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Incentives, Pro-social Preferences and Discrimination*

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Abstract. In this paper, I study how a principal can provide incentives, at minimal cost, to a group of agents who have pro-social preferences in order to induce successful coordination in the presence of network externalities. I show that agents' pro-social preferences - specifically a preference for the sum of the agents payoffs and/or for the minimum payoff - lead to a decrease in the implementation cost for the principal, a decrease in the payoff of each agent, and an increase in discrimination. The model can be applied in various contexts and it delivers policy implications for designing policies that support the adoption of new technologies, for motivating a group of workers, or for inducing successful coordination of NGOs.

JEL classification: D91, D62, D82, D86, 033

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1 Introduction

How to induce successful coordination among individuals who care about each other? Is it less costly than to induce successful coordination among individuals who don't? Do pro-social motivations lead to less or more discrimination? These questions arise when a regulator seeks to provide incentives to a group of agents to induce them to adopt a new technology, when a firm uses social networks to sell a product, when a manager seeks to motivate a team of workers, or when the United Nations has a mandate to coordinate NGOs.

Incentives are discriminatory when they involve non symmetric rewards even when all the agents are identical and the objective of the principal is to induce participation of all the agents. Optimal incentives can be discriminatory in various contexts, such as exclusionary contracts (Rasmusen et al., 1991, Innes and Sexton, 1994), introductory prices by a monopolist in the presence of consumption externalities (Farrell and Saloner, 1985, Katz and Shapiro, 1986), in general trade contracts (Segal, 2003), and in organizations (Winter, 2004). This literature has been exclusively focused on agents with standard preferences. However, in many contexts, the principal has to consider the fact that the agents care about each other. When designing an incentive policy for green technology adoption (such as electric vehicles), a government may have to take into account the pro-social preferences of the citizens. When setting differentiated prices (e.g. in using price discounts), a firm that uses social networks to sell its products may have to take into account the fact that groups of connected people care about each other. When setting wages for members of a group of workers, a manager may take into account the fact that these workers care about each other. When coordinating NGOs, the United Nations may take into account the fact that the members of an NGO care about the causes that are promoted by other NGOs.

In this paper, I study a situation in which a principal offers bilateral contracts to the members of a group of agents - who have pro social preferences - to participate in a project. Each agent's decision to participate to the project generates positive externalities for other participating agents (e.g. network externalities). The agents are assumed to have three different motivations: a selfish motivation, a collective motivation and a Rawlsian motivation, i.e. their utility is a weighted sum of their own payoff, the sum of the payoffs of the agents of the group and the minimum payoff among the members of the group.² The principal designs a set of incentive contracts to coordinate the group members' participation. Contracts are bilateral in the sense that the reward offered to one agent cannot depend on the other agents' decisions. The aim of the present paper is to analyze bilateral contracting with externalities when the agents have pro-social preferences and to study whether and how these preferences affect the optimal contract and the level of discrimination.

I first analyze the situation in which the principal seeks to induce participation of all the agents when coordination is not an issue, i.e. when he can coordinate agents on his preferred equilibrium (as in Segal, 1999). I thus characterize the optimal contract that implements the participation of all the agents as a Nash equilibrium ("partial implementation"). I show that the optimal contract specifies a transfer to each agent that depends on a one-dimensional individual index of pro-social preferences. This index is a ratio between the total weight the agent gives

¹See also Bensaid and Lesne (1996) and Cabral et al. (1999).

²This utility function corresponds to the one proposed in Charness and Rabin (2002), p. 852.

to the payoff of the other agents and the total weight he gives to his own payoff. The stronger the agent's pro-social preferences (i.e. the larger the index), the lower the transfer he receives. The optimal contract thus decreases the implementation cost. Moreover, when the agents are symmetric, they all receive the same transfer, i.e. there is no discrimination.

I then analyze the situation in which the principal designs bilateral contracts to induce successful coordination. I thus characterize the optimal contract that implements the participation of all the agents as a unique Nash equilibrium (unique implementation). I first show a neutrality result. When the agents' pro-social preferences are only Rawlsian (i.e. they give no weight to the sum of the payoffs of the agents), the optimal contract is the same as when the agents have no pro-social motivation. I then show that the optimal contract is not neutral with respect to collective motivation and that collective motivation increases discrimination, i.e. the difference between the transfers that the agents receive increases. I then study the case in which the agents have both kinds of pro-social preferences and find that the optimal contract depends on both collective and Rawlsian motivations. Moreover, I show that these pro-social preferences lead to an increase in discrimination. I also find, as in the case of partial implementation, that pro-social preferences decrease the transfer that each agent receives, thus they lead to a decrease in the implementation cost.

The intuition behind the results lies in the fact that the optimal contract is characterized by what I call the *decreasing divide and conquer property*. A contract has the divide and conquer property if each agent gets a reward that would convince him to participate when all the agents who precede him in an arbitrary ranking participate, and all subsequent agents abstain. A contract has the decreasing divide and conquer property if the payoff of each agent is lower than the payoff of the preceding agent in the ranking.³

This paper contributes to the literature on contracts with externalities. The seminal contribution by Segal (1999) shows, in a general setting, that partial implementation contracts are inefficient in the presence of multilateral externalities (assuming standard, selfish preferences). In a setting where the agents make participation decisions, I show that pro-social preferences affect the optimal partial implementation contract and that they decrease the implementation cost for the principal. Segal (2003) study, in a general setting, the property of the optimal unique implementation contract and shows that prohibiting the principal to propose discriminatory contracts aggravates inefficiencies when the agent's actions are strategic complements.⁴ In the present paper, I extend this literature and study the unique implementation contract when the agents have pro-social preferences in a situation where the agents make binary decisions (as in Winter, 2004) and their actions are strategic complements. This literature has not yet considered how pro-social preferences shape the relationship between incentives and discrimination, which is the purpose of the present paper.

