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WP 2020.06

Suggested citation:

S. Suzuki, H.F. Yamagami (2020). Optimism on Pollution-Driven Disasters and Asset Prices. *FAERE Working Paper, 2020.06.*

ISSN number: **2274-5556**

www.faere.fr

Optimism on Pollution-Driven Disasters and Asset Prices *

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February 25, 2020

Abstract

This study explores how investors' optimism about the likelihood of pollution-driven disaster occurrence affects asset prices. Environmental pollution resulting from economic activities raises the probability of disaster occurrence. However, the relationship between economic activities, pollution, and disaster occurrence is difficult to ascertain. Thus, investors make decisions based on subjective expectation; specifically, they subjectively evaluate the probability of disaster occurrence to be lower than its objective probability. As demonstrated in this study, the equity premiums under conditions of objective expectation are significantly higher than those under subjective expectation conditions only if a representative agent has high Intertemporal Elasticity of Substitution (IES). This discrepancy in asset returns is related to the propensity of individuals to discount events occurring in the "distant future" as described in existing literature.

JEL classification: E71, G12, Q54
Keywords: *Subjective Expectations, Disasters, Equity Premium Puzzles, Discount Rates, Climate Change.*

1 Introduction

A rare disaster is a potential resolution of the equity premium puzzle even in an exchange economy with a representative agent (cf. Rietz, 1988; Barro, 2006). Barro and Ursúa (2017) use historical cross-country data to define disasters as events that induce declines of more than 10% in cumulative consumption. Most of the disaster episodes observed in the past 100 years

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have been in the form of financial crises and wars. Recently, many researchers, Martin Weitzman in particular, have pointed out the potential risks brought on by disasters caused by environmental pollution. Greenhouse Gases (GHGs) and the associated climate effects are one representative example of pollution driving disaster occurrence. Accumulated GHGs contribute to changes in global climate, which in turn lead to frequent Category 5 hurricanes, severe droughts, and devastating bush fires. The rise of average global temperatures also contributes to the spread of infectious diseases and environmentally driven economic damages in industries such as agriculture and fishery (cf. IPCC, 2012; Auffhammer, 2018).

This study outlines the proposition that the probability of pollution-driven disasters is naturally time-varying, whereas investors do not perceive it as such. On the one hand, as discussed in Xepapadeas (2005) and Karydas and Xepapadeas (2019), environmental pollution is a by-product of production processes. This study considers an economy where economic activities endogenously determine the probability of pollution-driven disaster occurrence. Historically, high (low) economic growth rates have coincided with high (low) levels of emissions. Therefore, disaster occurrence probability is time-varying due to time-varying economic growth and, thereby, pollution levels.

On the other hand, the interaction among economic activities, emissions, pollution, and disaster occurrence is quite complicated and therefore difficult to ascertain. Thus, investors make decisions based on subjective, not objective, probability. Similar to Abel (2002) and Weitzman (2007), we evaluate asset prices, and how those may be determined by a representative agent model with a subjective probability. This study introduces a subjective or "optimistic" probability evaluation in the following two senses. First, investors subjectively perceive the probability of disaster occurrence to be lower than its objective probability. Second, though the objective probability of disaster occurrence is time-varying, investors perceive it to be a time-invariant Independent and Identically Distributed (IID) process.

The main findings of this study are as follows: (1) subjective and objective asset returns are different; (2) subjective risk-free rates are higher than objective ones; and (3) subjective equity premiums are lower (higher) than objective ones only if the representative agent has high (low) Intertemporal Elasticity of Substitution (IES). Following Weitzman (2007), we consider observed asset returns to be subjective. In contrast, we consider optimal expected asset returns to be objective.

The discrepancy in asset returns is related to the propensity of individuals to discount events occurring in the "distant future," as discussed by Weitzman (1998) and Weitzman and Gollier (2010). Weitzman and Gollier (2010, p. 350) state that "*the concept of discounting is central to economics, since it allows effects occurring at different future times to be compared by converting each future dollar into a common currency of equivalent present dollars.*" In the financial decision-making process, a risk-adjusted discount rate, which is calculated as the sum of the risk-free rate and the equity premium, is widely used for such discounting. For example, as discussed by

Dietz et al. (2018), an individual firm's cost of equity is calculated using the risk-free rate and the equity risk premium in the standard Capital Asset Pricing Model (CAPM). This study points out that the equity risk premium observed in financial markets fails to accurately price the risk associated with pollution-driven disasters. Thus, the risky asset should be discounted by the objective risk-adjusted discount rate; otherwise, the discounted value of distant future events may be biased.

