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Optimal climate policy when warming rate matters

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Abstract

Studies of the Social Cost of Carbon assume climate change is a stock externality for which damages stem from warming level. However, economic and natural systems are also sensitive to the *rate at which warming occurs*. In this paper, I study the optimal carbon tax when such a feature is accounted for. Damages caused by warming rates do not affect optimal long-term warming, but they delay the use of the same carbon budget. To achieve this, carbon price should start higher, and increase slowlier than in the case of damages stemming from the level of warming. This result holds numerically when controlling for the welfare loss from climate change. This suggests that mitigation strategies that overlook effects from warming rate might lead to too rapidly increasing temperature pathways.

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Introduction

Many human activities, in particular the burning of fossil fuels, release greenhouse gases that warm up the atmosphere and cause damage to the economy. These damages in economic analysis are typically considered to be a stock externality, driven by temperature anomaly, or the stock of atmospheric carbon dioxide, that is the "level" of climate change. However, economic and natural systems are not only sensitive to the level of change, but also to *the rate at which it occurs*, for instance because rapid changes constrain adaptation and thus induce greater damages. Failing to account for this sensitivity to warming rate may favor emission pathways for which global temperature increases too fast, given economies' ability to cope with the change.

There is evidence that the speed of change plays a key role in the way ecological, climate and human systems will be affected by temperature change. If ecosystems have been confronted to different climatic conditions in the past, what makes climate change so concerning is the never-seen rate at which it is occurring. More rapid rates of change limit the ability of natural systems to adapt (LoPresti et al., 2015; Hoegh-Guldberg et al., 2007; Gilman et al., 2008; Maynard et al., 2008; Malhi et al., 2009; Thackeray et al., 2010). Conversely, slower rates of change give ecosystem the time to adapt to new environmental conditions (either through behavioral or genetic changes) or to migrate in search for more favourable climates. A study suggests that for 30% of Earth, plant species would not be able to migrate to keep pace with projected climate change (Loarie et al., 2009). The importance of the rate of change holds in particular for systems with significant inertia, such as vegetation or soil carbon stores (Jones et al., 2009; Sihi et al., 2018). Coral reefs may also not be able to adapt to rapid rates of change (Maynard et al., 2008; Hoegh-Guldberg, 2009), because the rate of carbon absorption by the deep ocean is limited (Lenton et al., 2008).

Rapid rates of change can also contribute to trigger non-linear dynamics in the climate system, also referred to as 'tipping points' (Lenton, 2012; Levermann and Born, 2007; Steffen et al., 2018; Wieczorek et al., 2011). For instance, the stability of thermohaline circulation, as it involves water circulation flow and thus the melting rate of glacier, is sensitive to both warming level and rate of change (Stocker and Schmittner, 1997; Marotzke, 1996). A warming of 0.3 °C per decade sustained over a century could lead to a collapse in thermohaline circulation, while the same warming of 3°C reached with slower rates of change would only lead to a slowdown.

For economies too, climate damages may stem both from a changed climate and from a changing climate. Faster changes induce greater costs or less efficient adaptation (Huntingford et al., 2008; Stafford Smith et al., 2011; New et al., 2011; Smit and Wandel, 2006). For decisions involving long timescales, such as urbanisation plans, tranportation, building, or forestry, faster rates of change imply that infrastructures will be confronted to a larger range of climate conditions, which makes their design more difficult and construction more expensive (Hallegatte, 2009; Fankhauser and Soare, 2013). Slower rates of change also allow for more sequential decision making and to use capital more efficiently, while rapid change would force economies to retire productive capital sooner. Conversely, some of the damages may be reduced once the climate has stabilized, and that economies have adapted to new climate conditions, for instance, through the use of air conditioners, changes in crop varieties or behavioral adaptations such as changes in work hours. This is consistent with recent empirical analysis suggesting that economic damage is driven by a deviation from experienced temperatures in past decades, rather than by temperatures themselves (Kahn et al., 2019; Kalkuhl and Wenz, 2020), and thus may fade away once temperatures have stabilized. Finally, institutional barriers may also limit the ability of societies to react efficiently to rapid changes. Damages from warming rates reflect transitional adaptation costs of a changing climate, while damages from warming levels are persistent losses due to a changed climate.