This paper also contributes to the growing literature on behavioral contract theory (see Koszegi, 2014 for a review). More precisely, in considering a principal who contracts with multiple agents who have pro-social preferences, this paper is related to the contributions that have

³Segal (2003) and Bernstein and Winter (2012) find that the optimal contract has a similar structure in settings with standard preferences. Che and Yoo (2001) find that the optimal mechanism in a moral hazard in team problem has a similar structure.

⁴This model has been extended in several directions. See Bernstein and Winter (2012) and Sakovics and Steiner (2012) for contracting problems with heterogeneous externalities. See Bloch and Gomes (2006), Genicot and Ray (2006) and Galasso (2008) for dynamic models.

considered contracting with multiple agents. This literature has focused on the case of inequity averse and/or status seeking agents.⁵ An exception is Dur and Sol (2010) who focus on (endogenous) altruism.⁶ The present paper differs in several ways from this contribution. First, I assume that the agents give (exogenous) weights to the payoffs of the other agents and to the minimum payoff, while Dur and Sol focus on altruistic agents, i.e. agents who give weight to the utility level of the other agents. Second, my main focus is on the coordination problem and discrimination, while the aforementioned contribution asks whether incentives can help to generate altruism. Another exception is Sarkisian (2017) who study the role of altruism and Kantian morality when a principal seeks to motivate a team of two agents. The present paper differs from this contribution since I focus on bilateral contracting (and not on team incentives) and I tackle the issues of coordination and discrimination.

The remainder of the paper is organized as follows. The model is introduced in Section 2. In Section 3, I study the situation in which the principal can coordinate agents on his preferred equilibrium (partial implementation). Section 4 delivers the main results for the case in which the principal designs contracts in order to induce successful coordination (unique implementation). Finally, Section 5 concludes. All proofs are provided in an appendix.

2 The model

A principal offers bilateral contracts to several agents in an environment characterized by multilateral homogeneous externalities between the agents. The timing of the contract is as follows: first, the principal proposes a publicly observable contracting scheme to a set of agents; second, the agents observe the principal's proposition and simultaneously decide whether to accept or to reject their respective offers. Finally, the contracts are executed.

Formally, an agent who decides to participate to a project generates a positive externality w > 0 for the other agents who participate and no externality for the agents who do not participate to this project.⁷ An agent who decides not to participate gets his outside option c. The principal proposes a contracting scheme $v = (v_1, v_2, ..., v_n)$ to the agents in the set of agents N, with i = 1, 2, ..., n, in order to provide them incentives to participate to the project. The contracting scheme v is designed such that each agent receives a unique contract offer v_i , $i \in N$, i.e. the principal is able to use discriminative contracts. From the principal's perspective, the outcome is binary: either the project is a success or a failure. Success occurs only when all the agents decide to participate to the project. Therefore, the principal aims to reach agents' full participation at the lowest possible cost. The vector of agents' decisions is $x = (x_1, ..., x_n) \in \{0, 1\}^n$, where $x_i = 1$ means that agent i chooses to participate while $x_i = 0$ means that he chooses not to participate.

When agent i participates and m other agents participate too $(m+1=card\{j\in N: x_j=1\})$, agent i gets:

$$\pi_i(x) = mw + v_i,\tag{1}$$

and $\pi_i(x) = c$ if agent i does not participate $(x_i = 0)$.

We assume that the agents have social preferences and that they give weight to their own

⁵See Itoh (2004), Demougin et al. (2006), Neilson and Stowe (2008), Bartling and von Siemens (2010), and Bartling (2011).

⁶See also for the case of teams.

⁷This is a special case of increasing externalities as defined by Segal, 2003.

payoff (a "selfish" motive), to the sum of the payoffs of all the agents (a "collective" motive), and to the lowest payoff among the payoffs of all the agents (a "Rawlsian" motive). Formally, the utility of agent i is:

$$U_i(x) = \alpha_i \pi_i(x) + \beta_i \min\{\pi_1(x), ..., \pi_n(x)\} + \gamma_i \sum_{j \in N} \pi_j(x),$$
 (2)

where $\pi_i(x)$ is the payoff of player i and α_i, β_i , and γ_i are the (nonnegative) preference weights of agent i, with $\alpha_i + \beta_i + \gamma_i = 1$ and $\alpha_i > 0$.

In this setting, coordination can be an issue. Our objective will be to characterize the optimal contract that implements full participation as the unique Nash equilibrium of the game played by the agents when they face the optimal proposal of the principal.

3 Partial Implementation

We first characterize the partial implementation optimal contract, i.e. the least cost contract that implements full participation as one of the (possibly many) equilibria of the participation game played by the agents. Before stating the result, let me define the following index of pro-social motivation of agent i as follows:

$$S_i = \frac{(n-1)\gamma_i + \beta_i}{\alpha_i + \gamma_i}. (3)$$

This index is the ratio of the total weight that agent i gives to the payoff of the other agents $((n-1)\gamma_i + \beta_i)$ and the total weight he gives to his own payoff $(\alpha_i + \gamma_i)$. It will be a crucial element in the characterization of the partial implementation optimal contract.

