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Open space preservation in an urbanization context *

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Abstract

The objective of this paper is to address the question of open space preservation in an urbanization context. We study the possibility of preserving two different types of open spaces, large open spaces at cities' outskirt and small intra-urban open spaces. Thus we contribute to the debate of land sharing versus land sparing in a urban context. We analyze these questions by way of a theoretical microeconomics framework taking into account both households' preferences for open space and regulator's interest for the preservation of ecosystem services. We compare land use patterns at private equilibrium and when the social planner maximizes social welfare.

JEL classification : Q24, R14, R28
Keywords : Urbanization, Open space preservation, Public intervention

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1 Introduction

Since the publication of the report entitled "Urban sprawl : the ignored challenge" by the European Commission which concludes that urban sprawl is responsible for a lot of enrivonmental degradation in urban areas, fighting urban sprawl has become the number one objective of institutions and city planners (European Environment Agency, 2006). Creating compact cities appears as the best recommandation in the public debate, notably in order to avoid disturbance of natural open spaces at cities' outskirt. However, in the relentless fight against urban sprawl, the importance of green space inside the urban areas has been put aside. Nonetheless, open space located inside cities' boundaries represents a significant share of land and deserve to be taken into account in the analysis of environmental preservation and the search of an optimal city structure. Indeed, in Stokholm for example, given the European data from the Urban Audit, green urban areas appear to represent $24,1\%$ of land. Green urban areas include public green areas, predominant recreational use as gardens, zoos, parks, castle parks, and suburban natural areas that have become and are managed as urban parks. If we add natural land that are not managed as urban parks, the total share of intra urban open spaces rise to more than 35%. The proportion of intra urban open spaces is quite similar for the city of Berlin, is also very important for Paris (almost 25%) but is less for others cities like Amsterdam $(14,5\%)$(Eurostat, 2017). In France, public authorities have recognized the role of these open spaces, as illustrated by the circular dating from February 8th 1973 which fixed precise objectives regarding the accessibility of open spaces : each urban dweller should have access to $10m^2$ of "proximity open spaces" located inside the cities such as defined in the green urban spaces of urban audit, and $25m^2$ of "weekend open spaces" such as forests or natural land located farther from their home.

We see that a possible contradiction may arise between the willigness to fight urban sprawl in order to preserve large open spaces at city's outskirt, synonymous with the promotion of a compact city, and the preservation of intra-urban open spaces in small patches that could be responsible for an extension of the city's limit. This question fits in the recent land sharing versus land sparing framework. This framework was first proposed by Green et al. (2005) in the literature on the impact of agriculture on biodiversity and aimed at understanding the extent to which agriculture should be concentrated on intensively farmed land in order to conserve more biodiversity-rich natural spaces elsewhere (*land sparing*) or conversely, be more wildlife-friendly but less productive, hence conserving fewer natural spaces (*land sharing*). This framework was extended to the context of cities recently in the ecological literature: land sparing minimizes the spatial extent of developed areas, such that residential areas are developed as intensively as possible, enabling the maintenance of large open spaces. Under land sharing, development is more evenly but less intensively distributed, such that a larger land area is needed to accommodate a given number of households, and open spaces tend to be more fragmented but on average closer to residential areas (Brenda and Fuller, 2013; Soga et al., 2014; Stott et al., 2015). From an ecological perspective, the question of which urban structure is preferable for biodiversity conservation is far from being completely answered. To broaden the debate, we propose in this paper an economic analysis of these questions, in order to understand how the behavior of economic agents influences the city's structure and the existence and preservation of different types of open space.

Several papers have already studied the effect of open spaces in a spatial urban context. For example, Wu (2001) and Wu and Plantinga (2003) consider city formation when people have a taste for proximity to exogenously located open space, such as a park. Their analyses focus on the role of open spaces in city structure, but they do not consider the possibility for the available amount of open spaces to be modified by the choice of location of people, concealing the fact that people impose external costs on each other. Strange (1992) and Marshall (2004) consider the question of open space in city in a model with housing externalities but they do not model a land market. Walsh (2007) proposes a Tiebout model in which people have preferences over the characteristics of neighborhood landscape (the amount of open spaces in particular) but the Tiebout framework does not allow to analyse the micro-structure of urban development that we want to develop here. Justifying theoritically the fight against urban sprawl, Brueckner (2000) explains that the social value of open space is not taken into account by households when they make their choice of location, leading to an excessive extension of city. Hence, he recommends to limit urban sprawl in order to preserve open spaces that are located at the outskirts of cities, such as agricultural plains or forests. Several others papers focus instead on the role of intra-urban open spaces : in a theoretical framework, Turner (2005) analyzes the equilibrium and optimum city structure when households value local open space, and he shows that the optimal city is less compact than at the private equilibrium. According to him, public policies such as urban growth boundaries are not fitted when households value local open space. Cavailhès et al. (2004) also demonstrate that the sprawled pattern of cities and the existence of a periurban area is the consequence of households' preferences for natural amenities near their place of residence and thus is not necessarily an inefficient pattern of development. In a recent study, Caruso et al. (2015) analyse urban forms in a 2D microeconomic model where households value open space close to their location; they show that high preferences for green spaces increase both leapfrog development and the size of the leaps. Thus, to the extent of our knowledge, the question of the preservation of different types of open space in a sole model, and the analysis of land sparing and land sharing in a urban context is not yet covered in the economic literature.

