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Controlling an infectious disease and sustainability: lessons from Hartwick's Rule

Can Askan Mavi – Benteng Zou

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Controlling an infectious disease and sustainability: lessons from Hartwick's Rule[*]

Can Askan Mavi[1] and Benteng Zou[2]

[1]*University of Paris-Saclay, INRAE, AgroParisTech, Paris-Saclay Applied Economics, 91120, Palaiseau*
[2]*DEM, University of Luxembourg, Luxembourg*

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Abstract

This paper tries to understand how a pandemic disease changes the relationship between different production factors such as labor, capital and natural resource extraction under maximin criterion, that consists of providing a constant utility to each generation over time. We show that the prevention policy plays an important role to implement an optimal and sustainable economy. We also show that a public policy such as lockdown or social distancing may decrease or increase the natural resource extraction, depending on the cost of the public policy. Understanding these opposite cases is crucial to know how to create a sovereign wealth fund (i.e, capital accumulation) that is composed by natural resource rents. Another important result is that in an economy facing a pandemic, an increase in natural resource extraction does not mean a direct increase in capital accumulation as documented in the existing literature. In this sense, we also challenge the well-known Hartwick rule's "all resource rents invested in capital accumulation" idea.

Keywords: Prevention policy, Maximin principle, Intergenerational equity, Pandemic, Hartwick's rule.
JEL classification: Q01, Q3, D8, Q56.

1   Introduction

Many countries have accumulated significant amounts of debt because of the COVID-19 crisis (Kose et al., 2021). Countries such as Norway that invest natural resource rents into man-made capital, however, have remained resilient against the COVID-19 crisis thanks to their sovereign wealth funds (SWFs), which have allowed them to avoid increasing their public debt rates (Bortolotti and Fotak, 2020). In this sense, the pandemic has intensified the debate around the use of wealth funds for disasters.[1] [2] The question that motivates this study is the following: How should an economy build a wealth fund (i.e., capital accumulation) that helps mitigate economic losses due to harmful events such as pandemics? What is the impact of a pandemic on the allocation of production inputs such as the capital, labor and natural resource? How should these production factors change with respect to the dynamic features of a pandemic, in a way that an optimal constant utility is ensured over time (i.e, intergenerational equity)?

A sovereign wealth fund (SWF) can be efficiently created using Hartwick's rule so as to sustainably manage national wealth (van der Ploeg, 2017). According to this famous rule pioneered by Hartwick (1977), if an economy invests the rents stemming from the extraction of natural resources into the net accumulation of physical capital, it follows an equitable and sustainable growth path by maximizing the utility of the most deprived one (maximin criterion) (Solow, 1974a). Evidence also shows that countries applying Hartwick's rule are wealthier than those not following it (Hamilton et al., 2005).

In the context of a pandemic, the question of why Hartwick's rule is still important is crucial. It is evident that the pandemic has caused a reallocation of production factors such as labor, natural resources, and capital (Gromling, 2021). Thus, with the pandemic shock being accompanied by productivity changes, it is crucial to understand how we should extract natural resources and invest the associated rents in man-made capital so as to sustain utility and build an efficient wealth fund. Therefore, it is essential to outline a pandemic-modified Hartwick rule that encompasses the dynamics of the pandemic outbreak. In this sense, a pandemic-modified Hartwick rule is shown to provide new prescriptions for how to build a sovereign wealth fund in the context of a pandemic.

A large strand of the literature, starting with Hartwick (1977), strives to understand the connection between Hartwick's rule and sustainable development by ensuring a constant utility level over time (Hartwick, 1978; Dixit et al., 1980; Buckholtz and Hartwick, 1989; Hamilton, 1995; Asheim et al., 2007; Martinet, 2007; D'Autume and Schubert, 2008; D'Autume et al., 2010; Hartwick and Long, 2018). Cairns and Long (2006) investigate the maximin case under uncertainty, with events that stochastically affect the evolution of stock variables. They show that a constant utility level cannot be guaranteed due to shocks that occur. In the same vein, Butterfield (2003) concentrates on the uncertainty regarding the future prices of extracted natural resources, presenting a version of Hartwick's rule with uncertainty. Van Long and Tian (2003) investigate Hartwick's rule where the uncertainty comes from international trade. However, the existing literature does not make any connection between a disease that considerably changes the labor supply and sustainability through Hartwick's rule.

To provide answers to the questions posed above, we recall the Dasgupta–Heal–Solow–Stiglitz (DHSS) model with exogenous technological progress in which we embed the simple epidemiological susceptible–infected (SIS) model. In the context of a disease, the labor productivity is supposed to decrease if the prevalence rate

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[1]See https://www.ft.com/content/46a6bdf4-c965-48ff-be58-820067b04e81
[2]see https://bit.ly/3sVkizv

of the disease increases over time (see Goenka et al. (2014); Bosi et al. (2021); d'Albis and Augeraud-Veron (2021)). Labor productivity depends on the qualitative features of the disease, such as the transmission rate, the recovery rate (or removal), and prevention policy, which consists of measures such as lockdowns, social distancing, vaccination, etc. We adopt a similar approach to Bosi et al. (2021) for the modeling of prevention policy, but the economic literature also includes many recent contributions with policy instruments to mitigate the impacts of COVID-19 (Nævdal, 2020; d'Albis and Augeraud-Veron, 2021; Barbier, 2021). Note that our study does not focus on the optimal level of the prevention policy and the prevention policy is considered as an exogenous policy.

The contribution of the current study is threefold. First, to the best of our knowledge this is the first rigorous framework seeking to understand the connection between sustainability and a pandemic disease. Our framework helps understanding how the allocation of production factors (capital and resource extraction) should change in a such optimal way that the utility over time (intergenerational equity) is ensured. We also show that the policymaker should implement a prevention policy to control the ratio of infected/susceptible individuals (i.e., the prevalence rate). Otherwise, without control, Hartwick's rule may not be implemented and utility is not sustained over time. In this sense, our study legitimates in an analytical way the implementation of prevention policies to control a pandemic.

The second contribution is that we present a possible new trade-off between public health policies (such as prevention) and capital accumulation (i.e, SWF) in the context of Hartwick's rule. The mechanism is as follows: When the policymaker implements a prevention policy, labor productivity increases. This productivity increase naturally leads to lower natural resource extraction in the case where the prevention policy is not very costly[3]. It follows that there is a lower amount of rents stemming from the extraction of natural resources invested in physical capital accumulation. This result shows that sovereign wealth funds composed of natural resources rents may be at risk (Bortolotti and Fotak, 2020).[4] In other words, a policymaker faces an important trade-off between constructing a wealth fund and protecting the public health by a prevention policy. Thus, to get rid of this trade-off, policymakers should redesign wealth funds to be built from other sources of revenue such as trade surpluses. Of course, the scope of this study passes beyond the redesign of the sovereign wealth fund and leaves this important point for further research.

The third contribution is that we challenge the Hartwick's conventional result, stating that the utility can be maintained at a constant level if the natural resource rents are invested in capital accumulation. We show that a prevention policy seeking to mitigate the negative impacts of the pandemic increases the natural resource extraction (if the cost of the prevention is sufficiently low). However, we show that the increase in natural resource extraction does not lead to a higher capital accumulation. The reason is that a higher resource extraction and increased labor productivity due to the prevention policy gives room to a lower capital accumulation to keep the utility constant at a constant level.

The remainder of the paper is organized as follows. Section 2 introduces the economy with the expected pandemic. Section 3 illustrates the outbreak of a pandemic, where a SIS model is solved analytically, and the economic model are presented. The main analysis and results are given in Section 4. Section 5 offers some concluding remarks. All proofs are relegated to the end of the paper, in the Appendix.

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[3]We explain how the cost of the prevention policy may lead to a higher or lower extraction more in detail in the main text.
[4]Another risk mentioned by Bortolotti and Fotak (2020) is the sharp decrease in oil prices. Our study abstracts from the price dynamics of oil.

