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Optimal Abatement of Carbon Emission Flows

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Abstract:

We study optimal carbon capture and storage (CCS) from point sources, taking into account damages incurred from the accumulation of carbon in the atmosphere and exhaustibility of fossil fuel reserves. High carbon concentrations call for full CCS, meaning zero net emissions. We identify conditions under which partial or no CCS is optimal. In the absence of CCS the CO2 stock might be inverted U-shaped. With CCS more complicated behavior may arise. It can be optimal to have full capture initially, yielding a decreasing stock, then partial capture while keeping the CO2 stock constant, and a final phase without capture but with an inverted U-shaped CO2 stock. We also introduce the option of adaptation and provide a unified theory regarding the optimal use of CCS and adaptation.

Key words: Climate change, carbon capture and storage, adaptation, non-renewable resources

JEL codes: Q32, Q43, Q54

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1. Introduction

Carbon capture and storage (CCS) is generally expected to play a crucial future role in combating climate change. In a special report IPCC puts forward that "..the potential of CO2 capture and storage is considerable" (Metz et al. 2005). The European Union states "This (CCS) technology has significant potential to help mitigate climate change both in Europe and internationally, particularly in countries with large reserves of fossil fuels and a fast-increasing energy demand" (European Union, 2014). The Environmental Protection Agency argues "Carbon dioxide (CO_2) capture and sequestration (CCS) could play an important role in reducing greenhouse gas emissions, while enabling low-carbon electricity generation from power plants ... CCS could also viably be used to reduce emissions from industrial process such as cement production and natural gas processing facilities" (Environmental Protection Agency, 2014). And J. Edmonds (Joint Global Change Research Institute) puts forward: "meeting the low carbon stabilization limits that are being explored in preparation for the IPCC 5th Assessment Report are only possible with CCS" (Edmonds, 2008). The main rationale for this view is that the economy is still depending on the use of fossil fuels to a large degree and that it might be too costly to introduce renewables in the short to medium run. CCS would then offer the opportunity to keep on using fossil fuels while limiting the emissions of CO2 into the atmosphere.

CCS consists of several stages. In the first stage the CO2 is captured at point sources, mainly at coal-fired or natural gas-fired power plants, but also in the upgrading process of tar oil (see Shell's Quest project.[3]) Several technologies are available, including post-combustion capture, pre-combustion capture (oxidizing fossil fuel) and oxy-fuel combustion. In the second phase the CO2 is transported to a reservoir, where in the third phase the captured carbon is stored in for example deep geological CCS consists of several stages. A side effect of the latter could be the use of captured carbon for increasing the pressure in oil fields, thereby reducing the cost of future extraction, but at the same time increasing the profitability of enhanced oil extraction, with the subsequent release of carbon, unless captured[4]. As a fourth phase there is monitoring what is going on, once CO2 is in the ground. Each of these phases brings along costs. The economic attractiveness of capture depends on the cost of capture and storage and the climate change damage prevented by mitigation of emissions of carbon. Herzog (2011) and Hamilton et al. (2009) provide estimates of these costs and conclude that the capture cost are about $52 per metric ton avoided (from supercritical pulverized coal power plants), whereas for transportation and storage the costs will be in the range of $5-$15 per metric ton CO2 avoided. This leads to overall costs amounting to $60-$65 per metric ton. These numbers are more or less confirmed

[3] http://www.shell.ca/home/content/can-en/aboutshell/our_business_tpkg/business_in_canada/upstream/oil_sands/quest/about_quest/

[4] Herzog (2011) points out that already decades ago capture took place, but then the objective was to enhance oil recovery by injecting CO2 in order to increase the pressure in the well.

in ZEP (2011). The International Energy Agency (2011) reviews several studies concerned with technologies used on a large scale and finds cost per metric CO2 avoided $55 on average for coal-fired plants and $80 for gas-fired power plants[5]. At the present state of climate change policy CCS is obviously not profitable, but with a carbon price at present of $25 and rising by 4% per year, large scale CCS becomes a serious option before 2040. Nevertheless numerous obstacles remain and many questions are still unresolved. Some are of a regulatory and legal nature, for example the rights-of-way for pipelines[6], access to the formation where CO2 is injected[7], and how to make the transition from capture megatons in the present to capture gigatons in the future in order to have capture at a level that is substantial enough to combat climate change. Moreover, in Europe the success of CCS also depends of the prevailing CO2 permit price, which at present is low, and has induced Eon and GDF Suez to postpone investments in an EU funded demonstration project near Rotterdam, The Netherlands.

In the present paper we address not so much the development of the CCS technology but the optimal use of the technology once it is available. We only look at capture at point sources, and thereby abstract from geo-engineering, where carbon is captured from the atmosphere. We also assume that a storage technology is available, but that technology cannot be utilized for making fossil fuel reserves accessible at lower cost. The potentially limited availability of (costly) storage capacity (see e.g., Lafforgue et al., 2008a and 2008b) is not taken into account. Moreover, we neglect other important issues as well, such as the uncertainty surrounding the safety of storage over a very long period of time, due to the possibility of leakage. We consider both exhaustibility and non-exhaustibility of fossil fuels. British Petroleum (2013) estimates that world proved natural gas reserves at the end of 2012, 6,614 trillion cubic feet, are sufficient to meet 56 years of production. Roughly the same holds for oil. For coal the global reserves-to-production ratio is much higher: 109 years. Since climate change is an issue that needs to be addressed in the long term, the assumption of exhaustibility seems warranted, even for coal. However, one could argue that the technically recoverable amounts of gas and coal are much higher and that large part of it will become economically viable due to higher prices or extraction lower costs. For example, the U.S. Energy Information Administration (2013) estimates the technically recoverable amount of gas are huge: 25,000 trillion cubic feet, of which around 30% is shale gas. Given the fact that backstop technologies are becoming cheaper over time, we account for the possibility that not all recoverable resources will be used up, so that from an economic perspective exhaustibility is not taking place.

[5] Remarkably, the costs for a project in China are much lower

[6] See Jaakkola (2012) for problems that may arise in case of imperfect competition on the transportation network (offshore, in northwestern Europe).

[7] Feenstra et al. (2010) report on the public outcry when plans for storage in the village of Barendrecht (The Netherlands) were revealed.

The criterion for optimality that we use is discounted utilitarianism with instantaneous welfare being the difference between utility from energy use on the one hand and the capture cost and the damage arising from accumulated CO2 in the atmosphere on the other hand. In addressing optimality one needs to simultaneously determine optimal capture and storage of CO2. We make a distinction between constant marginal capture cost and increasing marginal capture costs (with marginal capture costs at zero capture zero or positive). Along the optimum a tradeoff has to be made between the direct instantaneous welfare of using fossil fuel on the one hand and the cost of capture and damage caused by the accumulated CO2 on the other.

The main contributions of the paper are twofold. First, we characterize the optimal use of CCS taking the exhaustibility of fossil fuels explicitly into account. The interplay between the atmospheric CO2 stock and the potential additional emissions through the existing fossil fuel stock is crucial. It is found that the stock of fossil fuels plays a crucial role in the degree to which it is desirable to employ CCS. For example, it could well be that initially there is no CCS, then CCS is only partial, whereas there is a final phase with zero CCS again. The possible optimal patterns of CCS also depend on the assumptions on capture and storage cost. The second contribution therefore is to show that different cost specifications lead to considerable differences in the combined optimal capture and storage and extraction regime, in the case of abundant fossil fuel reserves as well as when reserves are limited. We identify cases where in the presence of the CCS it is still optimal to let the CO2 stock increase before partial capture takes place[8]. The core of the paper is section 4 where we derive the optimum for the pivotal case of a finite resource stock and the availability of a CCS technology. There we show that it might be optimal to have full capture initially, then partial capture while keeping the CO2 stock constant, and a final phase with no capture but in which the CO2 stock increases initially, before decreasing eventually. Hence the CO2 stock is not inverted U-shaped, as in Tahvonen (1997). In addition to these main contribution we also analyze adaptation as a possible strategy to tackle the climate change problem. It will be shown that this option may lead to postponing CCS, at least CCS at the maximum rate.

The related literature is large. First of all there is the literature that highlights the interrelationship between the use of fossil fuels and climate change (see Plourde (1972), D'Arge and Kogiku (1973), Ulph and Ulph (1994), Withagen (1994), Hoel and Kverndokk (1996), for early contributions). Recently this literature was enriched by explicitly introducing backstop technologies (see e.g., Tsur and Zemel (2003, 2005)) with due attention to the Green Paradox, the problem that may arise if for political economy reasons an optimal carbon tax is infeasible and policy makers rely on a subsidy of

[8] We are aware of the fact that CCS requires large upfront investments, for example in creating the capacity to transport and store CO2. We neglect such costs, although we do allow for high marginal costs of the first unit of CCS. Fixed cost is subject to future research.

the renewable (see e.g. Van der Ploeg and Withagen (2012 and 2014)). Another step has been set by explicitly incorporating CCS in models with non-renewable natural resources.

We start by sketching two recent contributions by Amigues et al. (2012 and 2013), who give a nice up to date survey of the state of affairs and offer a generalization of Chakravorty et al. (2006) and Lafforgue et al. (2008a and 2008b). These papers come close to ours in several respects but at the same time our discussion here serves to highlight the essence of our work. Amigues et al. assume that there is a finite stock of fossil fuel, that can be extracted at constant marginal cost. In our case extraction is costless. This is without loss of generality, as the results hold for constant average extraction costs as well. They also assume the existence of a backstop technology that is produced at constant marginal cost, which may be high or low. The backstop is perfectly malleable with the extracted fossil fuel and yields utility, together with fossil fuel. In this paper we abstract from a backstop technology, but we shall argue that in the case of abundant fossil fuel reserves capture essentially functions as a backstop. Net accumulation of CO2 is the difference between on the one hand emissions, resulting from burning fossil fuel minus the amount captured and stored, and, on the other hand, the natural decay of the stock of CO2, which is a constant fraction of the existing stock. The average cost of capture may take several forms. It may depend just on the amount captured, but, alternatively, one could allow for learning or for scarcity effects. In the former case the average CCS cost is a decreasing function of amount already captured. The latter case captures the fact that with more CCS done in the past it gets more difficult to find new CO2 deposits. Stock dependent storage costs are not allowed for, but we do look at different capture cost constellations. Since we concentrate on capture at point sources and not on capture from the atmosphere, net emissions are bound to be non-negative. Apart from the cost aspect, a major difference is in the assumption regarding damages. Amigues et al. put an upper bound, sometimes called a ceiling, on the accumulated CO2 stock, whereas we allow for the stock to take any value in principle, but work with a strictly convex damage function. Conceptually a damage function is more appealing, because it can be constructed in such a way that it includes the ceiling, by taking the damage function almost flat until just before the presupposed ceiling is reached, from where on damage increases steeply. More importantly, Amigues et al. (2012 and 2013) show that for all specifications considered it is optimal not to start with CCS until the threshold is reached. But the main and usual motivation for choosing a ceiling is that it represents a threshold beyond which a catastrophe takes place. Given the many uncertainties surrounding the phenomenon of climate change, this evokes the question whether it is optimal indeed to capture only at the critical level of the atmospheric CO2 stock. One of the objectives of the present paper is to investigate this in detail. Our finding is that it might be optimal to do partial CCS at some threshold level, keeping the stock at this level. But after such a phase, the CO2 stock might increase for a while, without CCS taking place.

Other papers addressing CCS include Amigues et al. (2014) and Coulomb and Henriet (2010), who both acknowledge that demand for fossil fuel derives from different sectors of the economy. For example, one sector is the electricity production sector, whereas the other is the transport sector. In the latter capture is far less attractive than in the former. Also in these papers, a ceiling on the CO2 stock is exogenously imposed and capture only takes place at the ceiling in the most likely scenarios. We assume away the existence of a backstop in order to highlight these innovative aspects. Essentially our model is a simple theoretical Integrated Assessment Model of CCS, that also allows for an optimal carbon tax rule representing the social cost of carbon. Ayong Le Kama et al. (2013) is also closely related to our work, in the sense that they assume exhaustibility of the resource stock. They also have damages from accumulated atmospheric CO2. Moreover, they consider a limited capacity to store CO2. The model treated in this paper is more general in the sense that Ayong Le Kama et al. consider special functional forms, such as iso-elastic utility, constant marginal damages and quadratic CCS cost. Moreover, they are particularly focused on determining whether or not the constraint on storage capacity becomes binding in finite time.

