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The supply of non-renewable resources

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Abstract. There exists no formal treatment of non-renewable resource (NRR) supply, systematically deriving quantity as function of price. We establish instantaneous restricted (fixed reserves) and unrestricted NRR supply functions. The supply of a NRR at any date and location depends not only on the local contemporary price of the resource but also on prices at all other dates and locations. Besides the usual law of supply, which characterizes the own-price effect, cross-price effects have their own law. They can be decomposed into a substitution effect and a stock compensation effect. We show that the substitution effect always dominates: A price increase at some point in space and time causes NRR supply to decrease at all other points. Our new—although orthodox—setting takes into account not only NRR supply limitations, but also the heterogeneity of NRR deposits, and the endogeneity of their development and opening. Our analysis extends to NRRs the partial-equilibrium analysis of demand and supply policies. Thereby, it provides a generalization of results about policy-induced changes on NRR markets.

Résumé. L'offre de ressources non renouvelables. Aucune étude ne caractérise de manière formelle l'offre de ressources non renouvelables (RNRs), c'est-à-dire leur quantité produite en fonction de leur prix. Nous établissons les fonctions d'offre restreinte (à réserves données) et non-restreinte des RNRs. A chaque date et sur chaque marché, leur offre dépend non seulement du prix courant mais aussi du prix aux autres dates et sur les autres marchés. Au-delà de la loi de l'offre habituelle, qui caractérise l'effet prix propre, les effets prix croisés obéissent à leur propre loi. Ces derniers consistent en un effet substitution et en un effet compensation qui est lié au stock de ressource. Nous montrons que l'effet substitution domine toujours l'effet compensation : un accroissement du prix à une date sur un marché cause une réduction de l'offre d'une RNR aux autres dates et sur les autres marchés. Notre modèle est nouveau bien qu'il soit orthodoxe. Il tient compte non seulement de la disponibilité limitée des RNRs, mais aussi de l'hétérogénéité de leurs gisements et du fait que le développement et l'ouverture de ces derniers sont endogènes. Notre analyse étend l'analyse en équilibre partiel des politiques publiques au cas des RNRs. Ainsi, elle permet de généraliser des résultats existants quant aux effets des politiques publiques sur les marchés de RNRs.

JEL classification: Q38; D21; H22

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1 Introduction

The standard theory of supply treats price as exogenous. Curiously, such a treatment is missing for non-renewable resources (hereafter NRRs). We fill the gap in this paper, providing an analysis that can also be adapted to commodities that need to be produced before being allocated to various uses in space and time. The path of prices over time and their distribution in space are taken as parametric and we study the properties of supply functions, that is the effect on the quantity supplied at any date or location of a price change occurring at any date or location. This approach is highly orthodox. However, it is new and instructive.

The question of NRR supply has a long history and remains very contemporary. It was first addressed by Gray (1914) and formally undertaken by Burness (1976). They specifically inquired about the effect on the extraction path of changing the exogenous price, assumed to be constant throughout the extraction period. Sweeney (1993) later attempted to reconcile NRR supply with conventional supply theory, by deriving resource supply as a function of the contemporary producer price. He stated that "static supply functions, so typical in most economic analysis, are inconsistent with optimal extraction of depletable resources" (p. 780) but left the task unfinished.

This research is mainly motivated by the recent interest in the effectiveness of policies to reduce the use of carbon NRRs: more precisely, the effects across time and/or space of decisions implemented at various future dates and locations. First, the "green paradox" literature has examined taxation-induced intertemporal changes in NRR extraction. The initial contributions of Long and Sinn (1985), and Sinn (2008) assumed reserves to be homogeneous and given rather than heterogeneous and produced (by exploration and development). More recent papers relaxed these assumptions in various ways. For example, Hoel (2012), van der Ploeg and Withagen (2012a, 2012b, 2014) and Grafton, Kompas, and Long (2012), among others, considered that some part of the resource may be left unexploited after a demand-restricting policy change. Second, the "carbon leakage" literature has further considered interregional changes in NRR use. Fischer and Salant (2013), for example, extended the analysis of Eichner and Pethig (2011) to cases of incomplete NRR exhaustion. Third, some have focused on the NRR quantity left unexploited. Long-term effects of optimal carbon policies on "stranded NRR assets" have been examined by van der Ploeg and Rezai (2016). Harstad (2012) studied direct supply policies, showing that buying up and sterilizing NRR reserves will reduce consumption and emissions, whether supply is static as in the core of his paper, or dynamic as in his two-period extension (p. 97).

To address these currently important and complex issues, economists have hitherto focused on market equilibria responses, which combine supply and demand reactions, thus circumventing the preliminary separate treatment of supply and demand reactions which is customary in standard microeconomics. However, NRR demand being considered ordinary, NRR market equilibria responses are traditionally attributed to supply reactions.

By contrast, in this paper, we intend to identify general properties of NRR supply at various dates and locations—continuing the task undertaken by Gray (1914), Burness (1976), and Sweeney (1993). We then show how the obtained properties extend the partial-equilibrium analysis of demand and supply policies to NRRs, also illustrating how they can be used in the time-honored supply and demand diagram. The analysis has immediate applications to the green paradox, spatial carbon leakages and NRR extraction responses to various policies, including the response of stranded assets.

1.a *Our model of NRR supply and related literature*

In the textbook formulation of a supply function, producers take the price as given and choose production to maximize profits. Similarly, in the main part of this paper, supply at any given date and location depends on exogenous prices over time and at various locations. Such a highly orthodox approach is new for NRRs. The literature treats NRR prices as constant or as a single-parameter path, or alternatively studies the intersection of NRR supply and demand without any prior treatment of supply.

We examine a NRR that comes in a variety of deposits. For each deposit, the amount of exploitable reserves to be extracted needs to be previously developed at some cost: Producers first choose when and how much reserves are developed, and then how much to extract from developed reserves over time. As a matter of fact, NRR deposits not only differ by their size and their date of development and opening, but also by their geology, depth or quality, and location. This heterogeneity is reflected in the technologies underlying the cost of reserve development and the cost of extracting, transforming and transporting the production from each deposit.

This setup allows to study the flow of NRR supply at three basic levels: the contribution of each single deposit, aggregate supply, and the allocation of the latter to each outlet. At each level, we characterize supply as function of prices, and study their most fundamental properties. We also distinguish between short-run and long-run supply according to whether reserves are restricted or can be endogenously adjusted (McFadden, 1978).

Therefore, this model offers a highly general representation of NRR supply, combining three central aspects of NRRs that have given rise to a number of important contributions: ($\imath$) the multiplicity and heterogeneity of NRR sources, ($\imath\imath$) the endogeneity of the quantity developed and exploited from these sources, and ($\imath\imath\imath$) the endogeneity of deposits' opening. The following discussion explains how our model of NRR supply relates to other existing treatments.

Heterogeneity of deposits. NRRs often exist in various deposits, which differ by their size, location, geology, including the form that the resource takes. We give due and careful consideration to resource heterogeneity. The output supplied will be assumed homogenous, but the conditions of the resource development and extraction may vary, especially due to variations in the accessibility or quality of the geological reserves.

There are two main alternative approaches that have been used in the literature to deal with such a resource heterogeneity. One is to consider that the cost of resource production increases with the past cumulative extraction; this idea was already present in Hotelling

(1931), then addressed by Gordon (1967), and further perfected, among others, by Weitzman (1976) and Salant, Eswaran, and Lewis (1983).

Progressively rising extraction costs were introduced to refine the microeconomic representation of deposits' exploitation for some NRRs and extraction techniques. According to Gordon (1967), they are particularly relevant when the structure of a deposit requires that the resource of the highest quality or best accessibility be extracted first, before reaching worse units as far as quality and accessibility are concerned—e.g., metal mining—and when the pressure from remaining reserves within a deposit facilitates extraction from this deposit—see, e.g., the empirical evidence presented in Anderson, Kellogg, and Salant (2018) on the role of intra-deposit pressure in oil drilling.

This approach is often assumed to be also valid at the industry level—that is, across deposits rather than within. It has been used, for example, to model the aggregate supply of fossil fuels in the recent literature that investigates the green paradox. However, the increase in the industry's costs with cumulative extraction is controversial because it assumes that the relevant stock of reserves is the industry's aggregate and that, within the aggregate reserves, least-cost resources are used first irrespective of the deposit from which they are extracted.[1]

The second approach to resource heterogeneity is to consider a multiplicity of different and independent deposits which may all contribute to aggregate supply. This representation may be attributed to Herfindahl (1967) and gave rise to a series of papers on the optimum sequence and possible overlap of deposit exploitation—see, e.g., Amigues, Favard, Gaudet and Moreaux (1998), Gaudet and Lasserre (2011), and Salant (2013).[2]

[1] As Slade (1988) put it "The idea that the least-cost deposits will be extracted first is so firmly embedded in our minds that it is an often-made but rarely tested assumption underlying the construction of theoretical exhaustible-resource models." (p. 189). See also her references. According to Livernois and Uhler (1987), "if the sign of the relationship between the aggregate reserve base and aggregate extraction costs is ambiguous, then these models have very little to predict about the nature of optimal exploration or the likely shape of price paths."

[2] The computable version of the integrated assessment model due to Golosov, Hassler, Krusell, and Tsyvinski (2014), for example, uses three deposits with different costs of extraction and no (reserve) stock effect on these costs. However, they also assume reserves to be exogenous.

In the main part of our analysis, we adopt the second approach to resource heterogeneity. On the one hand, it is less restrictive in the sense that it does not assume that total reserves are extracted in any particular pre-determined order. On the other hand, this multi-deposit approach emphasizes that, for most NRRs, resource heterogeneity is manifest in differences between deposits—differences in size, exploration and development costs, and extraction costs mostly—probably more than in within-deposit differences.

Moreover, we extend our analysis to take into account that some NRRs exhibit intra-deposit heterogeneity. In this extension, we assume that each deposit's extraction cost depends on its own cumulative extraction. Our analysis of price effects in this context allows to verify that the presence of stock effects does not imply aggregation issues, thus establishing the consistency of the first approach to model aggregate NRR supply with progressively rising costs.

Development of exploitable reserves. In general, the stock of reserves to be exploited does not become available without some prior exploration and development efforts. The discovery of reserves and their development as exploitable reserves is an important factor of NRR supply that reacts to NRR prices—see, for example, Arezki, van der Ploeg, and Toscani's (2016) empirical findings on oil discoveries (p. 23).

At the aggregate level, exploration and exploitation often take place simultaneously as in Pindyck's (1978) and Quyen's (1988) models[3]—see Cairns (1990) for a comprehensive survey of related contributions.

At the microeconomic level of a deposit, however, they occur in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). We, therefore, adopt the latter representation for each deposit.

Opening of deposits. A fundamental factor determining the supply of NRRs is the timing of deposits' openings. In the case of oil, for example, the importance of this aspect was emphasized by Venables (2014) and Anderson et al. (2018), who consider that oil supply

[3]This is also the case in Arrow and Chang (1982) who further deal with the uncertainty surrounding reserve discoveries. For simplicity, our analysis does not address this aspect.

results from a sequence of investments in new fields.[4] Our model gives due consideration to the fact that deposits' development and opening is a matter of choice.

As explained above, we also consider that producers' decision to open a deposit is linked with expenditures in exploration and development that determine the quantity of resource that can ultimately be extracted from each deposit. In an extension, moreover, we also consider that these decisions are affected by technical progress in the technologies of exploration and development of the resource.

1.b *Structure and principal results*

In Section 2, we progressively develop our model of NRR supply. When a deposit's exploited reserves are given, an exogenous price change occurring at any date entails a *pure intertemporal substitution* effect. At the same time, a change in the resource price path faced by producers may also affect the quantity of reserves developed, and exploited, from each deposit. We call this the *stock-compensation effect* and we show that the substitution effect dominates the compensation effect, even when the latter is accompanied by a change in the deposit's opening date.

Consider an increase in price at some particular date leaving prices at all other dates unchanged; the pure intertemporal substitution effect increases supply at that date and reduces supply at all other dates; the stock effect results in an increase in ultimately extracted reserves. It follows that supply at the date of the price rise increases as the stock effect and the substitution effect work in the same direction; this is the NRR version of the law of supply. At all other dates, since the substitution effect dominates, it follows that supply diminishes despite the stock effect; this effect may be called the law of intertemporal substitution in NRR supply.

When the time dimension is combined with a space dimension, the same result applies to

[4]These studies do not, however, establish conventional oil supply as a function of oil price parameters. Venables (2014) assumes that the oil price grows at a constant rate. Anderson et al. (2018), for empirical purposes, only consider the spot price as a parameter, assuming that future prices are randomly distributed accordingly.

a price change occurring at a point in space and time. Thus, the intertemporal substitution effect is accompanied by an analogous spatial substitution effect: A price rise at any point in space and time reduces supply at all other points.

In the same section, we explain how our model and results differ both from standard treatments of NRR and from the conventional supply analysis of non-resource commodities.

It is customary to use the apparatus of supply and demand to study policies; this is the realm of partial-equilibrium analysis. Applying this apparatus to NRR markets requires taking into account the intertemporal nature of NRR supply and its properties. In Section 3, we explain how the properties of NRR supply established in Section 2 extends the partial-equilibrium policy analysis to NRRs. We provide two examples, involving a carbon NRR: first, demand taxation and the green paradox, and, second, reserve policies targeting stranded carbon assets.

In Section 4, we discuss three other aspects—with detailed analytical extensions in the Appendix—that affects some of our results. First, we consider technical progress in the technologies of exploration and development and show that our results carry over for an initial period of time in this context. Second, we assume resource heterogeneity not only across but also within deposits. In this context, a tedious analysis shows that the law of cross-price effects holds over cumulative quantities. The third extension addresses investments in extraction capacities.

We conclude by putting the results in perspective, reiterating their theoretical and policy relevance, and highlighting their applicability to all commodities that must be produced prior to being dispatched in space or time.

2 A model of non-renewable resource supply

There are various possible NRR sources $j = 1, ..., J$ (deposits, developed or not) that may contribute to the supply of a unique homogenous resource. Let $x_t^j \geq 0$ denote the quantity produced from deposit j at each of a countable set of dates $t = 0, 1, 2, ...$ Aggregate supply at each date t is

$$x_t = \sum_{j=1}^{J} x_t^j. \tag{1}$$

Deposit j may only come into production at and after the date $\tau^j \geq 0$ at which it is developed:

$$x_t^j = 0,\ t < \tau^j. \tag{2}$$

For simplicity, these deposits' opening dates are considered exogenous at this stage.[5] It is further assumed that $\tau^j = 0$ for at least one deposit.

Each source j is constrained by its own finite initial stock $X^j > 0$ of the resource. Since the resource is non renewable, the following exhaustibility constraint applies to the production of each deposit j:

$$\sum_{t \geq 0} x_t^j \leq X^j. \tag{3}$$

The present-value cost of producing a quantity x_t^j from deposit j is denoted by $C_t^j(x_t^j)$, where the function C_t^j may be time varying and deposit specific, is increasing and twice differentiable, and satisfies $C_t^{j\prime\prime}(x_t^j) > 0$. For simplicity, we also assume[6] $C_t^j(0) = 0$.

The stock of reserves to be exploited following a deposit's opening does not become available without some prior exploration and development efforts. For each deposit j, it is assumed that the development of initial exploitable reserves X^j is instantaneous and undertaken only once, at date τ^j, at a present-value cost $E^j(X^j)$; the E^j function is twice differentiable, increasing, strictly convex, and satisfies $E^j(0) = 0$ and, at least for one deposit, $E^{j\prime}(0) = 0$. The latter property that the marginal cost of reserve development is zero at the origin for some j is introduced because it will be is sufficient to ensure that a positive amount of reserves is developed; otherwise $E^{j\prime}(0) \geq 0$, so that a resource whose marginal exploration and development cost is too high for profitability does not need to be developed.

[5]The analysis will be extended to endogenous opening dates further below in this section.

[6]The qualitative results follow through in presence of a fixed cost or lump sum tax.

As a result, deposits differ by their size X^j, their geology, location and depth or quality, as reflected in the technologies underlying both extraction costs C_t^j and exploration costs E^j.

We will first establish the properties of NRR supply in the simplest and standard case in which the resource is supplied to a single outlet. This includes the well-known Hotelling case in which the resource is produced from fixed (restricted) reserves. The endogeneity of reserves and the dispatching of supply to a multiplicity of outlets will be examined thereafter.

2.a *NRR supply to a single outlet*

There is a single resource outlet at which the present-value producer price is denoted by $p_t \geq 0$. The stream of prices $p \equiv (p_t)_{t\geq 0}$ is taken as given by the producers and treated as exogenous at this stage.[7] We assume that $p_t > C_t^{j\prime}(0)$ for each deposit $j = 1, ..., J$ for at least one date, so that, the extraction of exploitable reserves, if any, is warranted.

Clearly, the problems of producers exploiting deposits $j = 1, ..., J$ are independent from each other. Consider any particular deposit j. Since the development of reserves is costly, the optimum plan of the producer will always bind the exhaustibility constraint (3). In other words, leaving part of any deposit's developed stock ultimately unexploited does not maximize cumulative net discounted revenues. For a given price sequence p, the cumulative value function corresponding to the optimum of the producer exploiting deposit j is

$$\max_{(x_t^j)_{t\geq 0}, X^j} \sum_{t\geq 0} \left(p_t x_t^j - C_t^j(x_t^j)\right) - E^j(X^j) \tag{4}$$

subject to (2) and to the binding exhaustibility constraint

$$\sum_{t\geq 0} x_t^j = X^j. \tag{5}$$

[7] In Section 3, we will examine the implications of our results in a partial-equilibrium setting in which prices are endogenously determined on markets.

Denoting by λ^j the Lagrange multiplier associated with constraint (5), the necessary first-order conditions characterizing the optimum extraction path are

$$p_t - C_t^{j\prime}(x_t^j) \leq \lambda^j \text{ with } \left(p_t - C_t^{j\prime}(x_t^j) - \lambda^j\right) x_t^j = 0, \ \forall t \geq \tau^j. \tag{6}$$

At dates when extraction is strictly positive, we have

$$p_t - C_t^{j\prime}(x_t^j) = \lambda^j, \ \forall t \geq \tau^j, \ x_t^j > 0. \tag{7}$$

Otherwise, $p_t - C_t^{j\prime}(x_t^j) \leq \lambda^j$ for all $x_t^j \geq 0$, i.e., equivalently, $p_t - C_t^{j\prime}(0) \leq \lambda^j$, and $x_t^j = 0$; indeed, if the price is too low at some date, production may be interrupted before exhaustion, and start again once prices are high enough.[8] For the choice of initial reserves, the first-order condition is

$$E^{j\prime}(X^j) = \lambda^j. \tag{8}$$

Expression (7) is the Hotelling rule stating that the marginal profit from extraction must be constant in present value over the period of active exploitation, equal to λ^j, the unit present value of reserves underground, called the Hotelling scarcity rent. (8) is a standard supply relationship that sets marginal cost equal to price. The price in this case is the unit scarcity rent and is defined implicitly; in other words, reserves are the output of a production process whose technology is described by the cost function E^j. However, reserves are not like conventional goods that can be produced under constant returns to scale, because of the scarcity of exploration prospects. The supply of reserves is thus a strictly increasing function of the rent:[9]

$$X^j = X^j(\lambda^j) \equiv E^{j\prime -1}(\lambda^j). \tag{9}$$

The Hotelling rule (7) implicitly defines the solution of problem (4)-(2)-(5) as a series

[8]The condition for supply interruptions must also hold after exhaustion.

[9]The finiteness of exploration prospects amounts to a fixed factor being imposed on the production process. Hence reserves are produced under rising marginal costs.

of functions x_t^j giving extraction at each date when it is strictly positive: The function

$$x_t^j = x_t^j(p_t, \lambda^j), \ \forall t \geq 0, \tag{10}$$

can be defined at all dates. It is strictly increasing in the current price p_t and strictly decreasing in the rent λ^j at all dates when $x_t^j > 0$. At other dates, extraction is given by the same function, which then takes a zero value: all dates $t < \tau^j$ prior to the deposit's opening, if any, during supply interruptions, if any, and after exhaustion.

As Sweeney (1993) noted, functions like (10) can be interpreted as conventional static supply functions, whose sole arguments are the price p_t of the extracted resource and the reserve price λ^j. However, λ^j is not a conventional price; unlike standard price parameters, it corresponds to the "shadow" or implicit valuation of reserve units and, therefore, is endogenous to the resource producer problem. Consequently, formulating regular supply functions further requires expressing the rent λ^j as a function of the vector of exogenous prices p.

