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Why health matters in the energy efficiency–energy consumption nexus? Some answers from a life cycle analysis

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Abstract

This paper shows that accounting for the growing interdisciplinary literature supporting the causality between energy efficiency and health and the empirical evidence re-assessing the importance of health on workforce productivity, could explain a part of the paradoxal relationship found between energy efficiency and energy consumption.

We build a 3-period overlapping generations model where we assume that residential energy inefficiency induces chronic disease for adults and bad health for elderly. We also assume that workers' health has an effect of their labor productivity. Our results suggest, in particular, that if mostly old (respectively young) people health is affected, the health impact of residential energy efficiency should have a backfire (resp. rebound) influence on residential energy consumption, by promoting precautionary saving (resp. by rising labor productivity).

In policy terms, by showing that the link between energy efficiency and energy consumption is far from being just associated with technical conditions about preferences and/or production technology, our research emphasizes how crucial and complex are for governments the discussion and policy action dealing with the connection between energy conservation policies, health insurance system and growth.

Keywords

Energy efficiency; Health; Precautionary saving; Labor productivity, Overlapping generations model.

***JEL* classification**

$D58$; $Q43$

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1 Introduction

The economic literature dealing with energy efficiency and energy consumption is large. The analysis of the issue was usually conducted from the angle of rebound effect: why energy conservation from energy efficiency policies is not as large as expected? What type of policy pathways are more effective in mitigating the rebound effect? Through analyzing the rebound effect, a number of contributions pointed out how difficult is to estimate the magnitude of rebound effect, thus, how complex are interactions between energy efficiency and energy consumption.

In this research, we study the role of health in understanding energy efficiency-consumption nexus and propose a new surrounding background to understand rebound effect persistance. We focus on residential energy efficiency. The contribution of this article is to originally build upon previous insights from two different bodies of literature in order to study how health contribute to the analysis of energy efficiency-consumption nexus. The first body of literature is empirical and interdisciplinary. It supports the causality assumption between energy efficiency and health. The second belongs rather to the economic growth field and focus on assessing the role of health, in particular on labor productivity, in the growth process.

We rely on fairly standard life-cycle model with 3-period overlapping generations under which we assume that residential energy inefficiency induces chronic disease for adults and bad health for elderly. We also assume that workers' labor productivity is dependent on their health status. Our theoretical model relies on Kotlikoff (1989) who introduces in a two-period OLG model an exogenous probability to be sick when old. We borrow also from Wang et al. (2015) who make endogenous the probability to be sick due to pollution and we assume rather that energy efficiency is the main determinant of illness. However, our framework differs from these studies in several aspects. First, while Kotlikoff (1989) as well as Wang et al. (2015) focus on old agents' health, our paper puts the spotlight on morbidity of both adults and elderly. This focus on adult morbidity is not simply an innocuous modeling variation. Instead, it is motivated by the observation that even empirical results support that many households living in an energy inefficient environment have low life expectancy, some other ones have high ill life expectancy especially when young and adult. Thus, the morbidity-energy linkage allows us to explore a novel inter-connection between health, savings and energy policies, in particular those related to the rebound effect phenomenon. Finally, another difference with respect to Kotlikoff (1989) and Wang et al. (2015) is that we consider that there are chronic effects of the disease meaning that the level of health does not return to its initial level even after health expenses.

Our results show that health channels we introduced could significantly shape the change in energy consumption induced by an energy efficiency variation, through their impacts on the propensity to save and on the revenue. In particular, our results suggest that if mostly old (resp. young) people health is affected, the health impact of residential energy efficiency should have a backfire (resp. rebound) influence on residential energy consumption, by promoting precautionary saving (resp. by rising labor productivity). Interestingly, our results also show that, if health externality on labor productivity exists,[1] an energy efficiency improvement should rise the energy consumption not only in the residential sector but also in production sector.

Our theoretical approach is based on an overlapping generation model. So, it allows to consider the age effect related to energy policy conservation. As far as we know, a study of how health impacts energy efficiency-energy consumption nexus in terms of the distinction between people's ages—in particular, young versus adults—has never been carried out before, although this issue is crucial for the implementation of policies that aim to promote energy conservation in connection to health and growth issues.

The paper unfolds as follows. In Section 2, we give two brief reviews of the literature. The first review is interdisciplinary and empirical and focuses on the impact of residential energy efficiency on health outcomes. The second review deals with the relationship between health and productivity as studied in the economic growth literature. In Sections 3 and 4, we present the basic theoretical model and we derive analytical results about the long-term equilibrium. In Section 5, we compute numerical simulations calibrated to the U.S. economy. In Section 6, we investigate two important extensions of

[1] As suggested by recent empirical evidence (see Section 2.2).

the basic model. The first extension deals with the influence of health-status on utility whereas the second considers the chronic dimension of bad health associated with low residential energy efficiency. Finally, in Section 7, we conclude and give some policy implications.

2 Related Literatures

In this section, we present two brief literature reviews on the two frameworks within which our analysis is constructed, namely the interdisciplinary empirical literature studying the impact of residential energy efficiency on health outcomes (Sub-section 2.1) and the economic growth literature focusing on the relationship between health, in particular labor productivity, and growth (Sub-section 2.2).

2.1 Residential energy efficiency and health

Housing thermal discomfort is one of the most important proxy of residential energy (in)efficieny[2] and there has been much research on its effects on health since the pioneering assessment of the cost of indoor cold and the definition of the 10% energy-poverty indicator by Boardman (1991). For instance, Baker (2001) produced a review of the evidence on the link between living in an energy inefficient dwelling and the increased risk of illness. This study showed in particular a strong association between low indoor temperatures and increased risk of strokes, heart attacks and respiratory illness. Other evidence shows cold stress causing cardiovascular strain and increased incidence of dust mites in poorly ventilated homes – in turn affecting asthma and eczema, especially in children.

Based on an epidemiological approach, the Large Analysis and Review of European Housing and Health Status (LARES) study[3] shows that there is a significant relationship between dwelling energy efficiency, as approximated by thermal discomfort level, and physical health (Ezratty et al., 2009). This relationship takes the form of a negative link between thermal discomfort and the risk of respiratory and cardiovascular diseases, hypertension or the presence of digestive disorders. LARES also shows the same type of negative link between thermal discomfort and mental and social well-being.

By the same, Wilkinson et al. (2001) showed that there is a credible chain of causation that links low indoor temperatures induced by residential energy inefficiency to cold-related deaths. In particular, there is a 23% excess of deaths from heart attacks and strokes. Indoor temperatures below 16^{o}C are a particular risk and are most likely to affect old and poorly heated housing with low-income residents. Also, Howden-Chapman et al. (2005) focused on analyzing the consequences of insulation measures[4] on health, the well-being of the occupants, as well as on their utilisation of health care.

By considering a discursive approach, Ezratty (2010) and Ormandy and Ezratty (2012) argue that housing conditions, in particular residential energy efficiency considerably affect physical and mental heath as well as social well-being. In the same context, based on a meta-analysis dealing with the impact of household energy efficiency measures on health, Maidment et al. (2014) argue that household energy efficiency interventions led to a small but significant improvement in the health of residents. Liddell and Guiney (2015), based on a literature review of nine intervention studies that outline the prevailing framework for understanding mental well-being in the fields of psychology and psychiatry, argue that living in an energy inefficiency dwelling *i.e.* cold and damp housing, contributes to a variety of different mental health stressors, including persistent worry about debt and affordability, thermal discomfort and worry about the consequences of cold and damp for health.

More recently, an effervescent literature considering different specific country-case studies confirms previous results and argues that there is a significant link between thermal energy efficiency and health. For example, Fisk et al. (2020) review empirical data from evaluations of the influence of thermal energy efficiency retrofits on indoor environmental quality conditions and self-reported thermal comfort and health. They show that average indoor temperatures during winter typically increased after retrofits and that dampness and mold almost always decreased after retrofits. They add that subjectively

[2] The other most used proxy in the literature is damp and mould growth.

[3] LARES is a pan-European housing and health survey that was undertaken from 2002 to 2003 in eight European cities at the initiative of the WHO European Housing and Health task force. It was designed to improve knowledge on the impacts of existing housing conditions on health and mental and physical well-being.

[4] Insulation measures affect indoor temperature, humidity, energy consumption and mould growth.

reported thermal comfort, thermal discomfort, non-asthma respiratory symptoms, general health, and mental health nearly always improved after energy efficiency retrofit.

In summary, a large body of empirical interdisciplinary studies, although using different approaches, provide vast evidence that indoor environmental quality, particularly residential energy inefficiency, have negative impacts on physical and mental health outcomes. Some of these studies particularly highlight the age's impact of energy efficiency.[5]

2.2 Health and labor productivity

The literature recognizing the impact of health on labor productivity is growing. In particular, there has been an important increase in the number of academic articles that focus on estimating the economic burden of illness, in particular, chronic deseases. This literature includes not only direct but also indirect costs of the disease (Li et al., 2006; Kirsten, 2010; Anis et al., 2010). Indirect costs are now widely referred to as productivity losses (Drummond et al., 2015; Gold et al., 2016). Zhang et al. (2011) argue that there is still a lack of detailed methodological guidance on how productivity loss should be measured. They review measurement issues and valuation methods for estimating productivity loss due to poor health and assert that in some cases, i.e. risk averse workers, job involving team production, unavailability of perfect substitutes, productivity loss is likely to be underestimated.

In the environmental economics field, in the vein of empirical works showing that pollution is a driver of bad health conditions, Graff Zivin and Neidell (2012) propose one of the first empirical studies which rigorously assess what they call the environmental productivity effect. More precisely, they assess the environmental pollution impact on worker productivity by linking the exogenous daily variations in ozone with worker productivity of agricultural workers, due to health deterioration. They find a significant evidence that ozone levels well below federal air quality standards have a significant impact on productivity. They argue that the empirical estimation of this relationship is complicated because obtaining clean measures of worker productivity is a perennial challenge and because the exposure to pollution levels is typically endogenous. Recently, Aguilar-Gomez et al. (2022) review the economic research investigating the causal effects of pollution on labor productivity, cognitive performance and multiple forms of decision making. Regarding labor productivity, they particularly show that air pollution reduces worker productivity and, in some cases, labor supply. However, they point-out that productivity estimates vary considerably and that there are several possible explanations for this divergence such as differences in occupations, setting, pollutant of interest and study design. Chang et al. (2016) also empirically show that particulate pollution have a negative effect on the productivity of workers at a pear-packing factory. Gibson and Shrader (2018) argue that the role that pollution may play in "sleep" disruption and its effects for labor productivity call for further investigations. By extension, they assert that a better understanding of who bears the costs of these effects would also help in identifying the incentives for private and public efforts to invest in, both, emissions control and exposure avoidance technologies.

Beyond the specific case of pollution as a driver of health conditions, Bhattacharya et al. (2008) studied impacts of chronic disease and severe disability among working-age populations and show that chronic conditions are recognized as an important cause of work disability. Their conclusion supports the assumption of a negative impact of health on labor productivity. By the same, Zhang et al. (2009) examine the impact of several chronic diseases, i.e. diabetes, cardiovascular diseases and mental illnesses on the probability of labour force participation using data from the Australian National Health Surveys. They show that the estimated effects are significant and that they differ by gender and age groups.

Considering a more general framework, Bloom et al. (2004, 2019), Dormont et al. (2010) and Weil (2007, 2014), amongst others, emphasize the relationship between health, labor productivity and economic growth. Bloom et al. (2004) show that good health has a positive, sizable and statistically significant effect on aggregate output and argue that the life expectancy effect in growth regressions appears to be a real labor productivity effect, rather than the result of life expectancy acting as a proxy

[5] It is important to mention that while energy efficiency measures can improve health outcomes (especially when targeting those with chronic respiratory illness), some of these measures can induce negative consequences on health (Sharpe et al., 2019).

for worker experience. Dormont et al. (2010) highlight that health could be seen as a labour-augmenting factor increasing the level of individual productivity, even if its effect could not be sufficient to generate a growth enhancing mechanism. At micro-level Weil (2007) calculates that an increase in adult survival rates of 10% raises labor productivity by about 6.7 points. At macro-level Bloom et al. (2019) estimate that an increase of 10% of the adult survival rate would lead to a 9.1% increase in labor productivity.

3 The basic model

The economy consists in an infinite sequence of overlapping generations. Each generation lives for three periods (de la Croix, 1996; De La Croix and Michel, 2002). The young generation has no decision to take, just lives with their parents and therefore benefits/suffers from residential energy conditions at parents' home. In the second period of her life, when she is adult, she supplies inelastically one unit of labor. She retires when old. Population evolves at a constant rate of growth $n \in]-1, +\infty[$ such that $N_{t+1} = (1+n)N_t$.

3.1 Health-status and energy efficiency

In a first step of the analysis, we consider the simple case where poor energy efficiency makes people sick when adult and/or old, but illness is cured during the period, thanks to healthcare expenditures. In a first simplifying step, we also consider that the probability to be sick when old is not related to previous health condition. In a second step, conducted in Section 6.1, we study a more realistic case where poor energy efficiency could lead to chronic diseases.

In our basic simplifying framework, each generation can be in "bad" health when adult with a probability π_t^{a} and when old with a probability π_{t+1}^{o}. Each probability is unrelated and it depends on the efficiency of energy services each generation lived with in the previous period. Then the probability of being sick for the adult generation at time t (respectively for the old generation at time $t+1$) is defined as:

$$\pi_t^{\mathrm{a}} = \pi^{\mathrm{a}}(\varepsilon_{r,t-1}) \qquad \text{and} \qquad \pi_{t+1}^{\mathrm{o}} = \pi^{\mathrm{o}}(\varepsilon_{r,t}) \tag{1}$$

with $\pi_t^{\mathrm{a}\prime}(\varepsilon_{r,t-1}) < 0$ and ${\pi_{t+1}^{\mathrm{o}}}'(\varepsilon_{r,t}) < 0$. $\varepsilon_{r,t}$ (resp. $\varepsilon_{r,t-1}$) denotes residential energy efficiency in period t (resp. $t-1$). The subscript "r" is used to denote the residential sector.

For convenience we normalize "good" health to unity and we assume that "bad" health of an adult generation (resp. old generation) is denoted by $h_t^{\mathrm{a}} < 1$ (resp. $h_{t+1}^{\mathrm{o}} < 1$). To restore her health, the adult generation (resp. old generation) with poor health incurs healthcare expenditures denoted by m_t^{a} (resp. m_{t+1}^{o}). We assume that the higher the detrimental effects of bad energy efficiency the higher healthcare expenditures requires to recover health. Therefore we define:

$$m_t^{\mathrm{a}} = m^{\mathrm{a}}(\varepsilon_{r,t-1}) \qquad \text{and} \qquad m_{t+1}^{\mathrm{o}} = m^{\mathrm{o}}(\varepsilon_{r,t}) \tag{2}$$

with $m^{\mathrm{a}\prime}(\varepsilon_{r,t}) < 0$ and $m^{\mathrm{o}\prime}(\varepsilon_{r,t+1}) < 0$. This is in accordance with Gutiérrez (2008) and Wang et al. (2015) who study the negative impact of ambient air pollution. We follow their assumption that healthcare expenditures enable agents to recover full health.[6] Nevertheless, conversely to these authors, we assume that health recovery takes time. When sick, agents need a portion $z^{\mathrm{a}}(\varepsilon_{r,t-1}) \in [0, \bar{z}^{\mathrm{a}}]$ of their adult lifetime (resp. $z^{\mathrm{o}}(\varepsilon_{r,t}) \in [0, \bar{z}^{\mathrm{o}}]$ when old) to get full health back, with $z^{j'}(\varepsilon_r) \leq 0$ and $\bar{z}^j < 1$ for $j = \mathrm{a}, \mathrm{o}$. As a consequence, remembering that full health is normalized to unity, "bad" health-status during each period of life is defined as:

$$h_t^{\mathrm{a}} = h^{\mathrm{a}}(\varepsilon_{r,t-1}) = 1 - z^{\mathrm{a}}(\varepsilon_{r,t-1}) > 0 \qquad \text{and} \qquad h_{t+1}^{\mathrm{o}} = h^{\mathrm{o}}(\varepsilon_{r,t}) = 1 - z^{\mathrm{o}}(\varepsilon_{r,t}) > 0 \tag{3}$$

In that sense, $z^j(\varepsilon_r)$ captures the severity of the illness associated with low energy efficiency.

[6] For the moment and to avoid complexity, we assume that health expenditures are targeted to back health to its initial level after detrimental impact of low energy efficiciency on health. In Section 6.2 page 20 we will relax this assumption and we will investigate the case of endogenously chosen health-status.

3.2 Households

At each period, adult and old generations consume non-energy goods and energy services whose efficiency may affect positively their health. The expected intertemporal utility of an adult is:

$$V_t = \pi_t^{\mathrm{a}}\Phi(h_t^{\mathrm{a}})U(\bar{c}_t^b) + (1-\pi_t^{\mathrm{a}})U(\bar{c}_t^g) + \beta\left[\pi_{t+1}^{\mathrm{o}}\Phi(h_{t+1}^{\mathrm{o}})U(\bar{d}_{t+1}^b) + (1-\pi_{t+1}^{\mathrm{o}})U(\bar{d}_{t+1}^g)\right] \tag{4}$$

where $U(x) \equiv (1-\sigma)^{-1}x^{1-\sigma}$ with $\sigma \in (0,1)$,[7] $\beta > 0$ is the subjective discounting parameter, and the composite consumption good of the adult generation and the old generation, respectively denoted by $\bar{c}^i$ and $\bar{d}^i$, are given by:

$$\bar{c}_t^i = \left[(1-\nu)c_t^{i\,\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}} + \nu\left(\varepsilon_{r,t}E_{r,t}^{\mathrm{a},i}\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}} \quad \text{and}$$

$$\bar{d}_{t+1}^i = \left[(1-\nu)d_{t+1}^{i\,\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}} + \nu\left(\varepsilon_{r,t+1}E_{r,t+1}^{o,i}\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}} \quad \text{with} \quad \nu \in]0,1[,\ i = (b,g)$$

c_t^i (respectively d_{t+1}^i) is the amount of non-energy goods the adult (resp. old) generation consumes with a health condition i (with $i = b$ when agents are in bad health and $i = g$ when agents are in good health). $E_{r,t}^{\mathrm{a},i}$ (respectively $E_{r,t+1}^{o,i}$) is the amount of energy services (whose efficiency is captured by ε_r) the adult (resp. old) generation consumes with a health condition i. $\sigma_{\mathrm{C,E}} \geq 0$ is the elasticity of substitution between non-energy goods and energy consumptions. Parameter ν is the share of energy consumption in the composite consumption good.