We can now state the following result:

Proposition 1: Let (re)order the agents such that $S_i \leq S_{i+1}$ for i = 1, ..., n-1. The optimal contract that implements full participation as a Nash equilibrium of the participation game is characterized as follows:

$$v_i^* = c - (n-1)w - S_i w$$
, for $i < n$, and $v_n^* = c - (n-1)w - S_n w + \beta_n [S_n - S_{n-1}] w$

This result shows that the principal can implement full participation as a Nash equilibrium. Notice that the reward vector v^* is unique, up to permutations of identical agents. Otherwise, the reward of all identical agents can be permuted. Interestingly, heterogeneity is only captured by the pro-social motivation index (S_i) , who is one-dimensional. The optimal transfer made to an agent equals his outside option minus the sum of the externalities he receives and the externality he generates per agent (w) weighted by his pro-social motivation index. When the agents have no pro-social motivation, their index is null $S_i = 0$ and the transfer they receive in this case is the same as in the case with standard preferences.⁸

The agent with the highest pro-social motivation index gets a (relative) premium $(\beta_n [S_n - S_{n-1}] w)$. The reason is the following. If he deviates from the full participation situation and chooses not to participate, he will get his outside option, which is larger than his participation payoff, and then he will no longer be the agent with the smallest payoff. The smallest payoff will be equal to

⁸See Bernstein and Winter (2012) for a characterization in the case with standard preferences.

the payoff of the agent with the second highest pro-social motivation index. This means that the smallest payoff will increase more (or decrease less) when the agent with the highest pro-social motivation deviates than when another agent deviates. This explains why he gets a (relative) premium.

Notice that if an agent has a higher pro-social motivation than an other agent, the optimal transfer to the former is lower than the optimal transfer to the latter. This is straightforward for the n-1 first agents. To see that this is also true for the agent with the highest pro-social motivation, simply notice that $v_{n-1}^* - v_n^* = (1 - \beta_n)(S_n - S_{n-1})w > 0$.

When the agents are homogenous, their pro-social motivation index levels are identical, i.e. $S_i = S$ for all i, and they all receive the same transfer, $v_i^* = c - (n-1)w - Sw$ for all i. Thus, there is no discrimination.

We deduce from Proposition 1 and the discussion above the following comparative static result:

Corollary 1: Pro-social motivations lead to a decrease in all the agents' payoffs and the stronger the pro-social motivation of an agent, the lower the transfer he receives, i.e. $v_i^* \geq v_j^*$ if and only if $S_i \leq S_j$.

Corollary 1 states that when some agents give some weight to the sum of the payoffs of all the agents and/or to the smallest payoff, the principal will be able to implement full participation (as one of the possibly many Nash equilibria) at a lower cost. The intuition is the following. When an agent decides to participate, he generates positive externalities for the other participating agents. Thus, an agent who gives more weight to the payoff of the other agents has stronger incentives to participate and the principal can decrease the transfer that he provides to this agent.

The main drawback of the optimal partial implementation contract is that the agents' participation decision subgame may have multiple Nash equilibria. Here, there are at least two equilibria: the situation in which all the agents participate (according to the definition of partial implementation) and the situation in which none of the agents participate (this can be easily checked because $v_i^* \leq c$ for all i). A principal's objective may thus be to provide incentives, at minimal cost, in order to induce successful coordination, i.e. to design the least cost contract that implements full participation as a unique Nash equilibrum. This is the purpose of the next Section.

4 Unique Implementation

In this section, I characterize the optimal unique implementation contract, i.e. the least cost contract that implements full participation as a unique Nash equilibrium of the participation game played by the agents. In order to highlight the role of each kind of motivation, I first consider the polar case in which the agents' pro-social motivation is purely Rawlsian, i.e they give weight to the minimum payoff and no weight to the sum of the payoffs of the group of agents ($\gamma_i = 0$ for all i). I then consider the other polar case in which the agents' pro-social motivation is purely a collective motive, i.e they give weight to the sum of the payoffs of the group of agents and no weight to the minimum payoff ($\beta_i = 0$ for all i). I finally consider the case in which the agents are symmetric and have both Rawlsian and collective motivations, i.e. they give weight to

both the minimum payoff and to the sum of the payoffs of the group of agents.

4.1 Rawlsian motivation

I assume here that the agents give no weight to the sum of the payoffs, i.e. $\gamma_i = 0$ for all i. The utility of agent i is:

$$U_i(x) = \alpha_i \pi_i(x) + \beta_i \min\{\pi_1(x), ..., \pi_n(x)\},\tag{4}$$

where $\alpha_i + \beta_i = 1$. In this case, we can show the following result:

Proposition 2: When the agents have Rawlsian preferences, the optimal contract that implements full participation as a unique Nash equilibrium of the participation game is the same as in the case where the agents have pure selfish preferences:

$$v^* = (c, c - w, c - 2w, ..., c - (n - 1)w).$$

This neutrality result is quite surprising. When the agents give weight to the minimum payoff, the optimal contract is not affected compared to the case in which they have standard preferences. This is also in contrast with partial implementation, since in this latter case the principal is able to take advantage of Rawlsian motivations, even if the agents have no collective motivation (since $S_i = \frac{\beta_i}{\alpha_i}$ when $\gamma_i = 0$).

The optimal contract is characterized by the divide-and-conquer property.⁹ The optimal contract is constructed by ranking agents in an arbitrary order, and by offering each agent a reward that would induce him to participate when all the preceding agents participate and all subsequent agents do not.

The principal is not able to make lower transfers than in the case of standard preferences because the optimal contract is characterized by a decreasing divide-and-conquer property. The optimal contract is such that each agent is the one who gets the lowest payoff among the set of participating agents when all preceding agents in the ranking participate while all the subsequent agents do not. The first agent in the ranking receives a transfer equal to the opportunity cost c. Now consider the case of the agent ranked second in the ranking when the first agent participates. If he does not participate, the two agents gets c. When the second agent participates, if he receives a transfer that is lower than the transfer he would get if he had standard preferences (c-w), his payoff is the minimum payoff (c-w < c) and then his utility from participating is lower than his utility from not participating. The same logic applies for all the subsequent agents.

Roughly speaking, the decreasing divide-and-conquer property implies that when all the previous agents in the ranking participate while all the subsequent agents do not, the agent is the one with the smallest payoff he participates or does not participates. Thus, the fact that he gives weight to both his own payoff and to the minimum payoff does not make a difference compared to the case where he only cares about his own payoff.

