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Smart Grids and Renewable Electricity Generation by Households*

Prudence Dato[†], Tunç Durmaz[‡] and Aude Pommeret[§]

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Abstract

The aim of the study is to analyze investments in intermittent renewable energy and energy storage by a household (HH). The novelty of our model accrues from the flexibility it assigns to a HH in feeding (purchasing) electricity to (from) the grid or storing energy from renewable energy installations. We study the consequences of demand-side management for a HH by accounting for three levels of equipment in smart grids. The first level refers to the possibility of feeding electricity to the grid, which can be achieved relatively simply by net metering. The second level concerns the installation of smart meters. The third level relates to energy storage. We analyze decisions concerning photovoltaic system and energy storage investments, and the consequences of energy storage and smart meters for electricity consumption and purchases of electricity from the grid. Additionally, we study the desirability of a smart meter installation and the implications of curtailment measures for avoiding congestion. Our results indicate that dynamic tariffs, which should encourage HHs to use the power system efficiently and, thus, to save energy, can lead to more reliance on the grid. Thus, the dynamic tariff structure needs to be carefully planned.

Keywords: *Renewable energy, Intermittency, Distributed generation, Smart solutions, Energy storage, Demand response*

JEL codes: *D24; D61; D81; Q41; Q42*

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1 Introduction

Fighting climate change requires a considerable reduction in the use of fossil fuels. Since electricity is expected to displace the use of fossil fuels in buildings, industries, and transportation in the near future, such a reduction can only be achieved through an energy transition toward clean and renewable sources of electricity generation. Decentralized electricity generation using renewables can also address outages and blackout problems arising in electricity-congested countries, such as the United States, following market deregulation. Furthermore, they can provide developing countries with better access to energy. This study investigates the integration of renewable electricity generation (e.g., solar and wind power). However, the fact that renewable sources of energy are inherently intermittent and unpredictable makes their integration challenging. This suggests that we cannot ignore energy storage opportunities and demand management. We, therefore, examine the optimal renewable energy investment decision for a household (HH), who can have access to the grid, as well as smart devices, such as smart meters and batteries.

Smart grids and the consequences of demand-side management for HHs have recently received much attention in academic literature (see De Castro and Dutra, 2013 or Hall and Foxon, 2014 and Bigerna et al., 2016) and in the media (see The Economist, 2009; The Telegraph, 2015b,a). Without smart grids, the lack of transparency on the distribution side of the system is particularly apparent to consumers. Most people do not know how much electricity they use (until they are presented with a bill), the proportions of energy generated by nuclear, coal, gas, and renewables, or the levels of emissions produced in the process. Moreover, a smart grid makes it easier to coordinate intermittent and dispersed sources of power, for example, from rooftop solar panels or backyard wind turbines. Accordingly, we model the installation of smart meters and energy storage devices as demand-side policies aimed at incentivizing agents to consume or store electricity when it is cheap. Such policies increase the substitutability between electricity in different periods, which can encourage HHs to use the power system efficiently and, thus, to save energy (NordREG, 2015).

This study focuses on the integration of renewable electricity generation (e.g., solar and wind power) at the HH level. However, the fact that renewable sources of energy are inherently intermittent and unpredictable makes their integration challenging. Therefore, a new approach that incorporates smart grid technologies into the economic analysis is necessary. A smart grid is an electricity network that uses advanced technologies to integrate power generators and consumers, and enables improved economic efficiency and reliability of supply (De Castro and Dutra, 2013; IEA, 2011).

We account for three levels of equipment when incorporating a smart grid into the analysis. The first level refers to the possibility of selling to the grid, which can be achieved relatively simply by net metering, as long as this does not conflict with the country's legislation.[1] Net metering is a billing system that allows a HH with a rooftop solar panel system, or other distributed generation system, to be credited at the full retail electricity rate for any excess electricity it generates and sells to the local electricity utility via the grid (EEI, 2016). The second level concerns the installation of a smart meter. A smart meter supports two-way communication between the utility and the consumer. This enables real-time pricing and, in turn, end-users in the electricity market to monitor and change their electricity consumption in response to changes in the electricity tariff (Durmaz, 2016, Borenstein and Holland, 2005, and Joskow and Tirole, 2007). Smart meters are relatively widely used in Europe (e.g., Linky in France). The third level relates to energy storage, that is, battery storage. Given the current storage technologies and their costs, energy storage is not used as widely as smart meters. However, with the development of better storage systems with larger storage capacities, energy storage devices are expected to have a wider use in the near future.[2]

In this study, we look for the optimal renewable energy investment of a HH that has access to the electricity grid and to smart devices, such as smart meters and batteries. Here, we examine whether energy storage and the use of a smart meter can influence a HH's electricity consumption and purchases from the grid. We also determine when it is optimal for a HH to install a smart meter. Additionally, we demonstrate how the HH investment decision, its electricity consumption, and its purchases from the grid change when a curtailment measure is in place to avoid congestion on the grid.

The novelty of our model accrues from the flexibility it assigns to a HH in feeding (purchasing) electricity to (from) the grid or storing energy from renewable energy installations. Our first result indicates that it is beneficial to install a smart meter when the expected electricity tariff is either sufficiently low or high. For a HH that is expected to purchase electricity from the grid in the absence of a smart meter, a lower uniform tariff reduces the gain in welfare from installing a smart meter. Alternatively, a higher uniform tariff reduces the welfare gain that would accrue from installing a smart meter when, on average, the HH feeds the grid. Our second result is that the objective of relying less on the grid by using a smart meter (NordREG, 2015) cannot be attained unless the expected tariff is sufficiently high. If the HH takes advantage of dynamic tariffs and consumes more, it will need to compensate for

[1]While the European Union and the United States allow net metering, Hong Kong and some African countries do not.

[2] Many countries provide financial support for the broader use of energy storage systems at the HH level. For example, in Germany, a public program has been in place since May 2013 that provides financial support for both solar PV and battery storage (IRENA, 2015).

the additional electricity consumption by using the grid, given that both storage and solar panels are used fully. Subsequently, the HH will cause increased grid activity. Note that grid activity refers to the amount of electricity produced by the utility company that is fed to the electricity grid. Both results crucially depend on the convexity of the value function with respect to the electricity tariff, which we show is large for sufficiently high or low tariffs. This result demonstrates that the level of electricity tariffs needs to be designed carefully if the aim is to depend less on the electricity grid.

Furthermore, we consider the congestion problem that can arise when too much electricity is fed into the grid, as well as grid curtailment to prevent this problem. Grid curtailment is a written contract with generators, based on a constraint at the grid connection point (Kane and Ault, 2014). More specifically, it limits the amount of electricity fed by the HH to the electricity grid at certain times. Our analysis demonstrates that curtailment measures to avoid congestion can discourage investment in renewable energy generation and energy storage capacities. When these investments are discouraged, our results show that (i) electricity generated and fed to the grid by the HH will be curtailed at the higher end of the tariff structure, and (ii) the HH will not necessarily purchase more electricity from the grid. We then show that curtailment measures can be used efficiently as an alternative to other mechanisms (e.g., zonal pricing, see Wu et al., 1996; Chao et al., 2000; Bjørndal and Jörnsten, 2007) when managing congestion problems.

Thus far, the literature on the penetration of renewables in the energy mix consists of two rather distinct fields. On the one hand, some models consider renewable resources as abundant and having a certain steady flow. These studies ignore variability and intermittency in renewable energy generation, and focus on the cost of generation and on technological progress (e.g., see the two-stage model of Fischer and Newell, 2008). The second strand of literature studies the optimal energy source mix for electricity generation (fossil fuels and renewables) when intermittency is taken into account (see Ambec and Crampes, 2012, 2015), or when storage takes care of peak electricity (see Gravelle, 1976; Crampes and Moreaux, 2010; Durmaz, 2016) or excess nuclear energy generation (Jackson, 1973). A recent survey on the economics of solar electricity (Baker et al., 2013) emphasizes the lack of economic analyzes of decentralized clean energy provision through renewable sources. We fill this gap by analyzing a model that accounts for intermittency, energy storage, and dynamic tariffs at the HH level. While Ambec and Crampes (2012) focus mainly on decentralizing the efficient mix of intermittent sources and fossil fuels (assuming that smart meters are already installed), we consider the problem from a HH perspective. We extend the setup in Ambec and Crampes (2012) by giving the HH the flexibility to use the electricity grid and energy storage as a backup, and to install smart meters. In particular, we show that smart meter installation, which would allow for a dynamic tariff, can indeed worsen the welfare of a HH. Furthermore, we show that installing a smart meter can induce more reliance on the electric grid.

The remainder of this paper is structured as follows. The model is presented in Section 2. We analyze the optimal investments in solar panels and storage devices in Section 3, and in Section 4, we study the consequences of energy storage and smart meters for purchases of electricity from the grid and electricity consumption. In Section 5, we discuss the desirability of smart meter installation. Section 6 studies the implications of curtailment measures to avoid congestion. Finally, Section 7 concludes the paper.

2 The model

We consider a model that focuses on HH decisions while the electricity utility always meets the demand. We assume a two-period economy. During the first period, the HH invests K_1 (e.g., in solar panels) to generate renewable energy (RE), the total usage cost of which for the two periods is rK_1.[3] Once the RE investment is made, it serves to produce K_1 kilowatt-hours (kWh) of electricity in the first period. RE generation during the second period depends on the two states of nature, which are "sun" and "no sun." Let P_s denote the probability that there will be sun in the next period. Conversely, $P_n = 1 - P_s$ denotes the probability that the weather will be cloudy, resulting in no solar power generation. Therefore, with probability P_s (or P_n), RE generation in the second period will be K_1 (or 0) kWh. In the first period, energy can be stored and transferred to the second period. Storing energy is costly owing to the loss of energy during the restoration process. Denoting the amount of energy stored in the first period by S_1, the amount of energy available that can be consumed in the second period is then ϕS_1. Here, $\phi < 1$ is the round-trip efficiency parameter.[4]

In addition to storing energy, we assume that the intermittent renewable energy source is integrated into the electricity grid such that the HH can feed (purchase) electricity to (from) the grid or store energy from a renewable energy installation. Consider the following probability tree diagram, which illustrates the state-dependent cost of purchases from the electricity grid.