2 The dynamics of a disease

We can model the dynamics of an epidemic with a SIS model which is quite appropriate for COVID-19 since the complete immunity is not possible and one may get infected many times. $S(t)$ and $I(t)$ represent the susceptible and infected people, respectively. The population $\bar{N}$ is assumed to be constant as in the baseline SIS model. The total number is given by

$$\bar{N} = S(t) + I(t) \tag{1}$$

and the share of the infected people in the society (i.e, prevalence rate) is given by

$$x(t) = \frac{I(t)}{\bar{N}} \tag{2}$$

We also integrate the prevention policy similar to Bosi et al. (2021).The regulator decides to apply a prevention policy (lockdown, vaccination or social distancing) which remains constant over time in order to mitigate (or eliminate) the epidemic. Denote $\lambda \in [0,1]$ which stands for the intensity of the prevention policy (such as the share of locked down people in the population). Of course, the regulator can also choose to apply a lock down to the whole population, in which case $\lambda = 1$.

$$(1 - \lambda) I(t) = (1 - \lambda) x(t) \bar{N} \tag{3}$$

$$(1 - \lambda) S(t) = (1 - \lambda) [1 - x(t)] \bar{N} \tag{4}$$

We assume that an individual meets v of people, meaning that an infected person meets $v(1 - x(t))$ susceptible people. The total number of meetings between susceptible and infected people is $v(1 - x(t))(1 - \lambda) I(t)$. When there is a contact between two individuals, there is a probability p of transmission of a disease. Therefore, the number of newly infected people is given by $pv(1 - x(t))(1 - \lambda) I(t)$. One may interpret λ as a policy parameter that helps to decrease the probability of a disease transmission, by referring to the term $p(1 - \lambda)$. The evolution of the number of infected people is given by

$$\dot{I}(t) = [\mu(1 - \lambda)[1 - x(t)] - r] I(t) \tag{5}$$

where $\mu = vp$ and r represent the transmission and recovery rates with μ and $r > 0$. After some simple algebra, we can describe the dynamics of the share of infected people $x(t)$

$$\dot{x}(t) = x(t) ([1 - x(t)] [\mu(1 - \lambda)] - r) \tag{6}$$

Assumption 1. $\mu > r$

Solving the equation (6) yields

$$x(t) = \frac{x_0 x_1}{x_0 + (x_1 - x_0) e^{\mu\left(\lambda - \left(1 - \frac{r}{\mu}\right)\right)t}}$$

where

$$x_1 = 1 - \left(\frac{r}{\mu(1-\lambda)}\right)$$

$$\lambda_1 = \left(1 - \frac{r}{\mu}\right)$$

Proposition 1. *There are two possible steady-states for the prevalence rate x^*; a disease-free steady state or an endemic steady state where there is a constant amount of infected people in the society.*

i) The disease-free steady state $x^{*,1} = 0$ if $\lambda > 1 - \frac{r}{\mu}$. In this case, $x(t)$ decreases and tends to the steady-state $x^{*,1}$ which is a globally stable.

ii) The endemic steady state $x^{*,2} \in [0,1]$ exists if $\lambda < 1 - \frac{r}{\mu}$. In this case, $x(t)$ increases (decreases) if $x_0 < x_1$ ($x_0 > x_1$) where x_0 is the initial number of the share of infected people in the total population.

Note that controlling the disease through a lockdown policy λ is necessary to increase the labor supply in the economy that we explain in detail in Section 2.1. In our epidemiological model, we abstract from population growth. Then, we do not analyze the impact of the over-mortality. The analysis of a variable population requires a different framework than what we use in order to analyze the impact of the disease and the prevention policy on production factors. The reason is that intergenerational equity with variable population requires specific population dynamics (see Asheim et al. (2021)).

2.1 The economic model

We suppose that labor supply changes depending on the prevalence rate of the disease. This specification is similar to d'Albis and Augeraud-Veron (2021). We suppose that the productivity of the labor decreases due to an increasing prevalence rate $x(t)$. We also consider a labor augmenting technological change. Then, we express the labor productivity as follows

$$L(t) = p(x(t)) \tag{7}$$

where $p(x(t))$ is a function that stands for the labor productivity that is proportional to the prevalence rate of the disease $x(t)$. In our specification, the impact of the pandemic disease is captured through the labor. For the sake of simplicity, we abstract from its impact on the capital destruction.

Assumption 2. *The labor productivity $p(x(t))$ is a decreasing function of the prevalence rate $x(t)$; $p(x(t)) > 0$, $p'(x(t)) < 0$*
i) $\lim_{x_t \to x^*} p(t) = \bar{L}_2$ where $\bar{L}_2$ is the level to which the labor converges when the economy reaches the endemic steady state level of the prevalence $x^{*,2}$.
ii) $\lim_{x_t \to x^*} p(t) = \bar{L}_1$ where $\bar{L}_1$ is the level to which the labor converges when the economy reaches the disease-free steady state level $x^{*,1}$.
iii) $\bar{L}_1 > \bar{L}_2$ implies that the labor supply in the disease-free steady state is always higher than in the endemic

steady-state.

Equation (7) implies that infected people are also supplying labor [5]. Moreover, the labor productivity of susceptible and infected individuals decreases with the prevalence rate. Assumption 2 indicates that the pandemic has a negative impact on production through a decrease in the labor supply. The prevention policy λ has a positive impact on the labor supply since it decreases the share of infected people in the population.

Note that another way of modeling the labor supply would be $L(t) = \bar{N} - I(t)$ as in Goenka et al. (2014); Bosi et al. (2021). However, with this specification, infected people (or asymptomatic infected) do not work. In the next subsections, we analyze the implications of the two different forms for the labor supply; first as in Goenka et al. (2014); Bosi et al. (2021) and also with a different functional form which is similar to d'Albis and Augeraud-Veron (2021). We show that our results are not sensitive qualitatively to the functional specifications and these two forms lead to same qualitative results.

In a world with a pandemic, the social planner aims to maximize the utility of the most "deprived" $\min u_t$ with the condition $u_t \geq \bar{u}$. Denote consumption as $C(t)$, and capital, non-renewable resources, labor and technological progress employed in production as $K(t)$, $R(t)$, $L(t)$ and $A(t)$, respectively. Consider output function $F(K(t), R(t), A(t) L(t))$ following a Cobb–Douglas form (Solow, 1974a):

$$Y = F\left(K\left(t\right), R\left(t\right), L\left(t\right)\right) = K\left(t\right)^{\alpha} R\left(t\right)^{\beta} \left(A\left(t\right) L\left(t\right)\right)^{\gamma} \left(1 - a\lambda\right),$$

where the term $a\lambda$ is the cost of the prevention policy (i.e, lockdown) that depends linearly on the prevention λ, in terms of the output loss. Parameters $\alpha, \beta, \gamma \in (0, 1)$ are the share of capital, natural resources, labor and technological progress, respectively, in the output. We assume that $\alpha + \beta + \gamma = 1$.

The technological progress is exogenous and labor augmenting. Also, the technological progress is labor-augmenting. To elaborate more on this, the current evidence regarding COVID-19 shows that labor supply after decreasing sharply at the very beginning of lockdowns,[6] increases significantly due to the accelerated digitalization and automation that allowed for remote work (Bloom et al., 2020; Chernoff and Warman, 2020; IMF, 2021; Petropoulos, 2021). One may think that the disease has only a transitory effect on the economy and the only thing that it matters is the technological progress in the long run. However, this is not true when there is a "positive" prevalence rate at the steady state. In next subsections, we will explain this more in detail. The labor-augmenting technological progress evolves as follows

$$\dot{A}\left(t\right) = \sigma A\left(t\right)$$

Hence, the technical progress can be found as $A(t) = A(0) e^{\sigma t}$. The policymaker's optimal control problem is similar to Van Long (1992), where the Bellman value function is defined as

$$\max_{C} V(K, S) = \int_0^{\infty} \rho \bar{u} e^{-\rho t} dt = \bar{u}, \tag{8}$$

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[5] Since we have $\bar{N} = S(t) + I(t)$, infected individuals are not excluded from the labor supply.

[6] The sharp decrease in labor productivity is mainly due to the lack of adaptation of many firms to remote work.

subject to

$$\dot{K}(t) = F(K(t), R(t), A(t) L(t)) - C(t), \tag{9}$$

$$\dot{S}(t) = -R(t), \tag{10}$$

$$U(C(t), x(t)) \geq \bar{u}, \tag{11}$$

$$\lim_{t \to \infty} S(t) = 0, \tag{12}$$

$$S(0) = S_0 > 0, \tag{13}$$

$$K(0) = K_0 > 0, \tag{14}$$

$$A(0) = A_0 > 0 \tag{15}$$

where positive parameter ρ is the pure rate of time preference. Equations (9) and (10) hold for the dynamics of capital accumulation and natural resource stock. Equation (11) is the condition that is to maximize the utility of the poorest individual (generation). K_0 , S_0 and A_0 are the initial conditions for the capital, natural resource stock and technical progress.

3   Analysis

Is it possible to sustain the utility when there exists a pandemic disease? Which kind of disease dynamics makes an economy unsustainable? What is the impact of a lockdown policy on the natural resource extraction and capital accumulation dynamics? In the next subsections, we seek to provide rigorous answers to these questions.

Besides the capital and resource stock dynamics, there are two different dynamic variables A and x to be taken into account in the maximization program. This means that one should consider two different additional state variables. However, incorporating the analytical forms of A and x in the model and defining the time variable t as a state variable allows to reduce the dimension of the system.

We reformulate the production function in the following way by incorporating the analytical forms of of A and x. In order to keep the notation simple, we define a function $f(t)$ that encompasses the time variable explicitly since we know the analytical form of $x(t)$ and the technological progress $A(t)$

$$F(K(t), R(t), A(t) L(t)) = K(t)^{\alpha} R(t)^{\beta} (f(t))^{\gamma} (1 - a\lambda),$$

where

$$f(t) = A(t) L(t)$$

In order to solve the optimization problem presented in the previous section, we employ the same argument as Cairns and Long (2006). First, we translate the non-autonomous system into an autonomous one since the function $f(t)$ implies the time t explicitly . To do so, we take time as a state variable by defining variable $W(t)$ as

$$W(t) = t.$$

Thus,

$$\dot{W} = 1$$

with initial condition $W(0) = 0$. Then, the Hamiltonian of the planner is defined as

$$\mathcal{H} = -p_1 R + p_2 \left(K^\alpha R^\beta \left(f\left(W \right) \right)^\gamma - C \right) + p_3,$$

where p_i $(i = 1, 2, 3)$ are costate variables of natural resources, capital, and time, respectively.