This paper expands the literature by developing a theoretical urban model which takes into account explicitly the existence of two different types of open spaces for which preservation strategies may be conflicting. Firstly, we study the impact of households' preferences for proximity open spaces - such as parks, greenways, public gardens and natural or agricultural land directly visible from their place of residence - on the equilibrium city structure. In a second step, we introduce the role of large open spaces outside of cities such as forests, meadows and agricultural land, which are valued by a social planner for their ecological interest. We study the optimal city structure when the social planner takes into account biodiversity conservation.

The remainder of the paper is organized as follows. Section 2 presents the general structure of the model. Section 3 provides an analysis of the equilibrium city structure. Section 4 develops the welfare analysis. Finally, section 5 concludes.

2 The model

2.1 Residential behavior

Consider a city lying on a uni-dimensional space $X = [-\infty, +\infty[$. The city is monocentric: all the firms locate at the central business district (CBD), located at 0 and which size is neglected [1]. At each location $x \in X$, the quantity of available land is equal to one. Our objective is to describe the pattern of the residential area in this city, that we assume closed: the number of households is fixed and the equilibrium utility level varies endogenously. The assumption of a closed city is relevant in order to study the possible allocations of different land-uses within the city.

All households are assumed to be homogeneous, meaning that each household's income level and utility function are identical. Households divide their entire income between the consumption of a composite good, a residential good, and commuting costs, proportional to the distance to the CBD where all firms are located.

The lot size of each house is assumed to be exogenously fixed; however a residential area is also characterized by the presence of local open space, which is inversely proportional to the level of development at each location. We consider that households have preferences for local open space, available directly at their place of residence. Natural amenities from local open space are considered as spatial attributes of housing, which affect the households'

[1] Several models where firms' location is endogenous exist in the economic literature (Fujita, 1989), the location of firms may matter when they affect environmental damage, as for instance air pollution (see (Regnier and Legras, 2017)). However we assume here that the location of firms is not relevant since their location choice is not influenced by open space.

utility function but not their budget constraint.

Following Fujita (1989), households' maximization program is given by :

$$\begin{cases} \max\limits_{m,x} & u(m, q, d(x)) \\ s.t. & p(x)q + m + t(x) \leq w \end{cases}$$

with:

$u(\cdot)$: the utility function

x : the distance from the CBD

m : the amount of the composite good, which price is the numéraire

q : the lot size of the house, assumed to be fixed

$d(x)$: the level of urban development at location x, with $0 \leq d(x) \leq 1$

$p(x)$: the rental price of a house at distance x

$t(x)$: commuting costs for a household located at distance x

w : the income

The amount of local open space is inversely correlated with the level of urban development at location x and thus is a decreasing function of $d(x)$.

At equilibrium, the utility level is given by u^*, and is equal among all households no matter their residential location as they are identical and mobile without cost. The household's demand function for the composite good $m^*(q, d(x), u^*)$ is obtained by solving:

$$u(m, q, d(x)) = u^* \tag{1}$$

We can now derive the residential bid-rent function $p(x)$, which indicates the maximum amount a household is willing to pay at location x while receiving the utility level u^* :

$$p(x) = \frac{w - t(x) - m^*(q, d(x), u^*)}{q} \tag{2}$$

The residential bid-rent is an implicit function of the income, the commuting cost, the lot size, and the level of urban development: $p(x) \equiv p(w, t(x), q, d(x), u^*)$, with $\frac{\partial p}{\partial w} > 0$, $\frac{\partial p}{\partial t(x)} < 0$, $\frac{\partial p}{\partial q} > 0$, and $\frac{\partial p}{\partial d(x)} < 0$. When prices vary accordingly across the city, households' utility is identical across locations and households have no incentive to move.

The bid-rent function reveals the difference between our model and the standard monocentric city model. In the standard model, natural amenities are assumed to be distributed uniformly across the landscape : residential rents always fall with the distance from the CBD, compensating residents for their increasing cost of commuting. However, with spatial variations in amenities, the spatial pattern of the rent is more complex. A household may be willing to sacrifice proximity to the workplace for amenities, with the result that the willingness to pay for housing may no longer be a monotonically decreasing function of the distance to the CBD.

Let's consider a linear utility function of the following form :

$$U(m, q, d(x)) = m + q + \gamma(1 - d(x)) \tag{3}$$

where γ is a positive constant, and linear transport costs $t(x) = tx$. Households' bid-rent function is given by:

$$p(x) = \frac{1}{q}(w - tx - u^* + q + \gamma(1 - d(x))) \tag{4}$$

It is decreasing with transport cost t according to: $\frac{\partial p}{\partial t} = \frac{-x}{q}$, and decreasing with the level of development at each x according to: $\frac{\partial p}{\partial d(x)} = \frac{-\gamma}{q}$. The impact of the distance from the

CBD will be analysed once the development level function is established in the developer's program.

