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Mitigation and adaptation are not enough: turning to emissions reduction abroad.*

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Abstract

In this paper we focus on a long-term dynamic analysis of the optimal adaptation/mitigation mix in the presence of a pollution threshold above which adaptation is no longer efficient. We account for accumulation in abatement capital, greenhouse gases, and adaptation capital in order to better capture the arbitrage between abatement and adaptation investments. Pollution damages arise from the emissions due to the country consumption but also from the emissions of the rest of the world (ROW). A pollution threshold is then introduced, above which adaptation is no longer efficient. We obtain that if this threshold is lower than the steady-state level of pollution, there is no way for the modelled economy to avoid it. In particular, such a situation will appear if the ROW's emissions are high. We then show that CDM may be a means to avoid a pollution threshold above which adaptation becomes of no use.

Keyword(s): climate change, mitigation, adaptation, CDM, pollution threshold.

JEL codes: Q5, Q52, Q56, Q58

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1 Introduction

The issue of climate change could be tackled at two levels: one can seek either to avoid it through mitigation or to avoid its consequences through adaptation. Mitigation refers to "an anthropogenic intervention to reduce the sources or enhance the sinks of greenhouse gases" (IPCC, 2007) while adaptation concerns measures that reduce the vulnerability of natural and human system. A usual example of adaptive measures is the construction of levees to prevent flooding of plain areas as a result of rising seawater but could also include the development of crop varieties better suited to the new climate characteristics. However, if climate change could become too large for adaptation to be effective, current mitigation and adaptation policies should be affected by this ineffectiveness threat. In this paper we focus on a long-term dynamic analysis of the optimal adaptation/mitigation mix in the presence of a pollution threshold above which adaptation is no longer efficient.

Both mitigation and adaptation are costly but an interesting difference between the two is that adaptation may be efficient even if implemented in a decentralized uncoordinated way. However, adaptation has long been left aside. Motivations were clear: it may be strategically dangerous to focus the attention on an alternative to mitigation while mitigation is already so difficult to organize because it deals with a global and intertemporal problem. Moreover, mitigation is viewed as a proactive policy while adaptation policies are rather reactive and could therefore always be considered in case mitigation had failed. This is probably why the economic and environmental science literature has largely analysed the cost and effectiveness of mitigation but adaptation to an already changed climate has only recently entered the picture. Having crossed the red line, it becomes more and more usual to consider them together as complements or substitutes. However, adaptation effectiveness is limited: it may be too costly or even impossible to adapt to a too large climate change. Studies available in the literature provide estimates (World Bank, 2009) that reach 100 billion USD per year for the adaptation cost between 2010 and 2050 to an only 2°C warmer world by 2050. Moreover, some catastrophic consequences often described (Keller *et alii.* Climatic change, 2008) seems difficult to cope with.

Only a few studies explicitly consider both adaptation and mitigation as policy responses to climate change in long term dynamic analysis.[1] Brechet *et alii* (2013) propose a theoretical framework with accumulation in physical capital, greenhouse gases, and adaptation capital, to show that the issue of substitutability between the two instruments depends on the stage of development. In particular, the relationship between adaptation and economic efficiency is inverted U-shaped implying that adaptation should be low or nil for poor or developed countries but of a significant size for medium-developed ones. Tsur and Withagen (2012) extend the framework to account for catastrophic (abrupt) uncertain climate change in a model with exogenous growth. The timing of this change depends on atmospheric GHG concentration that is assumed to be exogenous to a single country and there is therefore no pollution accumulation in the model. They obtain that for a stationary economy adaptation actions may be suboptimal depending on the discount rate, the aversion to intergenerational inequality, the catastrophic risk and the severity of the catastrophic damage. However a growing economy can more easily afford the adaptation investment cost and is more vulnerable to the climate change. It is therefore always optimal to invest in adaptation capital (up to the point where it is no longer effective) in such economies. Finally, Zemel (2014) consider optimal emission when adaptation is possible in the presence of an uncertain pollution damage. Less pollution reduces the risk of damage occurrence while adaptation capital reduces the size of this potential damage. The nature of the pollution and adaptation processes is studies. Cases are exhibited in which first only one of these processes evolves for some period of time and second pollution or adaptation is not monotonic.

In line with this literature this paper provides a long term analysis of the optimal mitigation/adaptation mix by considering the social planner problem of a polluting country. Following the first part of Tsur and Withagen (2012), we restrict our attention to economies with neither

[1] There exists however descriptive studies (e.g. Kane and Yohe,2000; Smit et al., 2000; Agrawal and Fankhauser, 2008; EEA,2007; UNFCCC, 2007), game-theoretic papers (Shalizi and Lecocq, 2009; Ikefuji, Mangnus and Sakamoto, 2014; Kane and Shogren, 2000 ; Buob and Stephan, 2010) or static modelling (Ingham, ma and Ulph, 2005).

growth nor technological progress[2] but we account for accumulation in abatement[3] capital, in greenhouse gases, and in adaptation capital in order to better capture the arbitrage between abatement and adaptation investments. Pollution damages arise from the emissions due to the country consumption but also from the emissions of the rest of the world (ROW). We obtain that adaptation and mitigation are in some way complementary since one should never use one without the other. Moreover, an increase in ROW's emissions unambiguously reduces mitigation and increases adaptation. Therefore as China pollutes more and more, European countries should consider to adapt more and mitigate less. In addition, we show that a less developed country should adapt more and mitigate less than a more advanced one. This can be reinterpreted in terms of timing for the policy mix: as a country develop, they should reduce their adaptation and increase their mitigation. Finally, a more efficient adaptation may lead to more adaptation, in contrast to what is obtained in Benchekroun (2011). We introduce a critical threshold of pollution (in the same vein as Prieur, 2009 or Naevdal, 2003), above which adaptation is longer efficient. We obtain that if this threshold is lower than the steady-state level of pollution, there is no way for the modelled economy to avoid it. In particular, such a situation will appear if the ROW's emissions are high. Next step is then to introduce another type of investment allowing for lower ROW pollution ie. emissions reduction abroad. In order to avoid international negotiations issues that are beyond the scope of this paper, we assume that this mitigation abroad takes the form of Clean Development Mechanisms. The model is therefore extended to capture the arbitrage between adaptation investment (necessarily domestic), domestic mitigation investment and investment in CDM that aims at reducing ROW's emissions. We obtain that CDM may be a means to avoid a pollution threshold above which adaptation becomes of no use.