The outline of the sequel is as follows. We set up the model in section 2. Section 3 deals with the case of an abundant resource, whereas section 4 treats the case of a limited resource. Section 5 discusses potential policy implications. Section 6 concludes.

2. The model and preliminary results.

We consider an economy that has a stock of fossil fuel stock denoted by $X(t)$ at instant of time t, running from zero to infinity. The initial stock by X_0. The extraction rate $x(t)$ and the stock are required to be non-negative. Hence, for all $t \geq 0$ and with dots denoting the derivative with respect to time, we have

(1) $\dot{X}(t) = -x(t), \quad X(0) = X_0.$

(2) $x(t) \geq 0, X(t) \geq 0.$

The accumulation of atmospheric CO2, $Z(t)$, is determined by three factors. The flow of generated CO2 emissions is proportional to fossil fuel use with factor of proportionality $\zeta > 0$: $\zeta x(t)$. Emissions of CO2 can be reduced through a CCS technology. With the rate of CCS denoted by $a(t)$, net emissions, added to the existing stock, are $\zeta x(t) - a(t).$ Finally, we assume that decay of atmospheric CO2 is linear at a constant and positive rate α [9]. Hence

[9] The process of decay is more complicated in reality, because of all kinds of possible feedbacks and because part of the CO2 stock stays in the atmosphere indefinitely. See Farzin and Tahvonen (1996) for an early economic contribution, basing themselves on Maier-Raimer and Hasselman (1987). For more recent work, see Archer (2005), Archer et al. (2009) and Allen et al. (2009). For a recent discussion of the potential consequences of the modelling of carbon cycle for economic policy, see Amigues and Moreaux (2013) and Gerlagh and Liski (2012).

(3) $\dot{Z}(t) = \zeta x(t) - a(t) - \alpha Z(t), \quad Z(0) = Z_0.$

Here, Z_0 is the given initial CO2 stock. A distinguishing feature of the model is that capturing of CO2 from the atmosphere is excluded. Only current emissions from point sources can be abated. The idea is that CO2 capture at power plants is far less costly than CO2 capture directly from the existing atmospheric stock, or from emissions due to transportation, for example. So, in addition to the non-negativity of capture we impose non-negativity of net current emissions.

(4) $a(t) \geq 0,$

(5) $\zeta x(t) - a(t) \geq 0.$

An alternative to assuming emissions from point sources only, is that an exogenously given constant fraction of emissions is due to point sources and its complement is due to non-point sources. That would not alter our results in a qualitative sense. In practice CCS requires more than capturing. Transportation, storage and the potential use of CO2 to increase the pressure in existing wells are important elements of the process as well, but they are neglected here. We consider a partial equilibrium model with an infinitely lived representative consumer who derives utility from consuming fossil fuels, $u(x(t))$. The accumulated stock of atmospheric CO2 causes damages to welfare, given by $h(Z(t)).$ We assume that climate damage affects social welfare directly. Alternatively, damage occurs in production (Nordhaus, 2008, and Rezai et al., 2012), but here production is not modelled explicitly by a production function so that the direct approach is appropriate. Emissions reduction through a CCS technology comes at a cost, depending on the amount of reduced emissions, $c(a(t))$. Social welfare is assumed separable in its three components, utility, CCS cost and damages, and given by

$$\int_0^\infty e^{-\rho t} \left[u(x(t)) - c(a(t)) - h(Z(t)) \right] dt .$$

Here, ρ is the constant rate of pure time preference, assumed positive. Regarding the functions involved we make the following assumptions.

Assumption 1.

Instantaneous gross surplus u is strictly increasing, strictly concave and satisfies $\lim_{x \downarrow 0} u'(x) = \infty$ and $\lim_{x \to \infty} u'(x) = 0$.

Assumption 2.

The damage function h is strictly increasing and strictly convex and satisfies $h(0) = 0$, $\lim_{Z \downarrow 0} h'(Z) = 0$ and $\lim_{Z \to \infty} h'(Z) = \infty.$

Assumption 3.

The capture and storage cost function $c(a)$ is strictly increasing and convex.

In the sequel we allow for different alternative properties within the class of CCS costs defined in assumption 3: Linear, as well as strictly convex (with zero or positive marginal costs at zero capture). We define $\psi \equiv c'(0).$ Formally, our modeling is equivalent to modeling abatement subject to the condition that net emissions are non-negative. Hence, we will interchangeably use the expressions CCS and abatement. In the sequel we omit the time argument where there is no danger of confusion.

The current value Hamiltonian corresponding with maximizing social welfare reads
$H(Z, X, x, a, \lambda, \mu, \gamma_a, \gamma_{xa}) = u(x) - c(a) - h(Z) - \lambda[\zeta x - a - \alpha Z] + \mu[-x].$
The Lagrangian is
$L(Z, X, x, a, \lambda, \mu, \gamma_a, \gamma_{xa}) = u(x) - c(a) - h(Z) - \lambda[\zeta x - a - \alpha Z] + \mu[-x] + \gamma_a a + \gamma_{xa}[\zeta x - a].$
Here λ is the shadow *cost* of pollution and μ is the shadow *value* of the stock of fossil fuels. The latter vanishes in case of an abundant resource. The variables γ_a and γ_{ax} are Lagrangian multipliers corresponding with the nonnegativity constraints (4) and (5), respectively. Since marginal utility goes to infinity as consumption of fossil fuel goes to zero, the non-negativity constraint on fossil fuel extraction is not explicitly mentioned. The fossil fuel stock is bound to be non-negative. There is no explicit lower bound imposed on the atmospheric CO2 stock, because it will actually never reach zero, in view of the fact that net emissions are non-negative. These latter two assumptions allow us to invoke Theorem 16 of Seierstad and Sydsaeter (1987, pp. 244-245), including a transversality condition on the shadow cost of pollution, to establish the following necessary conditions, in addition to (1)-(5).

(6) $$\frac{\partial L}{\partial x} = 0 : u'(x) = \mu + \zeta(\lambda - \gamma_{xa}).$$

(7) $$\frac{\partial L}{\partial a} = 0 : \lambda = c'(a) + \gamma_{xa} - \gamma_a.$$

(8) $$\gamma_a a = 0, \quad \gamma_a \geq 0, \quad a \geq 0.$$

(9) $$\gamma_{xa}[\zeta x - a] = 0, \quad \gamma_{xa} \geq 0, \quad \zeta x - a \geq 0.$$

(10) $$\frac{\partial H}{\partial X} = -\dot{\mu} + \rho\mu : \dot{\mu} = \rho\mu.$$

(11) $$\frac{\partial H}{\partial Z} = \dot{\lambda} - \rho\lambda : \dot{\lambda} = (\alpha + \rho)\lambda - h'(Z).$$

(12) $$\lim_{t \to \infty} e^{-\rho t} \lambda(t) = 0 .$$

Conditions (6)-(9) are necessary for the maximization of the Hamiltonian with respect to extraction and CCS, subject to (4) and (5). Equation (10) is the Hotelling rule. With a finite resource stock X_0

accumulated atmospheric CO2 is bounded from above (see (3)), so that also λ is bounded from above and $\lambda(t) \to 0$ as $t \to \infty$[10]. It then follows from (11) that

(13) $$\lambda(t) = e^{(\alpha+\rho)t} \int_t^{\infty} e^{-(\alpha+\rho)s} h'(Z(s)) ds .$$

This is the social cost of carbon, the cost of all properly discounted future marginal damages due to an increase of emissions at instant of time t. With this interpretation it is easy to see what (7) means. If at some instant of time CCS is *partial*, meaning strictly positive CCS and positive net emissions, equation (7) says that in an optimum, the marginal benefit of less pollution (i.e., the social benefit of carbon reduction) is equal to the marginal CCS cost ($\lambda = c'(a)$). However, it could well be that the social cost of carbon at some instant of time is lower than the marginal CCS cost, even for zero CCS. This is the case, for example, if the optimal atmospheric CO2 stock is low from some instant of time on and $c'(0)$ is large. In that case γ_a needs to be positive. On the other hand, if the social cost of carbon is high compared to the marginal CCS cost, e.g., because the initial CO2 stock is large and the marginal CCS cost is a small constant, then we need full CCS, meaning zero net emissions (and hence $\gamma_{ax} > 0$). In that case equation (6) shows that the marginal benefit of more extraction and, therefore, consumption equals the resource rent μ plus the marginal cost of abating the additional CO2 emissions $\zeta c'(\zeta x)$. Fossil fuel consumption is then disconnected from λ. Our concavity/convexity assumptions allow for the following well known result on sufficiency.

Lemma 1

Suppose a program satisfies the necessary conditions (1)-(12), and has $X(t) \to 0$ as $t \to \infty$. Then the program is optimal.

Proof

Our concavity/convexity assumptions imply that for any alternative feasible program, denoted by hats, and for any $t \geq 0$

$$\int_0^t e^{-\rho s} \{u(x(s)) - c(a(s)) - h(Z(s)) - (u(\hat{x}(s)) - c(\hat{a}(s)) - h(\hat{Z}(s))\} ds \geq$$

$$e^{-\rho t}\mu(t)(\hat{X}(t) - X(t)) - e^{-\rho t}\lambda(t)(\hat{Z}(t) - Z(t)) \geq -e^{-\rho t}\mu(t)X(t) = -\mu(0)X(t),$$

because any feasible CO2 stock is bounded above in view of the limited availability of the resource and $\lim_{t\to\infty} e^{-\rho t}\lambda(t) = 0$. Q.E.D.

3. CCS available, abundant resource.

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[10] With $v(t) = e^{-\rho t}\lambda(t)$ it holds that $\dot{v}(t)/v(t) = \alpha - h'(Z(t))/\lambda(t)$. If λ would be unbounded, $v(t)$ would not converge to zero, contradicting (12). Actually, we need $\lambda(t)$ to converge to zero, since Z, and therefore $h(Z)$ converge to zero.

The purpose of this section is twofold. First, we describe the optimum if the fossil fuel stock is abundant ($X_0 = \infty$ so that the shadow price μ is zero). The insights are useful for the analysis in subsequent sections. Second, we introduce notation that is used in the sections that follow. To determine the optimal paths when the CCS technology is available, it is useful to take as a reference the optimal path when this option is not available.

1. The no-CCS optimal policy

In the absence of the CCS technology there exists a unique optimal long run saddle point stable steady state atmospheric CO2, Z^*, defined by $u'(\alpha Z^* / \zeta) = \zeta h'(Z^*) / (\alpha + \rho)$, whereas the optimal long run saddle point stable steady state social cost of carbon is $\lambda^* = h'(Z^*) / (\alpha + \rho)$. They are the solution of $\dot{Z} = \zeta x(\lambda) - \alpha Z = 0$ and $\dot{\lambda} = (\alpha + \rho)\lambda - h'(Z) = 0$, with $x(\lambda)$ defined by $u'(x(\lambda)) = \zeta\lambda$. The isoclines $\dot{Z} = 0$ (downward sloping), $\dot{\lambda} = 0$ (upward sloping) and the steady states Z^* and λ^* are depicted in figures 1 and 2 below.

