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Marc Germain

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A simple degrowth model

Marc Germain*

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Abstract

While relying on the postulates of ecological economics, this paper proposes a Ramsey growth model with a natural resource and pollution. It studies the impact of voluntary degrowth policies on production and welfare. The instrument of these policies is a tax on the natural resource. These public policies are implemented after the downturn of the households' welfare following from the increased pollution.

Two kinds of policies are considered and rely either on an optimality criterion or on an intergenerational equity criterion. With respect to the laissez-faire case, they decrease both production and pollution on the one hand and increase welfare on the other hand. When the policy is based on an optimality criterion, a delayed reaction from the public authorities implies a higher tax rate during the first periods. Optimal degrowth paths appear to be non sustainable from an intergenerational point of view. Classes of sustainable degrowth paths characterized by time-constant or time-increasing tax rates are determined.

Key words: degrowth, steady state economics, pollution tax, intergenerational equity

JEL: O44, O49, Q57

*LEM-CNRS (UMR 9221), Université de Lille 3 et IRES, Université de Louvain. I would like to thank Géraldine Thiry and an anonymous referee for their helpful comments.

Introduction

Economics of degrowth has been developing as a new research topic for some years. Unsurprisingly, most contributors belongs to the school of ecological economics which, for a long time, have been interested in alternatives to unsustainable growth paths. By unsustainable paths, we mean paths that violate the biophysical limits of the economy but also paths that are undesirable from a social point of view (for example because welfare decreases or because social inequalities increase).

Although relatively recent, Economics of degrowth has been the subject of numerous contributions. In a review of the literature, Kallis et al. [1] classify the contributions in three streams of thoughts: (i) Steady-State Economics whose figurehead is Herman Daly, (ii) New Economics of Prosperity around Tim Jackson and (iii) Degrowth *à la* Serge Latouche and Joan Martinez-Alier[1].

If there are differences and even disagreements between these streams, they all consider that current economic growth is unsustainable and that another trajectory is desirable. Degrowth is then defined as the *voluntary and fair* transition from an unsustainable growth path to a stationary and sustainable state of the economy (O'Neill [3]). Moreover, even though the transition implies a decrease in production and consumption, it simultaneously aims at increasing welfare while complying to environmental constraints in the short and long terms (Schneider et al. [4]). It is thus a chosen process and it goes without saying that no author pleads for a perpetual degrowth that would lead to generalized misery.

If there is an abundant literature in Degrowth Economics, few contributions attempt to assess quantitatively the impacts of a degrowth transition. Bilancini et D'Alessandro [5] and Heikkinen [6] offer theoretical assessments. Bilancini et D'Alessandro contrast "unhappy growth" with "happy degrowth" in the framework of an endogenous growth model with externalities in consumption, leisure and production. The consumption externality is negative and leads to a competition between consumers in terms of social status. The leisure externality is positive and linked to the fact that leisure contributes to social activities that act like a public good and increase welfare. The third is linked to the accumulation of capital which stimulates knowledge and technical progress. The authors show that a decentralized economy is suboptimal from a welfare point of view. They however identify a "happy" transition toward an optimal path where all externalities are taken in account. This transition is characterized by (i) a temporary reduction in production and consumption and (ii) an increase in welfare, the decrease in consumption being more than compensated by the increase in relational activities allowed by a more extensive leisure time. Heikkinen [6] enriches the model by considering consumers with heterogeneous and time-varying preferences with respect to the importance of social status and voluntary simplicity. This one is defined as the deliberate choice of an agent to limit her consumption expenditures. The author shows that the weakening of status consumption increases aggregate welfare while decreasing the economy growth rate. Moreover, the voluntary simplicity adopted by a subset of consumers less sensitive to status competition has a positive impact on welfare.

Applied contributions include those of Peter Victor (see Victor [7] for an autobiographical note). In one of his contributions, Victor uses a macroeconomic model (called LowGrow) to assess how policies reducing GHG emissions would affect the Canadian economy, in particular growth, public spending and employment (Victor [8]). Among the considered scenarios, the author studies a degrowth scenario where the standard of living of the Canadians is more in line with the respect of the planet's limits. Using the methodological approach of societal metabolism[2], Sorman and Giampietro [10] analyze the implications of possible degrowth paths from an energetic point of view. The recent thesis of Briens [11] is also worth mentioning. On the basis of an input/output macroeconomic model, the author assesses different degrowth scenarios suggested by a series of interviews of people involved in the Degrowth movement (or interested by it) in order to obtain different detailed visions of what could be degrowth. As those of Victor [8] and Sorman and Giampietro [10], his results show that the degrowth required given the environmental constraints is likely to have a considerable impact on the economy and that it is barely conceivable without a

[1]The interested reader will find in Kallis et al. [1] the references to the contributions of these authors as well as many others. Another interesting survey dedicated to Degrowth is Petridis et al. [2].

[2]For an introduction to this literature, see Fischer-Kowalski and Haberl [9].

deep reorganization of society.

Despite their indisputable interest, the above mentioned papers suffer from different limitations. The theoretical contributions of Bilancini and D'Alessandro [5] and Heikkinen [6] develop growth models that ignore environmental and resource constraints. The analysis is done in terms of balanced growth paths and the degrowth phase is actually a transition from a suboptimal to an optimal growth path. If consumption and/or production decrease during the transition, they start to increase again once it is achieved. These two contributions also ignore the role that public policies could play. The applied models mentioned above are not growth models in the usual sense. They do not rely on a welfare approach, are not closed[3] (for example demand is exogenous) or suffer from other limitations (for example natural resources are ignored). Let us finally mention that most of the above mentioned contributions (either theoretical of applied) ignore sustainability issues.

The present paper develops a stylized theoretical model which aims at studying the impact of voluntary degrowth policies. Contrary to Bilancini and D'Alessandro [5] and Heikkinen [6], we explicitly consider environmental externalities. More precisely, we distinguish three types of externalities, linked respectively to the exploitation of a natural resource, to pollution and to production. In accordance with ecological economics, the model assumes that (i) substitution between natural and human factors is limited and (ii) technical progress in the use of the resource as well as in the treatment of pollution is bounded. Given that the resource is itself limited, infinite growth is impossible and the economy can at best converge to a stationary equilibrium. In the laisser-faire situation, the model generates after some time a decrease in the households' welfare which echoes the *threshold hypothesis* of Max-Neef [12]: beyond a certain GDP per capita level (the threshold), welfare (or quality of life) declines with economic growth. This welfare decrease motivates the public authorities' intervention and the implementation of a degrowth policy, whose instrument is a tax levied on the exploitation of the natural resource. Such a policy is first based on an optimality criterion (à la Ramsey), but because optimality does not guarantee sustainability, we also consider sustainable degrowth policies based on a (intergenerational) equity criterion (à la Brundtland).

The structure of the paper is as follows. Section 1 presents the equations of the model. We consider two institutional organizations of the economy depending on whether it is decentralized or centrally planned. The stationary equilibria of the economy are determined in Section 2. The dynamic paths (including the transitional phase) are computed in Section 3. Section 4 studies the impacts of voluntary optimal degrowth policies, with a focus on the role of the reaction time of the public authorities. The role of technical progress is also considered. Section 5 characterizes voluntary sustainable degrowth paths satisfying an intergenerational equity criterion. The conclusion summarizes the principal results and suggests several possible extensions.

1 The model

The economy enjoys an exogenous constant flow R of a renewable natural resource. There is a continuum of identical price-taking producers defined on the interval $[0, N]$. They use physical capital and the natural resource (NR) to produce final goods.

To produce y_t units of final output in time t, the representative firm needs a quantity $x_t = \mu_t y_t$ of NR. μ_t measures the quantity of NR per unit of final good and is assumed exogenous and bounded from below by a strictly positive value μ:

$$\mu_t \geq \mu > 0, \ \forall t. \tag{1}$$

It is thus never possible to produce a unit of final good with an infinitesimal quantity of NR, even if μ_t may be decreasing through time.

NR is a free common resource but its extraction/transformation process requires physical cap-

[3] Except the model of Victor [8].

ital. To handle the quantity x_t, the firm rents a capital stock k_t given by:

$$k_t = \frac{x_t^{1/\gamma}}{A(K_t, E_t)},\ 0 < \gamma < 1 \tag{2}$$

where γ is a parameter. $\gamma < 1$ indicates that returns to scale are decreasing at the level of the firm. Function A depends on K_t and E_t which denote respectively the aggregate stock of capital and the aggregate extraction rate of the resource. This rate is defined by $E_t = X_t/R$ where X_t is the total quantity of NR extracted by all firms. The relationship between A and its arguments reflects two external effects:
- $A'_K > 0$ means that the macroeconomic capital stock K_t has a positive influence on the firm's productivity;
- $A'_E < 0$ means that the capital necessary to extract one unit of NR increases with the extraction rate of the resource.

Because it belongs to a continuum, a firm has no significant impact on aggregate quantities so that it ignores all external effects. K_t and X_t (or E_t) are therefore exogenous at the firm level.

In each period, with the final good price chosen as numéraire, the profit maximization problem of the representative firm can be written as:

$$\max_{y_t, k_t, x_t} \pi_t = y_t - v_t k_t - \tau_t x_t \tag{3}$$

under the constraints $x_t = \mu_t y_t$ and (2). τ_t denotes the tax per unit of NR levied by the public authorities and v_t is the rental price of capital. Perfect competition is assumed so that all prices are exogenous at the firm level. First order optimality conditions lead to:

$$\frac{k_t}{y_t} = \frac{\gamma\left[1 - \mu_t \tau_t\right]}{v_t} \tag{4}$$

The product $\mu_t \tau_t$ represents the effective tax rate per unit of final good.