 $^{^9}$ Segal (2003), Winter (2004) and Bernstein and Winter (2012) find that the optimal contract has this property in settings with standard preferences.

4.2 Collective motivation

I assume here that the agents give no weight to the minimum payoff, i.e. $\beta_i = 0$ for all i. The utility of agent i is:

$$U_i(x) = \alpha_i \pi_i(x) + \gamma_i \sum_{j \in N} \pi_j(x), \tag{5}$$

where $\alpha_i + \gamma_i = 1$.

In this case, we can show the following result:

Proposition 3: When the agents have a collective motivation, the optimal contract that implements full participation as a unique Nash equilibrium of the participation game is:

$$v^* = (c, c - (1 + \gamma_2)w, c - 2(1 + \gamma_3)w, ..., c - (n - 1)(1 + \gamma_n)w),$$

where the agents are ranked such that $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$.

This result states that collective motivation, differently from Rawlsian motivation, affects the optimal contract. Moreover, collective motivation leads to an increase in discrimination among the agents, as the difference in the transfers between two subsequent agents i and i + 1 is w in the case of standard preferences while it is $w + ((i + 1)\gamma_{i+1} - i\gamma_i)w \ge w$ in the case where the agents have a collective motivation.

The effect of collective motivation on the optimal contract is quite intuitive. As in the case with standard preferences, the optimal contract is characterized by the divide-and-conquer property. An agent is indifferent between not participating and participating when all the preceding agents in the arbitrary ranking participate and all the subsequent agents do not. Compared to the situation with standard preferences, the principal can decrease the transfer made to the agent by the agent's valuation of the externalities he generates for the preceding agents, i.e. by $i(1+\gamma_{i+1})w$ for the agent ranked i+1 in the ranking.

4.3 Both Rawlsian and collective motivations

I now consider the case where the agents give weight to both the minimum payoff and to the sum of the payoffs when the agents are symmetric. The utility of agent i is:

$$U_i(x) = \alpha \pi_i(x) + \beta \min\{\pi_1(x), ..., \pi_n(x)\} + \gamma \sum_{j \in N} \pi_j(x),$$
 (6)

where $\alpha + \beta + \gamma = 1$.

In this case, I show the following result:

Proposition 4: When the agents have symmetric preferences and both Rawlsian and collective motivations, the optimal contract that implements full participation as a unique Nash equilibrium of the participation game is such that the agents are ranked according to an arbitrary order

(1, 2, ..., n without loss of generality) and:

$$v_{k+1} = c - kw(1+\gamma) - \gamma w \sum_{t=1}^{k} (k-t)\beta^{t},$$
(7)

for all $2 \le k+1 \le n$. The first agent gets a transfer equal to his outside option $v_1 = c$.

This result deserves several comments. First, the transfers differ from the case in which the agents have only a collective motivation and no Rawlsian motivation, meaning that Rawlsian motivations affect the optimal contract conditional on the fact that the agents have collective motivations. Second, compared to the case with only collective motivation, the agents get lower payoffs, thus the principal is able to take advantage of the presence of both pro-social motivations.

The optimal contract is still characterized by a decreasing divide and conquer property. The transfer made to an agent is such that he prefers to participate than not to participate when all the preceding agents in the ranking participate and all the subsequent agents in the ranking do not. Since the agents give weight to the payoffs of the other agents, the transfer to the k + 1th agent is lower than the transfer to the kth agent.

The decreasing divide and conquer property enables the principal to decrease the transfer to each agent up to make him indifferent between participating and not participating when all the preceding agents in the ranking participate and all the subsequent agents in the ranking do not. The principal is able to do so because a decrease in the transfer made to one of the preceding agents (weakly) increases the incentive for the subsequent agents to participate. Indeed, a decrease in the transfer made to one of the preceding agents does not affect the minimum payoff when the agent participate - since in this case the agent is the one who gets the minimum payoff - while it (weakly) decreases the minimum payoff when the agent does not participate.

The decreasing divide and conquer property also implies that an agent gets the minimum payoff when all the preceding agents in the ranking participate and all the subsequent agents do not participate. If the agent decides not to participate, the preceding agent in the ranking gets the minimum payoff. As a consequence, the surplus of the agent (the difference between his payoff and his opportunity cost) has to be equal to its valuation of the surplus of the preceding agent when the agent does not participate net of his valuation of the externalities he generates when he participates. Formally, we obtain:

$$v_{k+1} + kw - c = \beta(v_k + (k-1)w - c) - \gamma kw, \tag{8}$$

for k + 1 = 2, ..., n.

When the first agent participates alone, he gets no surplus, $v_1 - c = 0$. As a consequence, when the two first agents participate, the second agent gets a surplus $-\gamma w$ that does not depend on the weight he gives to the minimum payoff. Hence, for the following agents, the weight they give to the minimum payoff affect their surplus $(v_{k+1} + kw - c)$ only if $\gamma > 0$. This is the reason why the Rawlsian motivation affects the optimal contract only when the agents have a collective motivation.

Roughly speaking, the case with both collective and Rawlsian pro-social motivations differs from the case with purely Rawlsian pro-social motivations because collective motivations enable

the principal to provide incentives to the agents such that their payoff is lower than their opportunity cost. As a consequence, when all the previous agents in the ranking participate while all the subsequent agents do not, the agent is the one with the smallest payoff when he participates but not when he does not participate. Thus, in this case, the fact that he gives weight to the minimum payoff makes a difference compared to the case where he only cares about his own payoff.

I can now analyze how the weights affect the level of discrimination:

Corollary 2: When the agents have symmetric preferences and both Rawlsian and collective motivations, the difference in the transfers received by two subsequent agents is:

$$v_k - v_{k+1} = (1+\gamma)w + \gamma w \frac{\beta - \beta^k}{1-\beta},\tag{9}$$

for all $2 \le k+1 \le n$. The difference increases when γ or β increases and its cross derivative with respect to γ and β is positive.

