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Assessing and ordering investment in polluting fossil-fueled and zero-carbon capital

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Abstract

We study the transition from preexisting polluting fossil-fueled capital (coal power) to cleaner fossil-fueled capital (gas) and zero-carbon capital (renewable). We model exhaustible resources, irreversible investment, adjustment costs and a carbon budget; both fossil-fuel and renewable energy consumption are subject to capacity constraints. To smooth investment and spread costs, optimal investment in expensive renewable power may start before the cheaper fossil resources are exhausted. Gas power however operates as a bridge between coal and renewable power: it allows building less renewable at the beginning of the transition, moving some efforts from the short to the middle term. Finally, the popular criteria of the *levelized cost of electricity* is biased toward cheaper and lower-potential alternatives (gas instead of renewable) if computed against current prices. We provide numerical simulations of the European power sector based on the Commission's energy roadmap to 2050.

Keywords: climate change mitigation; path dependence; optimal timing; dynamic efficiency; early-scrapping;
JEL classification: Q54, Q58

The international community aims at stabilizing climate change to avoid important climate damages, which requires reaching near-zero greenhouse gas (GHG) emissions in the long term (Matthews and Caldeira, 2008; Steinacher et al., 2013). A recognized challenge to meet this goal is to abate GHG from energy-intensive sectors, such as power generation and private transportation. These sectors share various characteristics that are critical when determining optimal decarbonization pathways. First, energy-intensive sectors depend on long-lived capital. While their typical lifetime differs, a power plant and a per-

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sonal vehicle both represent a long-lived investment to their owners. Second, capital in these sectors frequently requires a specific fuel (e.g., most coal power plants may run on coal but not on oil). Finally, these sectors have developed in a possibly unsustainable way, because of environmental externalities not internalized in the price system when investment decisions were taken.

Today, electricity production and private transportation greatly rely on polluting fossil fuels. Stabilizing climate change will thus require replacing preexisting carbon-intensive capital by one or several types of cleaner capital. For instance, emissions from existing coal power plants may be reduced by replacing coal plants with new gas power plants (gas is less carbon-intensive than coal), or with more-expensive but almost-carbon-free options such as nuclear or renewable power. Also, decision makers can either wait for existing coal power plants to reach their natural lifetime, or decide to decommission them earlier to switch faster to cleaner energy sources.[1]

This paper addresses two policy-relevant questions: what is the optimal timing of the transition from coal power to gas and renewable power? and how should investments in different plants be assessed? We find that capacity constraints and adjustment costs affect the answers in three major ways. First, it may be optimal to start investing in expensive renewable power before apparently cheaper fossil resources are exhausted. Second, gas power plants operate as a bridge technology between coal and renewable, used to move some investment costs from the short to the middle term. Third, investment in different types of power plants cannot be assessed using only current prices, optimal decisions require to correctly anticipate all future prices in advance.

The order under which different exhaustible fossil fuel deposits should be exploited has been widely investigated since the seminal works by Herfindahl (1967), who found that deposit differing only by their extraction cost should be exploited sequentially, *cheapest first*. Recently, Chakravorty et al. (2008) investigated the impact of a climate policy on the extraction of different polluting nonrenewable resources. They find that the optimal strategy does not order exhaustible sources according to their carbon intensity: coal may be used first, followed by natural gas, and again by coal. In their framework, the clean backstop (e.g., renewable energy) should never be used before fossil fuels have been exhausted. We extend this literature by investigating the role of adjustment costs and capacity constraints that limit the extraction of both fossil and renewable energy. Our aim is to capture the implications of having to first invest in new capital — new coal, gas, or renewable power plants — before increasing production from the respective energy sources.

Our analysis suggests that it may be optimal to start building expensive renewable power (i) before exhausting polluting fossil fuels (coal and gas), (ii) before preexisting coal plants have depreciated, and (iii) before starting to invest in cheaper gas plants. Indeed, a finite carbon budget (or finite fossil fuel reserves) mean that production needs eventually to switch to renewable power. Besides, building renewable power plants requires skilled workers and appropriate cap-

[1] Similarly, private transportation relies on energy-intensive cars, but internal combustion engines and materials can be improved, or private transportation could switch to clean electricity in the future. While most results in these paper also apply to the transportation sector, we stick to the example of power generation for clarity.

ital, resulting in convex investment costs (frequently referred to as *adjustment costs*). Starting to build renewable power early and spreading investment over time therefore allows minimizing the discounted cost of the transition.

We thus confirm previous findings by Kemp and Long (1980), Amigues et al. (1998) and Holland (2003): if extraction from an expensive energy source is constrained, it may be optimal to use that source before cheaper alternatives are exhausted. The originality of our approach roots in the assumption that the extraction rate is limited by installed capital (power plants) which may be chosen endogenously by paying convex investment costs.[2] Amigues et al. (2013) also study investment with convex adjustment costs for a renewable substitute and (unconstrained) extraction of fossil resources, and prove that renewable investment can be independent from existing fossil resources. Our work differs in that we consider capacity constraints and adjustment costs for all energy sources, including fossil resources. While slightly reducing the analytical tractability, this allows investigating the role of investment in gas power plants, and the operational assessment of investment in different types of substitutes (e.g., gas and renewable power) to replace the preexisting dirty capital (e.g., coal plants).

The present analysis suggests that existing coal plants should be decommissioned before their natural end of life, to accelerate emission reductions. This result extends the analysis by Rozenberg et al. (2013), who analyze the trade-off between (i) early-scrapping pre-existing dirty capital to reduce emissions fast and (ii) using dirty capital at full capacity to increase short-term production. They find that early-scraping part of the dirty capital *built before climate policies were implemented* is optimal for stringent climate targets.[3] We find moreover that it may be optimal to build intermediate capital (gas power plants) in order to reduce short-term investment in renewable power, then to decommission these gas plants before their natural end of life and replace them with renewable power to reduce emissions further in the middle term. Gas plants thus operate as a temporary bridge between coal and renewable, used to move some efforts from the short to the middle term.

The last question we address is the operational assessment of investment in different types of power plants. An early literature on investment theory emphasized that firms may take optimal investment decisions based on current prices only (e.g., Arrow, 1964; Jorgenson, 1967, p. 145). It has been signaled, however, that this result critically depends on investment prices being exogenous: under endogenous investment costs, optimal investment decisions require to know all future prices (Gould, 1968; Mussa, 1977). Here, we warn that the

[2] Our paper may thus remind the strain of literature linking the theory of investment to the theory of the mine (Campbell, 1980; Gaudet, 1983; Lasserre, 1985), where capital directly limits the extraction rate of the minerals. After reviewing this literature, Cairns (1998) notes that "*there can be three phases in the exploitation of the mine, namely (1) a period of positive investment after time $t = 0$, in which production is at full capacity, then (2) a period in which investment is zero and production is at full capacity, and finally (3) a period of declining production*". We find a similar trajectory for gas investment and production; however, the addition of a positive depreciation rate means that investment is not zero during the whole second phase in our model.

[3] Both the present paper and Rozenberg et al. (2013) provide analytical frameworks. Early decommissioning of existing coal power plants has also been studied using numerical models (e.g., Rogelj et al., 2011).

levelized cost of electricity (LCOE), a popular metric in energy textbooks and studies (e.g. IPCC, 2007; Branker et al., 2011; Alok, 2011; Kost et al., 2012; EIA, 2013; Ueckerdt et al., 2013), suffers from the same drawback when adjustment costs are accounted for. The LCOE is the ratio of discounted costs of building and using a power plant, including fuel and GHG costs, over the discounted planned electricity production. It is generally admitted that technologies with lower LCOEs are superior to those with higher LCOEs, and that power plants should be built only when their LCOE is inferior to the electricity price.