Given the continuum of identical producers defined on $[0, N]$, total production in period t is equal to $Y_t = \int_0^N y_t(i)di = Ny_t$. Likewise:

$$X_t = Nx_t = \mu_t Y_t \tag{5}$$

and

$$K_t = Nk_t = N\frac{x_t^{1/\gamma}}{A(K_t, E_t)}$$

We assume the following functional form: $A(K_t, E_t) = hK_t^{\frac{1}{\gamma}-1}\left[1 - E_t\right]^{\frac{1}{\gamma}}$ where h is a positive constant. Then it is easy to show that (2) leads to:

$$K_t = a\frac{X_t}{1 - \frac{X_t}{R}} \tag{6}$$

where a is a positive parameter[4]. The denominator shows that the capital needed to extract NR tends to infinity when X_t approaches R. The above equation amounts to assuming that capital and NR are complementary inputs[5]. In this sense, it relies on the *strong sustainability hypothesis* which postulates that substitutability between natural and man-made inputs is limited. This assumption, combined with the fact that (i) the quantity of NR per unit of good is bounded from below (see (1)) and (ii) the flow of NR R is finite, implies that unlimited growth of Y_t is impossible in the framework of the present model.

[4]Indeed $K_t = N\frac{x_t^{1/\gamma}}{hK_t^{1/\gamma-1}[1-E_t]^{1/\gamma}} \Rightarrow K_t^{1/\gamma} = \frac{N^{1-1/\gamma}}{h}\frac{X_t^{1/\gamma}}{[1-E_t]^{1/\gamma}}$. Let $a^{1/\gamma} = \frac{N^{1-1/\gamma}}{h}$ and (6) follows.

[5]Indeed (6) can be rewritten as a CES production function of K_t and R with an elasticity of substitution equal to 1/2.

At the macroeconomic level, (4) becomes

$$\frac{K_t}{Y_t} = \frac{\gamma\left[1 - \mu_t \tau_t\right]}{v_t} \tag{7}$$

We assume that there is a one period lag between investment and the installation of capital. For the sake of simplicity we also assume a unitary depreciation rate of capital. In other words, the investment decided in $t-1$ is productive in t and scrapped in $t+1$. This last assumption supposes implicitly that a time period lasts several years.

Given the above assumptions and the fact that final output is allocated either to consumption or to investment, the equilibrium condition of the final good market can be written as follows:

$$Y_t = C_t + K_{t+1} \tag{8}$$

Let us define z_t as the inverse of the propensity to consume and s_t as the savings rate:

$$z_t = \frac{Y_t}{C_t} \text{ and } s_t = \frac{K_{t+1}}{Y_t} \tag{9}$$

Then output and the capital stock are linked by

$$K_{t+1} = \left[1 - \frac{1}{z_t}\right] Y_t = s_t Y_t \tag{10}$$

Final good production is accompanied by a global pollution flow P_t, which affects negatively households' utility. For the sake of simplicity, we assume that pollution is linearly proportional to production and does not accumulate. Formally:

$$P_t = \eta_t Y_t \tag{11}$$

where η_t is an exogenous parameter measuring the quantity of pollutant emitted per unit of final good. η_t is assumed exogenous and can possibly decrease (monotonically) through time because of technical improvements making production less polluting. For technical constraints however, we exclude the possibility to reduce pollution to zero. η_t is thus bounded from below by a strictly positive value:

$$\eta_t \geq \eta > 0,\ \forall t. \tag{12}$$

1.1 The decentralized economy

We consider a representative and long-living household who consumes the final good and invests in productive capital, which she lets to firms. Her instantaneous utility depends positively of consumption C_t and negatively of the pollution level P_t:

$$u_t = u(C_t, P_t) = \ln(C_t) - \sigma P_t \tag{13}$$

where $\sigma > 0$ is the marginal disutility of pollution. In each period, she receives the whole macroeconomic income which consists of the capital rent $v_t K_t$ and other incomes Ω_t (which include firms' profits and the receipt of the tax redistributed as a lump sum[6]). The household's preferences are represented by the intertemporal utility function $\sum_{t=1}^{T_f} \beta^t u_t$, where β is her discount factor ($0 < \beta \leq 1$) and T_f is the exogenous time horizon (possibly infinite). The representative household chooses her optimal consumption path by maximizing this function under her budget constraint: $C_t + K_{t+1} = \Omega_t + v_t K_t, \forall t \geq 1$. Given that the representative household has no direct influence on the pollution level, we obtain the well-known consumption smoothing behavior described by the following equation:

$$\frac{1}{C_t} = \beta v_{t+1} \frac{1}{C_{t+1}} \tag{14}$$

with $K_{T_f+1} = 0$ (or equivalently $Y_{T_f} = C_{T_f}$) as final condition.

Equations (7), (8), (14) lead to an interesting property.

[6]Firm profits are given by $\int_0^N \pi_t(i)di = N\pi_t$, where π_t is defined by (3).

Proposition 1 *In a decentralized economy, the path of the inverse of the propensity to consume* z_t *is given by:*

$$z_t = 1 + \sum_{\theta=t+1}^{T_f} \prod_{\varphi=t+1}^{\theta} \alpha_\varphi, \ t \in \{t+1, ..., T_f - 1\} \tag{15}$$

with $z_{T_f} = 1$ *as final value*[7]*.*

Proof. Indeed (8) and (14) imply $\frac{C_t}{C_{t-1}} = \beta\gamma [1 - \mu_t \tau_t] \frac{Y_t}{K_t}$, so that $\frac{K_t}{C_{t-1}} = \beta\gamma [1 - \mu_t \tau_t] \frac{Y_t}{C_t}$. Then given (8) : $\frac{Y_{t-1} - C_{t-1}}{C_{t-1}} = \beta\gamma [1 - \mu_t \tau_t] \frac{Y_t}{C_t}$. Let

$$\alpha_t = \beta\gamma [1 - \mu_t \tau_t] \tag{16}$$

Then given (9) we obtain the following first order difference equation:

$$z_{t-1} - 1 = \alpha_t z_t \tag{17}$$

This equation describes the evolution of the inverse of the propensity to consume z_t and its integration leads to (15). ■

Besides (5), (6), (10) imply:

$$Y_{t-1} \left[1 - \frac{1}{z_{t-1}}\right] = \frac{a \mu_t Y_t}{1 - \frac{\mu_t Y_t}{R}} \tag{18}$$

Given that μ_t is exogenous and the fact that $z_t, t \in \{1, ..., T_f\}$ is determined by (15), it is possible to solve (18) given the initial value of capital K_1 (which determines Y_1 via (5) and (6)).

1.2 The planned economy

The central planner is assumed to maximize the intertemporal utility function $\sum_{t=T+1}^{T_f} \beta_p^t u_t$, where u_t is the representative household's instantaneous utility defined by (13). β_p is the discount factor of the central planner ($0 < \beta_p \leq 1$) which may be different from the one of households. $T + 1$ ($0 \leq T < T_f$) is the period from which the central planner runs the economy, which may not coincide with the initial period $t = 1$. Given (9) and (11), the objective can be rewritten as:

$$\max_{\{Y_t, z_t\}_{t=T+1,...,T_f}} \sum_{t=T+1}^{T_f} \beta_p^{t-[T+1]} \left[\ln\left(\frac{Y_t}{z_t}\right) - \sigma \eta_t Y_t\right] \tag{19}$$

under constraint (18). (19) expresses that the central planner takes the pollution externality into account. Through the constraint (18), the central planner also takes care of the external effects linked to the exploitation of the NR and to the influence of aggregate capital on the productivity of firms.

We have the following result.

Proposition 2 *The path of the planned economy for* $t > T$ *is governed by equation (18) and by*

$$z_{t-1} - 1 = \beta_p [z_t - \sigma \eta_t Y_t] \left[1 - \frac{\mu_t Y_t}{R}\right], \ t = T+1, ..., T_f \tag{20}$$

with $z_{T_f} = 1$ *as final condition and* K_1 *(or* Y_1*) given as initial condition.*

(20) follows from the first order optimality conditions of problem (19) (see proof in Appendix A).

[7] The terminal condition $z_{T_f} = 1$ is equivalent to $K_{T_f+1} = 0$ for (14).

2 Stationary equilibria

Stationary states (or equilibria) are characterized by the constancy of all variables, in particular output, consumption and the flows of NR extraction and pollution. They describe asymptotic *zero growth paths* of the economy. Remember that the technological assumptions of the model exclude perpetual growth.

Stationary states imply the constancy of the exogenous parameters, either because they are always constant or because they have reached their limit values (in presence of technical progress). This concerns the quantity of NR per unit of final good μ_t and the quantity of pollutant per unit of final good η_t. The tax rate levied on the NR τ_t must also be constant. Let μ, η and τ be the constant (limit) values of these parameters.

We first consider the feasibility of the stationary states, that is the conditions these states must satisfy to be economically meaningful. Then we characterize successively the stationary states of the decentralized and of the planned economies.

2.1 Feasibility conditions

To be economically meaningful, a stationary equilibrium must satisfy certain conditions. In particular output is necessarily bigger than consumption so that, given (9), $z \geq 1$. Thus (6) and (10) imply at the equilibrium:

$$0 \leq s = 1 - \frac{1}{z} = \frac{a\mu}{1-E} \leq 1 \tag{21}$$

where $E = \mu Y/R$ is the equilibrium extraction rate. $0 < E \leq 1$ implies accordingly that $a\mu < 1$. Should this inequality be unverified, then the economy would be unable to sustain its output level even by allocating all production to investment.