In the presence of CCS we define $\tilde{Z}$ by $c'(0) = h'(\tilde{Z}) / (\alpha + \rho)$ [11]. It is the solution of $\dot{\lambda} = (\alpha + \rho)\lambda - h'(Z) = 0$ and $\lambda = c'(0)$. We use $\tilde{Z}$ to define *cheap* and *expensive* CCS technologies. Suppose that the economy without the CCS technology finds itself in the steady state Z^* and the CCS technology then becomes available. If $Z^* = \tilde{Z}$, the economy is indifferent between using and not using the CCS technology, because $c'(0) = \lambda^*$, so that the marginal CCS cost at zero capture is just equal to the social cost of carbon. If $\tilde{Z} > Z^*$ the CCS technology will not be adopted, because the marginal cost to reduce carbon is higher than the social cost of carbon. In that sense the CCS technology is *expensive*. If $\tilde{Z} < Z^*$ we say that CCS is *cheap*. In the sequel we will refer to $\tilde{Z}$ as the break-even CO2 stock. Finally, we define one additional pivotal CO2 stock. The threshold Z^h is the atmospheric CO2 stock corresponding with $\lambda = c'(0)$ on the stable manifold of the CCS-free economy leading to the steady state (Z^*, λ^*). This definition suggests that if $Z^h > Z^*$ no capture is needed as long as $Z < Z^h$. This is demonstrated below.

2. Expensive CCS (see figure1).

INSERT FIGURE 1 ABOUT HERE

[11] Due to assumption 2 $\tilde{Z}$ is well-defined. If the damage function would be linear the analysis requires a slight modification.

Since the stable manifold of the economy without the CCS technology lies below the curve $\dot{\lambda}=0$ for all $Z>Z^*$, we have $Z^*<\tilde{Z}<Z^h$. Suppose at instant of time t the economy with the CCS technology finds itself in $Z(t)\leq Z^h$. Then, even with the CCS technology available, it is optimal not to capture at all. The path followed by the economy not endowed with CCS leading to the steady state satisfies the necessary conditions (with $\mu(t)=0$) and is therefore optimal (lemma 1). Indeed, along the entire path the social cost of carbon, λ, is smaller than the marginal CCS cost, so that no CCS is needed. Next, consider the extraction rate x^m that solves $u'(x^m)=\zeta c'(\zeta x^m)$. It is the optimal extraction rate, given that there is full CCS: $\zeta x=a$ [12]. Then we define λ^m by $\lambda^m=c'(a^m)=c'(\zeta x^m)$. Clearly, there exists Z^m such that, starting in (λ^m, Z^m) the solution of the differential equations (3) and (11) with $c'(a(\lambda))=\lambda$ and $u'(x(\lambda))=\zeta\lambda$ exactly reaches $(c'(0), Z^h)$. Hence, for $Z^*<Z(t)<Z^m$ it is optimal to have partial CCS, whereas for $Z(t)>Z^m$ full CCS is optimal[13].

3. Cheap CCS (see figure 2).

INSERT FIGURE 2 ABOUT HERE

Due to assumptions 1-3 there exist $(\hat{Z},\hat{\lambda},\hat{a},\hat{x})$ with $\hat{a}>0$ and $\zeta\hat{x}-\hat{a}>0$, such that $h'(\hat{Z})/(\alpha+\rho)=\hat{\lambda}$, $u'(\hat{x})=\zeta\hat{\lambda}$, $\hat{\lambda}=c'(\hat{a})$ and $\zeta\hat{x}-\hat{a}=\alpha\hat{Z}$. Hence, if the economy initially finds itself in $Z_0=\hat{Z}$, it is optimal to stay there with partial capture, because all the necessary conditions are satisfied. Like in the previous case, it is possible to determine $Z^m>\hat{Z}$ such that for $\hat{Z}\leq Z_0\leq Z^m$ it is optimal to have partial capture throughout, and for $Z_0>Z^m$ full capture is required initially. It is also possible to define the critical level Z^h such that for $Z(t)\leq Z^h$ zero capture prevails, whereas for $Z^m\geq Z(t)>Z^h$ partial capture is in order.

The main finding of this section is that with an expensive CCS technology CCS will only be deployed for high CO2 levels. Moreover, the globally stable steady state coincides with the one in the absence of CCS. But with a cheap CCS technology there is a new globally stable steady state for atmospheric CO2 that is lower than before CCS was available. We end with three remarks.

[12] Given that u' is decreasing from ∞ for $x=0$ to 0 for $x=\infty$ and that c' is increasing, the solution of $u'(x)=\zeta c'(\zeta x)$ is well-defined and unique.

[13] For $x<x^m$, we have $u'(x)>\zeta c'(\zeta x)$. Hence, (6), (7) and (9) are satisfied for $\lambda-\gamma_{xa}=c'(\zeta x)$ with $\gamma_{xa}>0$.

1. With linear capture cost ($c(a) = \psi a$, where ψ is a positive constant) and optimality of a phase with partial capture we have from (7)-(9) that $\lambda = \psi$ so that, from (11), $Z = \tilde{Z}$. Hence, in figures 1 and 2 we have $Z^h = Z^m = \tilde{Z} = \hat{Z}$. Moreover, there cannot be an interval of time with partial capture if $\tilde{Z} > Z^*$. The reason is that along an interval with partial CCS we would then have $\lambda = c'(a) = \psi = h'(\tilde{Z}) / (\alpha + \rho) > h'(\tilde{Z}) / (\alpha + \rho) = \lambda^*$ from (7)-(9). Hence, with $\zeta x^* = \alpha Z^*$ we have $x < x^*$ from (6) with $\mu = \gamma_{ax} = 0$. But then from $\zeta x - a = \alpha \tilde{Z}$, we have $a < 0$, which is not allowed. If $\tilde{Z} < Z^*$ we have $\tilde{Z} = \hat{Z}$ and partial CCS is possible forever. Also, a discontinuity occurs in the capture rate once the steady state stock is reached.

2. With strictly convex CCS cost and $c'(0) = 0$, there is capture throughout, so that $Z^h = 0$.

3. The problem considered thus far, is essentially equivalent to the optimal use of a costly backstop technology. If we define $y = x + a / \zeta$ as total consumption, originating from the natural resource x and from a backstop a, properly scaled, and if the cost of producing the backstop is given by $c(a)$ then we have utility $u(y)$ and accumulation of pollution is given by $\dot{Z} = \zeta y - a - \alpha Z$. Hence, mathematically, the backstop problem is identical to the abatement problem. From the properties that have been established we can then infer that a cheaper backstop will always lead to less pollution, as long as the backstop cost is not prohibitively high. The equivalence result no longer holds if the natural resource is exhaustible, to which we turn now.

4. Optimal capture with a finite resource stock

4.1 General approach

Here we consider a finite fossil fuel stock. Also Tahvonen (1997) studies a world with a finite resource stock, but without CCS[14]. His work is an important benchmark since his assumptions on instantaneous utility and damages are equivalent to ours. Two properties of the optimum in his model are particularly relevant for the analysis of CCS. First, given the initial resource stock, for a low enough initial CO2 stock the shadow price of CO2 is inverted U-shaped over time, whereas otherwise it is monotonically decreasing. The shadow price approaches zero as time goes to infinity. Second, given the initial resource stock, for a low enough initial CO2 stock the CO2 stock is inverted U-shaped over time, and monotonically decreasing otherwise. The CO2 stock also approaches zero as time goes to infinity. The intuition is that with a low initial CO2 stock marginal damages from pollution are low compared to the marginal utility of consumption, so that it is welfare enhancing to consume a lot of fossil fuel initially, at the cost of a higher pollution stock. The possibility of non-monotonicity constitutes a relevant difference with the model with an abundant resource of the previous section where the shadow price and the atmospheric CO2 were monotonic. In our model we allow for CCS.

[14] Tahvonen (1997) allows for a backstop technology and for stock-dependent extraction cost. In describing Tahvonen's contribution we abstract from these issues, because our model does not incorporate them.

Three types of regimes are possible: full CCS ($\zeta x(t) = a(t) > 0$), no CCS ($a(t) = 0$) or partial CCS $\zeta x(t) > a(t) > 0$. The analysis will focus on finding the optimal sequence of these regimes. A first result is that full CCS is possible only for an initial period of time.

Lemma 2.

Suppose there exist $0 < T_1 < T_2$ such that $\zeta x(t) - a(t) = 0$ for all $t \in [T_1, T_2)$. Then $\zeta x(t) - a(t) = 0$ for all $t \in [0, T_2)$.

Proof.

See appendix A.

Next, we derive a useful benchmark from the Tahvonen economy without the CCS technology. For the sake of notation we denote variables of the Tahvonen economy by a superscript T. Moreover, for any variable y we denote its optimal value at instant of time t obtained with initial endowment (Z_0, X_0) by $y(t; Z_0, X_0)$. For any initial endowment (Z_0, X_0) the corresponding optimal social cost of carbon is

$$\lambda^T(0; Z_0, X_0) = \int_0^\infty e^{-(\alpha+\rho)s} h'(Z^T(s; Z_0, X_0)) ds ,$$

Now, suppose $Z_0 = \tilde{Z}$, the break-even CO2 stock. Then one may look for the initial resource stock, to be denoted by $\tilde{X}^{MW}$, such that, along the optimal Tahvonen path starting from $(\tilde{Z}, \tilde{X}^{MW})$, the social cost of carbon, λ^T , just equals the marginal CCS cost at zero abatement, $\psi \equiv c'(0)$. Figure 2 suggests that such a pivotal stock exists. The phase diagram shows that, in case of an abundant resource but in the presence of the CCS option, the initial shadow price of CO2, $\lambda^T(0)$, must be chosen larger than ψ if $Z_0 = \tilde{Z}$. In the Tahvonen economy the absence of the CCS option makes the social cost of carbon larger, but the finite resource has a dampening effect. Lemma 3 confirms that such a resource stock $\tilde{X}^{MW} > 0$ exists.

Lemma 3.

Suppose $c'(0) > 0$ and $\tilde{Z} < Z^*$. There exists $\tilde{X}^{MW} > 0$ such that

$$\lambda^T(0; \tilde{Z}, \tilde{X}^{MW}) = \int_0^\infty e^{-(\alpha+\rho)s} h'(Z^T(s; \tilde{Z}, \tilde{X}^{MW})) ds = c'(0) .$$

Proof.

See appendix A.

We now move to a full characterization of optimal CCS under different assumptions regarding the CCS cost function.

4.2 Constant marginal capture cost ($c'(a) = \psi > 0$ for all capture rates a).

A first result is stated in

Lemma 4.

Suppose CCS cost are linear and $Z(t) < \tilde{Z}$ for some $t \geq 0$. Then $a(t) = 0$

Proof

See appendix A.

The intuition is that for partial CCS to prevail the economy has to find itself in the break even CO2 stock, so that if the lemma would not hold, there is full CCS even for relatively low CO2 levels. We first assume that capture is *cheap*, meaning that $\psi = h'(\tilde{Z}) / (\alpha + \rho) < h'(Z^*) / (\alpha + \rho)$. With $\tilde{Z} < Z^*$ the optimum is depicted in figure 3, giving the optimal trajectories in (Z, X) space. We distinguish between three initial levels of the CO2 stock: at, below and above the break-even level ($Z_0 = \tilde{Z}$, $Z_0 < \tilde{Z}$ and $Z_0 > \tilde{Z}$, respectively).

1. $Z_0 = \tilde{Z}$.

Intuition tells that for low enough initial resource stocks CCS is not needed, in spite of the fact that it is cheap. The CO2 stock will then decrease even if CCS is not used. We show that this conjecture is correct. With an initial endowment $(Z_0, X_0) = (\tilde{Z}, \tilde{X}^{MW})$ total discounted damages incurred along the Tahvonen optimum equal $\psi = c'(0)$ (lemma 2). The optimal path in the Tahvonen economy is represented by the curve D starting in $(\tilde{Z}, \tilde{X}^{MW})$ and leading to $(Z, X) = (0,0)$. The resource stock decreases monotonically and the atmospheric CO2 stock increases initially, because otherwise it is below $\tilde{Z}$ forever, and total discounted marginal damages would be smaller than $\psi = c'(0)$. With this particular initial endowment the optimum of the Tahvonen economy is also optimal in the economy with CCS. Indeed, for this program all the necessary conditions are satisfied (in particular $\lambda(t) < \psi$ for all $t > 0$, so that $\gamma_a(t) > 0$ and $a(t) = 0$ for all $t > 0$). Also, the resource stock get asymptotically exhausted. Hence, lemma 1 on sufficiency applies. In figure 3 we have drawn the extended stable branch D as well. The point on D where $Z = 0$ is denoted by X_0^{MW} .

