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WP 2017.07

Suggested citation:

K. Constant *(2017).* Environmental policy and human capital inequality : A matter of life and death. *FAERE Working Paper, 2017.07.*

ISSN number: **2274-5556**

www.faere.fr

Environmental policy and human capital inequality: A matter of life and death*

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April, 2017

Abstract

This paper analyzes the economic implications of an environmental policy when we account for the life expectancy of heterogeneous agents. In a framework in which everyone suffers from pollution but health status also depends on individual human capital, we find that the economy may be stuck in a trap in which inequality rises steadily, especially when the initial pollution intensity of production is too high. We emphasize that such inequality is in the long run costly for the economy in terms of health and growth. Therefore, we study whether a tax on pollution associated with an investment in pollution abatement can be used to address this situation. We show that a stricter environmental policy may allow the economy to escape from the inequality trap while enhancing the long-term growth rate when the initial inequality in human capital is not too large.

JEL Classification: I14; O44; Q56; Q58

Keywords: Endogenous Growth, Environmental Policy, Human Capital, Inequality, Longevity.

1 Introduction

While average life expectancy has increased significantly in recent decades, health inequalities have persisted and even widened sharply in some countries. For example, Singh and Siahpush (2006) highlight that the absolute difference in life expectancy between the least-deprived groups and the most-deprived groups in the United States increased by over 60% between 1980 and

*I would like to thank François Salanié and an anonymous referee from the FAERE Working Papers Series for their helpful comments on an earlier draft. I am also grateful to Alain Ayong le Kama, Hassan Benchekroun, Mouez Fodha, Carine Nourry, Fabien Prieur, Natacha Raffin, Thomas Seegmuller, and participants in the conferences PET 2015, FAERE 2015, SURED 2016 and EAERE 2016 and the seminars of the Paris School of Economics, EconomiX (Nanterre) and LAMETA (Montpellier).

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2000. Such disparities in life expectancy represent a worldwide phenomenon. The OECD (2013) reports an average gap of 7.8 years in 2010 between men with the highest and lowest levels of education.[1] In addition to its cost in terms of well-being, health inequality may have crucial economic consequences, through increased health and social costs, reduced productivity, discouraged investments in education and savings, *etc.* Accordingly, it has become a major political issue that many countries explicitly seek to eliminate (see the report of the U.S. Department of Health and Human Services , 2000 for the United States).

In this paper, we study whether environmental policy can be a useful tool for mitigating existing inequalities in life expectancy. We focus on the role of the environment in this issue for two reasons. First, there is considerable evidence that pollution has a positive and significant effect on mortality (see, e.g., Bell and Davis , 2001 ; Pope et al. , 2002 ; Bell et al. , 2004 or Evans and Smith , 2005). At an aggregate level, studies estimate that 23 to 40% of all premature deaths can be attributed to environmental factors (see Pimentel et al. , 1998 and WHO , 2006), while air pollution alone was found to be responsible for approximately 7 million premature deaths in 2012 - representing 1 in 8 total global deaths (WHO , 2014).[2]

Second, a key feature of the health effects of pollution is their unequal distribution across the population. In this regard, it is often stated that "environmental degradation is everyone's problem but it is especially a problem for the poor, who are less able to respond effectively".[3] This observation is broadly supported by empirical studies, as they provide evidence that disadvantaged populations - in particular, those disadvantaged in terms of education - have an increased susceptibility to pollution-related mortality (see, e.g., Cifuentes et al. , 1999 ; Health Effects Institute , 2000 ; Pope et al. , 2002 ; O'Neill et al , 2003 ; Laurent et al. , 2007 or Cakmak et al. , 2011). For example, Zeka *et al.* (2006) reveal that low-educated individuals in the United States have more than twice the mortality risk associated with particulate matter PM_{10} of individuals with high education.[4] Those differences stem from the fact that more-educated individuals are more likely not only to live and work in better socioeconomic conditions but also to enjoy better information that leads to healthier behavior and to have better access to healthcare. Through

[1] On average, among 14 OECD countries for which data are available, at age 30 men with tertiary education can expect to live 7.8 years longer than men with less than upper secondary education.

[2] Specifically, air pollution plays an important role in the development of respiratory and heart diseases (asthma, cancer, stroke, etc.), which can be fatal.

[3] See, e.g., the Resources 2020 lecture by Joseph E. Stiglitz given in October 2012 (http://www.rff.org).