[3] Jackson (1973); Gravelle (1976); Ambec and Crampes (2012, 2015) make a similar assumption.

[4] For simplicity, we assume that the usage cost of storage is accounted for in this parameter.

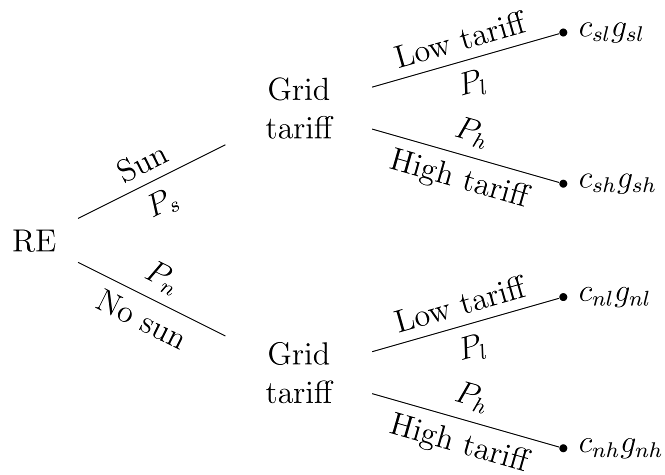

Figure 1: *Central grid electricity tariff*

In the diagram, P_l denotes the probability of a low tariff on the grid, while $P_h = 1 - P_l$ is the probability of a high tariff. In the first period, the electricity tariff is c_1. In the second period, the tariff on the grid will depend on the state. When sunlight is available and the tariff on the grid is low, the expenditure to purchase electricity will be $c_{sl}g_{sl}$, where g_{sl} is the quantity of electricity and c_{sl} the tariff. Similarly, when sunlight is available and the tariff is high, the total cost of purchasing electricity from the grid will be $c_{sh}g_{sh}$. The remaining entries on the diagram can be interpreted in a similar fashion.

At each period, the HH has an instantaneous (gross) surplus over energy consumption. For $j = s, n$ and $i = l, h$, let $u(K_1 + g_1 - S_1)$ and $u(\mathbb{1}_s(j)K_1 + \phi S_1 + g_{ji})$, where

$$\mathbb{1}_s(j) = \begin{cases} 1, & \text{if } j = s \\ 0, & \text{otherwise,} \end{cases}$$

denote these surpluses in the first and second periods, respectively. It is assumed that $u' > 0$ and $u'' < 0$, where u' and u'' are the first- and second-order derivatives of the surplus function, respectively.

3 Optimal investment in solar panels and storage devices

3.1 With smart meter devices

In this section, we consider the optimal decisions of a HH in terms of solar panel and energy storage investments, as well as purchases from and sales to the electricity grid. To do so, we consider that the HH is equipped with a smart meter that connects the home to the grid for two-way exchanges of information and energy. Given Figure 1, the HH solves the following program:

$$\max_{\{K_1,S_1,g_1,g_{ij}\}} u\left(K_1+g_1-S_1\right)-c_1g_1+\sum_j\sum_i P_jP_i\left[u\left(\mathbb{1}_s(j)K_1+\phi S_1+g_{ji}\right)-c_{ji}g_{ji}\right]-rK_1$$
$$\text{s.t.}\quad K_1\leq\overline{K}, S_1\geq 0, K_1\geq 0 \text{ and } S_1\leq\overline{S},$$

where $j=s,n$ and $i=l,h$, and $\bar{K}$ and $\bar{S}$ are available capacities of the solar photovoltaic and energy storage systems, respectively. The Lagrangian function is as follows:

$$\begin{aligned}\mathcal{L}(\cdot)&=u\left(K_1+g_1-S_1\right)-c_1g_1+\sum_j\sum_i P_jP_i\left[u\left(\mathbb{1}_s(j)K_1+\phi S_1+g_{ji}\right)-c_{ji}g_{ji}\right]-rK_1\\&+\nu_1(\overline{K}-K_1)+\nu_2S_1+\nu_3K_1+\nu_4(\overline{S}-S_1).\end{aligned}\tag{1}$$

We denote the optimal HH decisions using the "g" superscript. Then, the first-order conditions with respect to K_1, S_1, g_1, and g_{ji} yield

$$u'\left(K_1^g+g_1^g-S_1^g\right)+P_s\sum_i P_iu'\left(K_1^g+\phi S_1^g+g_{si}^g\right)-r=\nu_1-\nu_3,\tag{2a}$$

$$\phi\sum_j\sum_i P_jP_iu'\left(\mathbb{1}_s(j)K_1^g+\phi S_1^g+g_{ji}^g\right)-u'\left(K_1^g+g_1^g-S_1^g\right)=\nu_4-\nu_2,\tag{2b}$$

$$u'\left(K_1^g+g_1^g-S_1^g\right)=c_1,\tag{2c}$$

$$u'\left(\mathbb{1}_s(j)K_1^g+\phi S_1^g+g_{ji}^g\right)=c_{ji},\tag{2d}$$

respectively. Substituting the first-order necessary conditions for g_1^g and g_{ji}^g into Eqs. (2a) and (2b) gives

$$c_1+P_s\sum_i P_ic_{si}-r=\nu_1-\nu_3,$$
$$\phi\sum_j\sum_i P_jP_ic_{ji}-c_1=\nu_4-\nu_2.$$

The FOCs drop to the primitives of the model, that is, the tariffs. Different cases emerge depending on the cost of the solar panel installation relative to the grid tariff on the one hand, and the cost of storage (in terms of loss during the restoration process) relative to the grid tariff on the other hand. Here, we focus on the case where solar panels and storage are relatively cheap.[5] Thus,

$$c_1 + P_s \sum_i P_i c_{si} - r > 0, \tag{3a}$$

$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 > 0. \tag{3b}$$

Therefore, we have corner solutions, because, on the margin, the expected benefits from installing K_1 and S_1 are always higher. Consequently, $K_1^g = \overline{K}$ and $S_1^g = \overline{S}$. A similar analysis gives

$$\begin{aligned} g_1^g &> 0 \quad \text{if} \quad c_1 < u'(\overline{K} - \overline{S}), \\ g_1^g &\leq 0 \quad \text{otherwise.} \end{aligned} \tag{4}$$

Furthermore, the way the grid will be used in the second period will depend on

$$\begin{aligned} g_{ji}^g &> 0 \quad \text{if} \quad c_{ji} < u'(\mathbb{1}_s(j)\overline{K} + \phi\overline{S}), \\ g_{ji}^g &\leq 0 \quad \text{otherwise.} \end{aligned} \tag{5}$$

The optimal levels of the feed-ins to (or purchases from) the grid are then calculated as

$$\begin{aligned} g_1^g &= u'^{-1}(c_1) - \overline{K} + \overline{S}, \\ g_{ji}^g &= u'^{-1}(c_{ji}) - \mathbb{1}_s(j)\overline{K} - \phi\overline{S}. \end{aligned} \tag{6}$$

The optimality conditions given by Eqs. (4) and (5) dictate that the electricity will be purchased from (sold to) the electricity grid when it is sufficiently cheap (expensive). In particular, given K_1^g, when the energy storage capacity is sufficiently high, such that the marginal gross surplus is greater than the grid tariff in the first period, electricity will be purchased, and vice versa. A similar discussion follows for the second period. In contrast to the first period, previously stored energy (adjusted for the round-trip efficiency) will be used for consumption purposes, leading to a lower demand for grid electricity than otherwise would be the case. Note too that the demand for grid electricity will depend on the meteorological shock, that is, whether sunlight is available or not.

[5] We are convinced that this will be the case in the not-too-distant future. When solar panels and energy storage devices are sufficiently expensive, such that they are not utilized, then our analysis can be deemed as less useful. Nevertheless, it is certainly possible to analyze the other cases, and allow our study to be more exhaustive.

No storage devices ($\overline{S} = 0$)

In the absence of energy storage, we consider the optimal decision of a HH in terms of solar panel installations and purchases from and sales to the electricity grid. Without energy storage, the grid is the only backup available to the HH when it purchases electricity. We assume that the HH is still equipped with a smart meter, allowing it to be exposed to a dynamic tariff structure.

Since the absence of energy storage is a limited case of the general case we analyzed earlier, we set $\overline{S} = 0$ and consider that Eq. (3a) holds. Accordingly, the HH still has an incentive to undertake investments in solar panels when the usage cost of the solar panels (i.e., r) is lower than their expected benefits, which is the sum of the avoided marginal cost of electricity in the first and the second periods (i.e., c_1 and $P_s \sum_i P_i c_{si}$, respectively). Thus, it is optimal to use all available capacity to install the solar panels: $K_1^{gn} = \overline{K}$, where the superscript "gn" denotes the case of no storage devices. For $\overline{S} = 0$, the conditions that describe the grid activity in the first and second periods are given by Eqs. (4) and (5), respectively. Accordingly, the optimal levels of the feed-ins (or purchases) can be calculated from Eq. (6). The discussion on the grid activity is similar to that of the general case and, therefore, is omitted.