2.2 Development decision

On the supply side, housing is produced with land, labor and materials under constant returns to scale. The house size, q, is fixed, and outside-developers choose the level of development $d(x)$ at each location, which is equivalent to the number of dwellings per acre. We assume that at each location, only one developer is the landowner of the parcel and takes the development decision. Moreover, the cost of development $c(d(x))$ is assumed a linear function of the development density $d(x)$ such that $c(d(x)) = Cd(x)$ where C is a positive constant, meaning that the cost increases at the same rate as $d(x)$.

At each location, the developer chooses the development density to maximize profit :

$$\max_{d(x)} \pi(d, x) = p(w, t(x), q, d(x), u^*)d(x) - Cd(x)$$

The equilibrium development density is the solution of the first order condition given by:

$$\frac{\partial p(\cdot)}{\partial d(x)} d(x) + p(\cdot) = C \tag{5}$$

From the second order condition, we obtain that $\frac{\partial^2 \pi}{\partial d(x)^2} < 0$ as long as the rent is a decreasing and concave or decreasing and linear function of $d(x)$. Then the profit function reaches its maximum when $d(x)$ is the solution of the differential equation (5).

Using the implicit function theorem, we derive the variation of the level of development along the city :

$$\frac{\partial d(x)}{\partial x} = \frac{\frac{\partial^2 \pi}{\partial d(x) \partial x}}{\frac{-\partial^2 \pi}{\partial x^2}} \tag{6}$$

Using the second order condition, we know that the denominator of the above equation is always positive. Then the sign of (6) depends on the sign of the numerator :

$$\frac{\partial^2 \pi}{\partial d(x) \partial x} = \frac{\partial^2 p}{\partial d(x) \partial x} d(x) + \frac{\partial p}{\partial x} \tag{7}$$

The first part of the right hand side of (7) corresponds to a cross effect. It is interpreted as how the variation of the bid-price with respect to $d(x)$ changes along the city gradient. The sign of this term is a priori unknown and depends on the functional from specified in the household's program. The second term of (7) corresponds to the variation of the housing price along the city that does not depend on the level of development $d(x)$, it passes trough the commuting cost and it is always negative. As a consequence, the sign of the total variation of the development level with respect to the distance to the city center depends on the sign of the cross-effect. If the cross-effect is nul or negative, the level of development always falls along the city gradient. If the cross-effect is positive, the level of development might increase as we move away from the city center.

The first order condition is a second order differential equation of the variable $d(x)$, its solution is given by :

$$d^*(x) = \frac{p(w, t(x), q, d^*(x)) - C}{K} \tag{8}$$

where K is an unknown constant, as we have not defined any specifical functional form.

The development density is a function of the residential rent and, through it, the level of amenities at each location. Indeed, households have preferences for local open space, as indicated by (4), meaning that the development density is a disamenity for households and an increase in development density reduces households' willingness to pay for housing. In that case, the developer chooses the number of houses built by balancing households' taste for local open space and her own interest for high density. Thus, when households value local open space, the level of development at equilibrium is not maximal in each location x of the city. For some x, only a part of the parcel is converted to residential use. This result is fully developed in the analysis of the equilibrium land allocation in Section 3.

Using the linear utility function descrived in (3), the equilibrium development level writes as follows:

$$d^*(x) = \frac{1}{2\gamma}\left[w - tx - Cq + q - u^* + \gamma\right] \tag{9}$$

from which we can derive:

$$\frac{\partial d^*(x)}{\partial x} = \frac{-t}{2\gamma} \tag{10}$$

Under this specific functional form, at equilibrium, the level of development always decreases with distance to the city center. The negative slope of the development density results from a trade-off between the commuting cost and the level of open space in each x. This trade-off reflects households' preferences for open space, and it affects the equilibrium level of development chosen by the developer. Thus, this equilibrium development density is also the result of the developer's trade-off between maximizing the number of houses built and gaining the maximum possible rent from each house.

It is also interesting to note that by choosing the level of urban development, developers are able to influence the value of the residential rent: they have market power. Consequently, they make a positive mark-up (see equation (5)) leading to a market imperfection. In this respect, developers' decisions are made under imperfect competition, and this affects the land market. Comparing the development function derived above and the one obtained without market power (by setting $C = p(.)$), we obtain that land development without market power is twice than with market power.

3 The urban-periurban-rural equilibrium

Here we investigate in more details the possible configurations of the city at equilibrium, especially we demonstrate that a urban-periurban-rural equilibrium can arise.

At equilibrium, housing prices are bid up until no household has any incentive to move.