If $a\mu < 1$, then a stationary equilibrium exists if the corresponding extraction rate satisfies:

$$0 < E \leq 1 - a\mu \tag{22}$$

or equivalently if the corresponding savings rate satisfies:

$$a\mu < s \leq 1 \tag{23}$$

The *feasibility domain* of the economy is defined as the set of all possible stationary states satisfying (22) or (23). It is thus determined by the interval $]0, 1-a\mu]$ for the stationary extraction rate E or equivalently by the interval $]a\mu, 1]$ for the stationary savings rate s. It must be underlined that the feasibility domain does not depend on any institutional considerations and, in particular, on whether the economy is decentralized or planned.

2.2 The decentralized economy

Proposition 3 *The decentralized stationary equilibrium (DSE) is unique and characterized by the following values:*

$$s_* = \alpha = \beta\gamma\,[1 - \mu\tau] \tag{24}$$

$$z_* = \frac{1}{1-\alpha} \tag{25}$$

$$E_* = 1 - \frac{a\mu}{\alpha} \tag{26}$$

$$Y_* = \frac{E_* R}{\mu} = \left[1 - \frac{a\mu}{\alpha}\right]\frac{R}{\mu} \tag{27}$$

$$C_* = \frac{Y_*}{z_*} = [1-\alpha]\left[1 - \frac{a\mu}{\alpha}\right]\frac{R}{\mu} \tag{28}$$

See proof in Appendix B.1.

Production Y_* is proportional to the extraction rate of the NR E_*, which depends positively on the savings rate s_*. Now this rate depends negatively of τ. Accordingly, a higher tax on NR implies lower extraction and savings rates and a lower output level. The impact on C_* is however ambiguous[8].

For the DSE to be feasible, the extraction rate of the NR defined by (26) must satisfy (22), which implies:

$$\tau < \frac{1}{\mu}\left[1 - \frac{a\mu}{\beta\gamma}\right] \tag{29}$$

The term between brackets must be positive, which imposes that $a\mu < \beta\gamma$: households' discount factor must be high enough and the returns to scale at the firm level must not be too decreasing. Furthermore, if the previous inequality is satisfied, the tax rate on NR must not be too high. Indeed, as the tax discourages savings and investment, a too high tax rate would imply insufficient savings to maintain output at its equilibrium level.

2.3 The planned economy

Proposition 4 *(a) The optimal stationary equilibrium (OSE) is unique and characterized by the following values:*

$$\begin{aligned}
\sigma\eta\left[1 - \frac{a\mu}{s_0}\right]\frac{R}{\mu} &= \frac{1}{1-s_o}\left[1 - \frac{s_0^2}{\beta_p a\mu}\right] \\
z_o &= \frac{1}{1-s_o} \\
E_o &= 1 - \frac{a\mu}{s_o} \\
Y_o &= \frac{E_o R}{\mu} = \left[1 - \frac{a\mu}{s_o}\right]\frac{R}{\mu} \\
C_o &= \frac{Y_o}{z_o} = [1-s_o]\left[1 - \frac{a\mu}{s_o}\right]\frac{R}{\mu}
\end{aligned} \tag{30}$$

(b) $s_o \in \left]a\mu, \sqrt{\beta_p a\mu}\right]$
(c) The OSE belongs to the feasibility domain if the following inequality is satisfied:

$$a\mu < \beta_p \tag{31}$$

See proof in Appendix B.2.

Unfortunately (30) does not admit an explicit solution so that s_0 must be computed numerically. Knowing s_0, all other values characterizing the OSE can be determined.

It is easy to show that $s_o \in \left]a\mu, \sqrt{\beta_p a\mu}\right]$. This last property is useful to show that the OSE is unique. It is also useful to obtain (31), which means that the discount factor of the central planner should not be too small for the OSE to be feasible.

The OSE can be decentralized if there exists a tax rate that allows a decentralized economy to reach it, in other words if:

$$\exists\tau \in \left[0, \frac{1}{\mu}\right] : s_* = \alpha = \beta\gamma\left[1 - \mu\tau\right] = s_o \tag{32}$$

Then:

$$\tau_o = \frac{1}{\mu}\left[1 - \frac{s_o}{\beta\gamma}\right] \tag{33}$$

[8] Given the previous subsection, C_* is a function of α on the interval $]a\mu, 1]$. As C_* is positive on the interval and nil at the two boundaries, it must be a non monotonic function of α.

where s_o is the solution of (30). A necessary and sufficient condition that ensures (32) is that $s_o < \beta\gamma$. This inequality is not very helpful because s_o is not known explicitly. However we know by Proposition 4 that (i) $s_o \in \left]a\mu, \sqrt{\beta_p a\mu}\right]$ and (ii) s_* decreases from $\beta\gamma$ to $a\mu$ when τ increases from 0 to $\frac{1}{\mu}\left[1 - \frac{a\mu}{\beta\gamma}\right]$ (see (24)). Accordingly a sufficient condition for (32) to be satisfied is $\sqrt{\beta_p a\mu} < \beta\gamma$ or equivalently

$$\beta_p < \frac{\beta^2\gamma^2}{a\mu}$$

In words, the authorities should not have a too high discount factor if they want to decentralize the OSE.

3 Dynamics

3.1 The baseline scenario

In the particular case where α_t is constant (which requires that the tax rate per unit of final good $\mu_t\tau_t$ is constant), then (15) leads to:

$$z_t = \frac{1 - \alpha^{T_f+1-t}}{1-\alpha}. \tag{34}$$

Furthermore if the time horizon is infinite ($T_f \to \infty$) and given that $\alpha < 1$, z_t is constant and

$$z_t = \frac{1}{1-\alpha}. \tag{35}$$

In this case, $C_t = [1-\alpha]\,Y_t$ and the savings rate is constant and equal to $\alpha = \beta\gamma\,[1-\mu\tau]$ [9]. α is higher (i) the higher the discount factor of household β and (ii) the lower the elasticity of capital to production at the firm level $\left(\frac{y_t}{k_t}\frac{\partial k_t}{\partial y_t} = \frac{1}{\gamma}\right)$. Likewise α is an inversely proportional function of the tax rate per unit of final good $\mu\tau$.

Moreover, given (35), (18) becomes:

$$Y_{t-1} = \frac{a\mu}{\alpha}\frac{Y_t}{1 - \frac{\mu Y_t}{R}} \tag{36}$$

Then:

Proposition 5 *When the savings rate is constant, output is determined by the following expression:*

$$Y_t = \left[\left[\frac{1}{Y_1} - \frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}\right]\left[\frac{a\mu}{\alpha}\right]^t + \frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}\right]^{-1} \tag{37}$$

Proof. (36) may be rewritten as $\frac{1}{Y_{t-1}} = \frac{\alpha}{a\mu}\left[\frac{1}{Y_t} - \frac{\mu}{R}\right]$, or again $x_t = \frac{a\mu}{\alpha}x_{t-1} + \frac{\mu}{R}$, where $x_t = 1/Y_t$. The solution of the associated homogeneous equation is $c\left[\frac{a\mu}{\alpha}\right]^t$, where c is a constant of integration to be determined. A particular solution of the previous equation is $\frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}$, so that the general solution can be written as $x_t = c\left[\frac{a\mu}{\alpha}\right]^t + \frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}$. If $Y_t = Y_1$ in $t = 1$, then $c = \left[\frac{1}{Y_1} - \frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}\right]\frac{a\mu}{\alpha}$. Accordingly (37) follows. ■

In the Baseline Scenario (BS), there is neither technical progress nor resource tax. Figures 1.a-f illustrate the path of the principal variables for this scenario. The initial level of output

[9] It is important to underline that the constancy of the savings rate is not an assumption: it results from certain hypotheses of the model (logarithmic utility function and unitary depreciation rate of capital) and only holds when T_f is infinite.

Y_1 is assumed to be close to 0 and the output trajectory follows a logistic behavior (Figure 1.a). Because NR extraction is characterized by decreasing returns to scale, growth slows down and Y_t tends monotonically toward a stationary value Y_*. The inverse of the propensity to consume z_t is a constant in accordance with (35) and the same is true for the savings rate $s_t = \alpha$ (Figure 1.b-c). Accordingly, consumption $C_t = Y_t/z = [1-\alpha]\, Y_t$ follows the same pattern as output (Figure 1.d). By contrast, the instantaneous utility u_t behaves non monotonically because pollution increases with production (Figure 1.e). After a growing phase, u_t goes through a maximum and next decreases toward its stationary value u_*. In a laisser-faire framework, households benefit from a continuous increase of their consumption level, but the same does not apply in terms of utility because of the harmful impact of pollution.

The hump-shaped trajectory of the utility level described by Figure 1.e calls to mind the *threshold hypothesis* formulated by Max-Neef [12], which states that above a certain level of GDP per capita, welfare (or quality of life) is likely to decline with economic growth. This hypothesis is confirmed empirically by Kubiszewski et al. [13]. Applying the Genuine Progress Indicator as a measure of welfare[10] to the period 1950-2005 for a set of countries including more than the half of world population, these authors have shown that this indicator reached its maximum in 1978 (corresponding to a GDP per capita around 7000$US of 2005) and declined (or remained capped) afterward.

3.2 The planned economy

(18) and (20) describe the behavior of the centralized economy. These equations must be solved numerically. Figures 1.a-f compare BS to a simulation V0 where the central planner manages the economy since the beginning ($T = 0$), using a discount factor equal to the one of households ($\beta_p = \beta$).