3.2 No smart meter devices

This subsection is devoted to the optimal decisions of a HH that is not equipped with a smart meter device and, thus, cannot benefit from a dynamic tariff during the second period. Consider a tariff structure with c_1 and c_2 being the electricity tariffs on the grid in the first and second periods, respectively. For $c_{ji} = c_2$, the HH solves the same program as in Section 3.1. Therefore, the conditions that describe the incentives to invest in solar panels and in storage become

$$c_1 + P_s c_2 - r > 0 \text{ and} \tag{7a}$$

$$\phi c_2 - c_1 > 0, \quad \text{respectively.} \tag{7b}$$

An investment in solar panels is undertaken when its marginal benefit during the two periods ($c_1 + P_s c_2$) is greater than its marginal cost (r). The HH then optimally installs solar panels, given the available capacity $\bar{K}$. Thus, $K_1^o = \overline{K}$, where the superscript "o" denotes the case of no smart meters. Similarly, the HH has an incentive to store electricity in the first period when the avoided marginal cost of buying from the grid in the second period at a uniform tariff c_2 is higher than the marginal cost of storage (c_1/ϕ), that is, the opportunity cost of forgone electricity consumption in the first period, adjusted for the storage loss. It is then optimal to store as much as energy as possible so that $S_1^o = \overline{S}$. The way the grid electricity is used in the two periods is unchanged from Section 3.1. In other words, if the uniform tariff of electricity in the second period is low (high), electricity will be purchased (fed). The two conditions given by Eqs. (7a) and (7b) together yield:

$$r - P_s c_2 < c_1 < \phi c_2.$$

Simultaneous solar electricity production and storage is conditioned by the grid tariff in the first period, which should not be too high or too low. In fact, if the electricity tariff in the first period is too low (too high), this will prevent the HH from investing in solar energy (energy storage).

4 Grid activity

In this section, we discuss the implications of storage and smart meters for electricity consumption and the grid activity. Following the same parametric conditions that satisfy Eqs. (3a) and (3b), we first compare the cases with and without storage devices. This is followed by a comparison between the cases with and without smart meters in the presence of storage devices.

4.1 Storage vs. no storage

Recall that in the two cases (i.e., storage and no storage) and under the conditions given by Eqs. (3a) and (3b), it is always optimal to install solar panels and storage systems, up to the available capacities: $K_1^{gn} = K_1^g = \overline{K}$ and $S_1^g = \overline{S}$. Using Eq. (2c) and taking $\overline{S} = 0$ as the case of no storage, the difference in the grid activity is given by

$$g_1^g - g_1^{gn} = \overline{S} > 0.$$

This equation states that energy storage induces greater activity in the grid in the first period. Given that the HH can store energy, it will purchase more electricity from the grid in the first period, while keeping its first period electricity consumption the same.

Similarly, the grid activity in the second period can be calculated from Eq. (2d), as follows:

$$g_{ji}^{g} - g_{ji}^{gn} = -\phi\overline{S} < 0.$$

The negative difference states that storing energy will induce relatively lower grid activity in the second period. Accordingly, the HH takes advantage of the availability of the storage device by storing energy, up to its available capacity, in the first period, and then using this stored energy in the second period. Therefore, the storage device is used as a backup, allowing for less reliance on the electricity grid in the second period.

The two equations above allow us to deduce that the expected total grid activity,

$$(g_1^g - g_1^{gn}) + \sum_j \sum_i P_j P_i (g_{ji}^g - g_{ji}^{gn}) = (1-\phi)\overline{S} > 0,$$

will be higher when there is access to storage devices. This is because part of the additional grid electricity in the first period is lost to storage ($\phi < 1$).

4.2 Smart meters vs. no smart meters

Under the conditions given by Eqs. (3a) and (3b), as well as Eqs. (7a) and (7b), it is always optimal to install solar panels and storage systems, up to their available capacities: $K_1^g = K_1^o = \overline{K}$ and $S_1^g = S_1^o = \overline{S}$. The difference between the grid activities in the first period can be calculated from Eq. (2c), as follows:

$$g_1^g - g_1^o = 0.$$

During the first period, the grid activity is not affected by the use of smart meters. Nonetheless, this result can change in the second period, depending on the tariff structure. From Eq. (6), the difference between the expected grid activity when the tariffs are dynamic and when there is uniform tariff can be calculated as follows:

$$\sum_j \sum_i P_j P_i (g_{ji}^g - g_j^o) = \sum_j \sum_i P_j P_i u'^{-1}(c_{ji}) - u'^{-1}(c_2). \tag{8}$$

As shown, the difference depends on the margin between the expected electricity consumption when the tariffs are dynamic and when they are fixed.[6] Consequently, the availability of a smart meter induces high (low) grid activity when the expected electricity consumption with

[6] Note that in the case of no access to smart meters, that is, when the tariff is uniform, HH consumption is constant and, therefore, does not depend on the state of the weather.

a dynamic tariff is higher (lower) than the consumption with a uniform tariff. The result is that if the HH still consumes the same amount of electricity, having access to smart meter will not affect its grid activity. Conversely, given that both storage and solar panels are fully used, if the HH takes advantage of a dynamic tariff structure and consumes more, it will need to compensate for the additional electricity consumption by using the grid. Therefore, the HH will cause the grid activity to increase.

As Eq. (8) suggests, the difference between the grid activities in the two cases is affected by the tariffs in the two tariff structures (c_{ji} and c_2). In addition, this also depends on whether the HH is prudent.[7] Let $\mathbb{E}(x_{ji}) \stackrel{\text{def}}{=} \sum_j \sum_i P_j P_i x_{ji}$, where $\mathbb{E}$ is the expected value operator. Then, we have the following proposition.

Proposition 1. *Let $c_{ji} \equiv \mu + x_{ji}$, where $\mu > 0$, $\mathbb{E}[x_{ji}] = 0$, and $\mathrm{var}(x_{ji}) = \sigma^2$. Thus, μ and σ^2 correspond to the mean and variance of c_{ji}, respectively.*

a- If $c_2 = \mu$, there will be higher activity on the grid when the HH is prudent and is equipped with a smart meter; that is, $\mathbb{E}[g^g_{ji}] - \mathbb{E}[g^o_j] > 0$ when $u''' > 0$.

b- For a prudent HH, the expected grid activities in the dynamic and uniform cases can be the same only if the uniform tariff is strictly lower than μ. Let $\hat{c}_2 (< \mu)$ be the tariff such that $\mathbb{E}[g^g_{ji}] = \mathbb{E}[g^o_j]$. Then,

$$
\begin{aligned}
\mathbb{E}[g^g_{ji}] &> \mathbb{E}[g^o_j] \quad \text{if} \quad c_2 > \hat{c}_2, \\
\mathbb{E}[g^g_{ji}] &\leq \mathbb{E}[g^o_j] \quad \text{otherwise.}
\end{aligned}
\tag{9}
$$

Proof. The proof of Proposition 1-a follows from Jensen's inequality. Furthermore, because $\mathbb{E}[u'^{-1}(c_{ji})] - u'^{-1}(\mu) > 0$ and $\partial u'^{-1}/\partial c_2 < 0$, there exists a $\hat{c}_2 < \mu$, such that $\mathbb{E}[u'^{-1}(c_{ji})] - u'^{-1}(\hat{c}_2) = 0$. Consequently, if $c_2 \leq \hat{c}_2$, then $\mathbb{E}[u'^{-1}(c_{ji})] - u'^{-1}(\hat{c}_2) \leq 0$ and $\mathbb{E}[g^g_{ji}] - \mathbb{E}[g^o_j] \leq 0$, and vice versa. This proves Proposition 1-b. □

Figure 2 illustrates Proposition 1. If the objective is to rely less on the grid with a dynamic tariff structure, using smart meters, then Proposition 1 demonstrates that such an objective cannot be attained unless the expected dynamic tariff is sufficiently high. In particular, when the expected tariff is equal to the uniform tariff, the grid activity is higher when the tariffs are dynamic. This indicates that the discrepancy between low and high tariffs in the dynamic tariff structure needs to be considered carefully when the aim is to promote lower activity on the grid.

[7] A prudent HH is characterized by $u''' > 0$ (Kimball, 1990).

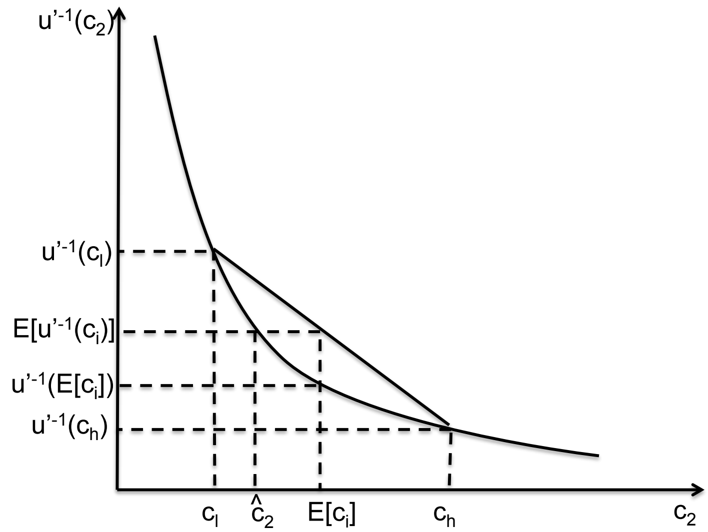

Figure 2: *Grid activity: Smart meter vs. no smart meter. For illustration purposes,* $c_{sl} = c_{nl} = c_l$ *and* $c_{sh} = c_{nh} = c_h$.