2 The Model and its Equilibrium

Following Mehra and Prescott (1985), among others, we model an economy in which a representative agent consumes fruit from a single Lucas tree. We use C_t and Y_t to denote consumption and output, respectively, in time period t. There are two financial assets: a risk-free asset f_t, and a risky equity share of Lucas tree e_t. q_t and Q_t denote, respectively, the risk-free asset and the equity share of Lucas tree prices in time period t. f_t and e_t indicate the outstanding positions of these two financial assets. Y_t is used to denote the dividend on Lucas tree in time period t. The representative agent has a time-additive utility $u(C_t)$ in time period t, where C_t denotes consumption. Given the above setup, the representative agent maximizes the following lifetime utility:

$$\max_{\{e_{t+1}, f_{t+1}\}_{t\geq 0}} E_0\Big[\sum_{t=0}^{\infty} e^{-\rho t} u(C_t)\Big],$$

subject to $C_t = Y_t e_t + Q_t(e_t - e_{t+1}) + f_t - q_t f_{t+1}$, where E_0 denotes the expectation operator conditional for $t = 0$. The Euler equations, or first-order conditions, for e_{t+1} and f_{t+1} are expressed as follows:

$$Q_t = E_t\Big[e^{-\rho}\frac{u'(C_{t+1})}{u'(C_t)}\big(Q_{t+1} + Y_{t+1}\big)\Big], \tag{1}$$

$$q_t = E_t\Big[e^{-\rho}\frac{u'(C_{t+1})}{u'(C_t)}\Big]. \tag{2}$$

Because the economy in this example is a closed one, market clearing conditions are given by $C_t = Y_t$, $e_t = 1$, and $f_t = 0$.

2.1 Environmental Pollution and Disasters

The two states, $s_t \in \{n, d\}$, are the normal and disaster states respectively. Output depends on the state the economy is in: $Y_t = Y(s_t)$. The stochastic process for the logarithm of output is:

$$\ln Y(s_{t+1}) = \ln Y(s_t) + g + \ln(1-b)\xi(s_{t+1}), \tag{3}$$

where g is the growth rate and b denotes the scale of the disaster. If there is a disaster in period $t+1$ (i.e., $s_{t+1} = d$), then $\xi(s_{t+1}) = 1$; otherwise (i.e.,

$s_{t+1} = n)$, $\xi(s_{t+1}) = 0$. Note that we do not add IID process shocks, which are typically included in standard consumption asset pricing models. These IID process shocks are considered to be business cycle shocks. That is, we focus on the effect the risk of pollution-driven disaster occurrence has on asset prices.

We use P_t to denote the pollution stock in period t, and Y_t to denote the output in period t. According to Xepapadeas (2005), the pollution accumulation equation is written as $P_{t+1} = mY_t + \hat{m}P_t$, where $\hat{m}$ denotes the pollution decay parameter and m denotes the parameter for emissions generated by production. For analytical tractability, we set $\hat{m} = 0$; therefore, the relationship between pollution and output can be written as:[1]

$$P_{t+1} = mY_t. \tag{4}$$

We assume that the pollution-output ratio, $p_t \equiv \frac{P_t}{Y_t}$, affects the probability of disaster occurrence $\Pr(s_{t+1} = d|s_t) = f(p_t)$, where $f'(p_t) > 0$ and $f''(p_t) > 0$. Since equation (4) can be rewritten as $p_{t+1} = m\frac{Y_t}{Y_{t+1}}$, the probability of disaster occurrence depends only on the previous output growth rate.

As described in (3), the output growth rate can only take two values, where $\frac{Y_{t+1}}{Y_t} = e^g$ if $s_{t+1} = n$ and $\frac{Y_{t+1}}{Y_t} = (1-b)e^g$ if $s_{t+1} = d$. Therefore, the pollution-output ratio can take only two values: $p_n = me^{-g}$ in state n and $p_d = \frac{me^{-g}}{1-b}$ in state d. The probability of disaster occurrence in state n is given by $\pi \equiv f(p_n)$, and that in state d by $\theta \equiv f(p_d)$. We denote $\varepsilon \equiv \theta - \pi$, and note that $\varepsilon > 0$ because $f'(p_t) > 0$. The stationary probability of the economy being in a disaster state is $\psi = \frac{\pi}{1-\varepsilon}$.