[4] This study considers the U.S. population in twenty cities between 1989 and 2000. Low education corresponds to less than 8 years of schooling, while high education refers to 13 years of schooling or more.

these channels, the human capital of an individual determines her/his exposure and susceptibility to pollution and hence how she/he is affected by it. Moreover, education alone is also identified as an important determinant of life expectancy (see, e.g., Elo and Preston , 1996 ; Lleras-Muney , 2005 ; Cutler and Lleras-Muney , 2010 or Miech et al , 2011), that is why it seems crucial to consider both pollution and human capital when dealing with health inequalities.

Thus far, from a theoretical perspective, there has been increasing interest in life expectancy and its interaction with human capital and/or pollution. The positive effect of human capital on longevity has been considered in papers such as Blackburn and Cipriani (2002), Castello-Climent and Domenech (2008) or Mariani et al. (2010), while the effect of pollution on mortality has been studied, for example, in Pautrel (2009), Jouvet et al. (2010), Mariani et al. (2010), Varvarigos (2010), Raffin and Seegmuller (2014) or Palivos and Varvarigos (2017).[5] By means of this health channel, these contributions have identified the existence of a poverty trap with low life expectancy or fluctuations in the development process and the role that environmental policy could play in this context and its consequences (positive or negative) for economic growth. However, no study has yet been conducted on the economic consequences of the unequal effects of pollution on longevity, and very little consideration has been devoted to the uneven distribution of health in general. Regarding the latter, Castello-Climent and Domenech (2008) analyze the relationship between inequality and a longevity index determined by parents' human capital. Focusing on a form of human capital that depends solely on a time investment, they find that an economy may converge to a long-term equilibrium in which two types of agents are highly unequal. Here, we extend this work by accounting for the role of pollution in health and a more complete relationship between longevity and inequality. We consider additional determinants of human capital accumulation that drive its convergence or divergence in a population, i.e., intergenerational transmission and the quality of the educational system (as in Tamura , 1991 or de la Croix and Doepke , 2003).

It is worth noting that two recent contributions analyze the interactions among health, inequality and pollution, i.e., Aloi and Tournemaine (2013) and Schaefer (2015). Aloi and Tournemaine (2013) formalize a model in which pollution has a direct effect on human capital accumulation (i.e., learning abilities) and find that a stricter environmental policy always reduces income

[5] Other contributions have considered morbidity instead of mortality by studying the negative effect of pollution on productivity or on human capital accumulation. See, e.g., van Ewijk and van Wijnbergen (1995), Aloi and Tournemaine (2011) or Raffin (2012).

inequality, as lower-skilled individuals are assumed to be more affected by pollution, and that this policy can also improve growth if the tax is not too high. Schaefer (2015) focuses on the effect of pollution on child mortality and finds that higher pollution implies widening inequality (due to different exposure) and hence that a higher proportion of the population will favor quantity over quality in their fertility decisions, thus hindering economic development. Here, we depart from these contributions in two major ways. First, we are interested in a different health mechanism, i.e., adult mortality, which is more relevant than childhood mortality, in our analysis of a developed economy because 99% of under-five deaths occur in developing countries (UNICEF , 2015). Second, we take into account endogenous and continuous disparities in the health effects of pollution (rather than having a threshold that defines two exposure levels). In this way, an agent's vulnerability to pollution depends at all times on her/his level of human capital, such that vulnerability can evolve with it, in accordance with empirical evidence that both human capital and pollution affect health. Such an assumption enables us to consider the dynamic nature of inequality and a broader spectrum of long-term behaviors of the economy, in which convergence and divergence among agents are possible.

To conduct our analysis, we formalize an overlapping-generations model, in which agents can live up to three periods depending on their survival probability when old. Their longevity is endogenously determined by their human capital and by pollution. Pollution is represented as a flow that stems from aggregate production, while human capital is the source of both endogenous growth and heterogeneity among households.