[2]Adding growth would prevent from analytically solving the model. Moreover, such an assumption together with the existence of a pollution threshold would lead to a degenerate solution.

[3]There exists a difference between mitigation and abatement: the former refers to a reduction in net emissions of greenhouse gases while the latter refers to a reduction in gross emissions. In this paper, we integrate only emission abatement opportunities as it is largely the case in the literature. In the rest of the paper we will indifferently refer to mitigation or abatement.

The rest of the paper is as follows. The general model is presented in section 2. It is solved for the optimal adaptation and abatement investments in section 3. CDM are considered in section 4. Section 5 concludes.

2 The model

We consider a country-level economy where consumption, c, is the source of polluting emissions. Households derive utility from consumption but are damaged by climate change due to GHG stock. However, damages occurring because of climate change of a given size depend on the adaptation efforts made by the country. To capture these features, the instantaneous utility function is defined over consumption and a damage function D whose arguments are the stock of pollution (that gives a measure of climate change) and the level of adaptation:

$$U(c, S, A) = u(c) - D(S, A) \tag{1}$$

where c is consumption, and S and A are respectively the stock of GHG and the adaptation capacity. Moreover, $u\prime > 0, u'' < 0, D_S > 0, D_{SS} \geq 0, D_A < 0, D_{AA} > 0, D_{AS} < 0$. Note also that, $D(0, A) = 0, D(S, 0) = D(S) > 0, D(\infty, A) = \infty, D(S, \infty) = 0$. Therefore mitigation and adaptation are substitutes in reducing the impact of climate change as in Buob and Stephan (2013). Note however that is not a generally admitted assumption as Parry et al. (2001), Klein et al. (2007) or Yohe and Strzepek (2007) consider that there is complementarity between adaptation and mitigation.

The stock of GHG evolves according to:

$$\dot{S}_t = \beta c_t - M(K_t) + \overline{E} - \sigma S_t \tag{2}$$

$\overline{E} \geq 0$ is exogenous and corresponds to the net emissions of the rest of the world. It can be seen as the difference between total emissions of the rest of the world and mitigation efforts of the rest of the world. σ is the rate of natural pollution decay. Note that we model a reduction in net emissions of greenhouse gases that could more rigorously be referred to as abatement rather than mitigation.[4] Note moreover that we focus on active mitigation policy meaning that we distinguish between optimal emissions (driven by consumption) and mitigation. In case the rest of the world would make very significant efforts, it is possible to have $\overline{E} = 0$. Finally $M(K)$ is the level of mitigation that depends on the stock of mitigation capital. An example of such a capital is the amount of renovations in buildings aimed at reducing energy consumption.

The economy is endowed with a fixed revenue Y and with a resource that is available in infinite quantity. Final consumption only comes from the free resource use that generates GHG emissions and contributes therefore to climate change. Revenue Y is used for expenses that have to be shared between mitigation and adaptation:

$$Y = I_{Kt} + I_{At}$$

where I_{Kt} and I_{At} denote investment in mitigation and adaptation respectively. The share of revenue financing mitigation, θ_t, is a control variable. The remaining share $(1 - \theta_t)$ is used to adapt the economy to climate change:

$$Y = \theta_t Y + (1 - \theta_t)Y \quad \text{with } \theta_t Y = I_{Kt} \text{ and } (1 - \theta_t)Y = I_{At} \tag{3}$$

Deriving the optimal θ_t will provide us with the optimal policy mix. Note however that there is some substitutability assumed for a constant revenue Y. Adaptation infrastructures are accumulated according to:

$$\dot{A}_t = [(1 - \theta_t)Y]^{\alpha_A} - \delta_A A_t \tag{4}$$

with δ_A being the depreciation rate of adaptation capital. Such a stock adaptation covers a large range of measures including storm warning and investment in infrastructures such as dykes (see Buob and Stephan 2011). Therefore there does not exist a uniform measure of this stock. Recall that the adaptation capital stock reduces the damage of a given pollution stock: D is function of A.

Investment in mitigation infrastructures (such as energy efficient investments in buildings) accumulates according to:

$$\dot{K}_t = (\theta_t Y)^{\alpha_M} - \delta_M K_t \tag{5}$$

[4]Mitigation rather refers to a reduction in gross emissions, ie. mitigation efforts would lower β in equation (2).

with δ_M being the depreciation rate of mitigation capital. Recall that the mitigation capital stock reduces the stock of pollution: $\dot{S}_t$ is a function of K.

In order to provide analytical results, we make the following simplifying assumptions:

$$
\begin{aligned}
u(c) &= c^{\omega},\ 0 < \omega < 1 \\
D(S, A) &= A^{-\gamma} S \quad \gamma > 0 \\
M(K) &= K^{m} \quad m \leq 1 \\
\alpha_A, \alpha_M &< 1; \quad \delta_A, \delta_M > 0
\end{aligned}
$$

3 Optimal adaptation and mitigation efforts

In this section, we solve for the optimal adaptation and mitigation efforts. We then perform some comparative statics to study the effect of ROW emissions, revenue and adaptation efficiency on the steady-state levels of adaptation and mitigation. We finally appraise whether a pollution threshold above which adaptation becomes inefficient can be avoided.