This condition is satisfied when housing prices are represented by (4) since the household's bid-rent function is their maximum willingness to pay for housing. At each location x, landowners choose the use that maximizes the return of their plot of land. Therefore, land is developed if the return in residential use exceeds the return of land in its previous state (agricultural or natural), which is supposed to equal zero.

The return per acre in residential use at a particular location x is given by the developers' profit at equilibrium and defined as :

$$r_c(w,t,q,u^*,d^*(x)) = p(w,t,q,u^*,d^*(x))d^*(x) - Cd^*(x) \tag{11}$$

The first closing condition of the model states that residential rent must equal the exogenous agricultural rent at the city boundary x_m :

$$r_c(w,t,q,u^*,d^*(x_m)) = 0 \tag{12}$$

The second closing condition requires that all households live in the city :

$$\int_0^{x_m} n(x)dx = N \tag{13}$$

Where $n(x)$ is the equilibrium number of households at any location and equals the equilibrium development density divided by floor space per household : $n(x) = d^*(x)/q$, and N is the total number of households in the city and is fixed exogenously. Moreover, the following condition must be satisfied : $Nq \leq L$, where L is the maximum boundary of the city (either for physical or administrative reasons). This conditions means that the total floor space occupied by city's residents cannot exceed the maximum boundary of the city.

The total value of land in the city, R, is given by the total value of land in residential use :

$$R = \int_0^{x_m} r_c(w,t,q,u^*,d^*(x))dx \tag{14}$$

3.1 Equilibrium level of development

The compact built-up area At equilibrium the level of urban development in each x is given by (9), reflecting the trade off between transport cost and the amount of open space in each x. It is possible that for some x, the transport cost is so low that the level of development x reaches its maximum level, so equal to 1 :

$$d^*(x_u) = \frac{1}{2\gamma}\left[w - Tx_u - Cq + q - u^* + \gamma\right] = 1 \tag{15}$$

$$\Leftrightarrow x_u = \frac{1}{T}\left(w - Cq + q - u^* - \gamma\right) \tag{16}$$

Thus, for every $x \leq x_u$, there is no open space left, and the level of development $d^*(x)$ is equal to 1. The parcel x_u delimitates the frontier of the compact built-up area in the city. We need to check under which condition this frontier x_u is greater than zero to ensure that there exists a compact built-up area at equilibrium :

$$x_u > 0 \tag{17}$$

$$\Leftrightarrow \gamma < (w - Cq) + q - u^* \tag{18}$$

If γ is low enough, meaning that households have moderate preferences for open space available at their place of residence, there exists a compact built-up area at equilibrium. Otherwise, if γ is too high, households have very strong preference for open space and there is no possibility for a compact built-up area to exist at equilibrium.

The periurban area The periurban area is where the level of development varies between 0 and 1. Recall that $\frac{\partial d(x)}{\partial x} = \frac{-1}{2}\frac{T}{\gamma}$, meaning that close to x_u, the level of development is high and close to 1, and it decreases along the city until it equals zero at the city's limit x_m.

The rural area The city's limit x_m is reached when the level of development $d^*(x)$ is equal to zero :

$$d^*(x_m) = \frac{1}{2\gamma}\left[w - Tx_m - Cq + q - u^* + \gamma\right] = 0 \tag{19}$$

$$\Leftrightarrow x_m = \frac{1}{T}\left(w - Cq + q - u^* + \gamma\right) \tag{20}$$

Thus, for every $x \geq x_m$, the level of urban development is null. The condition that $d^*(x) = 0$ is equivalent to the condition $R_c(x) = 0$, in other terms, when $x \geq x_m$ developers have no interest to develop houses because the residential return is no longer higher than the agricultural return, thus they let land in its agricultural state. The level of development along the city is depicted in Figure 1.

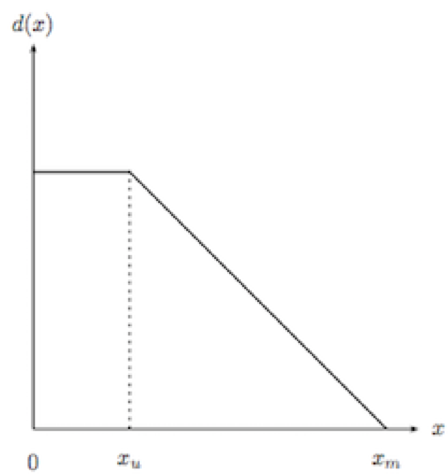

Figure 1 – Variation of urban development along the city

3.2 Total value of land at equilibrium

In the general framework, return per acre in residential use varies with distance to the city center according to :

$$\frac{\partial r_c}{x} = \underbrace{\left(\frac{\partial p}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial p}{\partial d^*}\frac{\partial d^*}{\partial x}\right) d^*(x)}_{\text{price effect}} + \underbrace{p(\cdot)\frac{\partial d^*}{\partial x} - C\frac{\partial d^*}{\partial x}}_{\text{size effect}}$$

We can decompose this expression as follows.