We confirm that on the optimal pathway, the LCOE computed against *future* energy and carbon prices should be equal to the average *future* electricity price. We investigate whether LCOEs computed against *current* prices may nonetheless provide an operational rule of thumb. We find that it is misleading to compare power plants projects using LCOEs computed against current prices. In a numerical calibration on the European electricity sector, the optimal LCOE is found to be much higher for new renewable power plants than for new gas power plants. If decision makers decided investment by comparing LCOEs to current electricity prices, they would thus build too much gas capacity, and not enough renewable capacity. These results remind the findings by Vogt-Schilb et al. (2013), who study optimal investment in abatement capital in different sectors. They find that sectors with larger abatement potential and sectors where abatement capital is more expensive should invest more dollars per abated ton than the others;[4] they also remind that optimal investment in clean capital requires a perfect knowledge and credibility of future carbon prices.[5]

The remaining of the paper is structured as follows. Section 1 details the model. Section 2 derives the first-order conditions, optimal *production* decisions at each point in time, and the resulting electricity price. This allows studying different types of phases through which the power sector may go during its transition to renewable power. Section 3 tackles *investment* decisions, completes the phases inventory, and studies the levelized cost of electricity. Section 4 provides numerical simulations calibrated with data from the European electricity sector. Section 5 concludes.

1. Model

A social planner controls the supply of an output, for instance electricity, using and investing in three different technologies: a preexisting high-carbon technology (h) e.g. coal power, a fossil-fueled low-carbon technology (ℓ), e.g. gas, and an inexhaustible zero-carbon technology (z), e.g. renewable power.

At each time t, the social planner chooses the physical investment $x_{i,t}$ in technology i. The investment adds to the installed capacity $k_{i,t}$, which depreciates at the constant rate δ (dotted variables denote temporal derivatives):[6]

$$\forall i, \quad \dot{k}_{i,t} = x_{i,t} - \delta k_{i,t} \tag{1}$$

[4] Here, renewable capacity is both more expensive and less carbon-intensive than gas capacity.

[5] Ploeg and Withagen (1991), Fischer et al. (2004) and Williams (2010) should also be mentioned as pioneers of clean capital accumulation theory.

[6] Through this paper, *capacity* is to be understood as equivalent capacity, e.g., in kWh/yr, unless otherwise specified. For instance if 2 kW of windmills are required to provide as much output per year as 1 kW of coal, then 2 kW of windmills are accounted as 1 kW_{eq}.

Physical investment is made at a positive, increasing and convex cost $c_i(x_{i,t})$:

$$c_i > 0, \; c_i' > 0, \; c_i'' > 0 \tag{2}$$

This captures the increasing opportunity cost to use scarce resources (skilled workers and appropriate capital) in order to build and deploy capacities faster. We assume that low-carbon capacity is cheaper than zero-carbon capacity in the sense that:

$$\forall x \quad c_\ell'(x) < c_z'(x) \tag{3}$$

Without loss of generality, we assume low-carbon and zero-carbon capacities to be nil at the beginning ($k_{\ell,t_0} = k_{z,t_0} = 0$).[7]

The social planner then chooses how much output to produce from each technology. We assume production exhibits constant returns to scale: two gas plants can produce twice as much as one gas plant. The positive production $q_{i,t}$ with technology i cannot exceed the installed capacity $k_{i,t}$:

$$\begin{aligned} \forall i, \quad 0 &\leq q_{i,t} \\ q_{i,t} &\leq k_{i,t} \end{aligned} \tag{4}$$

For analytical tractability, we assume the preexisting carbon-intensive capital is overabundant (such that (4) is not binding for coal); this assumption is relaxed (and confirmed) in the numerical application.

Using fossil fuel (gas or coal) requires to extract exhaustible resource from an initial stock, such that the current stock $S_{i,t}$ classically satisfies:

$$\begin{aligned} &S_{i,t_0} \text{ given} \\ &\dot{S}_{i,t} = -q_{i,t} \\ &S_{i,t} \geq 0 \end{aligned} \tag{5}$$

While it is convenient to use the above general notations (indexed by i), we will follow van der Ploeg and Withagen (2012) and focus on the case where the zero carbon technology is renewable and coal is overabundant ($S_{z,t_0} = S_{h,t_0} = \infty$). This is equivalent to recognize the carbon budget (see below) is more stringent than the scarcity of coal resources — as it will be discussed later, this assumption is likely to hold for gas resources as well, especially with the recent developments of non-conventional gas extraction methods..

Let R_i be the carbon intensity (or emission rate) of technology i. The high-carbon technology is more carbon-intensive than the low-carbon technology:

$$R_h > R_\ell > R_z = 0 \tag{6}$$

The social planner is subject to an exogenous so-called carbon budget, cumulative emissions cannot exceed a given ceiling $\bar{M}$:

$$\forall t, \quad m_t \leq \bar{M} \tag{7}$$

[7] This assumption does not result in a loss of generality as we study the transition from the existing situation to clean power.

Where cumulative emissions m_t grow with emissions $R_i\, q_{i,t}$:

$$\dot{m}_t = \sum_i R_i\, q_{i,t} \tag{8}$$

Cumulative emissions have been found to be a good proxy for global warming (Allen et al., 2009; Matthews et al., 2009). Some policy instruments, such as an emission trading scheme with unlimited banking and borrowing, set a similar constraint on firms.

Consumers derive a utility $u\left(\sum_i q_{i,t}\right)$ from consuming the output, where u is a classical utility function. The program of the social planner consists in determining the trajectories of investment $x_{i,t}$ and production $q_{i,t}$ that maximize discounted utility net from investment costs while complying with the carbon budget $\bar{M}$ and the various constraints:

$$\max_{x_{i,t}, q_{i,t}} \int_0^\infty e^{-rt} \left[u\left(\sum_i q_{i,t}\right) - \sum_i c_i(x_{i,t}) \right] dt \tag{9}$$

$$\begin{aligned}
\text{s.t. } & \dot{k}_{i,t} = x_{i,t} - \delta k_{i,t} && (\nu_{i,t}) \\
& q_{i,t} \leq k_{i,t} && (\gamma_{i,t}) \\
& q_{i,t} \geq 0 && (\lambda_{i,t}) \\
& \dot{m}_t = \sum_i R_i\, q_{i,t} && (\mu_t) \\
& m_t \leq \bar{M} && (\eta_t) \\
& \dot{S}_{i,t} = -q_{i,t} && (\alpha_{i,t}) \\
& S_{i,t} \geq 0 && (\beta_{i,t})
\end{aligned}$$

Where r is the constant discount rate and the Greek letters in parentheses are the costate variables and Lagrange multipliers (all chosen such that they are positive): among them, $\nu_{i,t}$ is the shadow cost of new power plants; $\gamma_{i,t}$, the social cost of the capacity constraint, also interprets as the shadow rental cost of capital or the marginal productivity of capital; μ_t is the shadow carbon price; and $\alpha_{i,t}$ the shadow price of resource i (all notations are gathered in Table A.5).

