others agents allows agent i to extract her or his peak consumption. The free-access lower bound for i writes $u_i = b_i(x^*_i) + t_i = b_i(\hat{x}_i) + t_i \geq b_i(\hat{x}_i)$ for every $i \in N$. It implies $t_i \geq 0$ for every $i \in N$ which, together with budget balancing $\sum_{i\in N} t_i = 0$, implies $t_i = 0$ for every $i \in N$. ■

The no-transfers solution assigns to any coalition its welfare with the efficient consumption plan. It is also the highest welfare that the coalition can achieve if outsiders extract the efficient consumption levels. If outsiders could extract more than the efficient consumption levels, less is left to the members of the coalition. It is precisely what users expect under free-access: outsiders extract all pools they are connected to up to satiation. Therefore the welfare of a coalition under free-access cannot be higher than with the no-transfers solution. The two welfares coincide if all users can consume their satiated consumption level, formally if $\boldsymbol{x}^* = \hat{\boldsymbol{x}}$. Hence any solution with strictly positive transfers would give to some users strictly more than their satiated benefits. It implies than others would obtain strictly less and thus less than their free-access welfare.

3.3 The equalitarian solution under the free-access lower bounds

We now consider the case of same preferences on the resource $b_i = b$ for all $i \in N$. Due to diminishing return in resource use, the maximal benefit would be achieved under equal division of the resource. It would also be fair as it would equalize welfare. Unfortunately, it is not feasible due to limited access: users connected to fewer pools are not able to consume and equal share of resource. Indeed, as already mention, the efficient consumption plan specify same consumption for users in subset N_k but less than for users in N_{k+1} for $k = 1, ..., K$. Nevertheless, differences in resource consumptions and, therefore, in benefits from resource use could be canceled out through transfers. Unfortunately, the egalitarian division of the welfare violates the free-access bounds. For instance, in the example form Figure 4, agent 1 refuses the egalitarian solution because he or she could get $b(8) > v(N)/3$ by extracting pool 1 he or she fully controls. Similarly, in the example in Figure 3, agent 1 and 2 would refuse any solution that yields then less that $b(7)$ for each of them, which is the welfare they can achieve by sharing the two pools they control.

We thus need to consider the free-access lower bounds as constraints on transfers designed under the aim of equalizing welfare. It is the well-known equalitarian solution under participation constraints (here the free-access lower bounds) proposed by Dutta and Ray (1989). A welfare distribution $\boldsymbol{u}$ is egalitarian under participation constraint if no other welfare distribution that satisfies the participation constraints Lorenz dominates the welfare distribution $\boldsymbol{u}$. Our participation constraints are the free-access lower bounds. We define the concept egalitarianism under the free-access lower bounds on solutions $(\boldsymbol{x}, \boldsymbol{t})$ rather than on welfare distributions. For a given solution $(\boldsymbol{x}, \boldsymbol{t})$, let us relabel $\boldsymbol{u}$ from the poorest to the richest agent: $u_1 \leq u_2 \leq .... \leq u_n$.

Egalitarian solution under the free-access lower bounds The solution $(\boldsymbol{x}, \boldsymbol{t})$ with welfare distribution $\boldsymbol{u}$ is egalitarian under the free-access lower bounds if here is no solution $(\boldsymbol{x}', \boldsymbol{t}')$ with welfare distribution $\boldsymbol{u}'$ that satisfies the free-access lower bounds and such that

$$\sum_{i=1}^{j} u'_i \geq \sum_{i=1}^{j} u_i$$

for $j = 1, ..., n$ with at least one strict inequality.

It turns out that the egalitarian solution under the free-access lower bounds forbids any redistribution of welfare.

PROPOSITION **2** *With homogeneous benefits, the no-transfers solution is the egalitarian solution under the free-access lower bounds.*