The two types of pro-social preferences lead to an increase in the difference between the transfer made to two subsequent agents, and then lead to stronger discrimination. Moreover, Rawlsian motivation acts as a complement to collective motivation and leads to an even larger level of discrimination.

5 Conclusion

In this paper, I have studied the role of pro-social preferences on the relationship between incentives and discrimination in a model in which a principal proposes bilateral contracts to a group of agents in order to induce successful coordination. I have shown that agents' pro-social preferences lead to a decrease in the implementation cost for the principal, a decrease in the payoff of each agent, and an increase in discrimination. These results suggest that a regulator can reduce the cost of an incentive policy for green technology adoption such as electric vehicles when the citizens have pro-social preferences, that a manager can propose lower wages to a group of workers when they care about each other, and NGO coordination bodies can reduce the implementation cost of a project when the members of the NGOs care about the causes that are promoted by the other NGOs.

There are several avenues for future research. Pro-social motivations are private information in many contexts. I have shown that pro-socially motivated agents get lower rewards than agents with standard (selfish) preferences when pro-social preferences are common knowledge. Thus, the agents may have incentives not to reveal their pro-social motivation to the principal. Extending the model to a situation with private information about pro-social preferences is an important extension that is left for future research. I have also shown that pro-social motivations lead to an increase in discrimination. This could discourage participation for if the agents are inequity averse besides having pro-social preferences. In the context of the present model, pro-social preferences and inequity aversion may act in opposing directions. This second possible extension is also left for future research.

6 Appendix

Proof of Proposition 1: The situation in which all the agents participate is a Nash equilibrium if and only if

$$\alpha_{i} (v_{i} + (n-1)w) + \beta_{i} min_{j} \{v_{j} + (n-1)w\} + \gamma_{i} \left(\sum_{j} v_{j} + n(n-1)w \right)$$

$$\geq \alpha_{i} c + \beta_{i} min \{ min_{j \neq i} \{v_{j} + (n-2)w\}, c\} + \gamma_{i} \left[\sum_{j \neq i} v_{j} + (n-1)(n-2)w + c \right],$$

or,

$$[\alpha_i + \gamma_i] (v_i + (n-1)w - c) + \beta_i [\min_j \{v_j + (n-1)w\} - \min\{\min_{j \neq i} \{v_j + (n-2)w\}, c\}] + \gamma_i (n-1)w \ge 0.$$
 (10)

There are three cases to consider:

- (i) If $\min_{j\neq i} v_j \leq v_i$ and $\min_{j\neq i} v_j < c (n-2)w$, then condition (10) becomes $v_i \geq c (n-1)w \frac{(n-1)\gamma_i + \beta_i}{\alpha_i + \gamma_i}w$.
- (ii) If $v_i < min_{j \neq i} v_j \le c (n-2)w$, then condition (10) becomes $v_i \ge c (n-1)w [1 + \gamma_i] \beta_i [c (n-2)w min_{j \neq i} v_j]$.
- (iii) If $c (n-2)w \le \min_{j \ne i} v_j \le v_i$ then condition (10) becomes $v_i \ge c (n-1)w \left[1 + \frac{\gamma_i}{\alpha_i + \gamma_i}\right] + \frac{\beta_i}{\alpha_i + \gamma_i} \left[\min_{j \ne i} v_j + (n-1)w c\right]$, which is always true here.
- (iv) If $c (n-2)w < v_i < \min_{j \neq i} v_j$ then condition (10) becomes $v_i \ge c (n-1)w \left[1 + \frac{\gamma_i}{\alpha_i + \gamma_i}\right]$, which is always true here.

Thus, in order to minimize costs, the principal can decide to choose a vector of transfers v^* such that condition for case (i) holds for all the agents. Assuming that the principal (re)order the agents as described in the statement of the Proposition, he will choose $v_i^* = c - (n-1)w - \frac{(n-1)\gamma_i + \beta_i}{\alpha_i + \gamma_i}w$ for all $i \neq n$ and $v_n^* = v_{n-1}^*$. The principal will never decide that the transfers to a subset of agents respect the condition for case (iii) or (iv). Indeed, this would imply larger transfers for these agents than in case (i). Moreover, this would not enable the principal to decrease the transfer of other agents.

The principal can also decide to choose a vector of transfers \widetilde{v} such that one agent (denoted s) gets a strictly smaller payment than the other agents. The transfer to agent s respects the condition for case (ii) while the transfer to the other agents respect the condition for case (i). The principal will thus set $\widetilde{v}_i = v_i^*$ for all $i \neq s$. Letting k(s) be an agent such that $\min_{j\neq s} \widetilde{v}_j = \widetilde{v}_{k(s)}$, according to the description of case (ii) above, the transfer \widetilde{v}_s must be such that $c - (n-1)w(1+\gamma_s) - \beta_s \left[1 + \frac{(n-1)\gamma_{k(s)} + \beta_{k(s)}}{\alpha_{k(s)} + \gamma_{k(s)}}\right] w \leq \widetilde{v}_s < \min_{j\neq s} v_j^* = v_{k(s)}^*$. This inequality characterizes a non empty set of transfers \widetilde{v}_s only if $\frac{(n-1)\gamma_{k(s)} + \beta_{k(s)}}{\alpha_{k(s)} + \gamma_{k(s)}} < \frac{(n-1)\gamma_s + \beta_s}{\alpha_s + \gamma_s}$. We conclude that the vector of transfers \widetilde{v} with s = n minimizes the cost of the principal. \square

Proof of Corollary 1: For all i < n - 1, we have $v_i^* - v_{i+1}^* = (S_{i+1} - S_i) w \ge 0$. For i = n, we have $v_{n-1}^* - v_n^* = (1 - \beta_n)(S_n - S_{n-1})w \ge 0$.