Parameter $\Phi(h^j)$ captures how health status enters utility function by affecting the marginal utility of consumption, as documented by Levy and Nir (2012) and Finkelstein et al. (2013), amongst others. We assume that:

$$\Phi(h^j) \equiv \left(h^j\right)^{\phi^j} \qquad \text{with} \qquad \phi^j \in (0,1) \tag{5}$$

for $j = (\mathrm{a},\mathrm{o})$. Because we assume that full health is normalized to unity, it does not appear in front of utility of healthy agents.

The program of each adult generation is to maximize intertemporal utility (4) under the following per-period budget constraints:

$$s_t + c_t^b + p_r E_{r,t}^{\mathrm{a},b} + m^{\mathrm{a}}(\varepsilon_{r,t-1}) = w_t^b \tag{6a}$$
$$s_t + c_t^g + p_r E_{r,t}^{\mathrm{a},g} = w_t^g \tag{6b}$$
$$d_{t+1}^b + p_r E_{r,t+1}^{\mathrm{o},b} + m^{\mathrm{o}}(\varepsilon_{r,t}) = R_{t+1}s_t \tag{6c}$$
$$d_{t+1}^g + p_r E_{r,t}^{\mathrm{o},g} = R_{t+1}s_t \tag{6d}$$
$$c_t^i \geq 0,\ d_{t+1}^i \geq 0,\ E_{r,t}^{\mathrm{a},i} \geq 0,\ E_{r,t+1}^{o,i} \geq 0 \tag{6e}$$

where $R_{t+1} = 1 + \rho_{t+1}$ (with ρ_{t+1} the real interest rate), w_t^i is real wage for individual with health condition i ($i = b, g$), p_r is energy price. Budget constraints (6a) and (6c) (respectively (6b) and (6d)) represent budget constraints of sick (resp. healthy) people respectively when adult and when old.

The resolution of the decision problem leads to (see Appendix B):

$$\begin{aligned} E_{r,t}^{\mathrm{a},i} &= \mathfrak{E}(\varepsilon_{r,t})c_t^i \\ E_{r,t+1}^{o,i} &= \mathfrak{E}(\varepsilon_{r,t+1})d_{t+1}^i \end{aligned} \quad \text{with} \quad \mathfrak{E}(\varepsilon_r) \equiv \left(\frac{\nu}{1-\nu}\right)^{\sigma_{\mathrm{C,E}}} p_r^{-\sigma_{\mathrm{C,E}}}\varepsilon_r^{\sigma_{\mathrm{C,E}}-1} \text{ and } i = (b,g) \tag{7}$$

and

$$\begin{aligned} \bar{c}_t^i &= \mathfrak{Z}(\varepsilon_{r,t})c_t^i \\ \bar{d}_{t+1}^i &= \mathfrak{Z}(\varepsilon_{r,t+1})d_{t+1}^i \end{aligned} \quad \text{with} \quad \mathfrak{Z}(\varepsilon_r) \equiv \left[(1-\nu) + \nu\left(\varepsilon_r\mathfrak{E}(\varepsilon_r)\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}} \tag{8}$$

[7] See Chakraborty and Das (2005, p.164) for a justification of $\sigma \in (0,1)$.

where $\partial\mathfrak{E}(\varepsilon_r)/\partial\varepsilon_r \leq 0$ for $\sigma_{\text{C,E}} \leq 1$ and $\partial\mathfrak{Z}(\varepsilon_r)/\partial\varepsilon_r > 0$ $\forall\sigma_{\text{C,E}}$.

Using budget constraints and equations (7) and (8), the expression of saving chosen by the adult generation is:

$$\left(\frac{\mathfrak{Z}(\varepsilon_{r,t})}{\mathfrak{Z}(\varepsilon_{r,t+1})}\right)^{-\frac{\sigma_{\text{C,E}}-1}{\sigma_{\text{C,E}}}}(1+p_r\mathfrak{E}(\varepsilon_{r,t}))\left\{\frac{\pi^{\text{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\text{a}})}{\left[w_t^b-s_t-m^{\text{a}}(\varepsilon_{r,t-1})\right]^\sigma}+\frac{1-\pi^{\text{a}}(\varepsilon_{r,t-1})}{\left[w_t^g-s_t\right]^\sigma}\right\}$$
$$-R_{t+1}\beta(1+p_r\mathfrak{E}(\varepsilon_{r,t+1}))\left\{\frac{\pi^{\text{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{o}})}{\left[R_{t+1}s_t-m^{\text{o}}(\varepsilon_{r,t})\right]^\sigma}+\frac{1-\pi^{\text{o}}(\varepsilon_{r,t})}{\left[R_{t+1}s_t\right]^\sigma}\right\}=0 \quad (9)$$

3.3 Firms

Firms produce an homogenous good, denoted Y, used to final and energy consumption in residential and producing sectors as well as to physical capital accumulation. They operate under perfect competition with a technology defined by the following nested Constant Elasticity Substitution (CES) production function:

$$Y_t = A\left[(1-\eta)\left(A_{\text{Q}}K_t^\alpha\tilde{L}^{1-\alpha}\right)^{\frac{\sigma_{\text{KL,E}}-1}{\sigma_{\text{KL,E}}}}+\eta\left(\varepsilon_f E_{f_t}\right)^{\frac{\sigma_{\text{KL,E}}-1}{\sigma_{\text{KL,E}}}}\right]^{\frac{\sigma_{\text{KL,E}}}{\sigma_{\text{KL,E}}-1}} \qquad \text{with} \qquad (\eta,\alpha)\in]0,1[,$$

where K_t denotes the aggregate stock of physical capital, $\tilde{L}$ is labor force expressed in efficiency terms, E_f the energy consumption in production and ε_f its efficiency. Physical capital fully depreciates during the period. We note $\sigma_{\text{KL,E}} \geq 0$ the elasticity of substitution between capital/labor and energy.[8]

Relying on the empirical evidence reported in Sub-section 2.2, we assume that the productivity of labor force, sick and healthy, relies on its health status. Then, remembering that fully health is normalized to unity, labor force expressed in efficiency terms is defined as:

$$\tilde{L} = B\left[(h_t^{\text{a}})^\psi L_t^b + L_t^g\right] \qquad \text{with} \qquad B>0,\ \psi\geq 1$$

where $h_t^{\text{a}} = 1-z^{\text{a}}(\varepsilon_{r,t-1})$ from (3), L^i represents the amount of workers with health condition i $(i=b,g)$, parameter ψ measures the intensity of health externality on labor productivity[9] and parameter B is a scale parameter which measures labor productivity when health does not influence labor productivity.

Firms maximize their profits $Y_t - R_tK_t - w_t^bL_t^b - w_t^gL_t^g - p_fE_{f_t}$, where p_f is the price of energy in production, leading to the following (inverse) demands:

$$R_t = \mathscr{R}(\tilde{k}_t,\varepsilon_{f,t}) \equiv \mathcal{A}(\varepsilon_{f,t})\alpha\tilde{k}_t^{\alpha-1} \quad (10a)$$
$$w_t^g = \mathscr{W}(\tilde{k}_t,\varepsilon_{f,t}) \equiv \mathcal{A}(\varepsilon_{f,t})(1-\alpha)\tilde{k}_t^{\alpha} \quad (10b)$$
$$w_t^b = (h_t^{\text{a}})^\psi\mathscr{W}(\tilde{k}_t,\varepsilon_{f,t}) \quad (10c)$$
$$E_{f_t} = \frac{A_{\text{Q}}}{\varepsilon_f\Omega(\varepsilon_f)}\tilde{k}_t^\alpha \quad (10d)$$

with $\tilde{k} \equiv K/\tilde{L}$ the ratio capital efficient labor and

$$\Omega(\varepsilon_f) \equiv \left(\frac{\left(\frac{p_f}{A\eta\varepsilon_f}\right)^{\sigma_{\text{KL,E}}-1}-\eta}{1-\eta}\right)^{\frac{\sigma_{\text{KL,E}}}{\sigma_{\text{KL,E}}-1}} > 0 \quad (11a)$$

$$\mathcal{A}(\varepsilon_{f,t}) \equiv (1-\eta)A_{\text{Q}}A\left(\frac{1-\eta\left(\frac{p_f}{A\eta\varepsilon_f}\right)^{1-\sigma_{\text{KL,E}}}}{1-\eta}\right)^{\frac{1}{1-\sigma_{\text{KL,E}}}} \quad (11b)$$

[8] We assumed that the technology linking physical capital and labor is Cobb-Douglas for convenience. Assuming that the elasticity of substitution between capital and labor is different from unity would not modify the qualitative results of the model. Proof upon request.

[9] Assuming that $\psi \geq 1$ is consistent with empirical evidence reported in Sub-section 2.2.

Lemma 1.

1. $\partial\Omega(\varepsilon_f)/\partial\varepsilon_f < 0$ *and* $\partial\Omega(\varepsilon_f)/\partial p_f > 0$, $\forall\sigma_{\text{KL,E}}$.

2. $\partial\mathcal{A}(\varepsilon_{f,t})/\partial\varepsilon_f > 0$ *and* $\partial\mathscr{R}(\tilde{k}_t,\varepsilon_{f,t})/\partial\tilde{k}_t < 0$ *and* $\partial\mathscr{R}(\tilde{k}_t,\varepsilon_{f,t})/\partial\varepsilon_f > 0$.

3. $\partial\mathscr{W}(\tilde{k}_t,\varepsilon_{f,t})/\partial\varepsilon_f > 0$ *and* $\partial\mathscr{W}(\tilde{k}_t,\varepsilon_{f,t})/\partial\tilde{k}_t > 0$ $\forall\sigma_{\text{KL,E}}$

Proof. Straightforward from expression of $\Omega(\varepsilon_f)$ in (11a) and Lemma 1.1. $\square$

From (3), (9), (10a) to (10c), saving is the solution of the following equation:

$$\begin{aligned}
&\left(\frac{\mathfrak{Z}(\varepsilon_{r,t})}{\mathfrak{Z}(\varepsilon_{r,t+1})}\right)^{-\frac{\sigma_{\text{C,E}}-1}{\sigma_{\text{C,E}}}}(1+p_r\mathfrak{E}(\varepsilon_{r,t}))\\
&\times\left\{\frac{\pi^{\text{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\text{a}})}{\left[(h_t^{\text{a}})^{\psi}\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\tilde{k}_t^{\alpha}-s_t-m^{\text{a}}(\varepsilon_{r,t-1})\right]^{\sigma}}+\frac{1-\pi^{\text{a}}(\varepsilon_{r,t-1})}{\left[\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\tilde{k}_t^{\alpha}-s_t\right]^{\sigma}}\right\}\\
&\qquad-\beta(1+p_r\mathfrak{E}(\varepsilon_{r,t+1}))\left[\alpha\mathcal{A}(\varepsilon_{f,t})\tilde{k}_{t+1}^{\alpha-1}\right]^{1-\sigma}\left\{\frac{\pi^{\text{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{o}})}{\left[s_t-\dfrac{m^{\text{o}}(\varepsilon_{r,t})\tilde{k}_{t+1}^{1-\alpha}}{\mathcal{A}(\varepsilon_{f,t+1})\alpha}\right]^{\sigma}}+\frac{1-\pi^{\text{o}}(\varepsilon_{r,t})}{s_t^{\sigma}}\right\}=0
\end{aligned}$$

Proposition 1.
Saving in the economy is defined as the following function:

$$s_t=\mathsf{S}\left(\underbrace{\underset{-}{\pi^{\text{a}}(\varepsilon_{r,t-1})},\underset{+}{\pi^{\text{o}}(\varepsilon_{r,t})},\underset{-}{\Phi(h_t^{\text{a}})},\underset{+}{\Phi(h_{t+1}^{\text{o}})}}_{\textit{Propensity to save effect}},\underbrace{\underset{-}{m^{\text{a}}(\varepsilon_{r,t-1})},\underset{+}{m^{\text{o}}(\varepsilon_{r,t})},\underset{+}{h^{\text{a}}(\varepsilon_r^{\star})^{\psi}}}_{\textit{Revenue effect}},\underset{+}{\varepsilon_{f,t}},\underset{-}{\varepsilon_{f,t+1}}\right)\tag{12}$$

Proof. From equation (9), (10a) to (10c). $\square$

Proposition 1 highlights that residential energy efficiency affects savings through different channels, both linked to health, for given $\tilde{k}$.[10] The first influence goes through the propensity to save income. It is associated to the probability to be sick when adults $\pi^{\text{a}}(\varepsilon_{r,t-1})$ and when old $\pi^{\text{o}}(\varepsilon_{r,t})$, as well as to the impact of bad health on utility when adult, $\Phi(h^{\text{a}}(\varepsilon_{r,t}))$, and when old, $\Phi(h^{\text{o}}(\varepsilon_{r,t+1}))$. The second influence is directly related to the revenue that agents will allocate to saving. It is associated to labor wage and revenue of saving. It depends on energy efficiency through health expenditures $m^{\text{a}}(\varepsilon_r^{\star})$ and $m^{\text{o}}(\varepsilon_r^{\star})$, as well through the link between good health on labor productivity (of sick adults captured by $h^{\text{a}}(\varepsilon_r^{\star})^{\psi}$).

It is worthy to note that health influence on saving is age dependent. An increase in the probability to be sick when adult will reduce savings, ceteris paribus, while an increase in the probability to be sick when old (similarly for health expenditures). As noted by Kotlikoff (1989), a higher probability to be sick when old (or a rise in old health expenditures) incites the adult to accumulate precautionary saving to face lower revenue or higher health expenditures when old, while a higher probability to be sick when adult (or a rise in adult health expenditures) will incite adults to reduce saving in order to face lower revenue or higher health expenditures when adult. Additionally in our framework, it will rise labor productivity (as well as the marginal utility of consumption) and therefore revenue.

[10]The positive or negative signs reported under variables in equation (12) represent the sign of the influence of the related variable.

3.4 The general equilibrium

Because each adult supplies one unit of labor and the size of adult population at time t is N_t, labor market equilibrium implies that:

$$L_t^b = \pi^{\mathrm{a}}(\varepsilon_{r,t-1})N_t \qquad \text{and} \qquad L_t^g = (1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))N_t \tag{13}$$

and we assume that the energy market is balanced at each time by imports of energy at exogenous price. Finally, the usual general equilibrium condition holds:

$$N_t s_t = K_{t+1} \tag{14}$$

Equations (12), (13) and (14) define the law of motion of per capita physical capital $k_t \equiv K_t/N_t$:

$$\begin{aligned}
&\left(\frac{\mathfrak{Z}(\varepsilon_{r,t})}{\mathfrak{Z}(\varepsilon_{r,t+1})}\right)^{-\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}} \left(\frac{1+p_r\mathfrak{E}(\varepsilon_{r,t})}{(1+p_r\mathfrak{E}(\varepsilon_{r,t+1}))}\right) \left\{ \frac{1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1})}{\left[\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\left(\frac{k_t}{\Psi(\varepsilon_{r,t-1})}\right)^{\alpha} - (1+n)k_{t+1}\right]^{\sigma}} \right.\\
&\left. + \frac{\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})}{\left[(h_t^{\mathrm{a}})^{\psi}\,\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\left(\frac{k_t}{\Psi(\varepsilon_{r,t-1})}\right)^{\alpha} - (1+n)k_{t+1} - m^{\mathrm{a}}(\varepsilon_{r,t-1})\right]^{\sigma}} \right\}\\
&= \\
&\beta\left[\alpha\mathcal{A}(\varepsilon_{f,t})\left(\frac{k_{t+1}}{\Psi(\varepsilon_{r,t+1})}\right)^{\alpha-1}\right]^{1-\sigma} \left\{ \frac{\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\mathrm{o}})}{\left[(1+n)k_{t+1} - \dfrac{m^{\mathrm{o}}(\varepsilon_{r,t})\left(\frac{k_{t+1}}{\Psi(\varepsilon_{r,t+1})}\right)^{1-\alpha}}{\mathcal{A}(\varepsilon_{f,t})\alpha}\right]^{\sigma}} + \frac{1-\pi^{\mathrm{o}}(\varepsilon_{r,t})}{[(1+n)k_{t+1}]^{\sigma}} \right\}
\end{aligned} \tag{15}$$

where $\Psi(\varepsilon_r) \equiv B\left[1-\pi^{\mathrm{a}}(\varepsilon_r)\left[1-h^{\mathrm{a}}(\varepsilon_r)^{\psi}\right]\right]$ captures the influence of health on labor productivity. Because the left-hand side of the equality is increasing in k_{t+1} and decreasing in k_t, according to the theorem of the implicit functions, this equation expresses k_{t+1} as an increasing function of k_t. We have $\partial\Psi(\varepsilon_r) \leq 1$ and $\partial\Psi(\varepsilon_r)/\partial\varepsilon_r \geq 0$.

