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A dynamic model of recycling with endogenous technological breakthrough*

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Abstract

We develop a growth model in which the use of a non-renewable resource yields waste. Recycling waste produces materials of poor quality. These materials can be reused for production only once a dedicated R&D activity has made their quality reach an exogenous minimum threshold. The economy then switches to a fully recycling regime. We refer to this switch as the technological breakthrough.

We analyze the optimal trajectories of the economy and present the Ramsey-Keynes and Hotelling conditions in this context. We characterize the determinants of the date of the breakthrough, which is endogenous, as well as the discontinuity in the variables' paths that is induced by this breakthrough. We show, in particular, that the availability of a recycling technology leads to a more intense exploitation of the resource and possibly to lower levels of consumption before the breakthrough.

Keywords: Recycling; Non-renewable resource; Technical change; Growth

JEL classifications: C61, O44, Q32, Q53

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1 Introduction

"Recycling is defined as any reprocessing of waste material in a production process that diverts it from the waste stream, except reuse as fuel. Both reprocessing as the same type of product, and for different purposes should be included. Recycling within industrial plants *i.e.* at the place of generation should be excluded." (United Nations[1]). By using waste as an input in the production process, recycling alleviates the scarcity of other resources. However, even if current levels of recycling greatly vary from sector to sector, recycling activity in the world is still low today. For instance, a UNEP report states that "many metal recycling rates are discouragingly low, and a "recycling society" appears no more than a distant hope" (UNEP, 2011). Geyer et al. (2017) underline that "between 1950 and 2015, cumulative waste generation of primary and secondary (recycled) plastic waste amounted to 6300 Mt. Of this, approximately (...) 600 Mt (9%) have been recycled".

The main reasons behind this low-level activity are first that it remains comparatively expensive – *i.e.* non-recycled materials remain relatively cheap. Regarding municipal solid waste, for example, "in some cases the value of recyclables are less than the extra costs associated with collecting the disturbed waste" (World Bank, 2012). Second, the recycled materials are not always perfect substitutes for the virgin materials, which entails reduced marketing possibilities – this issue will be central in our analysis. This substitution capacity is mainly driven by the degree of maturity of the recycling process itself and by the induced quality range of recycled goods. If recycling glass or pulp allows producing good-quality bottles and paper that are (almost) perfect substitutes for primary goods (see e.g. Alani et al., 2012 or ADEME, 2009), the recycling capacity of more sophisticated products can be constrained by a deterioration of the physical characteristics of the virgin product during the recycling process. This concerns certain types of plastics (thermosets), for instance. "Thermosets (...) are characterized by their high resistance to mechanical force, chemicals, wear and heat. The robust properties of thermosets make them more difficult to recycle and they cannot be re-melted down and reformed like thermoplastics" (OECD, 2018). Similarly, carbon fiber reinforced polymer (CFRP) waste yields materials that cannot yet have the same industrial use as the virgin materials, particularly in advanced technology sectors such as aeronautics (Oliveux, 2015). This means that the quality of the recycled material is fundamental in some industries, which justifies investments to improve the physical properties of the recycled material, and not only the efficiency of the process.

[1] United Nations – Environmental indicators: http://unstats.un.org/unsd/environment/wastetreatment.htm

The aim of the present paper is to understand how an economy will invest in research so that recycled waste can be used at a large scale and to study how the economy switches to a fully recycling regime. To do so, we consider a model in which a recycling technology is available, but the current quality of recycled materials makes them non-usable by the production process. The only way to trigger the recycling activity is thus to improve the quality of these materials. This can be done by investing in a specific type of research and development (hereafter R&D). After a certain threshold quality level has been reached, a technological breakthrough occurs in the sense that the production of consumption goods starts using as inputs both the virgin (primary) resource and the recycled (secondary) waste. We characterize the optimal trajectories of the economy and their properties; in particular, we study the discontinuity occurring at the date of the technological breakthrough.

Natural resources and waste recycling is an economic issue that has been addressed in dynamic contexts by many authors.[2] In some models, recycling is motivated as an option to mitigate the pollution generated by waste disposal. Smith (1972) follows this approach but focuses on the two stationary corner solutions where it is optimal to recycle either 100% of the waste flow or nothing, depending on the comparison of the private cost of recycling with the public disutility of waste. Hoel (1978) analyzes the long-term path of an economy that consumes a non-renewable resource and a recycled resource. He shows how the environmental impact of the use of these resources affects the optimal trajectories of the economy. In Huhtala (1999), consumption is based on flows of renewable resource as well as flows of recycled materials. The use of the renewable resource slows down the growth of its stock and the use of the two types or resources generates waste that negatively affects households' utility. At each date, the recycling activity is determined by labor allocation between conventional production and recycling. In an endogenous growth context, Di Vita (2001) studies an economy that uses both a non-renewable resource and recycled materials, the use of which harms the environment by producing waste. The model endogenizes the degree of recyclability of the accumulated waste: investing in a dedicated R&D sector allows improving recyclability. In an "AK" growth model without natural resource, Boucekkine and El Ouardighi (2016) introduce recycling to lighten the flow of waste generated both by capital accumulation and consumption. Waste storing is

[2] A large part of the studies on recycling can be found in the industrial organization literature (see Ba and Mahenc (2018) for a survey on strategic behaviors of recycling firms). We have chosen to focus here on (general-equilibrium) dynamic contexts.

however not considered. Sorensen (2017) considers a Ramsey model in which the economy can recycle households' waste as well as raw material used in production. Recycling allows reducing the impact of consumption and production on the environment. The recycling technology is initially mature and the intensity of recycling activities progressively increases as natural resources get scarcer and the stock of capital gets higher.

In other studies, as in the present one, recycling is justified only as a way to sustain production in the long run when the economy is constrained by the scarcity of non-renewable natural resources. André and Cerda (2006) study an economy that uses two types of natural resources, only one of which being recyclable. They show that if recycling may alleviate resource scarcity in the short term, its ability to prevent negative long-run growth depends on how much the economy depends on non-renewable and renewable resources. Di Vita (2007) focuses on the degree of substitutability between the non-renewable resource and recycled waste in the production process. He then analyses its impact on the economy's growth path and the time profile of resource extraction. Pittel et al. (2010) also use an endogenous growth model with non-renewable and recycled resources; they consider that the waste flow resulting from the use of these resources depends on the level of economic activity. They carefully take into account the material balance equation (see also Ayres, 1999) and they show how a market for waste, and subsidies to resource extraction and recycling allow restoring the social optimum.

Last, our study can be partially related to the literature on durable non-renewable resources (see for instance Salant and Henderson, 1978, Levhari and Pindyck, 1981, or Stewart, 1980, and subsequent contributions), in which resource extraction allows to increase an accumulated stock which is a productive input (see *e.g.* Atewamba and Gaudet, 2014). In the present framework, it is the instantaneous flow of extracted resource that is used in production.

In the aforementioned literature, the recycling technology is immediately available and recycled materials can be used by the economy, even when the technology is subject to endogenous improvements through R&D as in Di Vita (2001). In the present paper, however, we assume that the recycling technology initially produces materials of poor quality that cannot be used in the production process (*i.e.* at a large scale). Therefore, we consider a specific sector of R&D devoted to improving the technical properties of the recycled resource. Here, technological improvements are needed so that the recycled resource reaches a minimum quality threshold. When this quality level is attained, the secondary material can be used and the recycling activity starts.

The growth model we develop can be sketched as follows. The production of a consumption good requires (general-purpose) knowledge, labor, and raw material. Raw material corresponds to flows of a non-renewable resource, in its virgin or recycled form. Its use yields waste flows that add to a pre-existing stock. The flows of materials that will be recycled will be drawn from the accumulating stock of waste. This means that, here, we assume away the destruction (e.g. incineration) of waste; we implicitly focus on resources, like steel, that can be extracted from scrap (coming from the manufacturing of consumer goods as well as coming from spent consumer products) all over the world (Björkman and Samuelsson, 2014). The economy can invest in a dedicated research sector to improve the quality of recycled materials. Recycled waste starts being used as an input as soon as its quality meets a minimum threshold. This threshold is assumed to be exogenous for simplicity, but the date at which the recycling activity starts – referred to as the technological breakthrough – is endogenous. The main trade-offs faced by the economy are the following: the intertemporal management of the stock of non-renewable resource, the intertemporal management of the stock of waste, the use of the virgin or the recycled resource, and the allocation of efforts between output and R&D.

The general optimal conditions derived from the social planner's program feature a Ramsey-Keynes condition and a Hotelling rule. The possibility of recycling makes these conditions more complex than in standard dynamic resource models. The technological breakthrough entails a discontinuity in the trajectory followed by the economy. When it occurs, resource use jumps down and then declines at a slower pace. At the same time, the amount of labor dedicated to R&D falls down. Besides, consumption jumps upwards and follows a new trajectory with a higher growth rate.

We also study how the exogenous parameters affect the socially optimal trajectories and, in particular, the date of the technological breakthrough. Higher values of the social discount rate, of the output elasticity of the material input or of the efficiency of the research sector make the breakthrough occur earlier. A larger initial stock of waste or a higher maximal recycling rate also bring forward the date of the breakthrough. Conversely, this date is postponed with a higher growth rate of the total factor productivity, a higher required quality threshold or a larger initial stock of virgin resource.

We finally consider the impact of the recycling activity and its timing on the economy.

To do so, we compare the trajectories of the studied economy to those of a) an economy in which recycling is never possible and b) an economy in which recycling is immediately possible (the breakthrough occurs at date 0). We show that, as compared to both a) and b), the economy first exploits more intensely the virgin resource and then exploits it less intensely after the breakthrough has occurred. In other words, we show that the possible future occurrence of a recycling technology accelerates resource extraction before the breakthrough. The remaining stock of virgin resource is, therefore, lower at each date. The use of the virgin resource makes the waste stock higher than in the never-recycling economy, but it becomes lower after a finite interval of time following the breakthrough, to remain so forever after. The availability of a recycling technology has more complex effects on consumption. We show how it may reduce consumption before the breakthrough in some cases.

The general model is exposed in Section 2. Section 3 presents the optimal program of the economy. Then, we characterize the socially optimal trajectories and we study their properties in Section 4. In Section 5, we analyze how the availability of a recycling activity affects the economy. Section 6 concludes.