The first part of the right-hand side is a price effect. The price paid by households varies with distance according to variations of commmuting cost and urban development. They both act as negative forces on households' bid-rent function. However, commuting cost increases with distance, while the level of urban development can increase or decrease, as demonstrated above. If the level of development increases with distance, the price effect is always positive. However, if the level of development decreases with distance, the total effect on the price paid by households is unknown, and it depends of the rate of variation of commuting cost relative to that of urban development. This result is similar to the one demonstrated in the seminal paper by Richardson (1977), in which the variation of residential rent in the presence of externalities are analysed.

The second part of the right-hand side corresponds to a size effect: the return in residential rent depends on the share of land that developers choose to built ($d(x)$) which is decreasing with distance. Thus, the revenue of developers decreases with distance, but so do their costs. In a classic model of production the size effect is nul as the marginal revenue equals the marginal costs at equilibrium. However in our model the size effect is negative, because of the market power held by developers (see (5)).

The total variation of the residential return with respect to the distance to the city center is thus a priori unknown with no specific functional forms. Let's analyse it with a linear utility function. At equilibrium, the return of land in residential use is given by :

$$r_c(x) = \left[\frac{1}{q}(w - tx - (u^* - q) - \gamma(1 - d^*(x))\right] d^*(x) - Cd^*(x)$$

In the compact built-up area, the level of development $d^*(x)$ is equal to one. In that case, their is no difference with the classic urban economics model with no open space and fixed lot size, and the rent is linearly decreasing with the distance to the CBD according to the variation of transport cost :

$$\frac{\partial r_u^*(x)}{\partial x} = \frac{-t}{q} \tag{21}$$

In the periurban area, the level of development $d^*(x)$ varies between zero and one and it affects the equilibrium residential bid-rent. Here, we see that the slope of the residential return is given by :

$$\frac{\partial r_p^*(x)}{\partial x} = -\frac{d^*(x)t}{q} \tag{22}$$

The residential rent decreases with distance to the CBD, but at a slower rate than in the compact built-up area. This phenomenon is explained by households' preference for open space : when households locate far away from the center, they pay high transport cost, but they trade-off with the amount of open space enjoyed. Thus, the rent is decreasing slowly because open spaces create a positive force on the equilibrium rent. When households have preference for proximity open space, the city extends more than in the classic Muth-Mills model of urban economics. In our case however the slope of the rent remains negative, because the rate of variation in the amount of open space is not greater than the rate of variation in the transport cost.

Finally, in the rural area, for $x \geq x_m$, the equilibrium rent is equal to the land rent in its agricultural state, here equal to zero for simplicity. The variation of the residential return along the city is depicted in figure 2, where x_c represents the city's limit in the classic monocentric urban economics model.

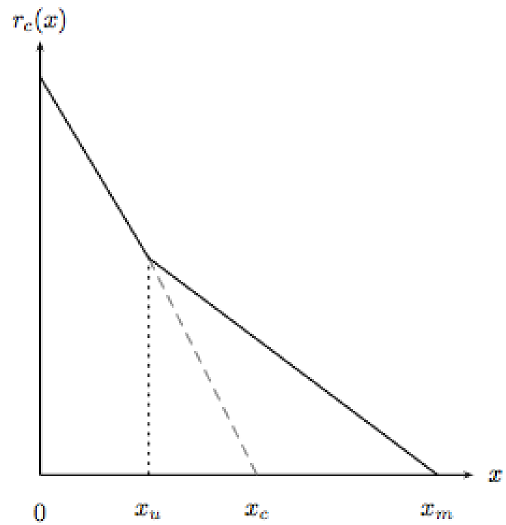

Figure 2 – Residential return gradient

4 Effects on biodiversity and welfare analysis

Until now, we have considered only the presence of intra-urban open space that are valued by households when they make their choice of location. However, the social planner cares also for big natural open space at the outskirts of city (Brueckner, 2000). Indeed, he values both type of open spaces because they offer provision and regulation ecosystem services, such as habitats for natural species. We define the level of biodiversity as follow :

$$B(d(x), x_m) = \int_0^{x_m} w_1[1 - d(x)]dx + w_2(L - x_m) \tag{23}$$

The first term of the right hand side of the equation represents the contribution of intra-urban open space, while the second term represents the contribution of the big open space at city's outskirts; w_1 and w_2 are the weights associated with each type of open space regarding biodiversity conservation and are assumed to be positive. These weights depend on the social planner's priorities on biodiversity conservation. Indeed, the presence of some species may be favored with small patches of intra-urban open space, while other species may prefer large open space.