4 The long-term equililibrium

Long-term equilibrium is such as $\varepsilon_{f,t} = \varepsilon_f^\star$, $\varepsilon_{r,t} = \varepsilon_{r,t+1} = \varepsilon_r^\star$ and $k_t = k_{t+1} = k^\star$. Equation (15) can be written as follows at the steady-state equilibrium:

$$\begin{aligned}
&\frac{1-\pi^{\mathrm{a}}(\varepsilon_r^\star)}{\mathcal{A}(\varepsilon_f^\star)(1-\alpha)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{\alpha} - (1+n)k^\star} + \frac{\pi^{\mathrm{a}}(\varepsilon_r^\star)\Phi(h^{\mathrm{a}}(\varepsilon_r^\star))}{\left[h^{\mathrm{a}}(\varepsilon_r^\star)^{\psi}\mathcal{A}(\varepsilon_f^\star)(1-\alpha)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{\alpha} - (1+n)k^\star - m^{\mathrm{a}}(\varepsilon_r^\star)\right]^{\sigma}}\\
&-\beta\left[\alpha\mathcal{A}(\varepsilon_f^\star)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{\alpha-1}\right]^{1-\sigma} \left\{ \frac{\pi^{\mathrm{o}}(\varepsilon_r^\star)\Phi(h^{\mathrm{o}}(\varepsilon_r^\star))}{\left[(1+n)k^\star - \dfrac{m^{\mathrm{o}}(\varepsilon_r^\star)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{1-\alpha}}{\mathcal{A}(\varepsilon_f^\star)\alpha}\right]^{\sigma}} + \frac{1-\pi^{\mathrm{o}}(\varepsilon_r^\star)}{[(1+n)k^\star]^{\sigma}} \right\} = 0
\end{aligned} \tag{16}$$

It implicitly defines the steady-state per capita physical capital as:[11]

$$k^\star = \mathsf{K}\left(\underset{-}{\pi^{\mathrm{a}}(\varepsilon_r^\star)}, \underset{+}{\pi^{\mathrm{o}}(\varepsilon_r^\star)}, \underset{+}{h^{\mathrm{a}}(\varepsilon_r^\star)^{\psi}}, \underset{-}{m^{\mathrm{a}}(\varepsilon_r^\star)}, \underset{+}{m^{\mathrm{o}}(\varepsilon_r^\star)}, \underset{-}{\Phi(h^{\mathrm{a}}(\varepsilon_r^\star))}, \underset{+}{\Phi(h^{\mathrm{o}}(\varepsilon_r^\star))}, \underset{+}{\varepsilon_f^\star}\right) \tag{17}$$

[11]The positive or negative signs reported under variables represent the sign of the influence of the related variable.

We obtain:

Proposition 2. *When energy efficiency positively affects health ($\pi^{\mathrm{a}} > 0$ and/or $\pi^{o} > 0$), the long-term equilibrium level of per capita physical capital is such as:*

$$k^{\star} = \kappa(\varepsilon_r^{\star}) \qquad \text{with} \qquad \kappa_{\varepsilon_r^{\star}}(\cdot) \gtreqless 0$$

Otherwise ($\pi^{\mathrm{a}} = 0$ and $\pi^{o} = 0$), it is independent from residential energy efficiency.

Proof. From equations (16), (1), (2). □

The negative effect of energy inefficiency on health makes the steady-state physical capital per capita dependent on residential energy efficiency. This dependance comes from two different channels that we already highlighted in the previous section. In particular, detrimental impact of ε_r on health when adult reduces the amount of revenue the agent allocates to saving in order to fund her health expenditures when adult. Conversely, the detrimental impact of ε_r on health when old is the source of precautionary saving in order to fund health expenditures when old. Even if the global impact of ε_r on savings and physical capital is hard to disentangle analytically, we can show intuitively that it relies on the relative importance of the influence of ε_r on young and old health-status and on the subjective discount rate β. For example, if we assume that ε_r does not affect old, then $\pi^{\mathrm{o}}_{t+1}(\varepsilon_r) = m^{\mathrm{o}}(\varepsilon_r) = 0$ and therefore, under the assumption that $\Phi(h^{\mathrm{a}}_t)$ is not too lower than unity, a rise in ε_r increases saving and therefore $k^{\star}$. Conversely, if we assume that ε_r does not affect young, then $\pi^{\mathrm{a}}_t(\varepsilon_r) = m^{\mathrm{a}}(\varepsilon_r) = 0$ and therefore, under the assumption that $\Phi(h^{\mathrm{o}}_{t+1})$ is not too lower than unity, a rise in ε_r reduces precautionary saving and, therefore, decreases $k^{\star}$. Furthermore, the health externality on labor productivity (captured by $h^{\mathrm{a}}(\varepsilon_r^{\star})^{\psi}$) introduces a further positive influence of $\varepsilon_r^{\star}$ on steady-state per capita physical capital because it rises the wage of adult and, therefore, their saving, ceteris paribus.

The expected per capita residential energy consumption at the steady-state (both, young and old of the previous generation) is:

$$E_r^{\star} = \underbrace{\frac{\mathfrak{E}(\varepsilon_r^{\star})}{1 + p_r \mathfrak{E}(\varepsilon_r^{\star})}}_{\text{Demand effect}} \left\{ \underbrace{\left[1 - \alpha + \alpha B\right] B^{-1} \mathcal{A}(\varepsilon_f^{\star}) \Psi(\varepsilon_r^{\star})^{1-\alpha} \kappa(\varepsilon_r^{\star}, \varepsilon_f^{\star})^{\alpha} - (1+n)\kappa(\varepsilon_r^{\star}, \varepsilon_f^{\star})}_{\text{General equilibrium effect}} \right.$$
$$\left. - \underbrace{\left(m^{\mathrm{a}}(\varepsilon_r^{\star}) \pi^{\mathrm{a}}(\varepsilon_r^{\star}) + \frac{m^{\mathrm{o}}(\varepsilon_r^{\star}) \pi^{\mathrm{o}}(\varepsilon_r^{\star})}{1+n} \right)}_{\text{Health expenditures effect}} \right\} \tag{18}$$

Equation (18) highlights the different channels through which long-run residential energy efficiency $\varepsilon_r^{\star}$ affects the total energy consumption in the presence of the detrimental health impacts:[12]

1. The first channel directly influences $E_r^{\star}$ via the consumers demand, independently from health effect (see equation (18)). It implies that, *Ceteris Paribus* an increase in residential energy efficiency would lead to a decrease in residential energy consumption if residential consumption is weakly substitutable with non-energy consumption $\sigma_{\mathrm{C,E}} < 1$. In the following we will call it the demand effect.

2. The second channel influence $E_r^{\star}$ through general equilibrium effects via the revenues of adult and old generations (the first term into curly brackets in equation (18) represents $w^{\star} - R^{\star}$ and the second term represents $s^{\star}$). These incomes are indirectly affected by $\varepsilon_r^{\star}$ through health (see Proposition 2 and equations (12), (10a) and (10b)).

[12] It is clear that what we call here "general equilibrium effect" and "health expenditures effect" are directly affected by the "saving propensity effect" and the "revenue effect" we previously highlighted.

3. The third channel arises from health expenditures associated with the detrimental effect of bad residential energy efficiency on health for adult and old generation (the last term into curly brackets in the second line of equation (18)). it is expected that this "*health expenditures effect*" will lead to positive impacts of $\varepsilon_r^\star$ on residential energy consumption in the long-term because a higher $\varepsilon_r^\star$ will reduce the expected healthcare expenditures for both generations and, then, everything being equal elsewhere, will increase the resources to spend in both types of consumption.

From (10d) and (13), steady-state per capita expected energy consumption in production is:

$$E_f{}^\star = \frac{A_{\mathrm{Q}}}{\varepsilon_f^\star \Omega(\varepsilon_f^\star)} \Psi(\varepsilon_r^\star)^{-\alpha} \kappa(\varepsilon_r^\star, \varepsilon_f^\star)^\alpha \tag{19}$$

Proposition 3. *Energy consumption in the economy is such as:*

$$E_r^\star = \mathcal{E}^r(\varepsilon_r^\star) \qquad \textit{with} \qquad \begin{cases} \mathcal{E}^r_{\varepsilon_r^\star}(\cdot) \gtreqless 0 & \textit{if } \pi^a > 0 \ \textit{ and/or } \ \pi^o > 0 \\ \\ \mathcal{E}^r_{\varepsilon_r^\star}(\cdot) < 0 & \textit{if } \pi^a = 0 \ \textit{ and/or } \ \pi^o = 0 \end{cases}$$

and

$$E_f^\star = \mathcal{E}^f(\varepsilon_r^\star) \qquad \textit{with} \qquad \begin{cases} \mathcal{E}^f_{\varepsilon_r^\star}(\cdot) \gtreqless 0 & \textit{if } \pi^a > 0 \ \textit{ and/or } \ \pi^o > 0 \\ \\ \mathcal{E}^f_{\varepsilon_r^\star}(\cdot) = 0 & \textit{if } \pi^a = 0 \ \textit{ and/or } \ \pi^o = 0 \end{cases}$$

Proof. From equations (18), (19), (1), (2). □

Proposition 3 shows that taking into account the detrimental health effect of low residential energy efficiency as well as the impact of health on labor productivity, may modify the link between residential energy efficiency improvement and energy consumption in, both, residential and production sectors. Because health impacts are difficult to disentangle analytically, we numerically simulate the basic theoretical model in the following section.

5 Numerical exercices

In this section, we present numerical simulations of our theoretical model that we calibrate on the U.S. economy for the period going from 1985 to 2018. In sub-section 5.1, we explain calibration of preference, technology and energy parameters. In sub-section 5.2, we discuss results.

5.1 Calibration

Preference parameters: The value of the elasticity of substitution between energy and non-energy consumption in utility comes from Lemoine (2020) and is very close to the value reported by de Miguel and Manzano (2011). Following De La Croix and Michel (2002) we assume that the quarterly psychological discount factor is equal to 0.99. The parameter β is thus equal to $0.99^{120} = 0.3$. Finally, we calibrate energy consumption share in utility, ν, in order to match the average value of the residential energy consumption to GDP ratio in the U.S. economy during the period 1985-2018. The residential energy consumption is extracted from the U.S. Energy Information Administration (EIA) State Energy data set. The real GDP comes from the Federal Reserve Economic Data (FRED).

Technology parameters: We follow the Real Business Cycle literature to set the share of physical capital in production a at 1/3. As for elasticities of substitution in the production function, we consider the elasticity of substitution between energy E_f and non-energy factors Z as well as between physical capital and labor. The question of what values to attribute to these elasticities is the subject of recurrent debate in the current state of empirical literature. As a consequence, in our model, we use as benchmark values the estimations made by van der Werf (2008) for the U.S. and we will also investigate alternative values estimated by other authors. Otherwise, we approximate the share of energy in industrial production a by the average value of the ratio energy expenditures in the industrial sector to GDP during the period 1985-2018, using the data from the U.S. EIA State Energy Data set. Following De La Croix and Michel (2002) and Wang et al. (2015), we set a steady-state target value of k around 1.2. With a steady-state quarterly interest rate equal to 1% (that is $R^\star = (1.01)^{120}$) from equation (10a), we get the labor productivity parameter $B = 1.45$ for scale parameters $A = 9.155$ and $A_{\mathrm{Q}} = 1.27$.

Energy parameters: Values of energy efficiencies are extracted from the 2016 American Council for an Energy-Efficient Economy (ACEEE) report. Energy prices are computed as the average value of prices (respectively for residential and for industrial sector) during the period 1985-2018. They come from the 2018 U.S. EIA State Energy Data set.

Health parameters: The impact of energy efficiency on health is less documented than the detrimental influence of air pollution especially with respect to the specification of the health risk function. That is why we make the simple assumption of a non linear sigmoidal negative influence of energy efficiency on the probability to becoming sick. We will investigate the influence of different sets of parameter value to check the robustness of numerical simulations. We discuss the shape of the health risk function in Appendix C. Especially, we assumed that when $\varepsilon_r < 0.2$ the probability to becoming sick is at its maximum ($\pi^{\mathrm{a}} = 1$) and when $\varepsilon_r > 0.8$, the probability to becoming sick is null ($\pi^{\mathrm{a}} = 0$).[13] We always assume that for a given ε_r, the probability to becoming sick is higher for the elderly. Following Gutiérrez (2008), we consider that health expenditures are linear with respect to energy efficiency, such that we define $m^{\mathrm{a}}(\varepsilon_{r,t-1}) \equiv \tilde{m}^{\mathrm{a}} \times (1-\varepsilon_{r,t-1})$ with $\tilde{m}^{\mathrm{a}} > 0$ and $m^{\mathrm{o}}(\varepsilon_{r,t}) \equiv \tilde{m}^{\mathrm{o}} \times (1-\varepsilon_{r,t})$ with $\tilde{m}^{\mathrm{o}} > 0$. Similarly, we assume that recovery time is linearly linked to energy efficiency: $z^{\mathrm{a}}(\varepsilon_{r,t-1}) \equiv \tilde{z}^{\mathrm{a}} \times (1-\varepsilon_{r,t-1})$ with $\tilde{z}^{\mathrm{a}} > 0$ and $z^{\mathrm{o}}(\varepsilon_{r,t}) \equiv \tilde{z}^{\mathrm{o}} \times (1-\varepsilon_{r,t})$ with $\tilde{z}^{\mathrm{o}} > 0$. Parameters $\tilde{z}^j$ ($j = \{\mathrm{a}, \mathrm{o}\}$) are arbitrarily fixed such that $\tilde{z}^{\mathrm{a}} = 0.1$ and $\tilde{z}^o = 0.2$ (elderly remain sick a longer time than adults) and parameters $\tilde{m}^j$ match steady-state targets. Finally, we set arbitrarily the intensity of health externality on labor productivity to the medium value $\psi = 1$ in accordance with empirical evidence (see section 2.2). For most of variables arbitrarily fixed, we will investigate how alternative values could modify our numerical simulations.[14]

In Table 1, we summarize benchmark parameter values and their sources.

Table 1: Benchmark parameter values

Parameter		Value	Source
Preference			
- Elasticity of substitution between energy and non-energy consumption	$\sigma_{\mathrm{C,E}}$	0.9	Lemoine (2020)
- Subjective rate of time preference	β	0.30	De La Croix and Michel (2002)
- Inverse of the intertemporal elasticity of substitution	σ	2/3	*Arbitrarily fixed*
- Energy consumption share	ν	0.37	*Matches steady-state targets*
Technology			
- Share of physical capital in production	α	0.30	De La Croix and Michel (2002)
- Scale parameter	A	9.155	*Matches steady-state targets*
- Scale parameter	A_{Q}	1.27	*Matches steady-state targets*
- Labor productivity parameter	B	1.45	*Matches steady-state targets*
- Share of energy in industrial production	η	0.017	*Matches steady-state targets*
- Elasticity of substitution between energy and non-energy in production	$\sigma_{\mathrm{KL,E}}$	0.5470	van der Werf (2008)
Energy			
- Energy efficiency in the residential sector	ε_r	0.65	ACEEE (2016)
- Energy efficiency in the industrial sector	ε_f	0.7	ACEEE (2016)
- Unitary price of residential energy services	p_r	16.83	U.S. EIA State Energy Data
- Unitary price of firm energy services	p_f	8.11	U.S. EIA State Energy Data
Health			
- Adult health expenditures (% of consumption)	μ^{a}	0.0673	U.S. BLS & Author calculus
- Old health expenditures (% of consumption)	μ^{o}	0.1270	U.S. BLS & Author calculus
- Adult recovery time constant	$\tilde{z}^{\mathrm{a}}$	0.1	*Arbitrarily fixed*
- Old recovery time constant	$\tilde{z}^{\mathrm{o}}$	0.2	*Arbitrarily fixed*
- Externality of health on labor productivity	ψ	1	*Arbitrarily fixed*
- Externality of health on utility	ϕ	0.5	*Arbitrarily fixed*

[13] That is the reason why, in subsequent figures, only the range of ε_r between 0.2 and 0.8 is of interest.

[14] Robustness analysis shows that our qualitative results are not modified when parameters take values different from benchmark. Proof upon request.

5.2 Main results

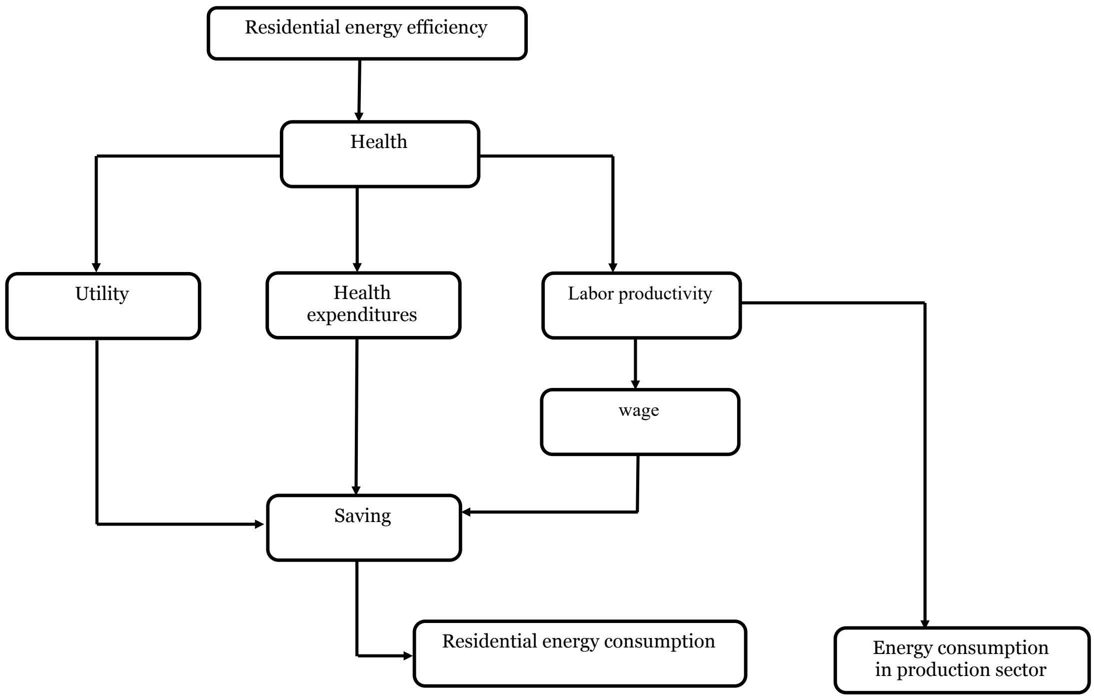


Figure 1. Summary of health channels through which energy efficiency relates to energy consumption

Figure 1 below summarizes health channels through which energy efficiency effect on energy consumption occurs. Below, we detail our results. In particular, different graphs in Figure 2 report the results of numerical exercice where the benchmark case is represented by blue plain curves and the "no health effect" case is represented by the red dashed curve. This figure reports the steady-state values of the physical capital, energy and non-energy consumptions (of adults and old) as well as welfare, with respect to the value of residential energy efficiency.