2 The model

We adopt the following conventional notations. We denote by φ_x the partial derivative of any function $\varphi(\cdot)$ with respect to variable x when this function contains more than one argument: $\varphi_x \equiv \partial\varphi(\cdot)/\partial x$. The expression g_x characterizes the growth rate of variable x: $g_x(t) \equiv \dot{x}(t)/x(t)$. Last, for simplicity, we drop the time index when this causes no confusion.

We consider an economy where a final consumption good Y is produced from a raw material M and from labor L_Y according to the technology f. Denoting by A_Y the total factor productivity ("TFP" thereafter), the quantity produced at any time t is then given by the following expression:

$$Y(t) = f(A_Y(t), M(t), L_Y(t)), \tag{1}$$

where the production function $f(\cdot)$ is increasing and concave in each argument. We also assume that labor and physical materials are essential in production: $f(A_Y, 0, L_Y) = f(A_Y, M, 0) = 0$.

For simplicity, we take the growth of the TFP as exogenous. Denoting by g_{A_Y} the growth rate (positive and constant) of A_Y and by $A_{Y0} \equiv A_Y(0)$ the initial TFP index, we have $A_Y(t) = A_{Y0}e^{g_{A_Y}t}$.

The physical input M is made up of two types of materials: a non-renewable resource X – which we will hereafter refer to as the virgin resource – and a recycled secondary material Z – which we will refer to as the recycled resource. A quality index is associated to each of these materials. Quality levels differentiate the two types of materials. We denote by A_X the quality of the virgin resource, and by A_Z that of the recycled resource. $A_X(t)X(t)$ and $A_Z(t)Z(t)$ must then be viewed as the augmented material inputs that enter the production process at time t. In order to focus on the recycling-related activities, we assume that the quality index of the virgin resource is fixed and exogenous: $A_X(t) = \bar{A} > 0$, $\forall t$. The quality index of the recycled resource is subject to improvements resulting from specific R&D activities. The production of the raw material is given by $M(t) = m(\bar{A}X(t), A_Z(t)Z(t))$, where $m(\cdot)$ is increasing and concave in each argument, such that $m(0,0) = 0$, $m(0, A_Z Z) > 0$ and $m(\bar{A}X, 0) > 0$. To simplify the notations, we denote by $m_1(\cdot)$ and $m_2(\cdot)$ the partial derivatives of function $m(\cdot)$ with respect to its first and second arguments, respectively. The quality level of the two resources is not necessarily identical and one could consider cases in which they are not perfect substitutes.[3] However, in Section 4, we restrict the analysis to the case of perfect substitution.

We assume that, as long as its quality is lower than a given fraction h of the quality of the virgin resource, the recycled material cannot be introduced into the production process. Certain recycled materials must indeed achieve minimum mechanical performance before some industrial sectors start using them. For instance, further research is needed before the recycling of carbon fiber reinforced polymer (CFRP) allows producing fibers of quality that fully matches the virgin fibers' quality (Pimenta and Pinho, 2014). Once A_Z has reached the minimum threshold $h\bar{A}$, with $h > 0$, then it can be used in combination with the virgin material:[4]

$$M(t) = \begin{cases} m(\bar{A}X(t), 0), & A_Z(t) < h\bar{A} \\ m(\bar{A}X(t), A_Z(t)Z(t)), & A_Z(t) \geq h\bar{A} \end{cases} . \tag{2}$$

[3]CFRP is more and more intensely used in aircraft industry, but, so far, recycled fibers can only be used in non-critical structural components, like the interiors of aircraft (Pimenta and Pinho, 2014).

[4]The minimum threshold $h\bar{A}$ is exogenous and constant for technical reasons. This assumption allows us to study the date of the technological breakthrough. This leads to eliminate the case in which a positive investment in research does not allow the breakthrough to occur at any point in time. In other words, if the effort in research is not nil, there will be a breakthrough, whose date is endogenous.

Starting from a given initial level $A_{Z0} \equiv A_Z(0)$, the quality index $A_Z(t)$ of the recycled resource can be improved through the following endogenous R&D process:

$$\dot{A}_Z(t) = \delta L_A(t) A_Z(t), \tag{3}$$

where $\delta > 0$ is a parameter of productivity and $L_A(t)$ is the quantity of labor invested in this R&D activity at time t. For the problem to be meaningful, we clearly must assume that $0 < A_{Z0} < h\bar{A}$.

The economy is endowed with a fixed labor amount L, which can be devoted either to production or to R&D:

$$L_A(t) + L_Y(t) = L. \tag{4}$$

The virgin resource is extracted from a non-renewable stock according to a one-to-one technology: one unit of extracted resource yields one unit of virgin material. We assume that the extraction cost is negligible and then, the virgin resource cost is only captured by its scarcity rent. Denoting by $S(t)$ the stock of resource at time t, and by $S_0 \equiv S(0)$ the initial reserves, we have the following standard depletion process:

$$\dot{S}(t) = -X(t). \tag{5}$$

The consumption of $C(t)$ units of final good generates an instantaneous utility $u(C(t))$ to consumers. The utility function $u(\cdot)$ satisfies the standard properties (increasing, concave, Inada conditions). Moreover, the utility flows are discounted by consumers at the social discount rate ρ, supposed to be positive and constant.

Resource use generates waste that can be saved and reused. In other words, the use of the resource at date t allows to produce more, reduces the remaining reserves, but also yields waste accumulation; the accumulated stock of waste constitutes a stock of future potential resource. We assume that recycling is instantaneous, meaning that waste production and dismantling occur instantaneously and at the same time. Within the production process, only the primary physical inputs – virgin and recycled resources – yield waste. For simplicity, the waste content rates of the virgin and recycled materials, α and β respectively, with $\alpha, \beta \in (0, 1)$, are taken as exogenous and constant. Moreover, we also assume that there is no natural degradation process. At any time, the incoming flow of waste is then $\alpha X(t) + \beta Z(t)$. Let $W(t)$ be the cumulative amount of waste at time t, and $W_0 \equiv W(0)$ the initial stock inherited from the past. As a flow of waste $Z(t)$ is eventually used by the recycling sector and thus removed from the accumulated stock $W(t)$, we can write:

$$\dot{W}(t) = \alpha X(t) - (1-\beta)Z(t). \tag{6}$$

Note that, in this model, we do not consider specific costs of resource extraction or recycling. One could have considered that storing waste so that it can be used in the future is costly. Such costs could be expressed in terms of consumption good, labor, or we could assume that storing additional waste partially degrades the existing stock of waste. Such a feature of the model would complexify the intertemporal management of the stock of the non-renewable resource. Indeed, beyond reducing the remaining stock available for future use, the use of the resource at each date would entail an instantaneous cost. We assume away such a cost in order to maintain the model tractable.

3 The optimal program

The social planner program consists in determining the trajectories of resource extraction, waste recycling and efforts in R&D and production, that maximize the discounted sum of utility flows subject to the set of technical constraints. However, the problem turns out to be discontinuous since the final output has two different expressions depending on whether the quality index of the recycled material is smaller or larger than the threshold $h\bar{A}$. In this section, we consider separately these two successive phases.

Let T be the (endogenous) time at which A_Z reaches $h\bar{A}$, *i.e.* the date at which recycling becomes operational. As $g_{A_Z} = \delta L_A \geq 0$, the trajectory of A_Z is always non-decreasing. Henceforth, if such a finite time T exists, then it is unique. We define respectively by $\mathcal{P}_1$ and $\mathcal{P}_2$ the social planner programs before and after time T, and we solve them backwards.

3.1 Recycling phase

Once the recycling option becomes available, *i.e.* after time T, the raw material is expressed as $M = m(\bar{A}X, A_Z Z)$ and the optimal program writes:

$$(\mathcal{P}_2) : \max_{\{X,Z,L_A,L_Y\}} \int_T^{\infty} u(f(A_Y, M, L_Y))e^{-\rho(t-T)}dt,$$

subject to the labor use condition (4), to the dynamic constraints (3), (5) and (6), and to the initial condition $A_Z(T) = h\bar{A}$.[5]

[5] We will check ex-post the conditions for non-negative control variables and for L_A and L_Y smaller than L in a specific analytical example.

Denoting by λ_A, λ_S and λ_W the co-state variables associated with A_Z, S and W respectively, the Hamiltonian of program $\mathcal{P}_2$ (in current value) writes $\mathcal{H}_2 = u(C) - (\lambda_S - \alpha\lambda_W)X - (1-\beta)Z\lambda_W + \delta L_A A_Z \lambda_A$. The first-order conditions are:

$$
\begin{aligned}
u'(C) f_M \bar{A} m_1 &= \lambda_S - \alpha \lambda_W && (7)\\
u'(C) f_M A_Z m_2 &= (1-\beta)\lambda_W && (8)\\
u'(C) f_{L_Y} &= \delta A_Z \lambda_A && (9)\\
\dot{\lambda}_S &= \rho \lambda_S && (10)\\
\dot{\lambda}_W &= \rho \lambda_W && (11)\\
\dot{\lambda}_A &= (\rho - \delta L_A)\lambda_A - u'(C) f_M Z m_2 \,. && (12)
\end{aligned}
$$

As usual in this kind of models, the transversality conditions can be expressed as:

$$
\lim_{t\to\infty} e^{-\rho(t-T)} \lambda_\kappa(t)\kappa(t) = 0, \quad \text{for } \kappa = \{A_Z, S, W\}\,. \tag{13}
$$

Conditions (7)-(9) state that the marginal social gain (in terms of utility) of one unit of input must be equal to its corresponding social marginal cost. More precisely, in (7), the marginal social gain of one unit of virgin resource equals the scarcity rent λ_S of the non-renewable resource stock, reduced by $\alpha\lambda_W$ to take into account that this unit generates waste up to α%, which accumulates into the stock W whose shadow value is given by λ_W. Note that, as no negative externality is associated with the stock of waste, λ_W works as a scarcity rent and is unambiguously positive. The same interpretation applies to (8) for the recycled resource, except that it does not involve the stock of natural resource but directly the stock of waste. Last, equations (9) is a standard static arbitrage condition relative to the labor allocation between production or R&D. The left-hand side reads as the marginal social gain (in terms of utility) of increasing labor in production by one unit while the right-hand side represents the marginal social cost (in terms of knowledge value) of this labor reallocation resulting from a diminution of the effort devoted to R&D.