Assumption 3. *The utility depends positively on consumption and negatively on the prevalence rate $x\left(t \right)$.*

i) $U_C\left(C, x \right) > 0$ and $U_x\left(C, x \right) < 0$

We assume that the social planner cares about the prevalence rate and considers that a high prevalence rate has a negative impact on the society(see Bosi et al. (2021), Desmarchelier et al. (2021)). La et al. (2020) also takes into account the prevalence rate as a social cost and seek to minimize the social cost by optimally using the tax incomes. Goenka and Liu (2010) also takes into account the share of susceptible individuals in the utility function.

In this sense, in order to ensure a constant egalitarian utility across generations, the social planner should provide an increasing consumption to individuals when the prevalence rate increases and vice versa. Given the inequality constraint $U\left(C, x \right) \geq \bar{u}$, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{H} + \Phi \cdot \left(U\left(C, x \right) - \bar{u} \right),$$

with Φ being the Kuhn–Tucker multiplier.

The first-order necessary condition for the choice variables (where the second-order condition holds as well) yields that

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial R} = -p_1 + p_2 F_R = 0, \\ \frac{\partial \mathcal{L}}{\partial C} = -p_2 + \Phi U^{'}\left(C, x \right) = 0, \\ \Phi \geq 0, \quad \Phi \cdot \left(U(C, x) - \overline{u} \right) = 0. \end{cases} \tag{16}$$

Obviously, the first equation

$$p_1 = p_2 F_R \tag{17}$$

states that the in situ price of a natural resource, p_1, is determined by its rental value employing $p_2 F_R$, which is the product of the rental price of capital p_2 and the marginal output of resource F_R. While the rental price of capital p_2 is given by the second equation

$$p_2 = \Phi U^{'}_C(C, x), \tag{18}$$

which is indeed the optimal value of the objective function coming from employing an extra unit of capital. Furthermore, this value is given by the product of the marginal utility and the shadow value, Φ. Thus, the rental value p_2 takes into account the trade-off between the minimum level of utility, $\bar{u}$, and its consequences on the marginal utility $U'(C)$.

Implicitly, as long as the rental price $p_2 > 0$, which should hold for $\forall t$, we must have

$$\Phi > 0, \tag{19}$$

given $U_C'(C, x) > 0$. In other words, along the maximin optimal trajectory, it is necessary that

$$U(C, x) = \bar{u}. \tag{20}$$

Furthermore, the first-order condition with respect to the three state variables S, K, and W, yields the following dynamics of the co-state variables:

$$
\begin{cases}
\dot{p}_1 = 0, & (a) \\
\dot{p}_2 = -p_2 F_K, & (b) \\
\dot{p}_3 = -p_2 \gamma \frac{f'(W)}{f(W)} Y - \Phi U_x x'(W), & (c) \\
-p_1 R + p_2 \dot{K} + p_3 = \mathcal{H} = 0, & (d)
\end{cases}
\tag{21}
$$

with transversality conditions $\lim\limits_{t \to +\infty} p_3 = 0$ and $\lim\limits_{t \to +\infty} e^{-\rho t} p_2(t) K(t) = 0$.

The first transversality condition simply indicates that the shadow value of time p_3 vanishes when $t \to +\infty$. The second transversality condition is a standard one stating that the discounted far future value of capital is zero.

Importantly, equation (a) states that the in situ price p_1 is constant over time for any $\forall t$. Equation (b) indicates that the growth rate of the shadow value of capital, $\frac{\dot{p}_2}{p_2}$, is negatively related to the marginal product of capital. Thus, given $p_1 = p_2 F_R$, the growth of marginal product, F_R, from employing natural resources must exactly compensate the growth of price p_2 in order for the product, $p_2 F_R$, to be constant over time.

The dynamic value of time, p_3, is determined not only by the production and rental price of natural resources but also by the transmission and infection of the pandemic disease, given the time explicitly entering the production process through the impact on labor supply that decreases due to the pandemic.

With the above preparation, we are ready to find the maximum achievable utility level, $\bar{u}$. To do so, we first propose special candidate solution inspired from Cairns and Long (2006) and then checking the necessary conditions for optimality. In the second part, we consider case by removing the assumptions we use for the special candidate solution. Unfortunately, we just offer a steady-state analysis since a complete analytic solution of the model is not possible. We also refer to a numerical analysis to get a complete understanding of the model.

To sum up, in the next subsections, we treat two different cases:

- *Special candidate solution*: we propose a special candidate solution that ensures a constant achievable utility across generations. In this section, the results are analytic and aim to show that controlling a disease by a prevention policy λ is the key in order to ensure a sustained utility over time across different generations. Due to the functional forms for the analytical tractability, we cannot analyze the disease-free steady and leave this for the general case.

- *General case* : we treat a general model without assumptions made on parameters to have analytical results for the special candidate solution. We also use other functional forms that allows us to analyze a disease-free economy. However, analytical results are impossible to obtain. Hence, we do a steady-state analysis and refer to a numerical exercise in order to confirm the insights we have from the special candidate solution.

3.1   A special candidate solution

The special candidate is defined as follows: (a) take $K(t) = K(0)$ along the optimal path, $\forall t$, thus $\dot{K} = 0$. In the general case, we relax this assumption and offer a general analysis. Hence, we have the equality $p_1 R = p_3$ that comes from (d) in equation (21). We provide a proof to show that the optimal solution is consistent with the constant utility in Appendix (A.1). We assume that the utility $U(C, x) = Cx^{-\epsilon}$. The utility increases with the consumption and decreases with the the share of infected people in the society. We define the labor productivity $p(t)$ as follows

$$p(t) = \frac{\bar{L}}{(x(t))^\phi}$$

The share of infected people in the society $x(t)$ (i.e, prevalence rate) decreases the labor productivity. For the candidate solution, we only treat the endemic steady state $x^* \in (0, 1)$ due to the functional form we choose for the analytical tractability. We exclude the disease-free steady-state since the labor supply would go to the infinity. This assumption is plausible form many diseases such as COVID-19 since a complete immunity against the disease is not possible and the disease never disappears even in the long run[7] . We define the value of $\bar{L}$ with respect to the initial value of the prevalence rate x_0 as follows

$$\begin{cases} p(t) = \frac{L_0(x_0)^\phi}{(x(t))^\phi} & \text{with } \bar{L} = L_0(x_0)^\phi \text{ if } x_0 < x^* \\ p(t) = \frac{L_0(x^*)^\phi}{(x(t))^\phi} & \text{with } \bar{L} = L_0(x^*)^\phi \text{ if } x_0 > x^* \end{cases}$$

This specification is important in order to scale the labor productivity such that it does not exceed its no-pandemic level $\bar{L}$ since $0 < x(t) < 1$.

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[7]https://www.nature.com/articles/d41586-021-00396-2
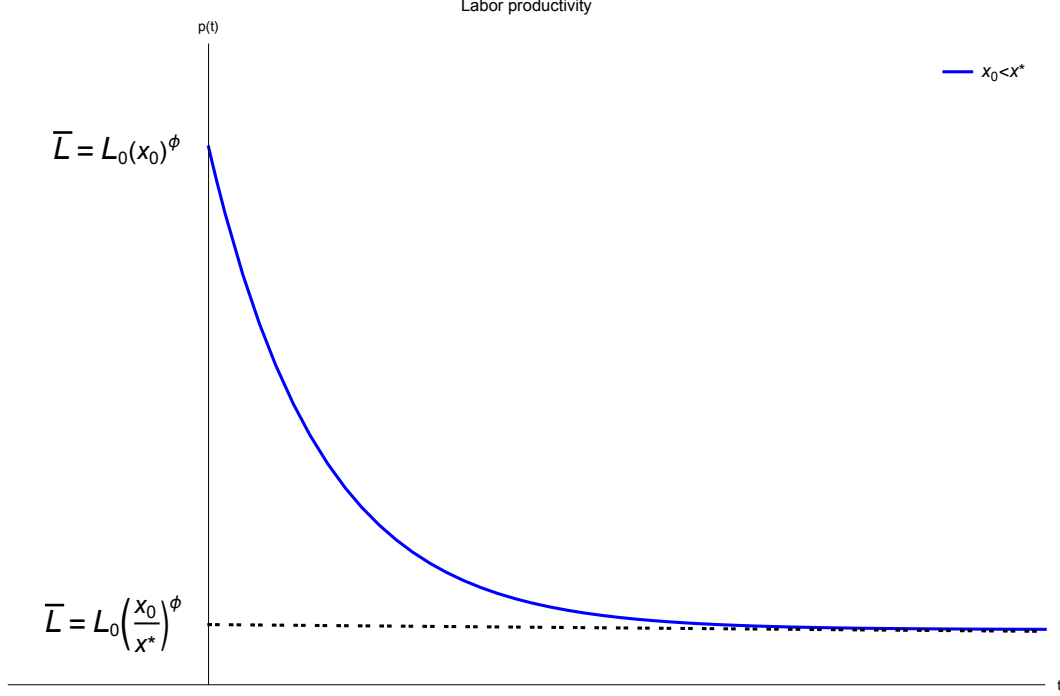
Figure 1: Labor productivity $p(t)$ over time with $x_0 < x^*$