Proof of Proposition 2: A necessary condition for a contract to implement full participation as a unique Nash equilibrium is that there exists at least one agent i_1 who prefers to participate than not to participate when no agent participates, one agent i_2 who prefers to participate when agent i_1 also participates and no other agent participate, etc. In other words, a necessary condition is that there exists an order $i_1, i_2, ..., i_n$ such that agent i_k prefers to participate when all preceding agents i_l with $l \leq k$ participate and no other agent participates. The rest of the proof has three steps. In step 1, I show that the necessary condition holds if and only if $v_{i_k} + (k-1)w \geq c$ for all k. In step 2, I show that the least cost contract that respects the necessary condition, i.e. such that $v_{i_k} + (k-1)w = c$ for all k, implements full participation has a unique Nash equilibrium.

Step 1: I use induction to prove that contract v respects this necessary condition if and only if $v_{i_k} + (k-1)w \ge c$ for all k. Since agent i_1 prefers to participate when no other agent participates, the result holds for k = 1, i.e. $v_{i_1} \ge c$. Now consider $k \le n-2$ and assume that $v_{i_l} \ge c - (l-1)w$ for all $l \le k$. Hence, $v_{i_l} + (k-1)w \ge c$ for all $l \le k$. Then, agent i_{k+1} participates when all the k preceding agents participate if and only if:

$$\alpha_{i_{k+1}}(v_{i_{k+1}} + kw) + \beta_{i_{k+1}} \min\{v_{i_{k+1}} + kw, c\} \ge \alpha_{i_{k+1}} c + \beta_{i_{k+1}} c, \tag{11}$$

Hence, we must have $v_{i_{k+1}} + kw \ge c$.

It remains to show that the result holds for agent i_n . Using $v_{i_k} + (k-1)w \ge c$ for all $k \le n-1$, we obtain that i_n prefers to participate when all the other agents also participate if and only if:

$$(\alpha_{i_n} + \beta_{i_n})(v_{i_n} + (n-1)w - c) \ge 0. \tag{12}$$

Hence, we must have $v_{i_n} + (n-1)w \ge c$.

Step 2: I show here that if $v_{i_k} + (k-1)w = c$ for all k, then agent i_k always prefers to participate when any other k-1 agents participate. Let P be the set of participating agents. We know from Step 1 that agent i_k prefers to participate than no to participate if P is such that there is no agent $i_l \in P$ such that l > k. Assume that P is such that there exists at least one agent $i_l \in P$ such that l > k. Let i_p denote the agent such that $min_{l \in P}\{v_{i_l}\} = v_{i_p}$. Notice that, using the result from Step 1, we must have p > k. Agent i_k with $k \le n-1$ prefers to participate when all the agents in P participate if and only if:

$$\alpha_{i_k}(v_{i_k} + (k-1)w) + \beta_{i_k} \min\{v_{i_p} + (k-1)w, c\} \ge \alpha_{i_k} c + \beta_{i_k} \min\{v_{i_p} + (k-2)w, c\},$$
 (13)

or, using $v_{i_p} = c - (p-1)w$,

$$\alpha_{i_k}(v_{i_k} + (k-1)w - c) + \beta_{i_k}w \ge 0,$$
(14)

which is always true since $v_{i_k} + (k-1)w = c.\Box$

Proof of Proposition 3: I make the proof for the more general case in which the agents have heterogeneous collective motivations, i.e. the α_i s and the γ_i s are not necessarily identical. Assume that there are q-1 agents $(1 \le q < n)$ in the set P and they all participate. An agent $i \in N \setminus P$

has an incentive to participate if and only if:

$$\alpha_i (v_i + (q-1)w) + \gamma_i \left(\sum_{j \in P \cup i} v_j + q(q-1)w \right) \ge \alpha_i c + \gamma_i \left(\sum_{j \in P} v_j + (q-1)(q-2)w + c \right),$$
 (15)

Using $\alpha_i + \gamma_i = 1$, we conclude that condition (15) is equivalent to:

$$v_i \ge c - (q - 1)w [1 + \gamma_i].$$
 (16)

Using Proposition 1 with $\gamma_i = \gamma$ and $\beta_i = 0$, we have that condition (16) also holds when q = n. Moreover, the previous reasoning implies that an agent participation decision is affected by the number of other participating agents and not by the identity of the other participating agents. Thus, in order to minimize his cost, the principal will (re)order the agents such that $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$ and will choose $v_1^* = c$, $v_2^* = c - w(1 + \gamma_2)$, $v_3^* = c - 2w(1 + \gamma_3)$, ..., $v_n^* = c - (n-1)w(1 + \gamma_n)$.

Proof of Proposition 4: I still assume, without loss of generality, that the agents are ordered such that i_{j+1} is the agent that has an incentive to participate when the first j agents participate. The corresponding formal condition for agent i_1 is $v_{i_1} \geq c$. Let $i_{p(j+1)}$ be the agent such that $\min_{l \leq j+1} \{v_l\} = v_{i_{p(j+1)}}$. For agent i_{j+1} with $2 \leq j+1 \leq n-1$ the corresponding formal condition can be written as follows:

$$(\alpha + \gamma)(v_{i_{j+1}} + jw - c) \ge -\beta \left(\min\{v_{i_{p(j+1)}} + jw, c\} - \min\{v_{i_{p(j)}} + (j-1)w, c\} \right) - \gamma jw. \tag{17}$$

And for agent i_n , the condition is:

$$(\alpha + \gamma)(v_{i_n} + (n-1)w - c) \ge -\beta \left(v_{i_{p(n)}} + (n-1)w - \min\{v_{i_{p(n-1)}} + (n-2)w, c\}\right) - \gamma(n-1)w.$$
(18)

Step 1: I show that any contract that is characterized by the divide and conquer property and that minimizes the implementation cost is such that an agent is indifferent between participating and not participating when all the preceding agents in the ranking participate and all the subsequent agents in the ranking do not. In other words, I show in this Step 1 that the least cost contract such that $v_{i_1} \geq c$, condition (17) for j + 1 = 2, ..., n - 1 and condition (18) hold is such that all these inequalities are binding.