To understand our results, we first investigate the case where the residential energy efficiency does not impact health (red dashed curves). In such case, our numerical exercise shows that the steady-state per capita physical capital and energy consumption in production are not affected by residential energy efficiency (see graphs 1 and 3 in Figure 2), which is consistent with Proposition 2 and equation (19). Welfare and non-energy consumption during adulthood and in the old-age are slightly positively influenced by the residential energy efficiency (see graphs 4 to 6 in Figure 2). Finally, residential energy consumption diminishes with residential energy efficiency (see graph 2 in Figure 2) because, with no health effect, the only impact of the residential energy efficiency goes through channel (I) in equation (18): higher residential energy efficiency leads to substitution from energy towards non-energy consumption in the residential sector (see graph 2 vs graphs 4 and 5 in Figure 2).

Now, we can study how the economy is affected when health is negatively impacted by residential energy efficiency (blue curves in Figure 2). A first remark is that all steady-state variables (blue plain curves in Figure 2) are lower with respect to the "no health effect" case. In fact, the probability of being sick induced by residential energy inefficiency reduces non-energy consumption and savings, i.e. health expenditures as well as lower productivity of labor reduce wage and interest rate, everything being equal. As a result, per capita physical capital, energy consumptions and welfare are reduced as well. Interestingly, variables which were not affected by, or just slightly affected by residential energy efficiency in the "no health" case are now significantly affected positively when health is taken into account. In particular, a rise in residential energy efficiency in the presence of detrimental health effect would increase energy consumption in both sectors (see graphs 2 and 3 in Figure 2).

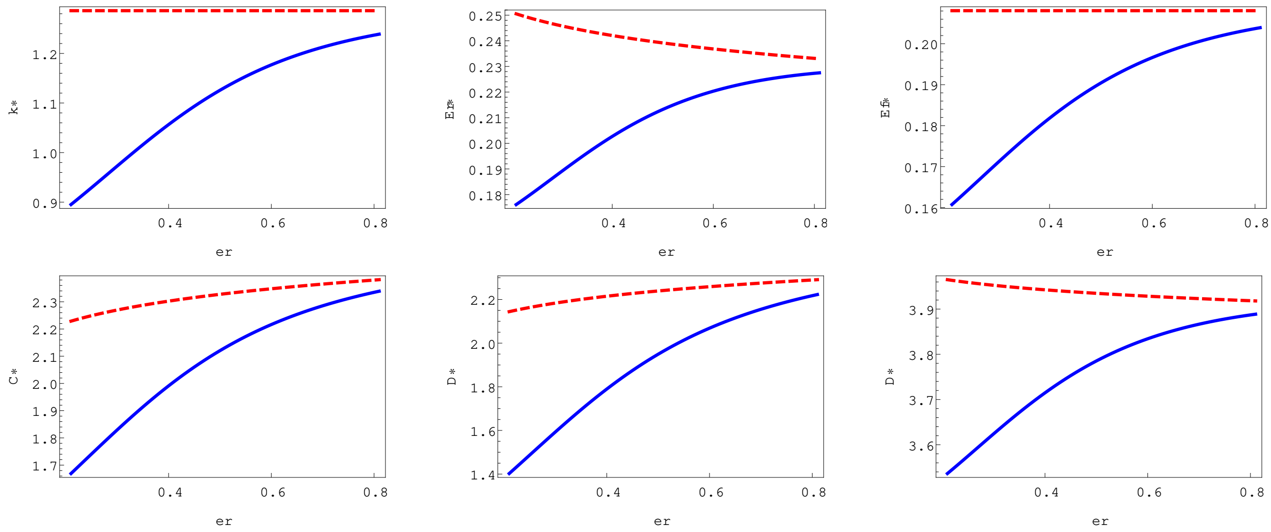


Figure 2: Influence of ε_r (Benchmark case (blue) vs no health effect (red large dashes))

Furthermore, graph 3 of Figure 2 shows how residential energy efficiency spills over energy consumption in production sector when health is taken into account. On the one hand, the detrimental health effect of low residential energy efficiency creates a channel of transmission of energy efficiency between the different sectors of the economy (as demonstrated in Proposition 3). It means that investigating the influence of energy efficiency improvements in a specific sector requires to enlarge the analysis of the consequences to the economy at large. On the other hand, our numerical exercise suggests that, according to the value of parameters chosen, an improvement in residential energy efficiency could rise energy consumption in the production sector while energy efficiency in production remains unchanged. Long-term energy consumption in the production sector is influenced by the energy inefficiency in the residential sector through the positive probability to be sick. This influence goes through two channels. A direct channel is associated with the health externality on labor productivity (captured by $\kappa(\varepsilon_r^\star, \varepsilon_f^\star)$). An indirect channel is linked to the "saving propensity effect" which affects savings and then long-term physical capital (captured by $\Psi(\varepsilon_r^\star)$).

It is worthy to note that when assuming that health is negatively impacted by residential energy efficiency, our results captures the fact that energy efficiency positively affects residential energy consumption. This means that the introduction of the two main empirical evidences highlighted in the Section 2, i.e. correlation between energy efficiency and health and between health and labor productivity, are relevant in terms of explaining the rebound effect occurrence. In particular, we show that this occurrence goes through the three channels highlighted in equation (18). In the next subsections 5.2.1 to 5.2.4, we will investigate how these channels operate.

5.2.1 Adulthood sickness vs old-age sickness

In his seminal article, Kotlikoff (1989) assumed that only elderly are sick. As a consequence, he demonstrated that elderly decide to save more when young, i.e. precautionary saving. Assuming as we did, that young individuals could be sick as well, and introducing health externalities on labor productivity and utility, has significant implications for saving decisions and for the influence of residential energy efficiency on energy and non-energy consumptions.

In Figure (3), we plot the results of numerical simulations when we assume either only elderly are sick because of residential energy inefficiency exposure when young (black dotted dashed line) or only young are sick (green tiny dashed line). We compare both situations to the conventional case with no detrimental health effect of low residential energy efficiency (red dashed line).

The first graph of Figure (3) shows the influence of illness on saving. If agents are sick only when old, they are expected to increase their saving for precautionary purpose in their adulthood, as shown by Kotlikoff (1989). As a consequence, we expect that the steady-state of physical capital is higher than its level with no health effect. Conversely, if agents are sick only when adult, we expect that they

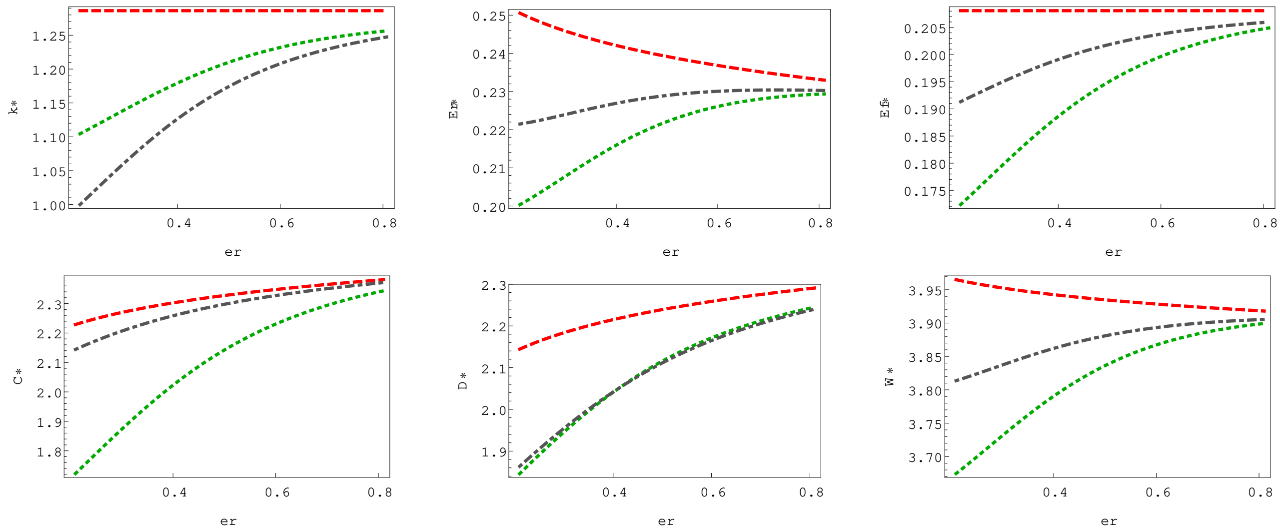


Figure 3: "Sickness by age" and influence of ε_r (*health effect during old age only (black dotted dashed), health effect during adulthood only (green tiny dashed) and no health effect (red large dashes)*)

reduce saving in order to fund their health expenditures. Therefore, the steady-state level of physical capital is expected to be lower than its level with no health effect. Nevertheless, the first graph of Figure (3) shows the reverse. First it is due to the introduction of health externality on labor productivity. In fact, by rising labor productivity, the increase in energy efficiency leads to an increase in wage when adults which reduces the accumulation of precautionary saving for old age.[15] Furthermore, the introduction of health externality on utility reduces the marginal utility of consumption when old and therefore the amount of goods agents want to consume when old. Both effects lead to a lower saving (translated in lower physical capital) when old agents are sick rather than healthy (illustrated by the red dashed line). The same rational exists for adult, that is the reason why green line (case where only old are sick) is below the dark line (case where only adult are sick).

It is interesting to note that whatever the period during which energy efficiency affects health (either adulthood or old age), this negative influence always leads to a positive relationship between the level of energy efficiency and the energy consumption in the residential sector (see black dotted dashed and green tiny dashed curves graph 2 in Figure (3)). Nevertheless, this positive link is slighter when only agents in their old age are concerned. This may be explained by the fact that, when old agents do not work, as a consequence externalities on labor productivity do not exist conversely to the case where only adult are affected by the negative influence of low energy efficiency in health.

5.2.2 The influence of health externality on labor productivity

We have taken into account the empirical evidence that labor productivity is affected by health-status. Therefore, in our basic framework, workforce productivity is positively affected by residential energy efficiency. This impacts the income of agents and their saving.

Here, we investigate how important is this transmission channel when analyzing the relationship between energy efficiency improvement and energy and non-energy consumption. For simplicity, we investigate the cases where there is no health externality on labor productivity ($\psi = 0$, thus, $\Psi(\varepsilon_r) = B$ in equations 16 and 18) and where the intensity of health externality on labor productivity is higher than the benchmark value ($\psi = 2 > 1$). We compare them to the benchmark case in Figure 4 where the green dotted curve illustrates the case $\psi = 2$ and the black dotted dashed curve illustrates the case $\psi = 0$.

The first insight of our numerical exercise is that the existence or not of health externality on labor productivity may not modify the positive link between energy efficiency and energy consumption in

[15] In Appendix D, we report the graphs when no health externalities exist and when only health externality on labor productivity exists to illustrate our explanations. The same Appendix also enables us to illustrate how energy efficiency relates to energy consumption.

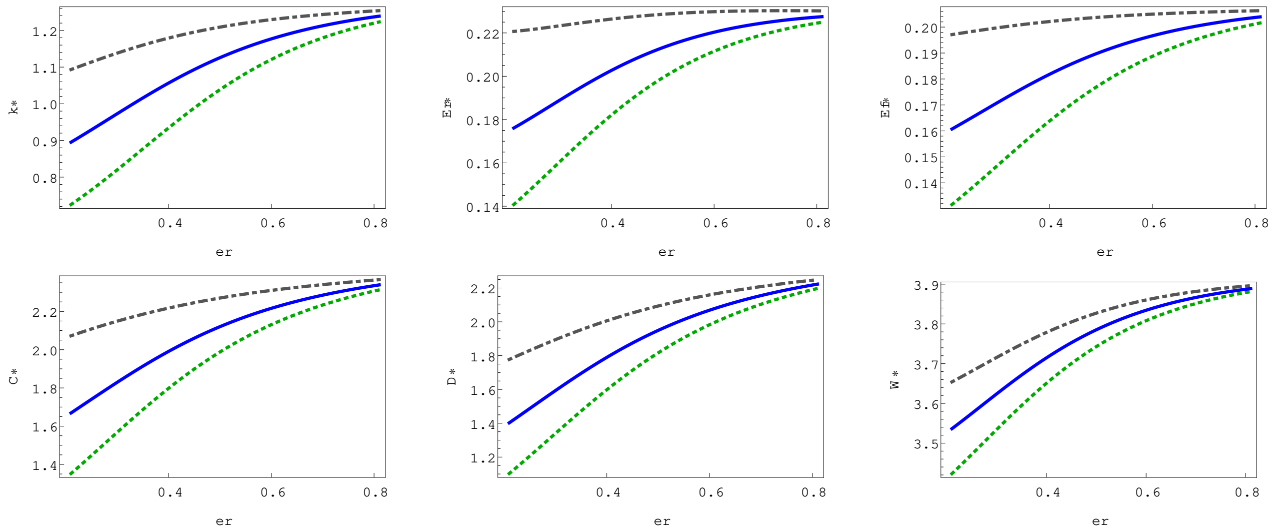


Figure 4: Health externality on labor productivity & influence of ε_r (*Benchmark case (blue) vs higher health externality (green dotted) vs no health externality* $\psi = 0$ *(black dotted dashed).*

the residential sector, when both adults and old are affected by health effects (see graph 2 in Figure (4)). Nevertheless, in the absence of health externality on labor productivity (the black dooted dashed curve in graph 2 in Figure (4)), the influence of energy efficiency is low because the ratio of capital efficient labor ($\tilde{k}$) and wages are independent from $\varepsilon_r^\star$. As a consequence, per capita physical capital (k defined by equation (16) with $\Psi(\varepsilon_r) = B$) is at its maximum level (with respect to the case where there is health externality on labor productivity). It comes from Proposition 1 and Lemma **??** that savings and $k^\star$ are at their highest level (see equation 16 where $\Psi(\varepsilon_r^\star)$ is independent from $\varepsilon_r^\star$). That is the reason why, in graph 1 of Figure 4, the steady-state value of physical capital is always higher than the "no health effect" case. It is the lower, the greater the health externality on labor productivity ($\psi = 1$).[16] This explanation is also consistent with the fact that the three curves representing energy consumption in graph 2 and 3 of Figure 4, tends to similar levels when energy efficiency is high (that is health effect of energy efficiency is minimal).

Generally, the influence of energy efficiency on key variables is slightly affected by the magnitude of health externality on labor productivity. The shape of the influence remains the same, while levels are higher in the absence of health externality on labor productivity, due to the fact that utility is higher in this case. As a result, agents choose a higher level of consumption when adults and when old, and experience a higher level of welfare (graphs 4 to 6 of Figure 4).

5.2.3 The influence of health externality in utility

Another channel through which energy efficiency and health affect energy consumptions is health externality in utility emphasized empirically by Levy and Nir (2012) and Finkelstein et al. (2013), amongst others. We investigate the cases where there is no health externality on utility ($\phi = 0$, thus, $\Phi(h^j) = 1$ in equation 16) and where the intensity of health externality on utility is higher than the benchmark value ($\phi = 1 > 0.5$). We compare them to the benchmark case in Figure 5 where the green dotted curve illustrates the case $\phi = 0$ and the black dotted dashed curve illustrates the case $\phi = 1$.

Numerical exercise confirms that detrimental health effect of low energy efficiency positively links energy efficiency to energy consumption (see graphs 2 and 3 of Figure 5). Obviously, the lack of health externality on utility leads to higher energy and non-energy consumptions because the marginal utility of consumption (both energy and non-energy) is higher with respect to case where there is health externality on utility inducing utility decrease (see graph 6 of Figure 5).

[16] Note that this is not true in the case where only adults are suffering from low residential energy efficiency. In this case, steady-state physical capital always remains under its "no health effect" case value (proof upon request).

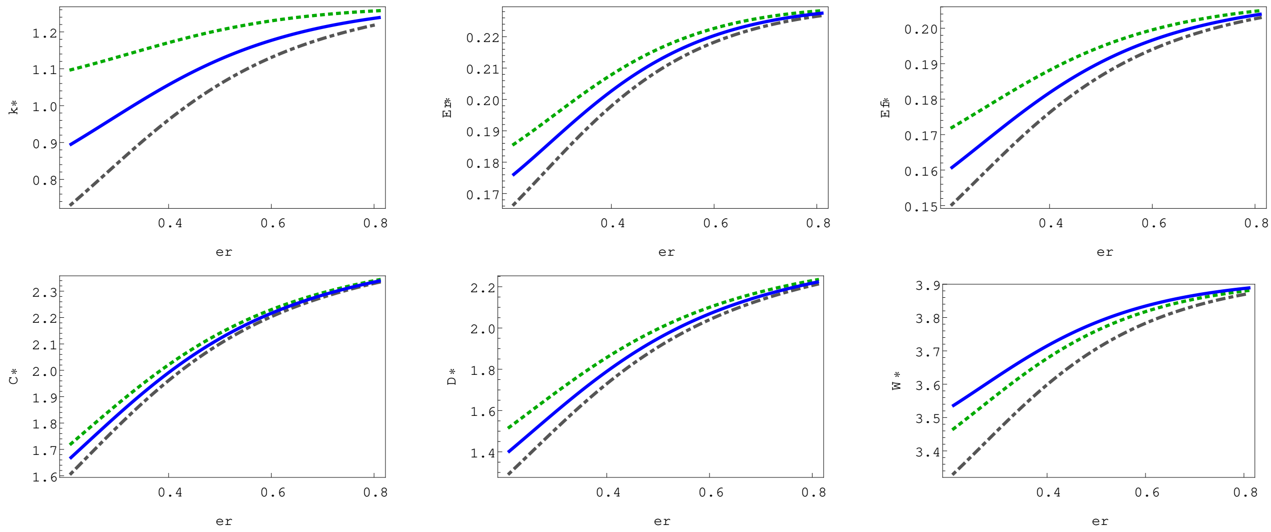


Figure 5: Health externality in utility& influence of ε_r (*Benchmark case (blue) vs no health externality tin utility $\phi = 0$ (green doted) vs greater health externalty in utility $\phi = 1$ (black dotted dashed)*).