Drawing upon the methodology of Fujita (1989), the problem of the social planner is to choose the level of urban development $d(x)$ at each x and the urban fringe distance x_m so as to maximize the sum of the total surplus and the biodiversity level while achieving the target utility u for N households.

$$\begin{cases} \max\limits_{d(x),x_m} \quad \int_0^{x_m} [(w-u+q) - td(x) - \gamma d(x)] \frac{d(x)}{q} dx - \int_0^{x_m} Cd(x)dx + B(d(x), x_m) \\ \\ s.t. \qquad \int_0^{x_m} \frac{d(x)}{q} = N \end{cases}$$

The Lagrangian function associated to the above maximization program is :

$$\mathcal{L}(d(x), x, \lambda) = \int_0^{x_m} [(w-u+q) - td(x) - \gamma d(x)] \frac{d(x)}{q} dx - \int_0^{x_m} Cd(x)dx \\ + w_1 \int_0^{x_m} (1-d(x))dx + w_2(L - x_m) + \lambda(N - \int_0^{x_m} \frac{d(x)}{q}) \quad (24)$$

The optimal conditions of this problem are given by :

$$d(x) = \begin{cases} 1 & \text{for } x \leq x_u \\ \frac{(w - Cq + q - u - tx - w_1 q + \lambda)}{2q\gamma} & \text{for } \quad x_u \leq x \leq x_m \\ \frac{w_2 - w_1}{p(\cdot) - C - w_1 + \lambda/q} & \text{at } \quad x = x_m \quad \text{boundary condition} \\ 0 & \text{for } x > x_m \end{cases} \quad (25)$$

$$\int_0^{x_m} \frac{d(x)}{q} = N \quad (26)$$

We now compare the optimal conditions with the equilibrium results. We see that the level of urban development inside the peri-urban area is different at the optimum than at equilibrium. Indeed, the optimal level of urban development decreases with w_1, which is the marginal contribution of the intra-urban open spaces to the preservation of biodiversity, and increases with λ which is linked to the population constraint and represents the marginal net cost of a household in the city.

Moreover we see that $\frac{\partial d(x)}{\partial x} = \frac{-t}{2q\gamma}$ for $x_u \leq x \leq x_m$, meaning that at the optimum the urban development decreases at a lower rate with the distance to the city center compared to equilibrium.

The second main difference between optimum and equilibrium comes from the boundary condition. At the optimum, the level of urban development does not longer equal zero at the urban fringe. The level of urban development at the urban fringe is greater than zero and depends on the marginal contributions of the two different types of open spaces captured by w_1 and w_2, and of the net cost of a household in the city captured by λ.

Solving this boundary condition, we obtain the optimal city's limit, that we call x_{m_o}:

$$x_{m_o} = \frac{(2q-1)(w_1 q + Cq - q - w + u + \lambda) - 2\sqrt{q^3\gamma(w_2 - w_1)(1-2q)}}{t(2q-1)} \tag{27}$$

Supposing that $q < \frac{1}{2}$ and that $w_1 q + Cq - q - w + u + \lambda > 0$ ensures that the optimal city's limit is positive. Using the first two equations of system (25), we also obtain x_{u_o}, the optimal limit of the compact built-up area :

$$x_{u_o} = \frac{1}{T}\left(w - Cq + q - u - \gamma - w_1 q + \lambda\right) \tag{28}$$

It is easy to compare the limit of the urban core at equilibrium x_u, with the one at optimum x_{u_o}. We see that two new terms appears in x_{u_o} : one related to the biodiversity provided by proximity open space, and the other related with the population constraint. More precisely, the compact built-up area becomes smaller when the marginal gain of biodiversity provided by local intra-urban open spaces increases, and becomes larger when the net cost of a households in a city increases. The size of the compact built-up area is not directly related to the marginal gain of biodiversity provided by big open spaces.

Comparing the optimal and the equilibrium limits of the whole city is more complicated, but we can see that at optimum, the city's limit x_{m_o} depends on w_1, w_2 and λ as follows :

$$\frac{\partial x_{m_o}}{\partial w_1} = -\frac{1}{t(2q-1)}(q-2q^2) + \frac{q^3\gamma}{t\sqrt{q^3\gamma(w_2-w_1)(1-2q)}} \tag{29}$$

If we suppose as previously that $q < \frac{1}{2}$, the variation of x_{m_o} with respect to w_1 is always positive. The higher the marginal contribution of intra urban open space to biodiversity, the larger the city.

$$\frac{\partial x_{m_o}}{\partial w_2} = \frac{-q^3\gamma}{t\sqrt{q^3\gamma(w_2-w_1)(1-2q)}} \tag{30}$$

The variation of x_{m_o} with respect to w_2 is always negative. The higher the marginal contribution of big outskirts open space to biodiversity, the smaller the city.

$$\frac{\partial x_{m_o}}{\partial \lambda} = \frac{1}{t} \tag{31}$$

The variation of x_{m_o} with respect to λ is always positive. The higher the net cost of a household in the city, the larger the city. The net cost of a household in the city, λ, can intuitively be interpreted as the strength of the population constraint. Thus, the stronger the population constraint, the larger the city to accommodate all the households.