INSERT FIGURE 3 ABOUT HERE

Suppose $X_0 < \tilde{X}^{MW}$. Then, obviously, it is optimal again to adopt the optimal Tahvonen program without any use of CCS.

Suppose $X_0 > \tilde{X}^{MW}$. It is to be expected that some CCS is needed. Indeed, the optimum now consists of two phases. A first phase, until an instant of time $\tilde{T} > 0$, has partial capture. The second phase, for $t \in [\tilde{T}, \infty)$, coincides with the optimum described above. In the first phase we have $\lambda(t) = \psi$ since $\gamma_a(t) = \gamma_{xa}(t) = 0$ (from (7)). Moreover, $u'(x(t)) = \mu(0)e^{\rho t} + \zeta\psi$. The CO2 stock remains constant at the $\tilde{Z}$ level (from (11)). The resource stock is reduced until it reaches $\tilde{X}^{MW}$, which occurs at $\tilde{T} > 0$. The initial co-state variable value $\mu(0)$ needs to be chosen such that $\int_0^{\tilde{T}} x(s)ds = X_0 - \tilde{X}^{MW}$. Moreover, the co-state λ is continuous so that $\zeta x(\tilde{T}) = \alpha\tilde{Z}$. These conditions yield $\tilde{T} > 0$. At $\tilde{T}$ the abatement rate is continuous as well, and equals zero. The proposed program satisfies all the necessary conditions, and it exhausts the resource. It is therefore optimal. Graphically, the path initially follows the curve denoted by *E* in figure 3. After $\tilde{T}$ we are in the Tahvonen economy with the property that the CO2 stock will first increase and then decrease. It is interesting to note that full CCS is not optimal, regardless of the size of the initial resource stock. In spite of a large resource stock the CO2 stock can be kept at an acceptable level over time using only partial CCS.

2. $Z_0 < \tilde{Z}.$

If the initial state of the economy (Z_0, X_0) is to the left of the part of curve *D* that leads to $(\tilde{Z}, \tilde{X}^{MW})$, the Tahvonen program is optimal: CCS is not needed. If the initial state is to the right of that part, it is optimal to have an initial period of time with zero capture (lemma 4). Then follows a period of time with partial capture, moving along *E*, and a final interval of time, starting when $(\tilde{Z}, \tilde{X}^{MW})$ is reached, with zero capture again, following *D* from the moment of the transition on. Note that at the moment where partial CCS starts, there is an upward discontinuity in the abatement rate.

3. $Z_0 > \tilde{Z}.$

It is possible now to have periods of time with full CCS. The trade offs are the following. For a given initial atmospheric CO2 stock $Z_0 > \tilde{Z}$ and a large resource stock, the economy wants to benefit from high consumption. At the same time it can also afford to have full CCS because it has a large endowment of the resource. However, for low initial resource stocks, the economy might not want to invest in CCS. Alternatively, for any given initial resource stock, there will be full CCS if the CO2 level is high enough. To describe the optimal trajectories precisely we introduce two additional dividing curves, denoted by F and G, and depicted in figure 3. The curve F is the locus of stocks such that in a regime with full abatement the economy exactly reaches $(\tilde{Z}, \tilde{X}^{MW})$. We can characterize points on F in detail as follows. Take some $Z' > \tilde{Z}$. Take this Z' as the starting point of a regime with full CCS. Then $\dot{Z}(t) = -\alpha Z(t), Z(0) = Z'$ and $\tilde{Z}$ is reached at some instant of time T'. Next we need to know how much of the resource is needed to get to $(\tilde{Z}, \tilde{X}^{MW})$ in an optimal way. From the previous case we know what is optimal from $t = T'$ on, with $(Z(T'), X(T')) = (\tilde{Z}, \tilde{X}^{MW})$. So, let us by μ^{MW} denote the shadow resource price corresponding with the optimal path starting from $(\tilde{Z}, \tilde{X}^{MW})$. Define $x'(T')$ as the optimal extraction rate starting from $(\tilde{Z}, \tilde{X}^{MW})$. So, $u'(x'(t)) = \mu^{MW} e^{\rho(t-T')} + \zeta\psi$. Then we have for the initial resource stock needed to get to $\tilde{X}^{MW}$:

$$X' = \int_0^{T'} x'(t)dt + \tilde{X}^{MW} .$$

Clearly, X' is well-defined, and so is therefore the curve F. It is upward sloping. For stocks to the right of F but above E it is optimal to have full CCS initially, until a point on E is reached.
The curve G is the locus of initial stock values, $X_0 < \tilde{X}^{MW}$ and $Z_0 > \tilde{Z}$, such that for higher initial stocks there is full CCS, whereas for lower initial stocks no CCS is taking place at all. The location of this curve can be derived from the optimal paths in the Tahvonen economy. Take some $X_0 < \tilde{X}^{MW}$. For every Z_0 we find the optimal Tahvonen path $Z^T(t; Z_0, X_0)$. For the given X_0 we can determine Z_0 such that $\int_0^{\infty} e^{-(\alpha+\rho)t} h'(Z^T(t; Z_0, X_0))dt = \psi$. For lower initial resource stocks we need higher initial CO2 stocks, since if the initial CO2 stock would not increase, total marginal damages caused from less emissions due to the lower fossil stock would become smaller. Hence, the curve G is decreasing.

The curves D, E, F and G G divide the space of stocks in several regions. For each initial configuration of initial stocks we can then describe the optimum. To formally state the results we introduce the following definitions.
For $0 \le X_0 \le \tilde{X}^{MW}$ let $Z^D(X_0)$ be the pollution stock corresponding with X_0 on the locus D.
For $0 \le X_0 \le \tilde{X}^{MW}$ let $Z^G(X_0)$ be the pollution stock corresponding with X_0 on the locus G.
For $X_0 \ge \tilde{X}^{MW}$ let $Z^F(X_0)$ be the pollution stock corresponding with X_0 on the locus F.

Proposition 1.

Suppose marginal CCS cost is constant (ψ) and the CCS technology is cheap ($\psi < h'(Z^*)/(\alpha+\rho)$).

1. Suppose $X_0 < \tilde{X}^{MW}$ and $Z_0 \le Z^G(X_0)$. Then it is optimal to have zero CCS throughout, with Z initially increasing for X_0 large enough.

2. Suppose $X_0 < \tilde{X}^{MW}$ and $Z_0 > Z^G(X_0)$. Then it is optimal to have an initial phase with full CCS, followed by a final phase with zero CCS. In the first phase Z is decreasing, in the second it might increase initially.
3. Suppose $X_0 > \tilde{X}^{MW}$ and $Z^D(X_0) < Z_0 \le \tilde{Z}$. Then it is optimal to have an initial phase with zero CCS, followed by a phase with partial CCS, and a final phase with zero CCS. Along the first phase Z is increasing to $\tilde{Z}$, where it stays during the second phase until the final phase starts.
4. Suppose $X_0 > \tilde{X}^{MW}$ and $\tilde{Z} < Z_0 \le Z^F(X_0)$. Then it is optimal to have an initial phase with full CCS, followed by a phase with partial CCS, and a final phase with zero CCS. Along the first phase Z is decreasing to $\tilde{Z}$, where it stays during the second phase until the final phase starts.
5. Suppose $X_0 > \tilde{X}^{MW}$ and $Z_0 > Z^F(X_0)$. Then it is optimal to have an initial phase with full CCS, followed by a final phase with zero CCS. Along the first phase Z is decreasing, in the second it might increase initially.

A lower marginal CCS cost, a lower rate of time preference and a lower rate of decay of atmospheric CO2 lower, decrease $\tilde{Z}$ and therefore lead to more CCS, or at least reduce the scope for no capturing at all.

We now move to the case of an *expensive* capture technology: $\tilde{Z} > Z^*$. The analysis is less complex now, because partial CCS can no longer occur. For partial capture it is necessary that $Z = \tilde{Z}, \lambda = \tilde{\lambda} = \psi.$ Hence, with partial CCS, $\zeta x = a + \alpha\tilde{Z} > \alpha\tilde{Z}$ so that

$$u'(x) = \mu(0)e^{\rho t} + \zeta\psi = \mu(0)e^{\rho t} + \zeta h'(\tilde{Z})/(\alpha+\rho) < u'(\alpha\tilde{Z}/\zeta).$$

This is incompatible with $\tilde{Z} > Z^*$ since $\zeta h'(Z^*)/(\alpha+\rho) = u'(\alpha Z^*/\zeta)$. Partial capture is therefore excluded. Moreover, according to lemma 4 there is no CCS for $Z(t) < \tilde{Z}.$ This leads to the following proposition.

Proposition 2.
Suppose marginal CCS cost is constant (ψ) and the CCS technology is expensive

$(\psi > h'(Z^*)/(\alpha+\rho))$.

Then there exists a critical level of the CO2 stock, larger than $\tilde{Z}$, and decreasing with the resource endowment X_0, such that:

-for initial CO2 stocks smaller than the critical level there is zero capture forever.

-for initial CO2 stocks larger than the critical level it is optimal to have full capture initially, before switching to a zero capture policy forever.

We end this discussion on constant marginal CCS cost with a remark. If climate change damages are incorporated in the model, not by means of a damage function in the social preferences but through a ceiling: $Z(t) \leq \bar{Z}$, then only necessary condition (11) changes. It becomes $\dot{\lambda} = (\rho + \alpha)\lambda - \pi,$ where $\pi(t) \geq 0, \pi(t)[\bar{Z} - Z(t)] = 0$. In the case of constant marginal capture cost capture only takes place at the ceiling. Indeed, suppose that at some instant of time we have $Z(t) < \bar{Z}$ and $a(t) > 0$. Then $\pi(t) = 0$ and $\dot{\lambda}(t) = (\rho + \alpha)\lambda(t)$, implying that λ is increasing. Since $\lambda(t) = c + \gamma_{xa}(t)$ it follows that $\gamma_{xa} > 0$ and increasing, so that there is full capture and the CO2 stock declines. This process goes on, and the threshold will never be reached. Moreover, consumption and capture both go to zero as time goes to infinity, whereas positive consumption, bounded away from zero, is feasible. Hence, there will only be capture at the ceiling. This poses a danger, if the ceiling is motivated by interpreting it as a threshold level, beyond which a catastrophe occurs and if there is uncertainty regarding the effect of capture. More importantly, our model without the ceiling allows for more complex behaviour of the CO2 stock, as outlined in proposition 1.

4.3 Strictly convex capture cost with $c'(0) = 0$

In this section we consider increasing marginal capture cost, with zero marginal cost at zero capture. Contrary to the case of positive constant marginal capture cost there will always be some CO2 capture. It can even be optimal to have full capture indefinitely. The intuition is that, because of the limited availability of the resource, the rate of extraction, and therefore the rate of emissions, is necessarily becoming smaller over time, so that the effort needed to capture all emitted CO2 gets smaller over time as well, and hence may be worthwhile. Let us study this possibility in some detail. In case of permanent full capture we have $\zeta x(t) = a(t)$ for all $t \geq 0$. Also $\lambda(t) = c'(a(t)) + \gamma_{xa}(t)$ for all $t \geq 0$ from (7) and (8). Hence, from (6), $u'(x(t)) = \mu(0)e^{\rho t} + \zeta c'(\zeta x(t))$ for all $t \geq 0$. Therefore, the extraction rate x is a function of time and the shadow price $\mu(0)$. It is monotonically decreasing over time, and so is the rate of capture. The resource constraint $\int_0^\infty x(t)dt = X_0$ uniquely determines $\mu(0)$.