3.1 Solving for the optimal adaptation and mitigation efforts

The policy maker of the economy chooses the optimal sequences of consumption $\{c_t\}_{t=0}^{\infty}$ and of the share of revenue devoted to mitigation $\{\theta_t\}_{t=0}^{\infty}$ in order to solve:

$$
\begin{aligned}
&\max_{\{c,\theta\}} \int_0^{+\infty} \exp^{-\rho t} U(c, S, A) dt \\
&\text{subject to(2), (4) and (5)} \\
&S_0, A_0, K_0 \text{ given}
\end{aligned}
$$

with $\rho \in (0, 1)$, the discount rate. The corresponding Hamiltonian is:

$$
\begin{aligned}
\mathcal{H} &= c^{\omega} - SA^{-\gamma} + \mu \left[\beta c - \sigma S - K^{m} + \overline{E}\right] \\
&\quad + \lambda_A \left[[(1-\theta)Y]^{\alpha_A} - \delta_A A\right] + \lambda_M \left[(\theta Y)^{\alpha_M} - \delta_M K\right]
\end{aligned}
$$

and associated FOCs lead to

$$\begin{aligned}
\dot{\mu}/\mu &= -\eta\dot{c}/c = (\rho+\sigma) - A^{-\gamma}\beta c^{1-\omega}/\omega && (6)\\
\frac{\dot{\theta}}{\theta} &= (1-\theta)\frac{\delta_A - \delta_M - \frac{1}{\lambda_M}\left[\gamma S A^{-\gamma-1}\frac{\alpha_A}{\alpha_M}\frac{(1-\theta)^{\alpha_A-1}}{\theta^{\alpha_M-1}}Y^{\alpha_A-\alpha_M} - \omega c^{\omega-1} m K^{m-1}/(\beta)\right]}{(1-\theta)(\alpha_M-1)+\theta(\alpha_A-1)} && (7)
\end{aligned}$$

where $\eta = -u''(c)c/u'(c) = \omega - 1$. A steady-state (SS) is such that $\dot{A}/A = 0$ $\dot{K}/K = 0$, $\dot{S}/S = 0$, $\dot{c}/c = 0$, and $\dot{\theta}/\theta = 0$. Equations (4), (5) and (6) gives:

$$\begin{aligned}
K^* &= (Y\theta^*)^{\alpha_M}/\delta_M = K^*(\theta^*) && (8)\\
A^* &= (1-\theta^*)^{\alpha_A}Y^{\alpha_A}/\delta_A = A^*(\theta^*) && (9)\\
c^* &= ((\rho+\sigma)\omega A^*(\theta^*)^{\gamma}/\beta)^{\frac{1}{1-\omega}} = c(\theta^*) && (10)
\end{aligned}$$

Note that $\theta^* \leq 1$ is equivalent to $K^* \leq \overline{K}$, with $\overline{K} = Y^{\alpha_M}/\delta$. Similarly, $\theta^* \geq 0$ is equivalent to $A^* \leq \overline{A}$, with $\overline{A} = Y^{\alpha_A}/\delta_A$. Moreover, (2) implies

$$\sigma S(\theta^*) = \beta\left(\frac{(\rho+\sigma)\omega\overline{A}^{\gamma}}{\beta}\right)^{\frac{1}{1-\omega}}(1-\theta^*)^{\frac{\alpha_A\gamma}{1-\omega}} - \overline{K}^m\theta^{*\alpha_M m} + \overline{E} \qquad (11)$$

Proposition 1 *In the current model with accumulation in pollution, in adaptation capital and in mitigation capital a necessary and sufficient condition for the existence and uniqueness of the steady-state denoted by* $(c^*, \theta^*, A^*, K^*, S^*)$ *is:* $\overline{E} > \overline{K}^m$. *Moreover* $0 < \theta^* < 1$, *meaning that adaptation and mitigation have to be used together.*

Proof. Note that (7) leads to

$$\sigma S(\theta^*) = \underbrace{\frac{\rho+\delta_A}{\rho+\delta_M}\frac{\alpha_M\delta_M}{\alpha_A\delta_A}\frac{\sigma m\overline{K}^m}{\gamma(\rho+\sigma)}(1-\theta^*)\theta^{*\alpha_M m-1}}_{G(\theta^*)}$$

with $G'(\theta^*) < 0$, $\lim_{\theta^*\to 0} G(0) = +\infty$, $G(1) = 0$ and $\sigma S'(\theta^*) < 0$ with $S(0) = \beta\left(\frac{(\rho+\sigma)\omega\overline{A}^{\gamma}}{\beta}\right)^{\frac{1}{1-\omega}} + \overline{E}$ and $\sigma S(1) = -\overline{K}^m + \overline{E}$. Note that $\overline{E} - \overline{K}^m > 0$ means that ROW emissions are too high for the country to be able to stabilize pollution even if all the revenue is used for mitigation and no adaptation is conducted. Assuming $\overline{E} - \overline{K}^m > 0$ ensures the existence and uniqueness results stated in proposition 1 and illustrated in figure 1. ■

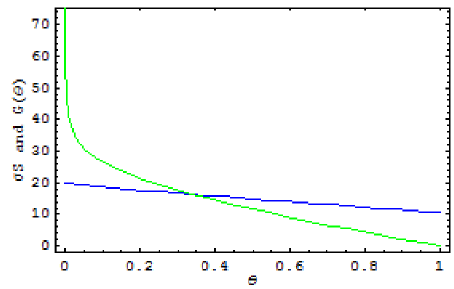

Figure 1

3.2 Comparative statics

Comparative statics results are summarized in proposition 2:

Proposition 2 *In the current model, an increase in the ROW emissions or a decrease in revenue unambiguously reduces the share devoted to mitigation and increases that used for adaptation. However, a more efficient adaption leads to more adaptation if:*

$$G(\widehat{\theta}) < \sigma S(\widehat{\theta}) \text{ with } \widehat{\theta} = 1 - \overline{A}^{-1/\alpha_A} \tag{12}$$

Proof.

- An increase in $\overline{E}$ affects the SS through $\sigma S(\theta^*)$ but has no effect on $G(\theta^*)$. Since $\partial(\sigma S(\theta^*))/\partial \overline{E} = 1$, it shifts the $\sigma S(\theta^*)$ curve upwards. An increase in the ROW emissions $\overline{E}$ generates a decrease in the steady-state level of the share of revenue devoted to mitigation thus increasing adaptation.

