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Assessing the sustainability of optimal pollution paths in a world with inertia

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Abstract

Most formal optimal pollution control models assume a constant natural assimilative capacity, despite the biophysical evidence on feedback effects that can degrade this environmental function, as it is the case with the reduction of ocean carbon sinks in the context of climate change. The few models that do consider this degradation establish a bijective relation between the pollution stock and the assimilative capacity, thus ignoring the inertia mechanism at stake. Indeed the level of assimilative capacity is not solely determined by the current pollution stock but by the history of this stock and by the time the ecosystem remains above the degradation threshold. We propose an inertia assessment tool that tests the sustainability of any benchmark optimal pollution path when the inertia of the assimilative capacity degradation process is taken into account. Our simulations show a strong sensitivity to both the inertia degradation speed and the discount rate.

Keywords: Optimal pollution control, Assimilative Capacity, Inertia, Ecosystem Services, Climate Change

JEL classification : D62 ; H23 ; Q01 ; Q5 ; Q54.

1. Introduction

Since the seminal articles on optimal pollution control (Keeler et al., 1972 ; Plourde, 1972), most standard stylized models in partial equilibrium assume a constant assimilative capacity of the environment. However this assumption ignores the degradation of this environmental function that can occur when the pollution stock crosses a critical threshold. This degradation is particularly significant in the case of greenhouse gases (GHG) accumulation. Ecological science, either through direct observation (Schuster and Watson, 2007) or ocean-climate modelization (Le Queré et al., 2007), has shown that the carbon uptake rate by oceans has been decreasing over the last decade. This decrease is due to feedbacks between climate and marine carbon cycle such as surface water warming, water stratification and thermohaline currents modification. Despite the scientific uncertainty characterizing the monitoring of carbon concentration, according to the International Climate Change Taskforce Report (2005, Chapter 10) there is "unanimous agreement among the coupled climate-carbon cycle models driven by emission scenarios run so far that future climate change would reduce

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the efficiency of the Earth system (land and ocean) to absorb anthropogenic CO_2". Plattner et al. (2001) forecasted a reduction in ocean carbon uptake by $7-10\%$ over the 21^{st} century compared to a constant climate scenario but the decrease might reveal much faster. Heinze et al. (2015) confirm this " reduction in carbon uptake efficiency with warming climate and rising atmospheric". This reduction is especially acute for the North Atlantic Ocean, as various studies (Watson et al., 2009 ; Schuster et al., 2009) show that its CO_2 assimilative capacity decreased by up to 50% over the 1995-2005 period while the atmospheric concentration of carbon increased. From an economic perspective, this alteration of the global carbon cycle might have a strong impact on the mitigation strategies as they demand increased emission reductions or sequestration in order to achieve the same stabilization objective.

Considering the risks at stake, especially in the case of climate change, it seems legitimate to expect that economic models of pollution control took the dynamics of assimilative capacity into account. The standard linear representation of the natural decay activity with a constant rate of assimilation has already been questioned by different authors such as Forster (1975), Tahvonen and Salo (1996), Tahvonen and Withagen (1996), Toman and Withagen (2000) or Hediger (2009) in stylized optimal pollution control models. These contributions, referred to as *bijective models* in the rest of this paper, have tried to introduce more realistic decay function, such as a concave-convex function and to allow for the irreversible annihilation of assimilative capacity beyond a critical threshold. The main conclusion from these contributions is the existence of multiple equilibria associated with either a positive or an irreversibly depleted assimilative capacity and the impossibility for the affected ecosystem to return to its initial state when it has reached an irreversible basin of attraction.

However these natural decay functions display an exclusive dependency [1] on the pollution stock variable Z that does not fit so well the ecological reality of GHG accumulation. Indeed, according to empirical ecological evidence it seems reasonable in a wide range of cases to assume that this assimilative capacity will not be only impacted by the absolute stock level, but also by the length of time spent above the degradation threshold. We define as the inertia effect the fact that the assimilative capacity available associated to the same pollution level will not be the same if the ecosystem has just reached this stock level or if it has spent a long period at this level. Consequently this assimilative capacity cannot simply increase back to a higher level if the pollution stock decreases as the models above assume. The bijective relation established in the pre-cited contributions forbids to link the assimilative capacity with the pollution "history" as it attributes the same level to any stock, regardless of when this stock was reached and how long it was sustained. The irreversibility of the degradation as well as the inertia effect should thus be reflected in the assimilative capacity autonomous dynamics. That is why we propose to test the sustainability of a benchmark optimal pollution path in a modified dynamic system accounting for inertia. Our outlook on environmental sustainability is the whole (strong) or partial (weak) preservation of a crucial ecosystem service, here the assimilative capacity of the environment (and of the oceans in the particular case of GHG emissions).

Our objective is thus to study the sustainability of a dynamic pollution system when the inertia effect is accounted for from both a theoretical and a numerical simulation perspective. We propose an inertia assessment tool that will highlight the conditions under which a benchmark optimal pollution paths can lead the assimilative function to an irreversible extinction. Relying on both analytical conditions and numerical simulations, this assessment tool can determine if a pollution path is sustainable in a world with inertia, in the sense that it preserves indefinitely a strictly positive level of assimilative capacity or unsustainable if it leads to the extinction of the assimilative function. Our goal is not to question the relevancy of the optimality criteria for pollution control policies, but rather to test the environmental sustainability of these policies.

1. The assimilative function $A(Z)$ is a bijective function of the pollution stock Z.

Moreover, our take on pollution control focusing on the dynamics of assimilative capacity can be interpreted as a case of ecosystem service management. Consequently our introduction of inertia in the degradation process of an ecosystem service might be a useful step to improve the economic analysis of other ecosystem services displaying similar dynamics with inertia.

Our analysis relies on the following steps. First we recall in Section 2 the main properties of the benchmark model of pollution control with constant assimilative capacity and we analyze the sensitivity of the steady state to this assimilative capacity parameter using functional forms. This analysis will be used in the inertia assessment tool later on. In Section 3 we present the most common bijective models that doe allow for a varying assimilative capacity and we underline their limitations when it comes to reflecting the time-dependent degradation mechanism of this capacity. To capture the inertia at work in some stock pollution problems, and especially acute in the case of GHG accumulation, we introduce a new system dynamic for the assimilative capacity variable and we discuss the sustainability issue raised by this inertia through zero-emission scenarios. Section 4 presents our assessment tool based on the analytical and numerical injection of a control variable sequence derived from a benchmark model into our dynamic system with inertia. We illustrate the application of this tool with the sustainability analysis of a standard benchmark set of parameters.