2. Optimal production decisions

When both production and investment with technology i are strictly positive, the first-order conditions simplify to (see Appendix A for the complete set of equations):

$$(\delta + r)\, c_i'(x_{i,t}) - \frac{d}{dt} c_i'(x_{i,t}) = u_t' - \alpha_{i,t} - \mu_t\, R_i \tag{10}$$

On the right hand side of (10), u_t' is the competitive price of the output (u_t' stands for $u'\left(\sum_i q_{i,t}\right)$), $\alpha_{i,t}$ is the fuel cost, R_i the carbon intensity of fuel i, and μ_t is the carbon price. The left hand side is what Vogt-Schilb et al. (2013) have called the *marginal implicit rental cost of capital* (MIRCC), extending the concept proposed by Jorgenson (1967) to the case of endogenous capacity prices. It is the efficient market rental price of capacities, where capitalists would be indifferent between (i) buy capital at t at a cost $c_i'(x_{i,t})$, rent it out during one period dt at the rental price, then sell the depreciated (δ) capacities at $t + dt$ at a price $c_i'(x_{i,t}) + \frac{d}{dt} c_i'(x_{i,t}) dt$ or (ii) simply lend money at the interest rate r.

Equation (10) can thus be seen as an application of the equimarginal principle. It provides a simple rule to arbitrate *production* decisions at each moment in an economy where there is a market for renting capacities. Equation (10) also allows determining the shadow price of electricity:

Lemma 1. *Once renewable capacity has been built, the output price u_t' is given by the variable costs of the marginal technology or the rental cost of renewable power plants.*

1. *When coal capacities are used at less than full capacity, the output price is given by variable costs from coal generation:*

$$0 < q_{h,t} < k_{h,t} \implies u_t' = \alpha_{h,t} + R_h \, \mu_t \tag{11}$$

2. *When gas capacities are used at less than full capacity, the output price is given by variable costs from gas generation:*

$$0 < q_{\ell,t} < k_{\ell,t} \implies u_t' = \alpha_{\ell,t} + R_\ell \, \mu_t \tag{12}$$

3. *If no resource is used under full capacity, the output price is set by the implicit rental cost of renewable capacity:*

$$\forall j, \; q_{j,t} = 0 \text{ and } \forall i, \; q_{i,t} = k_{i,t} \implies u_t' = (\delta + r) \, c_z'(x_{z,t}) - \frac{d}{dt} c_z'(x_{z,t}) \tag{13}$$

Proof. The results are straightforward once it is noted that $\gamma_{i,t}$ is zero when capacity i is underused, and that $\gamma_{i,t}$ equals the implicit rental cost of capital (Appendix A).[8] □

Lemma 1 implies that it is not possible that at a given date, gas and coal are both used under full capacity.[9] In other words, coal and gas cannot be the marginal power plant at the same time. Lemma 1 also implicitly defines three *types* of phases through which electricity production may go during the transition from coal-fired to renewable power. Fig. 1 illustrates this lemma: it shows a first phase when coal is underused, then a phase when coal is phased out and gas is used at full capacity,[10] followed by a phase when gas is underused, and a final steady-state phase when all production comes from renewable power.

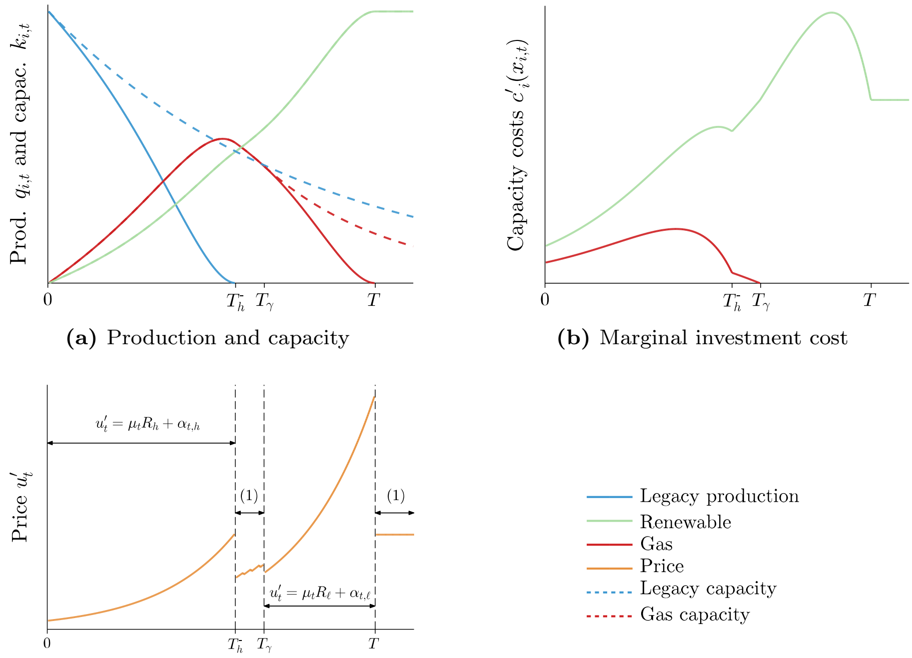
As discussed by Wang and Zhao (2013), the price may jump when the system changes phases.

At each time step, available capacity is given, and production decisions may be taken relying only on current prices. In the following we show that *investment* decisions, in contrast, require to know all future prices.

[8] Note that the implicit rental cost of capital is used to finance new capacities, and that investment in a particular technology is always null when capacity is underused. Demonstrating prices in the third case requires taking into account that gas cannot be underused unless gas is replaced by renewable power, meaning that if gas is underused then there is investment in renewable power (Appendix B)

[9] We disregard the case where fuel costs compensate exactly differences in carbon intensities $\alpha_\ell - \alpha_h = \mu \, (R_h - R_\ell)$ as it requires a very restrictive set of assumptions.

[10] During this phase, the implicit rental cost of both gas and renewable power plants is equal to the electricity price.



(a) Production and capacity

(b) Marginal investment cost

(c) Output price

Figure 1: Production, investment and output price during the three types of phases. (1): $u_t' = (\delta + r)\, c_z'(x_{z,t}) - \frac{d}{dt} c_z'(x_{z,t})$

3. Optimal investment decisions

3.1. General investment decision equation

If production and investment are strictly positive during a time interval (τ_i^+, τ_i^-), the optimal marginal investment cost $c_i'(x_{i,t})$ is the solution of the differential equation (10):

$$\forall t \in (\tau_i^+, \tau_i^-), \tag{14}$$
$$c_i'(x_{i,t}) = \int_t^{\tau_i^-} e^{-(r+\delta)(\theta - t)} (u_\theta' - \mu_\theta\, R_i - \alpha_{i,\theta}) d\theta + e^{-(r+\delta)(\tau_i^- - t)} c_i'(x_{i,\tau_i^-})$$

The right hand side decomposes in two terms: (i) the present value of all future revenues from selling the output minus costs from emission and resource usage $(u_t' - \mu_t\, R_i - \alpha_{i,t})$ produced by the depreciated marginal unit of capacity $(e^{-(r+\delta)(t-\theta)})$, plus (ii) the contribution of the marginal investment to the capacity installed at τ_i^-, valued at the replacement cost $c_i'(x_{i,\tau_i^-})$. Again, the optimal investment trajectory is given by a marginalist argument: the capacity cost is equal to the future discounted net revenues, including a resale value.

3.2. Ordering investment in low- and zero-carbon capacity

Equation (14) does not result in an operational criterion to compare investment in two technologies. The optimal investment cost of one technology can be superior or inferior to the other, depending on the dates when it is optimal to invest and use each technology (τ_i^+, τ_i^-), on the future price of inputs $\alpha_{i,t} + R_i\,\mu_t$ (and in particular on the stringency of the carbon budget), and on the resulting future price of electricity u'_t.