We can reformulate the dynamics of the natural resource extraction $R(t)$ by using the first-order condition (17) as follows

$$\frac{\dot{p_3}}{p_3} = \frac{\dot{R}}{R} = -\frac{\gamma}{\beta}\left(\frac{f'(W)}{f(W)}\right) + \frac{\epsilon}{\beta}\left(1 - \frac{x_0}{x_0 + (x_1 - x_0)e^{-\mu(\lambda-\lambda_1)W}}\right)\mu(\lambda - \lambda_1) \tag{22}$$

where $W = t$

$$\frac{f'(W)}{f(W)} = \left(\sigma + \phi\left(1 - \frac{x_0}{x_0 + (x_1 - x_0)e^{-\mu(\lambda-\lambda_1)W}}\right)\mu(\lambda - \lambda_1)\right) \tag{23}$$

To solve the differential equation (22), we assume $\frac{\gamma}{\beta} = \tilde{a}$ where $\tilde{a}$ is an arbitrary constant. Using $W(t) = t$, the analytic solution of the differential equation (22) is as follows

$$R(t) = \frac{e^{-\sigma t}\left(x_0 + (x_1 - x_0)e^{\mu(\lambda-\lambda_1)t}\right)^{-\left(\phi+\frac{1}{\beta}\right)}R(0)}{(x_1)^{-\left(\phi+\frac{1}{\beta}\right)}} \tag{24}$$

From the production function Y at $t = 0$, we can express the natural resource extraction at $t = 0$ when $x_0 < x^*$

$$R_0 = (\bar{u})^{\frac{1}{\beta}}(x_0)^{\frac{\epsilon}{\beta}}K_0^{-\frac{\alpha}{\beta}}(A_0 L_0)^{-\frac{\gamma}{\beta}}(1 - a\lambda)^{-\frac{1}{\beta}}$$

For the sake of simplicity, we assume $A_0 = 1$. In order to ensure that the candidate solution checks the

11

first-order conditions, we do a consistency check and give some conditions in Appendix (A.1). One should also guarantee that the natural resource extraction follows a decreasing path. Otherwise, an optimal solution cannot be implemented.

Proposition 2. *The policymaker should control the disease by ensuring $\lambda > (1 + \beta\phi)(\mu - r) - \beta\sigma$ in order to sustain the utility. Otherwise, the economy is not on an optimal sustainable path.*

Proof. See Appendix A.2

This result shows at which extent controlling a disease by a prevention policy[8] from the very beginning of the disease outbreak is crucial to ensure a constant and sustained utility to different generations. Thus, this proposition offers a theoretical basis on which to defend the "whatever it costs" strategy, even though it has an economic cost in terms of an output loss (see d'Albis and Augeraud-Veron, 2021 for a similar discussion). The next question to be asked is: what is the maximum achievable utility level? To answer this question, we first should find the analytical value of the maximum achievable constant utility. From the constraint $S_0 = \int_0^\infty R(t)\,dt$, we find the total stock S_0 as follows (see Appendix A.3)

$$S_0 = R(0) \frac{x_1}{\sigma x_0} \tau(.) \tag{25}$$

where

$$\tau(.) = {}_2F_1\left(1, 1 - \frac{1}{\beta} - \phi - \frac{\sigma}{\mu(\lambda - \lambda_1)}; 1 - \frac{\sigma}{\mu(\lambda - \lambda_1)}; -\frac{(x_1 - x_0)}{x_0}\right) \tag{26}$$

Once we have the analytical expression of the initial natural resource stock S_0, we can express the maximum achievable utility $\bar{u}$.

Proposition 3. *The maximum achievable utility $\bar{u}$*

$$V(\lambda) = \bar{u} = x_0^{-\epsilon} Y = x_0^{-\epsilon} K_0^\alpha \left[\frac{\sigma x_0 S_0}{x_1 \tau(.)}\right]^\beta (L_0)^\gamma (1 - a\lambda). \tag{27}$$

In order to have a positive natural resource stock $S_0 > 0$ and a positive maximum achievable utility $\bar{u}$, one may remark that we should ensure the condition $\lambda > (1 + \beta\phi)(\mu - r) - \beta\sigma$. Otherwise, the term $\tau(.)$ becomes negative. The implicit expression for the optimal level of the prevention policy may be expressed such as $\lambda^* = \arg\max_{\lambda \in (0,1)} V(\lambda)$. The analytical form does not yield unfortunately an explicit solution for the optimal λ^*. Note that the scoop of the paper is not the optimal level of the prevention policy.

Proposition 4. *The prevention policy λ decreases or increases the natural resource extraction depending on the cost of the prevention policy.*

Proof. see Appendix (A.4).

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[8]It is important to highlight that we do not consider very high values of λ such that the prevalence rate goes to zero, due to the functional forms we choose. Of course, in the general case, we do not need a such restriction.
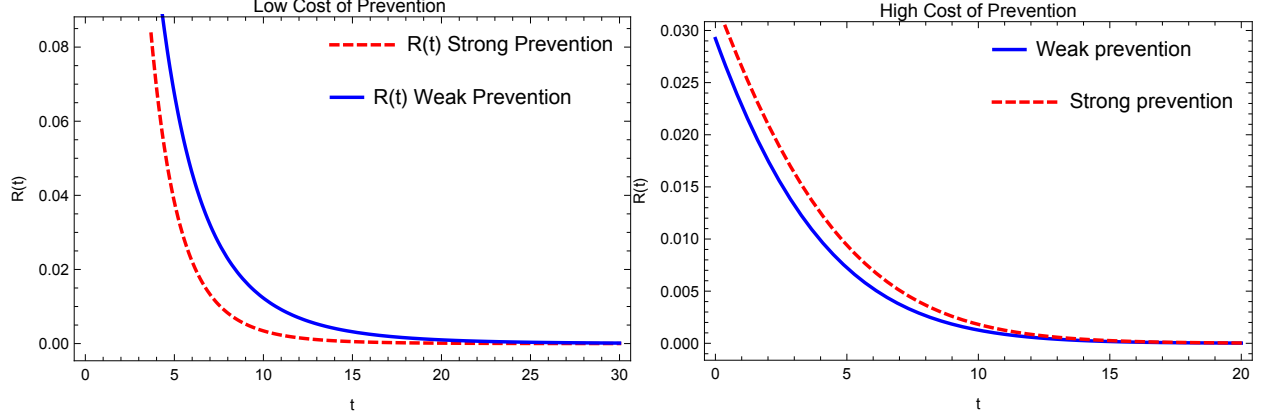
Figure 2: The impact of the prevention policy λ on the resource extraction $R\left(t\right)$ with a low (left) and high cost (right)

Figure (2)[9] shows that the prevention policy may decrease or increase the natural resource extraction $R\left(t\right)$. On the one hand, the prevention policy λ increases the labor supply since it decreases the prevalence rate x and this leads to a decrease of natural resource extraction. The reason is that the increase in labor supply substitutes for the natural resource extraction. In other words, with a higher labor productivity thanks to the prevention policy, the economy needs less natural resource to sustain the utility constant over time.

On the other hand, a stronger prevention policy leads to a higher output loss. Hence, the decrease in output due to the cost of the prevention is higher. Then, to compensate the decrease in production (so in utility), the natural resource extraction may be higher relative to a weaker prevention policy case (see the right panel on Figure (2)). The total impact of the prevention policy on the utility depends on the level of the unit cost of prevention a. We give analytically a threshold for the cost above which the natural resource extraction increases with a stronger prevention policy in Appendix A.4.

This result stipulates that the pandemic leads to a reallocation of production factors such as labor and natural resource. In addition, as one may remark, prevention policy's impact on the natural resource extraction is not straightforward.

3.2 General case

In this section, we propose a different functional form for the labor supply similar to Goenka et al. (2014) and Bosi et al. (2021) which allows us to do a disease-free analysis. We first reformulate the labor supply as a function of the infected/total population ratio (i.e, prevalence rate)

$$L\left(t\right) = S\left(t\right) = \bar{N} - \eta I\left(t\right) \tag{28}$$

where η is the rate for the infected people who cannot work remotely. If $\eta = 1$, All infected people are excluded from the labor supply. On the contrary, if $\eta = 0$, then all infected people can work remotely. In this study, we assume $0 < \eta < 1$. Reformulating equation (28) as a function the share of infected individuals

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[9]The numerical values are $a = 0.1$, $r = 0.1$, $\mu = 0.3$, $\alpha = 0.25$, $K_0 = 0.2619$, $\beta = 0.22$, $\phi = 1$, $\sigma = 0.5$, $\epsilon = 1$. The values of β and K_0 are found respecting the parameter restrictions given in Appendix A.1.We consider $a = 0.05$ as a low cost and $a = 1$ as a high cost of prevention. $\lambda = 0.35$ is for high prevention and $\lambda = 0.3$ low prevention

in the society (i.e, prevalence rate) $x(t)$ yields

$$L(t) = \bar{N}\left(1 - \eta\frac{I(t)}{\bar{N}}\right) = \bar{N}(1 - \eta x(t)) \tag{29}$$

The reason why we do not use this specification for the special candidate solution is that the analytical solution is not a possible outcome. With the current form in the general case, we have the same insights as in the special candidate solution case, that are also approved by a numerical exercise. We also make use of another utility function in order to make a disease-free analysis[10]

$$U(C, x) = C(1 - x)$$

Then, the utility is equal to C if the we have disease-free steady state.