We find that multiple balanced growth paths may exist. While there is always a long-term equilibrium without inequality, one or several long-term equilibria with inequality may also occur. A numerical illustration of the model reveals two possible scenarios.[6] First, the long-term equilibrium without inequality is the only one but is a saddle point, meaning that it defines a huge inequality trap representing a situation in which inequality is widening at each generation. Second, this long-term state without inequality is stable and coexists with a long-term equilibrium with inequality, which defines another inequality trap but one of a smaller size. Consequently, there always exists a trap in which the economy experiences steadily growing inequality. We highlight that the pollution intensity of production plays a crucial role in determining the long-term state of the economy. When it is too high, the economy will be stuck in the inequality trap even if

[6]In this illustration, we calibrate the model to match the features of the U.S. economy, which is characterized by substantial health inequalities and seeks to address this issue.

disparities are initially low. The underlying mechanism operates through the fact that longevity influences the return of individuals' investment in education. As vulnerability to pollution differs among individuals, disparities in terms of longevity are wide when pollution intensity is high, which implies that their investments in education are also very unequal. Therefore, the gap in terms of health and human capital is rising at each generation.

Our analysis also reveals the cost that inequality entails for the economy in the long run, notably in terms of economic growth and health (average life expectancy). The role of pollution in the threat of being caught in the inequality trap hence raises questions regarding the possible redistributive power of an environmental policy and the latter's effect on growth. We show that an increase in a tax on pollution, the revenue of which is invested in pollution abatement activities, reduces the size of the inequality trap and can thus enable the economy to escape from it. Such a result comes from the fact that an improvement in environmental quality further increases the return on education investment of poor individuals, who have less human capital and are therefore more vulnerable to the negative health effects of pollution, than that of rich agents. However, the policy may be insufficient to escape the trap. When human capital inequality is too wide, the improvement in the environment required to close the health gap would involve a very high tax rate on pollution, thereby preventing individuals from consuming and hence damaging their welfare. Therefore, we conclude that an environmental policy can be a useful tool to address health inequalities but that the government should implement it as soon as possible, before the gap among agents becomes too wide. Finally, we show that a stricter environmental policy enhances the long-term growth rate of the economy, through the positive effect of the decrease in pollution on life expectancy and the resulting incentive to invest more in education, which fosters human capital accumulation.

This paper is organized as follows. We establish the theoretical model in Section 2. We provide analytical results followed by a numerical illustration in Sections 3 and 4. Section 3 focuses on the long-term equilibria of the economy, while the implications of the environmental policy for the dynamics and growth of the economy are examined in Section 4. Finally, Section 5 concludes, and technical details are relegated to the Appendix.

2 The model

We consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2, ..., +\infty$. Households may live for three periods - childhood, adulthood, and old age - depending on a longevity index. At each date t, a new generation of N heterogeneous agents is born. We assume no population growth and, accordingly, normalize the number of births (N) to unity. Individuals are indexed by $i = p, r$, corresponding to the two groups of individuals in the economy: poor (p) and rich (r), which are of size ξ and $1 - \xi$, respectively. The two groups of agents differ in terms of family wealth because they are unequally endowed with human capital. More precisely, agents born in $t - 1$ differ only in the level of human capital of their parents ($h^p_{t-1} < h^r_{t-1}$).

2.1 Consumer's behavior

An individual of type i born in $t - 1$ cares about her/his consumption levels when an adult c^i_t and when old d^i_{t+1} and about the future human capital of her/his child h^i_{t+1} through paternalistic altruism. The preferences of this representative agent are represented by the following utility function:

$$\ln(c^i_t) + \pi^i_t \left[\beta \ln(d^i_{t+1}) + \gamma \ln(h^i_{t+1})\right] \tag{1}$$

with β and $\gamma > 0$.

The weight π^i_t represents the agent's longevity or her/his survival probability in old age.[7] A higher life expectancy enhances the welfare that individuals obtain from consuming when old and from the future human capital of their children. Indeed, we follow the "companionship" argument of Ehrlich and Lui (1991), who motivate parents' investment in their children by a combination of altruism and self-interest. When parents reach old age, they rely - at least partly - on their children for informal caregiving and material support. Consequently, we consider the parent's "satisfaction from mental security in her old age of having an educated - i.e. wealthier - caregiver and companion" as in Osang and Sarkar (2008).[8] While parents who live longer assign greater weight to the future (i.e., investments), parents with shorter life expectancies have a shorter time horizon and hence assign greater weight to the present (i.e., consumption when adult, which

[7] Since an individual i born in $t - 1$ lives $2 + \pi^i_t$, we interchangeably use the terms "life expectancy", "longevity" and "survival probability" in this paper. We also refer to it as health, although it is just one of many health indicators.