Let me first show that the claim holds for agent i_1 . A decrease in v_{i_1} - as long as $v_{i_1} \geq c$ - does not affect condition (17) for j+1=2,...,n-1. Let me now focus on condition (18). If $i_{p(n-1)} \neq i_1$ then $i_{p(n)} \neq i_1$ and then condition (18) does not depend on v_{i_1} . If $i_{p(n-1)} = i_1$ and $i_{p(n)} \neq i_1$, then condition (18) is more likely to hold when v_{i_1} decreases. If $i_{p(n)} = i_1$ then $i_{p(n-1)} = i_1$ and condition (18) becomes:

$$(\alpha + \gamma)(v_{i_n} + (n-1)w - c) \ge -\beta(v_{i_1} + (n-1)w - \min\{v_{i_1} + (n-2)w, c\}) - \gamma(n-1)w.$$
 (19)

If $v_{i_1} + (n-2)w \le c$, then condition (19) does not depend on v_{i_1} . If $v_{i_1} + (n-2)w > c$, condition

(19) becomes:

$$(\alpha + \gamma)(v_{i_n} + (n-1)w - c) \ge -\beta (v_{i_1} + (n-1)w - c) - \gamma(n-1)w. \tag{20}$$

Condition (20) can be rewritten as follows:

$$(\alpha + \gamma)(v_{i_n} - v_{i_1}) \ge -(v_{i_1} + (n-1)w - c) - \gamma(n-1)w. \tag{21}$$

The right hand side in (21) is negative because $v_{i_1} \geq c$ and the left hand side is positive because $i_{p(n)} = i_{k+1}$. We conclude that, if $v_{i_1} + (n-2)w > c$, then condition (18) is equivalent to $v_{i_1} \geq v_{i_1}$, which is more likely to hold when v_{i_1} decreases. This leads to conclude that $v_{i_1} = c$. This proves that the induction claim holds for agent i_1 .

Let me now show that the claim holds for $2 \leq j+1 \leq n-1$. Notice that condition (17) for j+1=k+1 is more likely to be binding when $v_{i_{k+1}}$ decreases. A sufficient condition for the principal to choose to bind condition (17) is thus that a decrease in $v_{i_{k+1}}$ does not make any other constraint less likely to hold. Let me first focus on condition (17) for $j+1 \leq n-1$. Let me show that $\Delta_{j+1} \equiv \min\{v_{i_{p(j+1)}} + jw, c\} - \min\{v_{i_{p(j)}} + (j-1)w, c\}$ for $j+1 \leq n-1$ and $j+1 \neq k+1$ does not increase when $v_{i_{k+1}}$ increases. First assume that $j+1 \leq k$. In this case, Δ_{j+1} does not depend on $v_{i_{k+1}}$. Second, assume that $k+2 \leq j+1 \leq n-1$. If $i_{p(j)} \neq i_{k+1}$ then $i_{p(j+1)} \neq i_{k+1}$ and then Δ_{j+1} does not depend on $v_{i_{k+1}}$. If $i_{p(j+1)} = i_{k+1}$ then $i_{p(j)} = i_{k+1}$ and $\Delta_{j+1} = \min\{v_{i_{k+1}} + jw, c\} - \min\{v_{i_{k+1}} + (j-1)w, c\}$. Hence, Δ_{j+1} decreases when $v_{i_{k+1}}$ increases. If $i_{p(j)} = i_{k+1}$ and $i_{p(j+1)} \neq i_{k+1}$, then Δ_{j+1} decreases when $v_{i_{k+1}}$ increases.

Let me now consider condition (18). If $i_{p(n-1)} \neq i_{k+1}$ then $i_{p(n)} \neq i_{k+1}$ and then condition (18) does not depend on $v_{i_{k+1}}$. If $i_{p(n-1)} = i_{k+1}$ and $i_{p(n)} \neq i_{k+1}$, then condition (18) is more likely to hold when $v_{i_{k+1}}$ decreases. If $i_{p(n)} = i_{k+1}$ then $i_{p(n-1)} = i_{k+1}$ and condition (18) becomes:

$$(\alpha + \gamma)(v_{i_n} + (n-1)w - c) \ge -\beta \left(v_{i_{k+1}} + (n-1)w - \min\{v_{i_{k+1}} + (n-2)w, c\}\right) - \gamma(n-1)w. \tag{22}$$

If $v_{i_{k+1}} + (n-2)w \le c$, then condition (22) does not depend on $v_{i_{k+1}}$. If $v_{i_{k+1}} + (n-2)w > c$, condition (22) becomes:

$$(\alpha + \gamma)(v_{i_n} + (n-1)w - c) \ge -\beta \left(v_{i_{k+1}} + (n-1)w - c\right) - \gamma(n-1)w. \tag{23}$$

Condition (23) can be rewritten as follows:

$$(\alpha + \gamma)(v_{i_n} - v_{i_{k+1}}) \ge -(v_{i_{k+1}} + (n-1)w - c) - \gamma(n-1)w.$$
(24)

The right hand side in (24) is negative and the left hand side is positive because $i_{p(n)} = i_{k+1}$. We conclude that, in this case, condition (18) is equivalent to $v_{i_n} \ge v_{i_{k+1}}$, which is more likely to hold when $v_{i_{k+1}}$ decreases. Hence, the principal chooses $v_{i_{k+1}}$ such that condition (17) for j+1=k+1 is binding if k+1 < n and such that condition (18) is binding if k+1 = n.