5 When to install smart meters

In this section, we analyze the conditions under which it is optimal to install a smart meter. We explained in Sections 3.1 and 3.2 that a corner solution case dictates that

$$c_1 + P_s \sum_i P_i c_{si} - r > 0 \quad \text{and}$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 > 0$$

for a dynamic tariff, and

$$c_1 + P_s c_2 - r > 0 \quad \text{and}$$
$$\phi c_2 - c_1 > 0$$

for a uniform tariff. Thus, for both the dynamic and uniform tariff cases, it is optimal to exhaust all investment possibilities for solar panels and energy storage systems.

The installation of the smart meter will be beneficial when the expected benefit (or the avoided cost) from its use is sufficiently high. Thus, we need to study the change in the difference between the two maximum value functions (i.e., $V^g - V^o$) with respect to the uniform tariff (c_2) on the grid. Let r^g denote the cost of installing the smart meter. This leads us to the following proposition.

Proposition 2. *Let* $c_{ji} \equiv \mu + x_{ji}$*, where* $\mu > 0$, $\mathbb{E}[x_{ji}] = 0$*, and* $\text{var}(x_{ji}) = \sigma^2$.

a- If $\mu = c_2$*, there exist two uniform tariffs,* $\underline{c}_2$ *and* $\bar{c}_2$*, where* $\underline{c}_2 \leq \bar{c}_2$*, such that smart* *meters will be installed if and only if* $c_2 \notin \left(\underline{c}_2, \overline{c}_2\right)$. *The size of the interval* $(\underline{c}_2, \overline{c}_2)$ *increases with* r^g.

b- *Given* μ, *if* $V^g - r^g \geq \min_{\{c_2\}} V^o(c_2)$, *then there exist two uniform tariffs,* $\underline{\underline{c}}_2$ *and* $\overline{\overline{c}}_2$, *where* $\underline{\underline{c}}_2 < \overline{\overline{c}}_2$, *such that smart meters will be installed if and only if* $c_2 \in \left(\underline{\underline{c}}_2, \overline{\overline{c}}_2\right)$.

Proof. Recall that the maximum value function for the dynamic tariff case, upon the installation of the smart meter, and for the uniform tariff case are

$$\begin{aligned} V^g &= u\left(\overline{K} + g_1^g - \overline{S}\right) - c_1 g_1^g + \mathbb{E}\left[u\left(\mathbb{1}_s(j)\overline{K} + \phi\overline{S} + g_{ji}^g\right) - c_{ji} g_{ji}^g\right] - r\overline{K} \quad \text{and} \\ V^o &= u\left(\overline{K} + g_1^o - \overline{S}\right) - c_1 g_1^o + \mathbb{E}\left[u\left(\mathbb{1}_s(j)\overline{K} + \phi\overline{S} + g_j^o\right) - c_2 g_j^o\right] - r\overline{K}, \end{aligned}$$

respectively.

It is optimal to install a smart meter if and only if the following is satisfied:

$$\begin{aligned} V^g - r^g \geq V^o \iff & \mathbb{E}\left[u\left(\mathbb{1}_s(j)\overline{K} + \phi\overline{S} + g_{ji}^g\right) - c_{ji} g_{ji}^g\right] - r^g \\ & \geq \mathbb{E}\left[u\left(\mathbb{1}_s(j)\overline{K} + \phi\overline{S} + g_j^o\right) - c_2 g_j^o\right] \qquad . \end{aligned}$$

Recall that the grid activities for the dynamic and uniform tariff cases in the second period are $g_{ji}^g(c_{ji}) = u'^{-1}(c_{ji}) - \mathbb{1}_s(j)\overline{K} - \phi\overline{S}$ and $g_j^o(c_2) = u'^{-1}(c_2) - \mathbb{1}_s(j)\overline{K} - \phi\overline{S}$, respectively. Let $f(c) \overset{\text{def}}{=} \left[u\left(u'^{-1}(c)\right) - c\big(u'^{-1}(c) - \mathbb{1}_s(j)\overline{K} - \phi\overline{S}\big)\right]$. The previous inequality can be rewritten as follows:

$$V^g - r^g \geq V^o \iff \mathbb{E}[f(c_{ji})] - r^g \geq f(c_2).$$

The first derivative of $f(c)$ with respect to c is

$$\frac{\partial f}{\partial c} = -u'^{-1}(c) + \mathbb{1}_s(j)\overline{K} + \phi\overline{S} < 0. \tag{10}$$

The second derivative gives:

$$\frac{\partial^2 f}{\partial c^2} = -\frac{1}{u''\big(u'^{-1}(c)\big)} > 0. \tag{11}$$

Because f is convex, $V^g \geq V^o$. For $r^g > 0$, there exist $\underline{c}_2(r^g)$ and $\overline{c}_2(r^g)$, such that $V^g - r^g > V^o$ when $\mu \notin [\underline{c}_2(r^g), \overline{c}_2(r^g)]$. Note that for any r^g, it is always possible to find a μ such that the slope of f is sufficiently steep to obtain $V^g - r^g > V^o$. This completes the proof of Proposition 2-a.

For Proposition 2-b, note that

$$\frac{\partial V^o}{\partial c_2} \leq 0 \;\text{ iff }\; c_2 \leq \tilde{c}_2 \;\; \Big(\text{i.e.,} \quad \mathbb{E}[g_j^o] \geq 0\Big),$$
$$\frac{\partial V^o}{\partial c_2} > 0 \;\text{ otherwise,}$$

with $\tilde{c}_2 \overset{\text{def}}{=} u'(P_s\overline{K} + \phi\overline{S})$, which is also the uniform tariff level for which the expected optimal grid activity is zero. When $V^g - r^g \geq \min_{\{c_2\}} V^o(c_2)$, $V^o(c_2)$ being convex, there exist $\underline{\underline{c}}_2$ and $\overline{\overline{c}}_2$, such that $V^o(\underline{\underline{c}}_2) = V^o(\overline{\overline{c}}_2) = V^g - r^g$ and $\underline{\underline{c}}_2 \leq \tilde{c}_2 \leq \overline{\overline{c}}_2$. Therefore, $V^g - r^g \geq V^o(c_2)$ if and only if $c_2 \in [\underline{\underline{c}}_2, \overline{\overline{c}}_2]$. □

The intuition behind the first part of Proposition 2 is as follows. The net expected grid activity is zero ($\mathbb{E}[g_j^o] = 0$) and V^o attains its minimum level at $\tilde{c}_2$. Thus, to the right (left) of $\tilde{c}_2$, the expected grid activity is positive (negative). The vicinity $\tilde{c}_2$ also corresponds to the points where V^0 is relatively flat. Given the probabilities and the convexity of the value function, this is also the value space where the additional expected value attained from the use of a smart meter device is relatively low for $\mu = c_2$. Consequently, the farther c_2 gets from $\tilde{c}_2$, the more the HH will benefit from the differentiated tariffs.

Figure 3 illustrates an example. For brevity, we restrict our attention to the positively sloped part of the V^o curve, and take $x_{sl} = x_{nl} = x_l$ and $x_{sh} = x_{nh} = x_h$. The discussion for the negatively sloped part of V^o is symmetrical.

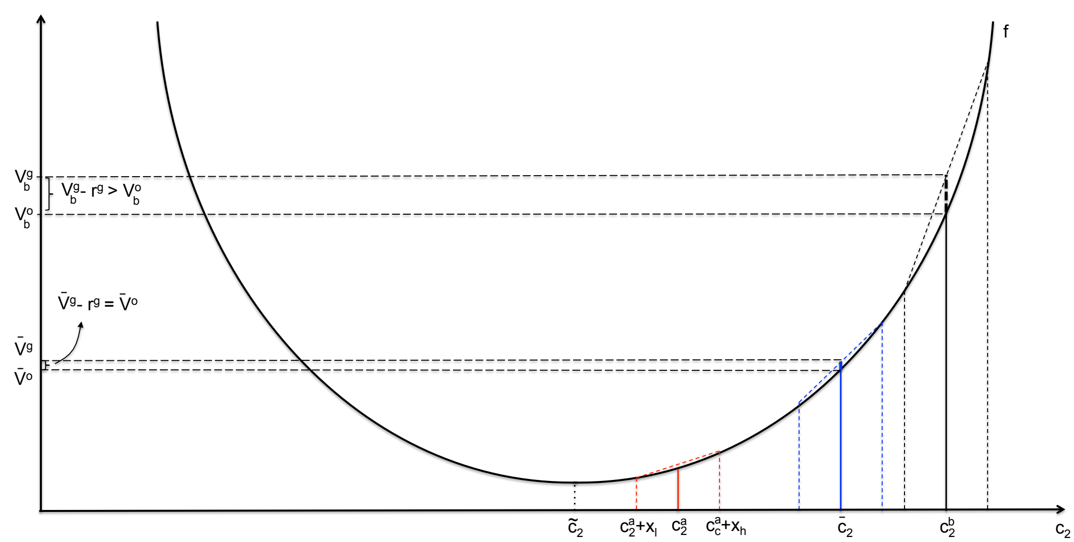

Figure 3: *Smart meter investment decision ($\mu = c_2$)*

As the figure shows, for a $\mu = c_2$ that is close to $\tilde{c}_2$, the tariff variation does not lead to a big difference between the two value functions, V^g and V^o. Given the cost of the smart meter, r^g, this makes it sub-optimal to invest. For a higher level of c_2, the convexity of the curve induces a disproportionate change in the value function corresponding to the high and low tariffs. When $c_2 = \bar{c}_2$, we see that the HH is indifferent as to the installation of the smart meter ($\bar{V}_g - r^g = \bar{V}^o$). However, for higher values of c_2, where the curve becomes steeper, the HH will increasingly benefit from installing the device. It is evident from the figure that a higher cost of installation will necessitate that $\bar{c}_2$ shifts rightward, leading to a larger wedge between the value function corresponding to the high tariff and that corresponding to the low tariff.