Proof: Let us denote the consumption of users in N_k as x^k in the efficient consumption plan $\boldsymbol{x}^*$ for $k = 1, ..., K$. We know from Proposition 1 that the no-transfers welfare distribution $u_i^0 = b(x_i^*)$ for $i = 1, ..., n$ satisfies the free-access lower bounds. I show that any welfare distribution with (non-zero) transfers that satisfies the free-access lower bounds is Lorenz-dominated by the no-transfers welfare distribution. Suppose not. Suppose $\boldsymbol{u}^0$ does not dominate a welfare distribution $\boldsymbol{u}'$ that satisfies the free-access lower bounds. Furthermore, assume without loss of generality that $\boldsymbol{u}'$ is not dominated by any other welfare distribution which satisfies the free-access lower bounds. Let us rank users according to their welfare in $\boldsymbol{u}^0$ and $\boldsymbol{u}'$ from the poorest 1 to the richest n. Then $\exists j$ such that

$$\sum_{i=1}^{j} u'_i > \sum_{i=1}^{j} u_i^0 \tag{10}$$

Since $\boldsymbol{u}^0$ and $\boldsymbol{u}'$ are welfare distributions, we have:

$$\sum_{i=1}^{n} u_i = \sum_{i=1}^{n} u_i^0 = v(N) = \sum_{k=1}^{K} |N_k| b(x^k). \tag{11}$$

Clearly j cannot be the richest user n because then (10) contradicts (11). Suppose $j = n - 1$. Equations (10) and (11) imply $u'_n < u_n^0 = b(x^K)$ where the last equality is due to the definition of $\boldsymbol{u}^0$. The free-access lower bound for coalition N_K yields:

$$\sum_{i \in N_K} u'_i \geq |N_K| b(x^K)$$

Then $\exists l \in N$ with $u'_l \geq b(x^K) > u'_n$ which contradicts that n is the wealthier user. Hence $u'_n \geq u_n^0 = b(x^K)$ and $j < n - 1$. Moreover, if $u'_n > u_n^0$, then one can define a welfare distribution $\boldsymbol{u}''$ with $u''_n = u_n^0$ and $u'_n - u_n^0$ distributed to the other poorest users so as to satisfy the free-access lower bounds. Then $\boldsymbol{u}''$ Lorenz dominates $\boldsymbol{u}'$ a contradiction. Therefore $u'_n = u_n^0 = b(x^K)$. The same argument shows that $u'_i = b(x^K) = u_i^0$ for the $|N_K|$ richest users and $j \leq n - |N_K|$. Hence, equation (11) becomes:

$$\sum_{i=1}^{n-|N_K|} u'_i = \sum_{i=1}^{n-|N_K|} u_i^0 = \sum_{k=1}^{K-1} |N_k| b(x^k)$$

Proceed as before to show that $u_i' = b(x^{K-1}) = u_i^0$ for the $|N_{K-1}|$ richest users and $j \leq n - |N_K| - |N_{K-1}|$. And so forth for $k = K-2, ..., 1$. ■

With same benefits, users extracting from same pools enjoy same resource consumption. More precisely, the efficient consumption plan divide the set of users N and resource pools N into subsets N_k and M_k for $k = 1, ..., K$ where users in N_k enjoy same consumption. They share pools in M_k. Moreover, users in k consume more than users in $k-1$ for $k = K, ..., 2$. Denoting x^k the consumption of users in N_k for $k = 1, ..K$. Users in N_K are the wealthiest. The benefit $b(x^K)$ of each member of N_K is higher then $b(x^{K-1})$ enjoyed by member of N_{K-1} who are wealthier than users in N_{K-2} and so forth for $k = K-3, .., 1$. On the other hand, according to Lemma 1, users in $\cup_{l=k}^{K} N_l$ are extracting resource only from resource pools they fully control for $k = 1, ..., K$. It means that, without transfer of welfare, they obtain exactly their free-access lower bounds. For instances, users in N_K enjoy together $|N_K| b(x^K)$ which is exactly what they would enjoy by sharing the pools in M_K they fully control. They would get strictly less if they transfer part of their welfare to the other users. Hence any welfare distribution with net positive transfers from users in N_K to the others would violate the free-access lower bounds. On the other hands, since users in N_K are the wealthiest, inequality is exacerbated if users in N_K receive net transfers from others. Equalizing the welfare among users in N_K would assign them exactly their benefit $b(X^K)$ which is precisely what the no-transfers welfare distribution prescribes. The same argument applies for coalition of users N_{K-1} to N_1.