5.2.4 The influence of health expenditures

Kotlikoff (1989) highlighted the role played by health expenditures in saving decision. We investigate here what is the influence of health expenditures in adulthood and old age. For that purpose we reported two polar cases. In the first case, the agent do not have any health expenditures when adult. In the second case, the agent do not have health expenditures when old.

Results are reported in Figure (6), the black doted dashed curve illustrates the first case, the green dotted curve illustrates the second case. Results suggest that, for chosen parameter values, the amount of health expenditures does not modify, both, the shape and the level of the influence of energy efficiency on key economic variables. In fact, even in the absence of health expenditures ($m^{\mathrm{a}}(\varepsilon_r^\star) = 0$ and $m^{\mathrm{o}}(\varepsilon_r^\star) = 0$), the steady-state per capita physical capital remains affected by residential energy efficiency.[17] Because the influence of $m^{\mathrm{a}}(\varepsilon_r^\star) = 0$ and $m^{\mathrm{o}}(\varepsilon_r^\star) = 0$ on $k^\star$ are respectively dependent from $\pi^{\mathrm{a}}(\varepsilon_r^\star)\Phi(h^{\mathrm{a}}(\varepsilon_r^\star))$ and $\pi^{\mathrm{o}}(\varepsilon_r^\star)\Phi(h^{\mathrm{o}}(\varepsilon_r^\star))$ which are small, $k^\star$ without health expenditures is not slightly different from its benchmark value. Furthermore, from equation (18), it appears that the contribution of expected health expenditures on residential energy consumption is also quite limited. Nevertheless, we recognize the result of Kotlikoff (1989) that health expenditures of old agent stimulates precautionary saving because in the absence of health expenditures for elderly, the level of physical capital (that is the amount of saving) is lower than the benchmark case (see the green dotted curve in graph 1 of Figure (6)).

6 Discussion

In this section, we extend the basic framework by considering two assumptions. The first allows to link the probability to be sick in adulthood with probability to be sick in great age, in Subsection 6.1. The second is about introducing health in utility function, in Subsection 6.2. We aim to analyse how these new assumptions would modify the results of the basic framework.

6.1 Chronic diseases

In this section, we assume that bad energy efficiency in the residential sector has a permanent effect on health. It creates chronic disease.[18] As a result, adult agents when sick never fully recover, i.e.

[17] This is quite different from Kotlikoff (1989) in which the absence of health expenditures entails no influence of health on savings. In our framework, the existence of health externalities on utility and labor productivity guarantee that health affects saving even if health expenditures are null.

[18] We use the modeling of chronic disease from Pautrel (2022).

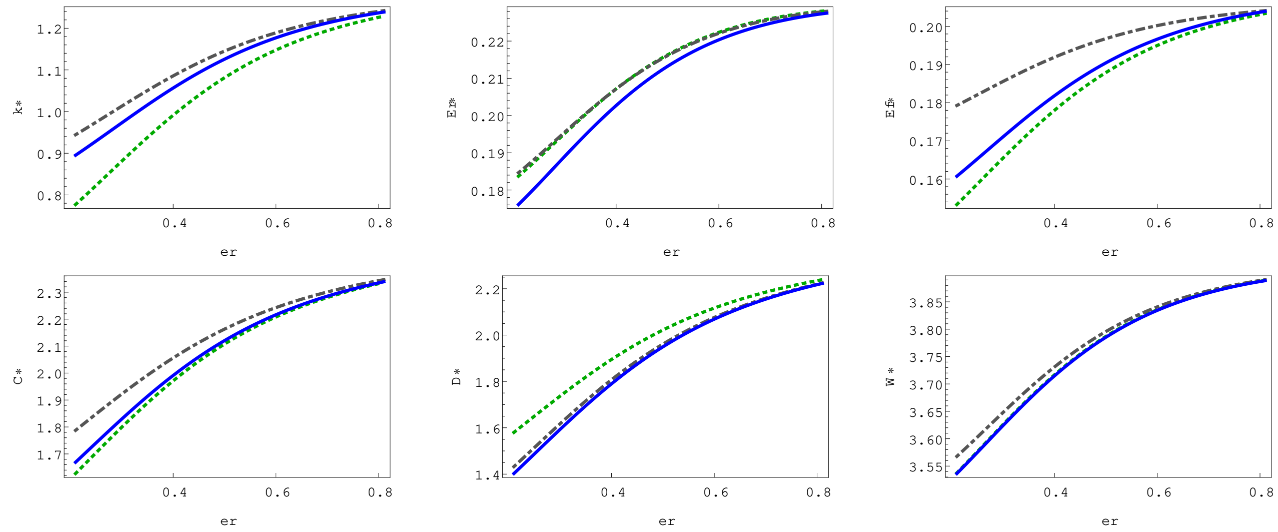
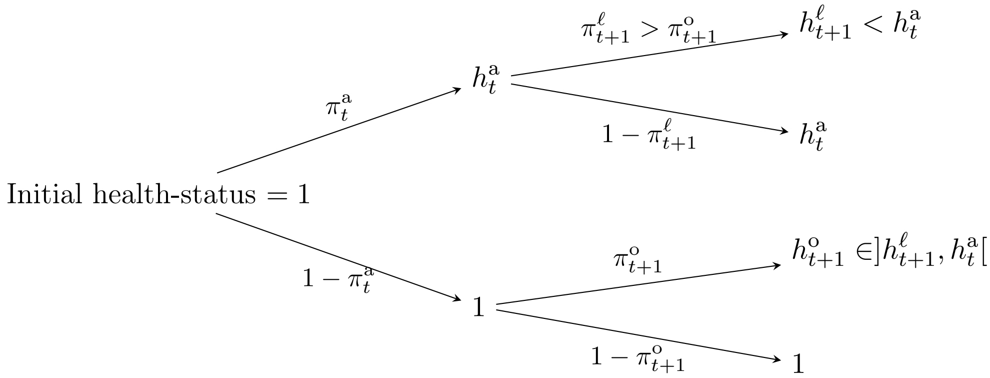


Figure 6: Health expenditures & influence of ε_r (*Benchmark case (blue) vs no elderly health expenditures (green doted) vs no adult health expenditures (black dotted dashed)*).

$z^{\mathrm{a}}(\varepsilon_{r,t-1}) = \bar{z}^{\mathrm{a}}$, and health condition when old is related to health condition when adult.

In particular, we firstly assume that an individual who was sick during adulthood will have a higher probability to be sick when elderly than an individual who wasn't sick during adulthood. We secondly assume that an individual who was sick during adulthood will suffer from a greater loss of health than an individual who wasn't sick during adulthood.

We represent this logical by the following tree:



where π_{t+1}^{ℓ} (" ℓ " for lifetime (chronic) disease) is the probability that old generation becomes sick when she has been sick during adulthood. The tree shows that the old generation can experience four different health conditions:

(i) 1 (perfect health) with a probability $(1-\pi_t^{\mathrm{a}})(1-\pi_{t+1}^{\mathrm{o}})$;

(ii) h^{a} (similar deteriorated health than during adulthood) with a probability $\pi_t^{\mathrm{a}}(1-\pi_{t+1}^{\ell})$;

(iii) $h^{\mathrm{o}} < h^{\mathrm{a}}$ (more deteriorated health than during adulthood) with a probability $(1-\pi_t^{\mathrm{a}})\pi_{t+1}^{\mathrm{o}}$;

(iv) $h^{\ell} < h^{\mathrm{o}} < h^{\mathrm{a}}$ (very deteriorated health with respect to adulthood) with a probability $\pi_t^{\mathrm{a}}\pi_{t+1}^{\ell}$.

Furthermore, we make the following realistic assumptions:

1. Elderly early sick when adult have a higher probability to be sick than elderly not sick when adult: $\pi_{t+1}^{\ell} > \pi_{t+1}^{\mathrm{o}}$.

2. Elderly not sick when adult have a higher probability to be sick than adult: $\pi_{t+1}^{\mathrm{o}} > \pi_t^{\mathrm{a}}$.

The expected intertemporal utility of an adult becomes:

$$V_t = \pi^{\mathrm{a}}(\varepsilon_{r,t-1})\left\{\Phi(h_t^{\mathrm{a}})U(\bar{c}_t^b) + \beta\left[\pi^\ell(\varepsilon_{r,t})\Phi(h_{t+1}^\ell)U(\bar{d}_{t+1}^{\ell b}) + (1-\pi^\ell(\varepsilon_{r,t}))\Phi(h_t^{\mathrm{a}})U(\bar{d}_{t+1}^{\ell g})\right]\right\}$$
$$+ (1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\left\{U(\bar{c}_t^g) + \beta\left[\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\mathrm{o}})U(\bar{d}_{t+1}^b) + (1-\pi^{\mathrm{o}}(\varepsilon_{r,t}))U(\bar{d}_{t+1}^g)\right]\right\}$$

with the following budget constraints:

$$\begin{aligned}
&s_t + c_t^b + p_r E_{r,t}^{\mathrm{a},b} + m^{\mathrm{a}}(\varepsilon_{r,t-1}) = w_t^b\\
&s_t + c_t^g + p_r E_{r,t}^{\mathrm{a},g} = w_t^g\\
&d_{t+1}^b + p_r E_{r,t+1}^{\mathrm{o},b} + m^{\mathrm{o}}(\varepsilon_{r,t}) = R_{t+1}s_t\\
&d_{t+1}^g + p_r E_{r,t}^{\mathrm{o},g} = R_{t+1}s_t\\
&d_{t+1}^{\ell b} + p_r E_{r,t+1}^{\ell b} + m^{\ell b}(\varepsilon_{r,t}) = R_{t+1}s_t\\
&d_{t+1}^{\ell g} + p_r E_{r,t}^{\ell g} + m^{\ell g}(\varepsilon_{r,t}) = R_{t+1}s_t\\
&c_t^i \geq 0,\ d_{t+1}^{i'} \geq 0,\ E_{r,t}^{\mathrm{a},i} \geq 0,\ E_{r,t+1}^{o,i'} \geq 0
\end{aligned}$$

where $m^{\ell i}$ with $i = b, g$ is health expenditures for old suffering from chronic disease.[19]

Because, in the presence of chronic disease, sick agents during adulthood never fully recover, it is not possible to define "bad" health-status following the same rationale than in equation(3). Nevertheless, we continue to assume that "bad" health-status is negatively affected by energy efficiency, such that:

$$h^j = \tilde{h}^j(\varepsilon_r) \qquad \text{with} \qquad \tilde{h}^{j'}(\varepsilon_r) > 0,\ \tilde{h}^j(0 < 1,\ \tilde{h}^j(1) = 1, \qquad j = \mathrm{a}, \mathrm{o}, \ell \tag{21}$$

The steady per capita physical capital in the presence of chronic disease is now defined by the following equation (see Appendix A):

$$\left\{\frac{1-\pi^{\mathrm{a}}(\varepsilon_r^\star)}{\left[\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\left(\frac{k^\star}{\Lambda(\varepsilon_r^\star)}\right)^\alpha - (1+n)k^\star\right]^\sigma} + \frac{\pi^{\mathrm{a}}(\varepsilon_r^\star)\Phi(\tilde{h}^{\mathrm{a}}(\varepsilon_r^\star))}{\left[\tilde{h}^{\mathrm{a}}\,(\varepsilon_r^\star)^\psi\,\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\left(\frac{k^\star}{\Lambda(\varepsilon_r^\star)}\right)^\alpha - (1+n)k^\star - m^{\mathrm{a}}(\varepsilon_r^\star)\right]^\sigma}\right\}$$
$$-\frac{\beta(1+p_r\mathfrak{E}(\varepsilon_{r,t+1}))}{\left[\alpha\mathcal{A}(\varepsilon_{f,t})\left(\frac{k^\star}{\Lambda(\varepsilon_r^\star)}\right)^{\alpha-1}\right]^{\sigma-1}}\left\{\underbrace{(1-\pi^{\mathrm{a}}(\varepsilon_r^\star))}_{I}\left[\frac{\pi^{\mathrm{o}}(\varepsilon_r^\star)\Phi(\tilde{h}^{\mathrm{o}}(\varepsilon_r^\star))}{\left[(1+n)k^\star - \dfrac{m^{\mathrm{o}}(\varepsilon_{r,t})\left(\frac{k^\star}{\Lambda(\varepsilon_r^\star)}\right)^{1-\alpha}}{\mathcal{A}(\varepsilon_f^\star)\alpha}\right]^\sigma} + \frac{1-\pi^{\mathrm{o}}(\varepsilon_r^\star)}{[(1+n)k^\star]^\sigma}\right]\right.$$
$$\left.\underbrace{+\,\pi^{\mathrm{a}}(\varepsilon_r^\star)\left[\frac{\pi^\ell(\varepsilon_r^\star)\tilde{h}^\ell(\varepsilon_r^\star)^\phi}{\left[(1+n)k^\star - \dfrac{m^{\ell b}(\varepsilon_r^\star)\left(\frac{k^\star}{\Lambda(\varepsilon_r^\star)}\right)^{1-\alpha}}{\mathcal{A}(\varepsilon_f^\star)\alpha}\right]^\sigma} + \frac{1-\pi^\ell(\varepsilon_r^\star)}{\left[(1+n)k^\star - \dfrac{m^{\ell g}(\varepsilon_r^\star)\left(\frac{k^\star}{\Lambda(\varepsilon_r^\star)}\right)^{1-\alpha}}{\mathcal{A}(\varepsilon_f^\star)\alpha}\right]^\sigma}\right]}_{II}\right\} = 0 \tag{22}$$

where $\Lambda(\varepsilon_r^\star) \equiv B\left[1 - \pi^{\mathrm{a}}(\varepsilon_{r,t})\left[1 - \tilde{h}^{\mathrm{a}}\,(\varepsilon_r^\star)^\psi\right]\right]$ captures the influence of health on labor productivity in the presence of chronic disease.

[19] In the case of chronic disease, even if health does not deteriorate with a probability $1 - \pi^\ell(\varepsilon_{r,t})$, agents remain sick with a health status equivalent to his deteriorated health status when young (h^{a}). As consequence, it is logical to assume that they continue to pay health expenditures because they do not recover full health, that is $m^{\ell g}(\varepsilon_{r,t}) > 0$. Assuming $m^{\ell g}(\varepsilon_{r,t}) = 0$ would not modify results (proof upon request).

The expected per capita residential energy consumption at the steady-state (both young and old of the previous generation) is now:

$$E_r^{\ell\star} = \frac{\mathfrak{E}(\varepsilon_r^\star)}{1+p_r\mathfrak{E}(\varepsilon_r^\star)} \Big\{ [1-\alpha+\alpha B]\, B^{-1}\mathcal{A}(\varepsilon_f^\star)\Psi(\varepsilon_r^\star)^{1-\alpha}\kappa^\ell(\varepsilon_r^\star,\varepsilon_f^\star)^\alpha - (1+n)\kappa^\ell(\varepsilon_r^\star,\varepsilon_f^\star)$$
$$- \left[\pi^{\mathrm{a}}(\varepsilon_r^\star)m^{\mathrm{a}}(\varepsilon_r^\star) + \underbrace{\frac{(1-\pi^{\mathrm{a}}(\varepsilon_r^\star))\pi^{\mathrm{o}}(\varepsilon_r^\star)m^{\mathrm{o}}(\varepsilon_r^\star) + \pi^{\mathrm{a}}(\varepsilon_r^\star)\left(\pi^\ell(\varepsilon_r^\star)m^{\ell b}(\varepsilon_r^\star) + (1-\pi^\ell(\varepsilon_r^\star))m^{\ell g}(\varepsilon_r^\star)\right)}{1+n}}_{\text{expected health expenditures in old age with chronic disease}} \right] \Bigg\} \tag{23}$$

Chronic disease has two main impacts on residential energy consumption, with respect to the benchmark case.

First, there is a general equilibrium effect which influences the steady-state value of per capita physical capital. On one hand, with respect to the benchmark case, the situation of being or not sick during old age is not certain but affected by the probability $1-\pi^{\mathrm{a}}(\varepsilon_r^\star)$ to not have being sick when adult (see the term I in equation (22)). On the other hand, a new situation (of suffering from chronic disease) appears affected by the probability $\pi^{\mathrm{a}}(\varepsilon_r^\star)$ to have being sick when adult (see the term II in equation (22)).

Second, there is additional expected health expenditures associated with chronic disease which rises the probability to be sick and the intensity of disease. Everything equal, the introduction of chronic disease affects residential energy consumption. We find an expression of the long-term residential energy consumption quite similar to the one obtained in equation (18), except the last term into square brackets at the second line of equation (18) became the last term into square brackets of the second and third lines of equation (23). This is explained by the fact that in the presence of chronic disease, the expected health expenditures when old is connected with the probability to be sick when adult ($\pi^{\mathrm{a}}(\varepsilon_r^\star)$).