2. Standard assimilative capacity in the benchmark optimal pollution control model

2.1. Basic properties of the benchmark model with constant assimilative capacity

In order to test the sustainability optimal pollution paths with regards to assimilative capacity conservation, we need to assert some properties of the following benchmark model :

$$\begin{aligned} &\max_{y(t)} \int_0^\infty [f(y(t)) - D(Z(t))]\, e^{-\rho t} dt \\ &\text{s.t.} \\ &\dot{Z}(t) = y(t) - \alpha_0 Z(t), \quad Z(0) = Z_0 > 0 \end{aligned}$$

with the standard properties : f the concave benefit function from pollution, D the environmental damage function (increasing and convex), ρ the discount rate and α_0 the constant assimilative capacity ($0 < \alpha_0$)

We thus get the following Hamiltonian with λ the co-state variable associated to Z

$$\mathscr{H}_c(t) = f(y(t)) - D(Z(t)) + \lambda(t)(y(t) - \alpha_0 Z(t)) \tag{1}$$

which yields the standard first order conditions

$$f'(y(t)) = -\lambda(t) \tag{2}$$

$$\dot{\lambda}(t) = (\rho + \alpha_0)\lambda(t) + D'(Z(t)) \tag{3}$$

2.1.1. Steady States

Given the dynamics of Z and λ and the properties of f and D, a unique Z_{ss} exists such that

$$\begin{aligned} &f'(\alpha_0 Z_{ss}) > 0 \\ &f'(\alpha_0 Z_{ss})(\rho + \alpha_0) - D'(Z_{ss}) = 0. \end{aligned} \tag{4}$$

Let us introduce the standard functional forms commonly found in the literature that we will use for further calculations

$$f(y) = cy - \frac{b}{2}y^2, \ with\ c,b > 0, \quad D(Z) = \frac{d}{2}Z^2, \ with\ d > 0.$$

With these functional forms, (4) is tantamount to

$$Z_{ss} = \frac{(\rho + \alpha_0)c}{(\rho + \alpha_0)b\alpha_0 + d}. \tag{5}$$

2.1.2. Characterization of the optimal path with functional forms

It is now possible to determine analytically the expression of $Z^*(t)$, $\lambda^*(t)$ and $y^*(t)$ along the optimal pollution path by solving the first order conditions (2, 3) for the quadratic linear functional forms.

$$Z^*(t) = e^{-\bar{\rho}t}(Z(0) - Z_{ss}) + Z_{ss}, \tag{6}$$

$$y^*(t) = \frac{c + \lambda}{b}, \tag{7}$$

$$\lambda^*(t) = b(-\rho + \alpha_0)e^{-\bar{\rho}t}(Z(0) - Z_{ss}) - \frac{d}{\rho + \alpha_0}Z_{ss}. \tag{8}$$

where $\bar{\rho} = \sqrt{(\rho/2 + \alpha_0)^2 + d/b} - \rho/2 > 0$. $\bar{\rho}$ is the speed of convergence of the dynamic.

Equations (6, 7) will be used in Section 4 to simulate numerically benchmark optimal pollution paths and to test their behaviour in presence of inertia.

2.2. Comparative statics of α and Z_{ss}

2.2.1. Completing the comparative statics result for our functional forms

The relation between Z_{ss} and α_0 will prove to be central in the sustainability assessment we carry out in Section 4. When it is mentioned in the literature (Plourde, 1972, page 124) or in textbooks (Pearman, 2010) , this relation is assumed to be increasing : a higher assimilative capacity leading systematically to a higher pollution stock at the steady state [2]. However using our functional forms and the solution (6 : 8) we show this relation is actually more ambiguous (to our knowledge, this ambiguity had been only noted by Forster (1973) and never dwelled upon).

Proposition 2.1 *With standard functional forms we have :*
– *for $\rho > \sqrt{d/b}$, Z_{ss} decreases with α_0*
– *for $\rho < \sqrt{d/b}$, Z_{ss} increases and then decreases with α_0 (see Figure 1)*

Proof. The proof is straightforward. According to (5) we have

$$\frac{\partial Z_{ss}}{\partial \alpha_0} = \left(c\frac{d - b(\alpha_0 + \rho)^2}{(d + \rho\alpha_0 b + \alpha_0^2 b)^2} \right).$$

2. An economic intuition often claimed to explain this relation is that the assimilative capacity level can be interpreted as a component of the discount factor, hence the higher α_0 the higher Z_{ss}.

hence defining $\bar{\alpha} = \sqrt{d/b} - \rho$

$$\frac{\partial Z_{ss}}{\partial \alpha_0} \gtreqless 0 \iff \alpha_0 \lesseqgtr \bar{\alpha}$$

– if $\rho > \sqrt{d/b}$, $\bar{\alpha} < 0$ and thus $\forall \alpha_0 > 0$ we have $\alpha_0 > \bar{\alpha}$ and $\frac{\partial Z_{ss}}{\partial \alpha} < 0$.
– if $\rho < \sqrt{d/b}$ then $\frac{\partial Z_{ss}}{\partial \alpha_0} > 0$ for $0 < \alpha_0 < \bar{\alpha}$ and $\frac{\partial Z_{ss}}{\partial \alpha_0} < 0$ for $\bar{\alpha} < \alpha_0$.
■
Since this result is not in line with the unambiguous comparative statics usually found in the litterature for general functions with the same basic properties as our specific functional forms, let us suggest an economic interpretation of Proposition 2.1.

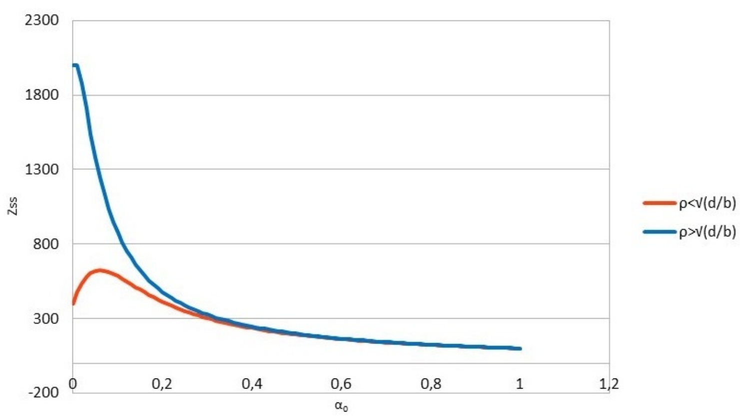

FIGURE 1: Z_{ss} as a function of α_0 depending on the value of ρ

2.2.2. Economic interpretation of Proposition 2.1
In order to sketch a clear interpretation of 2.1 let us distinguish two types of economies in which the variations of α_0 have a different impact on Z_{ss}.

Pollution-blind Economy. The case where $\rho > \sqrt{d/b}$ corresponds to a relatively high ρ and/or b and/or a relatively low d. This range of parameters characterizes a "pollution-blind" economy with little concern for the future (high ρ), little emission-efficiency in its production process (high b) and little sensitivity to pollution accumulation (low d). From the optimization standpoint, in such a setting the emissions $y^(t)$ will be mostly determined by the private benefit they generate in the present with little consideration for the resulting pollution accumulation in the future. As a result, the level of $y^*(t)$ along the optimal path will be close to the private static optimum $\hat{y}$ that would result from the sole static optimization of the benefit function. A change in the dynamics of the pollution stock, which are barely taken into account, will thus have no significant impact on y^*_{ss} which will remain close to $\hat{y}$. That is why a greater assimilative capacity will be confronted with a similar level of steady state emissions, which will naturally yield a lower steady state pollution stock (through the dynamics that are almost ignored from the heart of the optimization problem itself). This phenomenon can be read in the expression of $y_{ss} = \alpha_0 Z_{ss}$: since y_{ss} remains more or less constant and close to $\hat{y}$, a higher α_0 will be obviously compensated by a lower Z_{ss}.