Consequently, also $x(t)$ and $a(t)$ are determined for all $t \geq 0$. Moreover, with full capture from the start, we have

$$\lambda(t) = e^{(\rho+\alpha)t} \int_t^\infty e^{-(\rho+\alpha)s} h'(Z_0 e^{-\alpha s}) ds$$

for all $t \geq 0$. In order for permanent full capture to be optimal it must additionally hold that $\gamma_{xa}(t) \geq 0$ for all $t \geq 0$. By way of illustration consider the following functions: $c(a) = \frac{1}{2}\chi a^2$, $u(x) = x^{1-\eta} / (1-\eta)$ and $h(Z) = \frac{1}{2}\beta Z^2$, where $\chi > 0, \beta > 0$ and $0 < \eta < 1$ are constants. Then, in the proposed optimum with full CCS we have $\lambda(t) = \beta Z_0 e^{-\alpha t} / (\rho + 2\alpha)$. Moreover,

(14) $\quad x(t)^{-\eta} = \mu(0)e^{\rho t} + \zeta^{-2}\chi x(t)$.

It follows from (14) that $\dot{x}(t) / x(t) \to -\rho / \eta$ as $x(t) \to 0$. Moreover,

(15) $$\gamma_{xa}(t) = \lambda(t) - c'(a(t)) = e^{-\alpha t}\{\frac{\beta Z_0}{(\rho + 2\alpha)} - e^{\alpha t}\frac{\chi x(t)}{\zeta}\}.$$

So, a first necessary condition is that the rate of decay is small enough: $\alpha < \rho / \eta$, because otherwise $\gamma_{xa}(t)$ will become negative. Let us then assume that $\alpha < \rho / \eta$. Note that in case of full abatement throughout, the extraction rate $x(t)$ is independent of the initial CO2 stock Z_0 and the damage parameter β. So, if Z_0 and β are sufficiently large and X_0 is sufficiently small (in order not to have $x(0)$ too large), full CCS is optimal. If $\alpha > \rho / \eta$,the initial CO2 stock is small, damages are small or the initial resource stock is large, we either have partial CCS throughout or full CCS initially and partial CCS eventually. The first path will be optimal if the initial resource stock is large. In that case the initial shadow price $\mu(0)$ is small, and the initial extraction rate is large. That implies that initial abatement is small. With intermediate values of the resource stock there will be an initial phase with full CCS. These finding do not hinge on the specific functional forms of the example. Hence we have

Proposition 3.

Suppose capture costs are strictly convex with $c'(0) = 0$.

There exist an instant of time $T_1 \geq 0$ such that it is optimal to have full CCS until T_1 and partial CCS thereafter. The instant of time T_1 can take several values

i. $T_1 = 0$ for a relatively large initial resource stock.
ii. $0 < T_1 < \infty$ for intermediate values of the resource stock
iii. Necessary conditions for $T_1 = \infty$ are a high initial CO2 stock, a low decay rate ($\alpha < \rho / \eta$) and a low initial resource stock.

4.4 Strictly convex capture cost with $c'(0) > 0$

We finally consider increasing marginal capture cost and positive marginal cost at zero capturing. Compared with linear CCS cost, there is more scope for partial CCS now. But for the rest, the outcomes are qualitatively close to the case of linear CCS cost, because in both cases marginal costs are bounded from below by a positive constant. The optimal pattern of capture takes two possible forms. There is an initial phase with either full or zero CCS. Then there is partial CCS. And eventually there is no CCS.

Proposition 4.

Suppose capture costs are strictly convex with $c'(0) > 0$.

There are two candidates for optimality:

1. There exist instants of time $0 \le T_1 \le T_2$ such that there is full CCS for $t \in [0, T_1)$, partial CCS for $t \in [T_1, T_2)$ and no CCS for $t \in [T_2, \infty)$. If there is no phase with partial CCS ($T_1 = T_2$) then there is no phase with full CCS either ($T_1 = T_2 = 0$).
2. There exist instants of time $0 \le T_1 \le T_2$ such that there is no CCS for $t \in [0, T_1)$, partial CCS for $t \in [T_1, T_2)$ and no CCS again for $t \in [T_2, \infty)$.

Proof

There exists $T \ge 0$ such that $a(t) = 0$ for all $t \ge T$. If this wouldn't hold, then there exists $\varepsilon > 0$ such that for all T there exists $t > T$ with $a(t) = \varepsilon$. But at such t we have $\lambda(t) - \gamma_{ax}(t) = c'(a(t)) \ge c'(0)$ implying from (6) that $x(t)$ gets arbitrarily small for t large enough. This violates the constraint (5) $\zeta x(t) \ge a(t)$. Hence $a(t) = 0$ eventually. No transition is possible from full to zero abatement. If there would be a transition at some $T > 0$ then $\lambda(t) = c'(a(t)) + \gamma_{ax}(t)$ just before T and $\lambda(t) = c'(0) - \gamma_a(t)$ just after T. But this violates the condition that λ is continuous.

Finally, we have to exclude possibility of a sequence where there is partial abatement, then zero abatement and then again partial abatement. If this would be optimal, the shadow price λ would decrease at the end of the first interval and increase at the beginning of the third interval. However, it has been shown by Tahvonen that once λ starts decreasing, it will decrease forever. Tahvonen's proof can easily be extended to the case of CCS. So, if it is optimal to start with full CCS this initial phase is followed by a phase with partial CCS, which in turn is followed by a final phase with zero CCS. This is the sequence of part 1 of the proposition.

The sequence given in part 2 is the only alternative, because we have already excluded the possibility of a sequence where there is partial abatement, then zero abatement and then again partial abatement.

The conditions under which each of the candidates will prevail in an optimum closely resemble the conditions under which we get the optimal paths in propositions 2 and 3. Clearly, the first regime is in order with a high initial CO2 stock.

A final remark concerns the introduction of the option to adapt. Suppose that adaptation only requires a flow of money outlays $c_y(y)$, where y is the adaptation effort and c_y the convex adaptation cost function.[15]. The damage function becomes $\hat{h}(Z, y)$. As long as there is no adaptation that function is assumed identical to the our original function h. In addition we assume that damages are strictly increasing and convex in the atmospheric CO2 stock, they are decreasing at a decreasing rate in adaptation, and the higher adaptation expenditures the lower is marginal damage from the CO2 stock. The social planner's objective is to maximize

$$\int_0^\infty e^{-\rho t}[u(x(t)) - \hat{h}(Z(t), y(t)) - c(a(t)) - c_y(y(t))]dt,$$

subject to (1)-(5). One necessary condition for optimality is the minimization of the sum of damage cost and adaptation cost $\hat{h}(Z, y) + c_y(y)$. Hence, if adaptation takes place the marginal damage cost equals the cost of adaptation: $-\partial\hat{h}(Z, y) / \partial y = c_y^{'}(y)$. Under mild assumptions on $\hat{h}$ and c_y this yields optimal adaptation as a function of the existing CO2 stock: $y(Z)$. Moreover, there exists a threshold level Z_y defined by $-\partial\hat{h}(Z_y, 0) / \partial y = c_y^{'}(0)$, such that $y(Z) = 0$ for all $0 \le Z \le Z_y$ and $y(Z) > 0$ and $y'(Z) > 0$ for all $Z > Z_y$. Note that $y(Z)$ is continuous at $Z = Z_y$ but not differentiable. We may also define the net damage function by $\tilde{h}(Z) \equiv \hat{h}(Z, y(Z)) + c_y(y(Z))$. We are then essentially back in the model analyzed in the previous sections, since $\tilde{h}(Z)$ has the same properties as the function $h(Z)$, except for differentiability. The online appendix provides an example of the integration of CCS and adaptation in a unified framework, showing a.o. that adaptation may or may not fully replace CCS.

5 Policy implications

The implementation of the first-best outcome in a decentralized economy requires taking into account the social cost of carbon. If the resource extracting sector is competitive, generates the energy needed by the consumers and owns the CCS technology, then it suffices to impose a carbon tax corresponding with marginal damage, evaluated in the optimum. The problem of this firm is to maximize

$$\int_0^\infty e^{-\rho t}\left[p(t)x(t) - \tau(t)(\zeta x(t) - a(t)) - c(a(t))\right]dt$$

subject to the resource constraint and non-negativity constraints, (1), (2), (4) and (5). Here p is the given market price of energy and τ is the carbon tax on net emissions. The necessary conditions include:

(16) $p(t) = \mu(t) + \zeta(\tau(t) - \gamma_{xa}(t))$,

(17) $\tau(t) = c'(a(t)) + \gamma_{xa}(t) - \gamma_a(t)$

[15] Hence no specific capital is needed for adaptation (Tsur and Withagen (2013) and Zemel (2014) consider investment in adaptation capital).

where μ is the Hotelling rent and (8) and (9) hold. Hence, by taking the carbon tax equal to the optimal social cost of carbon, λ, the first best outcome is implemented in a market equilibrium. If there exists a separate CCS firm, the implementation is straightforward as well. The resource extracting firm does not consider abatement. It is still to be confronted with a carbon tax, $\hat{\tau}$, that now applies to gross emissions. Hence, we get

(18) $p(t) = \mu(t) + \zeta\hat{\tau}(t)$.

The tax imposed on the resource firm should be $\hat{\tau}(t) = \lambda(t) - \gamma_{xa}(t)$, where λ and γ_{ax} take their first-best values. The firm with the CCS technology maximizes its profits. It receives a price p_a per unit abated. Profit maximization, taking non-negativity of abatement into account, then yields

(19) $p_a(t) = c'(a(t)) - \gamma_a(t)$.

Hence, in order to implement the first best, it is needed to set $p_a = \lambda - \gamma_{xa}$, where λ and γ_{xa} are the first best values. Hence, the implementation of the first best requires setting a gross emission tax and a price of abatement. If these are properly imposed, in equilibrium the resource firm will choose the optimal production rate x and the CCS firm will buy and sequester the optimal part a of the resource firm's emissions, that is, either the whole emission flow, $a = \zeta x$ when $\gamma_{xa} > 0$, or only some part of it when $\gamma_{xa} = 0$.

If the model is interpreted as being a model of the global economy, and policy is of a global nature, there will be no intertemporal leakage, whereas spatial leakage is not occurring by definition. A multi-country setting requires a different model, with due attention to terms of trade effects and changes in the internationally ruling rate of interest. Nevertheless it is to be expected that the existences per se of the CCS technology in one country will not lead to an enhanced moral hazard problem in the sense that other countries might speculate that the CCS country will abate additional emissions caused by other countries. A reason is that there may indeed occur carbon leakage through the usual channel, but the CCS country is unable to reduce net emissions from other countries because it can abate only from point sources.

6. Conclusions

In this paper we have given a full account of optimal CCS under alternative assumptions regarding capture cost in the case of an abundant stock of fossil fuels that cause emissions of CO2. It has been shown that depending on initial conditions and the specification of carbon capture costs optimal policies may differ considerably. In the most realistic case of marginal capture cost bounded far away from zero, no capture is warranted at all. Otherwise, we might have full capture initially, if the initial CO2 stock is high. But eventually capture is partial.

If exhaustibility is taken into account, then in the case of positive marginal capture cost at zero capture, the picture changes. Any regime with capture comes to an end within finite time. With a high initial CO_2 stock it is optimal to have full use of CCS initially. The general picture that arises is that the CO_2 stock is inverted-U shapedWith a large initial resource stock it will initially increase, while CSS is not used, then follows a phase where CCS is used partially, whereas in a final phase no capture will take place. With constant marginal capture cost, the CO_2 stock is stabilized at a certain level as long as partial capture takes place, but then definitely the CO_2 stock increases for a period of time before approaching zero in the end. Compared with a world where for one reason or another an exogenous upper bound is set for the pollution stock, we find that, if we would put such an upper bound in addition to the damage function, it is well possible to have CCS use before the upper bound is reached.

The implementation of the first-best outcome in a decentralized economy is simple, at least from a theoretical perspective. If the resource extracting sector is competitive and also generates the energy needed by the consumers and owns the CCS technology, then it suffices to impose a carbon tax corresponding with marginal damage, evaluated in the optimum. If the extractive sector, the energy generating sector and the CCS sector are distinct industries, then the tax should account for the fact that net emissions are non-negative.