Proposition 1

1. *If the marginal gain of biodiversity provided by intra-urban open space, w_1, is large relatively to the marginal gain of biodiversity provided by big outskirts open space, w_2, such that $\frac{w_1}{w_2}$ is higher than $(\frac{w_1}{w_2})^h$, the optimal city structure is complete land sharing.*

2. *If the marginal gain of biodiversity provided by intra-urban open space, w_1, is low relatively to the marginal gain of biodiversity provided by big outskirts open space, w_2, such that $\frac{w_1}{w_2}$ is lower than $(\frac{w_1}{w_2})^l$, the optimal city structure is complete land*

sparing.

3. *If* $(\frac{w_1}{w_2})^h \geq \frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^l$, *the optimal city structure is a mixed between complete land sharing and complete land sparing. The closer* $\frac{w_1}{w_2}$ *is to* $(\frac{w_1}{w_2})^h$, *the more the land is spared, and the closer* $\frac{w_1}{w_2}$ *is to* $(\frac{w_1}{w_2})^l$, *the more the land is shared.*

See figure 3 for illustration and Appendix 1 for demonstration.

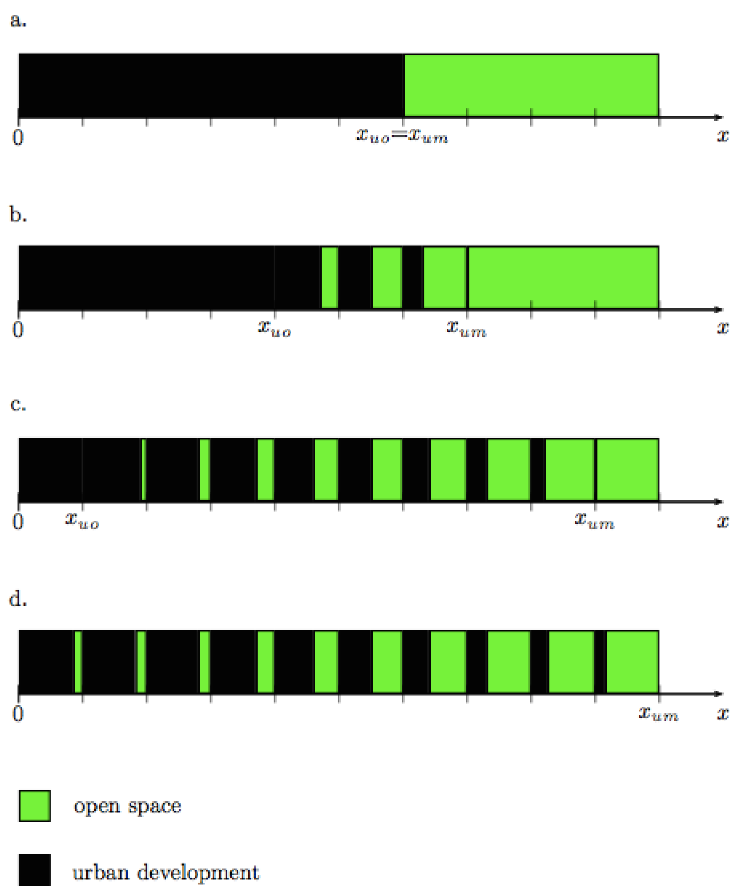

Figure 3 – Optimal city structure
a. $\frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^h$: complete land sparing. b. $(\frac{w_1}{w_2})^h \geq \frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^l$ and close to $(\frac{w_1}{w_2})^h$: land sparing. c. $(\frac{w_1}{w_2})^h \geq \frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^l$ and close to $(\frac{w_1}{w_2})^l$: land sharing. d. $\frac{w_1}{w_2} \leq (\frac{w_1}{w_2})^l$: complete land sharing.

Proposition 1 and figure 3 show that the optimal city structure can take several forms. The equilibrium city structure can either be too compact, or too sprawled, compared to the optimum, depending on the relative relative marginal contribution of the different types of open spaces on biodiversity conservation.

This result suggests that public policies must be carefully designed if the objective is to preserve biodiversity. In the case where optimum is obtained through land sparing configuration, a policy which provides incentives for infill development of vacant locations would be welfare increasing. Densification of the core city associated to creation of greenbelts by prohibiting development in wide regions of the periphery appears to be the best option to increase welfare.

However, the recommendation are different when intra urban open space provide a high marginal gain of biodiversity : in that case, land sharing should be promoted, and a policy aimed at fighting urban sprawl would be welfare decreasing. In that case, creating a series of urban parks is the best solution to enhance welfare.