Since the discount rate is the most flexible parameter compared to the more tangible d and b we can focus on ρ to assess the limit behaviour of the pollution-blind economy. As ρ tends towards infinity, the optimum pollution path tends towards the private static optimum, as if the future had no weight at all in the optimization process. In that case we have $\bar{\rho}^{\rho=\infty} = \alpha_0$, $Z_{ss}^{\rho=\infty} = \frac{c}{b\alpha_0}$.

Pollution-aware Economy. The case where $\rho < \sqrt{d/b}$ corresponds to a relatively low ρ and/or b and/or a relatively high d. This range of parameters characterizes a "pollution-aware" economy with a high concern for the future (high ρ), a high emission-efficiency in its production process (low b) and a high sensitivity to pollution accumulation (low d). In this "pollution-aware" economy, the level of emissions will be also be significantly guided by the negative externality of the accumulation of pollution over time. Given the properties of the private benefit and of the damage function, the impact of a greater assimilative capacity on the steady state pollution stock will be differentiated depending on the level of emissions at the steady state y_{ss}. Indeed as long as α_0, and thus y_{ss} are small enough so that the difference between the marginal benefit and the marginal damage yielded by an additional emission is positive, a higher α_0 will induce the social planner to increase $y_{ss} = \alpha_0 Z_{ss}$, which will in turn increase the corresponding social welfare as well as the pollution stock. Hence the first increasing part of the curve (which is often the only case envisioned in textbooks) in Figure 1, as long as $\alpha_0 < \bar{\alpha}$. However, once α_0 crosses the threshold $\bar{\alpha}$, y_{ss} is too high for an increase in emissions to add social welfare given the concavity of f and the convexity of D. On this side of the curve, a greater α_0 will thus not be used to enjoy much higher emissions but rather to reduce the pollution stock, which is at this point the most efficient way to increase social welfare.

2.2.3. Assimilative capacity and social welfare at the steady state

Out of curiosity we check that the intuitive positive effect of α on the social welfare at the steady state is actually true.

$$W(\alpha) = f(\alpha Z_{ss}(\alpha)) - D(Z_{ss}(\alpha))$$

hence taking into account (4)

$$\frac{\partial W}{\partial \alpha} = \left(\alpha \frac{\partial Z_{ss}}{\partial \alpha} + Z_{ss}(\alpha)\right) f'(\alpha Z_{ss}(\alpha)) - \frac{\partial Z_{ss}}{\partial \alpha} D'(Z_{ss}(\alpha)) = \left(-\rho \frac{\partial Z_{ss}}{\partial \alpha} + Z_{ss}(\alpha)\right) f'(\alpha Z_{ss}(\alpha)).$$

As by first order condition $f'(\alpha Z_{ss}(\alpha)) > 0$, when $\frac{\partial Z_{ss}}{\partial \alpha} < 0$ we have $\frac{\partial W}{\partial \alpha} > 0$. Nevertheless with our specific forms we can prove that it is always positive, in fact :

$$W(\alpha) = \frac{1}{2} \frac{c^2(\alpha + \rho)(d\alpha + \rho\alpha^2 b + b\alpha^3 - d\rho)}{(d + \rho\alpha b + b\alpha^2)^2}$$

and thus

$$\frac{dW(\alpha)}{d\alpha} = \frac{c^2 d(b(\rho + \alpha)^3 + d\alpha)}{(d + \rho\alpha b + b\alpha^2)^3} > 0.$$

As expected, a greater assimilative capacity offers a greater social welfare at the steady state. However this welfare gain can be obtained through two channels : either by allowing a higher level of emissions without increasing too much the pollution stock, or by reducing the pollution stock while maintaining a given emission level. The most efficient of these options will depend on whether the economy at stake is closer to the pollution-blind or to the pollution-aware features we have presented above.

3. Sustainability in a world with inertia

Let us now consider the alternative models and assess their sustainability when inertia is introduced in the assimilative capacity dynamics.

3.1. Bijective models and their limitations

As noted in the introduction, a strand of models have tried to correct the strong assumption on the constant assimilative capacity found in the benchmark model above. Most amended pollution control models (Tahvonen and Withagen, 1996, Prieur, 2009) that account for the variations in assimilative capacity do so through a bijective relation such that $\alpha(t) = \alpha(Z(t)) \; \forall \, t$, with a critical threshold Z^c such that $\alpha(t) = 0 \; \forall \, t > T$ if $\exists \, T$ such that $Z(T) \geq Z^c$. The image function $\alpha(Z(t))$ can take many forms : linear (Tahvonen and Withagen, 1996), quadratic (Prieur, 2009), etc. Although these models clearly improve the benchmark dynamics, they do not reflect important properties of the assimilative function.

First, the bijective relation implies associating systematically the same set of initial conditions $(Z_0, \alpha_0 = \alpha(Z_0))$ which prevents comparative studys of pollution problem with a similar dynamic but with different initial conditions.

Second, this dynamic does not allow the possibility of artificial restoration of the assimilative capacity, which can be a relevant tool in some pollution problems (Leandri, 2009 ; El Ouardighi, 2011). The only way to increase α is to change the pollution stock which does not reflect the opportunities offered by restoration ecology to deal separately with the assimilative capacity service. However, considering the current uncertainty on geo-engineering techniques, introducing assimilative capacity restoration in pollution control models with a climate change application in mind does not seem like a priority to us.

Third, and most importantly, the dynamics in the bijective models associate identical assimilative capacity rates to the same stock pollution level, whatever time the system has spent at that stock level. Whether it has just reached this stock Z from below for the first time or it has remained at this stock or higher for a very long time, $\alpha(Z(t))$ will be the same. This property is obviously a strong simplification of the ecological dynamics at stake and does not account for the important factor of inertia in the degradation of the ecosystem service of assimilation, especially acute in the case of the variations of the ocean carbon sinks. In particular some models (Tahvonen and Withagen, 1996) do not account completely [3] for the irreversibility of the degradation since returning to a lower pollution stock, even after a very long time spent a high pollution levels, will automatically increase the assimilative function back to higher levels, as if nothing happened before.