The intuition behind the latter part of Proposition 2, where the expected tariff does not correspond to the uniform one, is as follows. For a HH that is expected to purchase electricity from the grid in the absence of a smart meter device (i.e., $c_2 < \tilde{c}_2$ and $\mathbb{E}[g_j^o] > 0$), a lower uniform tariff translates into greater welfare, making it less attractive to install a smart meter. Alternatively, a rise in the uniform tariff increases the welfare of the HH when it is expected to feed the grid, $\mathbb{E}[g_j^o] < 0$. Thus, given μ, a higher uniform tariff makes it less attractive to install a smart meter.

Figure 4 illustrates the behavior of V^o, V^g, $\mathbb{E}[g_j^o]$, and $\mathbb{E}[g_{ji}^g]$ with respect to the uniform electricity tariff c_2. The y-axis on the left shows the values for $V^g - r^g$ and V^o. Values for the expected grid activity when the tariff is uniform and dynamic, $E[g_j^o]$ and $\mathbb{E}[g_{ji}^g]$, respectively, and the expected tariff, $\mathbb{E}[c_{ji}]$, appear on the right y-axis. The x-axis shows the uniform tariffs.[8] Note that the curve representing $\mathbb{E}[g_j^o]$ takes on positive values to the left of $\tilde{c}_2$, and vice versa.

As shown in the figure and in Proposition 2, the smart meter investment decision will not be optimal when the uniform tariff is sufficiently low (here, lower than $\underline{\underline{c}}_2$). This is because the welfare of the HH, V^o, becomes higher than the welfare that would be obtained once the smart meter is installed, $V^g - r^g$. When the tariff is sufficiently high (i.e., higher than $\bar{\bar{c}}_2$), the HH's welfare becomes superior to the one obtained from the installation of the smart meter. Consequently, for a tariff between $\underline{\underline{c}}_2$ and $\bar{\bar{c}}_2$, the smart meter will be installed. Note that both $\underline{\underline{c}}_2$ and $\bar{\bar{c}}_2$ are functions of r^g, the installation cost of the smart meter device, that is, $\underline{\underline{c}}_2(r^g)$ and $\bar{\bar{c}}_2(r^g)$. In particular, while $\partial\underline{\underline{c}}_2(r^g)/\partial r^g > 0$, $\partial\bar{\bar{c}}_2(r^g)/\partial r^g < 0$. Thus, the interval that calls for the installation of the smart meter expands with a lower installation cost, and vice versa.[9]

[8]In plotting the graph, we do not attempt to calibrate the model. The parameter values we use are $r = 0.03; \phi = 0.9; P_s = 2/3; P_l = 1/2; c_1 = 0.02; c_{sl} = 0.04; c_{sh} = 0.08; c_{nl} = 0.04; c_{nh} = 0.08, c_2 \in [0.025, 0.23]$, $\max(\overline{K}) = 2.1$, $\max(\overline{S}) = 1.9$, and $\gamma = 2$.

[9]For a HH that is not equipped with a smart meter, a rise in the uniform electricity tariff has different

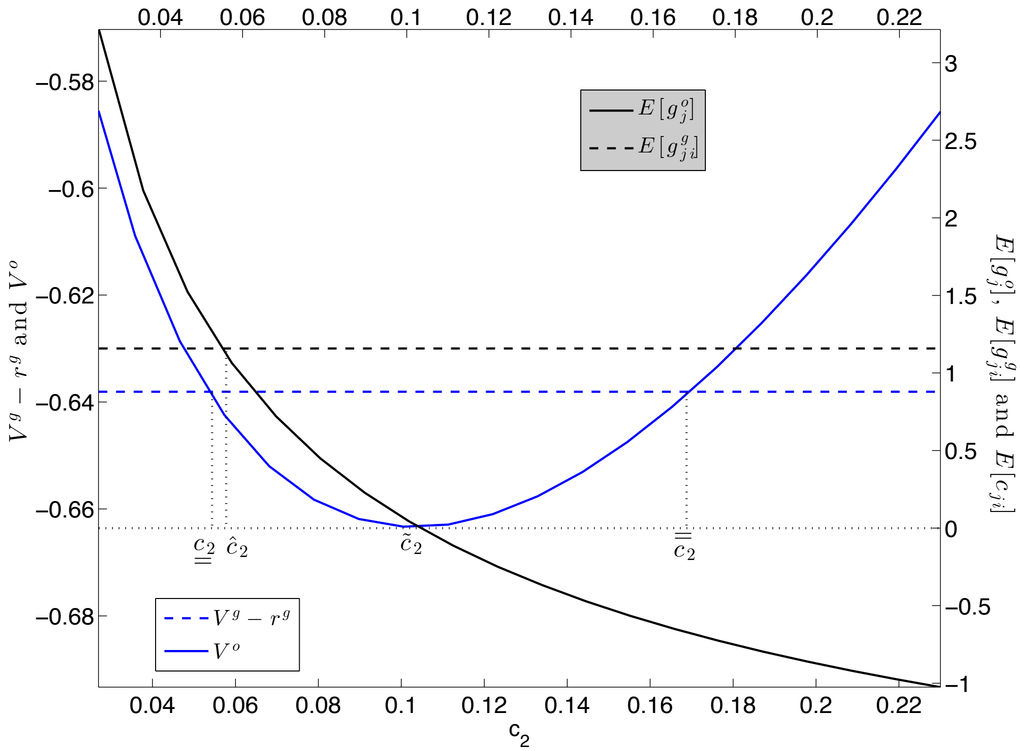

Figure 4: *Smart meter investment decision ($\mu \neq c_2$)*

The analysis thus far allows us to connect Proposition 2 with Proposition 1, and to explain the relationship between the decision to install a smart meter device and the expected grid activity and consumption. This is presented in the following corollary.

Corollary 1.

a- *For $\mu = c_2$, if $c_2 \notin [\underline{c}_2, \overline{c}_2]$, it is optimal to install a smart meter, leading to higher grid activity (and consumption).*

b- *Given μ, if $\underline{\underline{c}}_2 \gtreqless \hat{c}_2 < c_2 < \overline{\overline{c}}_2$, it is optimal to install a smart meter, leading to higher grid activity (and consumption). If $\underline{\underline{c}}_2 < c_2 < \hat{c}_2 \gtreqless \overline{\overline{c}}_2$, it is optimal to install a smart meter, leading to lower grid activity (and consumption).*

and opposite effects on its welfare, some of which cancel each other, overall. First, there is a negative effect (c_2/u''): an increase in the uniform tariff will reduce the total electricity consumption ($\partial u'^{-1}/\partial c_2 < 0$), resulting in a lower level of utility. Second, an increase in the uniform electricity tariff has two effects, coming from the total cost of grid electricity: (i) a direct and negative effect owing to the marginal increase in the tariff ($-\mathbb{E}[g_j^o]$), and (ii) an indirect and positive effect owing to the marginal change in the grid electricity ($-c_2 \partial g_j^o/\partial c_2 = -c_2/u''$). The effect on the HH's utility cancels the marginal change in grid electricity. Thus, the total effect depends negatively on the expected grid activity, $-\mathbb{E}[g_j^o]$. Therefore, the installation of a smart meter becomes attractive as the uniform electricity tariff increases (decreases) when the HH is expected to purchase from (feed) the electricity grid.

Proof. Considering Corollary 1-a, the optimal smart meter installation follows from Proposition 1-a and the grid activity from Proposition 2-a. The difference in grid electricity consumption with and without a smart meter is shown in Eq. (8).

With regard to Corollary 1-b, the optimal smart meter installation follows from Proposition 1-b and the grid activity from Proposition 2-b. The reader is referred to Eq. (9) for the difference in grid electricity consumption with and without a smart meter. $\square$

Corollary 1-a shows that a smart meter installation leads to a higher level of expected grid activity and electricity consumption, which, in turn, allows for a higher level of welfare. When the expected dynamic tariff is equal to the uniform tariff and is sufficiently low (i.e., $\mu < \underline{c}_2$), such that it is optimal to install a smart meter, the HH is expected to purchase more electricity from the grid and consume more when equipped with a smart meter device. Conversely, when the expected dynamic tariff (which is still equal to the uniform tariff) is sufficiently high (i.e., $\mu > \overline{c}_2$), such that it is optimal to install a smart meter, the HH is expected to sell less electricity to the grid and, therefore, consume more when equipped with a smart meter device.

Corollary 1-b shows that installing a smart meter can lead to a lower level of expected grid activity and electricity consumption, while also allowing for a higher level of welfare. Consider the uniform tariff ($\hat{c}_2$), which equates the expected grid activity with that of the dynamic tariff structure. Let's assume that this case corresponds to the one where, on average, the HH purchases electricity from the grid (Figure 4 illustrates an example of this case). For a lower uniform tariff (i.e., $c_2 \in [\underline{\underline{c}}_2, \hat{c}_2)$), the grid purchase and electricity consumption will be higher, on average. Conversely, the expected grid activity and electricity consumption in the uniform tariff case will be higher when the uniform tariff is greater than $\hat{c}_2$. Furthermore, when $\underline{\underline{c}}_2 = \hat{c}_2$, the expected grid activity will always be higher once a smart meter is installed.

Note too that $c_2 \in [\underline{\underline{c}}_2, \hat{c}_2[$ has an inverse relationship with the installation cost, r^g. The lower r^g is, the smaller the values the uniform tariff can take, such that it is optimal to install a smart meter. This implies relatively lower electricity purchases after installing a smart meter.