2.b *The benchmark Hotelling case: Restricted NRR supply with a single outlet*

The above model assumes that reserves are endogenous, to reflect that, in the long run, deposits' NRR capital can be increased by investment in exploration and development. In the short run, however, exploitable reserves are already established and, therefore, fixed. The resulting notion of supply is called restricted (McFadden, 1978) and its properties are now examined.

In fact, the restricted case of our model turns out to be the standard Hotelling description of deposits in which reserves are fixed. The properties of NRR supply are intuitive in this case and will be discussed further below.

Treating the stock of initial reserves as given at this stage and combining all relations (10) into (5), we obtain that deposit j's rent is a function increasing in all prices in $p \equiv (p_t)_{t \geq 0}$ and decreasing in stock X^j; we will denote that function with a tilde, and will do so for all functions of given reserves:

$$\lambda^j = \widetilde{\lambda}^j(p, X^j). \tag{11}$$

Note that $\widetilde{\lambda}^j$ is strictly increasing in p_t when extraction x_t^j is strictly positive. Otherwise a price change at a date when supply is zero may leave the rent unchanged. Substituting (11) into (10) gives the restricted supply functions, one at each date:[10]

$$x_t^j = \widetilde{x}_t^j(p, X^j) \equiv x_t^j\left(p_t, \widetilde{\lambda}^j(p, X^j)\right), \ \forall t \geq 0. \tag{12}$$

These functions do not make use of the first-order condition for initial reserves. Conditional on the initial reserve stock X^j and given the sequence p of prices, they determine how the supplier allocates extraction from the stock to different dates.[11]

The restricted NRR supply function $\widetilde{x}_t^j$ for any date $t \geq 0$ is increasing in X^j. Holding the reserve level unchanged, Appendix A examines the partial effects of prices, that is the *direct price effects*. It shows that a rise in price p_T at date T of the exploitation phase induces extraction to diminish at all exploitation dates $t \neq T$, and to increase at date T. The latter effect confirms that the law of supply obviously applies to restricted supply.

Our results about restricted supply from an individual NRR deposit, and their immediate implications for aggregate restricted NRR supply, are summarized in the following proposition.

Proposition 1 (Restricted NRR supply to a single outlet)

1. (Stock effect) *An exogenous rise in exploitable reserves X^j increases ($\imath$) restricted*

[10] A standard restricted supply function depends on the output price, on the prices of variable factors, and on the quantity of at least one restricted factor. Here, variable-factor prices are the prices of the factors entering the extraction technology, omitted from the notation for simplicity, and the restricted factor is the initial stock of reserves.

[11] Moreover, Hotelling's lemma is obtained from the optimized value function by use of the envelope theorem for constrained problems. That is, substituting (12) and (11) into the Lagrangian function associated with problem (4)-(2)-(5) and differentiating with respect to p_t, while holding the restricted level of X^j and its multiplier as well as all extraction rates constant, gives the restricted supply at t. Hotelling's lemma is obtained similarly in the case of non-restricted supply functions defined further below. The non-restricted value function is obtained by replacing the restricted level of X^j and the rent λ^j by their optimized values $X^{j*}(p)$ and $\lambda^{j*}(p)$ defined shortly below.

supply from each individual deposit at each of its exploitation dates, and, therefore, (ıı) aggregate restricted supply from all deposits at each date;

2. (Cross-price effects) *A price rise at any date T reduces (ı) restricted supply from each deposit at each of its exploitation dates $t \neq T$, and, therefore, (ıı) aggregate restricted supply from all deposits at each $t \neq T$.*

The effects just established are obviously non-zero if they involve dates when exploitation is active. To sum up, if $x_t^j > 0$, then restricted supply $\widetilde{x}_t^j(p, X^j)$ is strictly increasing in X^j and in p_t; if, furthermore, $x_T^j > 0$, $T \neq t$, it is strictly decreasing in p_T. Otherwise, stock or price changes may leave extraction unchanged, so that the above effects may in general be zero. The same will be true all along the paper for all effects that we will examine. For the sake of brevity, we will establish stock and price effects regardless of whether extraction is strictly positive or interrupted at the dates these effects involve, and, therefore, we will not repeat the above conditions under which they become non-zero.

2.c *Properties of supply in the Hotelling model*

The above restricted NRR supply corresponds to supply in the standard Hotelling model of NRR markets: In each deposit, reserves are established at a given level and are to be extracted over time. In this context, in particular, the resource literature has seldom considered exogenous price changes, and never in a systematic treatment of supply. One exception is Burness (1976) who forced prices to be constant in current value and investigated the effect on production of a simultaneous change in all prices.

Although proper supply functions have hitherto not been characterized and, therefore, examined in this case, their properties are now intuitive to economists of non-renewable resources and implicitly used to understand policy-induced NRR equilibrium market reactions. For example, to explain the green paradox policy-induced equilibrium change in NRR extraction, Sinn (2008) proposed the metaphor of a closed "pneumatic system of various pipes connecting various pistons" (p. 378): "If only one piston is pressed down, the others go up." Clearly, this metaphor applies even more implacably to the above supply setting than to Sinn's Hotelling market equilibrium setting. However, Sinn's analysis does not extend to unrestricted NRR supply, when reserves to be extracted are allowed to adjust. Accordingly, the green paradox has been questioned in setups in which reserves are endogenous, but only on the ground of particular cases.

As Section 3 will illustrate, these properties are fundamental to predict NRR market reactions to public policies. The remainder of this section aims at showing that the supply properties indicated by Proposition 1 carry over to much more general settings.

To start with, in the next subsection, we examine the case of unrestricted supply—endogenous reserves—allowing the properties of NRR supply to be compared with the supply of conventional goods that are not subject to supply limitations, unlike NRRs.

2.d *Unrestricted NRR supply with a single outlet*

Consider the choice of initial reserves. While (9) is a standard stock supply relation, deposit j's reserve price λ^j is not a standard exogenous price but an endogenous variable. The supply of reserves at the optimum of deposit j's producer in problem (4)-(2)-(5) can be expressed as a function of exogenous prices. The value of the unit rent at the producer's optimum satisfies $\lambda^j = \widetilde{\lambda}^j(p, X^j)$. By (9), the optimum amount of reserves satisfies $X^j = X^j(\lambda^j) = X^j\left(\widetilde{\lambda}^j(p, X^j)\right)$, which implicitly defines X^j and λ^j as functions of p:

$$X^j = X^{j*}(p) \text{ and } \lambda^j = \lambda^{j*}(p) \equiv \widetilde{\lambda}^j\left(p, X^{j*}(p)\right). \tag{13}$$

Thus, the supply of reserves depends positively on each of the whole sequence of resource prices, although this sequence can be summarized into one single rent.[12]

Restricted supply or factor demand as well as restricted cost or profit functions are

[12] As before, factor prices are omitted for notational simplicity from the reserve-supply function. They are the prices of the factors entering the extraction process because they affect the optimum rent, but also the prices of the factors entering the exploration and development process which are omitted arguments of the E^j cost function.

usually interpreted as representations of the short run. In the long run, the restricted factor is variable. This interpretation is adequate here, exploration and reserve development being analogous to capital investment. Just as capital goods are produced, reserves in (13) are the outcome of a production process. Then, they are used as a factor of production in the resource production process that generates the restricted supply (12).[13]

The optimal (unrestricted) NRR supply functions are defined as

$$x_t^{j*}(p) \equiv \widetilde{x}_t^j\big(p, X^{j*}(p)\big),\ \forall t \geq 0. \tag{14}$$

Appendix B presents the standard comparative supply analysis. Besides the usual law of supply that characterizes the own-price effect, our analysis establishes that, despite reserves' endogeneity, cross-price effects on extraction are systematically negative.

The following proposition summarizes the properties that are specific to NRR supply functions: the price effect on the stock, and the cross-price effect on extraction. Having shown these properties for each deposit, they follow through at the aggregate level.

Proposition 2 (Unrestricted NRR supply to a single outlet)

1. (Price effect on stock) *A price rise at any date T increases ($\imath$) unrestricted developed reserves from each deposit, and, therefore, ($\imath\imath$) aggregate unrestricted developed reserves from all deposits.*

2. (Cross-price effects) *A price rise at any date T reduces ($\imath$) unrestricted supply from each deposit at each of its exploitation dates $t \neq T$, and, therefore, ($\imath\imath$) aggregate unrestricted supply from all deposits at each $t \neq T$.*

[13] Although this is not usually modeled, capital does get depleted (worn out) by production at a rate that depends on the rate of production. However conventional capital can be replenished in a plant while this is not, or only partially, true of the reserves of a mine. On the related subjects of resource substitution and sustainability, see the huge literature initiated with the 1974 Symposium of the Review of Economic Studies.

2.e *NRR supply versus supply of conventional goods*

In the previous subsection, each deposit's unrestricted NRR supply corresponds to supply in the Gaudet and Lasserre's (1988) extension of the Hotelling setup to adjustable reserves. In this context, Proposition 2 indicates that the cross-price effect property of NRR supply, which is intuitive in the Hotelling model, extends to the case in which reserves can be adjusted—rather than fixed—as they are in the long run.

This case allows the comparison between NRR supply and the supply of conventional goods. Indeed, conventional goods in the classical theory of supply are producible without limit under conditions of constant returns to scale. By contrast, we have assumed decreasing returns to the development of reserves—increasing marginal cost of development, i.e., strict convexity of the cost function E^j. This assumption is inherent in the long-run production of NRR reserves as it reflects the finiteness of extraction and exploration prospects. It is also essential to the result. Suppose on the contrary that the development of reserves were subject to constant returns to scale, as would be the case for a conventional good first produced and then allocated to the different dates: $E^j(X^j) = e^j X^j$. As before, λ^j would give the present value of each reserve unit so that $\lambda^j = e^j$. The rent, thus determined by the technology, would then be insensitive to variations in prices p, and supply at t would only depend on current price by (12). Constant returns to scale in the development of X^j would make all cross-price effects on supply vanish, just like in the classical theory of supply under separable costs.

It follows that the supply of a NRR differs from a conventional supply function under identical standard technological assumptions of cost separability in that it not only depends on its own price, the current price, but also on the prices at all other dates. The comparative static properties of NRR supply are thus defined over a wider set of variables than those of a conventional supply function. With conventional supply functions, attention is usually limited to the law of supply, the effect of a change in the price of the good supplied.[14] With

[14] We ignore factor prices for simplicity.

NRRs, supply cross-price elasticities, the effect on supply at t of changes in prices at other dates, are also of theoretical interest: As the remainder of this section will show in a highly general model of NRR supply, they obey their own law. This law, moreover, is fundamental for the understanding of policy-induced equilibrium effects, as Section 3 will illustrate.

The decomposition of the change in NRR supply at t following a price change at $T \neq t$ into a pure substitution effect and a stock compensation effect may be reminiscent of the decomposition of Marshallian demand. However, these decompositions are certainly not isomorphic: In Appendix C, for the interested readers, we explain and graphically illustrate the difference.

2.f *Endogenous opening dates with a single outlet*

In fact, in the long run, producers not only choose their development efforts, but also the dates when these efforts are made. Assume now that each deposit j's development date τ^j is freely controlled by the producer: It may be zero, or, if it is an interior solution, is a strictly positive integer. Assume, moreover, that each deposit j's cost of reserve development $E^j(X^j)$ does not change with the development date τ^j.[15] The problem of the producer exploiting deposit j becomes

$$\max_{(x_t^j)_{t\geq 0},\tau^j,X^j} \sum_{t\geq 0} \left(p_t x_t^j - C_t^j(x_t^j)\right) - E^j(X^j) \tag{15}$$

subject to (2) and to the binding exhaustibility constraint (5).

The development of the resource may not be a necessity at early dates but become justified at later dates by the possibility of making a profitable exploitation. Here, however, an optimum extraction plan will not be constrained by the date of development of reserves to be extracted. Indeed, in absence of technical progress in reserve development, constraint (2) will never be active in the producer's optimum: If it were so for any development/extraction plan, the producer could obtain at least the same profit by producing the same amount of

[15]Technical progress in exploration and development will be examined in Section 4.

reserves at an earlier date, because this would imply the same present-value cost.

In this context, therefore, the development date of deposit j can be considered to be fixed at $\tau^j = 0$ without loss of generality. Since the development of reserves can take place at any sufficiently early date at no additional cost for producers, the model of this subsection turns out to be an extension of Herfindahl's (1967) influential multiple-deposit setup to the case of convex extraction costs and endogenous and costly reserve development.

Considering that $\tau^j = 0$ for all deposits $j = 1, ..., J$, the effects of a price change established in the previous subsection with exogenous development dates appear to follow through unchanged.

Proposition 3 (Unrestricted NRR supply to a single outlet with endogenous development dates)

1. (Price effect on stock) *A price rise at any date T increases ($\imath$) unrestricted developed reserves from each deposit, and, therefore, ($\imath\imath$) aggregate unrestricted developed reserves from all deposits.*

2. (Cross-price effects) *A price rise at any date T reduces ($\imath$) unrestricted supply from each deposit at each date $t \neq T$, and, therefore, ($\imath\imath$) aggregate unrestricted supply from all deposits at each $t \neq T$.*

However, the formulation of Propositions 2 and 3 hides a difference between the effect of a price change with fixed and endogenous development dates: In the context of this subsection, a price change may modify the date of development. Assume, for example, that the development/extraction plan for deposit j prior to the price change is such that reserves are developed at the first date where extraction is strictly positive. A future decrease in the price may cause extraction to become strictly positive at dates prior to the initial development date, requiring that the opening of the deposit and, therefore, its development be advanced. Whether development dates are endogenous or not, a price change at any future date always induces supply at earlier dates to move in the opposite direction.

2.g *NRR supply to many outlets*

For simplicity, we have focused so far on the dynamic interpretation of the NRR supply model. However, to the countable set of dates $t = 0, 1, 2, ...$ we may add a spatial dimension indexed by $l = 0, 1, 2, ..., \bar{l}$; in that formulation, the price p_{tl} is the present-value producer price at date t and location l. Location may then refer to a particular country or jurisdiction characterized by a particular price sequence, or an outlet commanding particular marketing efforts or transportation costs. The net spot revenue from selling the amount x_{tl}^j extracted from deposit j at location l at date t is $p_{tl}x_{tl}^j - c_{tl}^j(x_{tl}^j)$, where the function $c_{tl}^j(x_{tl}^j)$ gives the cost of selling specifically in location l at t the resource extracted from deposit j; it may be a transportation cost, a marketing cost, etc. It is assumed that c_{tl}^j is increasing and strictly convex. The net spot revenue from serving all locations $l = 0, ..., \bar{l}$ at date t is $\sum_{l=0,...,\bar{l}} \left(p_{tl}x_{tl}^j - c_{tl}^j(x_{tl}^j)\right) - C_t^j(x_t^j)$, where $x_t^j = \sum_{l=0,...,\bar{l}} x_{tl}^j$ is the quantity of NRR extracted from deposit j at date t and the function $C_t^j(x_t^j)$ gives the cost of extracting and otherwise processing this quantity before dispatching it to all locations $l = 0, ..., \bar{l}$. Like with a single deposit, we assume that $p_{tl} > C_t^{j\prime}(0) + c_{tl}^{j\prime}(0)$ for each deposit $j = 1, ..., J$ for at least one date and location.

The obtained model is a highly general model of NRR supply. It not only integrates a variety of heterogenous NRR sources and multiple outlets, in the spirit of Gaudet, Moreaux, and Salant's (2001) Hotelling extended setup, but also takes into account that deposits' reserves and opening dates are endogenous. In this context, the producer exploiting a deposit not only chooses the date of development/opening, his development efforts, and its extraction level, but also the dispatching of the latter to the set of outlets. In general terms, for a given matrix of prices, the cumulative value function corresponding to the optimum development, extraction and allocation of the resource for the producer exploiting deposit

j becomes

$$\max_{(x_t^j)_{t\geq 0},(x_{tl}^j)_{t\geq 0,l=0,\ldots,\bar{l}},\tau^j,X^j} \sum_{t\geq 0}\left(\sum_{l=0,\ldots,\bar{l}}\left(p_{tl}x_{tl}^j - c_{tl}^j(x_{tl}^j)\right) - C_t^j(x_t^j)\right) - E^j(X^j), \tag{16}$$

not only subject to (2) and to the binding exhaustibility constraint (5) as previously in this section, but also subject to the extraction-allocation constraint

$$\sum_{l=0,\ldots,\bar{l}} x_{tl}^j = x_t^j,\ \forall t\geq 0. \tag{17}$$

In this problem, like in previous subsections, control variables τ^j and X^j related to the development of the resource may be considered fixed in a short-run perspective, or free in the long-run. However, for the same reason as in Subsection 2.f, one can treat the opening date τ^j as exogenously set to zero without loss of generality: A producer can always advance the date of the deposit's development at no present-value cost; therefore, as far as supply reactions are concerned, the case of endogenous opening is similar to the case in which the opening takes place at date 0.

Defining υ_t^j as the Lagrange multiplier associated with the new constraint (17) and keeping the notation λ^j for the multiplier associated with the exhaustibility constraint (5), the necessary first-order conditions characterizing the producer's optimum are

$$p_{tl} - c_{tl}^{j\prime}(x_{tl}^j) = \upsilon_t^j,\ \forall t\geq 0,\ \forall l=0,\ldots,\bar{l}, \tag{18}$$

for the allocation of the production at any date t to all locations l, and

$$\upsilon_t^j - C_t^{j\prime}(x_t^j) = \lambda^j,\ \forall t\geq 0, \tag{19}$$

for the choice of extraction at date t. For simplicity, and with no consequence on the sign of the price effects to be established, we focus here on interior resource allocations in which

$x^j_{tl} > 0$. As far as the choice of initial reserves is concerned, it is determined by the same condition (8) as previously.

Condition (19) is the counterpart of (7) in the single-outlet model, in which the implicit value υ^j_t plays the role of the price at date t in the determination of the rate of extraction. υ^j_t can be interpreted as the after-extraction resource rent, i.e., the implicit value of the inventory to be dispatched. Condition (18) plays the role of allocating production to outlets by equalizing the contributions to υ^j_t from the various locations.

The analysis is presented in Appendix D: It unfolds in the same way as in the single-location treatment of Subsections 2.b-2.f, but in two stages rather than one. The following proposition summarizes the results obtained for a single deposit's reserves, extraction, and allocation of this extraction to each outlet. It also presents implications for aggregate supply from all deposits, which are straightforward.

Proposition 4 (NRR supply with multiple outlets)

1. (Stock effect) *A rise in exploitable reserves X^j increases ($\imath$) restricted supply from each individual deposit j at each of its exploitation dates and each location, and, therefore, ($\imath\imath$) total restricted supply from all deposits at each t and each l, as well as ($\imath\imath\imath$) aggregate restricted supply at each date t;*

2. (Price effect on stock) *A price rise at any date T and any location L increases ($\imath$) unrestricted developed reserves from each deposit, and, therefore, ($\imath\imath$) aggregate unrestricted developed reserves from all deposits, whether deposits' opening dates are endogenous or not.*

3. (Cross-price effects) *A price rise at any date T and location L reduces ($\imath$) restricted and unrestricted supply from each individual deposit j at date T at each location $l \neq L$, and at each date $t \neq T$ at each location l, and, therefore, reduces ($\imath\imath$) total restricted and unrestricted supply from all deposits at the same dates and locations, as well as*

(iii) aggregate restricted and unrestricted supply at each date $t \neq T$, whether deposits' opening dates are endogenous or not.

2.h *Summary for aggregate NRR supply*

In Section 3, we illustrate how the properties of NRR supply functions can be used to analyze policies in partial equilibrium. For ease of exposition, we do so using the properties of aggregate supply from the number of deposits, and consider that this supply serves a single outlet at which price is p_t at each date t. This means that we will focus on the intertemporal dimension of NRR supply, pushing into the background the allocation of extraction to various outlets.

From our previous results, however, it must be clear that aggregate supply to a single outlet reacts to a price change at any date in the same way as it reacts to a price change at any date at any of multiple outlets. The properties of aggregate NRR supply are summarized in the following corollary.