Starting with the same initial capital stock K_1, output Y_t first grows more quickly in V0 than in the case of the decentralized economy but it converges to a significantly lower stationary level (Figure 1.a). The initial value of the inverse of the propensity to consume z_t is very high reflecting a strong desire to grow at the beginning of the path (Figure 1.b). As a result, the initial consumption level C_1 is lower than in the decentralized economy (Figure 1.d). The same is true for the initial utility level u_1 since the pollution level is the same in the two scenarios BS and V0. We may thus say that the optimal policy of the central planner "sacrifices" the first generations of the dynasty. But the following generations enjoy a higher and monotonically increasing utility level thanks to the fact that the impact of pollution is taken into account (Figure 1.e).

The implementation of the central planner's choice in a decentralized framework implies a variable tax rate whose path is illustrated by Figure 1.f. During the first periods, this rate is negative: the public authorities must subsidize NR extraction[11]. Later, τ_t becomes positive and tends to the stationary value τ_o given by (33).

4 Optimal degrowth policies

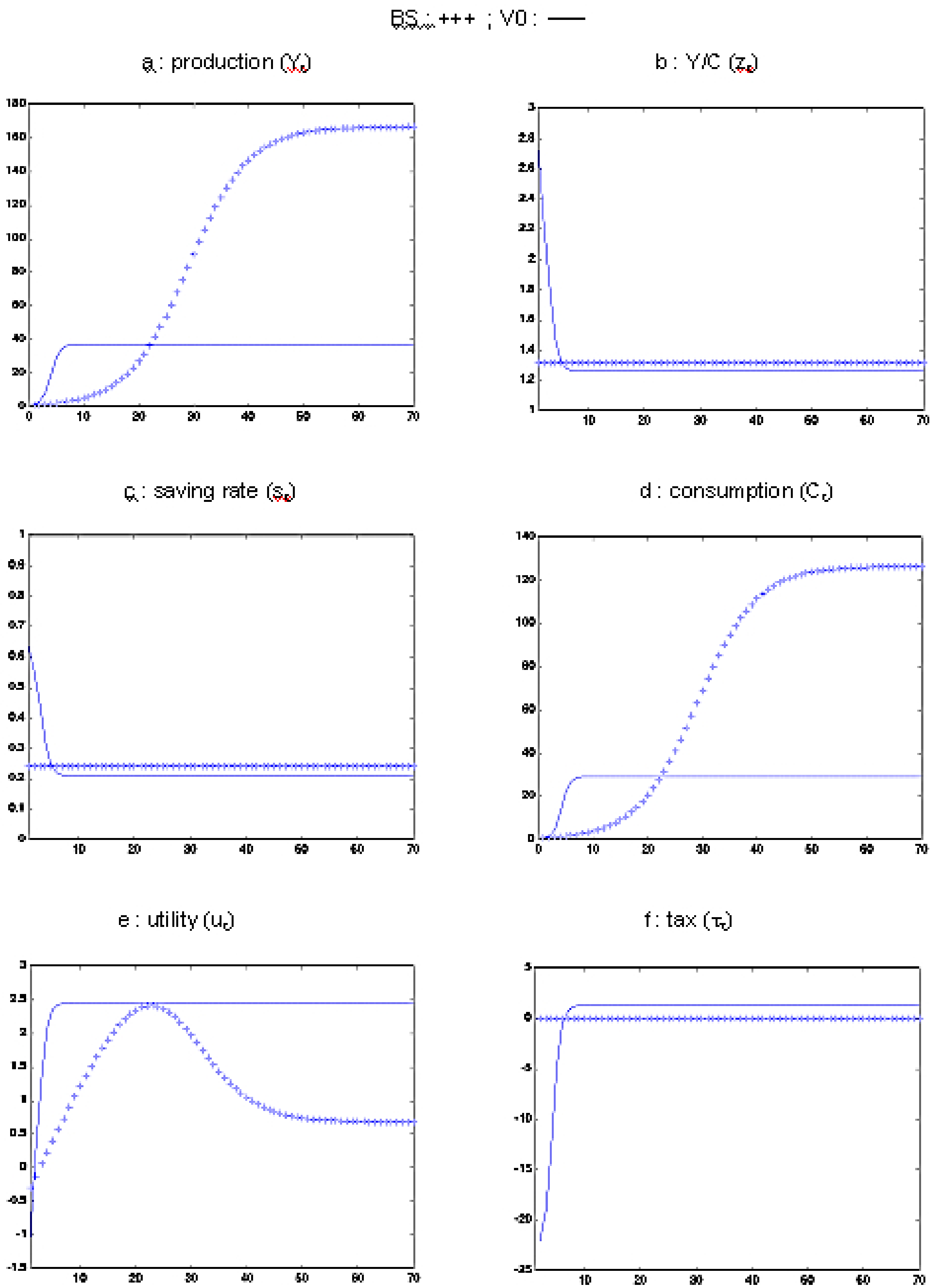
In this section, we study the economic impacts of *voluntary and optimal degrowth policies*. These policies rely on the use of a variable tax rate on resource extraction and are optimal, i.e. such that the tax rate is varied so as to replicate the optimal path chosen by the central planner (according to (19)).

These policies only make sense in a decentralized framework and public authorities implement them after observing a decrease in households' welfare. This implementation may take place several periods after the welfare peak. The analysis of the cause of such a delay goes beyond the scope

[10] This indicator, authored by Daly and Cobb [14], is obtained by correcting aggregate consumption via several positive or negative "adjustments", linked in particular to the distribution of income, environmental costs, domestic work... It is thus much wider than the concept of utility used here, which only depends on consumption and pollution levels.

[11] Because public finances are assumed to be balanced, the financing of the subsidy comes from a lump sum levy on households' income.

Figure 1: Comparison of simulations BS and V0



of this paper. We thus assume that the implementation delay is exogenous and analyze how its length impacts households' welfare. We also assume that the public authorities do not announce the degrowth policy so that it is not expected by households. This assumption has the merit to ease the model resolution since then the laisser-faire path (the one which precedes the public intervention) does not depend on the degrowth path (the one that follows the intervention). It can also be justified by the fact that the implementation of a degrowth policy is unprecedented and is thus surprise for households.

4.1 Comparison with the baseline

Figures 2.a-f compare BS with variant V1 where the public intervention takes place quite quickly after the utility peak (2 periods). W.r.t. BS, the tax implies a decrease in output Y_t already from $T+2$ (Figure 2.a). Indeed, the tax implementation decreases the firms' profitability, which reduces their demand for capital. Accordingly the rental price of capital decreases, which leads households to save less. The capital stock thus decreases, which implies in turn a decrease of output. Figure 2.d shows that it is also the case for consumption even though the investment cut initially allows a higher C_{T+1} (in V1 than in BS) at the initial level of production[12]. As shown by Figures 2.b-c, this positive variation of C_{T+1} (w.r.t. BS) leads to a noticeable decrease of z_{T+1} and s_{T+1} (w.r.t. BS), which is made possible by a sufficiently high tax rate at the start of the degrowth policy (see Figure 2.f). Later, the decrease in output (and thus in pollution) reduces the optimal tax rate and reduces the gap between the values of z_t, s_t and τ_t in V1 and in BS. As shown by Figure 2.e, the degrowth policy increases welfare in $T+1$ w.r.t. BS. In $T+1$, this welfare increase follows from the one in C_{T+1} (see above) at unchanged pollution level. For $t > T+1$, the variables tend monotonically to their respective stationary values, except u_t which first increases for a while, before decreasing monotonically to u_o. This "complex" behavior is understandable by the fact that utility depends of the difference between two variables (Y_t and C_t) that behave in the same way.

Figures 2.a and e show that the policy respects two main principles of a voluntary degrowth policy, namely a decrease in both output and pollution and a simultaneous increase in households' welfare (w.r.t. to the laisser-faire situation).

What happens if the authorities react later? Figures 2.a-f illustrate Variant V2 where the intervention of the authorities starts ten periods later than in V1. The results are quantitatively but not qualitatively different. As the authorities react later, production and consumption have increased more before the implementation of the tax (see Figures 2.a and d). The stationary equilibrium being the same for V1 and V2, the downward correction is accordingly stronger. This is also true for z_{T+1} and s_{T+1}, (see Figures 2.b and c). This is only possible via the imposition of a higher tax rate at the beginning of the intervention compared to V1 (see Figure 2.f).

Given that the authorities react later, households' utility has decreased more strongly before the intervention (see Figure 2.e). For the same reasons as in the previous paragraph, the ulterior utility rise toward the same stationary utility level as in V1 is thus stronger in V2. In the framework of our simple model, the time of tax implementation has no long run impact and leads to the same stationary equilibrium. However, in the short term, a later intervention implies a more aggressive degrowth policy with a higher initial tax rate.