[8] Without this assumption, we would have a counterfactual negative relationship between investments in education and longevity in the economy (see, *e.g.,* Jayachandran and Lleras-Muney , 2009 or Hansen , 2013).

includes their own consumption and, implicitly, that of their children).

Longevity is an index of health status that is assumed to depend on an individual's human capital h_t^i and pollution P_t, in accordance with the empirical evidence mentioned in the introduction. For the sake of simplicity, we assume the following functional form for survival probability in old age, which is in line with Blackburn and Cipriani (2002), Chakraborty (2004), Castello-Climent and Domenech (2008) or Raffin and Seegmuller (2014):

Assumption 1

$$\pi_t^i = \pi\left(\frac{h_t^i}{P_t}\right) = \frac{\sigma h_t^i/P_t}{1 + h_t^i/P_t} \tag{2}$$

with $\sigma \in (0, 1]$ *being the upper bound of longevity. Thus,* $\pi_t \in [0, 1]$, $\pi'(h_t^i/P_t) > 0$ *and* $\pi''(h_t^i/P_t) < 0$.[9]

During childhood, agents devote all of their time to the acquisition of human capital. After reaching adulthood, they are endowed with h_t^i units of human capital, which they allocate between labor force participation, remunerated at wage w_t per unit of human capital, and the education of their children. To ensure that a child achieves the level of education e_t^i that her/his parent selects, the latter has to invest $e_t^i \bar{h}_t$ units of human capital, where $\bar{h}_t$ represents the average human capital in the economy. For a given level of education e_t^i, the required investment in terms of human capital $e_t^i \bar{h}_t$ is the same for all types of agents, as it represents the given amount of knowledge to acquire in a standardized educational system. However, as parents do not use this efficient labor in production, educating children represents an opportunity cost equal to $e_t^i \bar{h}_t w_t$ that is relatively larger for poor parents. This is because poor agents are endowed with less human capital and are thus less efficient in teaching and in providing a given level of education. Note that our modeling of education is perfectly equivalent to that of de la Croix and Doepke (2003) in which education is provided by teachers with a level of human capital equal to the average in the economy. In their second period of life, adults finally allocate their income between consumption c_t^i and savings s_t^i. When old, agents only consume. In line with Yaari (1965), Blanchard (1985) or Chakraborty (2004), we assume a perfect annuity market to abstract from the risk associated with uncertain lifetimes. Therefore, households deposit their savings in a mutual fund, which invests these amounts in physical capital. In return, the mutual fund provides them with an

[9] We do not assume conditions on cross derivatives of π. While the vulnerability to pollution is decreasing with h^i in most of the cases, it is increasing when h^i is very low. The underlying intuition is that when h^i is very low, this agent's health is also very poor. In this particular case, when her/his h^i increases, she/he has now something to lose, which makes her/him more vulnerable to pollution.

actuarially fair annuity during retirement, corresponding to their savings increased by the gross return adjusted with respect to their life expectancy R_{t+1}/π_t^i. Consequently, the two budget constraints for an adult of type i born in $t-1$ are

$$c_t^i + s_t^i = w_t h_t^i - e_t^i \bar{h}_t w_t \tag{3}$$

$$d_{t+1}^i = \frac{s_t^i R_{t+1}}{\pi_t^i} \tag{4}$$

Besides formal education, knowledge accumulation is also determined by the transmission of cognitive and social abilities within the family. Following the endogenous growth models with education and inequality developed by Tamura (1991), Glomm and Ravikumar (1992) and de la Croix and Doepke (2003), the level of human capital of a child born in t h_{t+1}^i thus depends on education e_t^i, the parent's level of human capital h_t^i and the average human capital $\bar{h}_t$, which represents the quality of the educational system.

$$h_{t+1}^i = \epsilon (e_t^i)^\mu (h_t^i)^\eta (\bar{h}_t)^{1-\eta} \tag{5}$$

where $\epsilon > 0$ is the efficiency of human capital accumulation. The parameters μ, η and their sum $\mu + \eta$ are all $\in (0,1)$.[10] They are compatible with endogenous growth and capture the efficiency of education and the intergenerational transmission of human capital within the family relative to the transmission within the society, respectively.