Corollary 1 (Aggregate NRR supply)

1. (Stock effect) *A rise in exploitable reserves X^j increases aggregate restricted supply at each date t.*

2. (Price effect on stock) *A price rise at any date T, whether supply serves a single or multiple locations, increases aggregate unrestricted developed reserves from all deposits, whether deposits' opening dates are endogenous or not.*

3. (Cross-price effects) *A price rise at any date T, whether supply serves a single or multiple locations, reduces aggregate restricted and unrestricted supply at each date $t \neq T$, whether deposits' opening dates are endogenous or not.*

As will be explained further below, other interesting applications in the spirit of Section 3 but involving the spatial dimension of various outlets can be carried out using the same methodology.

Only after Section 3, and still focusing on aggregate NRR supply for simplicity, we will consider more complex setups, showing how the form of the properties of NRR supply functions established above are modified.

3 Partial equilibrium, policy analysis, and examples of application

Having defined and characterized NRR supply functions in the standard way opens the field of all applications that rely on the demand-supply schedule, in particular the partial-equilibrium analysis of economic policies. Policy-induced changes are more complex than the above analysis of supply for two main reasons. First, policy often affects equilibrium prices indirectly, because it affects the demand for, or the supply of, the NRR. Second, policy-related price changes usually take place over an extended period rather than at a single date.

Two examples are provided below, involving a carbon NRR: one on the taxation of demand, raising the issue of the green paradox; one on a reserve-reduction policy targeting stranded carbon assets. The equilibrium is determined by the intersection of supply with demand. Our applications emphasize the intertemporal dimension of NRR supply, thus pushing into the background its spatial dimension:[16] Therefore, we will consider a single outlet as in Subsections 2.b to 2.f. Precisely, we will assume that supply functions are those given in (12) for the short run and (14) for the long run, and use their properties established in Propositions 1, 2 and 3. Moreover, the deposit index will be dropped to mean that these functions reflect aggregate NRR supply from all deposits.

The properties of NRR supply are used to assess the equilibrium effects of changing a policy at some date. The following applications are particular examples that illustrate more general partial-equilibrium properties of NRR markets. These general properties are established in Appendix E where they are summarized by Proposition 5. In the same appendix, the issue of policy duration and timing is covered in Corollary 2 while the appendix ends

[16] It is for simplicity that we do away with the spatial dimension: With many outlets, Corollary 1 verifies that aggregate NRR supply has the same properties as supply to a single outlet.

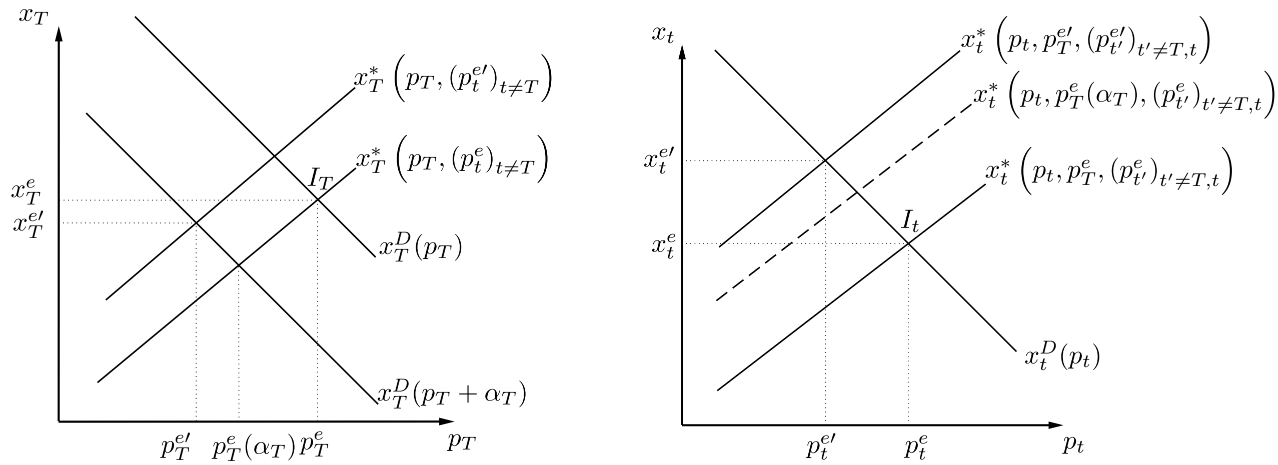
with a discussion of other possible applications of our results, for example, involving the spatial dimension of NRR supply.

3.a *An important application: Carbon taxation and the green paradox*

Assume that the demand for the NRR at date t is a function x_t^D that not only depends on the date, but is also decreasing and continuously differentiable in the consumer price. In this subsection, consider a tax α_t imposed on the use of the NRR at each date t. The tax may aim, for example, at penalizing the release of carbon emissions caused by the consumption of the resource. With the tax, the consumer price is $p_t + \alpha_t$ and the demand writes $x_t^D(p_t + \alpha_t)$. The path of the tax $(\alpha_t)_{t\geq 0}$ is exogenously given.

Assume that the tax is implemented at date T and only at that date, so that α_T rises from 0 to $\alpha_T > 0$ while $\alpha_t = 0$, $\forall t \neq T$; the partial-equilibrium effect is depicted in Figure 1 in which the horizontal axis represents the producer price p_t, exclusive of the tax. Prices p_t^e, $t \geq 0$, denote equilibrium prices with $\alpha_T = 0$; the corresponding market equilibrium is indicated by intersections I_t, $t \geq 0$. A producer price p_T being given, the implementation of the tax causes the demand at date T to shift down. Initially assuming that prices at dates $t \neq T$ are unchanged, date-T supply function does not change so that the drop in demand induces a move of date-T equilibrium down along the supply curve. The producer price p_T^e is reduced to $p_T^e(\alpha_T) < p_T^e$; quantity x_T^e is reduced accordingly (Figure 1(a)).

A lower producer price at T causes the supply curves at all other dates $t \neq T$ to shift up by Propositions 1, 2 and 3, i.e., in the short and long run, irrespective of whether deposits' opening dates are endogenous or not. This results in a drop in price at these dates. In turn, reduced producer prices at dates t', also different from T, further reinforce the upward shift in the supply curve at t (Figure 1(b)). Thus, fully adjusted prices $p_t^{e\prime}$ are lower and quantities supplied are higher at all dates $t \neq T$ than before the tax implementation, confirming the "green paradox" phenomenon. It follows that there is a feedback at T which shifts supply up to $x_T^*\left(p_T, (p_t^{e'})_{t\neq T}\right)$. This reinforces the initial drop in the price at T while also causing an increase in quantity (Figure 1(a)). However, this quantity response only partially offsets the initial downward reduction in quantity along the supply curve. Indeed, when reserves are fixed as in the short run, the increase in quantities at all dates $t \neq T$ implies that extraction must decrease at T; and in the long-run, following Section 2's analysis, exploitable reserves are lowered—stranded assets are increased—by the demand-reducing tax, so that extraction at T diminishes more than in the short run.



(a) Date T of policy implementation

(b) Any date $t \neq T$

Figure 1: Partial-equilibrium effects of demand reduction

This application illustrates how the general properties of NRR supply functions established in Section 2 can be used, for example, in the simple demand-supply schedule, to qualitatively assess policy-induced effects on NRR extraction. Specifically, it shows the generality of the green paradox phenomenon when ($\imath$) NRR supply results from the contribution of many deposits, ($\imath\imath$) allowing their developed and exploited reserves to adjust, ($\imath\imath\imath$) as well as their dates of opening.

Nevertheless, a quantitative assessment of the same policy effects obviously further requires that the effects be derived analytically, rather than graphically. For example, van der Ploeg (2016) examines the green paradox in a model that is comparable to, but more restrictive than, ours: There is a single deposit and two dates, the first date being the exogenous reserve development date, and the second date being when the policy is implemented. His results are in line with the above findings. Besides, his analysis allows to characterize the magnitude of the effect, depending on demand and supply elasticities, as well as the conditions under which the policy improves welfare (inclusive of environmental damages).

3.b *Another important application: Reserve policies and stranded carbon assets*

In this application, for simplicity, we consider that there is a single deposit[17] and that a supply policy aims at reducing exploitable reserves X from this deposit, for example, in the spirit of Harstad (2012), by buying up and sterilizing some of these reserves. For example, the policy aims at reducing the amount of a carbon NRR that will be ultimately exploited, or, equivalently, at increasing the amount of reserves ultimately left unexploited and called "stranded carbon assets." If initial exploitable reserves were fixed, resource supply would be directly given by restricted functions $\widetilde{x}_t(p, X)$ defined by (12)—the deposit index j is omitted—increasing in reserves according to Proposition 1. Thus any policy that reduces reserves from a level X to $X' = X - \Omega$ causes production to diminish at all dates. Partial-equilibrium implications turn out to be obvious, as illustrated in Figure 2 where point I_t represents the initial situation at date t. At all dates $t \geq 0$, instantaneous supplies $\widetilde{x}_t(p, X)$ meet instantaneous downward-sloping demands $x_t^D(p_t)$ and determine equilibrium prices p_t^e. For a change in exploitable reserves from X down to X' occurring at date zero, consider the market at any particular date $t \geq 0$. For unchanged prices $p_{t'}^e$ at dates $t' \neq t$, date-t supply is shifted down to $\widetilde{x}_t\left(p_t, (p_{t'}^e)_{t' \neq t}, X'\right)$, causing a rise in the equilibrium price. Since lower supplies at all other dates $t' \neq t$ similarly increase all equilibrium prices $p_{t'}^e$, supply curves at all dates are further shifted down. Fully adjusted equilibrium prices at all dates are $p_{t'}^{e\prime} \geq p_{t'}^e$, with equality if demand is infinitely elastic, and date-t supply curve becomes $\widetilde{x}_t\left(p_t, (p_{t'}^{e\prime})_{t' \neq t}, X'\right)$, lower than $\widetilde{x}_t\left(p_t, (p_{t'}^{e})_{t' \neq t}, X'\right)$.

When reserves are endogenous, reserve reduction may be partly compensated by the development of new reserves. In that case, the supply of exploitable reserves $X(\lambda)$ defined by (9) is shifted down to $X(\lambda) - \Omega$. For a more general supply-reduction policy

[17] The analysis accommodates many deposits and a reserve policy targeting one or all deposits.

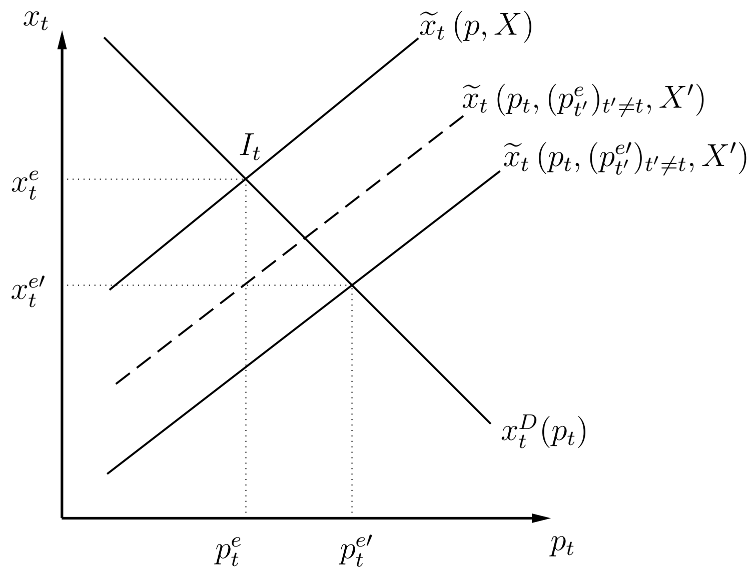


Figure 2: Partial-equilibrium effects of reserve reduction

whose stringency is indexed by Ω, the reserve supply function is redefined as $X\,(\lambda;\Omega)$, with $\frac{\partial X}{\partial \lambda} > 0$ and $\frac{\partial X}{\partial \Omega} < 0$.[18] Intuition suggests, and Proposition 5 will confirm, that an increase in the policy's stringency from Ω to $\Omega' > \Omega$ results in a greater equilibrium rent $\lambda^{e\prime} > \lambda^e$ and in lower equilibrium developed reserves $X^{e\prime} < X^e$, i.e., greater stranded carbon assets. Thus whether the reserve reduction is compensated (endogenous reserves) or not (restricted reserves), instantaneous supplies are reduced at all dates, and the partial-equilibrium implications of a rise in Ω are qualitatively the same as those of a fall in X described by Figure 2.

4 Other aspects of NRRs

The NRR supply model of Section 2 generalizes the framework of Hotelling (1931) and Herfindahl (1967) to take into account, besides the heterogeneity of NRR sources, three fundamental aspects considered important by more recent analyses: the determination of exploitable reserves by exploration and development, the timing of deposits' opening, and

[18] For example, suppose that exploitable reserves are reduced by an exogenous, policy induced, demand for reserves $X^D(\lambda;\Omega)$, decreasing in λ and increasing in the stringency index Ω. Then, the reserve supply function becomes $X(\lambda;\Omega) \equiv E'^{-1}(\lambda) - X^D(\lambda;\Omega) \leq E'^{-1}(\lambda)$.

the multiplicity of outlets. In this highly general model, NRR supply cross-price effects have proved to be robust: A price rise at any date induces a supply reduction at other dates.

For the sake of completeness, in this section—with details in the Appendix—we reexamine the above law of cross-price effects in presence of additional aspects of supply that are important for some NRRs. First, we consider technical progress in the development of deposits—see, for example, Managi et al. (2005) on improvements in the development of new oil reserves. Second, we assume intra-deposit heterogeneity, as when the cost of extracting the resource from a deposit increases with the deposit's depletion. For example, Anderson et al. (2018) show that deposits' pressure is an important factor driving oil extraction; they find, however, that the timing of deposits' opening is the most important determinant. Third, we consider that the extraction technology depends on prior investments in resource extraction infrastructure.

In this section again, for the sake of brevity, we focus on the intertemporal dimension of resource supply by assuming a single outlet.

4.a *Technical progress in NRR development*

Assume that each source $j = 1, ..., J$ is now characterized by its own exploration and development present-value cost $E_t^j(X^j)$, whose qualitative properties are the same as in Section 2, except that, for a given level of reserves X^j, both the cost $E_t^j(X^j)$ and marginal cost $E_t^{j\prime}(X^j)$ decrease over time. In this context, the development of a deposit's reserve need not be economic at early dates, but may become justified at later dates not only by high prices and/or low extraction costs, but also by technical improvements in the technology of reserve discovery and development. Obviously, the problem differs from Section 2 only in the long run, over which producers freely choose both the reserve development efforts and the dates τ^j when these efforts are made.

In this context, Appendix F examines the optimal choice of the date τ^j of development and opening. In general, with technical progress in reserve development, the producer is not indifferent as to the date when reserves are developed. Under standard conditions, we find that following a price rise at some date the planned opening of a deposit either remains unchanged or is modified in the direction of the date when the price rise occurs (Lemma 1). It follows that a price rise at any date $T > 0$ reduces aggregate unrestricted supply at each date $t \neq T$ of an initial period that extends at least until the date of the first postponed deposit development.

4.b *Stock effects or heterogeneity within NRR deposits*

As explained in the Introduction, for some NRRs and some extraction techniques, deposits' exploitation becomes increasingly costly with cumulative extraction. Assume that each source j can be exploited at a cost $C_t^j(x_t^j, X_t^j)$ that depends not only on extraction x_t^j in the same way as in Section 2, but also negatively on the remaining geological reserves X_t^j: Higher reserves means lower cumulative extraction, hence a lower cost.

Appendix H shows how the analysis is modified in presence of such an intra-deposit heterogeneity. The extension relies on restrictions due to Sweeney (1993), ensuring the consistency of the extraction cost function with our discrete-time model. For simplicity, the analysis assumes that dates of development/opening are fixed, but consider that the development of exploitable reserves is endogenous in the long run. Despite this simplification, the analysis turns out to be very tedious. Yet, the resulting Proposition 7 delivers a simple modification in the form of the results presented in Section 2: Negative cross-price effects survive but apply to cumulative extraction quantities. This has no implication as far as the first extraction date is concerned, since cumulative extraction at that date is identical to the extraction flow.

Since the result holds at the deposit level, it also applies at the industry level. Therefore, as far as the cumulative NRR extraction is concerned, our results ensure the consistency of the widely-used model of NRR supply that assumes that extraction costs rise at the aggregate level with aggregate cumulative extraction, even in absence of aggregate NRR reserves.

4.c *Costly adjustment in NRR extraction*

In general, deposits' exploitation requires extraction and transportation infrastructure; investment in capacity is a typical aspect of NRR extraction. Our formulation of the NRR supply problem is compatible with the use of capital as an input whose payment is included in extraction costs.[19]

Yet, this conventional formulation does not allow to take into account that adjustments in extraction capacity are limited or subject to frictions. A standard way to do so is to consider that capacity adjustments are done in the course of a resource's exploitation, and that adjustment costs are increasing with the change in the extraction rate from one date to another. Formally, for example, date-t extraction cost can be reinterpreted to include such adjustment costs, becoming a function

$$C_t(x_t, x_{t-1}),$$

with the same properties as in Section 2, except it also negatively depends on x_{t-1}. This standard modeling reflects that more extraction at date $t-1$ commands a capacity expansion, thus alleviating potential needs to scale up extraction capacity at the next date. This is in contradiction with the model of the previous subsection assuming that costs increase, rather than decrease, with past extraction. Therefore, if this standard modeling of adjustment costs was adopted here, it would intuitively tend to invalidate our results:[20] In a nutshell, a price rise at some date would call for a capacity expansion at earlier dates, thus justifying an increase in production at these dates.

At the level of an individual deposit, nevertheless, the investment in capacity is realized prior to the exploitation of the deposit and extraction capacity remains unchanged throughout the exploitation period as, for example, in Cairns' (2001) specific treatment of NRR exploitation in this context—see also the references therein. The initial investment

[19] For simplicity, we have omitted the prices of inputs mobilized to produce the extraction output.

[20] Thanks are due to an anonymous referee for this remark.

determines the extraction technology, including, perhaps, capacity constraints.

Therefore, in Appendix K, we adopt the latter approach to develop a simple extension of our analysis to initial deposit-specific investments in capacity that negatively affects the extraction cost function. The problem is only interesting in a long-run perspective over which both investment in capacity and reserve development efforts can be adjusted.[21] As noted by Cairns and Lasserre (1991), these two irreversible decisions—on extraction capacity and reserve development—are not redundant but interrelated.

In this context, we establish that NRR supply exhibits negative cross-price effects if returns to scale in capacity investment and/or in reserve development are sufficiently decreasing, as when the resource scarcity is sufficiently pronounced.

5 Conclusion

The supply of a NRR or any commodity that must be produced before being dispatched over time and space differs from conventional supply in that supply functions then depend on a vector of parametric prices rather than a single price. The inventory to be dispatched or, in the case of NRR supply, the reserves to be extracted, may be given. In that case, supply consists in optimally allocating a given stock (inventory or reserves) over time and space. The supply functions, called restricted because they are conditional on the given factor, are then functions of the parametric prices prevailing at all dates and locations, as well as the quantity of the restricted factor. This defines the short run. In the long run, the stock of reserves (or the production inventory) is chosen endogenously at the beginning of the exploitation period so that the (unrestricted) supply functions depend on prices only. The beginning of the exploitation period may itself be endogenous.

While the commodities produced have been treated as homogeneous throughout the paper, the restrictive factor (reserves or inventories) has been allowed to be heterogeneous. This is important in general, not only in the case of NRRs, but also in the case of non-

[21] They cannot be adjusted, however, after the opening of the deposit.

resource commodities as conditions of production may vary according to origin or due to technological change and other factors. Moreover, our setting allows to examine supply at three basic levels: production at the inventory or deposit, aggregate supply, and the allocation of the latter to each outlet.

The law of supply is only one property of the supply functions that we have characterized. This paper has focused on cross-price effects: the effect on supply at one date and location of changes in prices at other dates or locations. They were decomposed into substitution effects (across time or space) and long-run stock compensation effects.

Our main model offers a highly general representation of NRR supply and yields the following results. The substitution effect (across time or space) is always negative. It dominates the long-run stock compensation effect, which is positive. Consequently both the short-run and the long-run cross-price effects are negative.

Besides filling a gap in the analysis of supply functions, this result confirms or provides several policy results. We give the examples of the taxation of resource use—with implications for the green paradox—and reserve control policies, giving generality to results usually discussed on the basis of particular cases. In the context of limiting the use of carbon NRRs, for instance, our examples stress that supply responses in anticipation of future carbon-penalizing public policies should be expected to undermine policies' effectiveness, despite the role of stranded carbon assets in reducing the production of NRRs at all dates. Our analysis further implies that these effects are qualitatively robust to the modelling of NRR supply.