The pandemic-modified Hartwick rule

Hartwick's rule states that if the economy invests the rents obtained from the use of natural resources into man-made capital, the economy stays on a sustainable, constant path of utility, which implies equity across generations. In other words, if the economy is on a sustainable path, the genuine savings are always zero and the utility level is maintained as constant.

Under the current setting, a new pandemic-modified Hartwick rule can be obtained. To do so, we rely on the shadow values from the above analysis. We rename the shadow values p_1 and p_3 in terms of the shadow price of capital, p_2, as

$$q = \frac{p_1}{p_2} \text{ and } q_z = \frac{p_3}{p_2}.$$

Furthermore, from (d) in (21), the pandemic-modified Hartwick rule, $\mathcal{H} = 0$, can be rewritten as (dropping the time index)

$$\dot{K} = \frac{p_1}{p_2}R - \frac{p_3}{p_2} = qR - q_z. \tag{30}$$

Obviously, without a pandemic disease, it follows that $\dot{K} = qR > 0$. The rule shows that the presence of the pandemic may affect negatively the capital accumulation. However, this effect is not straightforward since it is important to understand the impact of the pandemic on the natural resource extraction over time $R(t)$.

We can now present the dynamics of the system under maximin in the general case, instead of the constant capital scenario. Denote $P(t) = S(t)(A(t)L(t))^{\frac{\gamma}{\beta}}$ as a deflated variable (see Appendix A.6 for details) in

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[10]The reason we do not use the functional form for the special candidate solution is that when the prevalence rate $x(t)$ goes to zero, the utility goes to infinity.

order to have an autonomous system. Then, the dynamic system becomes the following:

$$\begin{cases} \dot{x} = x \left([1-x] \left[\mu \left(1 - \lambda \right) - r \right] \right), \\ \dot{K} = \beta Y - q_z, \\ \dot{P} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} \left(1 - a\lambda \right)^{-\frac{1}{\beta}} + \frac{\gamma}{\beta} \left(\sigma - \left(\frac{\eta \dot{x}}{1-x} \right) \right) P, \\ \dot{q}_z = -\gamma \left(\sigma - \frac{\dot{x}}{1-x} \right) Y + r q_z + \left(\frac{\eta \dot{x}}{1-x} \right) \frac{\bar{u}}{1-x}. \end{cases} \tag{31}$$

where $r = \alpha \frac{Y}{K}$. Unfortunately, an analytical solution becomes impossible for this general case. Thus, we rely on numerical analysis to illustrate the main ideas.

Remark 1. *To have an optimal and sustainable economy, at each date t before converging to the steady-state, we should have $\frac{\dot{x}}{1-x} < \frac{\beta}{\gamma} \left[\sigma - \frac{\dot{P}}{P} \right]$ in order to ensure $\dot{S} < 0$. Otherwise, $\dot{S} > 0$ at some point in time, which violates the optimality since it is not possible to have $\dot{S} > 0$ due to the existence of a finite resource stock.*

From the variable $P(t) = S(t) \left(A(t) L(t) \right)^{\frac{\gamma}{\beta}}$, its time derivative gives $\frac{\dot{S}}{S} = \frac{\dot{P}}{P} + \frac{\gamma}{\beta} \frac{\dot{x}}{1-x} - \sigma$. For $\forall t$, $\frac{\dot{S}}{S} < 0$. This requires that there should be some control on the dynamics of the disease through the prevention policy λ since a higher increase in the prevalence rate $x(t)$ may violate the condition $\frac{\dot{S}}{S} < 0$. Unfortunately, we cannot analytically express the prevention policy λ to control the disease in order to ensure the optimality as in the previous section. However, the remark is useful to confirm the idea that the social planner should think of controlling the disease in order to ensure a constant optimal utility over time to each generation.

As mentioned before for the special candidate solution, a stronger prevention policy may decrease or increase natural resource extraction depending on the cost of the prevention. the prevention (in terms of the output loss) is sufficiently low, natural resource extraction decreases due to a higher labor supply. In other words, the prevention policy plays the role of an increase in productivity and substitutes for natural resource extraction.

When the cost of the prevention is sufficiently high, the increase in labor productivity cannot compensate the output loss. This is the reason why the natural resource extraction increases in order to maintain the utility constant over time. The following figure[11] shows the optimal extraction of natural resources over time

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[11]For the simulations we present, we use following numerical values $a = 0.1$, $r = 0.1$, $\mu = 0.25$, $\eta = 0.9$ $\alpha = 0.45$, $K_0 = 0.2619$, $\gamma = 0.15$ $\beta = 0.4$, $\phi = 1$, $\sigma = 0.5$, $\lambda = 0.35$, $\epsilon = 1$. High cost of prevention is $a = 1$. High prevention $\lambda = 0.04$. Low prevention $\lambda = 0.01$.
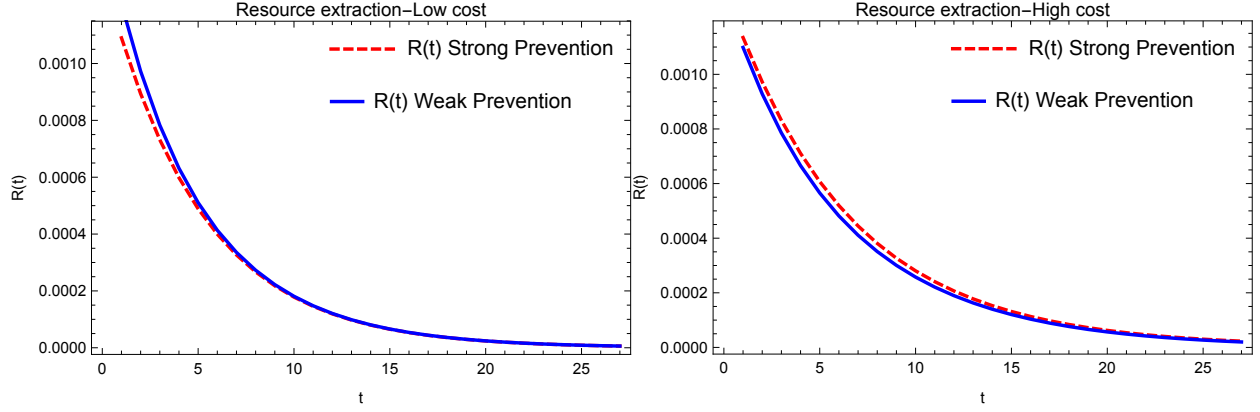
Figure 3: The impact of the prevention policy λ on the resource extraction $R(t)$ with a low (left) and high cost (right)

The findings in Figure (3) are consistent with the special candidate solution. Depending on the cost of the prevention policy, the optimal extraction path increases or decreases with the prevention policy λ. However, as it is shown in the Appendix A.6, the deflated long-term stock level P is always higher in the case of a strong prevention policy. In the next section, we present the disease-free steady state where $x^* = 0$

The optimal dynamics of the physical capital stock $K(t)$ are illustrated as follows;
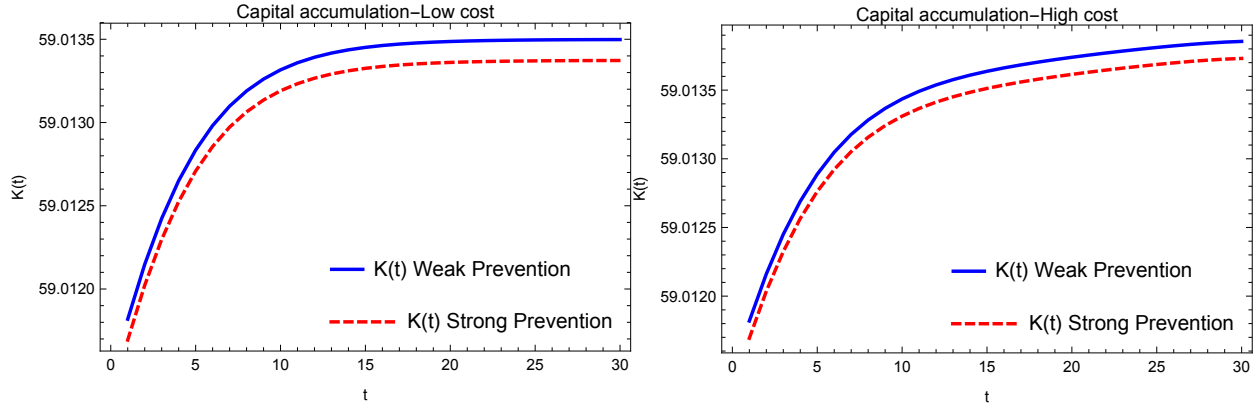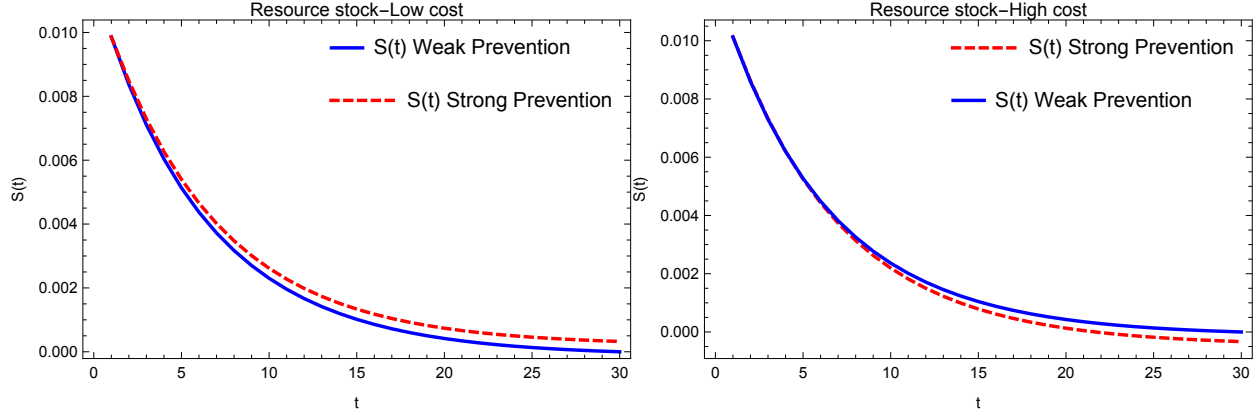