It has been argued in the scientific literature that climate change action should also seek to constrain the rate of change (O'Neill and Oppenheimer, 2004; Bowerman et al., 2011; Kallbekken et al., 2009). In the economic literature on climate change however, the role of the rate of change is rarely accounted for. Environmental externalities are usually considered as either a stock or a flow externality (Farzin, 1996; Ulph and Ulph, 1994; Van Der Ploeg and Withagen, 1991), with climate change belonging to the former category.

Both DICE, the most widely used numerical Integrated Assessment Model (IAM), and recent analytical models of the climate and the economy all assume that damages stem from the level of warming or the stock of atmospheric carbon dioxide (Golosov et al., 2014; Gerlagh and Liski, 2018; Dietz and Venmans, 2019), leaving aside the influence of the speed of warming. In other numerical IAMs, such as FUND (Tol, 1996) or PAGE (Hope et al., 1993), damage depend both on level and rate of change in some sectors, but the authors did not analyse how the combination of both types of damages affected the outcomes. A few studies in the 1990s have compared damage from warming level and warming rates, either in a numerical IAM (Peck and Teisberg, 1994) or in an analytical model (Tahvonen, 1995; Hoel and Isaksen, 1995), suggesting that both types of damages require different optimal climate policies. However, they do not look at the case of damages being caused by a combination of level and rate of change.

In this paper, I analyse how damage caused by both warming level and warming rate affect optimal climate policy. To do so, I use an analytical model of the climate and the economy building on Dietz and Venmans (2019), in which I add the feature that damage also depend on warming rate. I show that accounting for damages from warming rate does not change the long-term optimal temperature, compared to the case when damages depend solely on the warming level. However, it warrants different emission trajectories. When damages from warming rates are factored in, carbon price is greater, but increase less rapidly. Then, I explore combination of parameters for both types of damage leading to the same welfare loss from climate change. Less damage coming from the level of change results in higher long-term temperature. However, the effect on carbon price in the short run is offset by the countervailing influence of higher damages from warming rate, which provides incentives to slow down the warming. Thus, even when controlling for the welfare loss, in the short-run, damages from warming rates lead to higher carbon price.

In section 1, I present the model and derive optimal climate policy. In section 2, I explore numerically how damages from warming rate and warming level affect the outcomes. Section 3 discusses implications, perspectives and concludes.

1 Model

I build upon the model in Dietz and Venmans (2019) to analyze optimal climate policy when the warming rate induces damage. This choice is motivated by their representation of the climate system, which is in line with recent results from the climate science that after a short adjustement period of ten years, the ratio of warming on cumulated emissions is independent of both time and cumulated emissions (Matthews et al., 2009; Solomon et al., 2009; Mattauch et al., 2019).

1.1 Setting

Let us assume an economy, producing Q using three inputs, capital K, labour L and emissions E. Labour and total factor productivity grow exogenously, respectively at rate n and g. Warming T caused by emissions reduces production. In addition to the exponential quadratic-damage function of warming levels T, I consider a symmetrical damage factor capturing that warming rate $\dot{T}$ reduces output.

$$Q = e^{(n+g)t} f(K) exp\left(-\frac{\gamma}{2}T^2 - \frac{\alpha}{2}\dot{T}^2\right) exp\left(\Phi E - \frac{\varphi}{2}E^2\right) \tag{1}$$

α and γ determine the sensitivity of economies repectively to warming level and warming rate. The case $\alpha = 0$ is the special case of economies only affected by warming levels considered in Dietz and Venmans (2019), and more generally in the climate-economy literature.

Agents derive utility from their consumption $u(c)$, and the social planer, assumed to be utilitarian, seeks to maximize the present discounted social welfare, written as follows:

$$max_{c,E} W = \int_0^\infty e^{(n-\rho)t} u(c) dt \tag{2}$$

Where ρ is the rate of pure time preference, at which future utility is discounted, and utility is isoelastic, given by:

$$u(c) = \frac{c^{1-\eta}}{1-\eta} \tag{3}$$

η is the resistance to intertemporal substitution, which drives intergenerational inequality aversion.

As discussed above, in line with recent scientific findings, I assume quasi-linearity between cumulative emissions and warming:

$$\dot{T} = \epsilon(\zeta S - T) \tag{4}$$

where ϵ is the initial pulse-adjustment timescale, and ζ reflects the Transient Climate Response to Cumulative Carbon Emissions.