More importantly, the highly-orthodox theoretical apparatus developed in the paper extends the conventional treatment of competitive supply to commodities whose supply is determined across time and space according to parametric prices defined at dates and locations. By so doing, the paper extends to such supply situations the tool of partial-equilibrium analysis and the familiar method of assessing the effects of policies by analyzing shifts in demand and supply curves.

Other potentially important aspects have been discussed on the ground of detailed analyses that are presented in the Appendix. First, we address technical progress in the technology of production of NRR reserves or of the good to be dispatched. In this context, negative cross-price effects survive for an initial period of time. Second, besides the heterogeneity across inventories or reserves, we consider the heterogeneity of production within inventories and reserves. This aspect is probably more relevant for oil and minerals than for non-resource commodities. In this context, we find that negative cross-price effects hold but apply to cumulative supply rather than to the flow of supply, with no implication for the first date of production. Third, we consider that the cost of dispatching or extracting can be lowered by investment in related capacity. In that context, supply retains its central properties provided returns to scale are sufficiently decreasing as when the scarcity of a NRR is pronounced. However, especially as far as the green paradox is concerned, more research is needed to understand supply responses with irreversible investments in capacity.

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Online appendix to “The supply of non-renewable resources”

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A Proof of Proposition 1

The first point of the proposition is obtained in the main text preceding the proposition.

The second point is obtained as follows. Holding the reserve level unchanged, consider the partial effects of prices, that is the *direct price effects*. We will now show that $\widetilde{x}_t^j(p, X^j)$ is increasing in p_t and decreasing in any p_T, $T \neq t$. By (12),

$$\frac{\partial \widetilde{x}_t^j(p, X^j)}{\partial p_T} = \frac{\partial x_t^j\left(p_t, \widetilde{\lambda}^j(p, X^j)\right)}{\partial p_T} + \frac{\partial x_t^j(p_t, \lambda^j)}{\partial \lambda^j}\frac{\partial \widetilde{\lambda}^j(p, X^j)}{\partial p_T},$$

where the first term on the right is zero unless $T = t$, as $x_t^j(p_t, \lambda^j)$ is not directly dependent on prices other than the contemporary price. The second term is clearly negative whether $T = t$ or $T \neq t$ since x_t^j decreases in λ^j while $\frac{\partial \widetilde{\lambda}^j(p, X^j)}{\partial p_T}$ is clearly positive since a rise in the resource price at any date cannot reduce the rent. It follows that $\frac{\partial \widetilde{x}_t^j(p, X^j)}{\partial p_T}$ is negative for $T \neq t$ while a contemporary rise in price involves two effects working in opposite directions. However, if extraction diminishes at all dates $t \neq T$, it must increase at $t = T$ for otherwise reserves would not be exhausted, which would be suboptimal as already discussed: The law of supply obviously applies to restricted supply. Consequently, in case of a contemporary price rise, the direct price effect given by the first term must dominate the second term that operates via the resource rent.

B Proof of Proposition 2

The first point of the proposition is obtained in the main text preceding the proposition.

The second point can be shown as follows. To examine the effect of a change in price at date T on supply at date t, one must distinguish between a change at the same date $T = t$ and a change at $T \neq t$. From (14), this decomposes into a *direct price effect* and a *stock* *compensation effect*:

$$\frac{\partial x_t^{j*}(p)}{\partial p_T} = \frac{\partial \widetilde{x}_t^j(p, X^{j*}(p))}{\partial p_T} + \frac{\partial \widetilde{x}_t^j(p, X^{j*}(p))}{\partial X^j} \frac{\partial X^{j*}(p)}{\partial p_T}, \tag{B.1}$$

The first term on the right-hand side—the *direct price effect*—has been examined in isolation in Subsection 2.b. The second term—the new *stock compensation effect*—is positive since, by Proposition 1, the *stock effect* is positive and since resource prices always affect developed reserves positively:[22] $\frac{\partial X^{j*}(p)}{\partial p_T} \geq 0$.

When $T = t$, the total price effect may be called the *own price effect*; since $\widetilde{x}_t^j$ is increasing in both p_t and X^j, and as resource prices always affect developed reserves positively, the own price effect is positive. Expression (B.1) when $T = t$ indicates that the law of supply holds and illustrates the Le Châtelier principle, which says that the long-run (unrestricted) elasticity is higher than the short-run (restricted) elasticity.

When $T \neq t$, the direct price effect in (B.1) may be called the *pure substitution effect* as it reflects the reallocation of an unchanged reserve stock to extraction at a date different from T; (12) makes clear that this substitution effect only arises via the effect of the rent on the $\widetilde{x}_t^j$ function: $\frac{\partial \widetilde{x}_t^j(p,X^j)}{\partial p_T} = \frac{\partial x_t^j(p_t,\lambda^j)}{\partial \lambda^j} \frac{\partial \widetilde{\lambda}^j(p,X^j)}{\partial p_T}$. Also by (12), the *stock compensation effect* works in the opposite direction and can be itself decomposed into $\frac{\partial \widetilde{x}_t^j(p,X^j)}{\partial X^j} \frac{\partial X^{j*}(p)}{\partial p_T} = \frac{\partial x_t^j(p_t,\lambda^j)}{\partial \lambda^j} \frac{\partial \widetilde{\lambda}^j(p,X^j)}{\partial X^j} \frac{\partial X^{j*}(p)}{\partial p_T}$ so that the total cross-price effect can be factorized as follows:

$$\frac{\partial x_t^{j*}(p)}{\partial p_T} = \frac{\partial x_t^j(p_t, \lambda^{j*}(p))}{\partial \lambda^j} \left[\frac{\partial \widetilde{\lambda}^j(p, X^{j*}(p))}{\partial p_T} + \frac{\partial \widetilde{\lambda}^j(p, X^{j*}(p))}{\partial X^j} \frac{\partial X^{j*}(p)}{\partial p_T} \right], \; T \neq t, \tag{B.2}$$

where the term between brackets is in fact the total derivative of $\widetilde{\lambda}^j(p, X^j)$ with respect to p_T, decomposed into a direct price effect at constant initial reserves, and the effect on the rent of the change in initial reserves induced by the price change. Resource prices at all dates affect the rent positively, i.e., $\frac{\partial \lambda^{j*}(p)}{\partial p_T} \geq 0$, $\forall T$.[23] Consequently,

$$\frac{\partial x_t^{j*}(p)}{\partial p_T} = \frac{\partial x_t^j(p, \lambda^{j*}(p))}{\partial \lambda^j} \frac{\partial \lambda^{j*}(p)}{\partial p_T} \leq 0, \; \forall t \neq T, \tag{B.3}$$

implying that the stock compensation effect is never high enough to offset the pure substitution effect.

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[22]Formally, the definition of $X^{j*}(p) = X^j\left(\widetilde{\lambda}^j\left(p, X^{j*}(p)\right)\right)$ yields $\frac{\partial X^{j*}(p)}{\partial p_T} = \frac{X^{j\prime}(\lambda^j) \frac{\partial \widetilde{\lambda}^j\left(p,X^{j*}(p)\right)}{\partial p_T}}{1 - \frac{\partial \lambda^j(p,X^{j*}(p))}{\partial X^j} X^{j\prime}(\lambda^{j*}(p))}$, implying that the term between brackets in (B.2) can be factorized as $\frac{\partial \lambda^{j*}(p)}{\partial p_T} = \frac{\partial \widetilde{\lambda}^j\left(p,X^{j*}(p)\right)}{\partial p_T} \left(\frac{1}{1 - \frac{\partial \widetilde{\lambda}^j(p,X^{j*}(p))}{\partial X^j} X^{j\prime}(\lambda^{j*}(p))} \right)$, which is positive since $\frac{\partial \widetilde{\lambda}^j(p,X^j)}{\partial X^j}$ is negative. By (13), it also follows that $\frac{\partial X^{j*}(p)}{\partial p_T}$ is positive.

[23]See footnote 22.

C NRR supply versus Marshallian demand

The dependence on a vector of prices, as well as the substitution and compensation effects are reminiscent of demand theory: NRR producers allocate a stock of resource to different dates and outlets in a way that is comparable to the way consumers allocate their income to different expenditures on different goods. The time space and spatial space play a similar role as the good space in static demand. The stock of reserves is not unlike the budget constraint in demand theory, as both limit what can be allocated to alternative supplies or to expenditures on alternative goods; furthermore, these constraints are both affected by prices, although by different channels.

However, the law of supply always applies: Unlike the Giffen paradox, the supply of a NRR always increases if its price rises. Similarly, inferior goods have no counterpart in NRR supply: Given a price vector, supply does not diminish at any date if reserves are exogenously increased.

Similarly, although reminiscent of the decomposition of Marshallian demand, the decomposition of the change in NRR supply at t following a price change at $T \neq t$ into a pure substitution effect and a stock compensation effect is not isomorphic to the Slutsky decomposition. The substitution effect and the stock compensation effect of a resource price change are illustrated in Figure 3 for the case of a single deposit exploited over two periods, which corresponds to the two-good representation of demand theory. Assuming prices p_0 and p_1, point $O = (x_0, x_1)$ in Figure 3 depicts the producer optimum. Given a stock of reserves X, periods 0 and 1 extraction levels are chosen such that the producer reaches the highest possible two-period iso-extraction-profit curve for prices (p_0, p_1) (of level $\overline{\pi}$).[24] The optimum allocation (x_0, x_1) is thus at the point of tangency between the $\overline{\pi}$ iso-profit curve and the exhaustibility constraint, the -45 degree line which expresses the trade-off between quantities extracted in period 1 and quantities extracted in period 2 in such a way that $x_0 + x_1 = X$. Unlike the case of Marshallian demand, the slope of this linear constraint is not affected by changes in prices. Also, while prices do not affect iso-utility curves, they affect the slope of iso-profit curves: Iso-profit curves may cross at different prices.

Consider a rise in p_1 to $p_1' > p_1$. The price change implies that all iso-profit curves become flatter at any given feasible level of x_0. If the stock of reserves remains unchanged at X, the new tangency point is along the same exhaustibility constraint and along the iso-profit curve of level $\widetilde{\pi} > \overline{\pi}$, at point $\widetilde{O}$ above O, so that $\widetilde{x}_0 < x_0$ and $\widetilde{x}_1 > x_1$. The move from O to $\widetilde{O}$ represents the substitution effect.

However the rise in price leads producers to increase reserve development to X'. Taking this stock effect into account brings the new optimum to O'. It is clear that $x_1' > \widetilde{x}_1 > x_1$. Unlike the Slutsky decomposition, there is no possibility of a commodity analogous to a Giffen good, whose supply would diminish as a result of a rise in its price. Moreover, in the

[24]In Figure 3, the iso-profit curves correspond to the two-period extraction profit, conditional on X and before deduction of the sunk exploration cost $E(X)$: $\overline{\pi} = (p_0 x_0 - C_0(x_0)) + (p_1 x_1 - C_1(x_1))$. By (7), any optimum extraction is such that $p_t - C_t'(x_t) = \lambda$. Thus in a neighborhood of any optimum, $p_t - C_t'(x_t) > 0$. In a neighborhood of an optimum, it follows from the convexity of C_t that the slope $-\frac{p_0 - C_0'(x_0)}{p_1 - C_1'(x_1)}$ of any iso-profit curve at prices (p_0, p_1) is negative, increasing in x_0 and decreasing in x_1. In Figure 3, we focus on the relevant convex parts of the iso-profit curves. On other parts, they need not be convex.

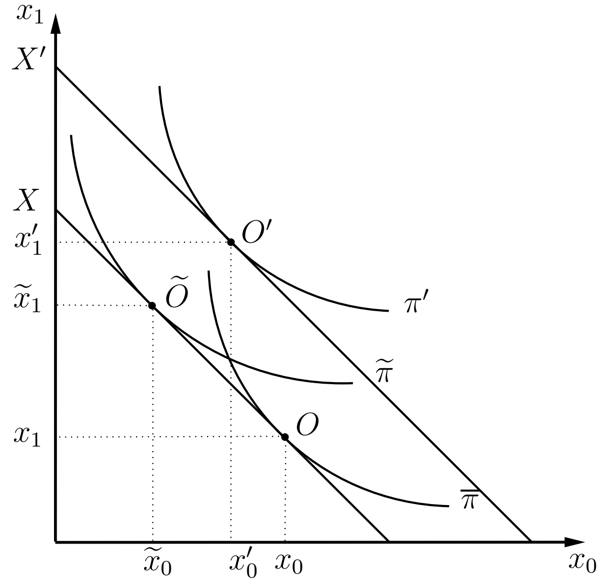

Figure 3: Price effect decomposition with $p_1' > p_1$

case of NRR supply, the substitution effect always dominates the compensation effect, so that, by (B.3), x_0' must be lower than x_0 following the rise in p_1. There is no such thing as NRR supply complements; quantities extracted at different dates are always substitutes.

D Proof of Proposition 4

The analysis shows similar results as in the single-location treatment of Subsections 2.b-2.f. However, it is slightly more complex, involving two stages rather than one, as follows.

The allocation rule (18) implicitly defines the solution as a series of functions giving the optimal quantity at each date and location

$$x_{tl}^j = x_{tl}^j(p_{tl}, \upsilon_t^j), \tag{D.1}$$

with the same properties as the functions defined by (10). The rule (19) similarly defines the total quantity across locations at each date t as a function

$$x_t^j = \bar{x}_t^j(\upsilon_t^j, \lambda^j). \tag{D.2}$$

The functions $\bar{x}_t^j$ also have properties with respect to the implicit value υ_t^j and the rent λ^j that are analogous to those of (10): $\bar{x}_t^j$ is increasing in the implicit price υ_t^j and decreasing in the rent λ^j.

Treating date-t extraction x_t^j to be dispatched across locations as given—in the same way as we took reserves X^j to be allocated across dates as given in Subsection 2.b—and combining all relations (D.1) into (17), we obtain the new multiplier υ_t^j as a function increasing in all components of the vector of date-t prices $p_t \equiv (p_{tl})_{l=0,\ldots,\bar{l}}$ and decreasing in

x_t^j:

$$\upsilon_t^j = \widetilde{\upsilon}_t^j(p_t, x_t^j). \tag{D.3}$$

Substituting (D.3) into (D.2) yields an implicit function identical to (10)

$$x_t^j = x_t^j(p_t, \lambda^j), \tag{D.4}$$

except that p_t is a vector rather than a single price. Results similar to the single-outlet model of the previous subsections follow.

A rise in p_{TL}, i.e., a rise at any date $T \geq 0$ and location $L \in [0, ..., \bar{l}]$, causes the rent λ^j to increase. This is true whether the size of the exploitable reserves X^j is restricted or not. Thus, by (D.4), extraction decreases at all dates t other than T. When reserves X^j are endogenous, they increase as a result of the price rise; since all x_t^j are reduced at all $t \neq T$, it follows that x_T^j must increase if the sum of all x_t^j is to use up reserves.

According to (D.1), quantities x_{tl}^j at all locations react in the same direction to the same change in implicit value υ_t^j. Since $x_t^j = \sum_{l=0,...,\bar{l}} x_{tl}^j$, the reduction in x_t^j at $t \neq T$ is only compatible with a rise in υ_t^j. Thus x_{tl}^j decreases for all $t \neq T$ and all l.

At date T where x_T^j increases as a result of the rise in p_{TL}, (D.2) makes clear that only a rise in υ_T^j is consistent with the increase in λ^j. The rise in υ_T^j in turn implies by (D.1) that x_{Tl}^j diminishes for all $l \neq L$. Since, total extraction x_T^j at date T rises, it must be that x_{TL}^j rises as a result of the price rise, which illustrates the law of supply for this spatio-temporal version of the model.

E Analysis of generic NRR supply and demand policies

The illustrations given in Section 3 are simple examples. For more general applications, generic supply and demand policies can be modeled as follows; their effects are described in Proposition 5 further below. For ease of exposition, we assume a single deposit.

A demand policy indexed by θ_t may reduce date-t demand for the NRR to $x_t^D(p_t; \theta_t)$ from its no-intervention level $x_t^D(p_t; 0)$. Demand-reducing policies may take various forms, such as consumer taxes of Subsection 3.a or support to NRR substitutes. Assuming that x_t^D is continuously differentiable and monotonic in both arguments, the inverse demand function at t, $P_t(x_t; \theta_t)$ is decreasing in x_t and in θ_t. In the sequel we will assume that stringency levels are chosen such that $P_t(0; \theta_t)$ is greater than the equilibrium price p_t^e for some dates, to avoid situations where policies do not warrant any production at all.

A supply policy may aim at extraction while affecting reserves only indirectly; or it may focus on reserves directly. We will refer to the former as extraction policy, while calling the latter a reserve policy. An extraction policy indexed by ξ_t reduces date-t marginal extraction profit $p_t - C_t'(x_t) - \frac{\partial G_t(x_t, p_t; \xi_t)}{\partial x_t}$, where the policy function G_t is positive and such that the policy-adjusted cost function $C_t + G_t$ inherits the properties of the original cost function: It is increasing, strictly convex and twice differentiable in x_t; it also satisfies $C_t(0) + G_t(0, p_t; \xi_t) = 0$ for all $t \geq 0$, $p_t > 0$ and ξ_t, and is such that $C_t'(0) + \frac{\partial G_t(0, p_t; \xi_t)}{\partial x_t}$ is lower than the equilibrium price p_t^e for at least one date where $P_t(0; \theta_t) > p_t^e$ also holds. Last, it is assumed that $\frac{\partial^2 G_t}{\partial x_t \partial p_t} < 1$ to eliminate ill-conceived policies under which marginal cost would increase more than marginal revenue as a result of a price increase. We define a more stringent extraction policy as one that reduces the marginal extraction profit: $\frac{\partial^2 G_t}{\partial x_t \partial \xi_t} > 0$, $\forall x_t \geq 0$. Modified this way, problem (4)-(2)-(5) yields necessary conditions

$$x_t = x_t(p_t, \lambda; \xi_t), \ \forall t \geq 0. \tag{E.1}$$

As their counterparts in (10), the new functions (E.1) are increasing in p_t and decreasing in the rent λ; they are further decreasing in the extraction policy index ξ_t.

Next, a reserve-reducing policy increases the marginal cost of developing exploitable reserves. The exploration and development cost $E(X)$ in the objective (4) is augmented by the function $F(X;\Omega)$, positive and increasing in its two arguments, where Ω reflects the stringency of the reserve-reducing policy. As in Section 2, assume that the total development cost $E(X) + F(X;\Omega)$ is twice differentiable, strictly convex and satisfies $E(0) + F(0;\Omega) = E'(0) + \frac{\partial F(0;\Omega)}{\partial X} = 0$. Modifying problem (4)-(2)-(5) accordingly, the necessary reserve-supply condition (8) is replaced by $E'(X) + \frac{\partial F(X;\Omega)}{\partial X} = \lambda$, which implicitly defines the policy-induced level of reserves as

$$X = X(\lambda; \Omega). \tag{E.2}$$

As its counterpart (9), this function is increasing in the rent λ; it is further decreasing in the reserve-policy index Ω. The NRR producer's supply behavior is summarized by the functions (E.1) and (E.2).

At this stage of the formalization in Section 2, the remaining step toward establishing supply functions was to replace the endogenous rent λ by an appropriate function of parametric prices. In the current partial-equilibrium analysis, prices are endogenous but one must recognize the dependency of λ on both the supply and the demand policy parameters. This is done as follows.

For any date $t \geq 0$, substituting the inverse demand function $P_t(x_t; \theta_t)$ into the supply function (E.1) implicitly defines date-t equilibrium extraction as a function

$$x_t = \widehat{x}_t^e(\lambda; \xi_t, \theta_t), \ \forall t \geq 0. \tag{E.3}$$

This function is decreasing in the rent λ, in the extraction-policy index ξ_t, and in the demand-policy index θ_t.

Treating the stock of initial reserves as given at this stage, and combining relations (E.3) at all dates into the exhaustibility constraint (5) defines the short-run equilibrium rent as a function $\widetilde{\lambda}^e(X; \Xi, \Theta)$ of initial reserves, of the vector of extraction-policy indices $\Xi \equiv (\xi_t)_{t\geq 0}$, and of the vector of demand-policy indices $\Theta \equiv (\theta_t)_{t\geq 0}$. As its counterpart (11), $\widetilde{\lambda}^e(X; \Xi, \Theta)$ is decreasing in X; it can be shown that it is also decreasing in all elements of the policy vectors Ξ and Θ.