Conditions (10) and (11) imply that $\lambda_S(t) = \lambda_S(T)e^{\rho(t-T)}$ and $\lambda_W(t) = \lambda_W(T)e^{\rho(t-T)}$. Consequently, as both resource stocks have a positive value at time T (*i.e.* $\lambda_S(T) > 0$ and $\lambda_W(T) > 0$ from (7) and (8)), the transversality conditions (13) associated with S and W reduce to $\lim_{t\to\infty} S(t) = \lim_{t\to\infty} W(t) = 0$. The stock of natural resource and the stock of waste must be asymptotically exhausted:

$$
S(T) = \int_T^\infty X(t)dt \quad \text{and} \quad W(T) = \int_T^\infty [(1-\beta)Z(t) - \alpha X(t)]dt. \tag{14}
$$

By replacing λ_W into (7) by its expression in (8), we obtain the following equation:

$$\frac{\lambda_S}{u'(C)} = TMP_X, \quad \text{where } TMP_X \equiv \left[\bar{A}m_1 + \left(\frac{\alpha}{1-\beta}\right) A_Z m_2\right] f_M \,. \tag{15}$$

This equation states that the marginal social gain of virgin resource use expressed in units of good, that is the left-hand side of (15), and the total marginal productivity of the virgin resource, expressed by the right-hand side of (15), must be equal. Note that the total marginal productivity of the resource, denoted by TMP_X, embodies the recycling possibility. Any unit of virgin resource extracted is indeed used a first time, which increases the output by $\bar{A}m_1 f_M$, i.e. the marginal productivity of X in the raw material production, multiplied by the marginal productivity of M. This unit then generates $\alpha\%$ of waste from which $(1-\beta)\%$ can be valued through recycling. The ratio $\alpha/(1-\beta)$ can be interpreted as the recyclability factor of the virgin resource. Multiplying this rate by the marginal productivity $A_Z m_2 f_M$ of the recycled material yields the second increase in production induced by the virgin resource through recycling.

We now use Equation (15) to derive the two main conditions that characterize the socially-optimal intertemporal use of the virgin resource. Denoting by $\sigma(C)$ the inverse of the elasticity of intertemporal substitution, *i.e.* $\sigma(C) \equiv -Cu''(C)/u'(C)$, the growth rate of the marginal utility can be simply expressed as $-\sigma(C)g_C$. Log-differentiating (15) with respect to time and using (10), we obtain the first following intertemporal arbitrage condition:

$$\rho + \sigma(C)g_C = \frac{T\dot{M}P_X}{TMP_X}. \tag{16}$$

This is the Ramsey-Keynes condition in the specific context of our economy. The standard Ramsey-Keynes condition characterizes the socially optimal arbitrage made between consumption and capital accumulation. Here, the arbitrage is made between consumption and the use of the virgin resource. What is new is that, as shown in (15), the total marginal productivity TMP_X of the virgin resource features the term $\left(\frac{\alpha}{1-\beta}\right) A_Z m_2 f_M$, which accounts for the fact that the waste induced by the use of the virgin resource is recycled and used as an input for consumption good production.

Assume that the social planner decides to save one unit of resource at date t and to use it at date $t+dt$ instead. Not using the resource at date t entails a decrease in consumption by TMP_X units of good. At date $t+dt$, the return to keeping the unit of resource in situ is the increase in its total marginal productivity: $T\dot{M}P_X$. Condition (16) simply states that this return is equal to the amount of consumption that compensates households for loss of consumption at date t, that is, $[\rho + \sigma(C)g_C]TMP_X$.

The second dynamic arbitrage condition is obtained as follows. By log-differentiating (9) and replacing $\dot{\lambda}_A$ by its expression in (12), and noting that $\lambda_A = u'(C)f_{L_Y}/(\delta A_Z)$ from (9), one obtains a new expression of the term $\rho + \sigma(C)g_C$. Inserting it in Condition (16) yields:

$$\frac{T\dot{M}P_X}{TMP_X} = \frac{\dot{f}_{L_Y}}{f_{L_Y}} + \frac{\delta A_Z Z m_2 f_M}{f_{L_Y}}\,. \tag{17}$$

Equation (17) can be seen as a Hotelling condition in the context of a dynamic general equilibrium framework, though modified in two ways. First, there is no physical capital but knowledge (intellectual capital) accumulation with intertemporal spillovers. Second, the resource is first used in its virgin form and then reused (after partial deterioration) in its recycled form. The economic reasoning behind this condition is the following.

Assume again that the social planner decides to save one unit of resource at date t and to use it at date $t+dt$ instead. At date t, the amount of labor that compensates for this lesser use of resource is given by $\Delta L_Y(t) = TMP_X/f_{L_Y}$, that is, the total marginal productivity of the resource expressed in terms of labor.[6] Note that, by Equation (4), having increased labor use in production by $\Delta L_Y(t)$ implies decreasing the research effort in recycling by $\Delta L_{A_Z}(t) = -\Delta L_Y(t)$. As mentioned above, the return to keeping the unit of resource in situ at date $t+dt$ is the increase in its total marginal productivity (that includes the use of the resource in its recycled form): $T\dot{M}P_X \equiv \Delta\tilde{Y}_1(t+dt)$. Meanwhile, the lesser effort in research entails less production of knowledge A_Z, and thus less output at date $t+dt$. Since $\delta A_Z Z m_2 f_M$ is the increase in output production induced by a marginal increase in labor devoted to research[7], the resulting loss of output at date $t+dt$ is given by $\Delta L_{A_Z}(t)\left[\frac{d(\delta A_Z Z m_2 f_M)}{dt} + \delta A_Z Z m_2 f_M\right] = \frac{TMP_X}{f_{L_Y}}\left[\frac{d(\delta A_Z Z m_2 f_M)}{dt} + \delta A_Z Z m_2 f_M\right] \equiv \Delta\tilde{Y}_2(t+dt)$. Since $\delta A_Z Z m_2 f_M = f_{L_Y}$, that is, the marginal productivity of labor is the same in production and research, then $\Delta\tilde{Y}_2(t+dt) = \left(\frac{TMP_X}{f_{L_Y}}\right)[\dot{f}_{L_Y} + \delta A_Z Z m_2 f_M]$. Resource use is efficient if $\Delta\tilde{Y}_1(t+dt) = \Delta\tilde{Y}_2(t+dt)$, that is, $T\dot{M}P_X = \left(\frac{TMP_X}{f_{L_Y}}\right)[\dot{f}_{L_Y} + \delta A_Z Z m_2 f_M]$. This is Condition (17).

[6] As the discrete changes in stocks at a given time t are nil, then $\Delta W = 0$ and $\Delta Z = \left(\frac{\alpha}{1-\beta}\right)\Delta X$ from (6). The discrete change in raw material M is $\Delta M = \bar{A}m_1\Delta X + (A_Z\Delta Z + Z\Delta A_Z)m_2$, with $\Delta A_Z = 0$. When $\Delta X = -1$, we can thus write $\Delta M = -\bar{A}m_1 - \left(\frac{\alpha}{1-\beta}\right)A_Z m_2$. As $\Delta Y = f_{A_Y}\Delta A_Y + f_M\Delta M + f_{L_Y}\Delta L_Y$, with $\Delta A_Y = 0$, then ΔL_Y must be equal to $-\frac{f_M}{f_{L_Y}}\Delta M = \frac{TMP_X}{f_{L_Y}}$ to maintain the same level of output, that is to have $\Delta Y = 0$.

[7] Indeed, δA_Z is the marginal productivity of labor in research (see (3)) and $Z m_2 f_M$ the marginal productivity of knowledge dedicated to the recycled resource.

3.2 Pre-recycling phase

Before time T, as the secondary material cannot be used for production yet, we have $M = m(\bar{A}X, 0)$. Denoting by V_2 the value function of program $\mathcal{P}_2$ at time T, we can write the initial program $\mathcal{P}_1$ as follows:

$$(\mathcal{P}_1): \max_{\{X, L_A, L_Y\}} \int_0^T u(f(A_Y, M, L_Y))e^{-\rho t}dt + e^{-\rho T}V_2(T, S(T), W(T)),$$

subject to the labor use constraint (4), to the dynamic constraints (3), (5), (6), and to the terminal condition $A_Z(T) = h\bar{A}$, given that the terminal date T is free. As the cumulative waste equation (6) is now reduced to $\dot{W} = \alpha X$, the trajectories of the resource reserves and of the waste stock are linked through the following relation:

$$W(t) = W_0 + \alpha(S_0 - S(t)),\ \forall t \in [0, T).$$

Keeping the same notations for the co-state variables, the Hamiltonian writes now $\mathcal{H}_1 = u(C) - (\lambda_S - \alpha\lambda_W)X + \delta L_A A_Z \lambda_A$. The first-order conditions of $\mathcal{P}_1$ are very similar to those of $\mathcal{P}_2$. Conditions (7), (9), (10) and (11) are the same. Condition (8) is no longer valid, whereas (12) becomes:

$$\dot{\lambda}_A = (\rho - \delta L_A)\lambda_A. \tag{18}$$

Note that even if some conditions have the same expression, the anticipation of the recycling option availability at time T is captured by the shadow prices, which may follow different trajectories than under $\mathcal{P}_2$. The transversality conditions at time T are:[8]

$$\mathcal{H}_1(T^-) = \rho V_2(\cdot) - \frac{\partial V_2(\cdot)}{\partial T} \tag{19}$$

$$\lambda_S(T^-) = \frac{\partial V_2(\cdot)}{\partial S(T)} \tag{20}$$

$$\lambda_W(T^-) = \frac{\partial V_2(\cdot)}{\partial W(T)}. \tag{21}$$

Last, the intertemporal trade-off condition writes:

$$\rho + \sigma(C)g_C = \frac{d(f_M m_1)/dt}{f_M m_1} = \frac{\dot{f}_{L_Y}}{f_{L_Y}}, \tag{22}$$

which means that, under program $\mathcal{P}_1$, the productivity of the resource and of labor must grow at the same rate.

[8]See Table 7.1 in Léonard and Long (1992) for a summary of the common transversality conditions.