5 Conclusion

We developed a model in which open space can be split into two categories : local intra-urban open space that are directly valued by households as cultural ecosystem services, and large open space at city's outskirts valued by the social planner for environmental reasons. Our aim was to understand how households' preferences affect the equilibrium city structure, we show that when households value local open space, the city is first composed of a pure urban core where all the land is developed, followed by a periurban area where a part of the land is not developed forming intra-urban open spaces. Finally a rural area completes the equilibrium land-use pattern. This result entails that the city extends more when households value local open spaces, which is directly responsible for the loss of large open spaces at city's outskirt. We then study what is the optimal city structure when a social planner maximizes social welfare taking into account biodiversity conservation and both types of open spaces. We show that the optimal structure of the city can be either land sharing or land sparing, depending on the relative marginal impact of each type of open spaces on biodiversity conservation. Thus, welfare maximization does not came necessarily with a bitter fight against uban sprawl, but we need to take into account the complex relation between urban form and nature preservation. This result is an invitation to undertake adequate researchs upstream in order to better grasp and foresee the potential perverse effects associated with the promotion of a single form of sustainable city, as is currently the case with the paradigm of the compact city. Our analysis is a first step in the land sharing vs. land sparing debate in an urban context with economic tools. However, several questions still need to be addressed. In particular, it would be particularly interesting to take into account the possibility of "vertical" densification, or to extend the model in a dynamic analysis to better understand the processus of urban development and not only the resulting city structure.

References

Brenda, B. L. and Fuller, R. a. (2013). Sharing or sparing? how should we grow the world's cities? *Journal of Applied Ecology*, 50:1161–1168.

Brueckner, J. K. (2000). Urban sprawl: diagnosis and remedies. *International regional science review*, 23(2):160–171.

Caruso, G., Cavailhès, J., Peeters, D., Thomas, I., Frankhauser, P., and Vuidel, G. (2015). Greener and larger neighbourhoods make cities more sustainable! a 2d urban economics perspective. *Computers, Environment and Urban systems*, 54:82–94.

Cavailhès, J., Peeters, D., Sékeris, E., and Thisse, J. (2004). The periurban city: Why to live between the suburbs and the countryside. *Regional Science and Urban Economics*, 34(6):681–703.

European Environment Agency (2006). Urban sprawl in Europe : The ignored challenge. Luxembourg office for official publications of European Communities.

Eurostat (2017). Urban Audit. http://appsso.eurostat.ec.europa.eu.

Fujita, M. (1989). *Urban Economic Theory: Land use and city size.* Cambridge university press.

Green, R. E., Cornell, S. J., Scharlemann, J. P. W., and Balmford, A. (2005). Farming and the fate of wild nature. *Science*, 307:550–555.

Marshall, E. (2004). Open-space amenities, interacting agents, and equilibrium landscape structure. *Land Economics*, 80:272–293.

Regnier, C. and Legras, S. (2017). Urban structure and environmental externalities. *Environmental and Resource Economics, online first: https://doi.org/10.1007/s10640-016-0109-0.*

Richardson, H. W. (1977). On the possibility of positive rent gradients. *Journal of Urban Economics*, 4:60–68.

Soga, M., Yamaura, Y., Koike, S., and Gaston, K. J. (2014). Land sharing vs. land sparing : does the compact city reconcile urban development and biodiversity conservation? *Journal of Applied Ecology*, 51:1378–1386.

Stott, I., Soga, M., Inger, R., and Gaston, K. J. (2015). Land sparing is crucial for urban ecosystem services. *Frontiers in Ecology and the Environment*, 13(7):387–393.

Strange, W. C. (1992). Overlapping neighborhoods and housing externalities. *Journal of Urban Economics*, 32(1):17–39.

Turner, M. a. (2005). Landscape preferences and patterns of residential development. *Journal of Urban Economics*, 57(1):19–54.

Walsh, R. (2007). Endogenous open space amenities in a locational equilibrium. *Journal of Urban Economics*, 61(2):319–344.

Wu, J. (2001). Environmental amenities and the spatial pattern of urban sprawl. *American Journal of Agricultural Economics*, 83(3):691–697.

Wu, J. and Plantinga, A. J. (2003). The influence of public open space on urban spatial structure. *Journal of Environmental Economics and Management*, 46(2):288–309.

Appendix 1

When x_{uo} equals zero, the city structure is complete land sparing. We have :

$$\begin{aligned} & x_{uo} = 0 \\ & \Leftrightarrow \frac{w_1}{w_2} \geq \frac{1}{qw_2}(w - Cq + q - u - \gamma + \lambda) \\ & \Leftrightarrow \frac{w_1}{w_2} \geq (\frac{w_1}{w_2})^h \end{aligned}$$

When x_{uo} equals to x_{um} the city structure is complete land sparing. We have :

$$\begin{aligned} & x_{uo} = x_{um} \\ & \Leftrightarrow \frac{w_1}{w_2} \leq \frac{1}{2(2q-1)qw_2}(2\gamma q + 4Cq^2 - 2u - 2Cq + 2\lambda + 2w - 4qw - \gamma q^2 + 2q + 4qu - 4q\lambda - 4q^2 - \gamma \\ & + \gamma q^2(4\gamma q - 2\gamma - 8q^2 + 8Cq^2 + 8q^2 w_2 - 4Cq + 4q - 8q\lambda - 4qw_2 - 8q + 4w))^{1/2} \\ & \Leftrightarrow \frac{w_1}{w_2} \leq (\frac{w_1}{w_2})^l \end{aligned}$$