- Following an increase in revenue, $G(\theta^*)$ shifts upwards and $\sigma S(\theta^*)$ rotates clockwise, therefore leading to a larger share devoted to mitigation.

- The effect of an increase in adaptation efficiency is less straightforward. $G(\theta^*)$ shifts downwards but σS rotates clockwise around $\widehat{\theta} = 1 - \overline{A}^{-1/\alpha_A}$. For an initially low θ^* (lower than $\widehat{\theta}$), a more efficient adaptation unambiguously decreases the share of revenue devoted to mitigation (see also the illustration in figure 4). For an initially high θ^*(larger than $\widehat{\theta}$), the result is ambiguous, meaning that a more efficient adaptation could lead to less adaptation.

■

Note that this latter possible result is consistent with the result of Benchekroun (2011). Intuition runs as follows. If adaptation becomes more efficient, it is then possible to invest less in adaptation, to reach a higher or equivalent level of A, therefore releasing resources for more mitigation. Finally, note that $\widehat{\theta}$ is independent from $\overline{E}$, while a change in the ROW emissions shifts the σS curve upwards. There exists a high enough value for these emissions such that the steady state is smaller than $\widehat{\theta}$, implying a non-ambiguous positive effect of adaptation efficiency on adaptation. One can find the condition on $\overline{E}$ that ensures $\theta^* < \widehat{\theta}$:

$$\begin{aligned} G(\widehat{\theta}) &< \sigma S(\widehat{\theta}) \\ \overline{E} &> \left(1 - \overline{A}^{-1/\alpha_A}\right)^{\alpha_M m - 1} \overline{K}^m \left[1 + \left(\frac{\rho + \delta_A}{\rho + \delta_M} \frac{\alpha_M \delta_M}{\alpha_A \delta_A} \frac{\sigma m}{\gamma(\rho + \sigma)} - 1\right) \overline{A}^{-1/\alpha_A}\right] - \beta \left(\frac{(\rho + \sigma)\omega}{\beta}\right)^{\frac{1}{1-\omega}} \end{aligned}$$

The set of parameters ensuring both the steady state existence and $\theta^* < \widehat{\theta}$ is obviously non-empty.

These results are obtained analytically but figures 2, 3 and 4 provide graphic illustrations.

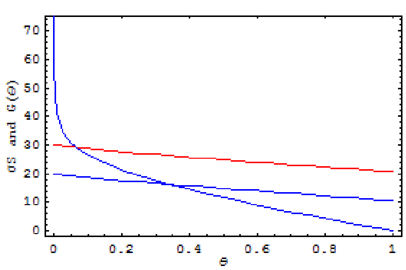

Figure 2: increase in ROW emissions (in red)

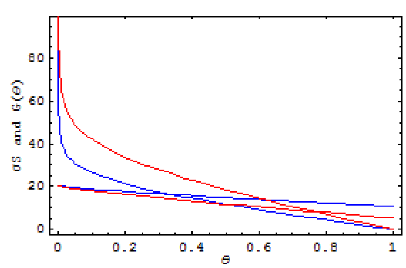

Figure 3: increase in revenue (in red)

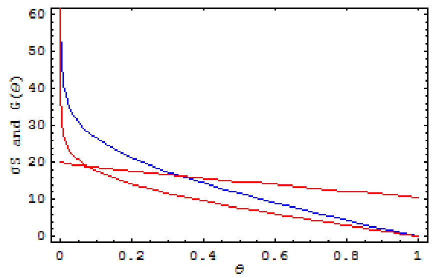

Figure 4: more efficient adaptation (in red)

Proposition 2 shows that as China pollution rises, European countries should mitigate less and adapt more. Moreover, a less developed country should adapt more and mitigate less than a more advanced one. This can be reinterpreted in terms of timing for the policy mix. As country develop, they should reduce their adaptation and increase their mitigation. In addition, mixing the two comparative statics suggest that a poor country suffering from high ROW emissions (such as Bangladesh) should clearly focus on adaptation. In contrast, rich polluting countries such as the US should favor mitigation. However, conclusions are mixed for poor polluting countries like China or rich countries with high ROW emissions like Hong Kong. Finally under condition (12), a more efficient adaptation leads to more adaptation: in our framework, there exist levels of ROW emissions such that the conclusion obtained by Benchekroun is not valid.

Remark: *Note that assuming $\alpha_A = \alpha_M = \alpha$ and $\delta_A = \delta_M = \delta$ does not affect the nature of the results since the steady state is then defined by:*

$$\beta\left(\frac{(\rho+\sigma)\omega\overline{K}^{\gamma}}{\beta}\right)^{\frac{1}{1-\omega}}(1-\theta^*)^{\frac{\alpha\gamma}{1-\omega}} - \overline{K}^{m}\theta^{*\alpha m} + \overline{E} = \frac{\sigma m\overline{K}^{m}}{\gamma(\rho+\sigma)}(1-\theta^*)\theta^{*\alpha m-1}$$

and the necessary and sufficient condition for existence and uniqueness given in proposition 1 is still valid. Moreover comparative statics as described in proposition 2 remain valid as well. Therefore, these simplifying assumptions will be made in the rest of the paper.

.

3.3 Introducing a deterministic threshold

In this subsection we take account of a threshold of pollution above which adaptation is no longer efficient. In this paper, the stock of pollution is a proxy for climate change and what we want to address here is the fact that for a too large climate change the effect of damages on welfare can no longer be reduced using adaptation investment. It may be too costly or even impossible to adapt to a too large climate change if costs of adaptation increase when carbon concentration rises (see Buob and Stephan, 2011). Studies available in the literature provide estimates (World Bank, 2009) that reach 100 billion USD per year for the adaptation cost between 2010 and 2050 to an only 2°C warmer world by 2050. Moreover, some catastrophic consequences often described (Keller *et alii.* Climatic change, 2008) seems difficult to cope with. For instance, Greenland ice sheet melting could generate a 7 meters sea-level rise (Gregory et al. 2004, and Hansen 2005) or a MOC[5] weakening would change significantly precipitations and temperatures (Gregory et al. 2004 and Link and Tol, 2004). This is also consistent with the findings of de Bruin et al. (2009) that showed that the higher the current value of climate damage, the less relevant adaptation is as a policy option. Therefore, for $S(t) > \overline{S}$, the adaptation alternative disappears. Note that this irreversibility only plays a role if the threshold is lower than the steady-state level of pollution derived in the previous section, ie. $\overline{S} < S^*$. However there exist two types of solution. Even if $\overline{S} < S^*$, it may be optimal, knowing the threshold, to change decisions in order to have pollution remaining below $\overline{S}$. Such a solution is said to be reversible. By contrast, a solution is irreversible if the pollution stock enters the irreversible region in finite time. We first show that the only reversible solution consists in reaching the threshold. Such a result suggests that it could be worth considering other climate policies such as devoting money to reduce emissions abroad (this will be considered in the next section).