4 Optimal trajectories of the economy

To illustrate the recycling problem with endogenous technical breakthrough and provide an example of optimal trajectories, we consider the following standard functional forms. Utility is characterized by a CES function: $u(C) = C^{1-\sigma}/(1-\sigma)$, with $\sigma > 0$. Production technology is described by a Cobb-Douglas function: $f(A_Y, M, L_Y) = A_Y M^{\epsilon} L_Y^{1-\epsilon}$, with $\epsilon \in (0,1)$. Last, we state that both types of resources are perfect substitutes (as in Di Vita, 2001, or Pittel *et al.*, 2010): $m(\bar{A}X, A_Z Z) = \bar{A}X + A_Z Z$.

Using these specified analytical forms, we study in this section the main qualitative properties of the optimal paths we have obtained. In particular, we explain their behavior at the time the economy switches from the pre-recycling to the recycling phases. We also analyze the sensitivity of the optimal variables to some key parameters of the model.

We can remark that, due to the assumption of perfect substitution between the two types of resources, $m_1 = m_2 = 1$. From the log-differentiation of (7)-(8) and using (10)-(11), we obtain that the marginal productivity of the natural resource and of the recycled material, respectively $\bar{A} f_M$ and $A_Z f_M$, must grow at the same rate ρ. The additional assumption of a constant quality index for the virgin resource then simplifies the analysis as it implies that the quality index of the recycled resource must also be constant. An immediate consequence is that no more effort in R&D is made once the quality of the secondary raw material has reached the required (minimum) threshold:

$$\forall t \geq T: \; A_Z(t) = h\bar{A}, \quad L_A(t) = 0 \quad \text{and} \quad L_Y(t) = L. \tag{23}$$

4.1 Qualitative properties of the optimal trajectories

The computational details of the social planner's solution are described in Appendix A.1. Moreover, this optimal solution must be such that $X(t)$ and $Z(t)$ are positive, and such that $L_A(t), L_Y(t) \in [0, L]$. As shown in Appendix A.1.4, this corresponds to a set of parameters that must satisfy conditions (A.26)-(A.28).

Labor allocation

Formal expressions of L_A and L_Y are given by:

$$L_A(t) = L - L_Y(t) = \begin{cases} L - L_{Y0} e^{-\tilde{k}t} & , \; t < T \\ 0 & , \; t \geq T \end{cases} \qquad \text{with } \tilde{k} \equiv \frac{\rho - (1-\sigma) g_{A_Y}}{\sigma} > 0 \, , \tag{24}$$

where the initial level L_{Y0} of productive labor is endogenously determined from the transversality conditions at time T (see expression (A.24) in Appendix A.1.4). Before time T, the effort L_A devoted to the improvement of the recycled resource quality continuously rises ($\tilde{k} > 0$ from (A.28)) and it stops once the required quality threshold is reached. The effort in R&D thus instantaneously falls to zero at time T, as depicted in Figure 1-a. Consequently, since the total labor supply L is constant (c.f. (4)), the effort in production L_Y declines throughout the first phase, then jumps to level L at date T, and remains at this level onwards.

The quality index of the recycled resource is exponentially increasing until T and then forever equal to $h\bar{A}$ (not illustrated, see expression (A.23)). The optimal breakthrough date T is endogenously determined in such a way that A_Z is continuous, i.e. $A_Z(T) = h\bar{A}$. We show in Appendix A.1 that this date is defined as the solution of Equation (A.25).

[Place Figure 1 here]

Virgin resource use

The optimal path of the virgin resource use is given by:

$$X(t) = \begin{cases} \tilde{k} S_0 e^{-\tilde{k}t} & , t < T \\ k S_0 e^{(k-\tilde{k})T - kt} & , t \geq T \end{cases} \qquad \text{with } k \equiv \frac{\rho - (1-\sigma) g_{A_Y}}{1 - \epsilon(1-\sigma)} > 0\,. \tag{25}$$

Resource use is always exponentially decreasing through time – at rate $\tilde{k}$ during the pre-recycling phase and at rate k during the recycling phase – and it asymptotically tends towards zero. In this sense, it follows a standard Hotelling depletion process, but discontinuous here. Let $\Delta X(T) \equiv X(T^+) - X(T^-)$ denote the magnitude of the jump made by X at time T. From (25), we have $\Delta X(T) = -(\tilde{k} - k) S_0 e^{-\tilde{k}T}$, which is negative as $\tilde{k} - k = (1-\sigma)(1-\epsilon)k/\sigma > 0$ from (A.26). This means that virgin resource use jumps down at time T and then follows a less sloping declining path, as illustrated in Figure 1-b. At that time indeed, the constraint on the virgin resource consumption is partially relaxed since i) recycling becomes operational, and ii) the whole labor flow is allocated to production (see above). Consequently, the resource stock S is continuously declining until its full exhaustion, but its trajectory is less steep declining after T than before T (see Figure 1-c).

Recycling activity and waste accumulation

The optimal trajectories of Z and W are given by:

$$Z(t) = \begin{cases} 0 & , t < T \\ k\Phi S_0 e^{-k(t-T)} & , t \geq T \end{cases} \quad \text{with } \Phi \equiv \frac{W_0 + \alpha S_0}{(1-\beta)S_0}, \tag{26}$$

$$W(t) = \begin{cases} W_0 + \alpha S_0(1 - e^{-\tilde{k}t}) & , t < T \\ \left[W_0 + \alpha S_0(1 - e^{-\tilde{k}T})\right] e^{-k(t-T)} & , t \geq T \end{cases}. \tag{27}$$

In (26), Φ can be interpreted as the maximal recycling rate of the virgin resource stock. Indeed, the total use of the recycled resource $\int_T^\infty Z(t)dt$ amounts to $(W_0 + \alpha S_0)/(1-\beta)$, which formally reads as the maximal quantity of waste that can be generated over the planning horizon divided by the net recycling rate of the secondary material. By dividing this expression by S_0, we obtain a ratio reflecting the maximal recycling potential of the virgin resource.

As shown in Figure 1-d, the flow of recycled material is first nil. At time T, it jumps upwards to its maximal value $Z(T) = k\Phi S_0$ and then behaves as $X(t)$ by following a trajectory that exponentially declines at rate k and that asymptotically converges towards zero. This partially explains the fact that, as previously mentioned (see Figure 1-b), virgin resource use is more intensive before T. During the pre-recycling phase indeed, resource use has two purposes: first, the immediate production of output in order to meet consumption needs; second, the accumulation of a stock of waste that will be used to produce recycled material during the second phase. Since the maximum of the recycling activity is reached at time T and then steadily declines, the waste stock needs to be high enough at this date.

The stock of waste, yielded by the use of the virgin and recycled resources, is first increasing before T and next declining until exhaustion (see Figure 1-e).

Consumption

As we show below, the optimal consumption trajectory can be either increasing or decreasing. This path is given by:

$$C(t) = \begin{cases} C_0 e^{\tilde{g}_C t} & , t < T \\ C_T e^{g_C(t-T)} & , t \geq T \end{cases}, \tag{28}$$

where $C_0 \equiv C(0) = A_{Y0}(\bar{A}\tilde{k}S_0)^\epsilon L_{Y0}^{1-\epsilon}$ and $C_T \equiv C(T^+) = A_Y(T)[\bar{A}kS_0(e^{-\tilde{k}T} + h\Phi)]^\epsilon L^{1-\epsilon}$. The growth rates of consumption during the pre-recycling and recycling phases, respectively denoted by $\tilde{g}_C$ and g_C, are:

$$\tilde{g}_C = \frac{g_{A_Y} - \rho}{\sigma} \quad \text{and} \quad g_C = \frac{g_{A_Y} - \epsilon\rho}{1 - \epsilon(1-\sigma)}. \tag{29}$$

It is easy to see that $g_C - \tilde{g}_C = \frac{(1-\epsilon)[\rho-(1-\sigma)g_{A_Y}]}{\sigma[1-\epsilon(1-\sigma)]}$ is positive from the existence condition (A.28). In other words, consumption grows faster – or decreases more slowly in case of negative rates – after date T.

We know that consumption is a combination of three factors (see (1)): TFP (A_Y), material input (M) and labor (L_Y). Its potential growth is only (exogenously) driven by A_Y, as L_Y is declining during the pre-recycling phase and constant afterwards and M is always decreasing (as a linear combination of virgin and recycled resources, both being declining as previously shown). More precisely, we show that $\tilde{g}_C$ and g_C can be positive or negative depending on the level of the TFP growth rate g_{A_Y} relative to the social discount rate ρ and the input substitution parameter ϵ. The three following cases can occur:

$$\begin{cases} \rho < g_{A_Y} & \Rightarrow \quad g_{\tilde{C}} > 0 \text{ and } g_C > 0\,, \\ g_{A_Y} \leq \rho < g_{A_Y}/\epsilon & \Rightarrow \quad \tilde{g}_C \leq 0 \text{ and } g_C > 0\,, \\ g_{A_Y}/\epsilon \leq \rho & \Rightarrow \quad g_{\tilde{C}} < 0 \text{ and } g_C \leq 0\,. \end{cases}$$

As usual, time impatience favors immediate consumption to the detriment of future consumption. Consequently, the larger the social discount rate, the weaker the consumption growth rate, with negative values below a given threshold. For intermediate values of ρ, the optimal growth path may be U-shaped (with a discontinuity at the bottom of the U): decreasing over time during the non-recycling phase, and then increasing during the recycling phase. For simplicity, and to reduce the number of scenarios, we only illustrate in Figure 1-f the case where both g_C and $\tilde{g}_C$ are positive, that is the case where the social discount rate is not too high.

Last, we turn to the discontinuity of C at time T. The combination of a downward jump in virgin resource and upward jumps in both recycled material and productive labor results, *a priori*, in an undetermined overall jump in consumption. This jump amounts to $\Delta C = \epsilon C \Delta M/M + (1-\epsilon) C \Delta L_Y/L_Y$. As previously discussed, $\Delta L_Y(T) > 0$, and then $\Delta M(T) > 0$ is a sufficient condition for consumption to jump upwards at time T. From $\Delta X(T) = -[(1-\sigma)(1-\epsilon)/\sigma] k S_0 e^{-\tilde{k}T}$ and $\Delta Z(T) = k\Phi S_0$ (see previous findings), it can easily be shown that:

$$\Delta M(T) = \bar{A}[\Delta X(T) + h\Delta Z(T)] = \bar{A} k S_0 \left[h\Phi - \frac{(1-\sigma)(1-\epsilon)}{\sigma} e^{-\tilde{k}T} \right] ,$$

which is positive from the existence condition (A.27). This means that, at time T, the instantaneous rise in recycled resource use overrides the fall in virgin resource use and the raw material production jumps upward. Then, the jump in consumption is also positive.