The part of production that is not consumed adds up to the capital stock k, but the stock also depreciates at rate δ. Thus, following the convention to write variables divided by effective labour $e^{(n+g)t}$ with a hat, capital follows the dynamical equation:

$$\dot{\hat{k}} = \hat{q} - \hat{c} - (\delta + n + g)\hat{k} \tag{5}$$

As in Dietz and Venmans (2019), it is reasonable, given the orders of magnitude at stake, to consider that the economy is on a balanced growth path with constant growth of output per capita as long as the damage from warming rates has a small effect on the growth rate.

1.2 Optimal path

To determine the evolution of optimal abatement, we can write the Hamiltonian of the welfare maximization problem:

$$H = \frac{\hat{c}^{1-\eta}}{1-\eta} - \lambda^S E - \lambda^T \epsilon(\zeta S - T) + \lambda^{\hat{k}} \left[\hat{q}(\hat{k}, E, T) - \hat{c} - (\delta + n + g)\hat{k}\right] \tag{6}$$

Optimality conditions lead to:

$$\lambda^S = \hat{c}^{-\eta}\hat{q}(\Phi - \varphi E) \tag{7}$$

$$\dot{\lambda}^S = (\rho - n + g(\eta - 1))\lambda^S - \epsilon\zeta\lambda^T - \hat{c}^{-\eta}\hat{q}\alpha\epsilon^2\zeta(\zeta S - T) \tag{8}$$

$$\dot{\lambda}^T = (\rho - n + g(\eta - 1) + \epsilon)\lambda^T - \hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T)) \tag{9}$$

$$\hat{q}_{\hat{k}} - \delta = \eta(\frac{\dot{\hat{c}}}{\hat{c}} + g) + \rho \tag{10}$$

Integrating equation 9 gives:

$$\lambda^T = \int_t^\infty e^{-(\rho - n + g(\eta-1) + \epsilon)(u-t)}\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))du \tag{11}$$

Given that the climate system adjusts quickly to emissions ($\epsilon \approx 0.5$), the discount rate applied to the marginal disutility of temperature change is high (around 50%). Thus, we can consider that the integral is dominated by the short-term of a few years, and over this period, $\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))$ is constant:

$$\lambda^T \approx \frac{\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} \tag{12}$$

Coming back to equation 8

$$\dot{\lambda}^S = (\rho - n + g(\eta - 1))\lambda^S - \epsilon\zeta\frac{\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} - \hat{c}^{-\eta}\hat{q}\alpha\epsilon^2\zeta(\zeta S - T) \tag{13}$$

Deriving the equation in λ^S, together with the assumption of a balanced growth paths, lead to:

$$\dot{\lambda}^S = (-\eta\frac{\dot{\tilde{c}}}{\tilde{c}} + \frac{\dot{\tilde{q}}}{\tilde{q}} - \frac{\varphi \dot{E}}{\Phi - \varphi E})\lambda^S \tag{14}$$

$$\dot{E} = [\rho - n + (\eta - 1)g]\,(E - \Phi/\varphi) + \epsilon\frac{\zeta}{\varphi}\frac{(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} + \frac{\alpha}{\varphi}\epsilon^2\zeta(\zeta S - T) \tag{15}$$

The climate system adjusts quickly to CO2, so I treat the growth rate of cumulative emissions as constant in the short run, $\theta = \dot{S}/S$, and I can approximate temperature as follows:

$$T \approx \frac{\epsilon}{\epsilon + \theta}\zeta S \tag{16}$$

Substituting into the equation in $\dot{E}$:

$$\dot{E} = [\rho - n + (\eta - 1)g]\,(E - \Phi/\varphi) + \frac{\zeta^2 S\epsilon^2}{\varphi(\epsilon + \theta)}\left(\frac{\gamma - \alpha\epsilon\theta}{\rho - n + \epsilon + g(\eta - 1)} + \alpha\theta\right) \tag{17}$$

Finally, since $\dot{S} = E$, we can write:

$$\ddot{S} = [\rho - n + (\eta - 1)g]\,\dot{S} + \frac{\zeta^2\epsilon^2}{\varphi(\epsilon + \theta)}\frac{\gamma + \alpha\theta(\rho - n + g(\eta - 1))}{\rho - n + \epsilon + g(\eta - 1)}S - [\rho - n + (\eta - 1)g]\,\frac{\Phi}{\varphi} \tag{18}$$

I obtain a second-order differential equation for cumulative emissions. $\rho - n + (\eta - 1)g$ is the discount rate applied to the marginal damages as a proportion of output. Compared to the case of level-only damages, the only coefficient that is different is the one before S.