A first way to illustrate the optimistic postulate behind the bijective models is to consider a subset of optimal pollution paths in the classic Tahvonen and Withagen (1996) setting. According to their proposition 1. (p. 1782) if $D'(Z^c) - \delta U'(0) \geq 0$ the optimal pollution path is reversible and converges towards a saddle point steady state. Among this set of reversible optimal paths, we can identify on the phase diagram in their Figure 2. a subset of monotonously decreasing optimal paths for all Z_0 such that $Z_0 \in [\hat{Z}, Z^c[$. We define $\hat{Z}$ such that $\dot{y}^T(\hat{Z}) = 0$ with $y = 0$, that is to say that $\hat{Z}$ is the intercept of the $\dot{y}$-isocline and the Z axis in the (Z, y)

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3. It must be noted that irreversibility is present once the pollution stock reaches Z^c since it is then impossible to reduce the pollution stock with a zero assimilative capacity and no external restoration.

plane. Along such an optimal path decreasing in $Z(t)$, a low optimal emission level $y^*(t)$ is necessary but the assimilative capacity $\alpha(Z(t))$ also comes into play. For Z_0 high enough (close enough to Z^c), the assimilative capacity $\alpha(Z(t))$ will be increasing along the path for a period given the properties of $\alpha(Z)$. In this case it is clear that the optimal path benefits from a higher assimilative capacity than the ecological processes should allow, as there is no inertia preventing the economy to enjoy once again a high assimilative capacity level after having reached (and possibly stayed long) a high pollution stock.

3.2. Introducing a new system dynamic to account for inertia

In order to better capture the essential inertia mechanism we suggest to modify the dynamics of the system by making $\alpha(t)$ an autonomous state variable of its own right, following its own dynamics and freed from the image function bijection with the pollution stock. Following the extensions of Pearce's intuitions (1976) carried out by Pezzey (1996) [4] and discussed by Godard (2006) and Leandri (2009), the dynamics of α are based on a degradation threshold $\bar{Z}$ (not to be confused with the prior mentioned Z^c) above which the assimilative capacity undergoes a degradation process. The new dynamics we will work with are the following :

$$\begin{aligned} \dot{\alpha}(t) &= -h(Z(t)) \\ \alpha(0) &= \alpha_0 \end{aligned} \tag{9}$$

where h, non decreasing and continuous in $\bar{Z}$, is the assimilative degradation function beyond the threshold $\bar{Z}$.
For the sake of clarity we will use the following specific forms for (9).

$$\dot{\alpha}(t) = -h(Z(t)) = -k(\max[Z-\bar{Z},0]) \text{ if } \alpha > 0 \tag{10}$$

$$\begin{aligned} &\dot{\alpha}(t) = 0 \; if \; \alpha = 0 \\ &\text{if } \exists\, t_1 \text{ s.t. } \alpha(t_1) = 0 \text{ then } \alpha(t) = \dot{\alpha}(t) = 0 \;\forall t \geq t_1 \end{aligned} \tag{11}$$

Obviously this very simple linear degradation mechanism does not capture all the complex ecological processes at stake in the evolution of oceans carbon uptake among others, but we need a simple form to highlight the inertia effect we are focusing on. What's more, we shall assume for the rest of the analysis that the only assimilative capacity available follows such a degradation mechanism whereas in global pollution problems such as GHG accumulation various forms of assimilative capacity come into play (oceans, forests, soil) but do not evolve in the same manner.

3.3. Highlighting the sustainability issue through a zero-emission scenario

The original feature of our dynamic system is best illustrated through an emission-free scenario. Observing the system's behaviour when emissions are always nil allows us to show clearly the specificity of our dynamics with inertia. In order to emphasize the significative effect of inertia on the sustainability of a pollution path, we shall give an overview of the behaviour of the bijective models in a zero emission world and comparing it with what takes place in a system with inertia.

4. Our model displays similarities with the work of El Ouardighi et al. (2011) as they both share the same inspirations in the literature. However we do not abide by their strong assumptions on assimilative capacity restoration that make their model less relevant to tackle the crucial case of GHG.

3.3.1. The behaviour of the bijective models with zero emissions

The optimistic postulate behind the bijective models appears quite clearly if we assess them in a zero emission world ($y(t) = 0 \; \forall \; t$). One of the decay function most commonly found in the literature is the inverted U-shape function[5] (Tahvonen and Withagen, 1996, Cesar and De Zeew, 1994). This function is obtained by a linear decreasing image function for the assimilative capacity :

$$\dot{Z} = -\alpha(Z)Z,$$
$$\alpha(Z) = \begin{cases} a(Z^c - Z) & \text{if } Z \leq Z^c, \\ 0 & \text{if } Z \geq Z^c. \end{cases} \tag{12}$$

Note that Z^c, the critical threshold in these models that implies the total extinction of assimilative capacity, must not be confused with our $\bar{Z}$, the threshold at which degradation of assimilative capacity starts. For the sake of clarity, let us rewrite (12) such that $\alpha(Z(t)) = \max(1 - aZ(t); 0)$ with $a = \frac{1}{Z^c}$. This simplified expression captures the same degradation mechanism but it also guarantees that $\alpha(t) < 1 \; \forall \; Z(t) > 0$ and that $\alpha = 0$ when $Z(t) = Z^c$. It can be easily anticipated that with this linear decay function any pollution stock starting below Z^c will decrease down to the threshold at which α stops increasing and further on down to zero. On the contrary any system starting with $Z_0 \geq Z^c$ will remain indefinitely in Z_0.

Using a set of arbitrary parameters [$a = 0,004$; $Z^c = 250$; $Z_0 = 200$], we can trace the path of a dynamic "bijective" pollution system with zero emissions when $Z_0 < Z^c$. The results, that would be identical with any set of parameters, show that as expected the pollution stock tends to zero while the assimilative capacity tends to its limit value 1 (Figure 2). This behaviour will be observed no matter how long the pollution stock has remained at high levels beforehand. Figure 3 illustrates this linear relation between α and Z along the path that starts in the top left hand corner and ends in the bottom right hand corner of the plane.

This "zero-emission" test reveals the optimistic assumption behind the bijective models' dynamics as the assimilative capacity will systematically be preserved and will allow the pollution stock to eventually converge towards zero, no matter the "history" of this pollution stock. We will see next that this is not the case when the dynamics allow for the inertia effect.

3.3.2. Pollution paths in a world with inertia and without emissions

Let us now observe the behaviour of a system allowing for inertia. As we noted in subsection 3.1 the forced correlation between the initial conditions Z_0 and α_0 that arises in the bijective model family is no longer present in a system with α as an autonomous state variable and we can thus explore a much wider set of initial conditions.

The dynamics with inertia are :

$$\dot{Z}(t) = -\alpha(t)Z(t),$$
$$\dot{\alpha}(t) = \begin{cases} -k(Z - \bar{Z}) & \text{if } Z \geq \bar{Z},\ \alpha > 0, \\ 0 & \text{if } Z < \bar{Z}, \text{ or } \alpha = 0. \end{cases}$$

5. it must be noted that we modify the notations of the authors to make them fit the ones we used so far and that here the inverted U-shape function refers to the total absolute assimilative capacity $\alpha(t) * Z(t)$, not to the image function $\alpha(Z(t))$.