The stochastic process considered here is similar to the one described by Saito and Suzuki (2014). However, they assume that investors can correctly determine the time-varying probability of disaster occurrence. Our analysis assumes that investors cannot perfectly understand the objective probability of disaster occurrence due to the complicated nature of the relationship between economic activities, emissions due to pollution, and disaster occurrences. Thus, investors make decisions based on subjective probability instead. We denote subjective probability as π^* and θ^*. In particular, we assume that investors underestimate the probability of disaster occurrence: $\pi^* < \pi = f(p_n)$ and $\varepsilon^* < \varepsilon = f(p_d) - f(p_n)$. In addition, investors do not know whether or how the damage induced by disasters affects the probability of disaster re-occurrence. Thus, they subjectively estimate that the probability of re-occurrence would be $\theta^* = \pi^*$ or $\varepsilon^* \equiv \theta^* - \pi^* = 0$. That is, investors consider that pollution-driven disasters follow an IID process. Therefore, we make the assumption that investors are *optimistic about the effects of pollution-driven disasters.*

Assumption 1 *The subjective probability is denoted as* $\pi^* < \pi = f(p_n)$

[1] If we set $\hat{m}$ to a positive value, the pollution-output ratio becomes a state variable and evolves over time. In this case, asset prices described below cannot be solved analytically. However, altering this assumption will not change the main result of this paper qualitatively. We will discuss this point in Section 4.

and $\theta^* = \pi^*$. *Thus,* $\varepsilon^* = 0$.

2.2 Risk-Free Rates and Equity Premiums Equilibrium

Like Weitzman (2007) and Suzuki (2014), we assume that actual asset prices are determined by the Euler equations under a subjective probability. On the other hand, optimal asset prices are determined by the Euler equations under an objective probability.[2] Thus, we compare asset prices under these two scenarios.

Suppose that x_{t+1} denotes the Stochastic Discount Factor (SDF). According to equations (1) and (2), the return on an asset i, denoted by R^i_{t+1}, must satisfy the following pricing equation:

$$1 = E_t[x_{t+1} R^i_{t+1}], \tag{5}$$

where $R^e_{t+1} = \frac{Q_{t+1}+Y_{t+1}}{Q_t}$ denotes the return on equity, and $R^f_{t+1} = \frac{1}{q_{t+1}}$ denotes the risk-free rate.

We assume that disaster episodes follow a Markov process, and that the representative agent has a power or a Constant Relative Risk Aversion (CRRA) utility. When these conditions are met, asset prices are a function of the current state of the economy. Hereafter, the expectation operator conditional on the current state s is denoted by $E_s[\cdot]$.

The risk-free rate conditional on state s is determined by the following equation: $R^f_s = \frac{1}{E_s[x_{ss'}]}$, where $x_{ss'}$ denotes the SDF as one moves from state s to s'. The unconditional safe asset return expectation is $R^f = (1-\psi)R^f_n + \psi R^f_d$.

The price of a Lucas tree in state s is Q_s. By using the price–dividend ratio in state s, defined as $\omega_s \equiv \frac{Q_s}{Y(s)}$, we can represent the ex-post return on equity as one moves from state s to s' as follows: $R^e_{ss'} = \frac{Y(s')}{Y(s)}\frac{\omega_{s'}+1}{\omega_s}$. From equation (5) and the above definition, we can derive the following equation:

$$\omega_s = E_s\Big[x_{ss'}\frac{Y(s')}{Y(s)}\Big(\omega_{s'}+1\Big)\Big]. \tag{6}$$

We can use this equation to compute the price–dividend ratio in each of the two states.

The conditional expected equity returns can be written as follows: $R^e_n = (1-\pi)R^e_{nn} + \pi R^e_{nd}$ and $R^e_d = (1-\theta)R^e_{dn} + \theta R^e_{dd}$. Thus, the unconditional expected equity return is $R^e = (1-\psi)R^e_n + \psi R^e_d$.

[2]Asset prices under objective probability are optimal in the following sense. Suppose that the social planner maximizes the representative agent's expected lifetime utility subject to the resource constraints. This maximized lifetime utility is considered as an optimal level. It is natural that the social planner uses the objective probability. Thus, the optimal level of lifetime utility equals the level of lifetime utility that results from the representative agent's optimization problem with the objective probability.