Substituting $\widetilde{\lambda}^e(X; \Xi, \Theta)$ into each extraction function (E.3) gives the restricted (short-run) NRR equilibrium extraction functions:

$$x_t = \widetilde{x}_t^e(X; \Xi, \Theta) \equiv \widehat{x}_t^e\left(\widetilde{\lambda}^e(X; \Xi, \Theta); \xi_t, \theta_t\right), \ \forall t \geq 0. \tag{E.4}$$

Each restricted NRR equilibrium extraction function is increasing in initial reserves X.

Holding X unchanged, the partial effects on equilibrium supply at t of changing policy intensities ξ_T and θ_T at any date $T \geq 0$ can be established as in Subsection 2.b's analysis of the effects of changing the price p_T on restricted supply. The only difference is that a rise in ξ_T or θ_T (a policy restriction) affects the quantity supplied in a direction opposite to that of a rise in price.[25]

Now consider the long term, allowing initial reserves to be endogenously determined. Substituting $\widetilde{\lambda}^e(X; \Xi, \Theta)$ into (E.2), implicitly defines unrestricted equilibrium reserves

$$X = X^e(\Xi, \Theta, \Omega). \tag{E.5}$$

In turn, substituting $X^e(\Xi, \Theta, \Omega)$ into $\widetilde{\lambda}^e(X; \Xi, \Theta)$ defines $\lambda = \lambda^e(\Xi, \Theta, \Omega) \equiv \widetilde{\lambda}^e\left(X^e(\Xi, \Theta, \Omega); \Xi, \Theta\right)$. It can be shown that X^e is decreasing in ξ_t and θ_t, for all $t \geq 0$, as well as in Ω, and that λ^e is decreasing in ξ_t, in θ_t, for all $t \geq 0$, but increasing in Ω. The unrestricted (long-run) equilibrium extraction level at date t is thus defined as the following function of all elements in the policy vectors Ξ and Θ as well as the policy index Ω:

$$x_t^e = x_t^e(\Xi, \Theta, \Omega) \equiv \widetilde{x}_t^e\left(X^e(\Xi, \Theta, \Omega); \Xi, \Theta\right). \tag{E.6}$$

The restricted equilibrium extraction functions (E.4) and the unrestricted equilibrium extraction functions (E.6) just established have the same comparative static properties with respect to the exogenous (supply and demand) policy parameters, *mutatis mutandis*, as their NRR supply counterparts have with respect to the exogenous prices, irrespective of whether opening dates are assumed exogenous or endogenous. This can be shown by adapting the steps followed in Section 2. In particular, the effects of supply- and demand-reducing policies on equilibrium extraction quantities can be decomposed into a stock compensation effect and a pure substitution effect as illustrated by (B.1) in the case of the price effect in Section 2. Those properties are summarized in the following proposition.

Proposition 5 (Policy-induced equilibrium changes in aggregate NRR extraction)
Whether opening dates are endogenous or not,

1. *A reserve-reducing policy decreases restricted and unrestricted equilibrium extraction at all dates t;*
2. *Any combination of extraction and demand-reducing policies at any date T*
   (a) *Reduces unrestricted developed reserves;*
   (b) *Reduces restricted and unrestricted extraction at date T;*
   (c) *Increases restricted and unrestricted extraction at all dates $t \neq T$.*

[25] For example consider an increase in θ_T at any date $T \geq 0$. By the definition (E.4) of the $\widetilde{x}_t^e$ restricted functions, extraction at all dates $t \neq T$ is only affected via the policy-induced change in the rent $\widetilde{\lambda}^e$ in $\widehat{x}_t^e$; $\widetilde{\lambda}^e$ is reduced as a consequence of the rise in θ_T, which in turn, by (E.3), increases $\widehat{x}_t^e$, for all $t \neq T$, and thus $\widetilde{x}_t^e$, for all $t \neq T$. Extraction being increased at all dates $t \neq T$ while exploited reserves are unchanged, equilibrium extraction must decrease at the date T of the policy change. The same analysis applies to a change in the supply-policy stringency ξ_T.

Proposition 5 focuses on the equilibrium of a single-outlet NRR market. It can be readily extended to multiple-outlet markets using the formalization leading to Proposition 4. Possible applications involving the spatial dimension of NRR supply include, for example, the analysis of a policy penalty on a carbon NRR use implemented unilaterally on a single of many outlets, raising the issue of carbon leakage.

The results stated in Proposition 5 can be used to study the effects of policies that reduce NRR demand and/or supply over some extended future period and in specific locations via various forms of restrictions to NRR extraction and use, or assistance to alternative sources of supply.

Let us again, for simplicity, on the dynamic dimension of NRR supply, and consider an increase in policy stringency at several dates T that form a set Δ. Proposition 5 establishes that such increases in policy stringency have the same qualitative effect on equilibrium NRR quantities at dates when no policies are implemented. Thus all effects combine to positively affect extraction at all dates $t \notin \Delta$.

When policies are unanticipated and not accompanied by any adjustment in the stock of reserves, they affect the restricted equilibrium supply $\widetilde{x}_t^e(X; \Xi, \Theta)$; when they are anticipated and associated with a drop in developed reserves, they affect the unrestricted equilibrium supply $x_t^e(\Xi, \Theta, \Omega)$. In either case, reductions, for example, in demand at $T \in \Delta$ increase extraction at all $t \notin \Delta$, confirming the validity of the green paradox in this more realistic context.

If the policy change occurs at a single date as in Proposition 5.2, the drop in price unambiguously causes a drop in extraction at that date. When Δ contains more than a single date, the reaction of NRR extraction at each $T \in \Delta$ depends on the magnitude of the price change occurring at that date relative to the changes occurring at other dates $T' \in \Delta$. However, the above analysis indicates that cumulative extraction over Δ is reduced. This is because X decreases while cumulative supply at all dates $t \notin \Delta$ increases.

Corollary 2 (Equilibrium changes in aggregate NRR extraction and policy over periods of time)
Whether opening dates are endogenous or not, an extraction or demand-reducing policy implemented at dates $T \in \Delta$

1. *Reduces unrestricted developed reserves;*
2. *Reduces restricted and unrestricted cumulative extraction over dates $T \in \Delta$;*
3. *Increases restricted and unrestricted extraction at all dates $t \notin \Delta$.*

As suggested already, using the properties of NRR supply involving the spatial dimension—and summarized in Proposition 4—other applications would follow, as, for example, the analysis of unilateral policies and resulting leakages. Indeed, leakages of NRR use (Gaudet, Moreaux and Salant, 2001; Fischer and Salant, 2017) work in the same direction spacewise, and timewise, in the short run and in the long run.[26] When the change in demand affects more than one region as with Fischer and Salant's (2017) technology-oriented policies, or

[26]The spatial version of Corollary 2 can be established using the formalization of Proposition 5; a proof is available on request.

takes place at more than one date as with their emission taxes, the reaction of resource supply to one region at one date depends on the magnitude of the price change occurring in that region and date relative to the changes occurring at other regions and dates. At dates and regions not concerned by the policies, short-run and long-run supply increases.

F Technical progress in NRR development

Assume that each source $j = 1, ..., J$ is now characterized by its own exploration and development present-value cost $E_t^j(X^j)$, whose qualitative properties are the same as in Section 2, except that it depends on time. In this context, the development of a deposit's reserve need not be economic at early dates, but may become justified at later dates not only by high prices and/or low extraction costs, but also by technical improvements in the technology of reserve discovery and development.[27]

We assume that technological progress on exploration and development is such that, for any date $t' > t$ and initial reserves $X^j \geq 0$,

$$E_t^j\left(X^j\right) \geq E_{t'}^j\left(X^j\right) \text{ and } E_t^{j\prime}\left(X^j\right) \geq E_{t'}^{j\prime}\left(X^j\right),\ \forall j. \tag{F.1}$$

As before, it is supposed that exploration and development are instantaneous and undertaken only once for each deposit; extraction may take place only after deposit development. Obviously, the problem differs from Section 2 only in the long run, over which producers freely choose both the reserve development efforts and the dates τ^j when these efforts are made. For deposit j, the producer solves the same problem as in Subsection 2.f, once the development cost is adjusted to depend on time:

$$\max_{(x_t^j)_{t\geq 0}, \tau^j, X^j} \sum_{t\geq 0}\left(p_t x_t^j - C_t^j(x_t^j)\right) - E_t^j(X^j) \tag{F.2}$$

subject to (2) and (5).

Like in Section 2, the choice of extraction is determined by condition (7). For simplicity, we assume that the evolution over time of resource price changes and extraction technology is such that, once initiated, production is not interrupted until exhaustion.[28] As far as the choice of developed reserves is concerned, it is determined by a condition similar to (8), except that the marginal cost of exploration and development varies with time:

$$E_t^{j\prime}(X^j) = \lambda^j. \tag{F.3}$$

We further assume that the problem is well behaved in the sense that the optima being characterized are global rather than local, at least in the neighborhood of the price vector under consideration. This rules out jumps from one local maximum to another local

[27]For some price and technology combinations, development occurs only at $\tau = 0$ if at all. Such is the case, for example, if (present-value) prices are non increasing while (present-value) extraction and development costs are non decreasing.

[28]This assumption facilitates the analysis while it eliminates situations of only minor economic interest such as temporary interruptions of production. It is satisfied if prices do not diminish too fast and technological change is such that extraction costs do not increase too fast over any part of the exploitation period.

maximum as a result of a small change in the price vector.

Since the exploitation of each deposit is independent of other deposits, let us focus on deposit j. The optimal development date $\tau^{j*}(p)$ of deposit j may be the corner solution $\tau^{j*} = 0$, or, if it is an interior solution, it is a non-zero integer within the set of possible dates.

At this preliminary stage, consider the development date τ^j as given. Conditional on τ^j, optimum extraction flows $x_t^j(p, \tau^j)$ for all $t \geq \tau^j$ are determined by (7) where $\lambda^j = \lambda^j(p, \tau^j)$ will be characterized shortly.

On the one hand, $\lambda^j(p, \tau^j)$ reflects the contribution of marginal reserves at the producer's optimum. Indeed, denoting deposit-j's value function conditional on τ^j by

$$\mathcal{V}^j\left(p, \tau^j, X^j\right) \equiv \max_{(x_t^j)_{t \geq \tau^j}} \sum_{t \geq \tau^j} p_t x_t^j - C_t^j(x_t^j) \tag{F.4}$$

$$\text{subject to} \quad \sum_{t \geq \tau^j} x_t^j \leq X^j, \tag{F.5}$$

and keeping in mind that $\lambda^j(p, \tau^j)$ is formally the Lagrange multiplier associated with constraint (F.5), the Envelope Theorem for constrained problems implies

$$\lambda^j(p, \tau^j) = \frac{\partial \mathcal{V}^j\left(p, \tau^j, X^j(p, \tau^j)\right)}{\partial X^j}, \tag{F.6}$$

which relates the optimum amount of reserves $X^j(p, \tau^j)$ with their implicit value $\lambda^j(p, \tau^j)$. In equation (F.6), $\frac{\partial \mathcal{V}^j\left(p, \tau^j, X^j\right)}{\partial X^j}$ is a decreasing function of reserves X^j by the assumption that extraction costs are strictly convex. Moreover, it is a decreasing function of τ^j, since exploiting unchanged reserves X^j over a smaller set of dates means that marginal reserves must optimally be extracted at higher costs.

On the other hand, (F.3) indicates that exploitable reserves $X^j(p, \tau^j)$ are optimally produced in such a way as to equate their marginal development cost $E_{\tau^j}^{j\prime}\left(X^j(p, \tau^j)\right)$ with their implicit valuation underground $\lambda^j(p, \tau^j)$. This reserve-supply relation is strictly increasing since the $E_{\tau^j}^j$ function is strictly convex by assumption.

Thus in optimum, for a given development date τ^j, the rent $\lambda^j(p, \tau^j)$ and reserves $X^j(p, \tau^j)$ are jointly determined by the combination of (F.6) with (F.3):

$$\frac{\partial \mathcal{V}^j\left(p, \tau^j, X^j(p, \tau^j)\right)}{\partial X^j} = \lambda^j(p, \tau^j) = E_{\tau^j}^{j\prime}\left(X^j(p, \tau^j)\right). \tag{F.7}$$

Using this result, Appendix G examines the optimal choice of the (endogenous) development date $\tau^{j*}(p)$ of each deposit j. It establishes the following result about the effect of a price change on development dates.

Lemma 1 (Endogenous opening dates with technical progress)

1. *A price rise at any date after the planned opening of a deposit either leaves the opening date of the deposit unchanged, or postpones it to a date closer and anterior to the date of the price rise.*

2. *A price rise at any date prior to the planned opening of a deposit either leaves the opening date of the deposit unchanged, or accelerates it.*

Consider a price rise at $T > 0$. Lemma 1 indicates that all deposits that were active before T have unchanged or postponed development dates. Clearly for deposits with unchanged development date the results of Proposition 3 carry over so that their combined supply diminishes at each date $t \neq T$. For deposits whose development is postponed their production becomes zero until the new development date, which is inferior or equal to T. However, it cannot be ruled out that their production in the new program at dates following the new development date might exceed their production at those dates under the initial price. Consequently, it is certain that aggregate production diminishes from date zero until the date of the first postponed deposit development if any.[29] In particular, a price rise at any date $t > 0$ always reduces aggregate supply at the present extraction date $t = 0$. The following proposition summarizes the above results.

Proposition 6 (Aggregate NRR supply with endogenous openings and technical progress)
(Cross-price effects) *A price rise at any date $T > 0$ reduces aggregate unrestricted supply at each date $t \neq T$ of an initial period that extends at least from date 0 until the date of the first postponed deposit development.*

G Proof of Lemma 1

The optimum rent $\lambda^j(p, \tau^j)$ and optimum reserves depend on the (at this stage) exogenous development date τ^j. As explained above, $\frac{\partial \mathcal{V}^j(p,\tau^j,X^j)}{\partial X^j}$ is decreasing in τ^j. Let $\tau^{j\prime} < \tau^j$ be two exogenous development dates: It must be that $\frac{\partial \mathcal{V}^j(p,\tau^{j\prime},X^j)}{\partial X^j} \geq \frac{\partial \mathcal{V}^j(p,\tau^j,X^j)}{\partial X^j}$. Moreover, by assumption (F.1), $E^{j\prime}_{\tau^{j\prime}}(X^j) \geq E^{j\prime}_{\tau^j}(X^j)$. It thus follows that (F.7) taken at τ^j and at $\tau^{j\prime} < \tau^j$ generates two different rents such that

$$\lambda^j(p, \tau^{j\prime}) \geq \lambda^j(p, \tau^j), \ \tau^{j\prime} < \tau^j. \tag{G.1}$$

For any given development date τ^j, the Maximum Theorem applies.[30] Thus price changes continuously affect all variables and functions. In particular, if τ^j is the optimum devel-

[29]Formally, a price rise at $T > 0$ reduces supply from deposit j at all dates $t \neq T$ if j is active at t in the initial program ($t \geq \tau^{j*}(p)$, where p is the initial price vector) in the two following cases:
1) The development date of the deposit is the same in the initial and new programs ($\tau^{j*}(p) = \tau^{j*}(p')$, where p' is the new price vector), implying that the deposit is still active at t after the price rise;
2) the deposit is inactive at t in the new program ($\tau^{j*}(p') > t \geq \tau^{j*}(p)$).

[30] When reserves are endogenously determined at the exogenous date τ^j, the optimal reserves $X^j(p, \tau^j)$ must be finite so that the extraction possibility set is bounded, and is evidently closed. Prices in p also affect the objective (F.2) continuously. Furthermore, this objective is strictly concave by assumption and the set of extraction possibilities is convex since the convex combination of two possible extraction paths satisfying the exhaustibility constraint (5) satisfies the same constraint. The Maximum Theorem thus applies: Given the development date τ^j, the optimum sequence of extraction $x^j_t(p, \tau^j)$ and the multiplier $\lambda^j(p, \tau^j)$ evaluated at the optimum are continuous functions of each price in the vector p.

opment date, small changes in price that do not require any change in that date have continuous effects.

Consider now that $\tau^j = \arg\max_{\tau \geq 0} \mathcal{V}^j\left(p, X^j(p,\tau), \tau\right) - E^j_\tau\left(X^j(p,\tau)\right)$, where, given τ, $X^j(p,\tau)$ is chosen optimally as described above. It follows that, for any $\tau^{j\prime} \neq \tau^j$,

$$\mathcal{V}^j\left(p, X^j(p,\tau^j), \tau^j\right) - E^j_{\tau^j}\left(X^j(p,\tau^j)\right) - \left[\mathcal{V}^j\left(p, X^j(p,\tau^{j\prime}), \tau^{j\prime}\right) - E^j_{\tau^{j\prime}}\left(X^j(p,\tau^{j\prime})\right)\right] \geq 0. \tag{G.2}$$

The first part of the lemma considers a price rise at some date T, posterior to the deposit-opening date τ^j. Assuming instead that $T \leq \tau^{j\prime} < \tau^j$, let us show now that no increase in price can cause (G.2) to be violated, so that a price rise cannot cause the development date to be accelerated. Precisely, consider an infinitesimal change in price at date $T > \tau^j$. Denoting by Δ the total derivative of the left-hand side of inequality (G.2), one obtains

$$\begin{aligned}\Delta &= \frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^j), \tau^j\right)}{\partial p_T} + \frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^j), \tau^j\right)}{\partial X^j}\frac{dX^j(p,\tau^j)}{dp_T} - E^{j\prime}_{\tau^j}\left(X^j(p,\tau^j)\right)\frac{dX^j(p,\tau^j)}{dp_T} \\ &- \left[\frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^{j\prime}), \tau^{j\prime}\right)}{\partial p_T} + \frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^{j\prime}), \tau^{j\prime}\right)}{\partial X^j}\frac{dX^j(p,\tau^{j\prime})}{dp_T} - E^{j\,\prime}_{\tau^{j\prime}}\left(X^j(p,\tau^{j\prime})\right)\frac{dX^j(p,\tau^{j\prime})}{dp_T}\right].\end{aligned}$$

Recalling the second equality in (F.7), we have

$$\frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^j), \tau^j\right)}{\partial X^j}\frac{dX^j(p,\tau^j)}{dp_T} - E^{j\prime}_{\tau^j}\left(X^j(p,\tau^j)\right)\frac{dX^j(p,\tau^j)}{dp_T} = 0$$

and similarly for $\tau^{j\prime}$. Thus, Δ may be rewritten

$$\Delta = \frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^j), \tau^j\right)}{\partial p_T} - \left[\frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^{j\prime}), \tau^{j\prime}\right)}{\partial p_T}\right],$$

where the Envelope Theorem applied to (F.4)-(F.5) implies

$$\frac{\partial \mathcal{V}^j\left(p, X^j(p,\tau^j), \tau^j\right)}{\partial p_T} = x^j_T(p,\tau^j),$$

for τ^j and for $\tau^{j\prime}$. Δ thus reduces to

$$\Delta = x^j_T(p,\tau^j) - x^j_T(p,\tau^{j\prime}).$$

Finally, the inequality (G.1) established above, together with the first-order condition (7) characterizing extraction at date T implies that $x^j_T(p,\tau^j) \geq x^j_T(p,\tau^{j\prime})$, which proves that

$$\Delta \geq 0.$$

Since a price rise positively affects the left-hand side of inequality (G.2), it cannot cause the earlier date $\tau^{j\prime} < \tau^j$ to become the optimal development date.

The second part of the lemma considers a price rise occurring at date T prior to the deposit-opening date τ^j. Under the initial prices, assume that extraction takes place at dates $t \geq \tau^j \geq T$ and reserves are optimally developed at τ^j; that must yield higher intertemporal profits (F.2) than opening the deposit at a later date. A postponing of the opening date τ^j to $\tau^{j\prime} > \tau^j$ would restrict extraction to dates $t \geq \tau^{j\prime} > \tau^j$, over which price conditions are unchanged by the rise in price at T. Clearly, that restriction cannot dominate the possibility of extracting at the larger set of dates $t \geq \tau^j$.

Having established the lemma, Proposition 6 follows, as indicated in Appendix F in the paragraph preceding the proposition.

H NRR supply with stock effects in extraction costs

In this appendix and the following Appendix J, we focus on a particular deposit for simplicity. For the sake of notational simplicity, we omit the deposit index j. Assuming that all deposits are modeled in the same way, the results obtained will indicate that their extension to the aggregate level is straightforward.