Figure 4: Strong prevention ($\lambda = 0.15$) (red) vs. weak prevention ($\lambda = 0.1$) (blue)

Figure 4 describes the optimal dynamics of capital stock K. A higher prevention policy decreases the physical capital accumulation. This fact can be understood through a lower natural resource extraction with a higher prevention policy that can be seen in Figure (2) (see left panel with low cost).

However, even though with a higher extraction rate, the capital accumulation is lower. This result implies that the conventional Hartwick rule stating that natural resource extraction feeding the capital accumulation is not always true in our model. One can understand this through the pandemic-modified Hartwick rule which is $\dot{K}(t) = qR(t) - q_z(t)$. The relative shadow price of time $q_z(t)$ has a positive sign and this makes possible a disconnection between natural resource extraction and capital accumulation. In this sense, the trade-off highlighted in the existing literature between natural resource extraction and capital accumulation (i.e, sovereign wealth fund) is irrelevant as shown in our configuration.

16

Overall, keeping utility constant over time is possible via the combination of variable capital and exploiting, instead of only constant capital as in the previous section.

Figure 5: The impact of the unit cost $a(\lambda = 0.15)$ (red) vs. weak prevention ($\lambda = 0.1$) (blue)

We express the steady-state of the economy

$$\begin{cases} K^* = \frac{\alpha\beta\bar{u}}{\sigma(1-x^*)}, \\ P^* = \frac{\beta\left[\frac{\alpha\beta\bar{u}(x^*)^\epsilon}{\sigma}\right]^{-\frac{\alpha}{\beta}}(1-a\lambda)^{-\frac{1}{\beta}}\bar{u}}{\sigma(1-x^*)}, \\ q_z^* = \beta C. \\ x^* = 1 - \frac{r}{\mu(1-\lambda)} \end{cases} \tag{32}$$

Proposition 5. *The prevention policy λ decreases on the one hand the steady-state level of the physical capital K^* and increases or decreases on the other hand the steady-state level of the deflated variable P^* defined for the natural resource stock, depending on the unit cost of the prevention policy a.*

Proof. See Appendix 5

It is easy to remark that the steady-state level of the physical capital decreases with a higher level of prevention policy λ. This is because the prevention policy λ increases the labor supply and hence decreases the resource extraction. Then, this leads to a lower capital accumulation.When the cost of prevention is high, the resource stock may exhaust more rapidly because the output loss due to the prevention is sufficiently high in this case and to keep the utility $\bar{u} = C(1-x)$, natural extraction should increase to keep the utility at a constant level.

It is important to remark that the natural capital converges to infinity and the deflated pollution stock P converges to zero (see (32)) if $\sigma \to 0$. This is exactly what happens in the classical maximin framework à la Solow Solow (1974b) without technical progress. Also, D'Autume and Schubert (2008) study the maximin case with technological progress and show that capital does not need to converge to infinity but to a constant value under technological progress. Differently from D'Autume and Schubert (2008), we analyze the impact of an epidemic disease on the capital, labor supply and natural resource extraction during the transition and in the long-run. In the next section, we present the disease-free steady state where $x^* = 0$

3.3 Disease-free economy

The steady-state of this economy can be easily written as

$$
\begin{cases}
K^* = \frac{\alpha\beta\bar{C}}{\sigma}, \\
P^* = \frac{\beta[K^*]^{-\frac{\alpha}{\beta}}(1-a\lambda)^{-\frac{1}{\beta}}\bar{C}}{\sigma}, \\
q_z^* = \beta\bar{C}. \\
x^* = 0
\end{cases}
\tag{33}
$$

Even though the disease ends and the prevalence rate goes to zero in the long-run, there is a cost of the prevention in terms of an output loss. To compensate the loss, there is a higher natural resource extraction during the transition.
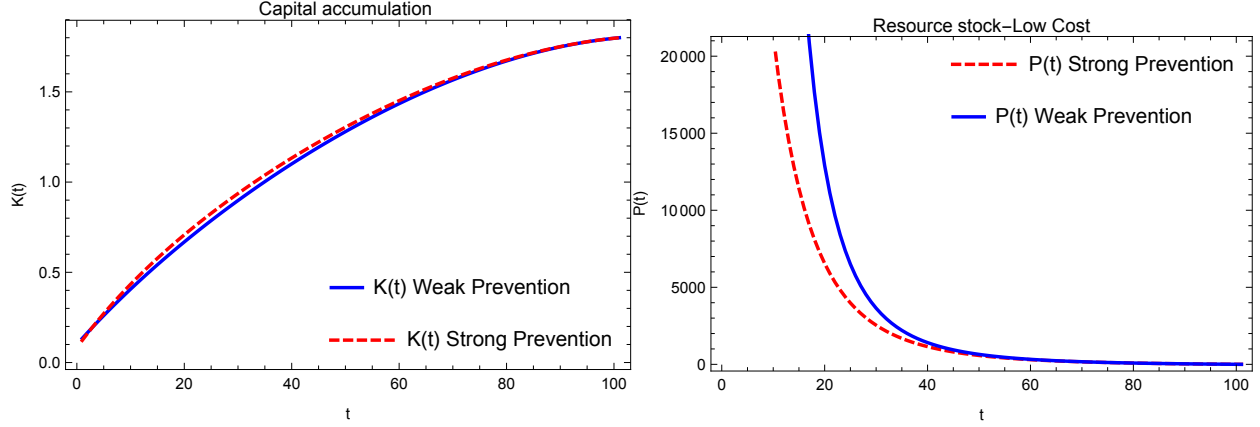
Figure 6: Optimal transition dynamics with the disease-free case

Numerical findings in Figure (6)[12] are also in line with the results in previous sections. We see that in the long run, weak and strong prevention policies leads to the same steady-state for the the capital accumulation since there is any disease in the long-run. However, with a stronger prevention policy, capital accumulation is higher during the transition. Also, a strong prevention policy leads to an accelerated natural resource extraction as it may be seen in the right panel in Figure (6) for the same reasons explained in previous sections.

4 Conclusion

This paper illustrates a sustainable economy that follows an equitable growth path in an economy facing a pandemic. In the first part, we analytically present an economy under the maximin criterion in order to understand the implications of the prevention policy for the sustainability. In this matter, we try to understand how the prevention policy qualitatively affects the natural resource extraction. We show that the impact is not straightforward; depending on the cost of the prevention, natural resource extraction may increase or decrease. The insights we offer for the special candidate solution is also confirmed by a general

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[12]The only different parameters relative to the general case is $\lambda = 0.8$ such that we have a disease-free steady-state.

analysis which relies on numerical results. Understanding the impact of the prevention policy is important when a policymaker designs a sovereign wealth fund.

An important finding in this study is that there exists a trade-off between the public health policy (i.e, prevention) and the construction of a wealth fund (i.e, capital accumulation) since it is shown that a strong prevention policy increases the labor productivity and decreases the exploitation of natural resources when the cost of the prevention is sufficiently low. There are two important channels to understand why the capital accumulation decreases with a strong prevention policy. First, the capital accumulation decreases due to a lower exploitation of natural resources (rents from the natural capital). However, even though the natural resource extraction increases, the physical capital accumulation decreases with a strong prevention. This is due to the fact that the utility of individuals increases with a lower prevalence rate, which gives room for a lower production to maintain a constant utility. Then, the economy needs less capital.

One of the limitations of this study is that the prevention policy is an exogenous variable that provides us analytical results. In future work, endogenous prevention policies (and optimal) are needed to capture the different phases of the pandemic's development and its impacts on Hartwick's rule. Moreover, the prevention policies also shape production activities, such that a threshold for prevention can be reached. Thus, more complex dynamics as well policy recommendations can be delivered without analytical results. Another limitation is that we do not analyze the impact of an infectious disease on the technological progress. Investigating these limitations are left for further research.