Proposition 1. *Depending on the parameters, investment phases may be ordered in three ways:*

1. *Two successive transitions. Gas first completely replaces coal, then renewable power replaces gas. In this case, investment in renewable power starts before coal and gas are phased out (Fig. 2a).*
2. *Gas and wind simultaneously replace coal. In this case, investment in zero-carbon capital starts before the fossil resources are phased-out (Fig. 2b).*
3. *Starting with investment in renewable power. In this case, investment in renewable power starts before investment in gas power (Fig. 2c).*

Proof. Appendix B lists all possible transition shapes. □

3.3. Assessing investment in different technologies

In the long term, the system reaches a steady state, when atmospheric carbon is stable, emissions are nil, and all the production comes from renewable power. During the steady state, all investment goes to maintain renewable power at its maximum capacity. There is thus a date, denoted T_h^-, after which production from coal is nil, and a date denoted τ_ℓ^- after which investment in gas is nil. After T_h^-, the output price is either the rental cost of renewable power, or variable costs from gas (Lemma 1). As a result, even if investment in renewable power starts after investment in gas power, when the social planner invests in both technologies, she spends more in renewable power:

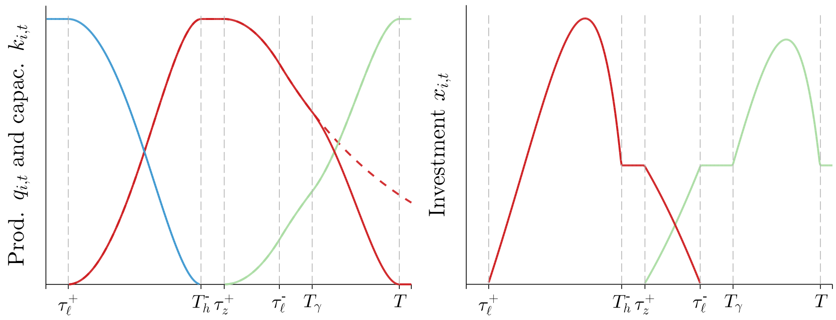
Proposition 2. *When the social planner invests in both the zero- and the low-carbon technology, it builds zero-carbon capacity at a higher marginal investment cost than low-carbon capacity.*

Proof. From (14), and denoting τ_i^+ the date when investment in capacity i starts:[11]

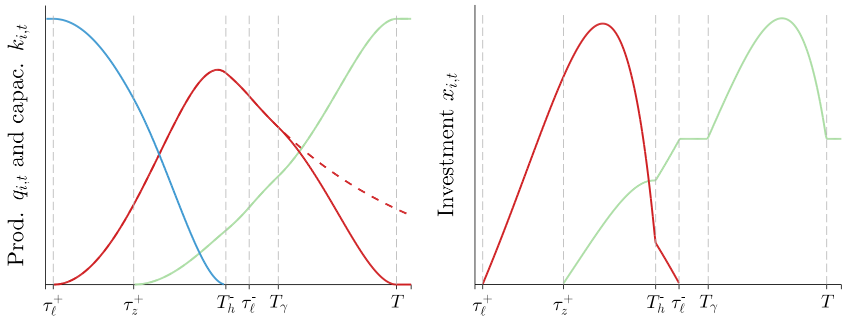
$$\forall t \in [\max_i (\tau_i^+)\,,\ \tau_\ell^-], \quad c'_z(x_{z,t}) - c'_\ell(x_{\ell,t}) = \underbrace{\int_t^{\tau_\ell^-} e^{-(r+\delta)(\theta-t)}\,(\mu_\theta\,R_\ell + \alpha_{\ell,\theta})\,d\theta}_{\Delta p} + \underbrace{\left(c'_z(x_{z,\tau_\ell^-}) - c'_\ell(0)\right)e^{(r+\delta)(t-\tau_\ell^-)}}_{\Delta c'} \tag{15}$$

Δp is the discounted value of emissions and fossil fuels that the marginal renewable capacity built at time t allows saving before τ_ℓ^- when compared to the marginal gas capacity built at time t.
$\Delta c'$ is the difference between the discounted values of the marginal capacities built at t which will depreciate from t to τ_ℓ^-. It is strictly positive, as $c'_z(x_{z,\tau_\ell^-}) > c'_z(0)$ by assumption (2) and $c'_z(0) > c'_\ell(0)$ by assumption (3). □

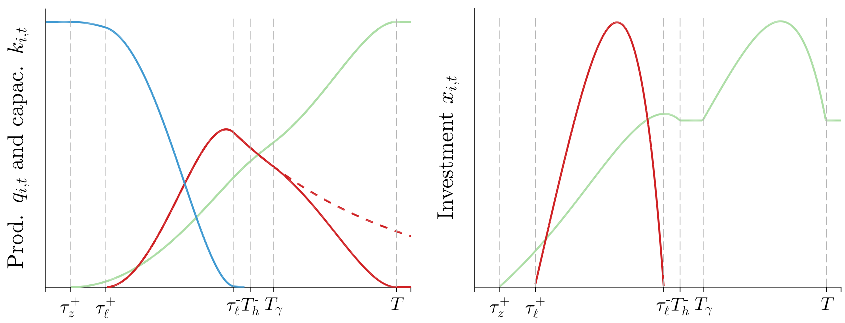
[11] Appendix B provides a more detailed resolution.



(a) Two successive transitions



(b) Gas and wind simultaneously replace coal



(c) Starting with investment in renewable power

Figure 2: Numerical simulations of three possible transition profiles. Figures on the left display capacity and production, figures on the right display optimal investment. The parameters used to produce these figures are gathered in Table B.6.

Along the optimal path, marginal investment costs for renewable power is higher compared to gas for three reasons: (i) renewable saves more GHG than gas; (ii) renewable saves fossil energy, compared to gas; and (iii) renewable power plants have a higher residual value at the end of the period (convex investment costs mean that this final condition impacts the optimal investment cost during all the period).

3.4. Levelized cost of electricity (LCOE)

Here we investigate the Levelized cost of electricity (LCOE) as an operational rule of thumb to assess investment in different power plants. Let us consider an incremental unit of capacity built at $t \in (\tau_i^+, \tau_i^-)$ during an interval when investment is positive. Dividing its optimal investment costs (14) with its discounted production over the same interval yields:

$$\forall t \in (\tau_i^+, \tau_i^-), \qquad (16)$$
$$\frac{\int_t^{\tau_i^-} e^{-(r+\delta)(\theta-t)} u'_\theta d\theta}{\int_t^{\tau_i^-} e^{-(\delta+r)(\theta-t)} d\theta} = \frac{\int_t^{\tau_i^-} e^{-(r+\delta)(\theta-t)} (\mu_\theta R_i + \alpha_{i,\theta}) d\theta}{\int_t^{\tau_i^-} e^{-(\delta+r)(\theta-t)} d\theta} + \frac{c'_i(x_{i,t}) - e^{-(r+\delta)(\tau_i^- - t)} c'_i(x_{i,\tau_i^-})}{\int_t^{\tau_i^-} e^{-(\delta+r)(\theta-t)} d\theta}$$

The expression on the right-hand side of (16) is the *levelized cost of electricity* (LCOE) produced with technology i. The first term is the average variable cost (i.e. the sum of the average carbon price and the average resource price) over the interval; the second term corresponds to the investment costs of the marginal unit of capacity at t minus its residual value at τ_i^-, over the discounted future production (the levelized cost of the capacity, net of its residual value). We recognize on the left-hand side the average output price during (t, τ_i^-).