Next, consider the case where the HH feeds electricity to the grid, on average. As before, let $\hat{c}_2$ be the uniform tariff that equates the grid activity to that with dynamic tariffs. To the left of $\hat{c}_2$, the expected sales of electricity to the grid will be higher and, in turn, electricity consumption will be lower for the dynamic tariff case. On the other hand, a uniform tariff higher than $\hat{c}_2$ will lead to lower expected sales to the grid, and a higher level of expected consumption. Accordingly, when $\overline{\overline{c}}_2 = \hat{c}_2$, the expected grid activity after the installation of a smart meter device will always be lower.

The interval $c_2 \in]\hat{c}_2, \bar{\bar{c}}_2]$ gets wider as the installation cost decreases. Nevertheless, the installation of the smart meter in the relevant domain leads to a smaller amount of sales of electricity to the grid for lower electricity tariffs.

6 Congestion

This section focuses on the curtailment measures designed to avoid the congestion problem that arises when too much electricity is fed into the grid. When selling to the grid is attractive, that is, $c_1 > u'(\overline{K} - \overline{S})$ or $c_{ji} > u'(\mathbb{1}_s(j)\overline{K} + \overline{S})$, and the HH can therefore feed a considerable amount into the grid, the distribution lines and transformers may become overloaded, reducing the quality of the electricity supply (Rui et al., 2014). There are a couple of mechanisms that can be used to avoid congestion. One current approach that can be used, until the grid expansion measures can be executed, is to curtail the feed-in from distributed generators, leading to waste in RE generation (Jacobsen and Schröder, 2012, Luhmann et al., 2015).[10]

To demonstrate the impact of curtailment on optimal decisions, we consider a threshold $\overline{g}$ ($\overline{g} \leq 0$) on the in-feeds. Accordingly, we impose $g_1 \geq \overline{g}$ and $g_{ji} \geq \overline{g}$. We are aware that, in reality, curtailment does lead to a waste of generated electricity, which means that the HH does not perfectly account for the curtailment. However, the correct model will become closer to that with perfect foresight as HHs become more aware of the curtailment problem (which, in turn, will eventually lead to greater welfare).0 Therefore, we focus on the case where they are fully aware of this measure.

6.1 Optimal investment decisions

Curtailment imposes two additional constraints on feeding the grid in the HH decision program: $g_1^m \geq \overline{g}$ and $g_{ji}^m \geq \overline{g}$. The two transmission constraints state that the grid activity should not exceed a negative threshold $\overline{g}$. Otherwise, the security and reliability of the grid would be disrupted. Let "m" denote the optimal value for the decision variables in the case where there is a threshold on feeding the grid. Then, the FOCs with respect to g_1 and g_{ji}

[10] Price management is another approach to solving this problem (Bjørndal and Jörnsten, 2007). When the market price induces capacity problems, the price can be adjusted to reduce the level of the electricity transmission from HHs to the grid.

are

$$u'\left(K_1^m + g_1^m - S_1^m\right) - c_1 = -\nu_5, \tag{12a}$$

$$u'\left(\mathbb{1}_s(j)K_1^m + \phi S_1^m + g_{ji}^m\right) - c_{ji} = -\nu_{ji}, \tag{12b}$$

respectively. Here ν_5 and ν_{ji} are the multipliers associated with the constraints on feeding in the grid. Substituting Eqs. (12a) and (12b) into the FOCs, wrt K_1 and S_1, yields

$$c_1 + P_s \sum_i P_i c_{si} - r = \nu_1 - \nu_3 + \nu_5 + P_s \sum_i P_i \nu_{si}, \tag{13a}$$

$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \nu_4 - \nu_2 - \nu_5 + \phi \sum_j \sum_i P_j P_i \nu_{ji}. \tag{13b}$$

Furthermore, as in the previous section, we consider that Eq. (3a) holds.

In light of these equations, several scenarios can emerge. For example, it is possible to face a scenario where it is optimal to use all available storage capacity, and yet not install any solar panels. We can also consider a case in which it is optimal to fully use the total capacity of solar panels, but not to store energy. It is also possible to think of a scenario in which the solar panel investment and energy storage decisions take interior values. Figure 5 illustrates various cases for investment and grid purchase decisions by considering different electricity tariffs on the grid in the first period. Without attempting to calibrate the model, the parameter values we use are $r = 0.03; \phi = 0.75; P_s = 2/3; P_l = 1/2; c_1 \in [0.0117, 0.0232]; c_{sl} = 0.01; c_{sh} = 0.06; c_{nl} = 0.01; c_{nh} = 0.06, \max(\overline{K}) = 8, \max(\overline{S}) = 8$, $\overline{g} = -1$ and $\gamma = 2$.

When the grid tariff in the first period is sufficiently low, the figure shows that it is optimal to store energy at full capacity by purchasing electricity from the grid only. In this case, there is no investment in solar energy. When energy is stored at full capacity, and the electricity tariff is high, we find that electricity is fed into the grid until the congestion threshold, $\bar{g}$, is met. For higher values of c_1, we see that both K_1 and S_1 take interior values. This regime changes when the electricity tariff on the grid in the first period becomes sufficiently high. In this case, all capacity for the solar panels is exhausted. However, as the grid tariff becomes sufficiently high, storing energy becomes suboptimal.

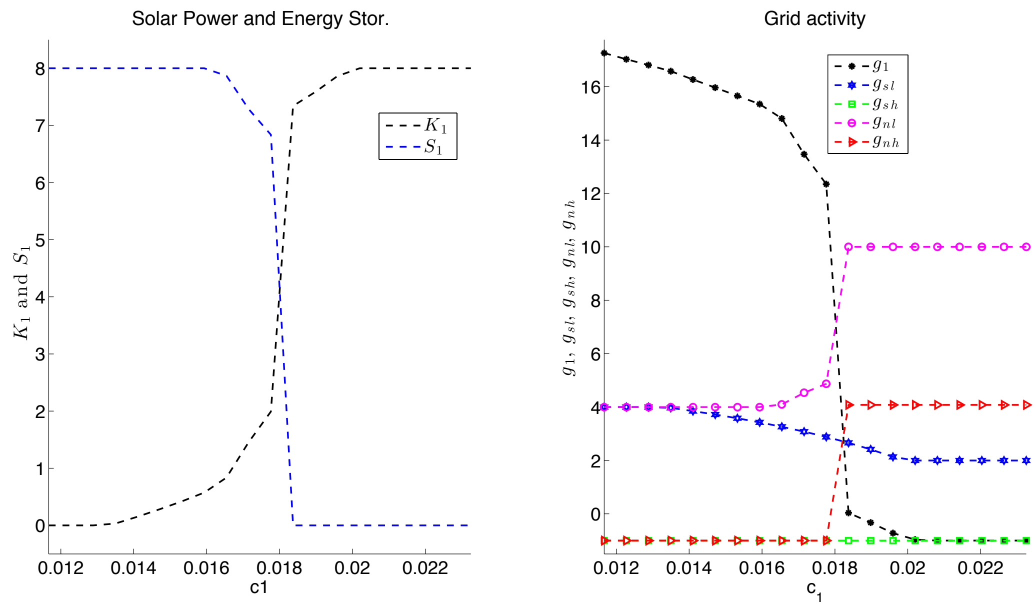

Figure 5: *A case for interior solution*

Interior solution

We first focus on the case with interior solutions, that is, $\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu_5 = 0$. Because it must always be true that

$$g_{nl}^m > g_{nh}^m > g_{sh}^m \text{ and } g_{nl}^m > g_{sl}^m > g_{sh}^m,$$

interiors solutions imply that $g_{sh}^m = g_{nh}^m = \overline{g}$.[11] From an analytical point of view, setting a limit on feeding the grid is equivalent to replacing two constraints, namely $K_1 \leq \overline{K}$ $S_1 \leq \overline{S}$, by two constraints on feeding the grid in the second period (i.e., $g_{sh}^m = g_{nh}^m = \overline{g}$).

The intuitive economic reasoning is as follows. If there is a limit on feeding the grid when sunlight is available and there is a high tariff on the electricity grid, then there is no incentive to buy an infinite number of solar panels. On the other hand, when there is no sunlight, the electricity tariff is high, and feeding the grid is technically limited, then there is no incentive to have an infinite amount of storage capacity.

The optimal levels of S_1 and K_1 can be calculated as follows. Using interior solutions,

[11]This is because assuming only $g_{sh} = \overline{g}$ leads to infinitely many solutions for S_1^m and K_1^m.

Eqs. (13a) and (13b) become

$$c_1 + P_s \sum_i P_i c_{si} - r = P_s P_h \nu_{sh}, \tag{14a}$$

$$\sum_j \sum_i P_j P_i c_{ji} - \frac{c_1}{\phi} = P_h \sum_j P_j \nu_{jh}. \tag{14b}$$

By replacing Eq. (12b) with ν_{ji} in Eqs. (14a) and (14b), the optimal levels of S_1 and K_1 can be calculated from the following system of equations:

$$c_1 + P_s(P_l c_{sl} + P_h u'(K_1^m + \phi S_1^m + \overline{g})) = r, \tag{15a}$$

$$P_n(P_l c_{nl} + P_h u'(\phi S_1^m + \overline{g})) + P_s(P_l c_{sl} + P_h u'(K_1^m + \phi S_1^m + \overline{g})) = c_1/\phi. \tag{15b}$$

The interpretation is as follows. Eq. (15a) shows that the marginal cost of a solar panel should be equal to (i) the avoided marginal cost of buying from the grid in the first period, (ii) the avoided marginal cost of buying from the grid when sunlight is available and the grid tariff is low, and (iii) the marginal benefit of consuming energy generated by the HH (i.e., $u'(K_1^m + \phi S_1^m + \overline{g})$) when sunlight is available and the tariff is high. On the other hand, Eq. (15b) indicates that the marginal cost of storage, c_1/ϕ, that is, the opportunity cost of forgone consumption in period 1 adjusted for the storage loss, should be equal to the expected avoided marginal cost of buying from the grid plus the expected marginal benefit of consuming energy generated by the HH.