A   Appendix

A.1   Proof for the consistency of the candidate solution

To check that the proposal candidate indeed establishess the first-order condition, differentiating two first-order conditions in (16) with respect to time, it follows that

$$\frac{\dot{p}_2}{p_2} = \frac{\dot{\Phi}}{\Phi} - \epsilon \frac{\dot{x}}{x} = -F_K. \tag{34}$$

Since we have $p_1 R = p_3$, by using the first order condition $p_1 = p_2 F_R$, we can write

$$p_3 = p_2 F_R R = p_2 \beta Y. \tag{35}$$

By using the first-order condition $p_2 = \Phi U_c^{'}(C, x)$, we have

$$p_3 = \Phi U_c^{'}(C, x)\, \beta Y. \tag{36}$$

By defining the utility function $U(C, x) = C x^{-\varepsilon}$. Then, differentiating (36) with respect to time yields

$$\frac{\dot{p}_3}{p_3} = \frac{\dot{\Phi}}{\Phi}. \tag{37}$$

Using equation (34) and (37), we express

$$\frac{\dot{p}_3}{p_3} = \epsilon \frac{\dot{x}}{x} - F_K. \tag{38}$$

where $F_K = \alpha \frac{Y}{K_0}$ with $Y = C$ due to the assumption $\dot{K} = 0$. On the other hand, the dynamics of the co-state equation from equation (21) gives

$$\frac{\dot{p}_3}{p_3} = \frac{\dot{R}}{R} = \frac{\gamma}{\beta} \left(\frac{f^{'}(W)}{f(W)} \right) \tag{39}$$

$$+ \frac{\epsilon}{\beta} \left(\frac{e^{(\lambda - \lambda_1)t} (x_1 - x_0)(\lambda - \lambda_1)}{x_0 + (x_1 - x_0) e^{(\lambda - \lambda_1)t}} \right)$$

where

$$\frac{f^{'}(W)}{f(W)} = \left(\sigma + \phi \left(1 - \frac{x_0}{x_0 + (x_1 - x_0) e^{-\mu(\lambda - \lambda_1)W}} \right) \mu(\lambda - \lambda_1) \right)$$

The candidate solution is consistent if and only if the solution of (39) and the solution of (38) are same. First, we solve analytically these equations. The general solution of (38) is the following by assuming $\epsilon = 1$;

$$p_3(t) = e^{-\frac{\bar{u}\alpha x_1}{K_0}t}\Psi_0\left(x_0 + (x_1 - x_0)e^{(\lambda-\lambda_1)t}\right)e^{\frac{\bar{u}\alpha x_1}{K_0\mu(\lambda-\lambda_1)}-1} \tag{40}$$

where

$$\Psi_0 = \frac{p_3(0)}{\left(x_0 + (x_1 - x_0)e^{\frac{\bar{u}\alpha x_1}{\mu K_0(\lambda-\lambda_1)}-1}\right)}$$

To solve the differential equation (39), we assume $\frac{\gamma}{\beta} = 1$. The analytic solution of the differential equation (39) is as follows

$$p_3(t) = \Upsilon_0 e^{-\frac{\xi}{\gamma}\sigma t}\left(x_0 + (x_1 - x_0)e^{\mu(\lambda-\lambda_1)t}\right)^{-\left(\phi+\frac{1}{\beta}\right)} \tag{41}$$

where
$$\Upsilon_0 = \frac{p_3(0)}{(x_1)^{-\left(\phi+\frac{1}{\beta}\right)}}$$

In order to ensure that equations (40) and (41) are same, we should impose some restrictions on parameters. This is the reason why we call this solution as a special candidate solution that respects the optimality conditions of the maximization program.

$$\begin{cases} -\left(\phi + \frac{1}{\beta}\right) = \frac{\bar{u}\alpha x_1}{K_0\mu(\lambda-\lambda_1)} - 1 \\ \sigma = \frac{\bar{u}\alpha x_1}{K_0} \end{cases} \tag{42}$$

With parameters given in (42), we have

$$p_3(t) = \frac{p_3(0)}{(x_1)^{-\left(\phi+\frac{1}{\beta}\right)}}e^{-\sigma t}\left(x_0 + (x_1 - x_0)e^{\mu(\lambda-\lambda_1)t}\right)^{-\left(\phi+\frac{1}{\beta}\right)}$$

Since, we know $\bar{p}_1 R = p_3$, we have the analytical solution of R.

$$R(t) = \frac{R(0)}{(x_1)^{-\left(\phi+\frac{1}{\beta}\right)}}e^{-\sigma t}\left(x_0 + (x_1 - x_0)e^{\mu(\lambda-\lambda_1)t}\right)^{-\left(\phi+\frac{1}{\beta}\right)}$$

A.2 Proof of Proposition 1

The proof is quite trivial. To ensure the condition $\lim_{t\to\infty}\mathcal{H} = 0$, we should have $\lim_{t\to\infty}p_3 = p_1 R = 0$. This implies that the natural resource extraction should be zero when time goes to infinity. In other words, the natural resource extraction should follow a decreasing path. Otherwise, the optimality cannot be ensured since the transversality condition is not respected, implying $\lim_{t\to\infty}p_3 \neq 0$ and $\lim_{t\to\infty}\mathcal{H} \neq 0$.

To understand this, we differenciate the analytical form of $R(t)$ with respect to time t

$$\frac{\partial R\left(t\right)}{\partial t}=-\frac{e^{-\sigma t}}{\left(x_{1}\right)^{-\left(\phi+\frac{1}{\beta}\right)}}\left[x_{0}+\left(x_{1}-x_{0}\right)e^{\mu\left(\lambda-\lambda_{1}\right)t}\right]^{-\left(1+\phi+\frac{1}{\beta}\right)}\left[\beta\sigma x_{0}+e^{\mu\left(\lambda-\lambda_{1}\right)t}\left(x_{1}-x_{0}\right)\left[\beta\sigma+\left(1+\beta\phi\right)\mu\left(\lambda-\lambda_{1}\right)\right]\right]$$

In case where $x_{0} < x_{1}$ where x_{1} is the steady-state level of the prevalence rate, when we have $\lambda > (1+\beta\phi)\left(\mu-r\right)-\sigma$, we have $\frac{\partial R(t)}{\partial t}<0$ for $\forall t$.

A.3 The initial resource stock S_0

In order to find the initial stock S_0 that is in line with the candidate solution.

$$S_{0}=\int_{0}^{\infty}R\left(t\right)dt=\int_{0}^{\infty}\left[\frac{e^{-\sigma t}\left(x_{0}+\left(x_{1}-x_{0}\right)e^{\mu\left(\lambda-\lambda_{1}\right)t}\right)^{-\left(\phi+\frac{1}{\beta}\right)}R\left(0\right)}{\left(x_{1}\right)^{-\left(\phi+\frac{1}{\beta}\right)}}\right]dt \tag{43}$$

Then, the solution of the integral (43) is

$$S_{0}=\left[-R\left(0\right)\frac{e^{-\sigma t}\left(x_{1}\right)^{\phi+\frac{1}{\beta}}\left(x_{0}+\left(x_{1}-x_{0}\right)e^{\mu\left(\lambda-\lambda_{1}\right)t}\right)^{-\left(\phi+\frac{1}{\beta}-1\right)}}{\sigma x_{0}}\upsilon\left(t\right)\right]_{0}^{\infty}$$

where

$$\upsilon\left(t\right)={}_{2}F_{1}\left(1,1-\frac{1}{\beta}-\phi-\frac{\sigma}{\mu\left(\lambda-\lambda_{1}\right)};1-\frac{\sigma}{\mu\left(\lambda-\lambda_{1}\right)};-\frac{e^{\mu\left(\lambda-\lambda_{1}\right)t}\left(x_{1}-x_{0}\right)}{x_{0}}\right)$$

The upper bound when $t=\infty$ is zero. Then, we have $\upsilon\left(0\right)=\tau\left(.\right)$ is the lower bound of the integral. So, we have

$$S_{0}=R\left(0\right)\frac{x_{1}}{\sigma x_{0}}\tau\left(.\right)$$

where

$$\tau\left(.\right)={}_{2}F_{1}\left(1,1-\frac{1}{\beta}-\phi-\frac{\sigma}{\mu\left(\lambda-\lambda_{1}\right)};1-\frac{\sigma}{\mu\left(\lambda-\lambda_{1}\right)};-\frac{\left(x_{1}-x_{0}\right)}{x_{0}}\right)$$

Hence, we prove the existence of a finite resource S_0 stock with the special candidate solution.