Example In the example represented in Figure 1, the no-transfers solution is egalitarian. It satisfies participation because the free-access individual welfare $v(A) = b(1)$ and $v(B) = v(C) = 0$ are always lower than the egalitarian welfare $b(5)$. Moreover, agents are worse-off by sharing the pools they fully control since they get $v(AB) = 2b(7/2)$, $v(BC) = 2b(4)$ and $v(AC) = b(1)$. It also satisfies the exclusive-access upper bounds because no users obtain more than her exclusive-access welfare $w(A) = b(7)$, $w(B) = b(14)$ and $w(C) = b(8)$. Moreover, no coalition of users obtain more than its exclusive-access welfare $w(AB) = b(7) + b(8) = w(AC)$ and $w(BC) = b(6) + b(8)$. In Figure 3, the egalitarian welfare distribution shares equally the total welfare $v(ABC) = b(1) + 2b(7)$, each agent obtains $1/3v(ABC)$. Agents A et B would block the egalitarian solution because they can achieve $v(AB) = 2b(7) > 2/3v(ABC)$ (the strict inequality is due to the concavity of b) by sharing optimally the two pools they fully control. Participation forces to assign at least $2b(7)$ to A and B. The egalitarian principle prescribes an equal split of $v(AB)$ so that each of then two player obtain its benefit from efficient extraction $b(7)$. Agent C gets the remaining welfare $v(ABC) - v(AB) = b(1)$ that is its efficient benefit.

Proposition 2 provides a characterization of the no-transfer solution welfare under homogeneous benefits. Under heterogeneous benefits, being egalitarian, is controversial because users might be (at least partly) responsible for their benefit function. For instance, they might be more productive in using resources because they have invested in the past in water saving technologies, energy efficiency or pollution abatement. They should therefore get a fair return from their investment. We now turn to the case of heterogenous and satiated benefits. We examine the resource division problem with a progressive supply of resource.

3.4 Sequential resource supply

In many real-words problem, natural resources are supplied sequentially across time. Surface and underground water are filled-up with random precipitations along the year. Water rights and quotas are defined within periods of time during the growing seasons and adjusted according to water supply. Similarly, emission allowances are issued for one phase of several years and then reset for the next phase. In the same vein, the world reserve of fossil fuels is evolving as new fields are discovered and exploited every year. Technological progress enlarge the set of resources that can be exploited from fossil fuel to minerals.

I extend the model by considering a resource supply evolving across time. I consider two criteria that account for the sequential supply of resource. The first one is a generalization of the free-access lower bounds to sequential resource supply. It requires that the solution satisfies the free-access lower bound not only in the current resource sharing problem but also for future ones. The second criteria is a consistency requirement on resources.[6] It states that the solution should not depend on the timing of resource supply. For both principles, I provide characterization results when benefits can be satiated in sense $\hat{x}_i < +\infty$ for all $i \in N$. To state formally those principles, I need a new peace of notation. For any given problem, $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ and its efficient consumption plan $\boldsymbol{x}^*$, let us denote by $\tilde{\boldsymbol{b}}$ the benefits function issues after assigning efficiently the amount of resource $\boldsymbol{e}$: $\tilde{b}_i(a) = b_i(x_i^* + a)$ for every $a \in \mathbb{R}_+$ for $i = 1, ..., n$.

Free-access lower bounds with sequential supply: A solution $(\boldsymbol{x}, \boldsymbol{t})$ to the problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ satisfies the free-access lower bounds under sequential supply if, for any $\boldsymbol{e}' \geq 0$, there exists a solution $(\boldsymbol{x}', \boldsymbol{t}')$ to the problem $\mathcal{P}(\tilde{\boldsymbol{b}}, \boldsymbol{e}')$, such that $(\boldsymbol{x}', \boldsymbol{t}')$ and $(\boldsymbol{x} + \boldsymbol{x}', \boldsymbol{t} + \boldsymbol{t}')$ satisfy the free-access lower bounds of the problems $\mathcal{P}(\tilde{\boldsymbol{b}}, \boldsymbol{e}')$ and $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e} + \boldsymbol{e}')$ respectively.

Proposition **3** *When benefits can be satiated, the no-transfer solution is the only one to satisfy the free-access lower bounds with sequential supply.*

Proof: Consider any problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$. Let $(\boldsymbol{x}, \boldsymbol{0})$ be the no-transfer solution to $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$. First, by Proposition 1, the no-transfer rule picks a solution to the problem $\mathcal{P}(\tilde{\boldsymbol{b}}, \boldsymbol{e}')$ that satisfies the free-access lower bounds. Second, by definition, if $\boldsymbol{x}$ and $\boldsymbol{x}'$ are efficient in the respective problems $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ and $\mathcal{P}(\tilde{\boldsymbol{b}}, \boldsymbol{e}')$, the consumption plan $\boldsymbol{x} + \boldsymbol{x}'$ is efficient in $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e} + \boldsymbol{e}')$. Therefore the solution $(\boldsymbol{x} + \boldsymbol{x}', \boldsymbol{0})$ obtained by assigning sequentially the no-transfer solution $(\boldsymbol{x}, \boldsymbol{0})$ and $(\boldsymbol{x}', \boldsymbol{0})$ is the no-transfer solution for $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e} + \boldsymbol{e}')$. By Proposition 1, it satisfies the free-access lower bounds in $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e} + \boldsymbol{e}')$.