The optimal levels for the number of solar panels and energy storage can be obtained by solving the following equations:

$$u'(\phi S_1^m + \overline{g}) = \frac{c_1/\phi - P_n P_l c_{nl} + c_1 - r}{P_n P_h}, \tag{16a}$$

$$u'(K_1^m + \phi S_1^m + \overline{g}) = \frac{r - c_1 - P_s P_l c_{sl}}{P_s P_h}. \tag{16b}$$

The grid activity is given by:

$$u'(K_1^m - S_1^m + g_1^m) = c_1,$$
$$g_{jh}^m = \overline{g},$$
$$u'(\mathbb{1}_s(j) K_1^m + \phi S_1^m + g_{jl}^m) = c_{jl}.$$

Constrained solar power

When the solar power is constrained by the available physical capacity and, therefore, $K_1^m = \overline{K}$, the following conditions for the multipliers,

$$\nu_2 = \nu_3 = \nu_4 = \nu_5 = 0, \text{ and } \nu_1 > 0,$$

allow us to write (cf. Eqs. (13a) and (13a))

$$c_1 + P_s \sum_i P_i c_{si} - r = \nu_1 + P_s \sum_i P_i \nu_{si} > 0,$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \phi \sum_j \sum_i P_j P_i \nu_{ji} > 0.$$

A necessary condition for an interior solution is $g_{sh}^m = \overline{g}$. This is because, on the margin, the benefit from storing energy at full capacity will be lower than its cost in the first period. Recall that this benefit would be higher than the cost if electricity can be fed into the grid.[12] One way to circumvent this problem is to pick a lower level of energy storage and avoid consuming from the grid in the state when sunlight is available and the tariff is high.

Constrained storage

When energy storage is constrained by the available capacity for the device and, thus, $S_1^m = \overline{S}$, we have the following conditions for the multipliers:

$$\nu_1 = \nu_2 = \nu_3 = \nu_5 = 0, \text{ and } \nu_4 > 0.$$

Eqs. (13a) and (13a) then allow us to write

$$c_1 + P_s \sum_i P_i c_{si} - r = P_s \sum_i P_i \nu_{si} > 0,$$
$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \nu_4 + \phi \sum_j \sum_i P_j P_i \nu_{ji} > 0.$$

Similar to the previous subsection, a necessary condition for an interior solution is $g_{sh}^m = \overline{g}$. Otherwise, using a higher number of solar panels or consuming from the grid when there is sunlight and a high grid tariff will lead to a lower expected marginal return from solar power

[12] The optimal solution dictates $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$ in the smart grid case so that some electricity can optimally be sold in both periods.

generation.

Both solar power and storage constrained

When the installation of both solar power and energy storage is constrained by the available physical capacity and, therefore, $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$, we have the following conditions for the multipliers:

$$\nu_2 = \nu_3 = \nu_5 = 0, \nu_1 > 0 \text{ and } \nu_4 > 0.$$

This leads to

$$c_1 + P_s \sum_i P_i c_{si} - r = \nu_1 + P_s \sum_i P_i \nu_{si}, \tag{17a}$$

$$\phi \sum_j \sum_i P_j P_i c_{ji} - c_1 = \nu_4 + \phi \sum_j \sum_i P_j P_i \nu_{ji}. \tag{17b}$$

As a result, there is no restriction on the use of the grid in the second period.

6.2 Electricity consumption and grid activity of unlimited feed-ins vs. limited feed-ins

In this section, we discuss the implications of unlimited and limited feed-ins (owing to the congestion problem) for electricity consumption and the grid activity. Following the same parametric conditions that satisfy Eqs. (3a) and (3b), we first compare the case with unlimited grid feed-ins and the case with limited feed-ins that can lead to interior solutions for solar panel and energy storage device installations. This is followed by specific cases of constrained solar power, constrained storage, and constrained solar power and storage.

Interior solution

Recall that the interior solution under unlimited feeding of the grid constitutes

$$\nu_1 = \nu_3 = \nu_3 = \nu_4 = \nu_5 = 0,$$

and the superscripts "g" and "m" denote the optimal decisions in the cases of unlimited and limited feeding of the grid, respectively.

From Eqs. $(12a)$ and $(2c)$, we have

$$g_1^m - g_1^g = (\overline{K} - K_1^m) - (\overline{S} - S_1^m).$$

Furthermore, in the second period, using Eqs. $(12b)$ and $(2d)$, we have

$$g_{ji}^m - g_{ji}^g \geq \mathbb{1}_s(j)(\overline{K} - K_1^m) + \phi(\overline{S} - S_1^m) \geq 0$$

, with the first inequality from the left being strict, at least for g_{sh}^m and g_{nh}^m.[13]

These two equations allow us to deduce that

$$\begin{aligned}(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) >& (\overline{K} - K_1^m) - (\overline{S} - S_1^m) \\ &+ \sum_j \sum_i P_j P_i \left[\mathbb{1}_s(j)(\overline{K} - K_1^m) + \phi(\overline{S} - S_1^m)\right]\end{aligned}. \tag{18}$$

Because K_1^m and S_1^m are optimal, the above inequality can be rewritten as:

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > (1 + P_s)(\overline{K} - K_1^m) - (1 - \phi)(\overline{S} - S_1^m). \tag{19}$$

A sufficient condition for buying less from the grid when the HH can feed an unlimited amount into the grid is therefore:[14]

$$\begin{aligned}(1 + P_s)(\overline{K} - K_1^m) \geq (1 - \phi)(\overline{S} - S_1^m) \\ \left(\text{or } (1 + P_s)\overline{K} - (1 - \phi)\overline{S} \geq (1 + P_s)K_1^m - (1 - \phi)S_1^m\right)\end{aligned}. \tag{20}$$

Consider the two periods. When the additional electricity that is expected to be generated by the solar panels exceeds that of the additional energy lost by the storage devices (i.e., $(1 - \phi)(\overline{S} - S_1^m)$), there will be less purchases from the grid in the unlimited feed-in case.

When the net amount of electricity generated in the unlimited feed-in case (i.e., $(1 + P_s)\overline{K} - (1 - \phi)\overline{S}$) is higher than it is in the limited feed-in case, then the HH will purchase a higher amount of electricity in the limited feed-in case. On the other hand, if the net amount of electricity generated in the unlimited feed-in case is lower (i.e., when Eq. (20) is not satisfied), the result is ambiguous. This is mainly because in the unlimited feed-in

[13] Otherwise, $g_{sh}^m - g_{sh}^g = 0$ and $g_{nh}^m - g_{nh}^g = 0$, which requires that $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$. From Eqs. (16a) and (16b), we can see that the likelihood of the two equalities holding simultaneously (or even individually) is extremely small and, therefore, negligible.

[14] Considering that $\nu_5 > 0$ and $g_1^m = \overline{g}$, Eq. (19) and the sufficient condition given by Eq. (20) will still be valid.

case, it is always optimal to store at capacity when Eq. (3a) holds. However, when $\overline{K}$ is sufficiently small, the necessary amount of energy that will be stored will be obtained from the grid. Even if there are less purchases from the grid in the second period, the purchases in first period can be sufficiently high to cause higher expected purchases from the grid in the unlimited feed-in case.

Figure 6 illustrates the differences between the grid purchases for the two cases. While the first graph from the right demonstrates the total purchases from the grid (i.e., $g_1^m - g_1^g + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$), the two figures from the left demonstrate the grid purchases in the first and second periods (i.e., $g_1^m - g_1^g$ and $\sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$, respectively). Here, we are only interested in the qualitative pattern. Therefore, we do not attempt to calibrate the model. The parameter values we use are $r = 0.05; \phi = 0.49; P_s = 2/3; P_l = 1/2; c_1 = 0.03; c_{sl} = 0.02; c_{sh} = 0.3; c_{nl} = 0.02; c_{nh} = 0.3, \overline{g} = -0.5, \min(\overline{K}) = 2.46, \max(\overline{K}) = 4.46, \min(\overline{S}) = 0.17, \max(\overline{S}) = 2.17$. ($K_1^m = 2.46$ and $S_1^m = .17$.)

In particular, if the accessible solar panel capacity is low (e.g., $\overline{K} = 2.46$) and the accessible energy storage capacity is rather large (e.g., $\overline{S} = 2.17$), having the possibility of feeding an infinite amount into the grid will generate an adverse effect by causing greater purchases from the grid. Figure 6 indicates that the difference between the grid purchases in the two cases is highest when the solar and storage capacities are low and high, respectively. In addition, a higher number of solar panels and a higher amount of stored energy lead to lower expected purchases from the grid.