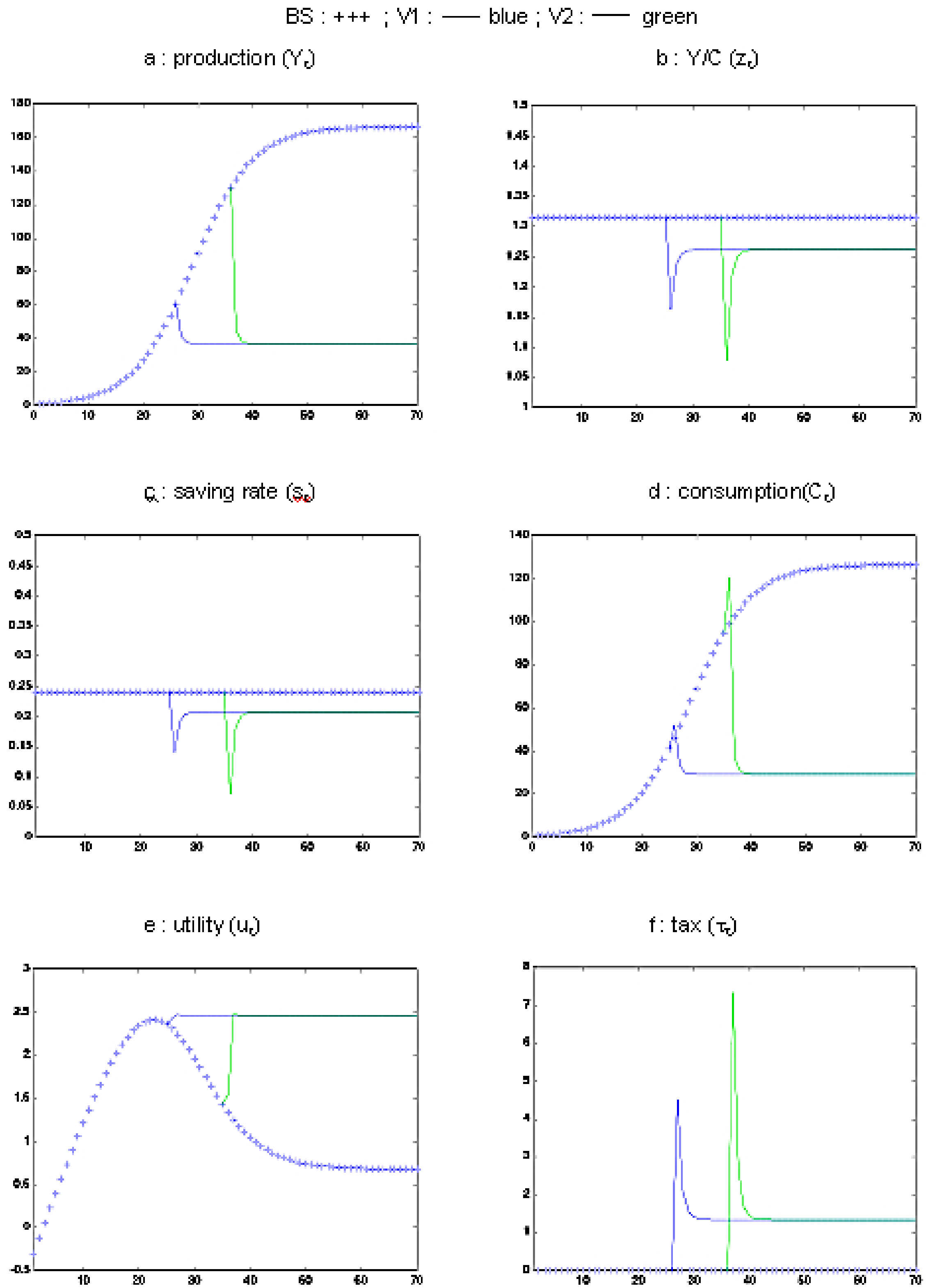
4.2 The impact of technical progress

The above results rely on the assumption that there is no technical progress, i.e. μ_t and η_t are constant. This assumption could a priori be considered as rather restrictive. However, as Germain [15] has shown in a companion paper, it does not change fundamentally the story. Hereafter we summarize his main conclusions w.r.t. two variants of technical progress.

In a first variant, technical progress reduces the resource intensiveness of output. However this technical progress is bounded in the sense that μ_t declines through time from μ_1 to $\mu = \lim_{t \to +\infty} \mu_t > 0$, where μ_1 and μ are respectively the initial and final values of the resource

[12] In $T+1$, output is determined by the capital stock inherited from the past (i.e. before the intervention of the public authorities). Thus $Y_{T+1}^{BS} = Y_{T+1}^{V1}$.

Figure 2: Comparison of simulations BS, V1 and V2



content of one unit of good. In a laisser-faire framework and w.r.t. to the situation where technical progress is absent, the (instantaneous) utility of households u_t grows initially more quickly, attains its peak sooner and finally decreases to a lower stationary equilibrium. Resource saving technical progress is thus beneficial only temporarily. It becomes deleterious in the long term because of a rebound effect implying an increase in output and pollution and because agents do not take care of the externalities linked to pollution. On the contrary, the implementation of a degrowth policy by the public authorities enables the economy to converge to a stationary equilibrium characterized by a higher utility level w.r.t. the level obtained in the absence of technical progress. In the long term, the impact of a resource saving technical progress is thus positive or negative depending on the institutional framework (i.e. on whether the public authorities "laissent faire" or regulates the economy).

In a second variant, technical progress makes production less polluting. Parameter η_t declines through time from η_1 to $\eta = \lim_{t\to+\infty} \eta_t > 0$, where η_1 and η are respectively the initial and final pollution levels per unit of good. For technological reasons or because of a complete treatment of pollution would have a prohibitive cost, η is strictly positive. Contrary to the case of a resource saving technical progress, improvements in the pollution treatment do not induce a rebound effect. W.r.t the situation where there is no technical progress, the (instantaneous) utility curve peaks at higher level and at a later period, before declining to a higher steady state. Thus in a laisser-faire economy, a technical progress that reduces pollution (ceteris paribus) has a positive impact both in the short and long terms. This does not prevent a degrowth policy to be beneficial by allowing the economy to reach a higher welfare level than in a laisser-faire economy. The welfare *gain* (w.r.t. to the laisser-faire situation) is however lower in the presence of a technical progress in the pollution treatment.

5 Sustainable degrowth policies

Optimality does not guarantee sustainability and vice versa (Bonneuil and Boucekkine [16])[13]. Now sustainability is a concept that "has been rather ambiguously defined" by the literature (Baranzini and Bourguignon [17], p.341). One possible definition relies on intergenerational equity in the sense that the welfare level of future generations should not be lower than the one of the contemporaneous generation. We exploit this idea hereafter.

Assuming an infinite horizon, intertemporal social welfare at time t is equal to:

$$V_t = \sum_{\theta=t}^{+\infty} \beta^{\theta-t} u_\theta \tag{38}$$

where u_t is the representative household's instantaneous utility defined by (13).

In our discrete time framework, the *Brundtland sustainability criterion* as formulated mathematically by Arrow et al. [18] stipulates that:

$$V_t \leq V_{t+1}, \ \forall t > T \tag{39}$$

In words, after the implementation of the degrowth policy from period T, social welfare as measured by V_t must not decrease over time.

To simplify our analysis, we assume that there is no technical progress so that μ_t and η_t are constant and equal respectively to μ and η. We have the following useful result.

[13] Using viability theory in the framework of the Ramsey growth model, Bonneuil and Boucekkine [16] characterize optimal viable, optimal non viable and viable optimal viable paths, where a path is said to be viable if it satisfies a minimum consumption condition coupled to a sustainability criterion.

Lemma 6 *A degrowth path such that*

$$u_t \leq u_{t+1}, \ \forall t > T \tag{40}$$

where

$$u_t = \ln\left(\frac{Y_t}{z_t}\right) - \sigma\eta Y_t \tag{41}$$

satisfies the Brundtland sustainability criterion (39).

See Appendix C for the proof.

(41) follows from (9), (11) and (13). Lemma 6 states that if after the implementation of the degrowth policy instantaneous utility is non decreasing over time, then the path of the economy is (Brundtland) sustainable. This result is useful because (40) is much easier to check than criterion (39).

Figure 2.e shows that the optimal degrowth policy analyzed in section 4 does not satisfy the criterion defined by (40). Indeed it does not lead to a monotonic evolution of households' instantaneous utility[14].

Hereafter we determine tax rates able to lead to *sustainable degrowth paths* (SDP) in a decentralized economy, i.e. paths that
(i) are governed by equations (17) and (18) (the economy is decentralized);
(ii) verify the sequence $Y_t \geq Y_{t+1}, \ \forall t > T$ (the path is a degrowth path);
(iii) verify the criterion (40) (so that the path is sustainable given Lemma 6).
Moreover α_t, z_t, Y_t must be positive along such paths.

First note that the sequence $Y_t \geq Y_{t+1}, \ \forall t > T$ is possible only if $Y_{T+1} \geq Y_*$, i.e. the initial level of production is higher or equal to the asymptotic level. Given (27), the last inequality implies that:

$$\left[1 - \frac{a\mu}{\alpha}\right]\frac{R}{\mu} \geq Y_{t+1} \tag{42}$$

where α is given by (24).

Characterizing all the SDPs that satisfy the above conditions is beyond the scope of this paper. In the sequel, we will limit our analysis to particular classes of SDPs. We proceed in two steps. We first determine SDPs that are generated by *time-constant* tax rates. We next determine SDPs that are generated by *time-increasing* tax rates.

5.1 SDPs with constant tax rates

If the tax rate is constant over time ($\tau_t = \tau, \ \forall t > T$) and if the horizon time is infinite ($T_f \to \infty$), we know (see subsection 3.1) that (i) the inverse of the propensity to consume is constant and given by (35) and (ii) the dynamic behavior of output is determined by:

$$Y_t = \left[\left[\frac{1}{Y_{T+1}} - \frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}\right]\left[\frac{a\mu}{\alpha}\right]^{t-(T+1)} + \frac{\mu}{R}\frac{1}{1-\frac{a\mu}{\alpha}}\right]^{-1}, \ \forall t > T \tag{43}$$

If (42) is satisfied, it is clear that Y_t decreases monotonically from Y_{T+1} to Y_*.

(43) implies directly a first result:

Lemma 7 *Consider two different tax rates τ^a and τ^b such that $\tau^a < \tau^b$. Then $Y_t^a > Y_t^b, \ \forall t > T$: the output path corresponding to the lower tax rate is everywhere above the one corresponding to the higher tax rate. In particular, the lower the tax rate, the higher the stationary output: $Y_*^a > Y_*^b$.*

Then the following result follows:

[14] The non monotonicity of u_t for simulations V1 and V2 is barely perceptible but is real. Now the non monotonicity of u_t could be more pronounced for other simulations. As the model is highly stylized, we pay more attention to the general shape of the trajectories than to the numerical values.