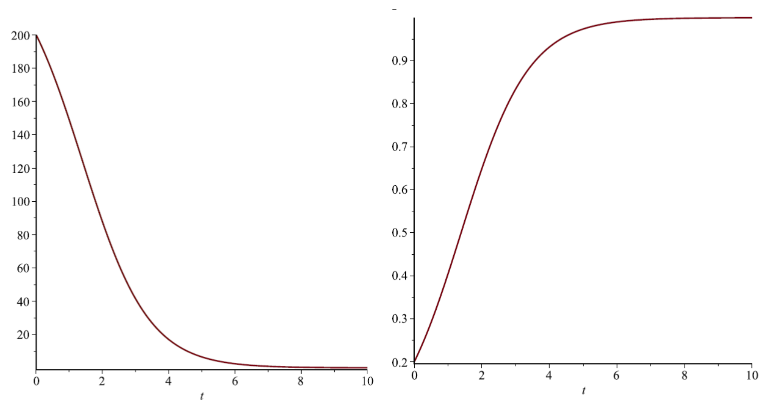

FIGURE 2: Pollution stock Z (left) and assimilative capacity α (right) along a zero-emissions bijective pollution path

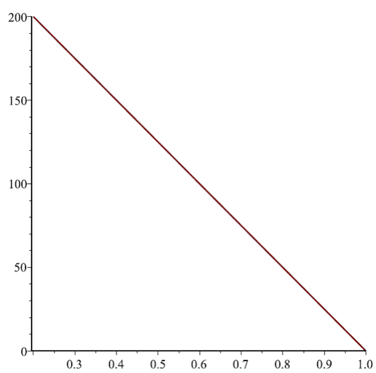

FIGURE 3: Z and α along a zero-emissions bijective pollution path

And at the steady state we have :

$$\dot{Z} = 0 \iff \alpha = 0 \text{ or } Z = 0$$

$$\dot{\alpha} = 0 \iff \alpha = 0\,,\text{ or } Z < \bar{Z}\,,\text{or } \{Z = \bar{Z} \text{and } \alpha > 0\}$$

Given the initial conditions $Z(0) = Z_0$ and $\alpha(0) = \alpha_0$:
– For all $Z_0 \leq \bar{Z}$ we have $\dot{\alpha} = 0$, and the steady state is thus $(\alpha, 0)$ for some α.
– For all $Z_0 > \bar{Z}$ we have two possibilities :

1. There exist $\hat{T} \leq \infty$ such that $\alpha(\hat{T}) = 0$, $Z(\hat{T}) \geq \bar{Z}$, then the steady state is $(0, Z(\hat{T}))$.
2. There exists $\bar{T}$ such that $Z(\bar{T}) = \bar{Z}$ with $\alpha(\bar{T}) > 0$, then the steady state is $(\alpha(\bar{T}), 0)$.

In conclusion there exists a curve $g(\alpha, Z) = 0$ in the (α, Z) plane with the function g such that $g(0, \bar{Z}) = 0$, $g(\alpha, \bar{Z}) > 0$ when $\alpha > 0$ and g decreasing in Z and increasing in α. This curve crosses the Z-axis at a finite time $\hat{T}$, which means that along a dynamic path following this curve there exists a time $\hat{T}$ such that $\alpha(\hat{T}) = 0$ and we can assess that
– In region V (below the curve $g(\alpha, Z) = 0$) the steady state is $(\alpha(\bar{T}), 0)$.
– In region D (above the curve $g(\alpha, Z) = 0$) the steady state is $(0, Z(\hat{T}))$.

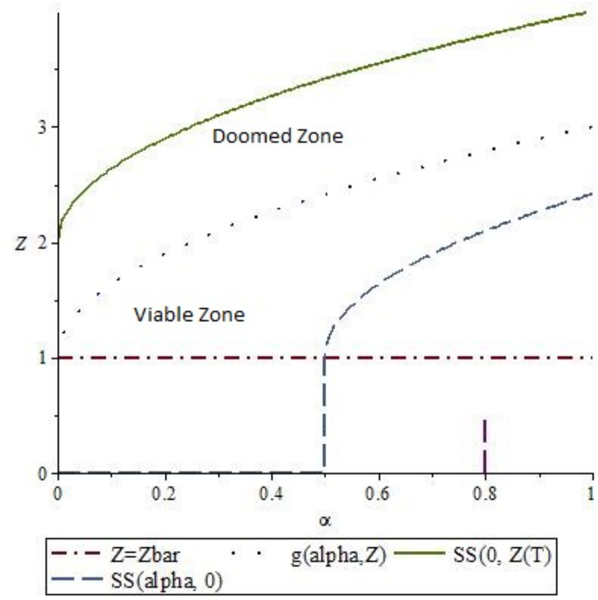

FIGURE 4: Sustainable and unsustainable paths in a world without emissions

In Figure 4, the paths of the dynamic system must be read from right to left as the pollution stock will never increase in the "natural" situation without emissions. The degradation threshold is marked by the red line $Z = \bar{Z}$. The dotted black curve $g(\alpha, Z) = 0$ separates the plane in two regions. In the doomed zone "D", above $g(\alpha, Z) = 0$, the system will naturally converge towards an unsustainable steady state $(0, Z(T))$. This reflects the fact that even without emissions this set of initial conditions is bound to lead to the total depletion of assimilative capacity as the pollution stock will not decrease fast enough to reach the threshold $\bar{Z}$ before α runs out. In the viable region "V", below $g(\alpha, Z) = 0$, the system starting with any set of initial conditions will converge towards a strongly or weakly sustainable steady state $(\alpha > 0, 0)$. The level of assimilative capacity preserved will depend on how long the pollution stock remains above $\bar{Z}$.

For instance, the green line $(SS(0, Z(T)))$ in the top left-hand corner is an example of trajectory starting in the D and converging towards a steady state $(0, Z(T))$ with $Z(T) > \bar{Z}$. The blue dotted curve $(SS(\alpha, 0))$ on the right is an example of trajectories that start in the V zone, above the threshold $Z = \bar{Z}$. This curve corresponds to a weakly sustainable path where both the pollution stock and the assimilative capacity decrease until the $\bar{Z}$ threshold is reached and the assimilative capacity remains constant afterwards. The purple line illustrates the simple case when the system starts below $\bar{Z}$ and thus converges towards $Z = 0$ without experiencing assimilative capacity degradation.

Note that if $\exists T$ such that $(Z(T), \alpha(T)) \in D$, $(Z(T), \alpha(T)) \in D \; \forall t \geq T$. As a consequence an economy with positive emissions located in the doomed zone at a given time will necessarily end up with a zero assimilative capacity. But of course the opposite is not true. An economy starting in the viable zone may also end up with a zero assimilative capacity if it chooses high levels of emissions that eventually lead it to the doomed zone. Or it can remain in the viable zone and reach a sustainable solution if it wishes.

The viable zone we have defined reminds of the concept of the viability kernel (Martinet & Doyen, 2007) : for any system located in the viability kernel/viable zone it is possible to find a path (not necessarily optimal) that remains in the viable zone (although it might imply a zero emission policy). In the case of zero-emission scenarios, this viability kernel corresponds to the constraints $y(t) \geq 0$ and $\alpha(t) > 0 \; \forall \; t$. Although the simulations would require much more complex algorithms, it could be theoretically possible to trace this kernel for a more realistic constraint on $y(t)$, such that $y(t) \geq \bar{y}$. This exploration of viability theory could be a relevant extension of our analysis in future work. What's more, it is straightforward that this viability kernel increases with $\bar{Z}$ and decreases with k.