4.2 Comparative dynamic analysis

We now perform some comparative dynamics so as to analyze the sensitivity of the most relevant variables, including the date T of occurrence of the technological breakthrough, to the exogenous parameters of the model. We do not intend to be exhaustive here, we rather focus on the main insights – a complete presentation of the comparative dynamics is provided in Appendix A.2, Table 1.

Parameters characterizing the preferences

An increase in the social discount rate ρ means that the representative household obtains more utility from current consumption relative to future consumption. In order to increase consumption today, the social planner increases early extraction, which means that future extraction gets lower. The extraction of the virgin resource before T is thus accelerated. Simultaneously, a higher ρ does not affect the initial labor allotment between production and R&D, but it has an impact on the dynamics of L_Y and L_A: it increases the growth rate of L_A. This increases the speed of convergence of A_Z towards the minimum quality threshold $h\bar{A}$. Hence, for a given initial quality A_{Z0}, the date at which the recycled resource reaches this threshold and starts to get used in the production process is put forward. In other words, a higher social discount rate makes the breakthrough occur earlier. Optimal consumption starts from a higher initial level (as X_0 increases) and grows slowlier.

An increase in the elasticity of marginal utility σ means that the representative household derives more utility from a uniform consumption path, *ceteris paribus*. Here, in order to achieve a flatter consumption path, the growth rates $\tilde{g}_C$ and g_C must both be reduced. However, the effects of an higher σ on the initial levels of the main variables and on the transition time are subject to further assumptions, as stated in Table 1.

Parameters characterizing the output production technology

In our framework, the total factor productivity (TFP) A_Y is taken as exogenous and growing at constant rate g_{A_Y}. A higher growth of TFP allows the planner to slow down resource extraction between 0 and T: less resource is used during the early periods and more in the future periods. In other words, g_X (which is equal to $-\tilde{k}$ for $t < T$ and to $-k$ for $t \geq T$) gets higher for all t. By complementarity in production (for a given ϵ), g_{L_Y} increases too. Consequently, the effort in research L_A grows slowlier at each date $t < T$.

Thus R&D is less intensive, the quality of the recycled resource grows slowlier, and the technological breakthrough is reached later. For a given level of A_{Y0}, the increase in g_{A_Y} entails higher A_Y for all t which pushes consumption up.

An increase in the output elasticity of raw material ϵ means that the material input M gets more productive (relative to labor). In this case, the economy relies more heavily on the flows of resource and hence the need for an additional source of material is more pressing. Therefore the economy transfers labor from production to research and by this makes the technological breakthrough happen sooner: T gets lower.

Parameters characterizing the research sector

Parameter $\bar{A}$ characterizes the quality level of the virgin resource. With a higher $\bar{A}$, the economy requires that the recycled resource reaches a higher quality threshold before it starts to get used in the production process. *Ceteris paribus* and, in particular, for a given investment in R&D, this postpones the date T of the breakthrough. The impact of A_{Z0}, the initial quality level of the recycled resource, is obviously opposite. A higher A_{Z0} means that the distance to (a given) level $h\bar{A}$ is shorter; the quality threshold is thus reached faster, and the technological breakthrough occurs at an earlier date.

A higher δ means that the R&D sector is more efficient and the quality of the recycled resource grows faster. The breakthrough occurs earlier here too.

Parameters characterizing the recycling sector

With a higher S_0, the initial stock of virgin resource is larger and the need for a complementary resource is thus less urgent. Hence the social planner can use more virgin resource at each date between 0 and T, that is X gets higher, and devotes more labor to production and thus less to research: L_A is lower. Thus the quality of the recycled resource grows less fast and the technological breakthrough occurs later.

The initial stock of waste W_0 and the waste content rates of the two resources, α and β have similar effects on the economy's trajectories and henceforth on the date of the technological breakthrough. A larger initial stock of waste W_0 means that, for a given path of resource use between 0 and T, the stock of recycled resource to be used from date T on is larger. This makes the technological breakthrough more significant. Hence the economy increases its effort in R&D, that is $L_A(t)$ gets higher for all $t < T$, and the recycled resource starts to get used sooner in the production process. Similarly, if the waste content of the virgin resource, α, and the waste content of the recycled resource, β, are higher, the planner gets an incentive to invest more in research, and the technological breakthrough occurs earlier. We have introduced $\Phi \equiv \frac{W_0+\alpha S_0}{(1-\beta)S_0}$ as the maximal recycling rate; a higher Φ thus also brings forward the date of the breakthrough.

5 Effects of the availability of a recycling technology

We analyze here how the availability of a recycling technology affects the trajectory followed by the economy. To do so, we consider three cases. First, an economy in which no recycling technology is available at any date t. Hereafter, we refer to this case as the "never-recycling economy". This is obviously a particular case of the economy studied in the last sections, in which date T tends towards infinity (equivalently, $h \to \infty$). Second, we consider an economy in which the recycling option is available at date 0. In other words, the quality threshold at which the recycled resource starts to be used is instantaneously met: $T = 0$, and thus $A_{Z0} = h\bar{A}$. We will refer to this case as the "always-recycling economy". The third case is the general case studied in Sections 2-4 in which recycling is possible after date T. We will refer to this case as the "T-recycling economy".

We first characterize the trajectories of the never and always-recycling economies. In both cases, we can easily show that the marginal social value of (A_Z-type) R&D is nil. Consequently, at any point in time, the total amount of available labor is allocated to production and the set of control variables reduces to the uses of raw material only. However, in this "cake-eating" problem, the nature and the availability of the stocks of resources differ in each case. In the never-recycling economy, only the virgin resource can be used and the final consumption good is produced according to the following technological form: $C = A_Y(\bar{A}X)^{\epsilon}L^{1-\epsilon}$. In the always-recycling economy, thanks to recycling activities, the initial stock of waste W_0 can be exploited at rate Z as a complementary resource: $C = A_Y(\bar{A}X + h\bar{A}Z)^{\epsilon}L^{1-\epsilon}$.

The optimal trajectories of such a program are the following: [9]

$$
\begin{aligned}
X_n(t) &= X_a(t) = kS_0e^{-kt} \quad \text{and} \quad S_n(t) = S_a(t) = S_0e^{-kt}, && (30)\\
Z_n(t) &= 0 \quad \text{and} \quad Z_a(t) = k\Phi S_0e^{-kt}, && (31)\\
W_n(t) &= W_0 + \alpha S_0\left(1-e^{-kt}\right) \quad \text{and} \quad W_a(t) = W_0e^{-kt}, && (32)\\
C_n(t) &= C_{n0}e^{g_C t}, \text{ with } \quad C_{n0} = A_{Y0}(\bar{A}kS_0)^{\epsilon}L^{1-\epsilon}, && (33)\\
C_a(t) &= C_{a0}e^{g_C t}, \text{ with } \quad C_{a0} = A_{Y0}(\bar{A}kS_0)^{\epsilon}(1+h\Phi)^{\epsilon}L^{1-\epsilon}, && (34)
\end{aligned}
$$

From (30), virgin-resource use in the never-recycling economy is identical to virgin-resource use in the always-recycling economy. As shown in Section 4.1, the emergence of a technological breakthrough at date $T > 0$, which characterizes the T-recycling economy, yields a discontinuity in the trajectory of resource use at date T. These three extraction paths are depicted in Figure 2-a. Before the breakthrough, we can see that the T-recycling economy (solid line) exploits more intensely the resource – as compared to the never and always-recycling economy (dashed line and doted line, respectively). Indeed, part of the labor flow L_Y directed to production in the never and always-recycling economies is affected to R&D in the T-recycling economy. The T-recycling economy thus compensates for this lower input use by using higher flows of resource X. Moreover, this yields additional flows of waste: the stock of waste that can be recycled from date T on is thus higher.

After the breakthrough, the whole amount of labor available L is devoted to production and the recycled resource Z starts to get used. This allows the T-recycling economy to use lower levels of virgin resource X at each date $t > T$. Figure 2-b illustrates the more intense exploitation of the virgin resource stock S by the T-recycling economy at each date $t > 0$.

[Place Figure 2 here]

Figure 2-c depicts the trajectories of the waste stock. From (32), the stock of waste W_n in the never-recycling economy grows over time and asymptotically tends to its upper limit level $W_0 + \alpha S_0$ (see the dashed curve in Figure 2-c). In other words, as the never-recycling economy uses the non-renewable resource, the associated waste adds to the existing stock until the whole resource stock is exhausted. The stock of waste W_a in the always-recycling economy exponentially declines from W_0 down to 0. This means that the amount of waste

[9] The subscript n denotes variables characterizing the never-recycling economy and the subscript a, variables characterizing the always-recycling economy.

produced is instantaneously re-used by the production process and that the stock of waste follows a continuous depletion process. The impact of the immediate availability of a recycling technology is therefore unambiguous: the stock of waste is lower at each date $t > 0$.

Equation (27) presents the trajectory of the stock of waste in the T-recycling economy. Before date T, W increases and is higher than W_n at each date. At date T, the stock reaches its maximum level $W_0 + \alpha S_0(1 - e^{-\tilde{k}T})$ and then starts to steadily decline, to asymptotically converge towards 0 since, in the long-run, the T-recycling economy uses waste until its stock gets exhausted (see Section 4.1). Conversely, the never-recycling economy keeps accumulating waste after date T and W_n gets higher than W at any date $\hat{T} > T$. In other words, the recycling option makes the stock of waste larger until date $\hat{T}$. After this date, the stock of waste is lower in the T-recycling economy.[10]

From the expressions (33) and (34), one can see that the growth rate of consumption is identical in the never-recycling and the always-recycling economies. Moreover, it is straightforward that $C_{a0} > C_{n0}$. One can thus conclude that the immediate availability of a recycling technology has a positive impact on consumption: $C_a(t) > C_n(t)$ for all t.