The steady-state per capita energy consumption in production is now:

$$E_f^{\ell\star} = \frac{A_{\mathrm{Q}}}{\varepsilon_f^\star\Omega(\varepsilon_f^\star)}\Lambda(\varepsilon_r^\star)^{-\alpha}\kappa^\ell(\varepsilon_r^\star,\varepsilon_f^\star)^\alpha \tag{24}$$

We report in Figure 7 the influence of energy efficiency on key economic variables in the presence of chronic disease (the black dotted dashed curve) compared to i) the benchmark case with no chronic disease (the plain blue curve) and ii) the case with no health effect (the dashed red curve).[20]

The main insight of this numerical exercise is that our key results are not modified in the presence of chronic disease. In particular, the positive link between residential efficiency energy and energy consumption in both sectors still exists and is slightly affected by the chronic disease assumption. For low level of energy efficiency, per capita physical capital is higher with respect to the benchmark case because the probability to be sick when adult ($\pi^{\mathrm{a}}(\varepsilon_r^\star)$) is higher and, therefore, the probability of suffering of chronic disease when old as well (graph 1 of Figure (7)). When the energy efficiency level rises, this probability diminishes as previously noted and, therefore, the "chronic disease" curve (the black dotted-dashed curve graph 1 of Figure (7)) is getting closer to the benchmark curve (the blue plain curve).[21] This also explains why the increasing relation between energy efficiency and energy consumption in both sector is flatted with respect to the benchmark case (see graphs 2 and 3 of Figure (7))

6.2 Endogenous health expenditures

In previous sections, we assumed that health expenditures only depends on energy efficiency to capture the fact that they are paid to rise the deteriorated health-status. We also assumed that the return to

[20]For the simulation of the benchmark chronic disease case, we assumed that chronic disease reduces of 20% the detrimental impact of disease on health and increases of 20% the probability to be sick when old with respect to the benchmark case.

[21]When $\pi^{\mathrm{a}}(\varepsilon_r^\star)$ is getting closer to 0 (that is when $\varepsilon_r^\star$ is high), the last term into square brackets in (23) is getting closer to the last term into square brackets in (18). That is why, when $\varepsilon_r^\star$ is high, the benchmark curve (the blue one) is closer to the "chronic disease" curve.

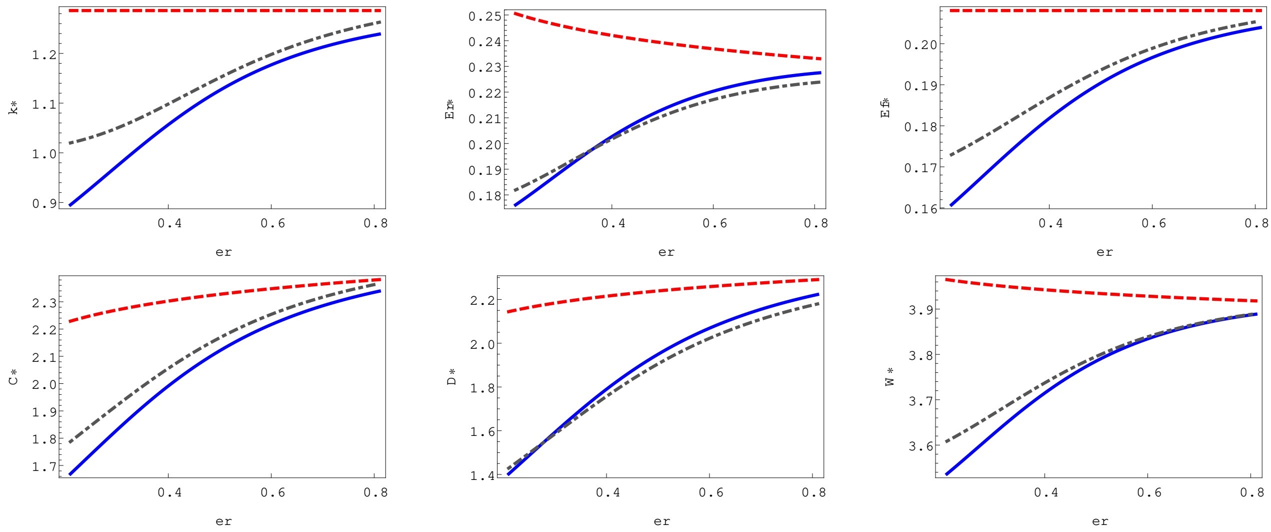


Figure 7: Chronic disease and influence of ε_r (*Benchmark case (blue) vs no health effect (red large dashed) vs chronic disease case (black dot-dashed)*)

full health took time and may not be achieved in the case of chronic disease.

Nevertheless, it is also possible to consider that health expenditures are endogenously chosen by utility-maximizing agents and directly affect the health-status in both periods. The purpose of this section is to investigate how this new assumption modifies the results of the benchmark model.

Health-status of individuals of type j ($j = \mathrm{a}, \mathrm{o}$) is now defined as:[22]

$$h_t^j \equiv \frac{1}{\gamma^j}\left[\eta^j m_t^j + \gamma^j \varepsilon_{r,t}\right] \tag{25}$$

where $\gamma^j \in (0,1)$ and $\eta^j \in (0,1)$ capture the positive influence of respectively energy efficiency ε_r and health expenditures m^j on health-status.[23]

Maximizing expected utility (4) under budget constraints given by equations (6a) to (6d), we obtain the same relationships (7) and (8) like in the benchmark model.

Proposition 4. *Health expenditures for adults and elderly depends on respective consumption and energy efficiency:*

$$m_t^{\mathrm{a}} = \mu^{\mathrm{a}}(\varepsilon_{r,t})c_t^b - \left(\frac{\gamma^j}{\eta^j}\right)\varepsilon_{r,t} \qquad \text{and} \qquad m_{t+1}^{\mathrm{o}} = \mu^{\mathrm{o}}(\varepsilon_{r,t+1})d_{t+1}^b - \left(\frac{\gamma^j}{\eta^j}\right)\varepsilon_{r,t+1} \tag{26}$$

with $\mu^j(\varepsilon_r) \equiv \frac{\phi^j}{\sigma(1-\nu)}\mathfrak{Z}(\varepsilon_r)^{\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}$ *, for j=a,o, which is increasing in* ε_r *for realistic values of* $\sigma_{\mathrm{C,E}} < 1$.

Proof. See Appendix B. □

Proposition 4 shows that endogenous health expenditures is influenced by energy efficiency in two opposite ways. The first influence is negative. It is the one assumed in our benchmark case with exogenous health expenditures. The second influence is rather positive. It is linked to utility maximization and the existence of a positive influence of health on utility associated with the fact

[22] An alternative specification of health-status could have been $h_t^j \equiv (1+1/\gamma^j)\left[1+\left(\eta^j m_t^j + \gamma^j \varepsilon_{r,t}\right)^{-1}\right]^{-1}$. Nevertheless, this form makes very difficult analytical and numerical resolution of the model (proof upon request). That is the reason why we don't use it despite its relevance.

[23] Note that this function is normalized in order to have $h^j = 1$ for $\varepsilon_r = 1$ and $m^j = 0$. When $\varepsilon_r < 1$, health expenditures are bounded by $m^{j\,\max} = \left(\frac{\gamma^j}{\eta^j}\right)(1-\varepsilon_r)$: the lower the energy efficiency, the higher is health expenditures required to restore full health. It means that our health function is compatible with the assumptions we made in Section 3.1 page 5.

that marginal utility of consumption is positively affected by energy efficiency ε_r. Because a higher energy efficiency rises marginal utility of consumption, it incites agents to invest in health because the marginal contribution of one unit of health expenditures is increased by energy efficiency.

Finally, utility maximization leads to the following expression of per capita physical capital at the steady-state in the presence of endogenous health expenditures:

$$\frac{\pi^{\mathrm{a}}(\varepsilon_r^\star)\mu^{\mathrm{a}}\left(\varepsilon_r^\star\right)^{\phi^{\mathrm{a}}}}{\mathcal{C}(\varepsilon_r^\star,k^\star)^{\sigma-\phi^{\mathrm{a}}}}+\frac{1-\pi^{\mathrm{a}}(\varepsilon_r^\star)}{\left(\mathcal{A}(\varepsilon_f^\star)(1-\alpha)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{\alpha}-(1+n)k^\star\right)^{\sigma}}$$
$$=\beta\mathcal{A}(\varepsilon_f^\star)\alpha(1+n)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{\alpha-1}\left[\frac{\pi^{\mathrm{o}}(\varepsilon_r^\star)\mu^{\mathrm{o}}\left(\varepsilon_r^\star\right)^{\phi^{\mathrm{o}}}}{\mathcal{D}(\varepsilon_r^\star,k^\star)^{\sigma-\phi^{\mathrm{o}}}}+\frac{1-\pi^{\mathrm{o}}(\varepsilon_r^\star)}{\left(\mathcal{A}(\varepsilon_f^\star)\alpha(1+n)k^{\star\alpha}\Psi(\varepsilon_r^\star)^{1-\alpha}\right)^{\sigma}}\right] \quad (27)$$

where $\mathcal{C}(\varepsilon_r^\star,k^\star)$ (respectively $\mathcal{D}(\varepsilon_r^\star,k^\star)$) is the solution of the budget constraint (6a) (respectively (6c)) when equations (25) and (26) are taken into account.

This expression differs from the benchmark case (equation (9) at the steady-state equilibrium) in two ways. First, health expenditures being endogenous and linearly connected to non-energy consumption, an increase in saving (and as a consequence a reduction in consumption for both adults and old) has two additional effects wit respect to the exogenous health expenditures case. On one hand, an increase in saving will reduce health expenditures by reducing consumption. On the other hand, it will reduce health status ceteris paribus and then the positive impact of health on utility. Those influences are captured by the parameter ϕ^{a} and ϕ^{o} in supercript of $\mathcal{C}(\varepsilon_r^\star,k^\star)$ and $\mathcal{D}(\varepsilon_r^\star,k^\star)$ respectively in equation (27). Second, a new term $\mu^j(\varepsilon_r^\star)^{\phi^j}$ appears in front of probability $\pi^j(\varepsilon_r^\star)$ in equation (27) (replacing $\Phi(h^j)$ in equation (9)) which depends on the structure of preferences and the price of energy in residential sector. As a consequence, the reaction of health expenditures of a variation of energy efficiency $\varepsilon_r^\star$ is linked to the elasticity of substitution between energy consumption and non-energy consumption in preferences ($\sigma_{\mathrm{C,E}}$) and to the inverse of the intertemporal elasticity of substitution (σ). It means that taking into endogenously chosen health expenditures reinforces the role consumer preferences play in the link between energy efficiency and energy consumption.

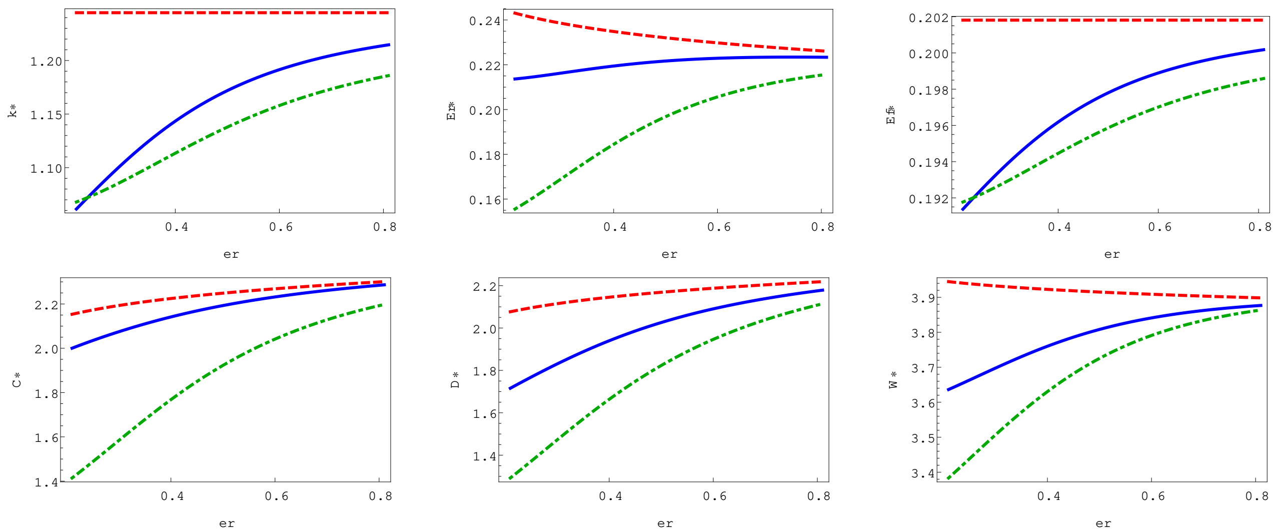


Figure 8: Influence of ε_r (Benchmark case with $\psi = 0$ (blue) vs endogenous health expenditures (green dashed) vs no health effect (red large dashes))

Figure (8) shows that the introduction of endogenous health expenditures does not modify our main result according to which taking into account the detrimental impact of low energy efficiency on health could rational the positive link empirically observed between residential energy efficiency and residential energy consumption (compare the green dashed line with the benchmark case plain blue curve in each graph of Figure (8)).[24] Nevertheless, the shape of this link is modified (the green dashed

[24]Note that in the reported numerical exercise here, parameters ϕ^j which the size of health externality in utility for

curves are all steeper and lower than the benchmark case curves). This means that the magnitude of the responses to health improvements associated with higher residential energy efficiency are more important when health expenditures are endogenous.

7 Conclusion and policy implications

This paper proposes a 3-periods overlapping generations model in order to investigate how health (via morbidity effects) influences the impacts of residential energy efficiency on energy consumption. Based on empirical evidences from two bodies of economic and interdisciplinary literatures, we assumed that these morbidity effects are induced by residential energy inefficiency and influence the workforce productivity.

Our results show that health channels we introduced could significantly shape the change in energy consumption induced by an energy efficiency variation, through their impacts on the propensity to save and on the revenue. In particular, our results suggest that if mostly old (resp. young) people health is affected, the health impact of residential energy efficiency should have a backfire (resp. rebound) influence on residential energy consumption, by promoting precautionary saving (resp. by rising labor productivity). Our results show that if health externality on labor productivity is strong enough, as suggested by recent empirical evidence, an energy efficiency improvement should rise significantly the energy consumption not only in the residential sector but also in production sector.

When our theoretical framework is extended to take into account the influence of health status on utility and to integrate chronic disease associated with residential energy inefficiency, our results interestingly show that chronic disease introduces two (additional) opposite effects. On the one hand, it reinforces the "propensity to save" effect by increasing precautionary saving. In fact, the probability to be sick when old increases due to chronic disease. On the other, the chronic disease reinforces the "disposal income" effect, because an improvement of residential energy efficiency induces a better health-status during adulthood, thus, a higher labor productivity. The global effect mainly depends on the magnitude of the health externality on labor productivity and on the probabilities of elderly of becoming sick.

In policy terms, this study adds to the debate about two main issues. The first deals with the rebound effect. From an empirical point of view, it is largely recognized that rebound effect is difficult to estimate. Usually, only a part of the rebound effect is estimated, i.e. direct effect. Although, theoretical studies give additional insights on this complex phenomena, several related questions are still open mainly regarding the interaction with the macroeconomic side. In our analysis, by introducing health and focusing on identifying health channels through which a variation in energy efficiency may affect energy consumption, we take an original tack to consider the rebound effect, thus, we introduce a new conceptual framework that may help defining new strategies to mitigate this phenomenon and designing new policy pathways. In fact, Font Vivanco et al. (2016) argue that policy inaction on rebound effect is partly explained by the unsuccessful push from academics. Our contribution particularly shows that considering health channels when analyzing the relationship between energy efficiency and energy consumption may help understanding the rebound effect by stressing some unexpected interactions with labor productivity and (precautionary) saving.

The second debate our study contributes to is about the intertwine between energy consumption (or more generally, environment), health and growth policies. In particular, from a microeconomic point of view, our research adds to the policy debate regarding the relationship between individual energy conservation policies and health (self-insurance) policies. We show in our model that -in some cases- energy inefficiency in the residential sector may increase the likelihood of precautionary saving in order to self-insure against future expected medical expenses. As a consequence, it is crucial to reconsider the question of households energy and health expenditures trad-offs and the subsequent question of impacts individual decisions may have on macroeconomic health, energy and growth programs. The international community recently pointed out the significant increase in energy bills during the global pandemic, i.e. lock-down periods and online working, and its dramatic consequences on households

age $j = (\mathrm{a}, \mathrm{o})$ is chosen such as $\sigma - \phi^j > 0$. Choosing $\sigma - \phi^j < 0$ would not modify the graphs reported in Figure (8), even if it changes the sign of the influence of $\mathcal{C}(\varepsilon_r^\star, k^\star)$ and $\mathcal{D}(\varepsilon_r^\star, k^\star)$ in equation (27) (proof upon request).

expenditures trade-offs. New rules of deprivation have been self-implemented and new forms of micro and macro-vulnerabilities have been revealed. These new observations reflect how crucial and complex are for governments the discussion and policy action dealing with the connection between energy conservation policies, health insurance system and economic growth.