This preliminary analysis will be used subsequently to sketch the "best scenario" behavior of the economy. Indeed Figure 4 gives us the final outcome of the system for any set of initial conditions when emissions are nil. We can thus deduce *a fortiori* that if at one time along the optimal path the system is in the doomed zone, it can never recover back to a sustainable solution, even less so if the emission level is strictly positive. At this point we cannot build systematically the separatrix between the two zones for any given set of parameters. Nevertheless further research could enable us to provide a tool that would immediately determine if the initial conditions are located in the Doomed or in the Viable zone, and maybe even allow us to test more realistic emissions constraints ($y(t) \geq \bar{y}$ instead of just $y(t) \geq 0$, see above the discussion of the viability kernel concept).

4. Assessing the importance of inertia through analytical and numerical injection

4.1. Injecting benchmark optimal emissions in our system with inertia

Now that we have highlighted the sustainability issue raised by inertia in pollution control paths we can present our inertia assessment tool. We shall test up to what extent an benchmark emission path (BP) such that ($y^*(t)$ ∀t) obtained in the benchmark model with constant assimilative capacity leads to the extinction of this assimilative capacity when it is injected in a system accounting for degradation with inertia. This assessment relies on a first analytical step followed if necessary by a numerical test.

The path under study will thus follow the dynamics below.

$$\dot{Z}(t) = y^*(t) - \alpha(t)Z(t) \tag{13}$$
$$\dot{\alpha}(t) = \begin{cases} -k(Z-\bar{Z}) & \text{if } Z \geq \bar{Z} \text{ and } \alpha > 0, \\ 0 & \text{if } Z < \bar{Z}, \text{ or if } \alpha = 0. \end{cases}$$
$$Z(0) = Z_0, \; \alpha(0) = \alpha_0$$

with $y^*(t) > 0 \; \forall t$ and Z^*_{ss} the optimal steady state level of pollution such that [6]

$$\alpha_0 Z^*_{ss} = \lim_{t\to\infty} y^*(t). \tag{14}$$

When the BP $(y^*(t))$ is implemented in the benchmark setting it yields a monotonic path for the state variable, $Z^*(t)$, but this pollution accumulation path might no longer be monotonic when we inject the BP in the dynamic system with inertia. This non-monotonocity can occur if the BP is decreasing in $Z(t)$, ie $y^*(t) < \alpha_0 Z^*(t) \; \forall t$. In that case once the BP is injected in the system with inertia α might decrease in such a way that at one point $y^*(t) > \alpha(t)Z(t)$, which means an increase in Z.

6. It must be noted that variables with no superscript refer to the variables in the system with inertia while variables with a * superscript refer to the variables obtained in the benchmark model without assimilative capacity degradation.

4.2. Analytical sustainability test

Let us now consider the outcomes in terms of steady states of a BP injected in the system with inertia. Once it is plugged in the dynamic system with assimilative capacity degradation, the BP might not lead to the steady state that was optimal in the benchmark setting since decrease in α can modify the dynamics of Z.

According to (13) and the dynamics of α, at the steady state we need the following conditions to be verified :

$$\dot{Z} = 0 \iff \lim_{t\to\infty} y^*(t) = \alpha_{ss} Z_{ss} \tag{15}$$

$$\dot{\alpha} = 0 \iff \alpha = 0 \text{ or } Z \leq \bar{Z} \tag{16}$$

with Z_{ss} the pollution stock at the steady state in the system with inertia and $\alpha_{ss} = \lim_{t\to\infty} \alpha(t)$.

Proposition 4.1 1. *if $\exists\ T$ such that $\alpha(T) = 0$ along the path in the inertia system, then $\alpha_{ss} = 0$ and (15) cannot be verified given the strict positivity of $y^*(t)$. No steady state can exist, the system tends towards a limit point $(\infty, 0)$.*

2. *if $\alpha(t) > 0\ \forall t$, a steady state exists if*

$$Z^*_{ss} \leq \frac{\alpha_{ss}}{\alpha_0} \bar{Z} \tag{17}$$

Proof.

1. is straightforward.
2. if a steady state with $\alpha > 0$ exists, we have

$$\alpha_{ss} Z_{ss} = \lim_{t\to\infty} y^*(t)$$

Hence, according to (14) :

$$Z_{ss} = \frac{\alpha_0}{\alpha_{ss}} Z^*_{ss}$$

A steady state exists if (16) is verified, given that $\alpha > 0$, ie if

$$Z_{ss} \leq \bar{Z} \quad \iff \frac{\alpha_0}{\alpha_{ss}} Z^*_{ss} \leq \bar{Z} \quad \iff Z^*_{ss} \leq \frac{\alpha_{ss}}{\alpha_0} \bar{Z} \tag{18}$$

■

Proposition 4.1 thus shows that for given α_0, a steady state can be reached if :

- α_{SS} is big enough, ie if the assimilative capacity has not been too degraded along the BP, which means a low enough degradation speed k
- Z^*_{ss} is small enough, ie if the benchmark optimal steady state pollution stock was not to high in the first place, which demands a high enough environmental damage parameter d and/or a low enough profitability of emissions captured by the ratio $\frac{c}{b}$ and/or a low enough discount rate ρ

The role of α_0 in this comparative statics is ambiguous since this parameter affects both the fraction on the right hand side of 17 and the level of Z^*_{ss} itself through the channels we unraveled in Section 2. According to Proposition 2.1, we know that in the case of a pollution aware economy ($\rho < \sqrt{\frac{d}{b}}$) and for low values of α_0, a higher α_0 yields a higher Z_{ss}. Since a higher α_0 also reduces the right hand side term of 17, it makes it harder to respect the condition $Z_{ss} \leq \bar{Z}$. However in a pollution-blind economy or for a high range of α_0 in a pollution-aware economy, the effect of α_0 remains ambiguous as Z_{ss} decreases with α_0 while the right hand side term of 17 also decreases. Although we cannot fully characterize the comparative statics of α_0 we have identified a counter-intuitive case where a higher α_0 increases the probability for the benchmark path to be unsustainable.

In order to draw an operative tool from Proposition 4.1 we need to introduce the following corollary :

Corollary 4.1 *If $Z^*_{ss} > \bar{Z}$, the BP is not sustainable.*

Proof.

– If $\alpha_{ss} = 0$, then according to Proposition 4.1 no sustainable steady state can be reached.

– If $\alpha_{ss} > 0$, since by definition $\alpha_{ss} \leq \alpha_0$, then $Z^*_{ss} > \bar{Z} \Rightarrow \frac{Z^*_{ss}}{\alpha_{ss}} > \frac{\bar{Z}}{\alpha_0}$
Condition (18) is not verified and no sustainable steady state can be reached.

■

Corollary 4.1 give us an a priori unsustainability condition for the BP under scrutiny that is easier to apply than 18. Indeed the latter requires to know the value of α_{ss} while the former only compares the ex ante values of Z^*_{ss} and $\bar{Z}$.