We have also paid attention to adaptation. It has been shown that adaptation can be represented by modifying the damage function in a straightforward way. Optimal adaptation can be decided upon independently of optimal deployment of CCS. Adaptation makes full scale CCS less desirable, in the sense that full CCS is needed for a shorter period of time, at least if adaptation is not prohibitively costly.

Future research on a large number of issues is in order. A crucial question is where the world's actual initial position is. It should be possible to accurately assess the amount of CO_2 that is in the atmosphere at present, as well as the CO_2 in the crust of the earth. But, to take the simple case of constant marginal CCS cost amounting to approximately \$60, we then still need to specify the global damage function. Estimates of marginal damages vary considerably among studies. Related to this is the fact that CCS and also some types of adaptation require huge set up cost, that we haven't taken into account here. We have treated energy as a commodity that yields utility directly, whereas it should play a role in production rather than in consumption. For the description of the carbon cycle we have followed an approach that is well established in economics, but, as we have stressed before (see footnote 9), that could be modified according to new insights from climatologists, according to which part of current emissions stay in the atmosphere indefinitely. With an abundant resource this would not lead to outcomes that qualitatively differ from what we found in section 3. We also conjecture that our results go through in case of decay being a strictly increasing and strictly convex function of the existing pollution stock. See Toman and Withagen (2000) on clean technologies and concavity of the self-regeneration function.

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Appendix A. Proofs.

Lemma 2.

Suppose there exist $0 < T_1 < T_2$ such that $\zeta x(t) - a(t) = 0$ for all $t \in [T_1, T_2)$. Then $\zeta x(t) - a(t) = 0$ for all $t \in [0, T_2)$.

Proof

Figure 4 illustrates the proof. The proof is by contradiction. Suppose that along the optimal path we have incomplete CCS before full CCS: $\zeta x(t) - a(t) > 0$ for all $t \in [0, T_1)$ and $\zeta x(t) - a(t) = 0$ for all $t \in [T_1, T_2)$ with $T_2 > T_1 > 0$. The stocks at T_1 and T_2 are depicted in figure 4.

INSERT FIGURE 4 ABOUT HERE

Take a large initial atmospheric CO2 stock $\breve{Z}_0$, such that, irrespective of the initial resource stock, it is optimal to start with full CCS. Such a CO2 stock exists. The reasoning is as follows. In view of $\dot{\lambda}(t) = (\alpha + \rho)\lambda(t) - h'(Z(t))$ (equation (10)) and $h'(\infty) = \infty$ (assumption 2), by taking Z_0 large, the optimal corresponding co-state $\lambda(0)$ can be made arbitrarily large, irrespective of the existing resource stock, in order to avoid that the co-state becomes negative. Now, if there would not be full CCS initially, we would have (7), $\lambda \le c'(a)$, that a is arbitrarily large (or in case of linear abatement cost, an immediate contradiction is obtained), and from (6), $u'(x) = \mu + \zeta\lambda$, that x is arbitrarily small, which contradicts $\zeta x - a > 0$. Next, for the high $\breve{Z}_0$ there exists an initial resource stock $\breve{X}_0$ such that starting from $(\breve{Z}_0, \breve{X}_0)$ it is optimal to arrive at exactly the stocks prevailing at T_2, $(Z(T_2), X(T_2))$, in the original program, at some instant of time $\breve{T}$ along a path with full CCS. It gives the curve (1) in the figure. Note that the first part of the original program, depicted as curve (2) in the figure, starting from (Z_0, X_0) and arriving at $(Z(T_1), X(T_1))$, is located below the first part of curve (1), connecting $(\breve{Z}_0, \breve{X}_0)$ with the same $(Z(T_1), X(T_1))$. The reason is that, starting from X_0, to arrive at $(Z(T_1), X(T_1))$ while partially abating requires a lower initial carbon stock than the initial carbon stock (denoted by $Z_0^{'}$) from which, with the same X_0, the economy is led to $(Z(T_1), X(T_1))$ while fully abating.

Then, hold $\breve{Z}_0$ fixed, so as to keep the necessity of starting with full CCS, and take a new initial stock $\breve{X}_0 + \delta$ slightly larger than $\breve{X}_0$. We get a new curve (denoted by 3) depicting the optimum, and it will have full CCS until some instant of time $\widehat{T}$. This path (3) is located below the path (1) and crosses the path (2) of the initial program at point A (see figure 4), provided that δ is sufficiently small. Thus, from the same initial endowment A (in both CO2 and resource stock) , we get different optimal paths:

The first best one along curve (3) and the second one along curve (2) initially up to point J and next along curve (1). But given the strict convexity assumptions made, the optimum is unique. A contradiction. Q.E.D.

Lemma 3

Suppose $c'(0) = \psi > 0$ and $\tilde{Z} < Z^*$. There exists $\tilde{X}^{MW} > 0$ such that

$$\lambda^T(0;\tilde{Z},\tilde{X}^{MW}) = \int_0^\infty e^{-(\alpha+\rho)s} h'(Z^T(s;\tilde{Z},\tilde{X}^{MW}))ds = \psi .$$

Proof

The proof proceeds in several steps. We first show that with an initial CO2 stock $Z_0 = \tilde{Z}$ in the Tahvonen economy there is an upper bound on extraction, irrespective of the initial resource stock. Then it is proven that $\mu(0;\tilde{Z},X_0)$ can be made arbitrarily small by taking X_0 large enough. The second step involves showing that $\lambda^T(0;\tilde{Z},\infty) > \psi$ and $\lambda^T(0;\tilde{Z},X_0) \le \psi$ for all $X_0 < \infty$ would imply that for X_0 large enough, consumption and hence accumulation of CO2 would be larger along the path with the finite, but large, resource stock. But then the social cost of carbon along the path with the finite stock cannot be smaller than ψ .

Step 1.

Suppose that for all $M_x > 0$ there exists $X_0 > 0$ and $t_1 > 0$ such that $x^T(t_1;\tilde{Z},X_0) > M_x$. This can only happen with $\lambda^T(t_1;\tilde{Z},X_0)$ close enough to zero since $u'(x^T(t;\tilde{Z},X_0)) = \mu^T(t;\tilde{Z},X_0) + \zeta\lambda^T(t;\tilde{Z},X_0)$ and $u'(\infty) = 0$. We also have $\dot{\lambda}^T(t;\tilde{Z},X_0) = (\alpha+\rho)\lambda^T(t;\tilde{Z},X_0) - h'(Z^T(t;\tilde{Z},X_0))$. Hence, in order to keep $\lambda^T(t;\tilde{Z},X_0)$ nonnegative it is necessary that $h'(Z^T(t_1;\tilde{Z},X_0))$ is arbitrarily close to zero. We have $h'(0) = 0$ so that we need $Z^T(t_1;\tilde{Z},X_0)$ arbitrarily close to zero. This is not the case if $t_1 = 0$, because $Z(0;\tilde{Z},X_0) = \tilde{Z}$. So, $t_1 > 0$ and, hence, $Z^T(t_2;\tilde{Z},X_0)$ must have been decreasing at some instant of time before t_1. It has been shown by Tahvonen that Z^T keeps on decreasing once it has started to decrease, so that $\dot{Z}^T(t_1;\tilde{Z},X_0)) < 0$. But also $\dot{Z}^T(t_1;\tilde{Z},X_0)) > 0$ since $x^T(t_1;\tilde{Z},X_0) > M_x$. So we obtain a contradiction. Since $x^T(t;\tilde{Z},X_0)$ is bounded from above by some $M_x > 0$ it follows from our concavity assumptions that

$$u(M_x)/\rho \geq \int_0^\infty e^{-\rho t}[u(x^T(t;\tilde{Z},X_0)) - u(\tfrac{1}{2}x^T(t;\tilde{Z},X_0))]dt \geq$$

$$\int_0^\infty e^{-\rho t}u'(x^T(t;\tilde{Z},X_0))\tfrac{1}{2}x^T(t;\tilde{Z},X_0)dt =$$

$$\int_0^\infty \mu^T(0;\tilde{Z},X_0))\tfrac{1}{2}x^T(t;\tilde{Z},X_0)dt = \tfrac{1}{2}\mu^T(0;\tilde{Z},X_0)X_0.$$

Hence, $\mu^T(0;\tilde{Z},X_0) \to 0$ as $X_0 \to \infty$.

Step 2.

The fact that $\lambda^T(0;\tilde{Z},\infty) > \psi = c'(0)$ under $\tilde{Z} < Z^*$ can be seen from figure 2, the phase diagram where we considered an infinite resource stock. Moreover, $\dot{\lambda}^T(t;\tilde{Z},\infty) > 0$ for all $t \geq 0$ because $\lambda^T(t;\tilde{Z},\infty) \to \lambda^*$ as $t \to \infty$. Now suppose that $\lambda^T(0;\tilde{Z},X_0) < \psi$ for all $X_0 < \infty$. This implies $\dot{\lambda}^T(0;\tilde{Z},X_0) < 0$ by the definition of $\tilde{Z}$ and (11). Moreover, $\dot{\lambda}^T(t;\tilde{Z},X_0) < 0$ for all $t \geq 0$, because, as shown by Tahvonen, λ^T keeps decreasing once it starts decreasing. Since

$$\lambda^T(0;\tilde{Z},\infty) = \int_0^t e^{-(\alpha+\rho)s}h'(Z^T(s;\tilde{Z},\infty)ds + \int_t^\infty e^{-(\alpha+\rho)s}h'(Z^T(s;\tilde{Z},\infty)ds > \psi$$

there exists T^* such that $\int_0^{T^*} e^{-(\alpha+\rho)s}h'(Z^T(s;\tilde{Z},\infty)ds = \psi$. Since $\lambda^T(t;\tilde{Z},\infty)$ is initially larger than ψ and monotonically increasing, and $\lambda^T(t;\tilde{Z},X_0)$ is initially smaller than ψ and monotonically decreasing, then by taking X_0 large enough, and therefore $\mu^T(0;\tilde{Z},X_0)$ small enough, we can make sure that $x^T(t;\tilde{Z},X\ \ ) > x^T(t;\tilde{Z},\infty)$ for all $0 \leq t \leq T^*$. Hence, also $Z^T(t;\tilde{Z},X_0) > Z^T(t;\tilde{Z},\infty)$ for all $0 \leq t \leq T^*$. But then $\lambda^T(0;\tilde{Z},X_0) > \int_0^{T^*} e^{-(\alpha+\rho)s}h'(Z^T(s;\tilde{Z},X_0)ds > \psi$, a contradiction.

Lemma 4.

Suppose CCS cost are linear and $Z(t) < \tilde{Z}$ for some $t \geq 0$. Then $a(t) = 0$

Proof

First, if $Z(t) < \tilde{Z}$ there is no partial CCS. So, suppose that at some instant of time t with $Z(t) < \tilde{Z}$ full CCS, $a(t) = \zeta x(t)$, prevails. Then $\lambda(t) \geq \psi$ from (7). It follows from (11) with $h'(Z(t))/(\alpha+\rho) < \psi = h'(\tilde{Z})/(\alpha+\rho)$ that λ increases over time. Hence, γ_{ax} increases, because $\gamma_a = 0$ along the interval of time with full CCS. At a transition to zero capture, which is the only transition possible, γ_{ax} has a downward discontinuity, which, in view of the continuity of λ must be compensated by a downward discontinuity of γ_a. But a negative γ_a is not allowed. Hence there will be full capture forever. But this is suboptimal. To see this consider figure 3 again. The optimal Tahvonen paths, with no CCS, have $0 \le Z^T(t) \le \tilde{Z}$ and $0 \le X^T(t) \le \tilde{X}^T$ for all t large enough. Moreover, $\lambda^T(t) < \psi$ eventually because $h'(Z^T(t))/(\alpha+\rho) \le h'(\tilde{Z})/(\alpha+\rho)$ eventually and λ^T has to decrease eventually. Hence, if at some $t_1 \ge 0$ the economy with CCS finds itself in the region with $0 \le Z(t_1) \le \tilde{Z}$ and $0 \le X(t_1) \le \tilde{X}^T$ it is optimal to follow the Tahvonen path, without making use of the CCS technology. Therefore, it is not optimal to have full CCS eventually. Hence, as long as $Z(t) < \tilde{Z}$ there is zero CCS. Q.E.D.