The unconditional expected equity premium is: $\Pi = R^e - R^f$. Simple manipulation of this expression yields the following unconditional expected equity premium:

$$\Pi = -\Big\{(1-\psi)\frac{\text{cov}_n[x_{ns'}, R^e_{ns'}]}{E_n[x_{ns'}]} + \psi\frac{\text{cov}_d[x_{ds'}, R^e_{ds'}]}{E_d[x_{ds'}]}\Big\}. \tag{7}$$

2.2.1 Asset Prices Under Objective Probability

Based on a power utility function, the SDF is given by $x_{ss'} = e^{-\rho}\left[\frac{Y(s')}{Y(s)}\right]^{-\gamma}$, where $-\rho$ denotes the subjective discount rate and γ denotes the coefficient of Relative Risk Aversion (RRA), which is equivalent to the reciprocal of the IES.

When the price of a risk-free asset is objectively evaluated, it is written as $q_n = \frac{e^{\rho+\gamma g}}{1-\alpha\pi}$ and $q_d = \frac{e^{\rho+\gamma g}}{1-\alpha(\pi+\varepsilon)}$, where $\alpha \equiv 1-(1-b)^{-\gamma}$. Note that $\alpha < 0$ if $\gamma > 0$. Thus, the unconditional risk-free rate is written as a function of π and ε:

$$R^f(\pi, \varepsilon) = \frac{e^{\rho+\gamma g}}{1-\alpha\pi}\frac{1-\alpha\pi-[1+\alpha(1-2\pi)]\varepsilon+\alpha^2\varepsilon}{1-\alpha\pi-[1+\alpha(1-\pi)]\varepsilon+\alpha^2\varepsilon}. \tag{8}$$

The equation (6) can be expressed by the following system of equations:

$$\begin{aligned}\omega_n &= (1-\pi)\beta(\omega_n+1)+\pi\beta\delta(\omega_d+1),\\ \omega_d &= (1-\pi-\varepsilon)\beta(\omega_n+1)+(\pi+\varepsilon)\beta\delta(\omega_d+1),\end{aligned}$$

where $\beta \equiv \exp\{-\rho+(1-\gamma)g\}$, and $\delta \equiv (1-b)^{1-\gamma}$. Thus, the price–dividend ratios are expressed as follows:

$$\omega_n = \beta\frac{1-(1-\delta)\pi-\beta\delta\varepsilon}{1-\beta+\beta(1-\delta)\pi-\beta(1-\beta)\delta\varepsilon}, \tag{9}$$

$$\omega_d = \beta\frac{1-(1-\delta)\pi-(1-\delta+\beta\delta)\varepsilon}{1-\beta+\beta(1-\delta)\pi-\beta(1-\beta)\delta\varepsilon}. \tag{10}$$

The associated unconditional equity premium is written as a function of π and ε:

$$\Pi(\pi, \varepsilon) = -e^g\frac{\alpha\pi}{\beta}\frac{a_4(\pi)\varepsilon^4+a_3(\pi)\varepsilon^3+a_2(\pi)\varepsilon^2+a_1(\pi)\varepsilon+a_0(\pi)}{h_4(\pi)\varepsilon^4+h_3(\pi)\varepsilon^3+h_2(\pi)\varepsilon^2+h_1(\pi)\varepsilon+h_0(\pi)}, \tag{11}$$

where the definitions of $a_4(\pi)$, $a_3(\pi)$, $a_2(\pi)$, $a_1(\pi)$, $a_0(\pi)$, $h_4(\pi)$, $h_3(\pi)$, $h_2(\pi)$, $h_1(\pi)$, and $h_0(\pi)$ are as listed in the Appendix.

2.2.2 Asset Prices Under Subjective Probability

Investors' subjective expectations of the probability of disaster occurrence are expressed by $\pi^* < f(p_n)$ and disaster occurrence follows an IID stochastic process $\varepsilon^* = 0$. Thus, the risk-free rate is written as follows:

$$R^{f*} = R^f(\pi^*, 0) = \frac{e^{\rho+\gamma g}}{1-\alpha\pi^*}. \tag{12}$$

The equity premium under subjective probability, Π^*, is expressed as follows:

$$\Pi^* = \Pi(\pi^*, 0) = -e^g \frac{\alpha b}{\beta} \frac{\pi^* - \pi^{*2}}{1 - [\alpha + (1-\delta)]\pi^* + \alpha(1-\delta)\pi^{*2}}.$$

Therefore, we put forward the following two propositions.