Once the threshold has been reached, the main assumption affected in our model specifications concerns the damage function, that becomes such that $D'_A = 0$, and consistent with this new assumption we now adopt $D(S, A) = S$. Since adaptation generates no benefits but is costly (it requires a fraction $(1-\theta)$ of revenue Y), it is of no interest to engage into any adaptation activities and variable A is simply removed from the model.

[5] Meridional Overturning Circulation.

If considering the reversible solution, the program is:

$$\max_{\{c,\theta\}} \int_0^{+\infty} \exp^{-\rho t} U(c, S, A) dt$$
$$\text{subject to(2), (4) and (5)}$$
$$S_0, A_0, K_0 \text{ given and } S_t \leq \overline{S} \text{ for all } t$$

The corresponding Hamiltonian is then:

$$\begin{aligned} \mathcal{H} &= c^{\omega} - SA^{-\gamma} + \mu\left[\beta c - \sigma S - K^m + \overline{E}\right] \\ &\quad +\lambda_A\left[[(1-\theta)Y]^{\alpha} - \delta A\right] + \lambda_M\left[(\theta Y)^{\alpha} - \delta K\right] + \xi(\overline{S} - S) \end{aligned}$$

The associated FOCs are the same as in the case without threshold except for the one with respect to S, and the additional condition related to the threshold constraint:

$$\begin{aligned} \dot{\mu}/\mu &= \rho + \sigma + A^{-\gamma}/\mu + \xi \\ \xi((\overline{S} - S) &= 0, \xi \geq 0 \end{aligned}$$

Two cases exist. In the first case, $S_t < \overline{S}$ then $\xi = 0$ and the solution reduces to the one without threshold. Since we have assumed that $\overline{S} < S^*$, the threshold cannot be avoided.

In the second case $S_t = \overline{S}$, and $\xi \geq 0$, then equation (2) becomes:

$$\beta c_t - K_t^m + \overline{E} - \sigma\overline{S} = 0$$

and consumption growth rate is:

$$\dot{c}/c = \frac{-1}{\eta}\left[(\rho + \sigma) - A^{-\gamma}\beta c^{1-\omega}/\omega + \xi\right]$$

where $\eta = -u''(c)c/u'(c) = \omega - 1$. The Lagrange multiplier ξ affects the arbitrage through the marginal value of the stock. Indeed $\dot{\mu}$ is larger and we expect μ_0 to be smaller since we have reached the threshold (see also Amigues and Moreaux, 2012 or Amigues, Lafforgue and Moreaux, 2014, among others). The threshold cannot be avoided. In particular, a steady-state with adaptation is no longer possible. Proposition 3 summarizes the effect of a pollution threshold:

Proposition 3 *In the current model with emissions generated by the ROW, accumulation in pollution, in mitigation capital and in adaptation capital, there is no way to avoid a threshold of pollution above which adaptation is no longer efficient if the steady state level of pollution (in a model that ignores this threshold) is higher than the threshold.*

The fact that the threshold cannot be avoided comes from ROW emissions (and pollution natural decay) that cannot be controlled for by the country. Devoting money to CDM would be a means to control for the ROW emissions. This is why it seems to be an especially relevant policy to consider. We turn to it in the next section.

4 Introducing ROW emissions reduction

We now consider that part of the resources of the country can be devoted to mitigation abroad leading to less ROW emissions. In order to avoid international negotiations issues that are beyond the scope of this paper, we assume that this mitigation abroad takes the form of Clean Development Mechanisms. Note moreover that for some countries, CDM are a non negligible part of their emission reduction to comply with the Kyoto target. In the European Union scheme country members are allowed to fulfil up to 50% of their emission reduction commitment using CDM or JI (Joint Implementation). For instance, CDM represented 15% of the Netherland's emission reduction in the commitment period 2008-2012.

In a first step, we simply focus on the effect of devoting resources to CDM on the trade-off between adaptation and mitigation, still using the previous framework. The fraction of Y used for CDM is then taken as exogenous and we do not consider any effect of these CDM on ROW emissions. Rationale for such an assumption is that little is known about the efficiency of CDM on global emissions. We obtain, in particular, that one cannot rule out the case for which less resources reduces so much more adaptation (and therefore consumption and emissions) than mitigation that the steady-state level of pollution is lower even if we ignore any effect on the ROW emission level. Analytical computation leads to

$$\frac{\partial S^*}{\partial Y} > 0 \text{ or } < 0 \text{ depending on the parameters}$$

Considering now some qualitatively negative effect on ROW emissions, the function $G(\theta^*)$ moves up but function $\sigma S(\theta^*)$ is not affected, that leads to a larger θ^*. Since $\partial A^*/\partial \overline{E} = \partial A^*/\partial \theta^* . \partial \theta^*/\partial \overline{E}$ with $\partial A^*/\partial \theta^* < 0$ adaptation is reduced. One can deduce, using equation (11), that following a decrease in the ROW emissions, the total stock of pollution is smaller, due to more mitigation, less adaptation and the direct effect of ROW emissions.

For parameters values such that $\partial S^*/\partial Y > 0$ investing in CDM is unambiguously a way to lower the steady-state level of pollution and therefore a means to avoid the threshold above which adaptation becomes ineffective. For parameters values such that $\partial S^*/\partial Y < 0$ two opposite effects exist and a lower S^* may only be reachable if CDM are in reality sufficiently efficient in reducing $\overline{E}$.