Eq. 16 states that at the optimum, the LCOE is equal to the average future price of the output. In other terms, the revenues priced at the optimal LCOE should cover for the variable costs and the investment cost.[12] In practice, using this relationship to assess investment requires to know in advance the future prices of additional capacity, resources, emissions and output. This may be challenging for actual agents willing to schedule or monitor investment in different types of power plants. For instance, it is recognized that governments have little capacity to commit in general, and in particular to future carbon prices (Helm et al., 2003).[13]

The question we address here is whether LCOEs computed against *current* prices may provide an operational rule of thumb to assess investment in different types of power plants. Such an *operational LCOE* expresses as:

$$\mathcal{L}_{i,t} = (r+\delta)\, c'_i(x_{i,t}) + \mu_t R_i + \alpha_{i,t} \qquad (17)$$

$\mathcal{L}_{i,t}$ is the sum of the annualized investment cost, the carbon cost and the fuel cost. In the next section we use numerical simulations calibrated on the

[12] We ignore here all intermittency issues. In a complete assessment of technologies, intermittency of renewable sources would have to be taken into account when comparing the value of technologies (Joskow, 2011; Ambec and Crampes, 2012; Ueckerdt et al., 2013).

[13] See also Spiro (2012) on the limited capacity of investors and regulators to anticipate prices over long time horizons.

Table 1: Technology sets considered in the numerical model

Set	Abbreviation	Description	Composition
High carbon technology	Legacy	Average current thermal production mix in 2008	40% gas , 50% coal , 10% oil (ENERDATA, 2012)
Low carbon technology	Gas	Efficient new generation fossil technologies	Efficient gas
Zero carbon technology	Wind	New generation renewable technologies	Onshore wind, biomass

Table 2: Fuel price trajectories of the fossil technology sets in \$/MWh (EU, 2011)

	2008	2025	2035	2050
Legacy	42.5	46	48	39
Gas	65	76	75	60

European power sector and find that renewable power should be built at an operational LCOE several times higher than new gas power plants.

4. Numerical application to the European electricity sector

4.1. Functional forms, data and calibration

Let us calibrate our model with data from the European power sector as described in the European 2050 Energy Roadmap (EU, 2011). In this numerical application, efficient gas power plants (the low-carbon technology) and renewable power (the zero-carbon technology), e.g. wind, are used to phase out the existing emitting capacities represented as the average legacy thermal production mix (Table 1).

To better fit the data, we express installed capacity $k_{i,t}$ in peak capacity (GW), and production $q_{i,t}$ in GWh/yr. Production is constrained by a maximum number of operating hours H_i. For instance, a given windmill will produce electricity only at the moments where it is windy, which expectedly happens a given number of hours per year.

We assume for simplicity that all technologies have the same depreciation rate δ, calibrated as $\delta = 1/\text{lifetime}$ assuming a lifetime of 30 years (IEA, 2010). We consider that Europe is price-taker on the fossil resources costs $\alpha_{i,t}$. The cost of resources follows the trajectory taken from the European 2050 Roadmap (see Table 2). Note that α_h corresponds to the average price of the resources of the legacy technology mix (cf. Table 1).

We assume quadratic investment costs. To calibrate the cost functions, we assume that when investment equals the actual average annual investment flow in Europe between 2009 and 2011 (X_i), the marginal investment cost C_i^m is

Table 3: Technology-specific data used in the numerical application

	Description	Unit	Legacy	Gas	Wind	Source
C_i^m	Nominal investment costs	\$/kW	1 800	1 200	2 000	IEA (2010)
X_i	Average annual new capacity in Europe	GW/yr	4.2	11	10	ENERDATA (2012)
H_i	Average annual operating hours	h/yr	7 500	7 500	2 000	IEA (2010)
R_i	Carbon intensity	gCO_2/kWh	530	330	0	ENERDATA (2012); Trotignon and Delbosc (2008)

Table 4: General parameter values used in the numerical application

	Description	Unit	Value	Source
r	Discount rate	%/yr	5	
$\bar{M}$	Carbon budget (central value)	$GtCO_2$	22	UE (2011)
D_0	Electricity demand in 2008	TWh/y	1 940	ENERDATA (2012); EU (2011)
G	Annual growth of demand	TWh/y	16.5	EU (2011)
δ	Depreciation rate	%/yr	3.33	IEA (2010)
A	Convexity parameter (central value)	·	0.1	

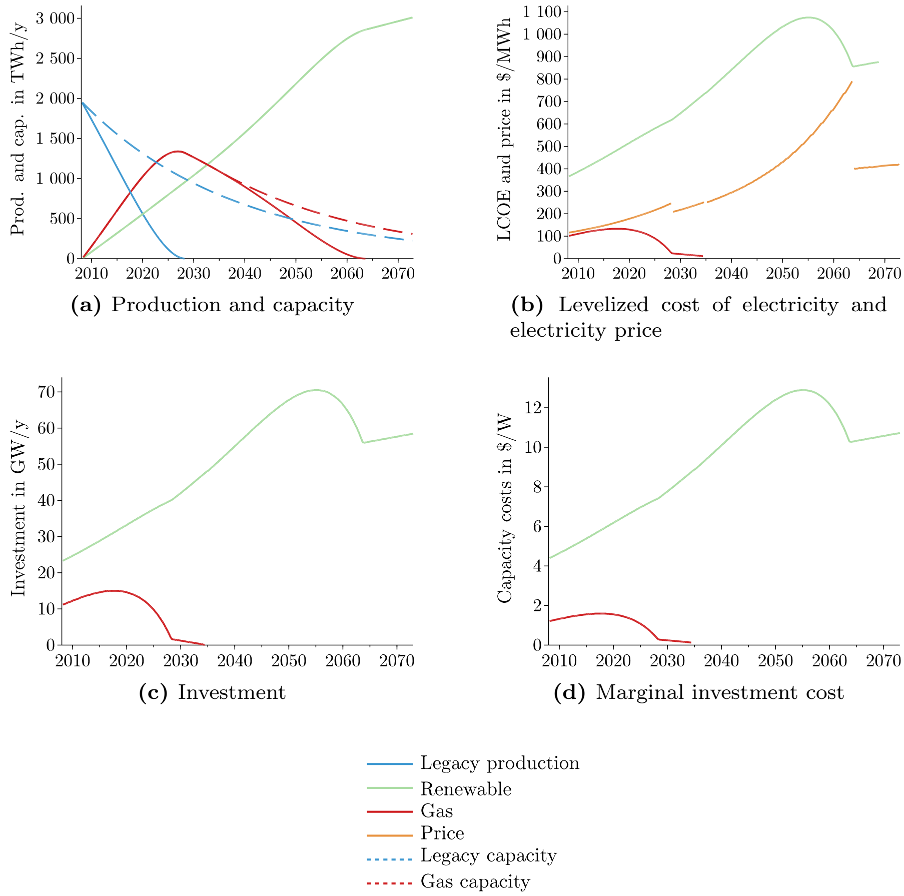
equal to the OECD median value for 2010 (as found in IEA (2010)). We write the cost function as:

$$c_i(x_{i,t}) = C_i^m \cdot X_i \cdot \left(A \, \frac{x_{i,t}}{X_i} + \frac{1-A}{2} \left(\frac{x_{i,t}}{X_i} \right)^2 \right) \tag{18}$$

$$\Longrightarrow c_i'(X_i) = C_i^m \tag{19}$$

$A \in (0,1)$ is a convexity parameter, equal across technologies. If $A = 1$, the marginal investment cost would be constant, there would be no adjustment cost for production capacity, and optimal investment pathways could exhibit jumps. If $A = 0$ the marginal cost curves would be linear: capacity accumulated at very low speed would be free ($\lim_{x_{i,t}\to 0; A=0} c_i'(x_{i,t}) = 0$), and the cost of new capacity would double when the investment pace doubles. An intermediate value $A \in (0,1)$ means that new capacity is always costly, and that its cost grows with the investment pace.We first assume $A = 0.1$, a relatively low convexity (investment cost doubles at 1.9 times the nominal pace). For instance, in the base year (2008), building one Watt of new renewable capacity at the pace of 10 GW/yr costs 2 \$/W. At 20 GW/yr, it would cost 3.8 \$/W. Later, we perform a sensitivity anaysis on A.