In the long-term, $\theta = 0$, so it is clear that the optimal cumulative emission and optimal peak warming is unchanged compared to a case where only level damage matter $S^* = c/b = S^*_{level}$. It follows that optimal temperature levels are also identical:

$$T^* = T^*_{level} = \frac{\rho - n + \epsilon + g(\eta - 1)}{\epsilon}\frac{(\rho - n + (\eta - 1)g)\Phi}{\zeta\gamma} \tag{19}$$

However, the dynamics of abatement changes when economies are also affected by warming rates. The factor in $\alpha\theta$ slows down the convergence to the long-term equilibrium, reflecting damages from warming rates as long as temperature changes.

In order to compare dynamics between our case and the classical case of damages depending solely on temperature level, I assume linearity between cumulative emissions and temperature in the next section.

1.3 Closed-form solution assuming no climate delay

In this section, I assume that temperature responds instantaneously to cumulative emissions, in order to obtain closed-form solutions. This simplifications rests on the fact that the climate system adjusts rapidly (within 10 years) to changes in cumulated emissions ($\epsilon = 0.5$). There is also evidence that the maximum of emissions levels is linked to the maximum warming rate (Bowerman et al., 2011), so a linear model could be an acceptable first-order representation for our purpose. $T = \zeta S$.

The damage factor describing the sensitivity of production to warming rate writes: $exp(-\frac{\alpha}{2}\dot{T}^2) = exp(-\frac{\alpha}{2}\zeta^2 E^2)$.

$$Q = e^{n+g} f(\hat{k}) exp(-\frac{\gamma}{2}T^2 - \frac{\alpha\zeta^2 + \varphi}{2}E^2 + \Phi E) \tag{20}$$

Appendix A demonstrates that the stock of carbon follows:

$$\ddot{S} = (\rho - n + (\eta - 1)g)\dot{S} + \frac{\zeta^2\gamma}{\varphi + \alpha\zeta^2}S - (\rho - n + (\eta - 1)g)\frac{\Phi}{\varphi + \alpha\zeta^2} \tag{21}$$

Writing the equation as $\ddot{S} = a\dot{S} + bS - c$, and comparing it the case of level-only damages, we have: $a = a_{level}$, $b = b_{level}\varphi/(\varphi + \alpha\zeta^2)$, and $c = c_{level}\varphi/(\varphi + \alpha\zeta^2)$. As expected, S convergences towards the same equilibrium, the time profile of cumulative emission is given by:

$$S_t = (S_0 - c/b) exp \frac{1}{2} t(a - \sqrt{a^2 + 4b}) + c/b \tag{22}$$

Thus, emissions write:

$$E_t = (c/b - S_0)\frac{1}{2}(\sqrt{a^2 + 4b} - a) exp \frac{1}{2} t(a - \sqrt{a^2 + 4b}) \tag{23}$$

The optimal carbon price is the optimal marginal abatement cost for producers, which do not internalize the climate change externality:

$$p^* = Q_0 e^{(\tilde{g}+n)t}(\Phi - \varphi E) \tag{24}$$

Initially the carbon price is given by:

$$p_0^* = Q_0 \left(\Phi - \varphi((\rho - n + (\eta - 1)g)\frac{\Phi}{\zeta^2\gamma} - S_0)\frac{\sqrt{(\rho - n + (\eta - 1)g)^2 + 4\frac{\zeta^2\gamma}{\varphi + \alpha\zeta^2}} - (\rho - n + (\eta - 1)g)}{2} \right) \tag{25}$$

Since $b < b_{level}$, initially, carbon price is higher than in the level-only case. However, it increases less rapidly to reach the same long-term trajectory as in the level-only case. The reverse occurs for emissions, with a lower start but slowlier decrease, so that the same carbon budget is just spread over time.