Comparing consumption in the T-recycling economy with consumption in the always and never-recycling cases involves numerous complex scenarios that depend on various parameters of the model, notably including Φ. Consumption grows more slowly in the T-recycling economy than in the two other cases before the breakthrough (remind that $\tilde{g}_C < g_C$); after the breakthrough, the growth rates of consumption are equal. Deriving clear and exhaustive conclusions on consumption levels is not possible at this stage. However, for high values of Φ, the initial consumption levels verify the following ranking: $C_{a0} > C_{n0} > C_0$.[11] In such a case, before the breakthrough, the T-recycling economy is characterized by consumption levels lower than those in the always and never-recycling economies.[12]

[10] Regardless of its date of occurrence, the maximal level reached by a stock of pollutant is a serious concern for many – since irreversible damages may occur after certain thresholds. Here, this maximal level $W(T)$ is reached earlier by the T-recycling economy; however, it is lower than the maximal level reached by the never-recycling economy. Indeed, the difference between the two is equal to $[W_0 + \alpha S_0(1 - e^{-\tilde{k}T})] - (W_0 + \alpha S_0)$, which is negative since $e^{-\tilde{k}T} > 0$.

[11] From (28) and (33), we obtain: $C_{n0} - C_0 = A_{Y0}(\bar{A}kS_0)^\epsilon L_{Y0}^{1-\epsilon}\left[(L/L_{Y0})^{1-\epsilon} - (\tilde{k}/k)^\epsilon\right]$. Replacing L_{Y0} by its expression (A.24), this expression is proved to be positive for $h\Phi \geq \frac{(1-\sigma)(1-\epsilon)}{\sigma}\left(\tilde{k}/k\right)^{\frac{\epsilon}{1-\epsilon}}$.

[12] The initial consumption C_0 in the T-recycling economy decreases with Φ (cf. Appendix A.2) as more initial effort is put in R&D instead of production.

6 Concluding remarks

The aim of this paper is first to study the socially optimal path of an economy that needs to invest in research so that recycling becomes operational. We also want to understand how the emergence of the recycling technology – what we call the technological breakthrough – affects the economy. To do so, we utilize a dynamic model with recycling and R&D, in which the use of recycled materials produced from waste requires a prior investment in research so that the quality of the recycled resource meets a certain standard.

We first present the general optimality conditions derived from the social planner's program. We characterize and comment a Ramsey-Keynes condition and a Hotelling condition that take new forms in the presence of a recycling activity. In particular, we show how these conditions tackle the joint dynamics of the remaining stock of virgin resource and of the accumulated stock of waste, the latter stock being partially renewable as it is fed by flows entailed by the virgin resource use.

We then study the socially optimal trajectories of the economy in a specific analytical case, and especially the discontinuity induced by the technological breakthrough. At the time of the breakthrough, resource use jumps down and then follows a less sloping declining path, whereas the R&D effort falls down to zero. If the social discount rate is not too high, consumption grows over time; it jumps upwards at the time of the breakthrough and then follows a new path which grows more rapidly than during the pre-recycling phase. We also consider the sensitivity of the optimal trajectories to the exogenous parameters. In particular, we study how these parameters affect the date of the technological breakthrough.

Next, we compare the general trajectories of this economy to two particular cases: an economy in which recycling is never possible and an economy where recycling is immediately operational. This allow us to analyze the impact of the recycling activity and its timing. We show that, as compared with these two particular cases, the economy exploits more intensely the virgin resource before the breakthrough, and then uses lower flows after this date. Besides, the waste stock is initially higher than in the never-recycling economy, but eventually becomes lower after a finite interval of time following the breakthrough and remains so forever after. The impact of the availability of a recycling technology on consumption is more complex. It results from combined effects on the different inputs. As previously mentioned, the virgin resource use is accelerated; in addition, part of the whole labor force is directed from production to research until the breakthrough occurs. According to the relative strengths of these effects, which depend on the exogenous parameters of the model (and notably the maximal recycling rate), the availability of the recycling technology may reduce consumption before the breakthrough.

To get a tractable framework, we have made simplifying hypotheses. First, the virgin resource is extracted from its finite stock at no cost. Similarly, recycled materials are costlessly drawn from the accumulating stock of waste. Furthermore, we consider that the initial stock of virgin resource is exogenous. Endogenizing this stock, as in Daubanes and Lasserre (2018), means that exploration and development efforts allow increasing reserves. The possibility to recycle the waste accumulated makes the resource more valuable. One can then think that if existing reserves were endogenous, this possibility would lead to further exploration, hence an increase in the resource supply and a more intense exploitation before the breakthrough, which would strengthen our result.

One major simplification is that we assume away environmental externalities. The use of many resources like, say plastics, yields flows of waste and their accumulated stocks are a major concern for public health and ecosystems - see for instance the "Great Pacific Garbage Patch" (UNEP, 2016). One characteristic of most recycling activities is that, besides from producing additional inputs for production, they allow reducing such pollution stocks and thus induce additional welfare gains. In other respects, it is well known that the use of many non-renewable resources in the production process yields greenhouse gas (GHG) emissions. However, using virgin materials to produce output does not yield the same amount of GHG than using recycled materials. "Producing new products with secondary materials can save significant energy. For example, producing aluminum from recycled aluminum requires 95% less energy than producing it from virgin materials." (World Bank, 2012). As mentioned before, the availability of a recycling technology accelerates resource use before the breakthrough. This means that, in the absence of environmental externality, the recycling option accelerates GHG emissions. This result may be nuanced in a framework in which, for instance, utility is negatively affected by carbon accumulation.

Finally, the introduction of environmental externalities in this framework paves the way for the study of pro-recycling policy that may be necessary for a decentralized economy to achieve the type of first-best outcomes analyzed in the present paper. The decentralization of the economy considered here could yield market incompleteness in the research sector because of knowledge spillovers. It could thus be interesting to study the impact of dedicated R&D-promoting policies (as in Grimaud and Rouge, 2008) on this economy and, in particular, on the date of the technological breakthrough. We leave this for future research.

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Appendix A.1 Computation of the optimal trajectories

A.1.1 Analytical solution of program $\mathcal{P}_2$

Given the specific functional forms introduced in Section 4 and given (23), the total marginal productivity of the resource, defined by (15), can be rewritten as $TMP_X = \epsilon\bar{A}\left[1+\left(\frac{h\alpha}{1-\beta}\right)\right]\frac{C}{M}$. Equations (16) and (17) thus simplify as:

$$\begin{aligned} \rho + \sigma g_C &= g_C - g_M && \text{(A.1)} \\ g_C - g_M &= g_C + \frac{\delta h\bar{A}L\epsilon Z}{(1-\epsilon)M}\,. && \text{(A.2)} \end{aligned}$$

From (A.1) and from $g_C = g_{A_Y} + \epsilon g_M + (1-\epsilon)g_{L_Y} = g_{A_Y} + \epsilon g_M$ as $g_{L_Y} = 0$ due to (23), we obtain the expressions of g_M and g_C:

$$g_M = \frac{(1-\sigma)g_{A_Y} - \rho}{1-\epsilon(1-\sigma)} \quad \text{and} \quad g_C = \frac{g_{A_Y} - \epsilon\rho}{1-\epsilon(1-\sigma)}\,. \tag{A.3}$$

As both g_C and g_M are constant, log-differentiating (A.2) with respect to time implies $g_Z = g_M$. Next, differentiating $M = \bar{A}(X + hZ)$ with respect to time yields $g_M M = \bar{A}(g_X X + h g_Z Z) = g_Z M$, and then we have $g_X = g_Z = g_M$.

Last, as X and Z grow at the same constant rate[13] g_M, we can easily solve the linear differential equation system (5)-(6), while taking into account the transversality condition (14), to get the optimal trajectories of resource extraction and of waste recycling. The entire solution of $\mathcal{P}_2$ is characterized in the following lemma.

Lemma 1 *For $t \in [T, \infty)$, the optimal trajectories of the model are:*

$$\begin{aligned} X(t) &= X(T^+)e^{-k(t-T)}; \quad X(T^+) = kS(T) \\ Z(t) &= Z(T^+)e^{-k(t-T)}; \quad Z(T^+) = \frac{k[W(T) + \alpha S(T)]}{(1-\beta)} \\ S(t) &= S(T^+)e^{-k(t-T)}; \quad W(t) \;=\; W(T^+)e^{-k(t-T)} \\ C(t) &= C(T^+)e^{g_C(t-T)}; \quad C(T^+) \;=\; A_Y(T)[\bar{A}X(T^+) + h\bar{A}Z(T^+)]^{\epsilon}L^{1-\epsilon} \end{aligned}$$

where $k \equiv -g_M$ and g_C are given by (A.3).

From the expression of C as given in Lemma 1 and noting that $k = \rho - (1-\sigma)g_C$, the optimal value of program $\mathcal{P}_2$ can be simply expressed as:

$$V_2(T, S(T), W(T)) = \int_T^{\infty} u(C)e^{-\rho(t-T)}dt \;=\; \frac{C(T^+)^{1-\sigma}}{(1-\sigma)k}\,, \tag{A.4}$$

[13] Note that, from (A.2), this rate must be negative as $g_M = -\frac{\delta h\bar{A}L\epsilon Z}{(1-\epsilon)M}$.

where $C(T^+) = A_{Y0} e^{g_{A_Y} T} M(T^+)^\epsilon L^{1-\epsilon}$ and $M(T^+) = \bar{A}k\,[S(T) + h(W(T) + \alpha S(T))/(1-\beta)]$. We can then compute the following derivatives:

$$\frac{\partial V_2}{\partial T} = (1-\sigma) g_{A_Y} V_2 \tag{A.5}$$

$$\frac{\partial V_2}{\partial S(T)} = (1-\sigma)\frac{\epsilon \bar{A} k}{M(T^+)}\left(1 + \frac{h\alpha}{1-\beta}\right) V_2 \tag{A.6}$$

$$\frac{\partial V_2}{\partial W(T)} = (1-\sigma)\frac{\epsilon \bar{A} k}{M(T^+)}\left(\frac{h}{1-\beta}\right) V_2\,. \tag{A.7}$$

A.1.2 Analytical solution of program $\mathcal{P}_1$

In what follows, growth rates with an upper tilde refer to the optimal trajectories under program $\mathcal{P}_1$, i.e. for $t \in [0, T)$. The intertemporal trade-off condition (22) directly implies that $\tilde{g}_X = \tilde{g}_{L_Y} = (1-\sigma)\tilde{g}_C - \rho$. As $C = A_Y(\bar{A}X)^\epsilon L_Y^{1-\epsilon}$, then $\tilde{g}_C = g_{A_Y} + \epsilon \tilde{g}_X + (1-\epsilon)\tilde{g}_{L_Y}$. From these last two expressions, it comes:

$$\tilde{g}_X = \tilde{g}_{L_Y} = \frac{(1-\sigma) g_{A_Y} - \rho}{\sigma} \quad \text{and} \quad \tilde{g}_C = \frac{g_{A_Y} - \rho}{\sigma}\,. \tag{A.8}$$

Knowing the growth rates of all the control variables, we can then solve the state equations (3), (5) and (6). We characterize the solution of program $\mathcal{P}_1$ as follows.