Proposition 1 *Suppose that π and ε are sufficiently low. The risk-free rate under the subjective probability is higher than the one under the alternate condition if, and only if, $\varepsilon < \sqrt{\pi - \alpha^{-1}}$ holds.*

Proof. Note that $\frac{\partial R^f(\pi,0)}{\partial \pi} < 0$ holds, and that

$$\frac{\partial R^f(\pi, 0)}{\partial \varepsilon} = \frac{\partial R^f(\pi, \varepsilon)}{\partial \varepsilon}|_{\varepsilon=0} = e^{\rho+\gamma g}\alpha\pi < 0$$

holds. Thus, based on the variational argument, we can prove that the value of $\frac{\partial R^f(\pi,0)}{\partial \pi}d\pi + \frac{\partial R^f(\pi,0)}{\partial \varepsilon}d\varepsilon$ is negative. (Q.E.D)

The reasoning behind Proposition 1 is quite straightforward. Increases in π and ε indicate that the consumption process is exposed to higher probabilities of disaster occurrence. When risk is high, the demand for assets and the price of the risk-free assets increase, and the risk-free rate decreases.

Proposition 2 *Suppose that π and ε are sufficiently small. The equity premium under the subjective probability is lower than the one under the alternate condition if $\delta < \delta^*$ holds, where $\delta^* > 1$ and is defined as follows:*

$$\delta^* \equiv \frac{2\pi - (1-3\pi)(1-b) + \sqrt{[2\pi + (1-3\pi)(1-b)]^2 + 8\pi(1-2\pi)b\beta^{-1}}}{4\pi}.$$

Proof. Based on the variational argument, we can prove that the value of $\frac{\partial \Pi(\pi,0)}{\partial \pi}d\pi + \frac{\partial \Pi(\pi,0)}{\partial \varepsilon}d\varepsilon$ is positive if $\delta < \delta^*$ holds. That is, marginal increases in both π and ε from subjective probabilities π^* and $\varepsilon^* = 0$ increase the equity premium. Because of $\frac{\partial \Pi(\pi,0)}{\partial \varepsilon} = \frac{\partial \Pi(\pi,\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0}$, we can demonstrate that $\frac{\partial \Pi(\pi,\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0} > (<)0$ if, and only if, $\delta < (>)\delta^*$. From equation (11), $\frac{\partial \Pi(\pi,\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0} = -e^g \frac{\alpha\pi}{\beta} \frac{a_1 h_0 - a_0 h_1}{h_0^2}$. Now that π is sufficiently small, we consider that π^j is approximately 0 when $j \geq 2$. Thus,

$$\mathrm{sign}\Big(\frac{\partial \Pi}{\partial \varepsilon}|_{\varepsilon=0}\Big) \simeq -2\pi\delta^2 - [(1-3\pi)(1-b) - 2\pi]\delta + (1-2\pi)b + (1-3\pi)\beta(1-b)\beta^{-1}.$$

$\mathrm{sign}\Big(\frac{\partial \Pi}{\partial \varepsilon}|_{\varepsilon=0}\Big) \simeq 0$ is a quadratic equation of δ, and the coefficient of the second order term is negative. This quadratic equation has two roots, one is positive δ^* and the other is negative. Thus, $\mathrm{sign}\Big(\frac{\partial \Pi(\pi,\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0}\Big) > (<)0$ is equivalent to $\delta < (>)\delta^*$, and $\delta^* > 1$. On the other hand, using the approximation of $\pi^j \simeq 0$ with $j \geq 2$, $\frac{\partial \Pi(\pi,0)}{\partial \pi} \simeq -e^g \frac{\alpha b}{\beta} \frac{1-2\pi}{1-2(1+\alpha-\delta)\pi} > 0$. Thus, the value of $\frac{\partial \Pi(\pi,0)}{\partial \pi}d\pi + \frac{\partial \Pi(\pi,0)}{\partial \varepsilon}d\varepsilon$ is positive if $\delta < \delta^*$. (Q.E.D.) .

Proposition 2 states that the magnitude of the relationship between Π^* and Π depends on the value of $\delta \equiv (1-b)^{1-\gamma}$. In particular, the threshold value of δ^* is greater than 1. It is well-known that the reciprocal of IES γ plays a crucial role in determining the equity premium. When $\gamma = 1$ and $\delta = 1$, the price–dividend ratio is independent of the current state: $\omega_s = \beta \frac{1-\beta\varepsilon}{1-\beta-\beta(1-\beta)\varepsilon}$. We can easily demonstrate that $\frac{\partial R^f}{\partial \varepsilon} < \frac{\partial R^e}{\partial \varepsilon} < 0$. Thus, $\text{sign}\left(\frac{\partial \Pi(\pi,\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0}\right) > 0$ holds if $\delta = 1$. On the other hand, when $\delta > 1$ and $\gamma > 1$, $\omega_d > \omega_n$ holds. That is, the ex-post return on equity from state n to state d R^e_{nd} increases due to an increase in the price–dividend ratio in state d, and the positive covariance between consumption growth and equity returns decreases. As a result, equity premiums decrease.