This result comes from the fact that damages from warming reduce the rate at which marginal productivity of emissions decreases (formally equivalent to a change in φ), because of the flow externality they represent. However, they do not change the marginal productivity of the first emission. Alternatively, if the damage factor was an exponential-linear function of the warming rate, it would decrease the marginal productivity of the first emission (akin to a change in Φ). Under such assumption, optimal long-term temperature would be lower.

To put things into perspective we can compare the dynamics of the carbon price to other models. In Golosov et al. (2014), with an exponential-linear level damage, carbon price grows as fast as the economy. With exponential-quadratic level damages, Dietz and Venmans (2019) find that this growth is enhanced in the short-run. Further assuming, as we do, that damages also depend on warming rate tends to raise initial carbon price, but moderate the short-term growth.

2 Application

2.1 Illustrative pathways

In this section, I propose a numerical application of the model, to evaluate the size of the effect. Assessing future level-damages is a challenging exercize, because of the diversity of impacts that climate change will induce and the many uncertainties surrounding them (Diaz and Moore, 2017; Auffhammer, 2018). The same limitation applies to the assessment of rate-dependent damages. The difficulty is compounded because impact assessments are typically based on damages at a given temperature level (see for instance recent review of impact estimates (Nordhaus and Moffat, 2017; Howard and Sterner, 2016; Tol, 2018)), and thus do not quantify the effect of the rate of change. However, we can use results from Kahn et al. (2019) to illustrate possible orders of magnitude. In the study, the impacts come from temperature deviation from its average in past decades, so it is equivalent to assuming that there is only a rate effect ($\gamma = 0$). In the central projections, output losses in 2100 due to a warming rate of 0.01 and 0.04 °C/year are respectively 1.1% and 7.2%. This is respectively consistent with α of 93 and 221. Note that the values of losses from warming rates considered in Peck and Teisberg (1994) correspond to α in the same order of magnitude, between 60 (a 2% loss brought by 0.015°C/year increase) and 180 (a 2% loss brought by a 0.025°C/year).

Figure 1 illustrates the influence of parameters specifying rate and level-damages on the optimal carbon price. Unless specified otherwise, all other parameters are calibrated as in Dietz and Venmans (2019). Figure 2 compares the temporal evolution of carbon prices to the case of damages solely based on warming levels (α=0). As we have seen, damage from warming level determine optimal long-term temperature, while damage stemming from warming rates affects the optimal path to reach the temperature. For the illustrative values proposed here, adding damages from warming rate raises initial carbon price by 25 to 55%, but the carbon price increases slowlier than in the case of level-only damage, so that both trajectories cross between 2200 and 2300. Thus, rate-damage lead to a significant delay in the use of the carbon budget to reach the same temperature target. Importantly, the temporal dynamics of impacts differ between both types of damages. The stock externality from level impacts rises over time as temperature increases. On the contrary, damages from warming rates hit economies early on and vanish as temperatures stabilize. Damages from warming rates therefore make carbon price less sensitive to discounting assumptions (see figure 3). For instance, for the central value of γ, when there is no damage from warming rates, moving the pure rate of time preference from 1 to 3% leads to a threefold reduction in carbon prices. However, if we assume $\alpha = 100$, the same change in time preference only leads to a 30% reduction of carbon prices.