A.4 Proof of Proposition 4

We differenciate the analytic solution of $R\left(t\right)$ expressed in equation (24) with respect to the prevention policy λ when $t=0$ in order to ease the calculations. First, we show that the natural resource extraction always decreases with the prevention policy if there is any cost of prevention in terms of an output loss. Note that we have the parameter restrictions explained in (42) in order to ensure the consistency. When we

differenciate $R(t)$ with respect to the prevention policy λ, we should also take into account the restrictions (42). All parameters are supposed to be constant and given. Free parameters are β and K_0. So, the change in λ leads to changes in β and K_0 in order to ensure the equalities in (42).

$$\frac{\partial R(t)}{\partial \lambda} = e^{-\sigma t} \left(1 - a\lambda\right)^{-\frac{1}{\beta}} \left(1 - \frac{r}{\mu(1-\lambda)}\right) K_0^{-\frac{\alpha}{\beta}} L_0^{\frac{\gamma}{\beta}} \left(\bar{u} x_0^\epsilon\right)^{\frac{1}{\beta}} \left(e^{-(\mu(1-\lambda)-r)t} \left(1 - \frac{r}{(1-\lambda)\mu} - x_0\right) + x_0\right)^{-\left(\phi+\frac{1}{\beta}\right)}$$

$$\left(\frac{a}{\beta(1-a\lambda)} - \frac{r(1+\beta\phi)}{\beta(1-\lambda)(\mu(1-\lambda)-r)} + \frac{e^{-(\mu(1-\lambda)-r)t}(1+\beta\phi)\left(r(1+t(1-\lambda)\mu) - t(1-\lambda)^2\mu^2(1-x_0)\right)}{\beta(1-\lambda)\left(\mu x_0(1-\lambda) - e^{-(\mu(1-\lambda)-r)t}(r - \mu(1-\lambda) + \mu(1-\lambda)x_0)\right)}\right.$$

(44)

$$\left. + \frac{\bar{u}\alpha}{K_0}\left(\frac{r}{(1-\lambda)^2}\right) + \frac{1}{(1-\lambda)^2}\frac{\bar{u}\alpha}{K_0}\left[\mathrm{Log}\,[x_1] - \mathrm{Log}\left[x_0 + (x_1 - x_0)e^{-(\mu(1-\lambda)-r)t}\right]\right]\right) \lessgtr 0$$

The first line in equation (44) is unambiguously positive. In any case, the second term in the second line is unambiguously positive. The denominator of the third term is also always positive. However, the numerator is negative when $\mu(1-\lambda)t\left[\frac{\mu}{r}(1-x_0) - 1\right] > 1$. This also holds when the time t tends to infinity if $\frac{\mu}{r}(1-x_0) - 1 > 0$. Also, the third line is always positive. We have $\frac{\partial R(t)}{\partial \lambda} > 0$ if the following condition holds

$$a > -\frac{\beta \tilde{z}}{1 - \beta \tilde{z}\lambda}$$

where

$$\tilde{z} = -\frac{r(1+\beta\phi)}{\beta(1-\lambda)(\mu(1-\lambda)-r)} + \frac{e^{-(\mu(1-\lambda)-r)t}(1+\beta\phi)\left(r(1+t(1-\lambda)\mu) - t(1-\lambda)^2\mu^2(1-x_0)\right)}{\beta(1-\lambda)\left(\mu x_0(1-\lambda) - e^{-(\mu(1-\lambda)-r)t}(r - \mu(1-\lambda) + \mu(1-\lambda)x_0)\right)}$$

$$+ \frac{\bar{u}\alpha}{K_0}\left(\frac{r}{(1-\lambda)^2}\right) + \frac{1}{(1-\lambda)^2}\frac{\bar{u}\alpha}{K_0}\left[\mathrm{Log}\,[x_1] - \mathrm{Log}\left[x_0 + (x_1 - x_0)e^{-(\mu(1-\lambda)-r)t}\right]\right]$$

This result is confirmed by the right panel in Figure (2).

A.5 Proof of Proposition 5

In order to understand the impact of the prevention policy on the steady-state level of K and P, we simply differenciate K^* and P^* with respect to the prevention policy λ

$$\frac{\partial K^*}{\partial \lambda} = \frac{\alpha\beta\bar{u}}{\sigma}\epsilon(1-x^*)^{-\dot{\epsilon}}\frac{\partial x^*}{\partial \lambda} < 0 \tag{45}$$

The marginal impact of the prevention policy on the steady-state level of K^* is always negative since $\frac{\partial x^*}{\partial \lambda} < 0$.

23

The impact of the prevention policy λ on the deflated natural resource stock level P is

$$\frac{\partial P^*}{\partial \lambda} = \underbrace{-\alpha \left[K^*\right]^{-1} \frac{\partial K^*}{\partial \lambda} \bar{u}}_{>0} + \underbrace{\frac{a}{(1-a\lambda)}}_{>0} + \underbrace{\frac{\beta}{(1-x)^2} \frac{\partial x^*}{\partial \lambda}}_{<0} \tag{46}$$

When $a < \dfrac{-\left(\alpha[K^*]^{-1} \frac{\partial K^*}{\partial \lambda} \bar{u} + \frac{\beta}{(1-x)^2} \frac{\partial x^*}{\partial \lambda}\right)}{1 - \lambda\left(\alpha[K^*]^{-1} \frac{\partial K^*}{\partial \lambda} \bar{u} + \frac{\beta}{(1-x)^2} \frac{\partial x^*}{\partial \lambda}\right)}$, we have $\frac{\partial P^*}{\partial \lambda} < 0$ and vice versa.

A.6 Long-term behavior

i) In order to characterize the long-run behavior, consider labor productivity as augmenting the natural resource stocks and diminishing physical capital. Then, the production function becomes

$$Y = \left(KL^{-m_K} A^{-\tilde{m}_K}\right)^{\alpha} \left(RL^{m_X} A^{\tilde{m}_X}\right)^{\beta} L^{m_L} A^{m_A},$$

with $J = L^{-m_K} A^{-\tilde{m}_K}$ and $P = SL^{m_X} A^{\tilde{m}_X}$ where L Thus, $-\alpha m_K + \beta m_X + m_L = \gamma = 1 - \alpha - \beta - \xi$ and $-\alpha \tilde{m}_K + \beta \tilde{m}_X + \tilde{m}_A = \xi = 1 - \alpha - \beta - \gamma$. Note that in the long run, $\frac{\dot{L}}{L} = -\frac{\eta \dot{x}}{1-x} = 0$ since the prevalence rate $x(t)$ converges to its steady-state x^*.

To guarantee that K and Y grow at the same rate in the long run, it must be that $m_K = m_A$. In the long run, $\dot{J} = 0$ and $\dot{P} = 0$ yield $g_K = \tilde{m}_K g_A$. From equation $Y = \frac{C - q_z}{1 - \beta} = \frac{\bar{u} x^{\epsilon} - q_z}{1 - \beta}$, it is easy to derive $g_Y = g_{q_z} = 0$ in the long-run. Note that $g_C = 0$ since $\dot{x} = 0$ in the long run. Furthermore, $g_K = g_Y$ implies $\tilde{m}_K = \tilde{m}_A = 0$. Then, it is easy to see that $\tilde{m}_X = \frac{1-\alpha-\beta-\gamma}{\beta} = \frac{\xi}{\beta}$ and $J = K$. So the dynamic system can be expressed as

$$\begin{cases} \dot{K} = \beta Y - q_z, \\ \dot{P} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} (1-a\lambda)^{-\frac{1}{\beta}} + \frac{\gamma}{\beta}\left(\sigma - \left(\frac{\eta \dot{x}}{1-x}\right)\right) P, \end{cases} \tag{47}$$

Other dynamic equations remain the same

$$\begin{cases} \dot{x} = x\left([1-x]\mu(1-\lambda) - r\right) \\ \dot{q}_z = -\gamma\left(\sigma - \frac{\dot{x}}{1-x}\right) Y + r q_z + \epsilon\left(\frac{\dot{x}}{1-x}\right)\frac{\bar{u}}{1-x} \end{cases}$$

where $q_z = \frac{p_3}{p_2}$.

ii) Thus, in the long run the above state and co-state variables can be expressed in terms of the constant consumption $\bar{C}$:

$$\begin{cases} K^* = \frac{\alpha\beta\bar{u}}{\sigma(1-x^*)}, \\ P^* = \frac{\beta[K^*]^{-\frac{\alpha}{\beta}}(1-a\lambda)^{-\frac{1}{\beta}}\bar{u}}{\sigma(1-x^*)}, \\ q_z^* = \beta C. \\ x^* = 1 - \frac{n}{\mu(1-\lambda)} \end{cases} \tag{48}$$

$P = S(AL)^{\frac{\gamma}{\beta}}$ where $L = \bar{N}(1-\eta x)$ yields that $\frac{\dot{P}}{P} = \frac{\dot{S}}{S} + \frac{\gamma}{\beta}\left(\sigma - \frac{\eta \dot{x}}{1-x}\right)$. Thus, in the long run, from $\dot{P} = 0$ it follows that $\frac{\dot{S}}{S} = -\sigma$. In other words, the natural resource stocks decrease at rate σ in the long run.

iii) When we have a disease-free case, the steady state becomes

$$
\begin{cases}
K^* = \frac{\alpha \beta \bar{C}}{\sigma}, \\
P^* = \frac{\beta [K^*]^{-\frac{\alpha}{\beta}}(1-a\lambda)^{-\frac{1}{\beta}}\bar{C}}{\sigma}, \\
q_z^* = \beta \bar{C}. \\
x^* = 1 - \frac{n}{\mu(1-\lambda)}
\end{cases}
\tag{49}
$$

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