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A The model with chronic disease

The expected intertemporal utility of an adult is:

$$
\begin{aligned}
U_t &= \pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})U(\bar{c}_t^b) + (1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))U(\bar{c}_t^g) \\
&+ \beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\left[\pi^{\ell}(\varepsilon_{r,t})\Phi(h_{t+1}^{\ell})U(\bar{d}_{t+1}^{\ell b}) + (1-\pi^{\ell}(\varepsilon_{r,t}))\Phi(h_t^{\mathrm{a}})U(\bar{d}_{t+1}^{\ell g})\right] \\
&\qquad + \beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\left[\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\mathrm{o}})U(\bar{d}_{t+1}^{b}) + (1-\pi^{\mathrm{o}}(\varepsilon_{r,t}))U(\bar{d}_{t+1}^{g})\right]
\end{aligned}
$$

where $U(x) \equiv \sigma^{-1}x^{\sigma}$ ($\sigma \geq 0$) and the composite consumption good of the adult generation and the old generation, respectively denoted by $\bar{c}^i$ and $\bar{d}^i$, are given by

$$
\bar{c}_t^i = \left[(1-\nu)c_t^{i\,\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}} + \nu\left(\varepsilon_{r,t}E_{r,t}^{\mathrm{a},i}\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}} \qquad \text{with} \qquad i = (b,g)\text{and}
$$

$$
\bar{d}_{t+1}^{i'} = \left[(1-\nu)d_{t+1}^{i'\,\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}} + \nu\left(\varepsilon_{r,t+1}E_{r,t+1}^{o,i'}\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}} \qquad \text{with} \qquad \nu \in]0,1[,\ i' = (\ell b, \ell g, b, g)
$$

c_t^i (respectively $d_{t+1}^{i'}$) is the amount of non-energy goods the adult (resp. old) generation consumes with a health condition i (resp. i'). $E_{r,t}^{\mathrm{a},i}$ (respectively $E_{r,t+1}^{o,i'}$) is the amount of energy services (whose efficiency is captured by ε_r) the adult (resp. old) generation consumes with a health condition i. $\sigma_{\mathrm{C,E}} \geq 0$ is the elasticity of substitution between non-energy goods and energy consumptions.

Per-period budget constraints are:

$$
\begin{aligned}
& s_t + c_t^b + p_r E_{r,t}^{\mathrm{a},b} + m^{\mathrm{a}}(\varepsilon_{r,t-1}) = w_t^b \\
& s_t + c_t^g + p_r E_{r,t}^{\mathrm{a},g} = w_t^g \\
& d_{t+1}^b + p_r E_{r,t+1}^{\mathrm{o},b} + m^{\mathrm{o}}(\varepsilon_{r,t}) = R_{t+1}s_t \\
& d_{t+1}^g + p_r E_{r,t}^{\mathrm{o},g} = R_{t+1}s_t \\
& d_{t+1}^{\ell b} + p_r E_{r,t+1}^{\ell b} + m^{\ell b}(\varepsilon_{r,t}) = R_{t+1}s_t \\
& d_{t+1}^{\ell g} + p_r E_{r,t}^{\ell g} + m^{\ell g}(\varepsilon_{r,t}) = R_{t+1}s_t \\
& c_t^i \geq 0,\ d_{t+1}^{i'} \geq 0,\ E_{r,t}^{\mathrm{a},i} \geq 0,\ E_{r,t+1}^{o,i'} \geq 0
\end{aligned}
$$

where $R_{t+1} = (1 + r_{t+1})$ with r the real interest rate, w_t is real wage, p_r is energy price. The maximization program is:

$$
\begin{aligned}
\mathcal{L} &= \pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})U(\bar{c}_t^b) + (1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))U(\bar{c}_t^g) \\
&+ \beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\left[\pi^{\ell}(\varepsilon_{r,t})\Phi(h_{t+1}^{\ell})U(\bar{d}_{t+1}^{\ell b}) + (1-\pi^{\ell}(\varepsilon_{r,t}))\Phi(h_t^{\mathrm{a}})U(\bar{d}_{t+1}^{\ell g})\right] \\
&+ \beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\left[\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\mathrm{o}})U(\bar{d}_{t+1}^{b}) + (1-\pi^{\mathrm{o}}(\varepsilon_{r,t}))U(\bar{d}_{t+1}^{g})\right] \\
&+ \lambda_1\left[w_t^b - s_t - c_t^b - p_r E_{r,t}^{\mathrm{a},b} - m^{\mathrm{a}}(\varepsilon_{r,t-1})\right]s + \lambda_2\left[w_t^g - s_t - c_t^g - p_r E_{r,t}^{\mathrm{a},g}\right] \\
&+ \lambda_3\left[R_{t+1}s_t - d_{t+1}^b - p_r E_{r,t+1}^{\mathrm{o},b} - m^{\mathrm{o}}(\varepsilon_{r,t})\right] + \lambda_4\left[R_{t+1}s_t - d_{t+1}^g - p_r E_{r,t+1}^{\mathrm{o},g}\right] \\
&\qquad + \lambda_5\left[R_{t+1}s_t - d_{t+1}^{\ell b} - p_r E_{r,t+1}^{\mathrm{o},\ell b} - m^{\ell b}(\varepsilon_{r,t})\right] + \lambda_6\left[R_{t+1}s_t - d_{t+1}^{\ell g} - p_r E_{r,t+1}^{\mathrm{o},\ell g} - m^{\ell g}(\varepsilon_{r,t})\right]
\end{aligned}
$$

First-order conditions give:

$$c^b \quad \Rightarrow \quad \pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})\left(\frac{c^b}{\bar{c}^b}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(c^b\right)^{-\sigma}=\lambda_1 \tag{A.1}$$

$$E_r^{\mathrm{a},b} \quad \Rightarrow \quad \pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})\nu\varepsilon_{r,t}\left(\frac{\varepsilon_{r,t}E_r^{\mathrm{a},b}}{\bar{c}^b}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(E_r^{\mathrm{a},b}\right)^{-\sigma}=\lambda_1 p_r \tag{A.2}$$

$$c^g \quad \Rightarrow \quad (1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\nu)\left(\frac{c^g}{\bar{c}^g}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}(c^g)^{-\sigma}=\lambda_2 \tag{A.3}$$

$$E_r^{\mathrm{a},g} \quad \Rightarrow \quad (1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\nu\varepsilon_{r,t}\left(\frac{\varepsilon_{r,t}E_r^{\mathrm{a},g}}{\bar{c}^g}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}(E_r^{\mathrm{a},g})^{-\sigma}=\lambda_2 p_r \tag{A.4}$$

$$d^b \quad \Rightarrow \quad \beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\pi_{t+1}^{\mathrm{o}}(\varepsilon_{r,t+1})\Phi(h_{t+1}^{\mathrm{o}})(1-\nu)\left(\frac{d^b}{\bar{d}^b}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(d^b\right)^{-\sigma}=\lambda_3 \tag{A.5}$$

$$E_r^{\mathrm{o},b} \quad \Rightarrow \quad \beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\pi_{t+1}^{\mathrm{o}}(\varepsilon_{r,t+1})\Phi(h_{t+1}^{\mathrm{o}})\nu\varepsilon_{r,t+1}\left(\frac{\varepsilon_{r,t+1}E_r^{\mathrm{o},b}}{\bar{d}^b}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(E_r^{\mathrm{o},b}\right)^{-\sigma}=\lambda_3 p_r \tag{A.6}$$

$$d^g \quad \Rightarrow \quad \beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\pi_{t+1}^{\mathrm{o}}(\varepsilon_{r,t+1}))(1-\nu)\left(\frac{d^g}{\bar{d}^g}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}(d^g)^{-\sigma}=\lambda_4 \tag{A.7}$$

$$E_r^{\mathrm{o},g} \quad \Rightarrow \quad \beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\pi_{t+1}^{\mathrm{o}}(\varepsilon_{r,t+1}))\nu\left(\frac{\varepsilon_{r,t+1}E_r^{\mathrm{o},g}}{\bar{d}^g}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}(E_r^{\mathrm{o},g})^{-\sigma}=\lambda_4 p_r \tag{A.8}$$

$$d^{\ell b} \quad \Rightarrow \quad \beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\pi^{\ell}(\varepsilon_{r,t})\Phi(h_{t+1}^{\ell})(1-\nu)\left(\frac{d^{\ell b}}{\bar{d}^{\ell b}}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(d^{\ell b}\right)^{-\sigma}=\lambda_5 \tag{A.9}$$

$$E_r^{\mathrm{o},\ell b} \quad \Rightarrow \quad \beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\pi^{\ell}(\varepsilon_{r,t})\Phi(h_{t+1}^{\ell})\nu\left(\frac{\varepsilon_{r,t+1}E_r^{\mathrm{o},\ell b}}{\bar{d}^{\ell b}}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(E_r^{\mathrm{o},\ell b}\right)^{-\sigma}=\lambda_5 p_r \tag{A.10}$$

$$d^{\ell g} \quad \Rightarrow \quad \beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})(1-\pi^{\ell}(\varepsilon_{r,t}))\Phi(h_t^{\mathrm{a}})(1-\nu)\left(\frac{d^{\ell g}}{\bar{d}^{\ell g}}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(d^{\ell g}\right)^{-\sigma}=\lambda_6 \tag{A.11}$$

$$E_r^{\mathrm{o},\ell g} \quad \Rightarrow \quad \beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})(1-\pi^{\ell}(\varepsilon_{r,t}))\Phi(h_t^{\mathrm{a}})\nu\varepsilon_{r,t+1}\left(\frac{\varepsilon_{r,t+1}E_r^{\mathrm{o},\ell g}}{\bar{d}^g}\right)^{\sigma-\frac{1}{\sigma_{\mathrm{C,E}}}}\left(E_r^{\mathrm{o},\ell g}\right)^{-\sigma}=\lambda_6 p_r \tag{A.12}$$

$$s \quad \Rightarrow \quad \lambda_1+\lambda_2=R_{t+1}\left(\lambda_3+\lambda_4+\lambda_5+\lambda_6\right) \tag{A.13}$$

From (A.1) to (A.8),we get

$$\begin{aligned} E_{r,t}^{\mathrm{a},i}&=\mathfrak{E}(\varepsilon_{r,t})c_t^i \\ E_{r,t+1}^{\mathrm{o},i}&=\mathfrak{E}(\varepsilon_{r,t+1})d_{t+1}^j \end{aligned} \qquad \text{with} \qquad \mathfrak{E}(\varepsilon_r)\equiv\left(\tfrac{\nu}{1-\nu}\right)^{\sigma_{\mathrm{C,E}}}p_r^{-\sigma_{\mathrm{C,E}}}\varepsilon_r^{\sigma_{\mathrm{C,E}}-1} \text{ and } i=(b,g),\ j=(\ell b,\ell g,b,g) \tag{7}$$

Furthermore

$$\begin{aligned} \bar{c}_t^i&=\mathfrak{Z}(\varepsilon_{r,t})c_t^i \\ \bar{d}_{t+1}^j&=\mathfrak{Z}(\varepsilon_{r,t+1})d_{t+1}^j \end{aligned} \qquad \text{with} \qquad \mathfrak{Z}(\varepsilon_r)\equiv\left[(1-\nu)+\nu\left(\varepsilon_r\mathfrak{E}(\varepsilon_r)\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}} \tag{8}$$

and $\varepsilon_r=\varepsilon_{r,t}$ for a and $\varepsilon_r=\varepsilon_{r,t+1}$ for o. Then

$$\begin{aligned} U(\bar{c}_t^i)&=(1-\sigma)^{-1}\left(\mathfrak{Z}(\varepsilon_{r,t})c_t^i\right)^{1-\sigma} \\ U(\bar{d}_{t+1}^{i'})&=(1-\sigma)^{-1}\left(\mathfrak{Z}(\varepsilon_{r,t+1})d_{t+1}^{i'}\right)^{1-\sigma} \end{aligned}$$

and first-order conditions (A.1)-(A.10) become:

$$\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})(1-\nu)\mathfrak{Z}(\varepsilon_{r,t})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}c^{b-\sigma}=\lambda_1 \tag{A.14}$$

$$(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\nu)\mathfrak{Z}(\varepsilon_{r,t})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}c^{g-\sigma}=\lambda_2 \tag{A.15}$$

$$\beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\pi^{\mathrm{o}}_{t+1}(\varepsilon_{r,t+1})\Phi(h^{\mathrm{o}}_{t+1})(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}d^{b-\sigma}=\lambda_3 \tag{A.16}$$

$$\beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\pi^{\mathrm{o}}_{t+1}(\varepsilon_{r,t+1}))(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}d^{g-\sigma}=\lambda_4 \tag{A.17}$$

$$\beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\pi^{\ell}(\varepsilon_{r,t})\Phi(h^{\ell}_{t+1})(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}d^{\ell b-\sigma}=\lambda_5 \tag{A.18}$$

$$\beta\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h^{\mathrm{a}}_{t})(1-\pi^{\ell}(\varepsilon_{r,t}))(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}d^{\ell g-\sigma}=\lambda_6 \tag{A.19}$$

Using equations (A.14) to (A.19) in (A.13), we get

$$\frac{\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h^{\mathrm{a}}_t)}{(c^b)^\sigma}+\frac{(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))}{(c^g)^\sigma}=\beta R_{t+1}\left[\frac{(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h^{\mathrm{o}}_{t+1})}{(d^b)^\sigma}\right.$$
$$\left.+\frac{(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\pi^{\mathrm{o}}(\varepsilon_{r,t}))}{(d^g)^\sigma}+\frac{\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\pi^{\ell}(\varepsilon_{r,t})\Phi(h^{\ell}_{t+1})}{(d^{\ell b})^\sigma}+\frac{\pi^{\mathrm{a}}(\varepsilon_{r,t-1})(1-\pi^{\ell}(\varepsilon_{r,t}))\Phi(h^{\mathrm{a}}_{t+1})}{(d^{\ell g})^\sigma}\right]$$

Using budget constraints and the different results previously found, we obtain the expression of per capita physical capital at the steady-state (see equation (22) in page 19).

B The model with endogenous health expenditures

In this section we present the resolution of the model with endogenous health expenditures.[25] The expected intertemporal utility of an adult is:

$$U_t=\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h^{\mathrm{a}}_t)U(\bar{c}^b_t)+(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))U(\bar{c}^g_t)$$
$$+\beta\left[\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h^{\mathrm{o}}_{t+1})U(\bar{d}^b_{t+1})+(1-\pi^{\mathrm{o}}(\varepsilon_{r,t}))U(\bar{d}^g_{t+1})\right]$$

where $U(x)\equiv(1-\sigma)^{-1}x^{1-\sigma}$ with $\sigma\in(0,1)$, and the composite consumption good of the adult generation and the old generation, respectively denoted by $\bar{c}^i$ and $\bar{d}^i$, are given by

$$\bar{c}^i_t=\left[(1-\nu)c_t^{i\,\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}+\nu\left(\varepsilon_{r,t}E^{\mathrm{a},i}_{r,t}\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}}\qquad\text{with}\qquad i=(b,g)\text{and}$$

$$\bar{d}^{i'}_{t+1}=\left[(1-\nu)d_{t+1}^{i'\,\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}+\nu\left(\varepsilon_{r,t+1}E^{o,i'}_{r,t+1}\right)^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}}\right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}-1}}\qquad\text{with}\qquad\nu\in]0,1[,\ i'=(\ell b,\ell g,b,g)$$

c^i_t (respectively $d^{i'}_{t+1}$) is the amount of non-energy goods the adult (resp. old) generation consumes with a health condition i. $E^{\mathrm{a},i}_{r,t}$ (respectively $E^{o,i'}_{r,t+1}$) is the amount of energy services (whose efficiency is captured by ε_r) the adult (resp. old) generation consumes with a health condition i. $\sigma_{\mathrm{C,E}}\geq 0$ is the elasticity of substitution between non-energy goods and energy consumptions.

Health-status is now defined as:

$$h^j_t\equiv\frac{1}{\gamma^j}\left[\eta^j m^j_t+\gamma^j\varepsilon_{r,t}\right] \tag{25}$$

[25]Note that the resolution of the basic model may be easily derived from this section.

Per-period budget constraints are:

$$
\begin{aligned}
& s_t + c_t^b + p_r E_{r,t}^{\mathrm{a},b} + m_t^{\mathrm{a}} = w_t^b \\
& s_t + c_t^g + p_r E_{r,t}^{\mathrm{a},g} = w_t^g \\
& d_{t+1}^b + p_r E_{r,t+1}^{\mathrm{o},b} + m_{t+1}^{\mathrm{o}} = R_{t+1} s_t \\
& d_{t+1}^g + p_r E_{r,t}^{\mathrm{o},g} = R_{t+1} s_t \\
& c_t^i \geq 0,\ d_{t+1}^{i'} \geq 0,\ E_{r,t}^{\mathrm{a},i} \geq 0,\ E_{r,t+1}^{o,i'} \geq 0
\end{aligned}
$$

where $R_{t+1} = (1 + r_{t+1})$ with r the real interest rate, w_t is real wage, p_r is energy price. The maximization program is:

$$
\begin{aligned}
\mathcal{L} = & \pi^{\mathrm{a}}(\varepsilon_{r,t-1}) \Phi(h_t^{\mathrm{a}}) U(\bar{c}_t^b) + (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) U(\bar{c}_t^g) \\
& + \beta \left[\pi^{\mathrm{o}}(\varepsilon_{r,t}) \Phi(h_{t+1}^{\mathrm{o}}) U(\bar{d}_{t+1}^b) + (1 - \pi^{\mathrm{o}}(\varepsilon_{r,t})) U(\bar{d}_{t+1}^g) \right] \\
& + \lambda_1 \left[w_t^b - s_t - c_t^b - p_r E_{r,t}^{\mathrm{a},b} - m_t^{\mathrm{a}} \right] + \lambda_2 \left[w_t^g - s_t - c_t^g - p_r E_{r,t}^{\mathrm{a},g} \right] \\
& \qquad + \lambda_3 \left[R_{t+1} s_t - d_{t+1}^b - p_r E_{r,t+1}^{\mathrm{o},b} - m_{t+1}^{\mathrm{o}} \right] + \lambda_4 \left[R_{t+1} s_t - d_{t+1}^g - p_r E_{r,t+1}^{\mathrm{o},g} \right]
\end{aligned}
$$