The demand is assumed inelastic in the short-term, and follows an exogenous long-term growth. Its value in 2008 is calibrated as the reference fossil



(a) Production and capacity

(b) Levelized cost of electricity and electricity price

(c) Investment

(d) Marginal investment cost

Figure 3: Numerical application to the European electricity sector

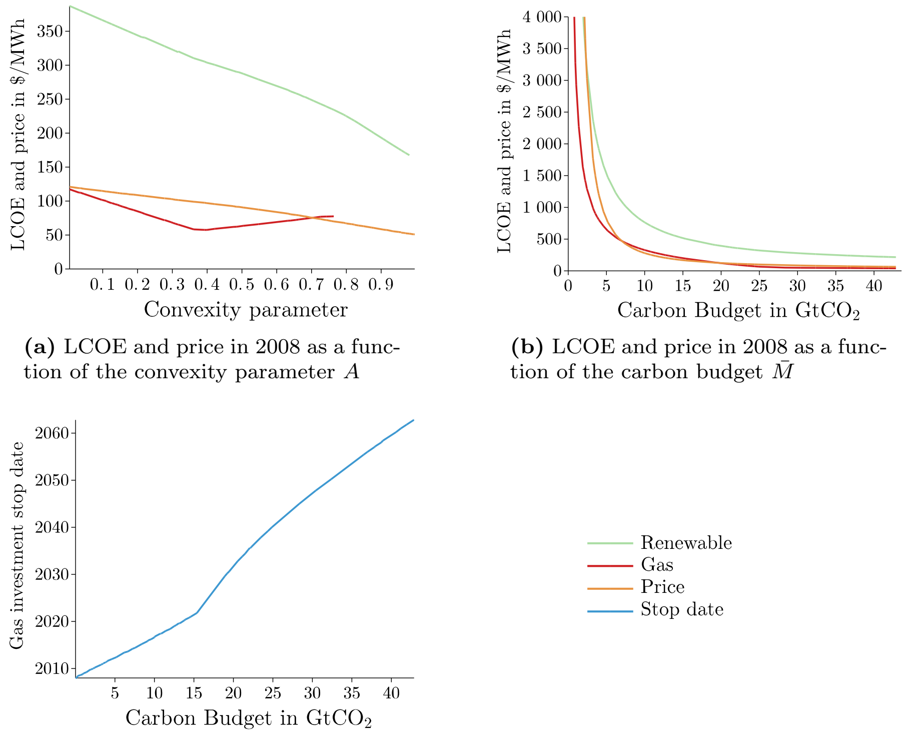
energy production (from coal, oil and gas), that is $D_0 = 1\,940$ TWh/yr (ENERDATA, 2012). The central scenario of the Roadmap forecasts a 700 TWh increase of electricity consumption between 2008 and 2050, corresponding to a linear increase of $G = 16.5$ TWh/yr^2. Demand after time t is thus:

$$D_t = D_0 + t \cdot G \tag{20}$$

The emission allowances allocated to the power sector amounted to $E_{\text{ref}} = 1.03$ GtCO_2/yr in 2008 (Trotignon and Delbosc, 2008), leading to a reference emission rate of 530 tCO_2/GWh. A linear decrease of these emissions until 2050, as planned in the Roadmap, yields a carbon budget of $\bar{M} = 22$ GtCO_2. A sensitivity analysis on $\bar{M}$ is performed later. We use $r = 5$ %/yr for the social discount rate.

4.2. Results

Fig. 3 shows production, investment, investment cost, the electricity price and the operational LCOE obtained in the numerical application. Despite lower



(a) LCOE and price in 2008 as a function of the convexity parameter A

(b) LCOE and price in 2008 as a function of the carbon budget $\bar{M}$

(c) Gas investment stop date as a function of the carbon budget $\bar{M}$

Figure 4: Sensitivity analysis on the carbon budget and the convexity parameter for the LCOE, the output price and the date when gas investment stops

fuel costs, the social planner does not invest in the legacy capacity, which is entirely phased out in 2028. Moreover, existing coal plants are decommissioned as soon as the climate policy is implemented (Fig. 3a). With our technology assumptions, the carbon budget is too lax to justify a complete transition by 2050, but instead phases out all fossil production by 2064.

As seen in Fig. 3c, investment in gas starts since the beginning of the transition, and stops at $\tau_\ell^- = 2035$ and gas is underused immediately after ($T_\gamma = T_h^- = 2036$). Fig. 4c illustrates how gas investment is sensible to the carbon budget. For any positive carbon budget, gas power plants are built at the beginning of the transition. Also, for any carbon budget, gas power plants are underused immediately after investment stops. Gas plants operate as a temporary bridge between coal and renewable, used to move some efforts from the short to the middle term. Only for very stringent carbon budget, lower than 17 $GtCO_2$, gas investment should stop before 2020 (remind the ambitious European roadmap implicitly sets a budget of 22 $GtCO_2$).

Investment in renewable power also starts from the beginning of the simulation (Fig. 3c). It grows over time, and only decreases for a short period from 2055 to 2064 when gas is phased out, as most of the power plants have already been replaced (Fig. 3a). Investment starts at 22 GW/yr in 2008, more than twice the actual average investment rate X_z, and reaches 70 GW/yr in

2060. Investment in renewable grow again after 2064 to maintain the installed renewable capacity and meet the growing demand.

Fig. 3d displays the resulting marginal costs for new capacities (gas and renewable) along the period. They essentially follow the investment trajectories. Gas investment costs decrease from 2015 on, as the average power plants becomes less and less carbon-intensive, making investment in low carbon capacity less and less profitable. Consistently with Prop. 2, the capacity cost is always higher for renewable than for gas.

Electricity prices are displayed in Fig. 3b. In a first phase (before 2028), the marginal capacity is the legacy dirty technology. The endogenous carbon price is 137 $/tCO_2$, representing 73 $/MWh or 63 % of the electricity price during the phase while the legacy power plant is marginal. After the dirty technology has been phased out, gas becomes the marginal technology and is under-used from 2036 to 2064. In between and after fossil technologies have been phased out, the electricity price equals the rental cost of renewable power plants (Lemma 1).