The consumer program is summarized as follows:

$$\max_{e_t^i, s_t^i}, U(c_t^i, d_{t+1}^i, h_{t+1}^i) = \ln c_t^i + \pi_t^i \left[\beta \ln(d_{t+1}^i) + \gamma \ln(h_{t+1}^i)\right] \tag{6}$$

$$s.t \quad c_t^i + s_t^i = w_t h_t^i - e_t^i w_t \bar{h}_t$$

$$d_{t+1}^i = \frac{s_t^i R_{t+1}}{\pi_t^i}$$

$$h_{t+1}^i = \epsilon (e_t^i)^\mu (h_t^i)^\eta (\bar{h}_t)^{1-\eta}$$

The maximization of this program leads to the optimal choices regarding education and sav-

[10] We assume that $\mu + \eta < 1$ to ensure that human capital convergence is possible. Education choice depends positively on h_t^i and negatively on $\bar{h}_t$ (representing the cost of education). Therefore, if $\mu + \eta > 1$, the return on h_t^i is always increasing and the return on $\bar{h}_t$ is negative, such that human capital convergence is impossible.

ings:

$$e_t^i = \frac{\pi_t^i \gamma \mu}{1 + \pi_t^i(\beta + \gamma\mu)} \frac{h_t^i}{\bar{h}_t} \tag{7}$$

$$s_t^i = \frac{\pi_t^i \beta}{1 + \pi_t^i(\beta + \gamma\mu)} w_t h_t^i \tag{8}$$

Rich households invest more in savings and in children's education than do poor households. The reasons for this are the following. First, longevity plays an important role in the optimal choices for education and savings. Rich individuals live longer and hence have higher returns on savings and on children's education, leading them to invest more. Second, there is a traditional income effect on savings. The total wage of a worker depends on the wage rate w_t, which is equal for all agents, and on her/his level of human capital h_t^i. Given that rich agents have a higher level of human capital, they have higher pay and can save more than poor agents. Third, regarding education, rich parents also benefit from a lower opportunity cost due to their higher level of human capital, which facilitates educating their children.

Note that the optimal choice in terms of education is determined by the relative human capital of parents (with respect to the average human capital in the economy) rather than by the parents' absolute human capital level. The rationale for this is that education implies an opportunity cost associated with the investment in human capital that agents have to make to educate their children. For a given e^i, this investment is the same for all agents and depends on the average human capital in the economy, to represent a standardized educational system (in which a unit of schooling time is equivalent for all types of agents). Consequently, the schooling time that parents choose for their children depends on their relative human capital and will be relatively more expensive for lower-skilled parents.

2.2 Production

The production of the consumption good is performed by a single representative firm. Output of this good is produced according to a constant returns to scale technology:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{9}$$

where K_t is the aggregate stock of physical capital, L_t is the aggregate efficient labor supply to production, i.e., the level of human capital used to produce the good, $A > 0$ measures the technology level, and $\alpha \in (0, 1)$ is the share of physical capital in production. Defining $y_t \equiv \frac{Y_t}{L_t}$ as the output per unit of labor and $k_t \equiv \frac{K_t}{L_t}$ as the capital-labor ratio, the production function per unit of labor is as follows:

$$y_t = Ak_t^{\alpha}$$

The government collects revenues through a tax of rate $0 \leqslant \tau < 1$ on production, which is the source of pollution. As a result, the firm chooses inputs by maximizing its profit $(1-\tau)Y_t - R_tK_t - w_tL_t$, such that

$$w_t = A(1-\alpha)(1-\tau)k_t^{\alpha} \tag{10}$$

$$R_t = A\alpha(1-\tau)k_t^{\alpha-1} \tag{11}$$

2.3 Pollution

The index of pollution that we consider in this paper principally embodies air pollution, which represents the world's largest single environmental health risk according to the WHO (2014). An interesting feature of such pollution is that its direct harmful effect on human health is due to its level before absorption, deposition or dispersion in the atmosphere. Moreover, the most significant health threats among air pollutants, i.e., particulate matter and ground-level ozone, remain only for short periods of time (from hours to weeks) in the atmosphere. Consequently, we choose to formalize pollution as the flow currently emitted in the economy rather than as a stock.[11]

Environmental degradation is a by-product of the current production process ($Y_t = y_tL_t$). The government can use the revenue from the pollution tax (τ) to reduce pollution by investing in public environmental maintenance $M_t > 0$. This maintenance, also called pollution abatement activities, represents a public investment in favor of the environment.[12] Finally, we multiply the pollution flow by the ratio of the total labor force $\bar{h}_t$ to the labor used solely in the production of the consumption good L_t (with $\frac{\bar{h}_t}{L_t} \geqslant 1$). The reason is that this weighting enables us to take into account the impact of all economic activity on pollution. More precisely, education is formalized

[11] The same choice is made by, *e.g.,* Pautrel (2009) and Aloi and Tournemaine (2013).