Corollary 4.1 can be translated with our functional forms into the following condition. Let us define ρ_u such that

$$\rho_u = \frac{d\bar{Z} - \alpha_0 c + b\alpha_0^2\bar{Z}}{c - b\alpha_0\bar{Z}}$$

We have

$$\rho > \rho_u \Rightarrow Z^*_{ss} > \bar{Z} \tag{19}$$

This condition allows us to deem unsustainable any BP such that $\rho > \rho_u$. Consequently we can focus our numerical simulations exclusively on potentially sustainable BP depending on the discount rate which remains the most "flexible" parameter of the model.

Corollary 4.1 provides the first component of our inertia assessment tool. If a BP does not respect condition 4.1 (or 4.3.1), we can determine a priori that it is unsustainable. However those conditions are sufficient and not necessary. A BP that respects condition 4.1 can nevertheless be unsustainable as well. To test the sustainability of those path we thus need to use the second (numerical) component of our inertia assessment tool.

4.3. Numerical sustainability test

The sustainability of a BP that respects condition 4.1 needs to be assessed with a numerical injection of the BP into a world with inertia. Mapple simulations built on our analytical results of Section 2 enable us to observe the behaviour of a given BP in a world with inertia and to test the sensitivity of the resulting path to the range of values of the main parameters. In order to illustrate this second component of our assessment tool we shall investigate the sustainability of a benchmark path produced by baseline parameters adapted from Schubert (2005). For the sake of clarity we estimate both the emission y and the stock Z in carbon gigatons (GtC) instead of converting the cumulated atmospheric carbon into ppm concentration as Schubert does. As far as the social discount rate is concerned we calibrate the initial system with $\rho = 3\%$ and we will discuss its impact further on. Let us recall that our model assumes that the entire assimilative capacity is subject to time-dependent degradation. This assumption overlooks the specific behavior of distinctive carbon sinks such as forests but it takes a conservative stand on the dynamics of this ecosystem service.

$$
\begin{aligned}
b &= 1,35 \\
c &= 10 \\
d &= 0,001 \\
\alpha_0 &= 0,004
\end{aligned}
$$

As most studies estimate this decrease to have begun roughly around the year 2000 for the Southern Ocean, which accounts for the highest carbon uptake on the globe, we shall set the value of $\bar{Z}$ as the carbon stock in the atmosphere for that year, rounding up the 369 ppm concentration (National Oceanic and Atmospheric Administration data) to 790 GtC using the standard 2,13 ppm/GtC conversion factor. The value of k is crucial to our analysis but to our knowledge it has never been calibrated for economic purposes. We have thus conducted a rough extrapolation of the estimations found in the climate and ocean science literature (Le Queré et al., 2007 ; Schuster and Watson, 2007), relying on our value of 790 GtC for Z_0. We are aware that the figure we use for k, like the mere concept of reducing the oceans' sink capacity to our linear degradation (10), is highly questionable for biologists and climate scientist. However it can nonetheless shed some light on the order of magnitude of the impact of inertia on the sustainability of economic climate policies. We will test the sustainability of the BP for much higher values of k to take a more conservative view on the issue. Finally we shall work with a current stock of atmospheric carbon Z_0 equal to 850 GtC, converting the 401 ppm concentration measured in june 2015 by the National Oceanic and Atmospheric Administration.

$$
\begin{aligned}
k &= \hat{k} = 8,508E^{-07} \\
\bar{Z} &= 790 \\
Z_0 &= 850
\end{aligned}
$$

Using these parameters and the results from section 2 (in particular Proposition **??**), we can easily obtain the numerical values for $y^*(t)$ at all time t, ie the optimal emission level for a benchmark pollution control model with constant assimilative capacity. We shall then apply this emission policy in a world with assimilative capacity degradation, ie in our dynamic system with inertia.

4.3.1. Ex ante unsustainability

Let us first check that the BP under study is not a priori unsustainable. In order to use condition , we calculate ρ_u with our parameters :

$$
\rho_u = \frac{d\bar{Z} - \alpha_0 c + b\alpha_0^2\bar{Z}}{c - b\alpha_0\bar{Z}} = 0,133774677 \tag{20}
$$

For any discount rate lower than 13,37%, the BP is not ex ante unsustainable and $Z_{ss}^* \leq \bar{Z}$. We can thus explore the standard range of values for the discount rate without falling into the ex ante unsustainable category. We shall start with $\rho = 3\%$ and run some sensibility tests.

4.3.2. Initial conditions and sustainability of the BP

In a world with inertia the sustainability of the benchmark path under scrutiny will depend heavily on the initial condition, and especially on the position of the initial pollution stock regarding the degradation threshold

$\bar{Z}$.

If $Z_0 \leq \bar{Z}$ then is straightforward that whether Z^*_{ss} is higher or lower than Z_0, it respects $Z^*_{ss} \leq \bar{z}$ nonetheless, and the system will always remain within the regime where α does not decrease, through either a decreasing (if $Z_0 > Z^*_{ss}$) or increasing (if $Z_0 < Z^*_{ss}$) monotonic path. It will thus behave exactly like the benchmark model, keeping the assimilative capacity at α_0. The BP will thus be sustainable and converge towards its expected steady state via a path identical to the setting without inertia.

We shall thus focus on the case $Z_0 > \bar{Z}$. Since $Z^*_{ss} \leq \bar{Z}$ we know that when when $Z_0 > \bar{Z}$ the BP will be decreasing. Figure 5-left shows the phase portrait of the system, with the baseline parameters and $Z_0 = 850$. The system starts with a pollution stock above the degradation threshold (upper part of the figure) but it nevertheless reaches a steady state below $\bar{Z}$ with a strictly positive α. The degradation undergone by the assimilative capacity is significant : about a 25% decrease (α decreases from 0,004 down to $\alpha(T) = 0.00287$=) after a degradation period such that $\bar{T} = 60,13$. What's more, a zero-emission path (solid red line in Figure 6) in the same setting would end up with an assimilative capacity $\alpha(T)$ significantly higher (0,0398) than the BP (dotted red line).