To conclude Step 1, it remains to show that the claim also holds for agent i_n . It is sufficient to observe that condition (18) is more likely to hold when v_{i_n} increases and that all the other constraints do not depend on v_{i_n} .

Step 2: Let me show that $v_{ij} + (j-1)w - c \le 0$ for all j. We know from Step 1 that the least cost contract that respects the necessary condition is such that $v_{i_1} = c$, and,

$$(\alpha + \gamma)(v_{i_{j+1}} + jw - c) = -\beta \left(\min\{v_{i_{p(j+1)}} + jw, c\} - \min\{v_{i_{p(j)}} + (j-1)w, c\} \right) - \gamma jw, \quad (25)$$

for all $2 \le j + 1 \le n - 1$, and:

$$(\alpha + \gamma)(v_{i_n} + (n-1)w - c) = -\beta \left(v_{i_{p(n)}} + (n-1)w - \min\{v_{i_{p(n-1)}} + (n-2)w, c\}\right) - \gamma(n-1)w.$$
(26)

Let me first consider $2 \leq j+1 \leq n-1$. Notice that the definition of p() implies that $v_{i_{p(j+1)}} \leq v_{i_{p(j)}}$. There are two cases to consider. If $v_{i_{p(j+1)}} + w \geq v_{i_{p(j)}}$, the right hand side in condition (25) is negative, and then $v_{i_{j+1}} + jw - c \leq 0$. If $v_{i_{p(j+1)}} + w < v_{i_{p(j)}}$, then we must have $v_{i_{p(j+1)}} = v_{i_{j+1}}$. There are two subcases to consider. If $v_{i_{p(j)}} + (j-1)w - c \geq 0$, condition (25) becomes:

$$(\alpha + \gamma)(v_{i_{j+1}} + jw - c) + \beta \min\{v_{i_{j+1}} + jw - c, 0\} = -\gamma jw, \tag{27}$$

and then $v_{i_{j+1}} + jw - c \le 0$. If $v_{i_{p(j)}} + (j-1)w - c < 0$, condition (25) becomes:

$$(\alpha + \gamma)(v_{i_{j+1}} + jw - c) + \beta \min\{v_{i_{j+1}} + jw - c, 0\} = \beta(v_{i_{n(j)}} + (j-1)w - c) - \gamma jw, \tag{28}$$

and then $v_{i_{j+1}} + jw - c \le 0$. The same reasoning can be used to show that $v_{i_n} + (n-1)w - c \le 0$. As a consequence, the least cost contract that respects the necessary condition is such that $v_{i_1} = c$ and,

$$(\alpha + \gamma)(v_{i_{j+1}} + jw - c) + \beta(v_{i_{p(j+1)}} + jw - c) = \beta(v_{i_{p(j)}} + (j-1)w - c) - \gamma jw,$$
 (29)

for all $2 \le j + 1 \le n$.

Step 3: Let me show that $v_{i_{j+1}} < v_{i_j}$ for all j. Since the choice of a transfer made to an agent does not depend on the choice of the transfers to the subsequent agents, the principal has an incentive to choose $v_{i_{j+1}} \le v_{i_j}$ for all j. I assume this is true and I will check that the solution respects this condition. We thus have $v_{i_{p(j+1)}} = v_{i_{j+1}}$ for all j. In this case, the least cost contract that respects the necessary condition is characterized by $v_{i_1} = c$ and the following recursive formula:

$$v_{i_{j+1}} + jw - c = \beta(v_{i_j} + (j-1)w - c) - \gamma jw, \tag{30}$$

for all j = 1, ..., n - 1. Solving for the recursive formula, we find:

$$v_{i_{j+1}} = c - jw(1+\gamma) - \gamma w \sum_{t=1}^{j} (j-t)\beta^{t}.$$
 (31)

The difference between two subsequent terms is then:

$$v_{i_{j+1}} - v_{i_j} = -(1+\gamma)w - \gamma w \frac{\beta - \beta^j}{1-\beta} < 0.$$
(32)

Step 4: Let me show that the necessary condition is also sufficient. It is sufficient to show that the least cost contract that respects the necessary condition is such that agent i_{j+1} prefers to

participate than not participate when j agents i_k with k>j+1 participate. Indeed, this enables to conclude that any situation in which $j\leq n-1$ agents participate cannot be a Nash equilibrium. Using the result from Step 3, we have that agent i_{j+1} with $2\leq j+1\leq n-1$ participates when j agents i_k with k>j+1 participate if and only if:

$$(\alpha + \gamma)(v_{i_{i+1}} + jw - c) \ge -\beta w - \gamma jw. \tag{33}$$

Using the result from Step 3, condition (25) can be written as follows:

$$(\alpha + \gamma)(v_{i_{j+1}} + jw - c) = \beta (v_{i_j} - v_{i_{j+1}} - w) - \gamma jw.$$
(34)

Hence, condition (33) is equivalent to $v_{i_j} - v_{i_{j+1}} \ge 0$, which is true according the result from Step 3.

Proof of Corollary 2: The expression of the difference between two subsequent terms comes from condition (32). The effect of γ on the difference is straightforward. The derivative of $v_k - v_{k+1}$ with respect to β is $\frac{1-\beta^{k-1}(k(1-\beta)+\beta)}{(1-\beta)^2}$. This derivative is positive if and only if $1 \le \beta^{k-1}(k(1-\beta)+\beta)$. The derivative of $\beta^{k-1}(k(1-\beta)+\beta)$ with respect to β is $k(k-1)(1-\beta)\beta^{k-2} \ge 0$. Moreover, when $\beta = 1$, we have $\beta^{k-1}(k(1-\beta)+\beta) = 1$. This leads to conclude that $v_k - v_{k+1}$ increases when β increases. The fact that the difference increases when γ and β increase simultaneously is thus straightforward. \square

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