First-order conditions give:

$$
c^b \quad \Rightarrow \quad \pi^{\mathrm{a}}(\varepsilon_{r,t-1}) \Phi(h_t^{\mathrm{a}}) (1 - \nu) \left(\frac{c^b}{\bar{c}^b} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} \left(c^b \right)^{-\sigma} = \lambda_1 \tag{B.1}
$$

$$
E_r^{\mathrm{a},b} \quad \Rightarrow \quad \pi^{\mathrm{a}}(\varepsilon_{r,t-1}) \Phi(h_t^{\mathrm{a}}) \nu \varepsilon_{r,t} \left(\frac{\varepsilon_{r,t} E_r^{\mathrm{a},b}}{\bar{c}^b} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} \left(E_r^{\mathrm{a},b} \right)^{-\sigma} = \lambda_1 p_r \tag{B.2}
$$

$$
c^g \quad \Rightarrow \quad (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) (1 - \nu) \left(\frac{c^g}{\bar{c}^g} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} (c^g)^{-\sigma} = \lambda_2 \tag{B.3}
$$

$$
E_r^{\mathrm{a},g} \quad \Rightarrow \quad (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) \nu \varepsilon_{r,t} \left(\frac{\varepsilon_{r,t} E_r^{\mathrm{a},g}}{\bar{c}^g} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} (E_r^{\mathrm{a},g})^{-\sigma} = \lambda_2 p_r \tag{B.4}
$$

$$
d^b \quad \Rightarrow \quad \beta (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) \pi^{\mathrm{o}}(\varepsilon_{r,t}) \Phi(h_{t+1}^{\mathrm{o}}) (1 - \nu) \left(\frac{d^b}{\bar{d}^b} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} \left(d^b \right)^{-\sigma} = \lambda_3 \tag{B.5}
$$

$$
E_r^{\mathrm{o},b} \quad \Rightarrow \quad \beta (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) \pi^{\mathrm{o}}(\varepsilon_{r,t}) \Phi(h_{t+1}^{\mathrm{o}}) \nu \varepsilon_{r,t+1} \left(\frac{\varepsilon_{r,t+1} E_r^{\mathrm{o},b}}{\bar{d}^b} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} \left(E_r^{\mathrm{o},b} \right)^{-\sigma} = \lambda_3 p_r \tag{B.6}
$$

$$
d^g \quad \Rightarrow \quad \beta (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) (1 - \pi^{\mathrm{o}}(\varepsilon_{r,t})) (1 - \nu) \left(\frac{d^g}{\bar{d}^g} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} (d^g)^{-\sigma} = \lambda_4 \tag{B.7}
$$

$$
E_r^{\mathrm{o},g} \quad \Rightarrow \quad \beta (1 - \pi^{\mathrm{a}}(\varepsilon_{r,t-1})) (1 - \pi^{\mathrm{o}}(\varepsilon_{r,t})) \nu \left(\frac{\varepsilon_{r,t+1} E_r^{\mathrm{o},g}}{\bar{d}^g} \right)^{\sigma - \frac{1}{\sigma_{\mathrm{C,E}}}} (E_r^{\mathrm{o},g})^{-\sigma} = \lambda_4 p_r \tag{B.8}
$$

$$
s \quad \Rightarrow \quad \lambda_1 + \lambda_2 = R_{t+1} \left(\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 \right) \tag{B.9}
$$

$$
m_t^{\mathrm{a}} \quad \Rightarrow \quad \pi^{\mathrm{a}}(\varepsilon_{r,t-1}) \Phi'(h_t^{\mathrm{a}}) U(\bar{c}_t^b) = \lambda_1 \tag{B.10}
$$

$$
m_{t+1}^{\mathrm{o}} \quad \Rightarrow \quad \beta \pi^{\mathrm{o}}(\varepsilon_{r,t}) \Phi'(h_{t+1}^{\mathrm{o}}) U(\bar{d}_{t+1}^b) = \lambda_3 \tag{B.11}
$$

From (B.1) and (B.2) and (B.3) and (B.4), and from (B.5) and (B.6) and (B.7) and (B.8),we get

$$
\begin{aligned}
E_{r,t}^{\mathrm{a},i} &= \mathfrak{E}(\varepsilon_{r,t}) c_t^i \\
E_{r,t+1}^{\mathrm{o},i'} &= \mathfrak{E}(\varepsilon_{r,t+1}) d_{t+1}^{i'}
\end{aligned}
\qquad \text{with} \qquad \mathfrak{E}(\varepsilon_r) \equiv \left(\tfrac{\nu}{1-\nu} \right)^{\sigma_{\mathrm{C,E}}} p_r^{-\sigma_{\mathrm{C,E}}} \varepsilon_r^{\sigma_{\mathrm{C,E}} - 1} \text{ and } i = (b, g),\ i' = (\ell b, \ell g, b, g) \tag{7}
$$

Furthermore

$$
\begin{aligned}
\bar{c}_t^i &= \mathfrak{Z}(\varepsilon_{r,t}) c_t^i \\
\bar{d}_{t+1}^{i'} &= \mathfrak{Z}(\varepsilon_{r,t+1}) d_{t+1}^{i'}
\end{aligned}
\qquad \text{with} \qquad \mathfrak{Z}(\varepsilon_r) \equiv \left[(1 - \nu) + \nu \left(\varepsilon_r \mathfrak{E}(\varepsilon_r) \right)^{\frac{\sigma_{\mathrm{C,E}} - 1}{\sigma_{\mathrm{C,E}}}} \right]^{\frac{\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}} - 1}} \tag{8}
$$

and $\varepsilon_r = \varepsilon_{r,t}$ for a and $\varepsilon_r = \varepsilon_{r,t+1}$ for o. Then

$$U(\bar{c}_t^i) = (1-\sigma)^{-1}\left(\mathfrak{Z}(\varepsilon_{r,t})c_t^i\right)^{1-\sigma} \tag{B.12}$$

$$U(\bar{d}_{t+1}^{i'}) = (1-\sigma)^{-1}\left(\mathfrak{Z}(\varepsilon_{r,t+1})d_{t+1}^{i'}\right)^{1-\sigma} \tag{B.13}$$

and first-order conditions (B.1)-(B.11) become:

$$\pi^{\mathrm{a}}(\varepsilon_{r,t-1})\Phi(h_t^{\mathrm{a}})(1-\nu)\mathfrak{Z}(\varepsilon_{r,t})^{1-\sigma+\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}c^{b-\sigma} = \lambda_1 \tag{B.14}$$

$$(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\nu)\mathfrak{Z}(\varepsilon_{r,t})^{1-\sigma+\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}c^{g-\sigma} = \lambda_2 \tag{B.15}$$

$$\beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))\pi^{\mathrm{o}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\mathrm{o}})(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{1-\sigma+\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}d^{b-\sigma} = \lambda_3 \tag{B.16}$$

$$\beta(1-\pi^{\mathrm{a}}(\varepsilon_{r,t-1}))(1-\pi^{\mathrm{o}}(\varepsilon_{r,t}))(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{1-\sigma+\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}d^{g-\sigma} = \lambda_4 \tag{B.17}$$

Using equations (B.12) to (B.17), first-order conditions (B.10)-(B.11) become:

$$\frac{\Phi'(h_t^{\mathrm{a}})}{\Phi(h_t^{\mathrm{a}})} = (1-\sigma)(1-\nu)\mathfrak{Z}(\varepsilon_{r,t})^{\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}c^{b-1}$$
$$\frac{\Phi'(h_{t+1}^{\mathrm{o}})}{\Phi(h_{t+1}^{\mathrm{o}})} = (1-\sigma)(1-\nu)\mathfrak{Z}(\varepsilon_{r,t+1})^{\frac{\sigma_{\mathrm{C,E}}-1}{\sigma_{\mathrm{C,E}}}-\sigma}d^{b-1}$$

From the expression of $\Phi^j(\cdot)$ and h^j we obtain:

$$m_t^{\mathrm{a}} = \mu^{\mathrm{a}}(\varepsilon_{r,t})c_t^b - \left(\frac{\gamma^{\mathrm{a}}}{\eta^{\mathrm{a}}}\right)\varepsilon_{r,t} \qquad \text{and} \qquad m_{t+1}^{\mathrm{o}} = \mu^{\mathrm{o}}(\varepsilon_{r,t+1})d_{t+1}^b - \left(\frac{\gamma^j}{\eta^j}\right)\varepsilon_{r,t+1} \tag{26}$$

with $\mu^j(\varepsilon_r) \equiv \frac{\phi^j}{(1-\sigma)(1-\nu)}\mathfrak{Z}(\varepsilon_r)^{\frac{1-\sigma_{\mathrm{C,E}}}{\sigma_{\mathrm{C,E}}}}$ increasing in ε_r for realistic values of $\sigma_{\mathrm{C,E}} < 1$.

At the steady-state, using (7), (26) and (5) and (10c), budget constraint of sick adult (6a) defines $c^{b\star}$ as a function of $\varepsilon_r^\star$ and $k^\star$, denoted by $\mathcal{C}(\varepsilon_r^\star, k^\star)$, solution of the following equality:

$$c^{b\star}\left(1 + p_r\mathfrak{E}(\varepsilon_r) + \mu^{\mathrm{a}}(\varepsilon_r^\star)\right) = \left(\mu^{\mathrm{a}}(\varepsilon_r^\star)c^{b\star}\right)^{\phi^{\mathrm{a}}\psi}\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^\alpha - s^\star + \left(\frac{\gamma^{\mathrm{a}}}{\eta^{\mathrm{a}}}\right)\varepsilon_r^\star$$

$$\text{where } \Psi(\varepsilon_r^\star) \equiv B\left[1 - \pi^{\mathrm{a}}(\varepsilon_r^\star)\left[1 - \left(\mu^{\mathrm{a}}(\varepsilon_r^\star)c^{b\star}\right)^{\phi^{\mathrm{a}}\psi}\right]\right].$$

Similarly, at the steady-state, budget constraint of sick old (6c) defines $d^{b\star}$ as a function of $\varepsilon_r^\star$ and $k^\star$, denoted by $\mathcal{D}(\varepsilon_r^\star, k^\star)$, solution of the following equality:

$$d^{b\star}\left(1 + p_r\mathfrak{E}(\varepsilon_r) + \mu^{\mathrm{o}}(\varepsilon_r^\star)\right) = (1+n)\mathcal{A}(\varepsilon_{f,t})\alpha\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^\alpha$$

Using equations (B.14) to (B.17) in (B.9), we get

$$\frac{\pi^{\mathrm{a}}(\varepsilon_r^\star)\Phi(h^{\mathrm{a}}(\varepsilon_r^\star))}{\mathcal{C}(\varepsilon_r^\star, k^\star))^\sigma} + \frac{(1-\pi^{\mathrm{a}}(\varepsilon_r^\star))}{\left(\mathcal{A}(\varepsilon_{f,t})(1-\alpha)\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^\alpha - (1+n)k^\star\right)^\sigma}$$
$$= \beta\mathcal{A}(\varepsilon_{f,t})\alpha\left(\frac{k^\star}{\Psi(\varepsilon_r^\star)}\right)^{\alpha-1}\left[\frac{\pi^{\mathrm{o}}(\varepsilon_r^\star)\Phi(h^{\mathrm{o}}(\varepsilon_r^\star))}{\mathcal{D}(\varepsilon_r^\star, k^\star)^\sigma} + \frac{1-\pi^{\mathrm{o}}(\varepsilon_r^\star)}{((1+n)\mathcal{A}(\varepsilon_{f,t})\alpha k^{\star\alpha}\Psi(\varepsilon_r^\star)^{1-\alpha})^\sigma}\right]$$

C The health risk function

In this section, we discuss the specification of the health function. First we denote $F^i(\varepsilon_r)$ the sigmoïd function related the probability, for the agent of type i ($i =$ a,o), to be sick according to the value of the energy efficiency ε_r. We define $F^i(\varepsilon_r)$ as:

$$F^i(\varepsilon_r) \equiv \frac{1}{1 + 0.01\left(\frac{a}{\varepsilon_r^{i,max}}\right)^\iota} - \frac{1}{1 + 0.01\left(\frac{a}{\varepsilon_r}\right)^\iota}$$

where $\varepsilon_r^{i,max}$ is the upper-bound of ε_r above which agent of type i can not be sick (here we assumed for adults $\varepsilon_r^{\mathrm{a},max} = 0.8$ then above 0.8 energy efficiency is high enough to prevent adults to be sick due to energy poverty). As a consequence $F^i(\varepsilon_r)$ is decreasing in ε_r and is negative when $\varepsilon_r > \varepsilon_r^{i,max}$. Parameters $a > 0$ and $\iota > 0$ defined the curvature of the sigmoïd and the inflection point. We choose here $a = 3$ and $\iota = 2.5$ for both types of agents.

We also assume that under a lower bound, denoted by $\varepsilon_r^{i,min}$, the probability of becoming sick due to energy poverty is maximal (but lower than unity) and independent from ε_r. This probability is then obtained when $\varepsilon_r = \varepsilon_r^{i,min}$ and is therefore given by $F^i(\varepsilon_r^{min})$. As a consequence, for type i agents, the health risk function associated to bad energy efficiency is defined by:

$$\pi_h^i = \min\left[F^i(\varepsilon_r), F^i(\varepsilon_r^{i,min})\right]$$

Agents of type i face a "non-energy" induced health risk, denoted by $s^i \in [0, 1[$ which adds to the energy induced health risk, and when energy efficiency is higher than $\varepsilon_r^{i,max}$ the only health risk that agents face is s^i. As a consequence, the probability to be sick is limited by s^i for the higher values of ε_r and by unity for the lower values of ε_r, such that:

$$\pi^i(\varepsilon_r) \equiv \min\left[\max\left[\pi_h^i + s^i, s^i\right], 1\right]$$

D Sickness by age and influences of health externalities

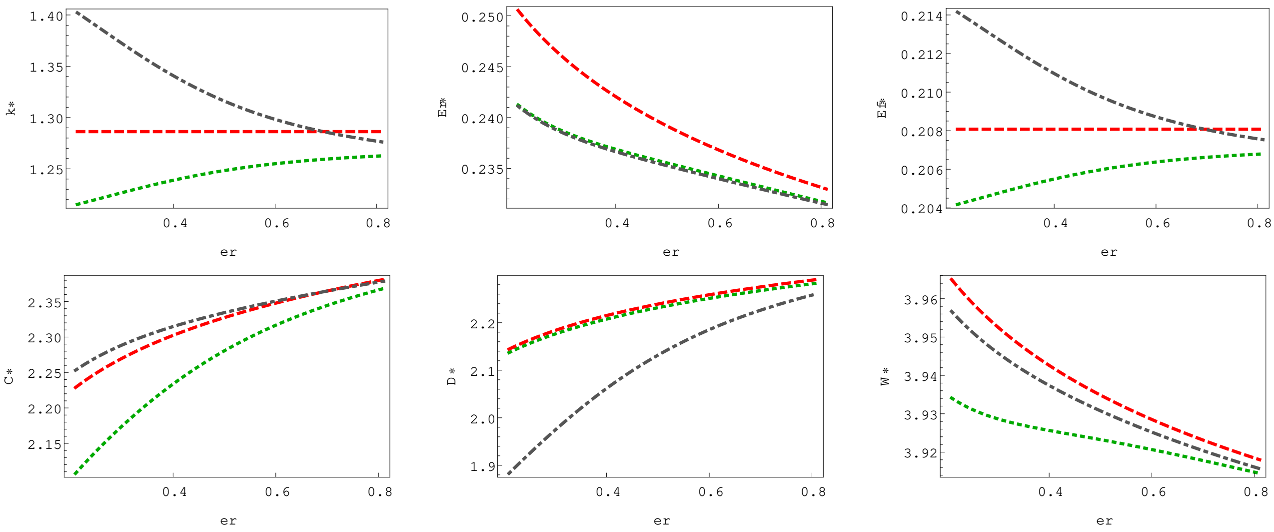

Figure D.1: "Sickness by age" and influence of ε_r in the absence of health externality (*health effect during old age only (black dotted dashed), health effect during adulthood only (green tiny dashed) and no health effect (red large dashes)*)

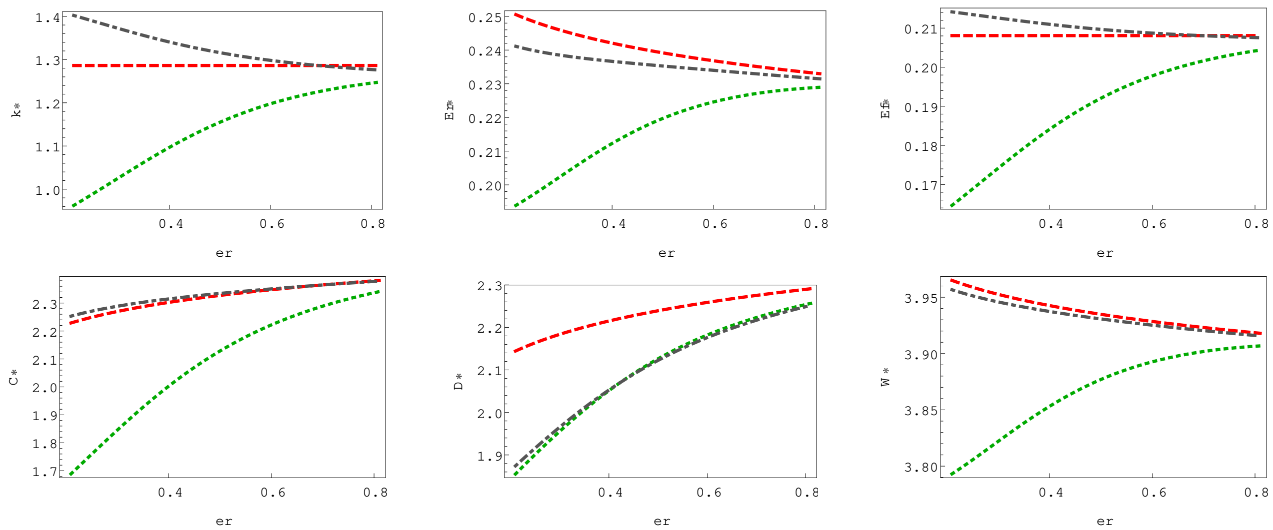

Figure D.2: "Sickness by age" and influence of ε_r with health externality only on labor productivity (*health effect during old age only (black dotted dashed), health effect during adulthood only (green tiny dashed) and no health effect (red large dashes)*)