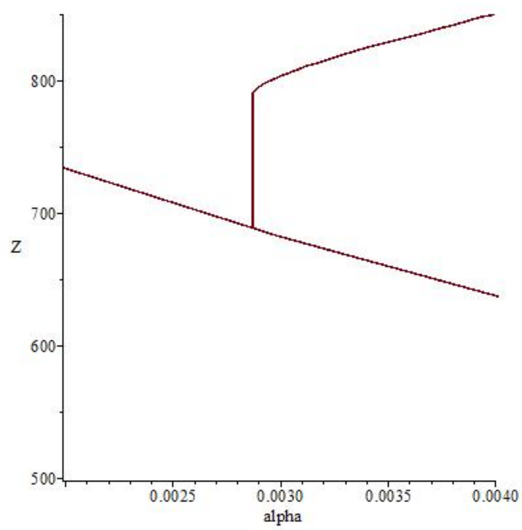

FIGURE 5: Phase portrait with baseline parameters, $k = \hat{k}$

Our simulations also show that the steady state pollution level Z_{ss} ($Z_{ss} = 688.33$ GT) reached in the system with inertia by the injected BP is higher than the steady state $Z^*_{ss} = \frac{y^*_{ss}}{\alpha_0}$ (=640 GT) it would have reached in the benchmark system without inertia. This result is rather intuitive since when the path starts above $\bar{Z}$, α_{ss} will be lower than α_0 following the degradation. The steady state level of pollution will thus settle at a level Z_{ss} such that

$$Z_{ss} = \frac{y_{ss}}{\alpha_{ss}} = \frac{y^*_{ss}}{\alpha_{ss}} > \frac{y^*_{ss}}{\alpha_0} = Z^*_{ss}$$

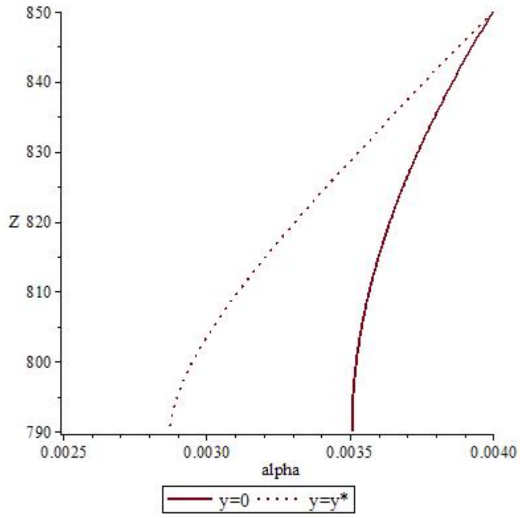

FIGURE 6: Phase portrait with baseline parameters, $k = \hat{k}$

4.3.3. Exploring the range of sustainability depending on the key parameters
In order to yield more robust results on the global impact of inertial, it is necessary to consider the outcome of the dynamic system for a more conservative range of values as far as the most questionable parameters are concerned.

Inertia degradation speed. Given the ecological uncertainty surrounding the parameter k, we need to assess the impact of higher values for k. Our sensitivity analysis shows that for any value of k higher than $1,1 * \hat{k}$, the pollution path is no longer sustainable and the pollution stock tends towards infinity. We illustrate this behaviour with the case $k = \hat{k} * 2$ in Figure 7.

The original value we had estimated for k is thus almost a limit value, all other parameters being equal. If the degradation speed of the assimilative capacity is slightly higher than our estimation, the pollution path becomes unsustainable. There is thus very little margin of error when considering the sustainability of a pollution path in presence of inertia.

Social discount rate. We have worked so far with a standard discount rate value of 3%. This social parameter can nonetheless be discussed as its effect on long-term outcomes is well known. We know from 20 that ρ must be inferior to $13,37\%$ for our test to be relevant (or else $Z^*_{ss} > \bar{Z}$ and the path is a priori unsustainable). Let us explore the consequences of a higher discount rate on the sustainability of the optimal path. Our numerical results show that for $\rho > 0,0305$, the new optimal path $y^*(t)$ causes the system to tip over to an unsustainable path that leads to the extinction of the assimilative capacity and the explosion of the pollution stock. Although the general effect of a higher discount rate is expected, the order of magnitude highlighted here draws attention : sustainability cannot be achieved when the BP is based on a discount rate slightly higher than 3%, whereas the choices for the discount rate in the optimal pollution control literature usually ranges up to 5%.

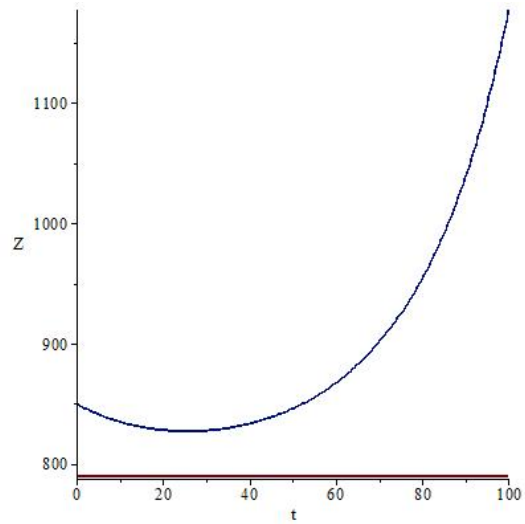

FIGURE 7: Phase portrait with baseline parameters, $k = 2 * \hat{k}$

5. Policy interpretation and conclusion

Through our assessment tool based on the injection of a benchmark path in a world with inertia we can establish an important result regarding environmental policies and especially climate change programs. Our numerical results based on robust parameters for aggregate climate change model show that inertia does matter. The ecological (k) and social (ρ) limit values that can lead to unsustainable pollution paths and subsequent explosive pollution accumulation are not far from the values found in common models. That is to say that a relatively innocuous pollution path obtained in a benchmark model can actually lead to the extinction of the assimilative capacity once we account for the effect of inertia. More importantly, our results highlight the very thin leeway of an environmental policy in presence of ecological inertia. If k is slightly higher than expected, which is not unlikely given the heavy uncertainty on this parameter, the path will no longer be sustainable.

Our frame of analysis can provide a useful inertia assessment tool for environmental policies applied to pollution problems affected by inertia. This tool relies consists of two steps :

1. An analytical sufficient condition (Corrolary 4.1) that determines the ex ante unsustainability of a BP
2. A numerical injection that determines whether or not the path is sustainable and assesses the margin of error the social planner can lean on when implementing such a path under the threat of assimilative capacity extinction.

In the light of the worrying evidence on the fast degradation of oceans' carbon sinks, we engaged in a process of complexifying the standard optimal pollution control model in order to reflect more accurately the dynamics of assimilative capacity. However our goal was not to build a more intricate model for the sake of it but to assess if this feature was a crucial one to take into consideration when it comes to actual implementation of climate economic policies. We believe our inertia assessment tool can shed some useful lights for policy-maker when it comes to the calibration of environmental policies in presence of inertia.

But our work also raises another environmental policy issue, namely the importance of monitoring better the natural assimilative capacity at play in crucial environmental problems such as climate change. The high uncertainty wrapping carbon sinks around the planet must be diminished through ambitious scientific programs in order to better understand and manage this ecosystem service. We must also keep in mind that the definition of sustainability we have worked with is restricted to keeping a positive level of assimilative capacity. We thus ignore the other ecological externalities that take place under the effect of GHG accumulation such as ocean acidification and its consequences on biodiversity. These other externalities should also be taken into consideration when we weigh in the consequences of climate change on oceans.

The mechanism of inertia in ecological dynamics is a crucial one. This paper sought to determine its importance in an economic framework and showed the thin margin of manoeuver available to policy makers for the case of oceans carbon uptake reduction in the case of climate change. We believe that inertia is a relevant concept that can help improve dynamic economic models, in particular in the field of resource and environmental economics. The management of other ecosystem services might benefit in future works from the introduction of inertia in the dynamic framework at play so as to avoid validating unsustainable policies.

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