In a second step, investment in CDM becomes an optimal choice. The corresponding model is presented in the next subsection that is followed by the corresponding comparative statics.

4.1 The model

Considering CDM as an optimal choice implies to formally introduce the benefits of CDM ie. to endogeneize ROW emissions E_t that become dependent on the amount devoted to CDM:

$$E_t = \overline{E} - [(1 - \phi_t)Y]^s$$

where ϕ is the share of revenue devoted to domestic management of pollution (i.e. either adaptation or mitigation). s is the CDM efficiency in reducing ROW emissions. We assume $s > 1$ to ensure the concavity of the pollution accumulation process. We impose $(1 - \phi_t)Y > 1$ such that a larger s, by rising mitigation efficiency abroad, effectively reduces ROW emissions. Moreover, we assume that $Y^s < \overline{E}$, meaning that even if all the revenue is devoted to CDM, emissions cannot be negative. The share ϕ is a new control variable. The adaptation and mitigation processes become then:

$$\dot{A}_t = [\phi_t(1 - \theta_t)Y]^\alpha - \delta A_t \quad (13)$$

$$\dot{K}_t = (\phi_t \theta_t Y)^\alpha - \delta K_t \quad (14)$$

and the corresponding Hamiltonian writes:

$$\begin{aligned}\mathcal{H} &= c^{\omega} - SA^{-\gamma} + \mu\left[\beta c - \sigma S - K^{m} + \overline{E} - [(1-\phi)Y]^{s}\right] \\ &\quad + \lambda_A\left[[\phi(1-\theta)Y]^{\alpha} - \delta A\right] + \lambda_M\left[(\phi\theta Y)^{\alpha} - \delta K\right]\end{aligned}$$

FOCs with respect to consumption, θ, pollution adaptation and mitigation stocks remain unaffected by the fact that ROW emissions are endogeneized. However, there is a new FOC, with respect to ϕ :

$$-\mu Y^{s}\frac{(1-\phi)^{s-1}}{\alpha\phi^{\alpha-1}} = \lambda_M \theta^{\alpha-1} Y^{\alpha}$$

The new dynamic equation and steady-state expressions are provided in the Appendix. Using the pollution accumulation process and the fact that at the steady-state $\dot{S}/S = 0$, the steady-state level of pollution is now such that:

$$\sigma S^{*}(\theta^{*}) = \beta\left(\frac{(\rho+\sigma)\omega}{\beta}\phi^{*\gamma\alpha}(1-\theta^{*})^{\gamma\alpha}\overline{K}^{\gamma}\right)^{\frac{1}{1-\omega}} - (\phi^{*}\theta^{*})^{m\alpha}\,\overline{K}^{m} + \overline{E} - (1-\phi^{*})^{s}Y^{s}$$

Moreover, using the dynamic equation for ϕ (see the Appendix) and the fact that at the steady-state $\dot{\phi}/\phi = 0$ (and knowing that at this steady-state we also have $\dot{\theta}/\theta = \dot{c}/c = 0$):

$$\frac{(\rho+\delta)}{\alpha m}Y^{s-\alpha}\,(1-\phi^{*})^{s-1}\,\overline{K}^{1-m} = (\phi^{*}\theta^{*})^{\alpha m-1} \tag{15}$$

$$\Rightarrow \frac{d\phi^{*}}{d\theta^{*}} > 0 \text{ iff } \phi^{*} > \widetilde{\phi} \text{ with } \widetilde{\phi} = \frac{1-\alpha m}{s-\alpha m} < 1$$

For a large (*resp.* small) ϕ^{*},there exists some substitutability (*resp.* complementarity) between home emissions reduction and ROW emissions reduction : a higher (*resp. lower)* ϕ^{*}, that means less (*resp. more*) ROW emissions reduction, goes with a higher (*resp. lower*) θ^{*}, that means a larger (*resp.* smaller) share of home mitigation with respect to adaptation). Moreover

$$\lim_{\phi^{*}\to 0}\theta^{*} \to +\infty \quad \text{and} \quad \lim_{\phi^{*}\to 1}\theta^{*} \to +\infty$$

The share devoted to home investments ϕ^{*} can never be equal to zero or unity, meaning that it is never optimal to use all revenue only for CDM or only for home climate policies. We have

$\theta(\phi) > 0\ \forall\phi$, and the condition ensuring that there exist $\theta(\phi) \leq 1$ is

$$\theta(\widetilde{\phi}) \leq 1 \Leftrightarrow Y^{s-\alpha} \geq \frac{\alpha m}{(\rho+\delta)} \overline{K}^{m-1} \widetilde{\phi}^{\alpha m-1} (1-\widetilde{\phi})^{1-s}$$

The revenue must be sufficiently high so that it is worth devoting some money to home and foreign climate policies. From now on we restrict the definition domain for ϕ^* to be $[\underline{\phi}, \overline{\phi}]$ with $\underline{\phi}$, and $\overline{\phi}$ implicitly determined by $\theta^*(\phi) = 1$.