Since the resource is not homogeneous, it is important to make a distinction between geological and economical reserves. Let X_t represent the stock of geological reserves remaining at t, and relabel the stock of initial geological reserves X^0, with $X_0 = X^0$. Units of measurement are chosen such that the flow of extraction is homogeneous over time, e.g., barrels of oil of constant energy content, viscosity, refining cost, etc. Cumulative extraction between dates 0 and t is equal to $X_0 - X_t$, with

$$X_{t+1} = X_t - x_t,\ X_0 = X^0 \text{ given.}$$

The extraction cost function may be written as $C_t(x_t, X_t)$. Under such a techno-geological constraint on extraction, the net present-value extraction revenue $p_t x_t - C_t(x_t, X_t)$ not only depends on x_t and p_t as in Section 2, but also on remaining reserves X_t. It is still true at each date t that $\frac{\partial C_t(x,X)}{\partial x} > 0$ and $C_t(0, X) = 0$, $\forall X \geq 0$. Also, we assume that $\frac{\partial C_t(0,X^0)}{\partial x} < p_t$ for at least one date so that some exploitation is warranted. The dependency of extraction cost on cumulative extraction implies that the cost depends on X negatively: Higher current reserves imply lower cumulative extraction, hence a lower cost. We assume

$$\frac{\partial C_t(x,X)}{\partial X} < 0 \text{ and } \frac{\partial C_t(x,X)}{\partial x \partial X} < 0, \tag{H.1}$$

for all $t \geq 0$ and for all $x > 0$ and $X \geq 0$.

We also assume that $C_t(x, X)$ is strictly convex in its two arguments.[31] We owe James Sweeney (1993) a thorough investigation of the discrete version of the Hotelling-Gordon model; as he showed, the existence of an underlying continuous-time representation of the technology implies restrictions on the partial derivatives of allowable discrete-time cost

[31] Under the maintained assumption that the cost function is convex, the objective function for this problem is concave and the feasible set is convex. Thus the first-order necessary conditions are sufficient for optimality and multiple local unconnected optima cannot exist (Sweeney, 1993, p. 771). We further assume the strict convexity of the cost function in order to avoid having to deal with supply correspondences rather than supply functions.

functions; precisely, it must be true that

$$\frac{\partial^2 C_t\left(x, X\right)}{\partial x^2} + \frac{\partial^2 C_t\left(x, X\right)}{\partial x \partial X} > 0, \tag{H.2}$$

a property called the dominance of extraction rate on marginal cost[32] which will also be assumed to hold here.[33]

As in the simple model of Section 2, the producer must identify and develop the reserves to be exploited before extraction begins. At date 0, a portion $X^0 - X^F$ is chosen within the initial stock X^0 of geological reserves and undergoes a costly exploration and development process that makes it suitable for exploitation. No development expenditure is applied to reserves that are not deemed economical, implying that the stock $X^F \geq 0$ of geological reserves left undeveloped will be left unexploited at the end of the extraction process.

The amount $X^0 - X^F$ defines economic reserves. Economic reserves can be increased at date zero by reducing X^F. It is sensible to assume decreasing returns to exploration and development on the ground, as argued before, that the best prospects are developed first. Redefining the function E, we thus assume that the cost of developing an initial stock of exploitable economic reserves $X^0 - X^F$ when geological reserves are X^0 is $E\left(X^0 - X^F\right)$,[34] with $E(0) = 0$ and $E'(0) = 0$.[35]

The problem faced by a NRR producer under such conditions is (see the Appendix for details of the resolution)

$$\max_{(x_t, X_t)_{t\geq 0}} \sum_{t\geq 0} \left(p_t x_t - C_t(x_t, X_t)\right) - E(X^0 - X^F) \tag{H.3}$$

subject to

$$X_{t+1} = X_t - x_t, \ \forall t \geq 0, \tag{H.4}$$

[32] Salant et al. (1983) also imposed this assumption, although not in reference with any underlying continuous-time technology.

[33] As a simple way to focus on the rise in extraction cost with cumulative extraction, the extraction cost of the underlying continuous-time cost function is sometimes assumed to depend on the remaining-reserve stock, but not on the extraction rate (e.g., van der Ploeg and Withagen, 2012a, among many others); i.e., total cost is assumed linear in extraction rate. However, this linearity assumption in a continuous-time model implies a discrete-time representation in which marginal extraction cost is a strictly increasing function of extraction rate and a decreasing function of the remaining stock (Sweeney, 1993). Thus the discrete-time model presented here encompasses both continuous-time versions of Gordon's model that treat the marginal cost of extraction as constant or as strictly rising.

[34] We write the amount of reserves developed for exploitation as $X^0 - X^F$ rather than merely X as in Section 2 in order to emphasize an important property of the model. The marginal unit of reserves being developed at date zero is the unit that will be extracted last, not first. In models of homogeneous resources, this does not matter; in Gordon's model, the sequence of reserve development and extraction is a geological and technological assumption, although it is often justified on economic grounds. Increasing the stock of developed reserves at date zero means reducing the amount of geological reserves X^F to be left unexploited at the closure date. As a result, the cost of extraction at date zero is the same whatever the amount of developed reserves.

[35] As in Section 2, the property $E'(0) = 0$ is introduced because it is sufficient to ensure that a positive amount of reserves is developed. It plays no other role than ruling out uninteresting situations where resource prices do not warrant the production of any reserves.

$$x_t \geq 0, \ \forall t \geq 0, \tag{H.5}$$

$$X_0 = X^0 \text{ given} \tag{H.6}$$

$$X_t \geq X^F \geq 0, \ \forall t \geq 0, \tag{H.7}$$

where economic reserves $X^0 - X^F$, and thus also geological reserves X^F left undeveloped at the end of exploitation, are fixed in the short run but endogenous in the long run.

We denote by S, which may be infinite, the last date at which strictly positive extraction occurs. Thus if S exists, $X_t = X_{t+1}$ for all $t \geq S+1$. Clearly, extraction may also be null occasionally before S.

Because of resource heterogeneity, the present-value resource rent measured by the Lagrangian multiplier associated with (H.4) is not constant over time. It is sometimes called a Ricardian resource rent and diminishes as reserves diminish; we denote it μ_t rather than λ, the Lagrangian multiplier associated with the same constraint in Section 2, to emphasize that this rent is different from a pure Hotelling scarcity rent.

Pure scarcity arises in this model at two levels, associated with the two inequality constraints in (H.7). The right-hand side inequality addresses the possibility that the totality of geological reserves be worth exploiting. It has been well investigated (e.g., Levhari and Liviatan, 1977) and will be considered in Appendix I. The left-hand side inequality recognizes the assumption that only those reserves that have been previously discovered and developed at date zero may be exploited. As already argued, exploration and reserve development are costly, so that no reserves are developed to be ultimately left unexploited. Prior to the exhaustion of economic reserves, $X_t > X^F$ and the constraint is not binding; at the date of exhaustion, the constraint becomes binding and we denote the associated multiplier by λ with no time index because the date of exhaustion S is endogenous; λ may be interpreted as the pure Hotelling component of the resource rent in a short-run perspective, or, in a long-run perspective, as a quasi-rent reflecting both pure Hotelling scarcity and expenditures sunk in exploration and development.

The first-order condition for strictly positive extraction at date t is

$$p_t - \frac{\partial C_t(x_t, X_t)}{\partial x} = \mu_t, \ x_t > 0, \ \forall \ t = 0, ..., S. \tag{H.8}$$

The resource rent evolves according to the first-order condition

$$\begin{aligned} \mu_t &= \mu_{t-1} + \frac{\partial C_t(x_t, X_t)}{\partial X}, \ \forall \ t = 0, ..., S, && \text{(H.9)} \\ &\text{with} \\ \mu_S &= \lambda \text{ and } \lambda = E'\left(X^0 - X^F\right) \text{ in the long run if } X^F > 0. && \text{(H.10)} \end{aligned}$$

According to (H.9), the present-value rent diminishes as the stock of reserves diminishes and extraction cost increases. At the end of operations, all pre-developed reserves are exhausted so that remaining geological reserves equal X^F. When those remaining reserves are not null, the constraint that $X^F \geq 0$ is not binding and the resource rent equals the cost of finding and developing the marginal unit of economic reserves, as stated by the right-hand equality in (H.10).

The following proposition characterizes the effect on restricted supply of a change in restricted reserves, and the effect on restricted and unrestricted supply at any date of a change in price occurring at any other date of the exploitation phase. It extends, in the context of the Hotelling-Gordon model, Propositions 1 and 2 which apply to a homogeneous resource within each deposit. Intertemporal cross-price effects are negative, whether reserves are endogenous or not; the impact of a price change on reserves does not dominate the direct substitution effect. However, this result holds in terms of cumulative, rather than instantaneous, supply.[36]

Proposition 7 (NRR supply with intra-deposit heterogeneity)
Assume that each deposit is heterogenous in the way just described.

1. (Stock effect) *An exogenous rise in exploitable reserves increases* cumulative *restricted supply at all dates.*

2. (Cross-price effects) *A price rise at any date $T \leq S$ reduces* cumulative *restricted and unrestricted supply at all dates $t \neq T$, where cumulative supply if $t > T$ is defined to exclude the supply at date T.*

The extension to aggregate NRR supply is straightforward.

I NRR supply with stock effects in extraction costs: Problem statement and preliminaries

The problem under investigation is (H.3)-(H.7). In this problem, (H.4) and (H.5) imply that $X_{t+1} \leq X_t$ for all $t \geq 0$. Thus it is sufficient that (H.7) be imposed when $t \to \infty$ or at the highest date considered. Let us call X_∞ the value of X_t at that date or its limit when $t \to \infty$. Then (H.7) can be replaced by

$$X_\infty \geq X^F \geq 0. \tag{I.1}$$

We denote μ_t and η_t, the Lagrange multipliers respectively associated with date-t constraints (H.4) and (H.5); the Lagrange multipliers associated with $X_\infty \geq X^F$ and $X^F \geq 0$ are λ and ε. The Lagrangian is

$$\begin{aligned} \mathcal{L} &= \sum_{t\geq 0} \left(p_t x_t - C_t(x_t, X_t) + \eta_t x_t\right) - E(X^0 - X^F) \\ &+ \sum_{t\geq 0} \mu_t \left(X_t - x_t - X_{t+1}\right) + \lambda \left(X_\infty - X^F\right) + \varepsilon X^F. \end{aligned} \tag{I.2}$$

It must be maximized with respect to x_t and X_t at all dates in both the long-run and the short-run versions of the problem. In the long-run version it is also maximized with

[36] Thus the results of Section 2 hold in terms of cumulative supply when extraction costs change with the stock of remaining reserves. They hold in terms of flows at date zero, the current date of extraction of the planning horizon. Indeed, from the present perspective of the producer, current supply and cumulative supply are identical.

respect to X^F, resulting in the long-run supply and cumulative supply functions $x_t^*(p)$ and $X_t^*(p)$. In the short-run version X^F, or equivalently developed reserves $X^0 - X^F$, is taken as given, resulting in the short-run (or restricted) supply and cumulative supply functions $\widetilde{x}_t\left(p, X^0 - X^F\right)$ and $\widetilde{X}_t\left(p, X^0 - X^F\right)$.

$\mathcal{L}$ is strictly concave in X^F by the convexity of E; $\mathcal{L}$ is strictly concave in x_t and X_t by the convexity of C_t; the latter means that

$$\frac{\partial^2 C_t(x,X)}{\partial x^2}, \ \frac{\partial^2 C_t(x,X)}{\partial X^2}, \text{ and } \frac{\partial^2 C_t(x,X)}{\partial x^2}\frac{\partial^2 C_t(x,X)}{\partial X^2} - \left(\frac{\partial^2 C_t(x,X)}{\partial x \partial X}\right)^2, \tag{I.3}$$

are all strictly positive for all $t \geq 0$, for all $X \geq 0$ and all $x > 0$.

Differentiating (I.2) with respect to x_t and X_t gives the following first-order conditions:

$$\mu_t - \eta_t = p_t - \frac{\partial C_t(x_t, X_t)}{\partial x}, \ \eta_t \geq 0, \ x_t\eta_t = 0, \ \forall t \geq 0 \tag{I.4}$$

and

$$\mu_{t-1} = \mu_t - \frac{\partial C_t(x_t, X_t)}{\partial X}, \ \forall t \geq 0. \tag{I.5}$$

Let S be defined as the last date at which extraction is strictly positive; S is endogenous; we assume for simplicity that $S \geq 1$. Also for simplicity, we assume that $x_0 > 0$.[37] Thus,

$$x_0 > 0, \ x_t \geq 0, \ \forall t = 1, ..., S-1, \ x_S > 0 \text{ and } x_t = 0, \ \forall t > S, \tag{I.6}$$

and, by (H.4),

$$X_t = X_{S+1}, \ \forall t \geq S+1. \tag{I.7}$$

Since the development of reserves is costly, the optimum plans of the producer will bind the first exhaustibility constraint, $X_\infty \geq X^F$, in (I.1). Precisely, starting at $t = S+1$, $X_t = X_{t+1}$ by (I.7) so that X_t must reach $X_\infty = X^F$ at $S+1$ if it is to equal X^F at the end of the program. Consequently,

$$X_t = X^F, \ x_t = 0, \ \eta_t \geq 0, \ \forall \ t \geq S+1; \tag{I.8}$$

$$X_t > X^F, \ x_t \geq 0, \ \eta_t \geq 0, \ \eta_t x_t = 0, \ \forall \ t \leq S. \tag{I.9}$$

It follows from (I.4) that

$$\mu_t = p_t - \frac{\partial C_t(x_t, X_t)}{\partial x}, \ x_t > 0, \ \forall t = 0, ..., S, \tag{I.10}$$

which is expression (H.8) in the text. If $x_t = 0$, either

$$\mu_t \geq p_t - \frac{\partial C_t(0, X_t)}{\partial x}, \ \forall t \leq S, \tag{I.11}$$

[37] The proofs of this appendix extend to the case where the deposit is not exploited at date 0, or only exploited at date 0. The results are also valid when $S \to \infty$.

or

$$\mu_t \geq p_t - \frac{\partial C_t(0, X^F)}{\partial x}, \ \forall t \geq S+1. \tag{I.12}$$

Since $C_t(0, X) = 0$ for all $X \geq 0$, $\frac{\partial C_t(0,X)}{\partial X} = 0$ for all $X \geq 0$. Therefore, before S, (I.5) implies

$$\mu_{t-1} = \mu_t, \text{ if } x_t = 0, \ \forall t < S. \tag{I.13}$$

Beyond S, since $x_t = 0$, (I.5) reduces to

$$\mu_{t-1} = \mu_t, \ \forall t \geq S+1. \tag{I.14}$$

(I.5) is unchanged at $t \leq S$ when extraction is strictly positive:

$$\mu_{t-1} = \mu_t - \frac{\partial C_t(x_t, X_t)}{\partial X}, \ x_t > 0, \ \forall t = 0, ..., S, \tag{I.15}$$

which is expression (H.9) in the text.

Since, starting at $t = S+1$, $x_t = 0$ and $X_t = X_{t+1}$ by (I.6) and (I.7), the sum $\sum_{t \geq 0} \mu_t (X_t - x_t - X_{t+1})$ in the Lagrangian (I.2) reduces to $\sum_{t=0,\ldots,S} \mu_t (X_t - x_t - X_{t+1})$. Consequently, the Lagrangian may be rewritten

$$\begin{aligned} \mathcal{L} &= \sum_{t \geq 0} (p_t x_t - C_t(x_t, X_t) + \eta_t x_t) - E(X^0 - X^F) \\ &+ \sum_{t=0,\ldots,S} \mu_t (X_t - x_t - X_{t+1}) + \lambda \left(X_\infty - X^F \right) + \varepsilon X^F, \end{aligned} \tag{I.16}$$

where (I.7) also implies that X_{S+1} in the term $\mu_S (X_S - x_S - X_{S+1})$ equals X_∞. It follows that the first-order condition to the choice of X_∞ requires the equality $\mu_S = \lambda$. Since μ_t remains constant starting at $t = S$ by (I.14), we obtain

$$\mu_t = \lambda, \ \forall t \geq S \tag{I.17}$$

This gives the equality on the left-hand side of (H.10) in the text.

When X^F is treated as endogenous (long run) and the constraint $X^F \geq 0$ in (I.1) is binding, the first-order condition associated with X^F in (I.2) is[38]

$$E'(X^0) = \lambda - \varepsilon, \ X^F = 0, \ \lambda > 0, \ \varepsilon \geq 0; \tag{I.18}$$

when $X^F > 0$, it must satisfy

$$E'(X^0 - X^F) = \lambda, \ X^F > 0, \ \lambda > 0, \ \varepsilon = 0. \tag{I.19}$$

This establishes the right-hand-side equality in (H.10).

[38]Given our assumptions on prices and costs, a strictly positive stock of reserves $X^0 - X^F$ is developed and exploited. Thus $E'(X^0 - X^F) > 0$, which will imply that $\lambda > 0$.

Let us define the short-run value function

$$\mathcal{V}\left(p, X^0 - X^F\right) \equiv \sum_{t\geq 0} \left(p_t x_t - C_t(x_t, X_t)\right), \tag{I.20}$$

where x_t and X_t are solutions $\widetilde{x}_t\left(p, X^0 - X^F\right)$ and $\widetilde{X}_t\left(p, X^0 - X^F\right)$ to the restricted ($X^0 - X^F$ fixed) version of Problem (H.3)-(H.7), i.e. satisfy (I.10), (I.11), (I.12), (I.13), (I.14) and (I.15). $\mathcal{V}\left(p, X^0 - X^F\right)$ denotes the total present-value revenue derived from the exploitation of the developed reserves $X^0 - X^F$. By standard interpretation, $\frac{\partial \mathcal{V}\left(p, X^0 - X^F\right)}{\partial (X^0 - X^F)}$ is the implicit value of the marginal extracted unit, which is also the implicit value λ of the marginal developed reserve unit:

$$\lambda \equiv \frac{\partial \mathcal{V}\left(p, X^0 - X^F\right)}{\partial (X^0 - X^F)} > 0. \tag{I.21}$$

By the assumption of cost convexity, $\mathcal{V}$ is increasing and strictly concave in $X^0 - X^F$.

In the long run, when X^F is endogenous, so that (I.18) and (I.19) hold; it follows

$$\begin{aligned} E'(X^0) &= \frac{\partial \mathcal{V}\left(p, X^0\right)}{\partial (X^0)} - \varepsilon, \;\; X^F = 0; \;\; \varepsilon \geq 0, \lambda > 0; \\ E'(X^0 - X^F) &= \frac{\partial \mathcal{V}\left(p, X^0 - X^F\right)}{\partial (X^0 - X^F)}, \;\; X^F > 0, \;\; \varepsilon = 0, \;\; \lambda > 0. \end{aligned}$$

When the constraint $X^F \geq 0$ on the availability of geological reserves is not binding, reserves are developed in such a way that the cost of developing the marginal unit is equal to the contribution of this unit to the intertemporal profit. However, in the case where the marginal cost of developing the totality of geological reserves falls short of the value of the marginal reserve unit, it is optimal to set $X^F = 0$, with $0 < E'\left(X^0\right) = \lambda - \varepsilon < \lambda$.

In the short run, when $X^0 - X^F$ is given, $(I.21)$ holds, but conditions (I.18) and (I.19) are not necessarily satisfied.

J Proof of Proposition 7

1. Stock effect on restricted cumulative supply

Assuming that exploitable reserves $X^0 - X^F$ are parametric, with $X_{S+1} = X^F$ as per (I.8), consider an increase $d(X^0 - X^F) > 0$, i.e. a reduction $dX^F < 0$ (X^0 is given); this requires $X^F > 0$. In the sequel, we establish the effect on the restricted supply and restricted cumulative supply functions $\widetilde{x}_t\left(p, X^0 - X^F\right)$ and $\widetilde{X}_t\left(p, X^0 - X^F\right)$ of this exogenous reserve increase; $\widetilde{x}_t\left(p, X^0 - X^F\right)$ and $\widetilde{X}_t\left(p, X^0 - X^F\right)$ are the values of X_t and x_t in the solution of Problem (H.3)-(H.7), whose Lagrangian is (I.2), when x_t and X_t are endogenous but X^F is exogenous.