[12] It may correspond, *e.g.*, to clean air strategies implemented to reduce the use of fossil fuels through investments in renewable energy or green transportation subsidies.

in our model as a separate sector that is not included in aggregate output Y. However, education has positive and negative effects on the environment that are important to consider. On the one hand, like any other good or service, its production implicitly implies energy consumption and transportation, which are the largest contributors to early deaths related to particulate matter and ozone (see Caiazzo et al. , 2013). On the other hand, education also allows the economy to improve the efficiency of its pollution abatement activities by promoting technological advances and eco-friendly behaviors (see, e.g., Nelson and Phelps , 1966 and Franzen and Meyer , 2010). Therefore, this weighting allows us to consider both effects and hence to extend the scale effect of pollution to all economic activity.[13] We define the pollution flow as

$$P_t = (ay_tL_t - bM_t)\frac{\bar{h}_t}{L_t} \tag{12}$$

where the parameters $a > 0$ and $b > 0$ correspond to the emission rate of production and the efficiency of environmental maintenance, respectively. The government budget being balanced in each period, the level of public environmental maintenance is equal to $M_t = \tau y_t L_t$. Thus, the pollution flow in a period t can be rewritten as

$$P_t = (a - b\tau)y_t\bar{h}_t \tag{13}$$

Pollution is composed of two elements. While production per unit of efficient labor y_t (which depends on capital intensity k_t) represents an index of the pollution intensity of production, aggregate human capital $\bar{h}_t$ corresponds to the scale effect.

To ensure that human activities lead to a positive pollution flow regardless of the tax rate, we assume that

Assumption 2 $a > b$

[13] Without the weighting, an increase in education would automatically decrease current pollution, thus neglecting its externalities and making the model much more complex. Note that an alternative way of considering the impact of the education sector on the scale effect of pollution is to model a polluting output that includes education, with a cost of education in goods (see, e.g., Mariani et al. , 2010 or Aloi and Tournemaine , 2013).

3 Equilibrium

The market clearing conditions for capital and labor are given by

$$K_{t+1} = \xi s_t^p + (1-\xi) s_t^r \tag{14}$$

and

$$L_t = \xi[h_t^p - e_t^p \bar{h}_t] + (1-\xi)[h_t^r - e_t^r \bar{h}_t] \tag{15}$$

The presence of $e_t^i \bar{h}_t$ illustrates the human capital of parents that does not enter the production of the consumption good. The values of e_t^i and s_t^i are given by the optimal choices of consumers (7) and (8) while the wage w_t corresponds to (10). Thus, the market clearing conditions can be rewritten as follows:

$$K_{t+1} = A(1-\alpha)(1-\tau)k_t^\alpha \left[\xi h_t^p \frac{\pi_t^p \beta}{1+\pi_t^p(\beta+\gamma\mu)} + (1-\xi) h_t^r \frac{\pi_t^r \beta}{1+\pi_t^r(\beta+\gamma\mu)}\right] \tag{16}$$

and

$$L_t = \bar{h}_t \left[\xi x_t^p \frac{1+\pi_t^p \beta}{1+\pi_t^p(\beta+\gamma\mu)} + (1-\xi) x_t^r \frac{1+\pi_t^r \beta}{1+\pi_t^r(\beta+\gamma\mu)}\right] \tag{17}$$

Following de la Croix and Doepke (2003), we introduce the variable $x_t^i \equiv \frac{h_t^i}{\bar{h}_t}$ corresponding to the relative human capital of an individual i in period t. Using (5), the relative human capital of her/his child is described by

$$x_{t+1}^i = \epsilon \left(\frac{\pi_t^i \gamma\mu x_t^i}{1+\pi_t^i(\beta+\gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^i)^\eta \tag{18}$$

with $g_t \equiv \frac{\bar{h}_{t+1}}{\bar{h}_t}$ being the growth factor of average human capital. From the definition of $\bar{h}_t$ $(= \xi h_t^p + (1-\xi) h_t^r)$, we can deduce the expression for the growth of human capital:

$$g_t = \epsilon(\gamma\mu)^\mu \left[\xi \left(\frac{\pi_t^p}{1+\pi_t^p(\beta+\gamma\mu)}\right)^\mu (x_t^p)^{\mu+\eta} + (1-\xi)\left(\frac{\pi_t^r}{1+\pi_t^r(\beta+\gamma\mu)}\right)^\mu (x_t^r)^{\mu+\eta}\right] \tag{19}$$