Fig. 3b shows the levelized costs of electricity computed against current prices (*operational LCOEs*) and compares them with the corresponding electricity price. The LCOE is higher than the electricity price for renewable plants, and lower for gas plants. As shown in Fig. 4, for large ranges of values for the convexity parameter of the investment cost function and for the carbon budget, the operational LCOE of is 2 to 5 times higher for renewable power than for gas. These simulations suggest that if decision makers decided investment in new capacity for the European electricity market by comparing LCOEs to current electricity prices, they would build too much gas capacity, and not enough renewable capacity.

5. Conclusion

Our analysis makes several simplifications. For instance, it has been shown using general equilibrium settings — extending the framework by Dasgupta and Heal (1974) — that knowledge accumulation and directed technical change play a key role on the optimal transition from fossil to renewable energy (Tahvonen and Salo, 2001; Acemoglu et al., 2012; André and Smulders, 2014). Knowledge spillovers would tend to increase investment in renewable on the optimal trajectory (Rosendahl, 2004; del Rio Gonzalez, 2008; Slechten, 2013), their effect would thus add to the effect of adjustment costs studied here. We do not model either the link between extraction cost and cumulative extraction, the effect of exploration, adjustment lags, ore quality, or market power, all mentioned by Krautkraemer (1998) as important factors to understand nonrenewable resource scarcity. Finally, we disregard the possibility to retrofit existing plants (notably with carbon capture and storage), and the possibility to use cleaner fuels (in particular derived from biomass) in existing power plants.

Notwithstanding these limitations, our analysis suggests that capacity constraints and adjustment costs play an essential role in the transition to different types of energy-intensive capital, and therefore in the dynamics of fossil fuel extraction. In particular, investment in renewable power should start before coal and gas resources are exhausted, and should always be higher than investment in gas power.

In terms of policy recommendations, we warn that the levelized cost of electricity, a good metric in theory, is biased against renewable power in practice, if it is computed against current prices. It should therefore be used cautiously when monitoring investment in the power sector. Indeed, optimal investment decisions in different types of capital require properly anticipating all the future prices of fossil fuels, capital costs, and the carbon tax. This stresses the importance that governments announce credible climate policies well in advance, and let a well-functioning market invest in consequence.

If governments are not able (or willing) to credibly commit to long-term climate targets, a pragmatic way to assess the transition to renewable energy may be to compare actual investment with normative scenarios provided by numerical models of the electricity sector (e.g., Seebregts et al., 2002; Loulou, 2008). Wider integrated assessment models can also be used, as they have been used to study the extraction of exhaustible resources (Waisman et al., 2012) and, at least implicitly, the optimal transition to clean capital (e.g., Weyant, 2004; Kriegler et al., 2014a,b). Such models usually take into account the limited ability to switch quickly from high- to low-carbon capital, by building on maximum investment speeds and maximum penetration rates (Wilson et al., 2013). As optimal investment pathways are in this case largely driven by such exogenous growth constraints, a key question for public policy is to discuss explicitly their calibration (Wilson et al., 2013; Vogt-Schilb and Hallegatte, 2014; Vogt-Schilb et al., 2014; Morris et al., 2014).

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A. First-order conditions and complementarity slackness conditions

The present value Hamiltonian associated with Problem (9) reads:

$$\begin{aligned}\mathcal{H} = e^{-rt}\Bigg(&u\left(\sum_i q_{i,t}\right) - \sum_i c_i(x_{i,t}) - \sum_i \nu_{i,t}\,(\delta\, k_{i,t} - x_{i,t}) \\ &- \mu_t \sum_i R_i\, q_{i,t} - \eta_t\left(m_t - \bar{M}\right) - \sum_i (\alpha_{i,t}\, q_{i,t} - \beta_{i,t}\, S_{i,t}) \\ &- \sum_i \gamma_{i,t}\,(q_{i,t} - k_{i,t}) + \sum_i \lambda_{i,t}\, q_{i,t}\Bigg)\end{aligned} \tag{A.1}$$

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial x_i} = 0 \iff c_i'(x_{i,t}) = \nu_{i,t} \tag{A.2}$$

$$\frac{\partial \mathcal{H}}{\partial q_i} = 0 \iff \lambda_{i,t} - \mu_t R_i - \alpha_{i,t} + u_t' = \gamma_{i,t} \tag{A.3}$$

$$\frac{\partial \mathcal{H}}{\partial k_i} = -\frac{\partial\left(e^{-rt}\nu_{i,t}\right)}{\partial t} \iff (\delta + r)\,\nu_{i,t} - \dot{\nu}_{i,t} = \gamma_{i,t} \tag{A.4}$$

$$\frac{\partial \mathcal{H}}{\partial m_t} = -\frac{\partial\left(e^{-rt}\mu_t\right)}{\partial t} \iff \dot{\mu}_t - r\,\mu_t = -\eta_t \tag{A.5}$$

$$\frac{\partial \mathcal{H}}{\partial S_i} = -\frac{\partial\left(e^{-rt}\alpha_{i,t}\right)}{\partial t} \iff \dot{\alpha}_i - r\,\alpha_{i,t} = -\beta_{i,t} \tag{A.6}$$

Where the first derivative of utility is denoted u_t'. The complementary slackness conditions are:

$$\forall i, t, \quad \lambda_{i,t} \geq 0, \quad q_{i,t} \geq 0 \qquad \text{and} \qquad \lambda_{i,t}\, q_{i,t} = 0 \tag{A.7}$$

Table A.5: Variables and parameters notations used in the model.

	Description	Possible units
i	technology index	
h	high-carbon technology (h)	
l	low-carbon technology (ℓ)	
z	zero-carbon technology (z)	
$k_{i,t}$	capacity of technology i at time t	GW
$q_{i,t}$	production of technology i at time t	GW
$x_{i,t}$	physical investment in technology i at time t	GW/yr
$\nu_{i,t}$	shadow price of new capacity i	\$/GW
μ_t	cost of emissions	\$/tCO_2
$\alpha_{i,t}$	cost of resource used by technology i	\$/MWh
$\gamma_{i,t}$	shadow rental cost of existing capacity i	\$/(GW· yr)
u'_t	output price	\$/GWh
$c_i(x_{i,t})$	monetary investment in technology i at time t	\$/yr
m_t	stock of atmospheric carbon	tCO_2
δ	depreciation rate	yr^{-1}
r	discount rate	yr^{-1}
R_i	emission rate of technology i	tCO_2/GWh
$\bar{M}$	carbon budget	tCO_2
$u\left(\sum_i q_{i,t}\right)$	consumer utility	\$/yr

$$\forall i,t, \quad \eta_t \geq 0, \quad \bar{M} - m_t \geq 0 \quad \text{and} \quad \eta_t\left(\bar{M} - m_t\right) = 0 \tag{A.8}$$

$$\forall i,t, \quad \beta_{i,t} \geq 0, \quad S_{i,t} \geq 0 \quad \text{and} \quad \beta_{i,t}\, S_{i,t} = 0 \tag{A.9}$$

$$\forall i,t, \quad \gamma_{i,t} \geq 0, \quad k_{i,t} - q_{i,t} \geq 0 \quad \text{and} \quad \gamma_{i,t}\left(k_{i,t} - q_{i,t}\right) = 0 \tag{A.10}$$

The transversality condition is replaced by the terminal condition that at some point the atmospheric carbon reaches its ceiling (7).