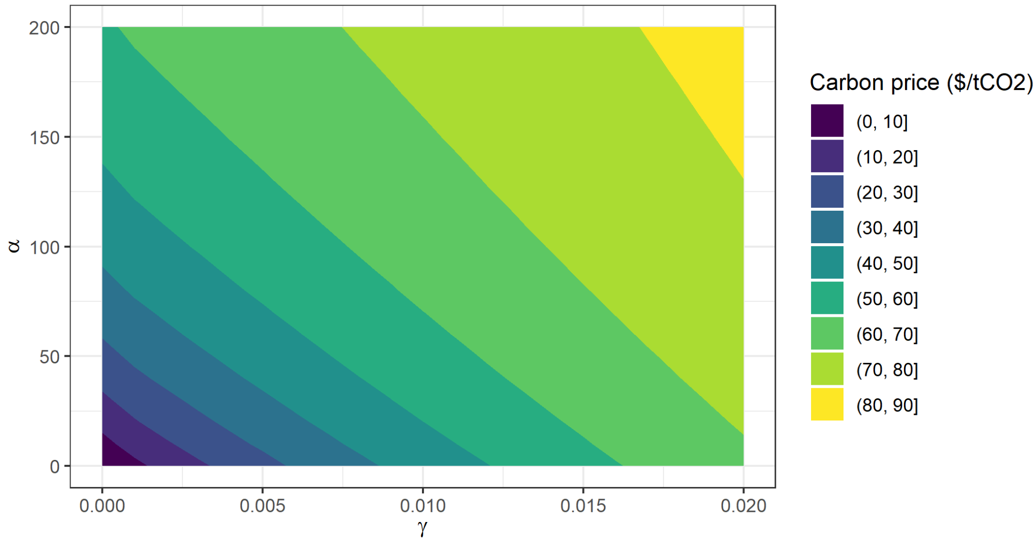

Figure 1: Carbon price for different values of parameters defining damages from warming level (γ), and warming rate (α)

2.2 Level vs Rate damages: a controlled comparison

To highlight how the balance between the two channels of damage influence optimal policy, I explore different sensitivities of the economy to the level and speed of warming affects, keeping 'total damages' constant. This allows to disentangle the structural nature of damages (from warming level or warming rate) from the increased magnitude of damages. To do so, I explore the results under combinations of damage parameters leading to the same welfare loss under a *laissez-faire* scenario. This metric is used notably in Stern (2007), and applied in Guivarch and Pottier (2018) to compare the effect of damage

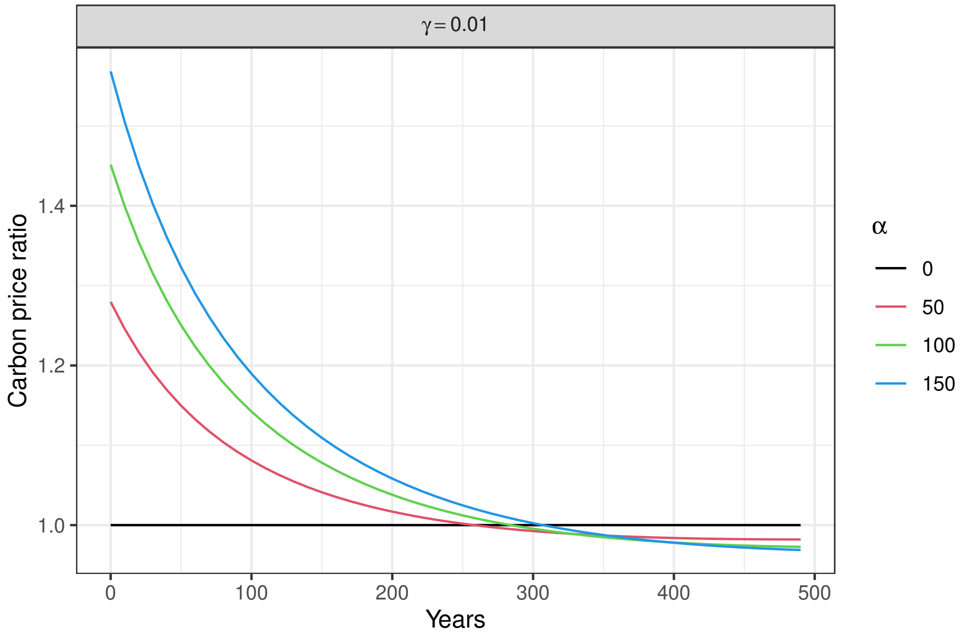

Figure 2: Evolution of carbon price for different values of parameters defining damages from warming level (γ), and warming rate (α)