I now show uniqueness. Suppose that $(\boldsymbol{x}, \boldsymbol{t})$ is a solution to $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ that satisfies the free-access lower bounds with sequential supply. For $\boldsymbol{e}' = \boldsymbol{0}$, it implies that $(\boldsymbol{x}, \boldsymbol{t})$ satisfies the free-access lower bounds and, therefore, the consumption plan $\boldsymbol{x}$ is efficient. Take $\boldsymbol{e}' = \hat{\boldsymbol{x}} - \boldsymbol{e}$ so that the efficient consumption plan with $\boldsymbol{e} + \boldsymbol{e}'$ is the satiated levels $\hat{\boldsymbol{x}}$. By Proposition 1, the solutions $(\boldsymbol{x}', \boldsymbol{t}')$ and

[6] For a discussion on the philosophy foundations of the consistency principles see Thomson (2012).

$(\boldsymbol{x}+\boldsymbol{x}', \boldsymbol{t}+\boldsymbol{t}')$ to $\mathcal{P}(\tilde{\boldsymbol{b}}, \boldsymbol{e}')$ and $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e}+\boldsymbol{e}')$ must be the no-transfer solutions. It implies $\boldsymbol{t}' = \mathbf{0}$ and $\boldsymbol{t}+\boldsymbol{t}' = \mathbf{0}$ which, in turn, imply $\boldsymbol{t} = \mathbf{0}$. ■

Although the free-access lower bounds with sequential supply are required for a given solution $(\boldsymbol{x}, \boldsymbol{t})$ to a problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$, resource consistency is defined for allocation rules. An allocation rule ϕ is a mapping from the set of resource sharing problems to the set of solutions. It prescribes a solution for any problem. With a slight abuse of notation, we denote ϕ as a function only of $(\boldsymbol{b}, \boldsymbol{e})$. The solution prescribed to problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$ by the rule is thus denoted $\phi(\boldsymbol{b}, \boldsymbol{e}) = (\boldsymbol{x}, \boldsymbol{t})$.

Resource Consistency: Consider any arbitrary resource sharing problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$. The allocation rule ϕ satisfies resource consistency if, for any $\boldsymbol{e}' \geq 0$, $\phi(\boldsymbol{b}, \boldsymbol{e}) + \phi(\tilde{\boldsymbol{b}}, \boldsymbol{e}') = \phi(\boldsymbol{b}, \boldsymbol{e}+\boldsymbol{e}'')$.

PROPOSITION **4** *When benefits can be satiated, the no-transfers rule ϕ^0 is the only consistent rule that prescribes solutions satisfying the free-access lower bounds.*

Proof: Consider any arbitrary resource sharing problem $\mathcal{P}(\boldsymbol{b}, \boldsymbol{e})$. Suppose that ϕ satisfies the free-access lower bounds and resource consistency. By the free-access lower bound for coalition N, the consumption plan prescribed by ϕ must be efficient. I show that transfers must be nil. Suppose not. Suppose that $t_i > 0$ for some agent i. Take $\boldsymbol{e}' = \hat{\boldsymbol{x}} - \boldsymbol{e}$ so that the efficient consumption plan with $\boldsymbol{e}+\boldsymbol{e}'$ is the satiated levels $\hat{\boldsymbol{x}} = \boldsymbol{x}'' = \boldsymbol{x}'$ where the last equality is due to the definition of $\tilde{\boldsymbol{b}}$. By Proposition 1, we must have $\boldsymbol{t}' = \boldsymbol{t}'' = \mathbf{0}$. Resource consistency implies $\boldsymbol{t}'' = \boldsymbol{t}+\boldsymbol{t}'$ which, combined with the previous equalities, implies $\boldsymbol{t} = \mathbf{0}$. It is straightforward no show that the no-transfer rule satisfies consistency. ■

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