Lemma 2 *For $t \in [0, T)$, the optimal trajectories of the model are:*

$$\begin{aligned}
X(t) &= \tilde{k} S_0 e^{-\tilde{k}t}; \quad S(t) = S_0 e^{-\tilde{k}t}; \quad W(t) = W_0 + \alpha S_0 (1 - e^{-\tilde{k}t}) \\
L_Y(t) &= L_Y(0) e^{-\tilde{k}t}; \quad L_A(t) = L - L_Y(t) \\
A_Z(t) &= A_{Z0} \exp\left[\delta L t - \frac{\delta L_Y(0)}{\tilde{k}}(1 - e^{-\tilde{k}t})\right] \\
C(t) &= C(0) e^{\tilde{g}_C t}; \quad C(0) = A_{Y0}\left(\bar{A}\tilde{k} S_0\right)^\epsilon L_Y(0)^{1-\epsilon},
\end{aligned}$$

where $\tilde{k} \equiv -\tilde{g}_X$ and $\tilde{g}_C$ are defined by (A.8), and where $L_Y(0)$ and T are endogenous variables that must be determined from the set of continuity and transversality conditions at time T.

A.1.3 Transversality conditions at time T

Given the expression of the state variables provided by Lemma 1 and 2, we can deduce the following continuity conditions at time T:

$$S(T) = S_0 e^{-\tilde{k}T} \tag{A.9}$$

$$W(T) = W_0 + \alpha S_0 (1 - e^{-\tilde{k}T}) \tag{A.10}$$

$$A_Z(T) = h\bar{A} \Leftrightarrow \delta L T = \frac{\delta L_Y(0)}{\tilde{k}}\left(1 - e^{-\tilde{k}T}\right) + \ln\left(\frac{h\bar{A}}{A_{Z0}}\right). \tag{A.11}$$

Next, we analyze the first transversality condition (19). From (7) and (9), we get for any time t:

$$\lambda_S(t) - \alpha\lambda_W(t) = \frac{\epsilon \bar{A}}{M(t)} C(t)^{(1-\sigma)} \tag{A.12}$$

$$\lambda_A(t) = \frac{(1-\epsilon)}{\delta A_Z(t) L_Y(t)} C(t)^{(1-\sigma)} . \tag{A.13}$$

From these expressions, and noting that $M(t) = \bar{A}X(t)$ for any $t < T$ and $L_A = L - L_Y(t)$, the Hamiltonian of program $\mathcal{P}_1$ can be rewritten as follows:

$$\mathcal{H}_1(t) = \frac{C(t)^{1-\sigma}}{(1-\sigma)} \left[\sigma + (1-\sigma)(1-\epsilon)\frac{L}{L_Y(t)}\right] ,$$

where the expressions of $C(t)$ and $L_Y(t)$ are determined in Lemma 2. Taking the expression of the Hamiltonian at time T, using (A.4) and (A.5), and rearranging some terms, transversality condition (19) becomes:

$$\left[\frac{C(T^+)}{C(T^-)}\right]^{1-\sigma} = \frac{\sigma + e^{\tilde{k}T}(1-\sigma)(1-\epsilon)L/L_Y(0)}{1-\epsilon(1-\sigma)} . \tag{A.14}$$

The other two transversality conditions, (20) and (21), can be combined as follows:

$$\lambda_S(T^-) - \alpha\lambda_W(T^-) = \frac{\partial V_2}{\partial S(T)} - \alpha\frac{\partial V_2}{\partial W(T)} .$$

Using (A.4), (A.6), (A.7) and (A.12), this last equation yields:

$$\left[\frac{C(T^+)}{C(T^-)}\right]^{1-\sigma} = \frac{M(T^+)}{M(T^-)} . \tag{A.15}$$

Using the first two results of Lemma 1 and the continuity conditions (A.9)-(A.10), we can express $M(T^+) = \bar{A}[X(T^+) + hZ(T^+)]$ as:

$$M(T^+) = \bar{A}kS_0\left(e^{-\tilde{k}T} + h\Phi\right) , \tag{A.16}$$

where $\Phi \equiv (W_0 + \alpha S_0)/[(1-\beta)S_0]$. Given that $M(T^-) = \bar{A}\tilde{k}S_0 e^{-\tilde{k}T}$ and from (A.16), the transversality condition (A.15) can finally be rewritten as follows:

$$\left[\frac{C(T^+)}{C(T^-)}\right]^{1-\sigma} = \frac{\sigma(1 + h\Phi e^{\tilde{k}T})}{1-\epsilon(1-\sigma)} . \tag{A.17}$$

The two tranversality conditions (A.14) and (A.17) allow the determination of the optimal initial level of labor in production. Last, given this optimal value of $L_Y(0)$, the optimal switching time T is obtained as the solution of the continuity equation (A.11).

A.1.4 Optimal trajectories: Summary

The optimal solution is characterized by the following trajectories:

$$X(t) = \begin{cases} \tilde{k} S_0 e^{-\tilde{k}t} & , \quad t < T \\ k S_0 e^{(k-\tilde{k})T - kt} & , \quad t \geq T \end{cases} \tag{A.18}$$

$$Z(t) = \begin{cases} 0 & , \quad t < T \\ k \Phi S_0 e^{-k(t-T)} & , \quad t \geq T \end{cases} \quad \text{with } \Phi \equiv \frac{W_0 + \alpha S_0}{(1-\beta) S_0} \tag{A.19}$$

$$S(t) = \begin{cases} S_0 e^{-\tilde{k}t} & , \quad t < T \\ S_0 e^{(k-\tilde{k})T - kt} & , \quad t \geq T \end{cases} \tag{A.20}$$

$$W(t) = \begin{cases} W_0 + \alpha S_0 (1 - e^{-\tilde{k}t}) & , \quad t < T \\ \left[W_0 + \alpha S_0 (1 - e^{-\tilde{k}T}) \right] e^{-k(t-T)} & , \quad t \geq T \end{cases} \tag{A.21}$$

$$L_Y(t) = L - L_A(t) = \begin{cases} L_{Y0} e^{-\tilde{k}t} & , \quad t < T \\ L & , \quad t \geq T \end{cases} \tag{A.22}$$

$$A_Z(t) = \begin{cases} A_{Z0} \exp\left[\delta L t - \frac{\delta L_{Y0}}{\tilde{k}} (1 - e^{-\tilde{k}t}) \right] & , \quad t < T \\ h \bar{A} & , \quad t \geq T \end{cases} \tag{A.23}$$

where the initial level of productive labor is given by:

$$L_{Y0} \equiv L_Y(0) = \frac{(1-\sigma)(1-\epsilon) L}{\sigma h \Phi}; \tag{A.24}$$

and where the optimal switching time T is solution of the following equation:

$$\delta L T - \ln\left(\frac{h \bar{A}}{A_{Z0}} \right) = \frac{\delta L_{Y0}}{\tilde{k}} \left(1 - e^{-\tilde{k}T} \right). \tag{A.25}$$

Moreover, the following conditions must be satisfied: $L_{Y0} \in (0, L)$, $k > 0$ and $\tilde{k} > 0$. This corresponds to a set of parameters that verify the following conditions:

$$\sigma < 1, \tag{A.26}$$

$$\frac{(1-\sigma)(1-\epsilon)}{\sigma h} < \Phi, \tag{A.27}$$

$$(1-\sigma) g_{A_Y} < \rho. \tag{A.28}$$

Condition (A.26) says that the elasticity of substitution of intertemporal consumption, i.e. $1/\sigma$, must be larger than one to guarantee a positive labor share devoted to production. From condition (A.27), the maximal recycling rate Φ of the virgin resource must be large enough to also guarantee a positive labor allocation to R&D in recycling. Last, condition (A.28) states that, to justify resource extraction, the social discount rate must be large enough as compared with the exogenous trend parameter of technical progress (this condition guarantees that both k and $\tilde{k}$ are positive).

Appendix A.2 Comparative dynamic analysis

We conduct a sensitivity analysis of the key variables of the model, with respect to the set of parameters. The results are described in Table 1. Each box indicates the sign of the partial derivative of the variable mentioned in line with respect to the parameter given in column. This sign can be positive ("+") or negative ("−"). An empty box means that there is no relation between the variable and the parameter whereas "?" indicates that the sign is ambiguous.

[Place Table 1 here]

Partial differentiation of the growth rates k, $\tilde{k}$, g_C and $\tilde{g}_C$, and of the initial values $X(0)$, $Z(T)$, L_{Y0} and $C(0)$ are mostly trivial so that their computations are not detailed here. The sensitivity of the initial consumption $C(0) = A_{Y0}(\bar{A}\tilde{k}S_0)^{\epsilon}L_{Y0}^{1-\epsilon}$ with respect to σ is less obvious as:

$$\frac{\partial C(0)}{\partial \sigma} = \left[\frac{\epsilon}{\tilde{k}}\frac{\partial \tilde{k}}{\partial \sigma} + \frac{(1-\epsilon)}{L_{Y0}}\frac{\partial L_{Y0}}{\partial \sigma}\right] C(0) = \left[(g_{A_Y} - \rho) - \sigma(g_{A_Y} - \epsilon\rho)\right] \frac{C(0)}{\sigma^2(1-\sigma)\tilde{k}} ,$$

and it depends on the value of σ but also on the respective signs of g_C and $\tilde{g}_C$ as explained by note (c) in Table 1. Moreover, the sign of $\partial C(0)/\partial \epsilon$ is ambiguous:

$$\frac{\partial C(0)}{\partial \epsilon} = \left[\ln\left(\frac{\bar{A}\tilde{k}S_0}{L_{Y0}}\right) + (1-\epsilon)\frac{\partial L_{Y0}/\partial \epsilon}{L_{Y0}}\right] C(0) = \left[\ln\left(\frac{\bar{A}\tilde{k}S_0}{L_{Y0}}\right) - 1\right] C(0) .$$

Next, the sensitivity analysis of the switching date T is not immediate as we cannot get a closed-form expression for T. We simply know that T is characterized by the implicit function (A.25). We define the following functions i and j:

$$\begin{aligned} i(t) &= Lt - \frac{1}{\delta}\ln\left(\frac{h\bar{A}}{A_{Z0}}\right) \\ j(t) &= L_{Y0}\left(\frac{1-e^{-\tilde{k}t}}{\tilde{k}}\right) . \end{aligned}$$

These functions are depicted in Figure 3. Given their analytical properties, we can observe graphically that the solution T to the equation $i(T) = j(T)$ is unique. Moreover, for this solution to exist, we must have $i'(T) > j'(T)$.