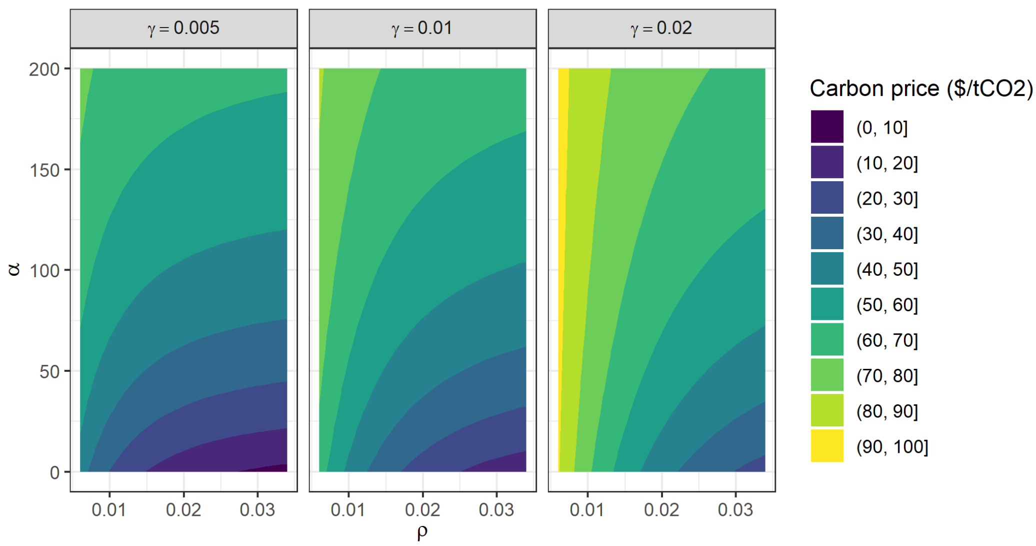

Figure 3: Carbon price sensitivity to pure rate of time preference (ρ), for different values of parameters defining damages from warming level (γ), and warming rate (α)

falling on GDP level and GDP growth. Welfare losses need to be assessed in a laissez-faire scenario so they only reflect damage, and not mitigation costs.

I compute values of α and γ leading to the same welfare loss. I consider three damage strengths corresponding to the case of damages from warming level γ in 0.005, 0.01,0.02. These values make climate damages correspond to a loss of consumption, now and forever, of respectively around 1% (low),2% (medium) and 4% (high). Note that attributing losses to warming rates in the low and medium damage case correspond to values of α of respectively 100 and 200, which are consistent with the orders of magnitude from Kahn et al. (2019).

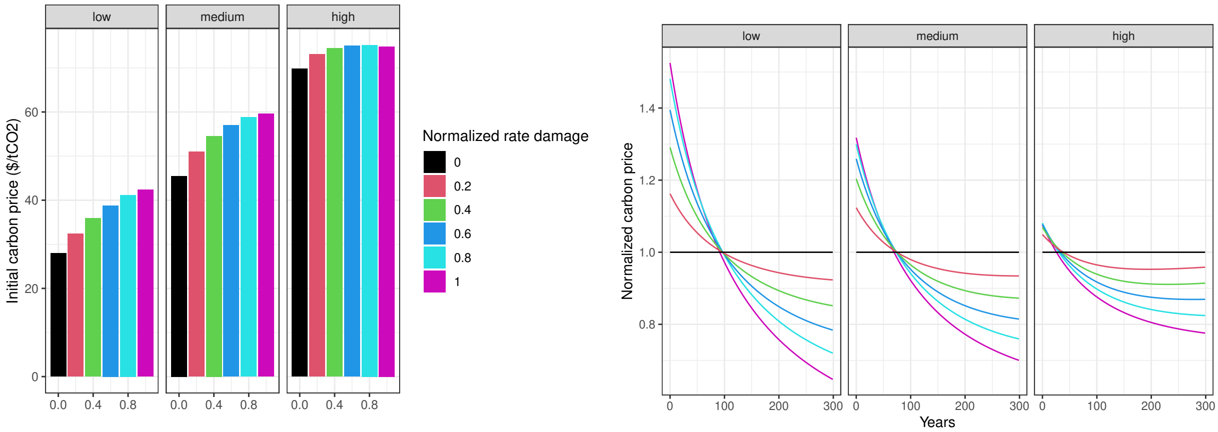

Figure 4: Initial value and evolution of carbon price for a given level of damage strength

In the long-term carbon prices are always lower when damages stem from warming levels (a higher α combined with a lower γ). Indeed, as emissions decrease and temperature gradually stabilize, the level effect dominates (see figure 4). This is directly linked to the fact that damages from warming rates generate a flow externality and are transitional, while damages from warming levels are permanent and so lead to more stringent long-term targets.