2.3 Recursive utility analysis

We compute asset prices using recursive utility, as discussed by Weil (1989) and Epstein and Zin (1991). Let χ denote the reciprocal of the IES. The recursive utility function indicates that the SDF can be written as $x_{ss'} = e^{-\rho\frac{1-\gamma}{1-\chi}} \left[\frac{Y(s')}{Y(s)}\right]^{-\chi\frac{1-\gamma}{1-\chi}} R_{ss'}^{e\,\frac{\chi-\gamma}{1-\chi}}$. Because there are no closed form solutions of equity premiums under the recursive utility, we must compute the asset prices numerically.

3 Numerical Results

We conduct a numerical evaluation to investigate how optimism about the likelihood of pollution-driven disaster occurrence alters asset prices. However, it is difficult to determine the size and frequency of damage from pollution-driven environmental disasters because such events have not been observed enough. Thus, we do not argue that our pollution-driven disaster model quantitatively explains the behaviors of risk-free rates and equity premiums. Nonetheless, we argue that the calibrated standard representative agent model is vulnerable to small changes in disaster occurrence probability π and ε. In other words, observed asset prices could be far from optimal ones.

3.1 Calibration Parameters

Barro and Ursúa (2017) use Barro-Ursua macroeconomic data to define disasters as events that induce declines of 10% or more in cumulative consumption. However, it is difficult to use this data to estimate the extent of damage from pollution-driven environmental disasters, because most disaster episodes observed in the past 100 years are in the form of financial crises and wars. Thus, we have selected calibration parameters in order to ensure our numerical results are comparable to those in existing literature on the equity premium puzzle. In particular, we set the value of b to 0.325 so that a disaster occurrence changes consumption by approximately -0.33.[3] We also set the value of π^* to 0.02 and the value of ε^* to 0.00 in the benchmark case, while π is set to 0.03 and ε to 0.03 in the high probability of disaster case. Following Barro (2006), we assume that $\rho = 0.03$, $g = 0.025$. For the CRRA rate, we use $\gamma = 1$, as the log utility function, and $\gamma = 5$, which is widely used in asset pricing literature. The IES coefficients χ^{-1} are set to 1 and 0.2, respectively. In addition, we compute asset prices under the recursive utility, where $\gamma = 5$ and $\chi^{-1} = 2$. These preference parameters are widely employed outside of this study (cf. Bansal et al., 2014).

3.2 Quantitative Results

Columns (1a)–(1d) of Table 1 report results based on $\gamma = 1$ and $\chi^{-1} = 1$ (i.e., results based on the log utility function). Columns (2a)–(2d) of Table 1 report results based on the CRRA utility function with high RRA ($\gamma = 5$) and low IES ($\chi^{-1} = 0.2$). Columns (3a)–(3d) of Table 1 report results based on the recursive utility function with high RRA ($\gamma = 5.0$) and high IES ($\chi^{-1} = 2$). Columns (1a), (2a), and (3a) of Table 1 report results based on low π of 0.02 and low ε of 0.00. Columns (1b), (2b), and (3b) of Table 1 report results based on low π of 0.02 and high ε of 0.03. Columns (1c), (2c), and (3c) of Table 1 report results based on high π of 0.03 and low ε of 0.00. Columns (1d), (2d), and (3d) of Table 1 report results based on high π of 0.03 and high ε of 0.03.

[Table 1 is inserted here.]

Based on the log utility function, pollution-driven disasters with low π and low ε (outlined in Column (1a) of Table 1) generate equity premiums as high as 0.32%. The equity premiums are even higher if π or ε increases in value. For example, the model with low π and high ε (outlined in Column (1b)) generates equity premiums of 0.33%; the model with high π and low ε (outlined in Column (1c)) generates equity premiums of 0.47%, and the model with high π and high ε (outlined in Column (1d)) generates equity premiums of 0.49%. This is consistent with Proposition 1 because $\delta = 1$ when $\gamma = 1$.