Appendix B. Adaptation

As an illustration of adaptation let us consider an example, with linear CCS cost and linear adaptation cost: $c(a) = \psi a, c_y(y) = \kappa y$. Moreover, $\hat{h}(Z, y) = \beta Z^2 / 2(y + \varphi)$.

Hence, $Z_y = \varphi\sqrt{\dfrac{2\kappa}{\beta}}$ and

if $Z < Z_y$, then $\tilde{h}(Z) = \dfrac{\beta Z^2}{2\varphi}$ and $\tilde{h}'(Z) = \dfrac{\beta Z}{\varphi}$;

if $Z > Z_y$, then $\tilde{h}(Z) = Z\sqrt{2\beta\kappa_y} + \varphi\kappa$ and $\tilde{h}'(Z) = \sqrt{2\beta\kappa}$.

Marginal damages are linear initially and then become constant. We make a distinction between two cases, with linear CCS cost.

Case a. $\psi > \dfrac{\sqrt{2\beta\kappa}}{\rho+\alpha}$

Hence, $\psi \ge \tilde{h}'(Z)/(\alpha+\rho)$ for all $Z \ge 0$. Intuitively, taking account of adaptation the marginal CCS cost is higher than the maximal marginal damages from atmospheric CO2 for all possible CO2 levels. Hence, CCS is expensive relative to adaptation and will never be deployed. The formal argument runs as follows. Suppose $a > 0$ along some interval of time. Then, along that interval, $\lambda = \psi + \gamma_{xa} \ge \psi$. We also have $\dot{\lambda} = (\rho+\alpha)\lambda - \tilde{h}'(Z)$. Moreover, $\tilde{h}'(Z) \le \sqrt{2\beta\kappa} < (\rho+\alpha)\psi$ so that $\dot{\lambda} > (\rho+\alpha)(\lambda-\psi) > 0$. Hence λ increases, so that also γ_{xa} increases and there is full capture. This phase will never come to an end because of the continuity of λ. But this contradicts that eventually we are in the Tahvonen economy with zero capture at low enough pollution stocks. Hence, the optimum is characterized by adaptation prevailing as long as the pollution stock is high, whereas there will be no adaptation if it gets below the certain threshold Z_y.

Case b. $\psi < \dfrac{\sqrt{2\beta\kappa}}{\rho+\alpha}$

The solution $\tilde{Z}$ of $\psi = \tilde{h}'(Z)/(\rho+\alpha)$ satisfies $\tilde{Z} < Z_y$. This $\tilde{Z}$ equals the $\tilde{Z}$ defined in the previous section because $\tilde{h}'(Z) = h'(Z)$ for $Z \le Z_y$. Let us consider several possibilities.

Suppose $Z^* > \tilde{Z}$. Then CCS is cheap. It will be used if the technology would become available in the resource abundant economy's steady state. Moreover, CCS is cheap relative to adaptation. We can reproduce figure 3 and insert Z_y on the vertical axis. This yields figure 5.

INSERT FIGURE 5 ABOUT HERE

The existence of the adaptation option is mainly reflected in the slope of the *G*-curve, the curve along which there was indifference between full and zero capture. Clearly, any path that is optimal in the economy without the adaptation option and that has $Z(t) \le Z_y$ for all $t \ge 0$ is also optimal in the economy with the adaptation option, because this option is not used. Next, consider optimal paths without adaptation where there is no capture at all, but where the CO2 stock is larger than Z_y at some instant of time. This holds for example if we would start at the old *G*-curve at a point with $Z_0 > Z_y$. With the adaptation option in place, it would be used, and, of course, carbon capture will never be optimal. Finally, consider optimal paths in the economy without the adaptation option that will start with full capture and have a CO2 stock larger than Z_y at some instant of time. This holds for example if we start to the right of the old *G*-curve with $Z_0 > Z_y$. The aim of the economy is to reduce the CO2 stock as quickly as possible, in order to reduce damages. In the new situation there will be adaptation initially. This mitigates the damages and therefore also the need to reduce the CO2 stock. Hence, typically, there will be full capture initially, but for a shorter period of time than before. Another way of looking at this is to say that the *G*-curve becomes steeper. To illustrate this, let us fix the initial resource stock and assume that we are in an initial state on the old *G*-curve. Then there is zero capture throughout. In order to have full capture initially, we need a higher initial CO2 stock, i.e. we need to be in a point above the *G*-curve, for the same initial resource stock. Also the *F*-curve, the path that has full capture and leads to $(\tilde{Z}, \tilde{X}^{MW})$, changes. Note, first of all, that $\tilde{X}^{MW}$ may change itself. Recall that $\tilde{X}^{MW}$ is defined as the initial state from where it is optimal to have zero capture forever and an initial increase of the CO2 stock at the same time, assuming $Z_0 = \tilde{Z}$. We have $\lambda^{MW} = h'(\tilde{Z})/(\alpha+\rho) = c_a$. It could well be that the curve starting in $(\tilde{Z}, \tilde{X}^{MW})$ has $Z(t) > Z_y$ at some instant of time. If so, total discounted marginal damages will be smaller than c_a, so that the new $\tilde{X}^{MW}$ is larger. The shift to the right is then needed to have total discounted marginal damages equal to c_a. But, let us assume for the sake of exposition that $\tilde{X}^{MW}$ is unaffected by the adaptation option. Of course, there is a path with full capture leading to $(\tilde{Z}, \tilde{X}^{MW})$. This is still the *F*-curve in figure 3. However, starting from a point (Z_0, X_0) on the *F*-curve with $Z_0 > Z_y$, it is optimal now to switch to zero capture after $\tilde{X}^{MW}$ is reached, hence to cross the (new) *G*-curve.

Suppose, as a final case, $Z^* < \tilde{Z}$. Here CCS is expensive, but still cheaper than adaptation. Essentially we have the same result as in proposition 2. There will never be partial capture. Zero capture prevails for small enough CO2 stocks and full capture for large enough CO2 stocks. The effect of adaptation is a reduction of the time for which full capture is needed.

Concluding, we can say that adaptation can easily be included in the CCS framework. In our setting the decisions on CCS and adaptation can be separated in the sense that the adaptation strategy can be decided upon independently of the CCS strategy. The analysis of optimal CCS can then be conducted along the lines of the previous section. We generally find that CCS efforts need to be less strong in the presence of adaptation.

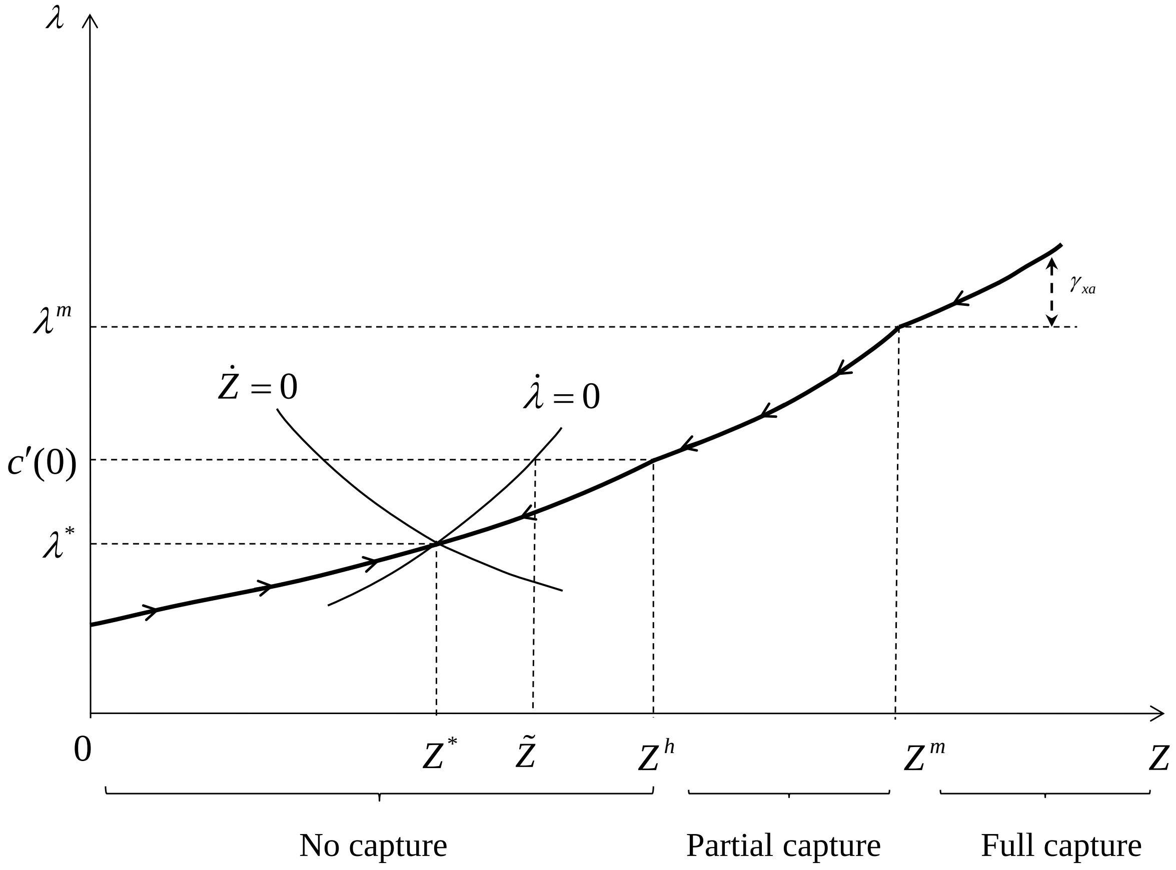

Figure 1. Phase diagram. Expensive CCS $\left(\tilde{Z} > Z^{*} \text{ or } c'(0) > \lambda^{*}\right)$ and an abundant non-renewable polluting resource $\left(X_0 = \infty \text{ or } \mu = 0\right)$.