Proposition 8 *A degrowth path generated by a constant tax rate and satisfying the inequality* $Y_* \geq \frac{1}{\sigma\eta}$ *is sustainable.*

Proof. Given (41), we have $\frac{\partial u_t}{\partial Y_t} = \frac{1}{Y_t} - \sigma\eta \leq 0$ if and only if $\frac{1}{\sigma\eta} \leq Y_t$. Given that $Y_* = \min_{t>T}\{Y_t\}$, if $Y_* \geq \frac{1}{\sigma\eta}$ then $Y_t \geq \frac{1}{\sigma\eta}, \forall t > T$. Then, the sequence $Y_t \geq Y_{t+1}$, $\forall t > T$ and the fact that z_t is constant imply obviously (40). Then sustainability follows from Lemma 6. ■

In words, Proposition 8 states that a degrowth path generated by a constant tax rate must be characterized by a sufficiently high stationary output to be sustainable. On the contrary, if the inequality $Y_* \geq \frac{1}{\sigma\eta}$ is not satisfied, there is a period after which the decrease of output due to the (too high) taxation of the natural resource is no more compensated by the corresponding decrease of pollution, so that households' instantaneous utility decreases and the criterion (40) is not verified.

We now consider two particular SDPs. The first one, Λ_l, is characterized by the stationary output $Y^l = \frac{1}{\sigma\eta}$. Let α^l be the value of α that generates Λ_l. It is easily computable from (27): $\left[1 - \frac{a\mu}{\alpha^l}\right]\frac{R}{\mu} = \frac{1}{\sigma\eta} \Rightarrow \alpha^l = \frac{a\mu}{1-\frac{\mu}{\sigma\eta R}}$. Then using $\alpha = \beta\gamma\left[1 - \mu\tau\right]$, we obtain the tax rate that generates the path Λ_l :

$$\tau^l = \frac{1}{\mu} - \frac{a}{\beta\gamma\left[1 - \frac{\mu}{\sigma\eta R}\right]} \tag{44}$$

The second SDP, Λ_h, is characterized by a constant output level equal to Y_{T+1}, i.e. $Y_t = Y_{T+1}$, $\forall t > T$ [15]. Let α^h be the value of α that generates Λ_h. From (43), it follows that

$$\alpha^h = \frac{a\mu}{1 - \frac{\mu Y_{T+1}}{R}} \tag{45}$$

which is the particular value of α that makes (42) an equality. The corresponding tax rate is:

$$\tau^h = \frac{1}{\mu} - \frac{a}{\beta\gamma\left[1 - \frac{\mu Y_{T+1}}{R}\right]} \tag{46}$$

Proposition 9 *A constant tax rate* τ *such that*

$$\tau^h \leq \tau \leq \tau^l \tag{47}$$

generates a SDP.

Proof. Given Lemma 7, if $\tau^h \leq \tau$, then the path generated by τ is everywhere under (or equal to) the (constant) path Λ_h. It follows in particular that $Y_* \leq Y_{T+1}$, so that (42) is satisfied. Given (43), the path is monotonously decreasing and is thus a degrowth path.

Given Lemma 7, if on the other hand $\tau \leq \tau^l$, then the path generated by τ is everywhere above (or equal to) the path Λ_l and therefore satisfies the condition $Y_* \geq \frac{1}{\sigma\eta}$.

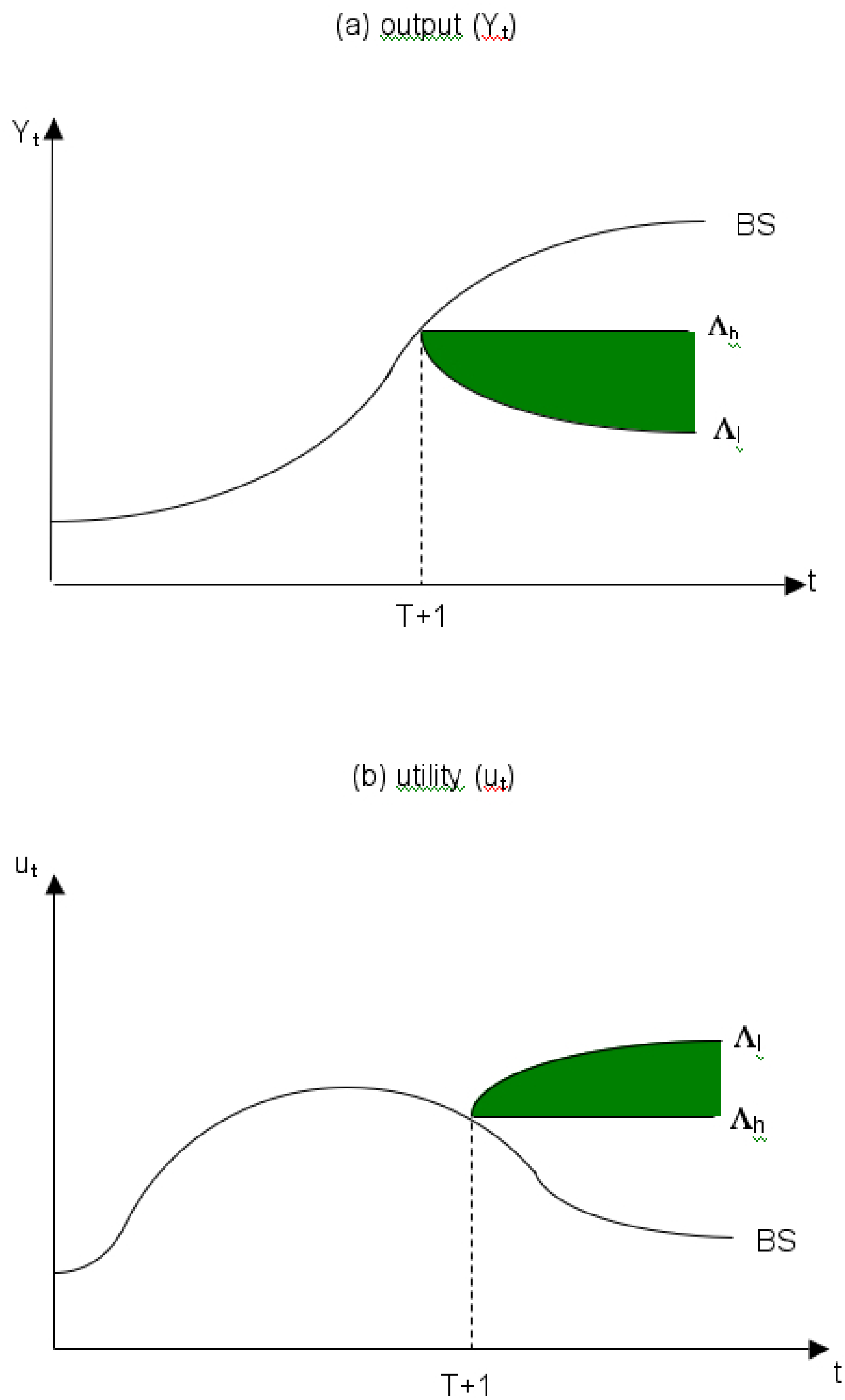
Then the path generated by τ is a SDP according to Proposition 8. ■

Figures 3.a-b illustrate output and utility characterizing the Baseline Scenario (BS) and the SDPs generated by a constant tax rate. As for Figure 2, when a public intervention occurs in $t = T + 1$, BS lasts from $t = 1$ to $t = T$ and is followed by a degrowth path. Starting with an initial output (resp. utility) level Y_{T+1} (resp. u_{T+1}), a SDP generated by an admissible constant tax rate (i.e. belonging to the interval $\left[\tau^h, \tau^l\right]$) is between path Λ_l and path Λ_h. Together, all admissible SDPs generate the green surfaces in Figures 3.a-b[16].

[15] The path Λ_h is indeed a SDP because z_t and Y_t constant ensure that u_t is constant (given (41)), so that (40) is satisfied.

[16] Contrary to Figures 1 and 2, Figure 3 is purely illustrative and does not rely on numerical computations.

Figure 3: Sustainable degrowth paths



5.2 Sustainable degrowth paths with increasing tax rates

For a degrowth path to be sustainable, the tax rate must not necessarily be constant. In the sequel, we characterize a class of SDPs that are associated to sequences of increasing tax rates that verify:

$$\tau_{T+2} \leq ...\tau_t \leq \tau_{t+1}... \leq \tau_* \tag{48}$$

where $0 < \tau_{T+2}$ and $\tau_* \in \left]0, \frac{1}{\mu}\right[$. τ_* is the asymptotic tax rate and the condition $\tau_* < 1/\mu$ ensures that $\alpha_t > 0, \forall t > T+1$.[17]

We have the following lemma:

Lemma 10 *A sequence of increasing tax rates such as (48) leads to the sequence:*

$$z_{T+1} \geq ...z_{t-1} \geq z_t... \geq z_* \tag{49}$$

where $z_* = \frac{1}{1-\alpha_*}$ (> 1).