However, in the short run, the balance between level and rate damage is *a priori* ambiguous on the carbon price, because α and γ both increase the carbon price. A lower γ is associated with a greater carbon budget, which decreases carbon price. However, a greater α gives more incentives to reduce emissions in the short-run. In the range of values considered, the rate-effect dominates, and thus more damages from warming rates lead to greater carbon price in the short-run. The balance between both types of damages matters more when the damage strength is low, with discrepancies reaching 45% of the value for the initial carbon price. Conversely, whether damages come from warming rates or warming levels only leads to 10% differences in initial carbon price under the assumption of high damage strength. Indeed, under strong welfare impacts, there is a strong incentive to limiting emissions in the short-run, which also limits the rate of warming in the short-run. On the other hand, if the welfare losses from climate damages are rather low, there can be a strong discrepancy between the case of level-only damages (in which warming increases pretty fast in the short-run), and the case of rate-only damages (in which warming rate is contained).

3 Perspective and conclusion

In this article, I argue that the rate of warming plays an important role in assessing damages from climate change. I review the literature to show that both natural and economic systems have limited ability to adapt to rapid changes, thus suggesting that damages depend not only on warming levels, but also on warming rates.

Using an analytical model of the climate and the economy, I show that the damages from the rate of change do not affect optimal long-term temperature change, compared to a case when damages only depend on warming level. This is due to the marginal productivity of the first emission being unchanged under exponential-quadratic damages from the rate of warming. However, the use of the same carbon budget is spread over time. Damages from warming rate warrant higher carbon price in the short-run, but carbon price should increase slowlier. When controlling for the welfare losses under a business as usual scenario, damages from warming rate still require higher carbon price in the short run, even if reduced damages from warming levels ultimately leads to lower carbon price and higher temperature levels in the long-term.

This suggests that mitigation strategies only seeking to contain global temperatures below a certain level, as specified by the Paris Agreement, overlook crucial issues on the timing at which the target should be reached to minimize damages. Although damages from warming rate are only transitional, compared to permanent 'level damages', they are crucial to understand optimal mitigation in the short run. This opens up research avenues to further refine the representation of how different warming rates affect economic and natural systems. I acknowledge that both the functional form of damages and the calibration I use is questionable. For instance, as stated above, assuming that the damage factor from warming rates is exponential-linear, rather than exponential-quadratic, reduces optimal long-term warming. In the formulation I use, warming rate affects output in the next period, while the effects could be more persistent.

Accounting for the sensitivity of economies to warming rate has crucial implications for other climate policy questions, which the simplicity of the model I use does not allow me to deal with. First, the possibility to rely on negative emissions in the future raises the question of assessing overshoot temperature trajectories, in which Earth warms up to a peak before decreasing significantly (Bowerman et al., 2011). Overshoot trajectories have a very different temperature dynamics, in particular with a strong rate of temperature change in the short-term. Thus, accounting for damages from warming rates would probably affect the evaluation of such pathways. Second, given that different greenhouse gas have different lifetimes in the atmosphere, rate-dependent damages can change the trade-off between greenhouse gas (Manne and Richels, 2001), and would provide a stronger case for abating short-lived atmospheric components in the near-term, in order to slow warming rates.

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A Solution for the no-delay

The Hamiltonian of the welfare maximization problem, with this time only two state variables and two control variables is:

$$H = \frac{\hat{c}^{1-\eta}}{1-\eta} - \lambda^S E + \lambda^{\hat{k}} \left[\hat{q}(\hat{k}, E, S) - \hat{c} - (\delta + n + g)\hat{k} \right] \tag{26}$$

Optimality conditions give us:

$$\lambda^S = \hat{c}^{-\eta} \hat{q}(\Phi - (\varphi + \alpha\zeta^2)E) \tag{27}$$

$$\dot{\lambda}^S = (\rho - n + g(\eta - 1))\lambda^S - \hat{c}^{-\eta}\hat{q}\gamma\zeta^2 S \tag{28}$$

Derivating the expression in λ^S:

$$\dot{E} = (\rho - n + (\eta - 1)g)(E - \frac{\Phi}{\varphi + \alpha\zeta^2}) + \zeta^2\gamma S/(\varphi + \alpha\zeta^2) \tag{29}$$

$\dot{S} = E$ leads to the differential equation.