The strict concavity of $\mathcal{V}$ in (I.21) implies that the rise in reserves causes a strict reduction in λ. The final extraction date S may be modified as a result of the reserve change. In what follows, we will denote by S the date at which extraction endogenously stops once the reserve change is taken into account.

Notation 1 *If S is modified by any parametric change, S denotes in these proofs the date at which extraction endogenously stops once the parameter change is taken into account. For example, if S changes from S_0 to S_1 as a result of a change in X^F from X_0^F to X_1^F, the notation x_S signifies x_{S_1} and dx_S signifies $x_{S_1}|_{X^F=X_1^F} - x_{S_1}|_{X^F=X_0^F}$.*

The proof makes use of several lemmas.

Lemma 2 *As a result of a reserve change $d(X^0 - X^F) > 0$, $d\lambda < 0$, $d\mu_S < 0$ and $dX_{S+1} < 0$.*

Proof. $d\lambda < 0$ is shown above.

If S is unchanged, $\mu_S = \lambda$ and $X_{S+1} = X^F$ before and after the reserves' change; the lemma immediately holds in that case.

If S is postponed, for all $t \geq S$, $\mu_t = \mu_S = \lambda$ by (I.17) and $X_{t+1} = X_{S+1} = X^F$, before and after the change; the lemma also immediately holds.

Examine now the case where S is advanced. Before the change, $\mu_t > \lambda$ by (I.15) and $X_{t+1} > X^F$ for all t strictly preceding the initial last-extraction date; in particular at the date S which will be the last extraction date after the change, remembering Notation 1, $X_{S+1} > X^F$. After the change, since $\mu_S = \lambda$ and $X_{S+1} = X^F$ while the change implies $d\lambda < 0$ and $dX^F < 0$, it follows that $d\mu_S < 0$ and $dX_{S+1} < 0$. QED

The following lemma will be used later to exploit Lemma 2.

Lemma 3 *If, as a result of the increase in reserves, $d\mu_t \geq 0$ and $dX_{t+1} \geq 0$ for some $t = 0, ..., S-1$, then $d\mu_{t+1} \geq 0$ and $dX_{t+2} \geq 0$.*

Proof. For some date $t = 0, ..., S-1$, assume that the change in reserves causes changes $d\mu_t \geq 0$ and $dX_{t+1} \geq 0$.

If x_{t+1} was null and remains so as a result of the change in reserves, then $\mu_{t+1} = \mu_t$ by (I.13); it follows that $d\mu_{t+1} \geq 0$. Also, $dx_{t+1} = 0$ implies by (H.4) that $dX_{t+2} = dX_{t+1} \geq 0$.

Consider now that x_{t+1} was strictly positive and remains so after the change, so that (I.10) holds. Assume, as a premise to be contradicted, that $d\mu_{t+1} < 0$. On the one hand, total differentiation of (I.10) at date $t+1$ then implies

$$dx_{t+1} > \frac{-\frac{\partial^2 C_{t+1}}{\partial x \partial X}}{\frac{\partial^2 C_{t+1}}{\partial x^2}} dX_{t+1}. \tag{J.1}$$

On the other hand, totally differentiating (I.15) and (I.10), taken at $t+1$, and substituting yield

$$d\mu_t = -\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right] dx_{t+1} - \left[\frac{\partial^2 C_{t+1}}{\partial X \partial x} + \frac{\partial^2 C_{t+1}}{\partial X^2}\right] dX_{t+1}. \tag{J.2}$$

The first term between brackets is strictly positive by assumption (H.2). Using (J.1), and simplifying it follows that

$$d\mu_t < \left(\frac{\partial^2 C_{t+1}}{\partial x^2}\right)^{-1} \left[\left(\frac{\partial^2 C_{t+1}}{\partial x \partial X}\right)^2 - \frac{\partial^2 C_{t+1}}{\partial x^2}\frac{\partial^2 C_{t+1}}{\partial X^2}\right] dX_{t+1},$$

where assumption (I.3) implies that the term multiplying dX_{t+1} is negative. Thus the lemma's assumption $dX_{t+1} \geq 0$ implies that $d\mu_t < 0$. This contradicts the other lemma's assumption $d\mu_t \geq 0$. We conclude that the maintained assumption $d\mu_{t+1} < 0$ cannot hold and that $d\mu_{t+1} \geq 0$ when x_{t+1} remains positive as a result of the change.

Consider finally the (intermediate) cases where extraction is zero before the reserve change and becomes strictly positive as a result of the change, or *vice versa*, where extraction is strictly positive and becomes zero. We just showed that $d\mu_{t+1} \geq 0$ if extraction is null and remains so, and also if extraction is strictly positive and remains so. By the Maximum Theorem μ_{t+1} is continuous across these cases, so that we also have $d\mu_{t+1} \geq 0$.[39]

We have shown that $d\mu_{t+1} \geq 0$; now consider dX_{t+2}. Consider first that extraction x_{t+2} was null before the reserve change and remains so after the change. In that case, (H.4) implies that $dX_{t+2} = dX_{t+1}$, which is positive as lemma assumption. Consider now that extraction x_{t+2} was and remains strictly positive. In that case, (I.10) holds before and after the reserve change, so that the rise $d\mu_{t+1} \geq 0$ implies a reduction in $\frac{\partial C_{t+1}}{\partial x}$, that is by total differentiation of (I.10)

$$dx_{t+1} \leq \frac{-\frac{\partial^2 C_{t+1}}{\partial x \partial X}}{\frac{\partial^2 C_{t+1}}{\partial x^2}} dX_{t+1}. \tag{J.3}$$

By assumptions (H.1) and (H.2), the coefficient of dX_{t+1} in the above inequality is positive and lower than unity. Thus the inequality implies that $dx_{t+1} \leq dX_{t+1}$, which yields $dX_{t+2} \geq 0$. The continuity argument made earlier to invoke the Maximum Theorem also applies here for cases where extraction x_{t+2} becomes positive or null as a result of the change in reserves. QED

The combination of Lemma 2 and Lemma 3 will give the following result.

Lemma 4 *As a result of the reserve increase, $d\mu_0 < 0$ and $dX_1 < 0$, and for all $t = 1, ..., S$, either $d\mu_t < 0$ or $dX_{t+1} < 0$.*

Proof. This lemma follows from Lemma 2 and the contrapositive of the series of implications in Lemma 3. Indeed, the final implication of Lemma 3 that $\mu_S \geq 0$ is contradicted by Lemma 2. Thus for all $t = 0, ..., S$, $d\mu_t \geq 0$ and $dX_{t+1} \geq 0$ do not hold at the same time: either $d\mu_t < 0$ or $dX_{t+1} < 0$, where the relation "or" is not exclusive.

For $t = 0$, $x_0 > 0$ by (I.6) so that (I.10) holds. It follows that $d\mu_0 < 0$ is equivalent to $dx_0 > 0$ since X^0 is given, which is equivalent by (H.4) to $dX_1 < 0$. Thus the proposition that "either $d\mu_0 < 0$ *or* $dX_1 < 0$ holds" is equivalent to "$d\mu_0 < 0$ *and* $dX_1 < 0$ hold." QED

[39] The Maximum Theorem applies as follows to the restricted problem under study. The exploitable reserves parameter $X^0 - X^F$ continuously affects the extraction possibility set defined by (H.4), (H.5) and (H.7), and continuously affects the objective (H.3). Since $X^0 - X^F$ must be finite by (H.7) and by the finiteness of X^0, the extraction possibility set is bounded, and is evidently closed. Furthermore, the objective (H.3) is strictly concave by assumption and for any given reserves level $X^0 - X^F$, the set of extraction possibilities is convex since the convex combination of two possible extraction paths satisfying the exhaustibility constraint satisfies the exhaustibility constraint. The Maximum Theorem thus applies, implying that the optimum extraction path $(x_t, X_t)_{t \geq 0}$ is a continuous function of the reserve level $X^0 - X^F$. In turn, because all multipliers μ_t, $t \geq 0$, are defined as continuous functions of x_t and X_t by (I.10) and (I.14), it follows that in optimum they are continuous functions of $X^0 - X^F$.

The result that $d\mu_0 < 0$ and $dX_1 < 0$ will be later exploited by use of the following lemma, whose proof partly relies on the other part of Lemma 4.

Lemma 5 *If, as a result of the reserve increase,* $d\mu_t < 0$ *and* $dX_{t+1} < 0$ *for some* $t = 0, ..., S-1$, *then* $d\mu_{t+1} < 0$ *and* $dX_{t+2} < 0$.

Proof. For some $t = 0, ..., S-1$, assume that $d\mu_t < 0$ and $dX_{t+1} < 0$ hold simultaneously.

Suppose, as an assumption to be contradicted, that $dX_{t+2} \geq 0$.

There are several possibilities as far as extraction x_{t+1} is affected by the change in reserves. First consider the case where $x_{t+1} = 0$ before and after the change. Then by (H.4), $dX_{t+2} = dX_{t+1} < 0$, which contradicts the maintained assumption.

Consider now the case where extraction x_{t+1} was strictly positive before the change in reserves and remains so after the change. By Lemma 4, $dX_{t+2} \geq 0$ implies

$$d\mu_{t+1} < 0. \tag{J.4}$$

On the other hand, by (H.4), $dX_{t+2} \geq 0$ implies

$$dX_{t+1} \geq dx_{t+1}. \tag{J.5}$$

Differentiating (I.10) at $t+1$ gives

$$d\mu_{t+1} = -\frac{\partial^2 C_{t+1}}{\partial x^2} dx_{t+1} - \frac{\partial^2 C_{t+1}}{\partial x \partial X} dX_{t+1},$$

where the coefficient of dx_{t+1} is strictly negative by assumption (I.3). Substituting for dx_{t+1} by use of inequality (J.5) implies

$$d\mu_{t+1} \geq -\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right] dX_{t+1},$$

where the term between brackets is strictly positive by (H.2). Since $dX_{t+1} < 0$ by assumption of the lemma, the inequality implies that $d\mu_{t+1} > 0$, which contradicts (J.4).

Therefore, whether extraction at $t+1$ remains zero or remains strictly positive as a result of the reserve change, it must be true that $dX_{t+2} < 0$. Finally consider the (intermediate) cases where x_{t+1} was zero and becomes strictly positive or, *vice versa*, was strictly positive and becomes zero. By the Maximum Theorem (see Footnote 39 on how it applies here), for any $t \geq 0$, X_{t+2} is continuous in reserves. Hence, the result that $dX_{t+2} < 0$ also applies across the cases where extraction remains zero or strictly positive, that is in the intermediate cases.

We conclude that, whether extraction was and remains zero or becomes strictly positive, or else was strictly positive and remains so or falls to zero at date $t+1$,

$$dX_{t+2} < 0.$$

It follows by (H.4) that

$$dX_{t+1} < dx_{t+1}. \tag{J.6}$$

Let us now show that $d\mu_{t+1} < 0$. Consider first that extraction x_{t+1} was null before the reserve change and remains so after the change. In that case, $\mu_{t+1} = \mu_t$ before and after the change by (I.13) or (I.14). It follows that $d\mu_{t+1} = d\mu_t < 0$ by the lemma's assumption that $d\mu_t < 0$. Consider now that extraction x_{t+1} was and remains strictly positive. In that case, (I.10) holds at $t+1$ before and after the reserve change, which gives $d\mu_{t+1} = -\frac{\partial^2 C_{t+1}}{\partial x^2} dx_{t+1} - \frac{\partial^2 C_{t+1}}{\partial x \partial X} dX_{t+1}$, where the coefficient of dx_{t+1} is strictly negative by (I.3). Substituting for dx_{t+1} by use of inequality (J.6) implies

$$d\mu_{t+1} \leq -\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right] dX_{t+1},$$

where the term between brackets is strictly positive by (H.2). Since $dX_{t+1} < 0$ by assumption of the lemma, the inequality implies that $d\mu_{t+1} < 0$. The continuity argument invoked earlier (Maximum Theorem; see Footnote 39) also applies here for cases where extraction x_{t+1} becomes positive or null as a result of the change in reserves. QED

The following lemma concludes the proof of Proposition 7.1.

Lemma 6 *As a result of the increase in reserves, $d\mu_t < 0$ and $dX_{t+1} < 0$ for all $t = 0, ..., S$.*

Proof. By Lemma 4, $d\mu_0 < 0$. Since $x_0 > 0$ by assumption (I.6), (I.10) holds, where X_0 is fixed. It follows from $d\mu_0 < 0$ in (I.10) that $dx_0 > 0$. In turn, (H.4) implies $dX_1 < 0$.

$d\mu_t < 0$ and $dX_{t+1} < 0$ are thus simultaneously verified for $t = 0$, which implies by Lemma 5 that $d\mu_{t+1} < 0$ and $dX_{t+2} < 0$, for all $t = 0, ..., S-1$. Lemma 6 is obtained by recurrence, thus completing the proof of Proposition 7.1. QED

2. Cross-price effects on restricted and unrestricted supply

The following proof will simultaneously establish the effects of a price change on restricted cumulative supply $X^0 - \widetilde{X}_t\left(p, X^0 - X^F\right)$ and on unrestricted cumulative supply $X^0 - X_t^*(p)$. $X_t^*(p)$ is the value of X_t in the solution of Problem (H.3)-(H.7), whose Lagrangian is (I.2), when x_t, X_t and X^F are treated as endogenous; $\widetilde{X}_t\left(p, X^0 - X^F\right)$ is the value of X_t in the solution of Problem (H.3)-(H.7) when x_t and X_t are endogenous but X^F is exogenous.

We will consider a strict price rise $dp_T > 0$ at some date $T \geq 1$ that differs from the pre-change date of last strictly-positive extraction, and is such that x_T is strictly positive before the change; the proof can easily be extended to $T = 0$ and to a price rise occurring at the pre-change date of last extraction. The restriction that $x_T > 0$ rules out the uninteresting possibility that the price rise has no effect on λ because it applies to a null extraction base. It also implies that the price rise does not occur at a date posterior to the pre-change date of last extraction.

When the size of reserves is restricted, according to (I.21), the value of the marginal unit of reserves is $\lambda = \frac{\partial \mathcal{V}\left(p, X^0 - X^F\right)}{\partial (X^0 - X^F)}$, with $\frac{\partial^2 \mathcal{V}\left(p, X^0 - X^F\right)}{\partial (X^0 - X^F) \partial p_T} > 0$. Thus the price rise $dp_T > 0$ at some date of the extraction phase implies $d\lambda > 0$.

When reserves are not restricted, there are two possibilities. When $X^F > 0$, (I.19) and (I.21) hold: Developed and exploited reserves $X^0 - X^F$ and their marginal value λ are jointly determined by the equality of the strictly rising marginal development cost function

$E'(X^0 - X^F)$ with the decreasing marginal value of reserves $\frac{\partial \mathcal{V}(p, X^0 - X^F)}{\partial (X^0 - X^F)}$. In that context, a price rise $dp_T > 0$ at some date of the extraction phase causes a rise $d\lambda > 0$ and an increase in developed and exploited reserves $d(X^0 - X^F) > 0$ or, equivalently, $dX^F < 0$. When $X^F = 0$ before the price rise, the above effect does not take place: $d(X^0 - X^F) = 0$ despite the absence of reserve restriction so that $d\lambda > 0$ as when reserves are restricted.

The following lemma gathers those results.

Lemma 7 *A price rise $dp_T > 0$ at some date $T \geq 1$ such that $x_T > 0$ and T strictly precedes the last date of strictly positive extraction, causes the value of the marginal reserve unit to increase strictly ($d\lambda > 0$) and developed reserves to increase ($dX^F \leq 0$).*

The possibility that $dX^F = 0$ ensures that Lemma 7 holds whether supply is restricted or unrestricted and, in the latter case, whether the constraint $X^F \geq 0$ is binding or not.

The second step of this proof involves the backward recurrence described in Lemma 8. This recurrence will be used to assess the effect of the price change on quantities at the terminal date S and then at all dates between T and S.

Lemma 8 *If, as a result of the date-T price rise, $d\mu_t > 0$ and $dX_t \leq dx_t \leq 0$ for some date $t \leq S$, $t \neq T$, then $d\mu_{t-1} > 0$ and $dX_{t-1} \leq dx_{t-1} \leq 0$, $t - 1 \neq T$.*

Proof. Assume that for some date $t \neq T$, $1 < t \leq S$, $d\mu_t > 0$ and $dX_t \leq dx_t \leq 0$.

First, consider the case where x_t was null before the price change and remains so afterwards. Then, $dx_t = 0$, so that $\mu_{t-1} = \mu_t$ by (I.13). By the maintained assumption $d\mu_t > 0$, it follows that $d\mu_{t-1} = d\mu_t > 0$ in that case.

Second, consider the situation where x_t was strictly positive before the change in price, and remains so with the change. (I.10) holds in that case; totally differentiating (I.10) at date t and using $d\mu_t > 0$ yields

$$dx_t < \frac{-\frac{\partial^2 C_t}{\partial x \partial X}}{\frac{\partial^2 C_t}{\partial x^2}} dX_t. \tag{J.7}$$

Assume now that $t - 1 \neq T$. Replacing μ_t in (I.15) by its expression as per (I.10) and totally differentiating give

$$d\mu_{t-1} = -\left[\frac{\partial^2 C_t}{\partial x^2} + \frac{\partial^2 C_t}{\partial x \partial X}\right] dx_t - \left[\frac{\partial^2 C_t}{\partial X^2} + \frac{\partial^2 C_t}{\partial X \partial x}\right] dX_t,$$

where the first term between brackets is strictly positive by assumption (H.2). Substituting dx_t by use of inequality (J.7) and rearranging, we obtain

$$d\mu_{t-1} > \left(\frac{\partial^2 C_t}{\partial x^2}\right)^{-1} \left[\left(\frac{\partial^2 C_t}{\partial x \partial X}\right)^2 - \frac{\partial^2 C_t}{\partial x^2}\frac{\partial^2 C_t}{\partial X^2}\right] dX_t, \tag{J.8}$$

where by assumption (I.3) the term between parentheses is positive and the term between brackets is negative. Since $dX_t \leq 0$ by the assumption of this proof, it follows from (J.8) that $d\mu_{t-1} > 0$ also in that case.

One can conclude that the rise in price yields $d\mu_{t-1} > 0$, whether x_t decreases and remains strictly positive or x_t is and remains zero. The Maximum Theorem implies that μ_{t-1} is continuous[40] across these cases, so that $d\mu_{t-1} > 0$ also holds as a result of the price rise when x_t decreases from a strictly positive level to zero.

As far as date $t-1$ is concerned, there are two possibilities. If x_{t-1} was and remains zero following the price rise, $dx_{t-1} = 0$ so that (H.4) implies $dX_{t-1} = dX_t \leq 0$ by the maintained assumptions. Therefore, $dX_{t-1} \leq 0 = dx_{t-1}$; the lemma applies in that case.

When $x_{t-1} > 0$, the maintained assumption $dX_t \leq 0$ implies by (H.4) that

$$dX_{t-1} \leq dx_{t-1}. \tag{J.9}$$

On the other hand, the differentiation of (I.10) at $t-1$ gives $d\mu_{t-1} = -\frac{\partial^2 C_{t-1}}{\partial x^2} dx_{t-1} - \frac{\partial^2 C_{t-1}}{\partial x \partial X} dX_{t-1}$, where $\frac{\partial^2 C_{t-1}}{\partial x \partial X}$ is strictly negative by assumption (H.1). Substituting for dX_{t-1} by use of inequality (J.9) thus yields

$$d\mu_{t-1} < -\left[\frac{\partial^2 C_{t-1}}{\partial x^2} + \frac{\partial^2 C_{t-1}}{\partial x \partial X}\right] dx_{t-1},$$

where the term between brackets is positive by assumption (H.2). We have shown above that $d\mu_{t-1} > 0$, so that dx_{t-1} must be strictly negative. (J.9) thus implies $dX_{t-1} \leq dx_{t-1} < 0$.

In the case where x_{t-1} decreases with the price rise in such a way that it becomes zero, the latter inequality must be adjusted to $dX_{t-1} \leq dx_{t-1} \leq 0$; the lemma also applies. QED

Let us now examine the effect of the price change at the last extraction date S.