Proposition 4 *In the current model with accumulation in pollution, in adaptation capital and in mitigation in the home country and abroad, sufficient conditions for the existence of a steady-state denoted* $(c^*, \theta^*, \phi^*, A^*, K^*, S^*)$ *are:*

$$\overline{E} > \max[\left(\underline{\phi}\right)^{m\alpha} \overline{K}^m + (1-\underline{\phi})^s Y^s, \left(\overline{\phi}\right)^{m\alpha} \overline{K}^m + (1-\overline{\phi})^s Y^s]$$

$$\textit{and } G(\widetilde{\phi}) > F(\widetilde{\phi}) \qquad \textit{with } \widetilde{\phi} = \frac{1-\alpha m}{s-\alpha m}$$

Proof. Using the fact that $\dot{\theta}/\theta = 0$ we obtain:

$$\underbrace{\sigma S(\theta(\phi^*), \phi^*)\phi^{*-\alpha m}}_{F(\phi^*)} = \underbrace{\frac{\sigma m \overline{K}^m}{\gamma(\rho+\sigma)} (1-\theta(\phi^*))\, \theta(\phi^*)^{\alpha m-1}}_{G(\phi^*)}$$

with $G(\theta^*)$ that is the same as previously (without CDM). Since $G'(\phi^*) = \frac{\partial G(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial \phi^*}$, $G(\phi^*)$ is hump-shaped with a max in $\widetilde{\phi}$. Moreover

$$\begin{aligned} \lim_{\phi^* \to \underline{\phi}} G(\phi^*) &= 0 \text{ and } \lim_{\phi^* \to \overline{\phi}} G(\phi^*) = 0 \\ \lim_{\phi^* \to \underline{\phi}} F(\phi^*)\underline{\phi} &> 0 \text{ and } \lim_{\phi^* \to \overline{\phi}} F(\phi^*) > 0 \end{aligned}$$

since we assume[6] $\overline{E} > \max[\left(\underline{\phi}\right)^{m\alpha} \overline{K}^m + (1-\underline{\phi})^s Y^s, \left(\overline{\phi}\right)^{m\alpha} \overline{K}^m + (1-\overline{\phi})^s Y^s]$. Therefore there exist at least two steady-states[7] if and only if $G(\widetilde{\phi}) > F(\widetilde{\phi})$ that will require $\overline{E}$ not to be too large (see comparative statics with respect to $\overline{E}$ below). ■

[6] It means that if doing no adaptation, home mitigation and international agreements are not enough to induce a pollution stock that is decreasing in time.

[7] In the special case $\alpha m = 1$, which means that income Y has a linear impact on pollution accumulation, $G'(\theta^*) < 0$ and $F'(\theta^*) < 0$ that leads to a unique steady state and ϕ^* is constant. Moreover, it provides hints for the selection of the steady-state that should be considered for combinations of α and m such that αm is not too far from unity.

The existence and uniqueness results established in proposition 4 are illustrated in figure 5.[8].

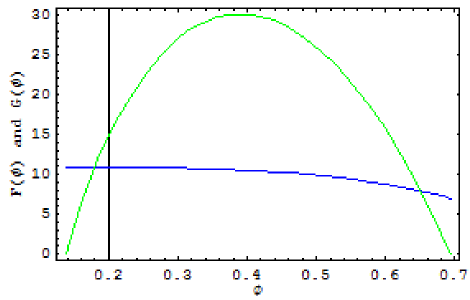

Figure 5

4.2 Comparative statics

Having characterized the SS, we can now turn to some comparative statics. An increase in BAU (Business As Usual) shifts $F(\phi)$ upwards. As a result, a steady-state ϕ^* located in the decreasing (resp. increasing) part of $G(\phi)$ will be reduced (resp. increased). The effect of BAU ROW emissions i.e. ROW emissions that would prevail in the absence of CDM can be analyzed as follows:

$$\frac{\partial \phi^*}{\partial \overline{E}} = \frac{\phi}{\sigma S}\left(\varepsilon_{G/\phi} \underbrace{-(\varepsilon_{S/\phi} - \alpha m)}_{FE}\right)$$

where $\varepsilon_{G/\phi}$ is the elasticity of G with respect of ϕ, and $\varepsilon_{S/\phi}$ is the elasticity of S with respect of ϕ. Note that $\frac{\partial \phi^*}{\partial \overline{E}} > 0$ iff $\varepsilon_{G/\phi} > 0$ since FE is only a feedback effect potentially reducing or reinforcing a direct positive effect (see figure 5). This means that larger pre-CDM ROW's emissions leads to less revenue devoted to CDM (ie. a larger ϕ^*) if and only if a large fraction of Y was initially used for CDM. This translates then into a smaller share of mitigation in home policy irrespective of the initial level of foreign mitigation. Note that the introduction of CDM reverses the effect of BAU ROW on home mitigation. Figure 6 provides a numerical illustration. The effect of a more efficient adaptation is even more complicated to study than in the absence of CDM since the relationship between ϕ^* and θ^* is not straightforward. Moreover, a larger γ shifts $G(\phi)$ downwards but $\partial F(\phi)/\partial\gamma > 0$ iff $\phi^*(1-\theta(\phi^*)) > \overline{K}^{-1/\alpha}$. In particular, $\partial F(\phi)/\partial\gamma|_{\phi=\underline{\phi}} = 0$ and $\partial F(\phi)/\partial\gamma|_{\phi=\overline{\phi}} = 0$. But interestingly in the case described in figure 7, starting from a

[8] For clarity of the figure, we have chosen parameters such that $F'(\phi) > 0$ but the same reasoning applies for a non-monotonous $F(\phi)$ that could lead to more than 2 steady-states.

low ϕ^*, a more efficient adaptation generates more home policies and more adaptation (since $d\theta^*/d\phi^* < 0$). Starting from a high ϕ^*, more efficient adaptation generates less home policies and more adaptation (since $d\theta^*/d\phi^* > 0$). Therefore the result obtained by Benchekroun (2011) is not valid for parameters used to design figure 7.

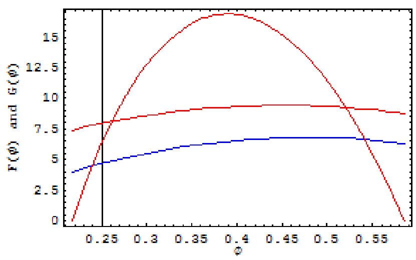

Figure 6: effect of $\overline{E}$

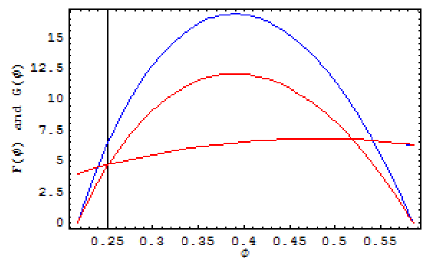

Figure 7: effect of a more efficient adaptation

We also study the effect of an increase in CDM efficiency. We obtain that more efficient CDM unambiguously shifts $F(\phi)$ downwards and $G(\phi)$ upwards. Focusing then on a small ϕ^* (such that $\phi^* < \widetilde{\phi}$), an increase in CDM efficiency unambiguously reduces the SS share of revenue devoted to home policies and increases the share of mitigation in home policy. For an initially large ϕ^*, less revenue is used for policy abroad if the efficiency of this policy rises, and this comes with more domestic mitigation as a share home policy (and in level as well).