Proof. Indeed (16) and (48) imply:

$$(1 >)\ \beta\gamma > \alpha_{T+2} \geq ...\alpha_t \geq \alpha_{t+1}... \geq \alpha_* \tag{50}$$

where $\alpha_* = \beta\gamma\left[1 - \mu\tau_*\right]$. (15) $\Rightarrow$

$$\begin{aligned} z_t &= 1 + \alpha_{t+1} + \alpha_{t+1}\alpha_{t+2} + \alpha_{t+1}\alpha_{t+2}\alpha_{t+3} + ... \\ z_{t-1} &= 1 + \alpha_t + \alpha_t\alpha_{t+1} + \alpha_t\alpha_{t+1}\alpha_{t+2} + ... \end{aligned}$$

Then obviously (50) $\Rightarrow$ (49). ■

Proposition 11 *If the initial tax rate τ_{T+2} and the final tax rate τ_* are chosen such that*

$$\tau^h \leq \tau_{T+2} \leq \tau_* \leq \tau^l \tag{51}$$

where τ^l and τ^h are defined by (44) and (46), a sequence of increasing tax rates such as (48) generates a SDP.

Proof. (i) The first step proves that a sequence of increasing tax rates verifying (48) and $\tau^h \leq \tau_{T+2}$ generates a degrowth path, satisfying:

$$Y_{T+1} \geq ...Y_{t-1} \geq Y_t... \geq Y_* \tag{52}$$

See Appendix D for the details.

(ii) The second step proves that the path is sustainable. First we know by Lemma 10 that (48) imply sequence (49).

On the other hand, given (24) and (27), $\tau_* \leq \tau^l \Rightarrow \alpha_* \geq \alpha^l \Rightarrow Y_* \geq Y^l$. Now τ^l generates the stationary output $Y^l = \frac{1}{\sigma\eta}$, so that $Y_* \geq \frac{1}{\sigma\eta}$.

Finally, given (41), we have $\frac{\partial u_t}{\partial Y_t} = \frac{1}{Y_t} - \sigma\eta \leq 0$ if and only if $\frac{1}{\sigma\eta} \leq Y_t$. Given that $Y_* = \min_{t>T}\left\{Y_t\right\}$, if $Y_* \geq \frac{1}{\sigma\eta}$ then $Y_t \geq \frac{1}{\sigma\eta}, \forall t > T$.

It is then clear that the sequences (49) and (52) imply (40), which ensures sustainability given Lemma 6. ■

The first inequality of (51) ensures that the path degrows from the beginning. The increasing sequence of tax rates defined by (48) shows that the degrowth policy is enhanced over time. But as shown by the last inequality of (51), it is not too much enhanced in order to avoid the detrimental effects of a too high tax rate on output (see the comment following Proposition 8).

[17] The sequence (48) starts indeed at $T+2$ and not at $T+1$ (first period of the degrowth phase) because z_t is function of α_{t+1} through (17) (and thus of τ_{t+1} given (16)).

6 Conclusion

This paper innovates by studying voluntary degrowth policies, which is rather unusual in the economic literature, in particular in growth theory. In the framework of a Ramsey growth model with natural resource and pollution and technological assumptions consistent with ecological economics, this paper analyzes the impacts of such policies on output, consumption and welfare. The instrument of these policies is a tax on the natural resource.

We begin by comparing the dynamics and final states of the economy when it is decentralized or governed by a central planner. In the decentralized case, economic agents do not care of the externalities present in the economy (in particular those which are linked to pollution). Output and consumption increase monotonically and converge to their respective stationary levels. Besides, households' utility first increases, reaches a peak and then declines due to the harmful impact of pollution. On the contrary, by taking all externalities into account, the central planner rules the economy in such a way that output, consumption and utility increase monotonically to their stationary levels. The asymptotic production (resp. utility) level is lower (resp. higher) than in the decentralized case.

We then study the implementation of optimal degrowth policies, i.e. voluntary degrowth policies that aim at replicating in a decentralized economy the optimal path chosen by the central planner. The path of the economy thus consists of two phases: (i) a first phase characterized by laisser-faire followed by (ii) a second phase generated by the degrowth policy. With respect to the laisser-faire situation, the main impact of these degrowth policies is to decrease production and pollution but to increase welfare. It is also shown that a later public intervention implies a more aggressive policy in the short term, in the sense that the tax rate must be higher in the first periods.

Unfortunately the above mentioned optimal degrowth policies appear to be unsustainable, at least on intergenerational equity grounds. Relying on a sustainability criterion that states that social welfare should be non decreasing over time, we study the implementation of sustainable degrowth policies in a decentralized economy. We first determine the interval of time-constant tax rates that generate sustainable degrowth paths. Secondly we extend the preceding set of solutions by characterizing a class of sustainable degrowth paths associated to increasing tax rates.

This paper relies on several assumptions that have the merit to simplify the model but which also limits its implications. Several extensions are thus possible and we mention three of them hereafter. First the natural resource as well as pollution have been modeled as flows. The model could be made more realistic by modeling them as stocks, which would also enrich the variety of possible paths of the economy. For example a decrease in output (and not only in utility) would then be possible in a laisser-faire framework (Germain [19]). A second extension could introduce consumption externalities as in Bilancini et D'Alessandro [5]. This would allow us to assess whether the degrowth policies considered here have positive impacts when social competition via consumption is present[18]. Finally it would be interesting to remove the assumption of a representative household in order to study the sustainability of degrowth policies from an intragenerational equity point of view.

7 References

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8 Appendix

A. Proof of Proposition 2

Given (19) and taking the logarithm of constraint (18), the Lagrangian writes:

$$\begin{aligned}\mathcal{L} &= \sum_{t=T+1}^{T_f} \beta_p^{t-[T+1]} \left[\ln(Y_t) - \ln(z_t) - \sigma\eta_t Y_t\right] ... \\ &\quad + \sum_{t=T+1}^{T_f-1} \lambda_t \left[\ln\left(1 - \frac{1}{z_t}\right) + \ln(Y_{t+1}) - \ln(a\mu_{t+1}Y_{t+1}) + \ln\left(1 - \frac{\mu_{t+1}Y_{t+1}}{R}\right)\right]\end{aligned}$$

where λ_t is the multiplier associated to the constraint at time t. To be acceptable, a path should verify $Y_t \geq 0$ and $z_t \geq 1$. We ignore temporarily these conditions and verify them numerically a posteriori.

First order conditions for an interior maximum lead to:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial z_t} &= -\beta_p^{t-[T+1]}\frac{1}{z_t} + \lambda_t \frac{1}{1-\frac{1}{z_t}}\frac{1}{z_t^2} = 0 \\ &\Rightarrow \lambda_t \frac{1}{z_t - 1} = \beta_p^{t-[T+1]} \\ &\Rightarrow \lambda_t = \beta_p^{t-[T+1]}\left[z_t - 1\right],\ t = T+1, ..., T_f - 1\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial z_{T_f}} = -\beta_p^{T_f-[T+1]}\frac{1}{z_{T_f}} < 0 \Rightarrow z_{T_f} = 1 \tag{53}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial Y_t} &= \beta_p^{t-[T+1]}\left[\frac{1}{Y_t} - \sigma\eta_t\right] + \lambda_{t-1}\left[-\frac{1}{Y_t} + \frac{1}{1-\frac{\mu_t Y_t}{R}}\frac{-\mu_t}{R}\right] + 1_{t<T_f}\lambda_t \frac{1}{Y_t} = 0 \\ &\Rightarrow \beta_p^{t-[T+1]}\left[\frac{1}{Y_t} - \sigma\eta_t\right] - \beta_p^{t-1-[T+1]}\left[z_{t-1} - 1\right]\left[\frac{1}{Y_t} + \frac{1}{\frac{R}{\mu_t} - Y_t}\right] + 1_{t<T_f}\beta_p^{t-[T+1]}\left[z_t - 1\right]\frac{1}{Y_t} = 0 \\ &\Rightarrow \beta_p\left[\frac{1}{Y_t} - \sigma\eta_t + 1_{t<T_f}\frac{z_t - 1}{Y_t}\right] = \left[z_{t-1} - 1\right]\frac{\frac{R}{\mu_t} - Y_t + Y_t}{Y_t\left[\frac{R}{\mu_t} - Y_t\right]} \\ &\Rightarrow \beta_p\left[\frac{z_t}{Y_t} - \sigma\eta_t\right]Y_t = \left[z_{t-1} - 1\right]\frac{1}{1 - \frac{\mu_t Y_t}{R}} \qquad (\Leftarrow (53)) \\ &\Rightarrow \beta_p\left[z_t - \sigma\eta_t Y_t\right]\left[1 - \frac{\mu_t Y_t}{R}\right] = z_{t-1} - 1,\ t = T+1, ..., T_f - 1\end{aligned}$$

where $1_{t<T_f} = 1$ if $t < T_f$ and 0 if $t = T_f$,

B. Stationary states

B.1 Proof of Proposition 3

(14) $\Rightarrow 1 = \beta v_* \Rightarrow$

$$v_* = \frac{1}{\beta}$$

(17) $\Rightarrow z_* - 1 = \alpha z_* \Rightarrow$

$$z_* = \frac{1}{1-\alpha} \tag{54}$$

$z_t = \frac{Y_t}{C_t}$ and (8) $\Rightarrow Y_t = \frac{Y_t}{z_t} + K_{t+1} \Rightarrow K_* = \left[1 - \frac{1}{z_*}\right] Y_*$ at the DSE. Given (54),

$$K_* = \alpha Y_*$$

At the DSE, $\alpha = \beta\gamma\left[1-\mu\tau\right]$ coincides with the savings rate.