B. Investment phases

B.1. Carbon budget constraint and steady state

Let T_η be the date when the ceiling on atmospheric carbon is reached. The carbon-free atmosphere can be seen as a non renewable resource depleted by GHG emissions. In this context, the optimal current carbon price follows Hotelling's rule, i.e. grows at the discount rate r (A.5,A.8):

$$\forall t < T_\eta, \quad \mu_t = \mu e^{rt} > 0 \tag{B.1}$$

where μ is the carbon price at $t = 0$. Eq. B.1 reflects the fact that abatement realized at any time contributes equally to meet the carbon budget. We focus on the case where the carbon budget is binding, so that the carbon price μ_t is strictly positive.

Let T be the date when the system reaches a steady state. During the steady state, the clean backstop produces all the output. Indeed, atmospheric carbon is stable, hence emissions and production from high- and low-carbon fossil fuels must be nil (4,7,8):

$$\forall t \geq T,\ \dot{m}_t = 0 \implies q_{\ell,t} = q_{h,t} = 0 \tag{B.2}$$

When the carbon budget is binding, the date when the ceiling on atmospheric carbon is reached and the steady state necessarily coincide: $T = T_\eta$.

B.2. Output production and fossil fuel deposits

Initially, all the output is produced from coal by assumption. Coal production is eventually replaced by gas or renewable power. Let T_h^- be the date when high-carbon production stops:

$$\forall t \geq T_h^-,\ q_{h,t} = 0 \tag{B.3}$$

This happens necessarily before the steady state:

$$T_h^- \leq T \tag{B.4}$$

Similarly, let T_i^+ be the date when production of technology i (ℓ or z) starts, and T_ℓ^- the date when gas production stops. Note that only low-carbon production eventually stops, since zero-carbon energy is used indefinitely during the steady state.

Let T^β be the date when the low-carbon fossil fuel deposit is depleted. If it exists (i.e. if the gas deposit is eventually depleted), T^β coincides with the end of production:

$$\exists T^\beta \implies T_\ell^\beta = T_\ell^- \tag{B.5}$$

Fossil fuel prices then follow the Hotelling rule: their current price $\alpha_{\ell,t}$ is increasing at the discounting rate before they are exhausted (A.6,A.9):

$$\forall t < T^\beta,\ \alpha_{\ell,t} = \alpha_\ell\, e^{rt} > 0 \tag{B.6}$$

B.3. Investment and under-utilization of capacities

For coal to be phased out, production in one of the alternative technologies must have started:[14]

$$T_h^- \geq \min(T_\ell^+, T_z^+) \tag{B.7}$$

If gas is used during the transition, production from gas must decline before the steady state, either because gas reserves are depleted, or to reduce GHG emissions to zero. If it declines faster than the depreciation rate of capacities,

[14]We are classically assuming that the marginal utility tends to infinity when energy consumption tends to zero, granting that energy production is never nil.

installed gas capacities may be underused. Let $T_\gamma \in [T_\ell^+, T]$ be the date when gas production is lower than its capacity:

$$\forall t \geq T_\gamma, k_{\ell,t} > 0 \implies q_{\ell,t} < k_{\ell,t} \tag{B.8}$$

Let $\tau_i^+ \leq T_i^+$ be the date when investment in capacity i starts, and τ_i^- the date when it stops. Note that only the investment in gas stops eventually, since renewable capacity is used indefinitely during the steady state and must be maintained as $\delta > 0$.

If the low-carbon capacity is underused, investment in new low-carbon capacity is not optimal, since instead of investing the utilization rate could be increased first:

$$\tau_\ell^- \leq T_\gamma \tag{B.9}$$

We assume that coal capacity is overabundant at the beginning of the transition, meaning that its capacity is underused from the start. In other words, coal is marginal during the first phase of the transition. As two technologies cannot be marginal at the same time (Lemma 1), gas can be the marginal technology (and gas capacity gets underused) only after coal is completely phased out:

$$T_h^- \leq T_\gamma \leq T_\ell^- \tag{B.10}$$

If gas is used, it's thus used from the moment there are some gas capacities

$$\exists (T_\ell^+, \tau_\ell^+) \implies T_\ell^+ = \tau_\ell^+ \tag{B.11}$$

B.10 also means that if gas is used, once gas production ends, emissions are null. The end of gas production thus coincides with the steady state:

$$\exists T_\ell^- \implies T_\ell^- = T \tag{B.12}$$

If production does not decrease with time, the moment when gas starts being underused necessarily happens after wind started to produce:

$$T_z^+ \leq T_\gamma \tag{B.13}$$

Finally, because variable costs from renewable are nil, its never optimal not to use renewable at full capacity:

$$T_z^+ = \tau_z^+ \tag{B.14}$$

B.4. Summary of the transition phases

This leaves us with three possible transition profiles:

1. Two successive transitions. Gas first completely replaces coal, then renewable power replaces gas.

$$\overbrace{\tau_\ell^+ = T_\ell^+}^{(B.11)} \leq T_h^- \leq \underbrace{\overbrace{\begin{matrix} \tau_z^+ = T_z^+ \leq \tau_\ell^- \\ \tau_\ell^- \leq \tau_z^+ = T_z^+ \end{matrix}}^{(B.14)} \overbrace{\leq T_\gamma}^{(B.10)}}_{(B.13)} \leq \underbrace{T_\ell^- = T}_{(B.12)} \tag{B.15}$$

2. Gas and wind simultaneously replace coal, starting with gas.

$$\underbrace{\tau_\ell^+ = T_\ell^+}_{\text{(B.11)}} \leq \underbrace{\tau_z^+ = T_z^+}_{\text{(B.14)}} \leq \overbrace{\begin{matrix} T_h^- \leq \tau_\ell^- \\ \tau_\ell^- \leq T_h^- \end{matrix}}^{\text{(B.9,B.10)}} \leq \underbrace{T_\gamma \leq \overbrace{T_\ell^- = T}^{\text{(B.12)}}}_{\text{(B.10)}} \tag{B.16}$$

3. Gas and wind simultaneously replace coal, starting with renewable power.

$$\underbrace{\tau_z^+ = T_z^+}_{\text{(B.14)}} \leq \underbrace{\tau_\ell^+ = T_\ell^+}_{\text{(B.11)}} \leq \overbrace{\begin{matrix} T_h^- \leq \tau_\ell^- \\ \tau_\ell^- \leq T_h^- \end{matrix}}^{\text{(B.9,B.10)}} \leq \underbrace{T_\gamma \leq \overbrace{T_\ell^- = T}^{\text{(B.12)}}}_{\text{(B.10)}} \tag{B.17}$$

Fig. 2 illustrates the three possible transition profiles, by using following cost function (using parameters gathered in Table B.6):

$$c_i'(x_{i,t}) = C_i^m \cdot A \cdot I + C_i^m \cdot (1 - A) \frac{x_{i,t}}{X_i} \tag{B.18}$$

Table B.6: Parameters used to produce Fig. 2

	Fig. 2c	Fig. 2b	Fig. 2a		Fig. 2c	Fig. 2b	Fig. 2a
δ	.0333	.0333	.0333	α_z	0	0	0
$\bar{M}$	42	38	40	α_h	.055	.055	.055
D	1940	1940	1940	α_ℓ	.1	.06	.06
r	.05	.05	.05	C_z^m	75000	75000	80000
R_z	0	0	0	C_h^m	18000	18000	18000
R_h	.00063	.00063	.00063	C_ℓ^m	35000	12000	12000
R_ℓ	.0004	.0004	.0003	X_z	.00003	.0005	.001
H_z	7500	7500	7500	X_h	.005	.005	.005
H_h	7500	7500	7500	X_ℓ	.0001	.001	.01
H_ℓ	7500	7500	7500	A	.9	.9	.9
				I	12	12	2