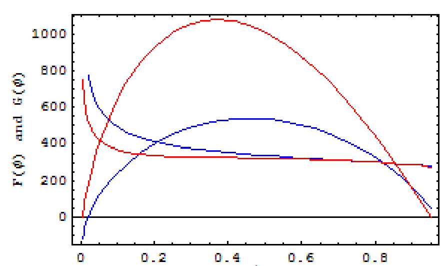

Figure 9: More efficient CDM (in red)

The final question consists in appraising whether CDM are a means to avoid the "adaptation threshold". To address it, we compare steady-state pollution with foreign mitigation (denoted by S^*_ϕ) and without such mitigation (denoted by S^*):

$$\sigma S^*_\phi - \sigma S^* = \underbrace{\beta \left(\frac{(\rho+\sigma)\omega \overline{K}^\gamma}{\beta} (1-\theta^*)^{\gamma\alpha} \right)^{\frac{1}{1-\omega}} (\phi^{*\gamma\alpha} - 1)}_{<0} + \underbrace{\theta^{*m\alpha} \overline{K}^m (1-\phi^{*m\alpha})}_{>0} \underbrace{-(1-\phi^*)^s Y^s}_{<0}$$

A sufficient condition for $\sigma S^*_\phi < \sigma S^*$ is :

$$\theta^{*m\alpha}\overline{K}^m(1-\phi^{*m\alpha}) < (1-\phi^*)^s Y^s \tag{16}$$

since $\frac{\partial \theta^*}{\partial s} < 0$ (see equation (15)), a sufficiently high value for s ensures that condition (15) is satisfied. Therefore, we can conclude that CDM are an effective way to lower the probability of reaching a pollution threshold provided foreign mitigation is sufficiently efficient. Comparative statics results are summarized in proposition 5:

Proposition 5 *In the current model, conducting mitigation abroad is a means to avoid a pollution threshold provided foreign mitigation is sufficiently efficient. Moreover, comparative statics are the following:*

- *an increase in BAU ROW reduces the share of mitigation in home policy; it increases foreign mitigation if and only if foreign mitigation was initially low;*
- *a more efficient adaptation does not necessarily result in less adaptation;*
- *a more efficient foreign mitigation increases the share of mitigation in home policy; it reduces foreign mitigation if and only if it was initially low therefore leading to a further lower level of foreign mitigation.*

5 Conclusion

This paper provides a long term analysis of the optimal mitigation/adaptation/CDM mix by considering the social planner problem of a polluting country. We account for accumulation in mitigation capital, in greenhouse gases, and in adaptation capital. Notably, we obtain that a more efficient adaptation does not necessary lead to a substitution of adaptation for mitigation that contradicts the results in Benchekroun (2011). In our framework, if a pollution threshold exists, above which adaptation is no longer efficient we show that there is no way for the modelled economy to avoid it. We therefore extend our framework to allow a fraction of revenue to be devoted to CDMs and we show that they may be a means to avoid the pollution threshold.

Moreover, our results concerning the effect of adaptation efficiency are robust to the introduction of CDM. Finally we show that more efficient CDMs generate an expansion in foreign mitigation only if this policy was already largely developed.

Some extensions of the paper could be considered. In particular, the pollution threshold above which adaptation is of no use is not perfectly known. Studying the consequence for mitigation and adaptation of an uncertain threshold would be very interesting since we know that uncertain thresholds lead to dynamics that drastically differ from those that prevail with deterministic thresholds (see Ayong Le Kama *et alii*, 2013). Further research could also extend the model to allow for a share of revenue used to develop a non polluting resource such that consumption would be fully free. This would clearly affect the mitigation/adaptation arbitrage as well as the risk to reach the pollution threshold.

6 Reference

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7 Appendix

The FOCs lead to

$$\frac{\dot{\phi}}{\phi}\left(\alpha-1+\frac{(s-1)\phi}{1-\phi}\right)=(1-\alpha)\frac{\dot{\theta}}{\theta}-(1-\omega)\frac{\dot{c}}{c}-(\rho+\delta)+\frac{\omega m}{\beta\lambda_M}c^{\omega-1}K^{m-1} \tag{17}$$

A steady-state is such that $\dot{A}/A=0$ $\dot{K}/K=0$, $\dot{S}/S=0, \dot{c}/c=0$, $\dot{\phi}/\phi=0$ and $\dot{\theta}/\theta=0$. Equations (13), (14) and (6) gives:

$$K^*(\theta^*,\phi^*) = (\phi^*\theta^*)^{\alpha}\,\overline{K} \tag{18}$$

$$A^*(\theta^*,\phi^*) = \phi^{*\alpha}(1-\theta^*)^{\alpha}\overline{K} \tag{19}$$

$$c^*(\theta^*,\phi^*) = \left(\frac{(\rho+\sigma)\omega}{\beta}\phi^{*\gamma\alpha}(1-\theta^*)^{\gamma\alpha}\overline{K}^{\gamma}\right)^{\frac{1}{1-\omega}} \tag{20}$$

Using the pollution accumulation process and the fact that at the steady-state $\dot{S}/S=0$, we have:

$$\sigma S^*(\theta^*)=\beta\left(\frac{(\rho+\sigma)\omega}{\beta}\phi^{*\gamma\alpha}(1-\theta^*)^{\gamma\alpha}\overline{K}^{\gamma}\right)^{\frac{1}{1-\omega}}-(\phi^*\theta^*)^{m\alpha}\,\overline{K}^{m}+\overline{E}-(1-\phi^*)^{s}Y^{s}$$