In the case of CRRA utility, where the coefficient of RRA is equal to 5 and the coefficient of IES to 0.2, pollution-driven disasters with low π and low ε (outlined in Column (2a)) generate equity premiums as high as 3.78%. In the context of the equity premium puzzle, these values of approximately 4.0% are considered to indicate good performance. It is well known that the asset pricing model with IID disaster occurrence process generates high equity premiums. However, the risk-free rate amounts to 4.00%, which is higher than the historically observed value of 1.0%. As discussed in Proposition 1, because $\frac{\partial \Pi(\pi,0)}{\partial \pi} > 0$, the model with high π and low ε (outlined in Column (2c)) generates much higher equity premiums (5.13%). On the

[3] Suzuki (2014) employs this value as a severe scenario of this sort of disaster as seen in Table 2, column (2) and (6) of page 273.

other hand, because $\frac{\partial \Pi(\pi,0)}{\partial \varepsilon} < 0$, the model with low π and high ε (outlined in Column (2b)) generates much lower equity premiums (2.89%). The model with high π and high ε (outlined in Column (2d)) generates equity premiums of 3.91%, close to those of pollution-driven disasters with low π and low ε (outlined in Column (2a)).

In the case of recursive utility, where the coefficient of RRA equals 5.0 and the coefficient of IES equals 2.0, the equity premium equals 3.67%, and the risk-free rate equals 0.95% (outlined in Column (3a)). That is, these values are quite favorable in terms of the equity premium and the risk-free rate. The equity premiums are higher if π or ε increases. For example, in the model with low π and high ε (outlined in Column (3b)) equity premiums are 4.38%; in the model with high π and low ε (outlined in Column (3c)) equity premiums are 5.18%; and in the model with high π and high ε (outlined in Column (3d)) equity premiums are 6.11%. These high equity premiums are mainly driven by a reduction in the risk-free rates. That is, the risk-free rates in the scenarios outlined above are 0.95%, 0.36%, -0.45%, and -1.20% respectively.

We must note the risk-free rate and the price–dividend ratio results. On the one hand, as argued in Proposition 1, the risk-free rate decreases due to increases in π and ε. In particular, as shown in (2c), (2d), (3c), and (3d) in Table 1, the risk-free rate takes negative value. The risk-free rate is reciprocal of the price of risk-free asset; the risk-free price is high when investors have strong demand for assets. High π and ε cause investors hold risk-free asset more and raise the price of risk-free asset above one, which results in the negative risk-free rate. In other words, investors would like to pay insurance premium to hold risk-free asset.

On the other hand, Table 1 summarizes the positive price–dividend ratios, confirming that a unique competitive equilibrium exists in all cases. As shown in (2b) and (2d) in Table 1, price-dividend ratios in state d is higher than that in state n. These are consistent with Proposition 2 because IES is lower than 1. In addition, how are these results related to welfare should be explained, provided that there is well known relationship between the price–dividend ratio and welfare as explained by Weitzmann (2007). In our model, the sum of the price–dividend ratio and one $\omega_s + 1$ equal the lifetime utility to current utility ratio. Since utility level take negative value when utility function is CRRA and IES is higher than 1 ($\gamma > 1$), lifetime utility also takes negative value. Thus, higher value in the lifetime utility to current utility ratio means that lifetime utility takes lower values in the case of high IES. Opposite is true when IES is lower than one.

4 Discussion and Conclusions

We explore the effect of investors' optimism about the likelihood of occurrence of pollution-driven disasters on asset prices. We theoretically demonstrate that this optimism causes an increase in risk-free rates and a decrease in equity premiums. We confirm these quantitative asset pricing results using a calibrated standard representative agent model. In particular, the disaster model with recursive utility and high IES performs well. However, these results are vulnerable to small changes in the probability of disaster occurrence. That is, there are discrepancies in the risk-free rates and equity premiums depending on the current condition. These discrepancies have significant implications in terms of the behaviors of long-term discount rates: under an objective probability, eligible discount rates do not equal equity premiums, and whether eligible discount rates are higher than market equity premiums depends on the IES.