[Place Figure 3 here]

We apply now the implicit function theorem to i and j. For any parameter x, we obtain:

$$\frac{dT}{dx} = \frac{\partial j/\partial x - \partial i/\partial x}{i'(T) - j'(T)} \quad \Rightarrow \quad \text{sign}\left(\frac{dT}{dx}\right) = \text{sign}\left(\frac{\partial j}{\partial x} - \frac{\partial i}{\partial x}\right). \tag{A.29}$$

This equation, together with the computations of $\partial i/\partial x$ and $\partial j/\partial x$, thus allows identifying the sign of the derivatives of T with respect to any parameter x.[14]

[14] We also use the fact that $\frac{\partial j(t)}{\partial \tilde{k}} = \frac{L_{Y0}}{\tilde{k}^2}\left[(1+\tilde{k}t)e^{-\tilde{k}t} - 1\right] \leq 0 \ \forall t$. Indeed, the function $f(x) = (1+x)e^{-x} - 1$ defined for $x \in \mathbb{R}^+$ is proved to be continuously decreasing from $f(0) = 0$ to $f(+\infty) = -1$.

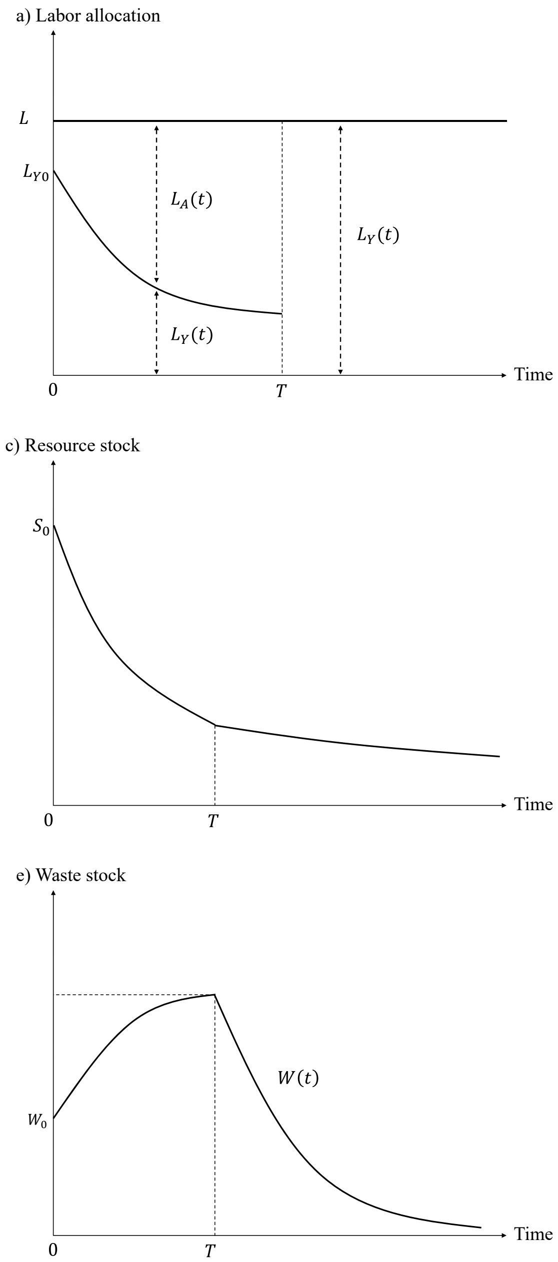

Figure 1: Optimal trajectories of the model

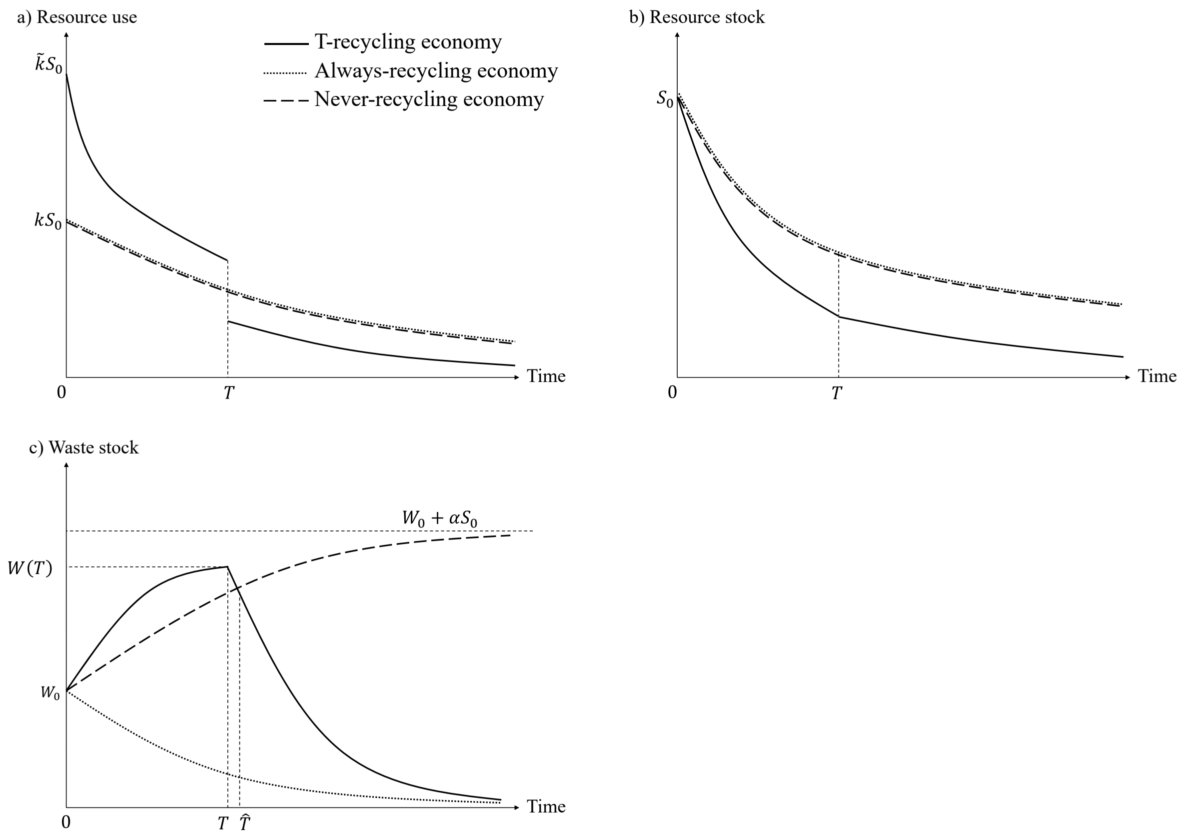

Figure 2: Effect of the availability of the recycling technology

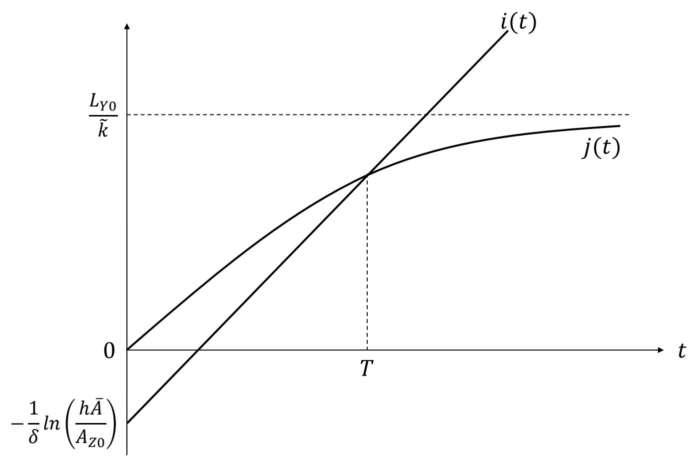

Figure 3: Graphical identification of T

Table 1: Comparative dynamic analysis

	ρ	σ	g_{A_Y}	ϵ	$\bar{A}$	δ	S_0	W_0	α	β	Φ
k	$+$	(a)	$-$	$+$							
$\tilde{k}$	$+$	(b)	$-$								
g_C	$-$	$-$	$+$	$-$							
$\tilde{g}_C$	$-$	$-$	$+$								
Φ							$-$	$+$	$+$	$+$	
$X(0)$	$+$	(b)	$-$				$+$				
$Z(T)$	$+$	(a)	$-$	$+$			$+$	$+$	$+$	$+$	$+$
L_{Y0}		$-$		$-$			$+$	$-$	$-$	$-$	$-$
$C(0)$	$+$	(c)	$-$	$?$	$+$		$+$	$-$	$-$	$-$	$-$
T	$-$	(d)	$+$	$-$	$+$	$-$	$+$	$-$	$-$	$-$	$-$

(a) $+$ if $g_{A_Y} > \epsilon\rho$, i.e. if $g_c > 0$, $-$ otherwise.
(b) $+$ if $g_{A_Y} > \rho$, i.e. if $\tilde{g}_c > 0$, $-$ otherwise.
(c) $+$ if $\tilde{g}_C > 0$ and $\sigma < \frac{g_{A_Y}-\rho}{g_{A_Y}-\epsilon\rho}$, or if $g_C < 0$ and $\sigma > \frac{g_{A_Y}-\rho}{g_{A_Y}-\epsilon\rho}$, $-$ otherwise.
(d) $-$ if $\tilde{g}_C > 0$, ? otherwise.