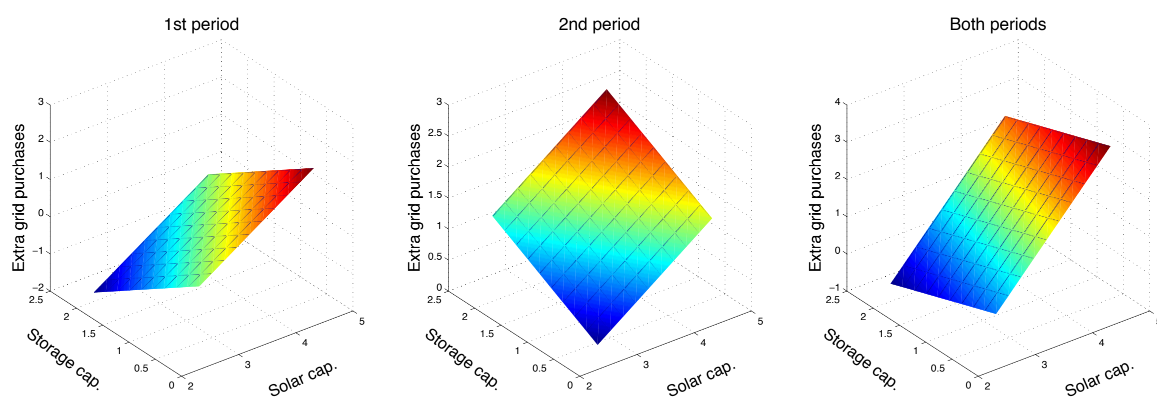

Figure 6: *Difference in purchases from the grid.*

Solar power and/or storage constrained

When the solar power is constrained by the available physical capacity and, therefore, $K_1^m = \overline{K}$, the difference between the expected total purchase of electricity in the unlimited feed-in case and the limited feed-in case can be expressed as follows:

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > -(1-\phi)(\overline{S} - S_1^m). \tag{21}$$

Because the RHS of Eq. (24) is negative, the result is ambiguous. On the other hand, when the solar power is constrained by the available physical capacity and, therefore, $S_1^m = \overline{S}$, the average level of grid purchases will be higher in the limited feed-in case. Lastly, the average level of grid purchases will be higher in the limited feed-in case when both the solar power and the storage are constrained. For further details, see Appendix A.

The following proposition summarizes the results thus far:

Proposition 3.

a- *Curtailment measures can discourage investment in generating and storage capacities. In particular, in contrast to the case with unlimited feed-ins, K_1 and S_1 can take interior values, even when Eqs.* (3a) *and* (3b) *are satisfied.*

b- *When investments are discouraged, electricity generated and fed by the HH to the grid will be curtailed at the higher end of the tariff scheme; that is, $g_{sh}^m = g_{nh}^m = \overline{g}$.*

c- *When solar power and storage take interior values, the HH will not necessarily purchase more electricity from the grid.*

7 Conclusion

Climate change, congested electricity grids in developed countries, and a lack of access to electricity in developing countries are problems that can be mitigated by the further use of renewables (e.g., wind and solar power). Nonetheless, the intermittent nature of renewables coupled with consumers who are non-reactive to short-term fluctuations in electricity provision suggest we should implement new levels of equipment, such as the possibility of selling to the grid, installing smart meters, and using energy storage.

In this study, we analyze the optimal investments in solar panels and storage devices, and evaluate the consequences of energy storage and smart meters for purchases of electricity from the grid and for electricity consumption. In addition, we discuss the desirability of smart meter installations, and investigate the implications of curtailment measures that aim to avoid congestion. Our first result indicates that it is beneficial to install a smart meter that enables the HH to benefit from electricity tariff variations when the expected electricity tariff is either sufficiently low or high. Our second result is that the objective of relying less on the grid by using a smart meter cannot be attained unless the expected tariff is sufficiently high. If this is not the case, the reliance on the grid will be higher, leading to further emissions. This indicates that electricity tariffs need to be considered carefully when the objective is to rely less on the grid by deploying a smart grid. We also consider the congestion problem that can arise when too much electricity is fed in to the grid. Our results show that curtailment measures aimed at avoiding such congestion can discourage investment in generating renewable energy and in energy storage. In this case, we find that (i) electricity generated and fed into the grid by the HH will be curtailed at the higher end of the tariff scheme, and (ii) the HH will not necessarily purchase more electricity from the grid.

Our framework has the potential for additional research. For example, we can appraise the suitability of smart grids when there is a blackout risk, as encountered in developed countries, such as the United States, and in developing countries, such as India. In addition, we can explore cases where solar panels or storage investments are so expensive that related investments are only beneficial when complemented with additional smart grids. Finally, our results can serve as the basis for developing environmental policies at both the HH and smart grid levels.

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Appendices

A Electricity consumption and grid activity: unlimited vs. limited feed-ins

Solar power constrained

When solar power is constrained by the available physical capacity and, therefore, $K_1^m = \overline{K}$, we have the following conditions for the multipliers:

$$\nu_2 = \nu_3 = \nu_4 = \nu_5 = 0, \text{ and } \nu_1 > 0.$$

Considering the first period, an interior solution for g_1 implies

$$u'(\overline{K} + g_1^m - S_1^m) = u'(\overline{K} + g_1^g - \overline{S}). \tag{22}$$

Because the marginal utility is decreasing with consumption (i.e., $u'' < 0$, $g_1^m < g_1^g$), a higher level of energy storage will lead to greater amounts of electricity being purchased from the grid in the case of unlimited feed-ins in the first period:

$$g_1^m - g_1^g = -(\overline{S} - S_1^m) < 0.$$

In the second period, the expected difference between grid purchases in the smart meter and smart grid cases is

$$\sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) \geq \phi(\overline{S} - S_1^m) > 0. \tag{23}$$

This indicates that the expected purchase in the unlimited feed-in case will be higher in the second period.

Summing the two inequalities, the difference between the expected total purchase of electricity in the unlimited and limited feed-in cases can be expressed as follows:

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > -(1-\phi)(\overline{S} - S_1^m), \tag{24}$$

Because the RHS of Eq. (24) is negative, the inequality given by Eq. (24) is no longer a sufficient condition to buy less from the grid in the unlimited feed-in case. The intuition behind this is as follows. Because the parametric condition dictates that it is optimal to store at maximum capacity, the grid purchases in the first period can be high enough that they cause greater purchases from the grid in the unlimited feed-in case.

Figure 7 shows the differences between the grid purchases in the unlimited and limited feed-in cases when $K_1^m = \overline{K}$. The first two graphs from the left show the purchases from the grid in the first and the second periods, respectively. The last figure shows the expected sum of the grid purchases in the two periods. In other words, the three figures, from left to right, show $g_1^m - g_1^g$, $\sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$, and $g_1^m - g_1^g + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g)$, respectively. We are only interested in the qualitative pattern and, thus, do not attempt to calibrate the model. The parameter values that we employ are $r = 0.05; \phi = 0.49; P_s = 2/3; P_l = 1/2; c_1 = 0.030; c_{sl} = 0.02; c_{sh} = 0.3; c_{nl} = 0.02; c_{nh} = 0.3, \overline{g} = -0.5, \overline{K} = 2.45, \min(\overline{S}) = 0.15, \max(\overline{S}) = 2.15$. ($K_1^m = 2.45$ and $S_1^m = 0.15$.)

In line with our reasoning above, the last figure shows that lower values of energy storage capacity will allow for greater grid activity in the limited feed-in case. Nevertheless, with higher storage capacities, which allow for larger amounts of energy to be stored in the first period (see Fig. 7a), the total amount of energy purchased from the grid increases. This

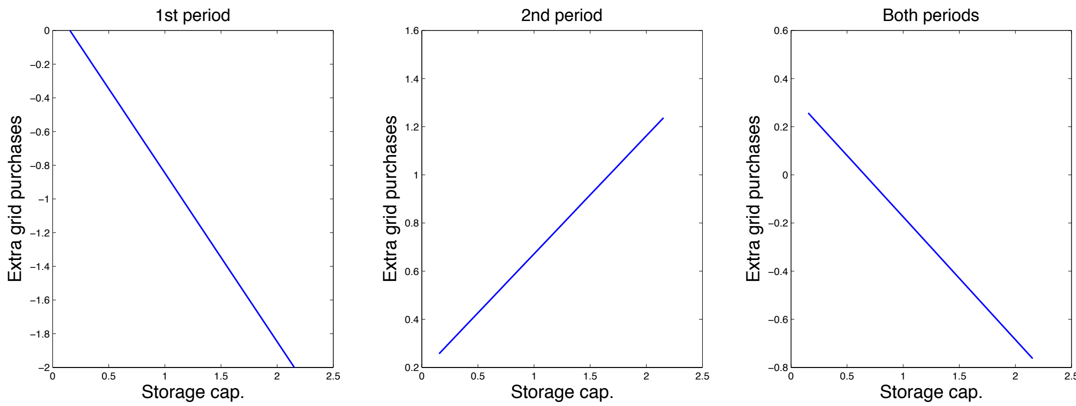

Figure 7: *Difference in purchases from the grid* ($K_1^m = \overline{K}$).

happens even if the grid purchases are lower in the second period in the unlimited feed-in case.

Storage constrained

When the solar power is constrained by the available physical capacity and, therefore, $S_1^m = \overline{S}$, we have the following conditions for the multipliers:

$$\nu_1 = \nu_2 = \nu_3 = \nu_5 = 0, \text{ and } \nu_4 > 0.$$

From Eq. (19), the difference between the purchase of electricity in the limited and unlimited feed-in cases can be expressed as follows:

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > (1 + P_s)(\overline{K} - K_1^m). \tag{25}$$

Because $\overline{K} > K_1^m$, the LHS is strictly positive. Therefore, the average level of grid purchases will be higher in the limited feed-in case.

Solar power and storage constrained

Recall that we have the following conditions for the multipliers when the installation of solar power and energy storage are both constrained by the available physical capacity (i.e., $K_1^m = \overline{K}$ and $S_1^m = \overline{S}$):

$$\nu_2 = \nu_3 = \nu_5 = 0, \nu_1 > 0 \text{ and } \nu_4 > 0.$$

The difference between the purchases of electricity in the unlimited and limited feed-in cases can now be expressed as follows:

$$(g_1^m - g_1^g) + \sum_j \sum_i P_j P_i (g_{ji}^m - g_{ji}^g) > 0. \tag{26}$$

Therefore, the average level of grid purchases will be higher in the limited feed-in case.