Our primary purpose is to relate economic activity to the probability of disaster endogenously. The assumption that the pollution stock depends only on the previous output level makes the stochastic process tractable. If we consider the case that pollution stock accumulates over time, the pollution-output ratio becomes a state variable and evolves over time. Thus, the objective probability of disaster is time-varying even if the disaster has never occurred. In this case, because asset prices under objective expectation depend on the distribution of the pollution-output ratio, they have no analytical solutions. However, we have to note that the qualitative results in Proposition 1 and 2 are unchanged. This is because we consider that investors cannot understand such a complicated stochastic process correctly and the optimistic subjective probability determines the asset prices. On the other hand, accumulated pollution stock would have some significant quantitative implications. This should be the issue in the future research.

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Appendix: Definition of notations

$a_4(\pi) \equiv \eta_2\lambda_2$, $a_3(\pi) \equiv \eta_1\lambda_2 + \eta_2\lambda_1$, $a_2(\pi) \equiv \eta_0\lambda_2 + \eta_1\lambda_1 + \eta_2\lambda_0$, $a_1(\pi) \equiv (\eta_0\lambda_1+\eta_1\lambda_0$, $a_0(\pi) \equiv (1-\pi)b(1-\alpha\pi)[1-(1-\delta)\pi]$, $h_4(\pi) \equiv -\mu_3$, $h_3(\pi) \equiv \mu_3-\mu_2$, $h_2(\pi) \equiv \mu_2 - \mu_1$, $h_1(\pi) \equiv \mu_1 - \mu_0$, $h_0(\pi) \equiv (1-\alpha\pi)^2[1-(1-\delta)\pi]^2$, $\eta_0 \equiv (1-\pi)b$, $\eta_1 \equiv \alpha\beta(1-b)(1-\pi)-b$, $\eta_2 \equiv -\alpha\beta(1-b)$, $\lambda_0 \equiv (1-\alpha\pi)[1-(1-\delta)\pi]$, $\lambda_1 \equiv \delta(1-\beta)-\alpha+\alpha\{2(1-\delta)+\beta\delta\}\pi-\alpha(1-\delta)\pi^2$, $\lambda_2 \equiv \alpha(1-\delta)(1-\pi)-(1-\alpha)\beta\delta$, $\mu_0 \equiv \lambda_0^2$, $\mu_1 \equiv -(1-\alpha\pi)\{1-(1-\delta)\pi\}\{1+\alpha-\delta+2\beta\delta-2\alpha(1-\delta+\beta\delta)\pi\}$, $\mu_2 \equiv (1-\alpha\pi)[(1-\delta+\beta\delta)(\alpha+\beta\delta)+\alpha\beta\delta-\{\alpha(1-\delta+\beta\delta)^2+(1-\delta)\alpha\beta\delta\}\pi]$, and $\mu_3 \equiv -(1-\alpha\pi)\alpha\beta\delta(1-\delta+\beta\delta)$.

Table 1: Asset Price Results

(1) CRRA utility with high IES: $\gamma = 1$				
	(1a)	(1b)	(1c)	(1d)
R^f (%)	4.65	4.62	4.15	4.10
Π (%)	0.32	0.33	0.47	0.49
ω_n	33.84	33.84	33.84	33.84
ω_d	33.84	33.84	33.84	33.84
(2) CRRA utility with low IES: $\gamma = 5$				
	(2a)	(2b)	(2c)	(2d)
R^f (%)	4.00	3.70	-1.39	-1.80
Π (%)	3.78	2.89	5.13	3.91
ω_n	18.23	22.16	46.84	148.43
ω_d	18.23	24.71	46.84	165.52
(3) Recursive utility: $\gamma = 5$, $\chi^{-1} = 2$				
	(3a)	(3b)	(3c)	(3d)
R^f (%)	0.95	0.36	-0.45	-1.20
Π (%)	3.67	4.38	5.18	6.11
ω_n	36.96	35.21	31.71	29.95
ω_d	36.96	34.72	31.71	29.54

γ is used to denote the coefficient of RRA, and χ the reciprocal of IES, R^f the unconditional risk-free rates, and Π the equity premiums. ω_n is the welfare measure in state n, and ω_d is the welfare measure in state d. Calibration parameters are as follows: $\rho = 0.03$ is the subjective time discount rate, $g = 0.025$ is the trend growth rate, and $b = 0.325$ is the shock size of disasters. $\pi = 0.02$ and $\varepsilon = 0.00$ in (1a), (2a), and (3a), $\pi = 0.02$ and $\varepsilon = 0.03$ in (1b), (2b), and (3b), $\pi = 0.03$ and $\varepsilon = 0.00$ in (1c), (2c), and (3c), and $\pi = 0.02$ and $\varepsilon = 0.03$ in (1d), (2d), and (3d).