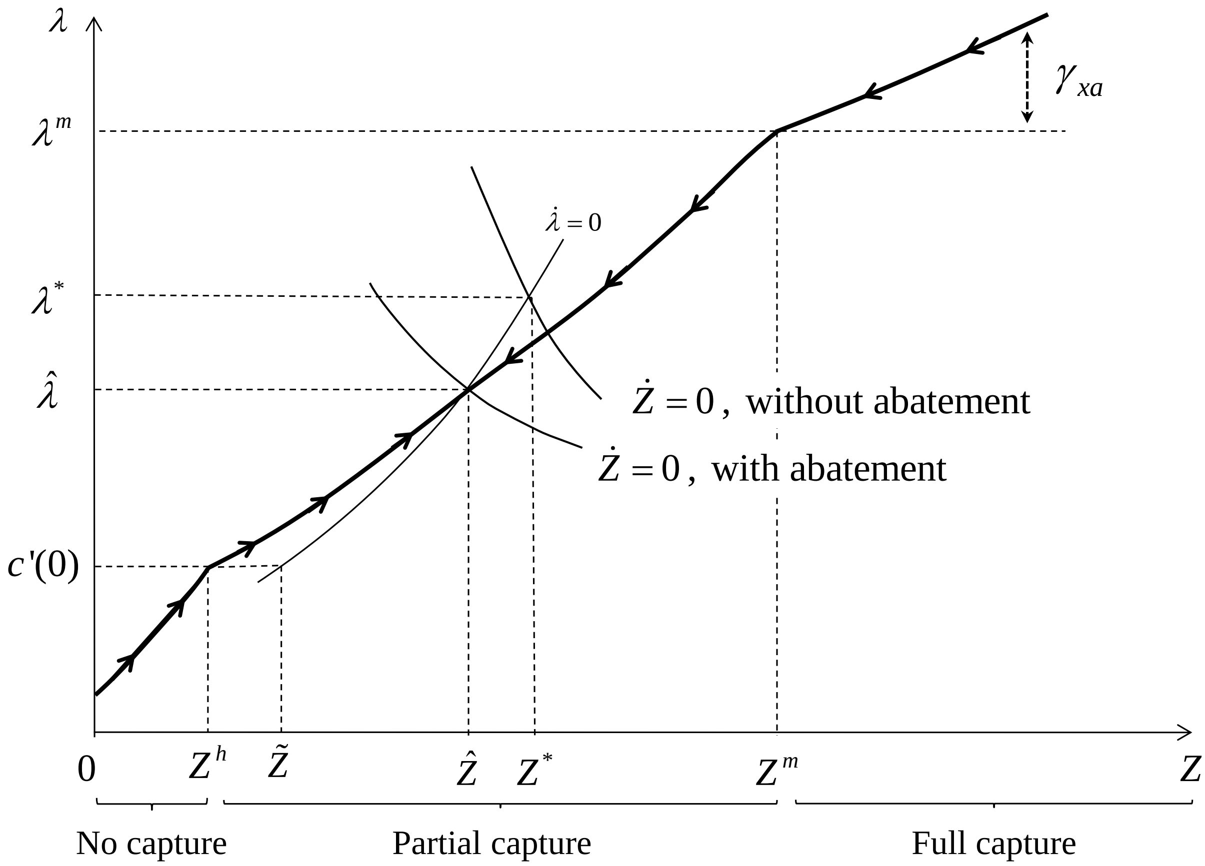

Figure 2. Phase diagram. Cheap CCS $\left(\tilde{Z} < Z^* \text{ or } c'(0) < \lambda^*\right)$
and an abundant non-renewable polluting resource $\left(X_0 = \infty \text{ or } \mu = 0\right)$.

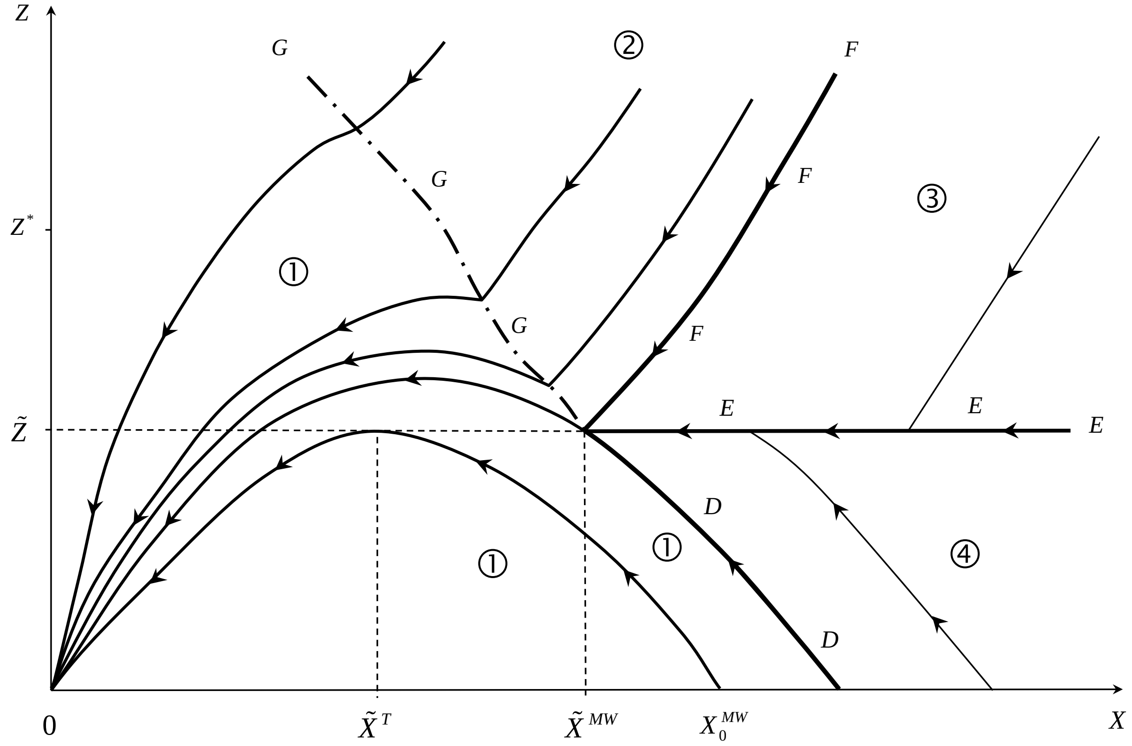

Figure 3. Phase diagram. Cheap CCS $\left(\tilde{Z} < Z^* \text{ or } c'(0) < \lambda^*\right)$ and a finite non-renewable polluting resource.

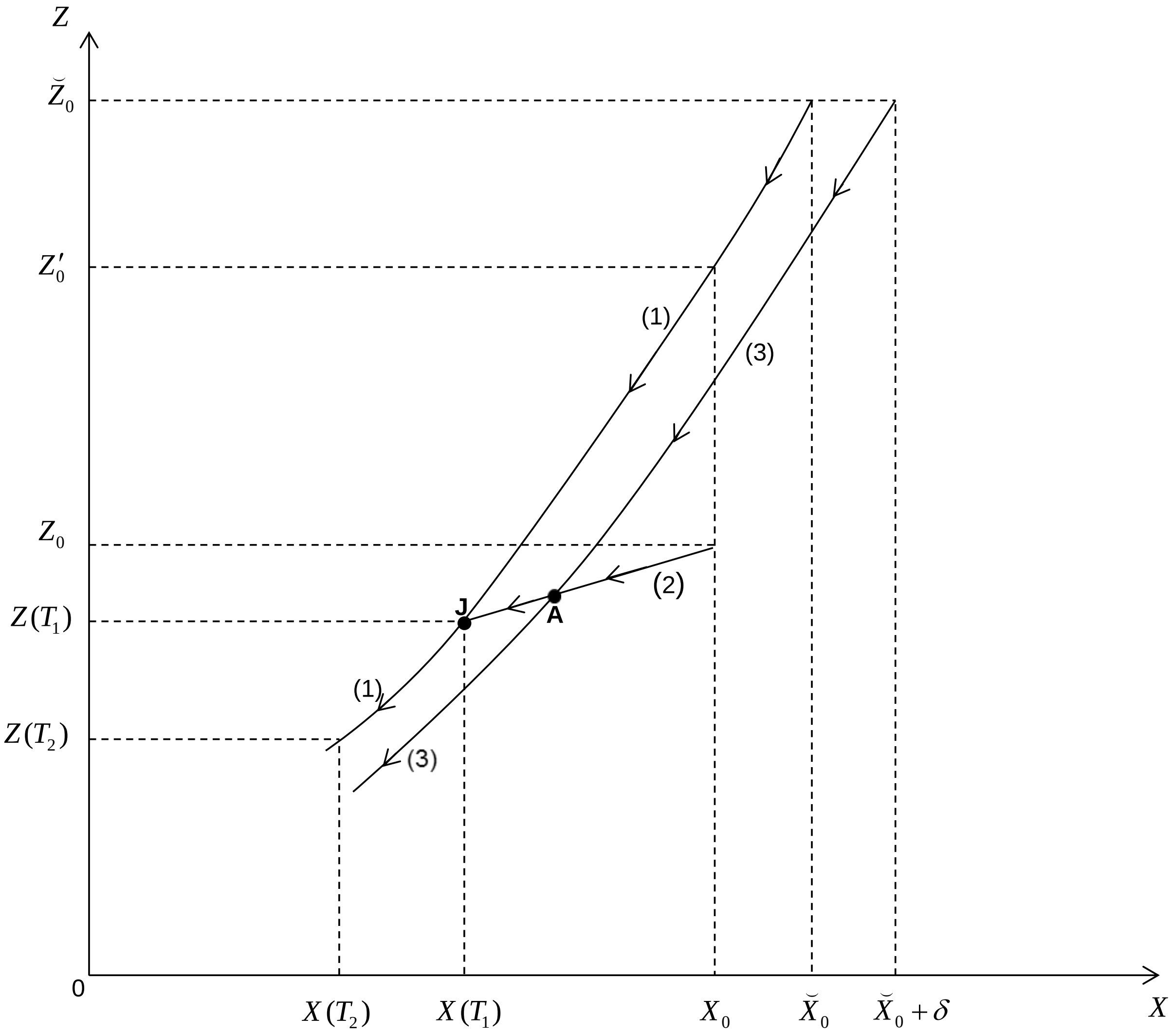

Figure 4. Illustration of lemma 2

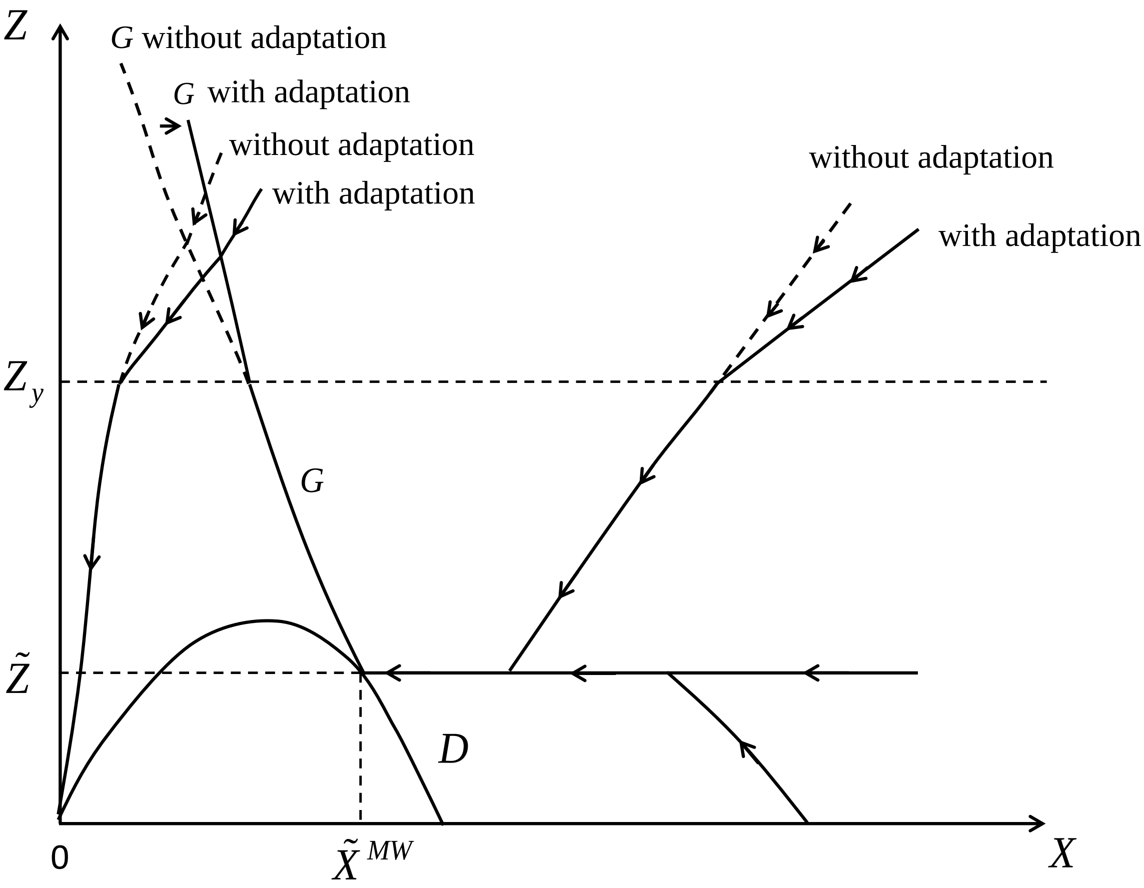

Figure 5. Phase diagram. Finite stock of the non renewable resource, low constant marginal capture costs and high adaptation costs.