(6) $\Rightarrow 1 - \frac{X_*}{R_*} = a\frac{X_*}{K_*} = a\frac{\mu Y}{\alpha Y} = \frac{a\mu}{\alpha} \Rightarrow$

$$E_* = 1 - \frac{a\mu}{\alpha}$$

which implies:

$$\begin{aligned} X_* &= E_* R = \left[1 - \frac{a\mu}{\alpha}\right] R \\ Y_* &= \frac{E_* R}{\mu} = \left[1 - \frac{a\mu}{\alpha}\right] \frac{R}{\mu} \end{aligned}$$

B.2 Proof of Proposition 4

(18) $\Rightarrow 1 - \frac{1}{z_o} = \frac{a\mu}{1-\frac{\mu Y_o}{R}} \Rightarrow 1 - \frac{\mu Y_o}{R} = \frac{a\mu}{1-\frac{1}{z_o}} \Rightarrow$

$$1 - \frac{\mu Y_o}{R} = \frac{a\mu z_o}{z_o - 1} \tag{55}$$

(20) $\Rightarrow z_o - 1 = \beta_p\left[z_o - \sigma\eta Y_o\right]\left[1 - \frac{\mu Y_o}{R}\right] \Rightarrow \beta_p\left[z_o - \sigma\eta Y_o\right] = \frac{z_o-1}{1-\frac{\mu Y_o}{R}} = \left[z_o - 1\right]\frac{z_o-1}{a\mu z_o} \Rightarrow z_o - \sigma\eta Y_o = \frac{\left[z_o-1\right]^2}{\beta_p a\mu z_o} \Rightarrow$

$$z_o - \sigma\eta\left[1 - \frac{a\mu z_o}{z_o - 1}\right]\frac{R}{\mu} = \frac{\left[z_o - 1\right]^2}{\beta_p a\mu z_o} \tag{56}$$

which allows us to compute z_0.

It is possible to rewrite (56) in terms of the savings rate s_0. (56) $\Rightarrow \frac{1}{1-s_o} - \sigma\eta\left[1 - \frac{a\mu}{s_o}\right]\frac{R}{\mu} = \frac{\left[\frac{s_o}{1-s_o}\right]^2}{\beta_p a\mu\frac{1}{1-s_o}} = \frac{s_o^2}{\beta_p a\mu\left[1-s_o\right]} \Rightarrow \sigma\eta\left[1 - \frac{a\mu}{s_o}\right]\frac{R}{\mu} = \frac{1}{1-s_o}\left[1 - \frac{s_o^2}{\beta_p a\mu}\right]$, and we obtain indeed (30).

Knowing s_0, all other values characterizing the OSE can be obtained in a similar manner as for the DSE.

Because the left-hand side of (30) is positive, the term between brackets at the right-hand side must be positive, which implies $s_o^2 \leq \beta_p a\mu \Rightarrow s_o \leq \sqrt{\beta_p a\mu}$. Given (23), the solution of (30) thus belongs to the interval $\left]a\mu, \sqrt{\beta_p a\mu}\right]$, which is result (b). To ensure that this interval is not empty, it is necessary that $a\mu < \sqrt{\beta_p a\mu} \Rightarrow \left[a\mu\right]^2 < \beta_p a\mu \Rightarrow a\mu < \beta_p$, which is (31).

Let us now show the uniqueness of the OSE. Let $LM(s_0) =_{def} \sigma\eta\left[1 - \frac{a\mu}{s_o}\right]\frac{R}{\mu}$ and $RM(s_0) =_{def} \frac{1}{1-s_o}\left[1 - \frac{s_o^2}{\beta_p a\mu}\right]$ be the left- and right-hand member of (30) respectively. Because of result (b), we study the behavior of these functions on the interval $\left]a\mu, \sqrt{\beta_p a\mu}\right]$, with $\sqrt{\beta_p a\mu} < 1 \Leftarrow a\mu < \beta_p \leq 1$. $LM(s_0)$ is a monotonically increasing function, nil for $s_0 = a\mu$ and equal to $\sigma\eta\left[1 - \sqrt{\frac{a\mu}{\beta_p}}\right]\frac{R}{\mu}$ for $s_0 = \sqrt{\beta_p a\mu}$. On the other hand $RM(s_0)$ is equal to $\frac{1}{1-a\mu}\left[1 - \frac{a\mu}{\beta_p}\right]$ for $s_0 = a\mu$ and nil for $s_0 = \sqrt{\beta_p a\mu}$. Because the two functions are continuous, they admit at least one intersection and thus at least one OSE exists.

We now show that there is only one intersection, by looking at the behavior of the derivative of $RM(s_0)$.

$$\begin{aligned} RM'(s_0) &= \frac{-\frac{2}{\beta_p a\mu}s_0}{1 - s_o} + \frac{1 - \frac{s_o^2}{\beta_p a\mu}}{\left[1 - s_o\right]^2} \\ &= \frac{-\frac{2}{\beta_p a\mu}s_0\left[1 - s_o\right] + 1 - \frac{s_o^2}{\beta_p a\mu}}{\left[1 - s_o\right]^2} \\ &= \frac{1}{\beta_p a\mu}\frac{s_o^2 - 2s_0 + \beta_p a\mu}{\left[1 - s_o\right]^2} \end{aligned}$$

The two roots of the numerator are $s_0 = \frac{2 \pm \sqrt{4-4\beta_p a\mu}}{2} = 1 \pm \sqrt{1-\beta_p a\mu}$. They are real because $\sqrt{\beta_p a\mu} < 1$. The bigger one is inadmissible because it is higher than 1. The lower one is also inadmissible because it is lower than $a\mu$. Indeed $1 - \sqrt{1-\beta_p a\mu} < a\mu \Leftrightarrow (0 <)\ 1 - a\mu < \sqrt{1-\beta_p a\mu} \Leftrightarrow 1 - 2a\mu + [a\mu]^2 < 1 - \beta_p a\mu \Leftrightarrow -2 < -a\mu - \beta_p$, which is true because $a\mu < \beta_p \leq 1$. So $RM(s_0)$ is monotonically decreasing on the interval. There is thus only one intersection and one OSE.

C. Proof of Lemma 6

Given (38), (39) may be rewritten as $V_t = u_t + \beta V_{t+1} \leq V_{t+1}$ or again:

$$u_t \leq [1-\beta] V_{t+1}$$

Now if criterion (40) is satisfied, then:

$$\begin{aligned}
u_t &\leq [1-\beta] \frac{u_{t+1}}{1-\beta} \\
&= [1-\beta] \left[1 + \beta + \beta^2 + ...\right] u_{t+1} \\
&= [1-\beta] \sum_{\theta=t+1}^{+\infty} \beta^{\theta-[t+1]} u_{t+1} \\
&\leq [1-\beta] \sum_{\theta=t+1}^{+\infty} \beta^{\theta-[t+1]} u_\theta \qquad (\Leftarrow (40)) \\
&= [1-\beta] V_{t+1}
\end{aligned}$$

D. Proof of step (i) of Proposition 11

First note that (18) $\Rightarrow$

$$\frac{1}{Y_t} = \frac{a\mu}{1 - \frac{1}{z_{t-1}}} \frac{1}{Y_{t-1}} + \frac{\mu}{R} \tag{57}$$

We first check that $Y_{T+1} > Y_{T+2}$. Given (57), this last inequality $\Rightarrow$

$$\begin{aligned}
\frac{1}{Y_{T+2}} &= \frac{a\mu}{1 - \frac{1}{z_{T+1}}} \frac{1}{Y_{T+1}} + \frac{\mu}{R} > \frac{1}{Y_{T+1}} \\
&\Leftrightarrow \frac{\mu Y_{T+1}}{R} > 1 - \frac{a\mu}{1 - \frac{1}{z_{T+1}}} \\
&\Leftrightarrow \frac{a\mu}{1 - \frac{1}{z_{T+1}}} > 1 - \frac{\mu Y_{T+1}}{R} \\
&\Leftrightarrow \frac{a\mu}{1 - \frac{\mu Y_{T+1}}{R}} > 1 - \frac{1}{z_{T+1}} \\
&\Leftrightarrow \frac{1}{z_{T+1}} > 1 - \frac{a\mu}{1 - \frac{\mu Y_{T+1}}{R}} \\
&\Leftrightarrow z_{T+1} < \frac{1}{1 - \frac{a\mu}{1 - \frac{\mu Y_{T+1}}{R}}}
\end{aligned} \tag{58}$$

Now (15) $\Rightarrow$

$$\begin{aligned}
z_{T+1} &= 1 + \alpha_{T+2} + \alpha_{T+2}\alpha_{T+3} + ... \\
&\leq 1 + \alpha_{T+2} + \alpha_{T+2}^2 + ... = \frac{1}{1-\alpha_{T+2}}
\end{aligned}$$

the last inequality resulting from (50). A sufficient condition for (58) to be satisfied would be $\frac{1}{1-\alpha_{T+2}} \leq \frac{1}{1 - \frac{a\mu}{1 - \frac{\mu Y_{T+1}}{R}}}$ or $\alpha_{T+2} \leq \frac{a\mu}{1 - \frac{\mu Y_{T+1}}{R}} = \alpha^h$ (recall (45)). Given that $\alpha_{T+2} = \beta\gamma[1 - \mu\tau_{T+2}]$

and $\alpha^h = \beta\gamma[1 - \mu\tau^h]$, this last inequality is equivalent to $\tau_{T+2} \geq \tau^h$, which is precisely assumed in (51).

We now check that $Y_{t-1} > Y_t \Rightarrow Y_t > Y_{t+1}$. (57) $\Rightarrow$

$$\frac{1}{Y_{t+1}} - \frac{1}{Y_t} = \frac{a\mu}{1 - \frac{1}{z_t}} \frac{1}{Y_t} - \frac{a\mu}{1 - \frac{1}{z_{t-1}}} \frac{1}{Y_{t-1}} > 0$$

This inequality is indeed satisfied because of $Y_{t-1} > Y_t$ and (49).