Lemma 9 *Following the date-T price rise, at the date S of last strictly positive extraction, $d\mu_S > 0$ and $dX_S \leq dx_S < 0$.*

Proof. The date of last extraction may change as a result of the price rise considered in Lemma 7. However it cannot be postponed. Date T of the price rise does not occur at a date with no extraction; thus T is not posterior to the pre-change last extraction date. Therefore if S is a date at which extraction had already stopped before the price rise, condition (I.12) had to hold before the price rise; considering (I.17), this implies that $\mu_t = \lambda > p_t - \frac{\partial C_t(0,X^F)}{\partial x}$, where by Lemma 7 the price change induces $d\lambda > 0$ and $dX^F \leq 0$. It follows from (H.1) that $dX^F \leq 0$ does not reduce the marginal cost $\frac{\partial C_t(0,X^F)}{\partial x}$; as a result, no rise $dx_t > 0$ can

[40] The Maximum Theorem applies to price changes in the context of this proof as explained shortly below. The continuity of multipliers μ_t, for all dates $t \geq 0$ follows because those multipliers are continuous functions of x_t and X_t variables and price parameters by (I.10), (I.13) and (I.14). Whether Problem (H.3)-(H.7) is restricted ($X^0 - X^F$ given) or not (X^F free), the Maximum Theorem applies as follows, when the parameters of interest are prices. The extraction possibility set defined by (H.4), (H.5) and (H.7), is independent of price parameters, hence continuous. Price parameters also continuously affect the objective (H.3). Even in the unrestricted problem, the finiteness of geological reserves X^0 ensures that developed reserves $X^0 - X^F \geq 0$ are finite. Thus the extraction possibility set is bounded, and is evidently closed. Furthermore, the objective (H.3) is strictly concave by assumption and for any vector of prices, the set of extraction possibilities is convex since the convex combination of two possible extraction paths satisfying the exhaustibility constraint satisfies the exhaustibility constraint. The Maximum Theorem thus applies, implying that the optimum extraction path $(x_t, X_t)_{t\geq 0}$ is a continuous function of any component of the price vector.

cause equality (I.10) to be satisfied. However, following the price rise, (I.10) must hold at S by definition of the after-change last *strictly-positive* extraction date.

Thus S can only be advanced or left unchanged by the price rise. Then the equalities $\mu_S = \lambda$ and $X_{S+1} = X^F$ hold as per (I.17) and (I.8) respectively. Thus Lemma 7 implies $d\mu_S > 0$ and $dX_{S+1} \leq 0$; the latter is equivalent by (H.4) to $dX_S \leq dx_S$. Totally differentiating (I.10) and substituting dX_S by use of the latter inequality yield

$$d\mu_S \leq -\left[\frac{\partial^2 C_S}{\partial x^2} + \frac{\partial^2 C_S}{\partial x \partial X}\right] dx_S, \tag{J.10}$$

where the term between brackets is positive by assumption (H.2). Since $d\mu_S > 0$ in this case, it follows from inequality (J.10) that $dx_S < 0$, which remains compatible with $x_S > 0$. Therefore, $dX_S \leq dx_S < 0$. QED

Lemma 10 immediately follows from the combination of Lemma 9 with Lemma 8's recurrence.

Lemma 10 *As a result of the date-T price rise, $d\mu_t > 0$ and $dX_t \leq dx_t \leq 0$, for all $t = T+1, ..., S$.*

Let us now examine the effects of the price rise at the date T when it occurs. We will establish the following lemma.

Lemma 11 *As a result of the date-T price rise, $dx_T > 0$, $dX_T \leq dx_T$, and $d\mu_T > 0$.*

Proof. By Lemma 10,

$$dX_{T+1} \leq dx_{T+1} \leq 0, \tag{J.11}$$

where $dX_{T+1} \leq 0$ implies

$$dX_T \leq dx_T.$$

The law of supply prevails at date T, which implies, under Lemma 7's assumption that $x_T > 0$, the strict inequality

$$dx_T > 0. \tag{J.12}$$

From Lemma 10, $d\mu_{T+1} > 0$. Let us now show that $d\mu_T > 0$.

Consider first the case $x_{T+1} = 0$ before and after the price change; by (I.13), $\mu_T = \mu_{T+1}$. It follows that $d\mu_T = d\mu_{T+1} > 0$.

Now consider the case where x_{T+1} was strictly positive before the price change and remains so. Then, (I.10) holds; $d\mu_{T+1} > 0$ implies

$$dx_{T+1} < \frac{-\frac{\partial^2 C_{T+1}}{\partial x \partial X}}{\frac{\partial^2 C_{T+1}}{\partial x^2}} dX_{T+1}. \tag{J.13}$$

Taking (I.15) at $t = T+1$ and using (I.10) to eliminate μ_{T+1} gives

$$d\mu_T = -\left[\frac{\partial^2 C_{T+1}}{\partial x^2} + \frac{\partial^2 C_{T+1}}{\partial x \partial X}\right] dx_{T+1} - \left[\frac{\partial^2 C_{T+1}}{\partial X^2} + \frac{\partial^2 C_{T+1}}{\partial X \partial x}\right] dX_{T+1},$$

where the first term between brackets is strictly positive by assumption (H.2). Substituting dx_{T+1} by use of inequality (J.13) and rearranging, one obtains

$$d\mu_T > \left(\frac{\partial^2 C_{T+1}}{\partial x^2}\right)^{-1} \left[\left(\frac{\partial^2 C_{T+1}}{\partial x \partial X}\right)^2 - \frac{\partial^2 C_{T+1}}{\partial x^2}\frac{\partial^2 C_{T+1}}{\partial X^2}\right] dX_{T+1},$$

where by assumption (I.3) the term between parentheses is strictly positive and the term between brackets is strictly negative. Since $dX_{T+1} \leq 0$, the latter inequality implies that $d\mu_T > 0$.

Finally consider the situation where x_{T+1} was strictly positive before the change and decreases so as to become null with the price change. By the Maximum Theorem (see Footnote 40), the continuity of x_{T+1} with the price guarantees that the above results apply in that situation. QED

The analysis will now turn to the effect of the price change on quantities at dates that precede the date T of the change. We will first establish the following recurrence, that will be used shortly below.

Lemma 12 *If, as a result of the price rise, $d\mu_t \leq 0$ and $dX_{t+1} \leq 0$ for some $t = 0, ..., T-2$, then $d\mu_{t+1} \leq 0$ and $dX_{t+2} \leq 0$.*

Proof. For some $t = 0, ..., T-2$, assume that $d\mu_t \leq 0$ and $dX_{t+1} \leq 0$ as a result of the price rise.

Consider first the case where x_{t+1} was zero before the price rise and remains so afterwards. In that case, by (I.13), $\mu_t = \mu_{t+1}$. It follows that $d\mu_{t+1} = d\mu_t \leq 0$. Moreover, differentiating (H.4) gives $dX_{t+2} = dX_{t+1} - dx_{t+1}$. With $dx_{t+1} = 0$ in that case and $dX_{t+1} \leq 0$ by the maintained assumption, it follows $dX_{t+2} \leq 0$.

Second, consider the case where extraction x_{t+1} initially was and remains strictly positive with the price rise so that (I.10) holds. Taking (I.15) at $t+1$, substituting for $d\mu_{t+1}$ using (I.10), and differentiating, we obtain

$$d\mu_t = -\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right] dx_{t+1} - \left[\frac{\partial^2 C_{t+1}}{\partial X^2} + \frac{\partial^2 C_{t+1}}{\partial X \partial x}\right] dX_{t+1},$$

where the first term between brackets is strictly positive by assumption (H.2). Thus $d\mu_t \leq 0$ implies

$$dx_{t+1} \geq \frac{-\left[\frac{\partial^2 C_{t+1}}{\partial X^2} + \frac{\partial^2 C_{t+1}}{\partial X \partial x}\right]}{\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right]} dX_{t+1}. \tag{J.14}$$

On the other hand, the differentiation of (I.10) at $t+1$ yields

$$d\mu_{t+1} = -\frac{\partial^2 C_{t+1}}{\partial x^2} dx_{t+1} - \frac{\partial^2 C_{t+1}}{\partial x \partial X} dX_{t+1}, \tag{J.15}$$

where $-\frac{\partial^2 C_{t+1}}{\partial x^2}$ is strictly negative by (I.3). Substituting for dx_{t+1} using (J.14) and rearranging, one obtains

$$d\mu_{t+1} \leq \frac{\left[\frac{\partial^2 C_{t+1}}{\partial x^2}\frac{\partial^2 C_{t+1}}{\partial X^2} - \left(\frac{\partial^2 C_{t+1}}{\partial x \partial X}\right)^2\right]}{\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right]} dX_{t+1},$$

where the coefficient of dX_{t+1} is strictly positive by (H.2) and (I.3). Thus the assumption $dX_{t+1} \leq 0$ implies $d\mu_{t+1} \leq 0$.

With $d\mu_{t+1} \leq 0$, (J.15) implies

$$dx_{t+1} \geq \frac{-\frac{\partial^2 C_{t+1}}{\partial x \partial X}}{\frac{\partial^2 C_{t+1}}{\partial x^2}} dX_{t+1},$$

where the coefficient of dX_{t+1} is positive and lower than unity by (H.2), while $dX_{t+1} \leq 0$ by the maintained assumption. It follows that $dx_{t+1} \geq dX_{t+1}$, which by (H.4) implies $dX_{t+2} \leq 0$.

Last, by continuity (see Footnote 40 on the Maximum Theorem), the above results also hold across cases, when x_{t+1} becomes strictly positive or becomes zero following the price rise. QED

We can now show the following result.

Lemma 13 *As a result of the price rise, $d\mu_0 > 0$ and, for all $t = 1, ..., T$, either $d\mu_t > 0$ or $dX_{t+1} > 0$.*

Proof. As a result of Lemma 12's recurrence, if $d\mu_t \leq 0$ and $dX_{t+1} \leq 0$ for some $t \leq T-2$, then $d\mu_{T-1} \leq 0$ and $dX_T \leq 0$. Suppose, as an assumption to be contradicted, that $d\mu_{T-1} \leq 0$ and $dX_T \leq 0$. Differentiating (I.15) at $t = T$, we have

$$d\mu_{T-1} = d\mu_T - \frac{\partial^2 C_T}{\partial X \partial x} dx_T - \frac{\partial^2 C_T}{\partial X^2} dX_T.$$

On the left-hand side, $d\mu_{T-1} \leq 0$ by assumption. On the right-hand side, the last term is positive by (I.3) and by the maintained assumption $dX_T \leq 0$; $d\mu_T > 0$ by Lemma 11; and the second term is strictly positive by Lemma 11 and (H.1). Thus the right-hand side is strictly positive while the left-hand side in non positive. This contradiction implies that $d\mu_{T-1} \leq 0$ and $dX_T \leq 0$ do not hold simultaneously so that either $d\mu_{T-1} > 0$ or (non exclusive) $dX_T > 0$.

Now use the following contrapositive of Lemma 12's implication: If as a result of the price rise, for some $t = 0, ..., T-2$, $d\mu_{t+1} > 0$ or (non exclusive) $dX_{t+2} > 0$, then either $d\mu_t > 0$ or (non exclusive) $dX_{t+1} > 0$. By backward recurrence, starting from the result established above that either $d\mu_{T-1} > 0$ or (non exclusive) $dX_T > 0$, it follows that, for all $t = 0, ..., T-1$, either $d\mu_t > 0$ or (non exclusive) $dX_{t+1} > 0$. At date $t = 0$, we have already shown by differentiation of (I.10) where X_0 is given, that $d\mu_0 > 0$ is equivalent to $dx_0 < 0$, also equivalent by (H.4) to $dX_1 > 0$. Therefore, "$d\mu_0 > 0$ or (non exclusive) $dX_1 > 0$" is equivalent to "$d\mu_0 > 0$ (and $dX_1 > 0$)." QED

The following lemma will conclude the proof of Proposition 7.2.

Lemma 14 *As a result of the date-T price rise,* $d\mu_t > 0$ *and* $dX_{t+1} > 0$*, for all* $t = 0, ..., T-1$.

Proof. By Lemma 13, $d\mu_0 > 0$, and therefore, $dX_1 > 0$. Assume that for some $t = 0, ..., T-2$, $d\mu_t > 0$ and $dX_{t+1} > 0$; we will show that this implies $d\mu_{t+1} > 0$ and $dX_{t+2} > 0$.

First consider the situation where $x_{t+1} = 0$ before and after the change in price. In that case, by (I.13), $\mu_t = \mu_{t+1}$, which immediately shows that $d\mu_{t+1} > 0$. Also, the differentiation of (H.4) with $dx_{t+1} = 0$ implies $dX_{t+2} = dX_{t+1} > 0$.

Second, consider that x_{t+1} was strictly positive and remains so after the change. In that case, (I.10) holds at $t+1$. Suppose, as an assumption to be contradicted, that $d\mu_{t+1} \leq 0$. On the one hand, by the total differentiation of (I.10) at date $t+1$, $d\mu_{t+1} \leq 0$ is equivalent to

$$dx_{t+1} \geq \frac{-\frac{\partial^2 C_{t+1}}{\partial x \partial X}}{\frac{\partial^2 C_{t+1}}{\partial x^2}} dX_{t+1}. \tag{J.16}$$

On the other hand, totally differentiating (I.15) at $t+1$ and substituting for $d\mu_{t+1}$ from (I.10) yield

$$d\mu_t = -\left[\frac{\partial^2 C_{t+1}}{\partial x^2} + \frac{\partial^2 C_{t+1}}{\partial x \partial X}\right] dx_{t+1} - \left[\frac{\partial^2 C_{t+1}}{\partial X \partial x} + \frac{\partial^2 C_{t+1}}{\partial X^2}\right] dX_{t+1}, \tag{J.17}$$

where the first term between brackets on the right-hand side is strictly positive by assumption (H.2). Using (J.16) and simplifying it follows that

$$d\mu_t \leq \left(\frac{\partial^2 C_{t+1}}{\partial x^2}\right)^{-1} \left[\left(\frac{\partial^2 C_{t+1}}{\partial x \partial X}\right)^2 - \frac{\partial^2 C_{t+1}}{\partial x^2}\frac{\partial^2 C_{t+1}}{\partial X^2}\right] dX_{t+1}, \tag{J.18}$$

where assumption (I.3) implies that the coefficient of dX_{t+1} is strictly negative. Thus the lemma's assumption that $dX_{t+1} > 0$ implies $d\mu_t < 0$, which contradicts the other lemma's assumption that $d\mu_t > 0$. Thus the maintained assumption $d\mu_{t+1} \leq 0$ implies a contradiction and one must conclude that $d\mu_{t+1} > 0$.

We still have to show that $dX_{t+2} > 0$. By (I.10), the rise $d\mu_{t+1} > 0$ requires a strict reduction in $\frac{\partial C_{t+1}}{\partial x}$, that is

$$dx_{t+1} < \frac{-\frac{\partial^2 C_{t+1}}{\partial x \partial X}}{\frac{\partial^2 C_{t+1}}{\partial x^2}} dX_{t+1}. \tag{J.19}$$

By assumptions (H.2) and (I.3), the coefficient of dX_{t+1} in the above inequality is strictly positive and lower than unity. Since $dX_{t+1} > 0$ as maintained assumption, it follows from (J.19) that $dx_{t+1} < dX_{t+1}$. The latter inequality finally implies by (H.4) that $dX_{t+2} > 0$.

Given that the maintained assumption $d\mu_t > 0$ and $dX_{t+1} > 0$ is satisfied at $t = 0$ (Lemma 13), Lemma 14 has been proven both when extraction x_{t+1} is and remains null or when it is and remains strictly positive. By continuity (see the application of the Maximum Theorem in Footnote 40), it follows that Lemma 14 also holds across cases, that is when x_{t+1} becomes null or becomes strictly positive following the price rise. QED

Let us now sum up the results. As a consequence of the rise in p_T, X_t increases by Lemma 14, implying that cumulative supply $X^0 - X_t$ decreases, at all dates $t = 1, ..., T$. By the law of supply (Lemma 11), date-T instantaneous supply x_T not only increases but increases in such a way that the subsequent cumulative supply $X^0 - X_{T+1}$ is higher than before the price rise. However instantaneous supply x_t decreases at all subsequent dates $t > T$ by Lemma 10. Hence, defining cumulative supply as excluding date-T supply yields the second point of Proposition 7.

K Costly adjustment in NRR extraction

For simplicity, we focus on a single deposit whose reserves X are developed at date 0. The deposit is identical to those described in Section 2, except that, at the development date 0, a capital investment $K \geq 0$ can be made by the producer so as to lower the extraction cost over the exploitation period. The extraction cost becomes

$$C_t(x_t, K),$$

with the same properties as in Section 2, except that

$$\frac{\partial C_t(x,K)}{\partial K} < 0, \frac{\partial^2 C_t(x,K)}{\partial x \partial K} < 0 \text{ and } \frac{\partial^2 C_t(x,K)}{\partial K^2} > 0.$$

The first two assumptions reflect that more investment in extraction capacity makes extraction less costly, while the third one means that this investment is subject to decreasing returns to scale.

In a long-run perspective, not only the developed reserves X but also the capital investment K are free. In that context, the problem of the deposit's competitive producer is

$$\max_{(x_t)_{t\geq 0}, X, K} \sum_{t\geq 0} \left(p_t x_t - C_t(x_t, K)\right) - E(X) - K \tag{K.1}$$

subject to the binding exhaustibility constraint

$$\sum_{t\geq 0} x_t = X. \tag{K.2}$$

Assuming that the problem is well-behaved, the first-order conditions associated with this problem are

$$p_t - \frac{\partial C_t(x_t, K)}{\partial x_t} = \lambda, \ \forall t \geq 0 \tag{K.3}$$

for the choice of extraction, where λ denotes the co-variable associated with (K.2),

$$E'(X) = \lambda \tag{K.4}$$

for the choice of developed and exploited reserves, and,

$$-\sum_{t\geq 0}\frac{\partial C_t(x_t,K)}{\partial K}=1 \tag{K.5}$$

for the choice of capacity.

Consider the latter first. It tells that the profit-maximizing investment in capacity equates the marginal cost to the sum of cost reductions generated by the marginal capacity unit. Assume, for simplicity, that these reductions are proportional to the extracted quantity at each date:

$$\frac{\partial^2 C_t(x_t,K)}{\partial K\partial x_t}\equiv\beta<0. \tag{K.6}$$

In this context, the total differentiation of (K.5) implies

$$\sum_{t\geq 0}\frac{\partial^2 C_t(x_t,K)}{\partial K^2}dK=-\sum_{t\geq 0}dx_t,$$

where the binding exhaustibility constraint (K.2) implies that $\sum_{t\geq 0} dx_t 0 dX$. It follows a positive relationship between a capacity change dK and a change dX in developed reserves:

$$dK=\frac{-\beta}{\sum_{t\geq 0}\frac{\partial^2 C_t(x_t,K)}{\partial K^2}}dX. \tag{K.7}$$

At the same time, the reserve supply condition (K.4), the same as in Section 2 in absence of capacity choice, implies the following positive relationship between a change dX in developed reserves and a change $d\lambda$ in the implicit reserves' value:

$$dX=\frac{1}{E''(X)}d\lambda. \tag{K.8}$$

(K.7) and (K.8) means that not only developed reserves X and their implicit price λ—as in Section 2—but also capacity K evolve hand in hand.

Let us now consider (K.3), which implicitly defines an extraction supply function of the same kind as (10), except that it now positively depends on K:

$$x_t=x_t(p_t,\lambda,K),\ \forall t\geq 0. \tag{K.9}$$

In this context, cross-price effects are driven by changes in λ and K, holding the own price p_t unchanged. The differentiation of (K.3) yields the relationship between the function's variables

$$dx_t=\frac{d\lambda+\beta dK}{-\frac{\partial^2 C_t(x_t,K)}{\partial x_t^2}},$$

which, using (K.7) and (K.8), becomes

$$dx_t = \frac{1}{-\frac{\partial^2 C_t(x_t,K)}{\partial x_t^2}} \left(1 - \frac{\beta^2}{E''(X) \sum_{t \geq 0} \frac{\partial^2 C_t(x_t,K)}{\partial K^2}} \right) d\lambda.$$

Clearly, a price rise at any date T induces a rise in reserves' implicit value λ. Therefore, the function (K.9) exhibits negative cross-price effects if and only if

$$\frac{\beta^2}{E''(X) \sum_{t \geq 0} \frac{\partial^2 C_t(x_t,K)}{\partial K^2}} < 1.$$

This condition is more likely to be satisfied when $E''(X)$ and $\partial^2 C_t(x_t, K)/\partial K^2$ are high, reflecting rapidly decreasing returns to scale in X and K, as when reserves limitations are difficult to overcome by investments in the development of new reserves or capacity.