As the pollution flow corresponds to

$$P_t = (a - b\tau) A k_t^\alpha \, \bar{h}_t \tag{20}$$

we can rewrite the longevity given in (2) in terms of individual relative human capital and the capital-labor ratio:

$$\pi_t^i = \pi\left(\frac{x_t^i}{P_t/\bar{h}_t}\right) = \frac{\sigma x_t^i}{(a-b\tau)Ak_t^\alpha + x_t^i} \tag{21}$$

From equations (16) to (21), we can define the dynamics of the economy as follows:

Definition 1. *Given the initial conditions $K_0 \geqslant 0$, $h_0^p \geqslant 0$ and $h_0^r \geqslant 0$, the intertemporal equilibrium is the sequence $(k_t, x_t^p, x_t^r)_{t\in\mathbb{N}}$ such that the following dynamical system is satisfied for all $t \geqslant 0$.*

$$\begin{cases} k_{t+1} &= \frac{A(1-\tau)(1-\alpha)k_t^\alpha}{g_t}\left[\xi x_t^p \frac{\pi_t^p\beta}{1+\pi_t^p(\beta+\gamma\mu)} + (1-\xi)x_t^r\frac{\pi_t^r\beta}{1+\pi_t^r(\beta+\gamma\mu)}\right] \\ & \left[\xi x_{t+1}^p \frac{1+\pi_{t+1}^p\beta}{1+\pi_{t+1}^p(\beta+\gamma\mu)} + (1-\xi)x_{t+1}^r\frac{1+\pi_{t+1}^r\beta}{1+\pi_{t+1}^r(\beta+\gamma\mu)}\right]^{-1} \\ x_{t+1}^p &= \epsilon\left(\frac{\pi_t^p\gamma\mu}{1+\pi_t^p(\beta+\gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^p)^{\mu+\eta} \\ x_{t+1}^r &= \epsilon\left(\frac{\pi_t^r\gamma\mu}{1+\pi_t^r(\beta+\gamma\mu)}\right)^\mu \frac{1}{g_t}(x_t^r)^{\mu+\eta} \end{cases} \tag{22}$$

where g_t and π_t^i are given by (19) and (21), respectively.

The evolution of the economy is summarized by the laws of motion of the physical capital-labor ratio k and of the relative human capital of poor and rich individuals x^p and x^r, respectively. We can rewrite the dynamical system (22) by substituting the growth of average human capital by its expression given in (19). Moreover, from the definition of average human capital, we can express the relative human capital of rich agents x_t^r as a function of the relative human capital of poor individuals: $x_t^r = \frac{1-\xi x_t^p}{1-\xi}$. After some computations, it follows that the dynamical system given in Definition 1 can be simplified into a two-dimensional system in terms of the capital-labor ratio in consumption good production k and the relative human capital of poor agents x^p.

$$
\begin{cases}
k_{t+1} &= \frac{A(1-\tau)(1-\alpha)k_t^{\alpha}}{\epsilon(\gamma\mu)^{\mu}}\left[\xi x_t^p \frac{\pi_t^p \beta}{1+\pi_t^p(\beta+\gamma\mu)} + (1-\xi x_t^p)\frac{\pi_t^r \beta}{1+\pi_t^r(\beta+\gamma\mu)}\right] \\
& \left[\xi x_{t+1}^p \frac{1+\pi_{t+1}^p \beta}{1+\pi_{t+1}^p(\beta+\gamma\mu)} + (1-\xi x_{t+1}^p)\frac{1+\pi_{t+1}^r \beta}{1+\pi_{t+1}^r(\beta+\gamma\mu)}\right]^{-1} \\
& \left[\xi \left(\frac{\pi_t^p}{1+\pi_t^p(\beta+\gamma\mu)}\right)^{\mu} (x_t^p)^{\mu+\eta} + (1-\xi)\left(\frac{\pi_t^r}{1+\pi_t^r(\beta+\gamma\mu)}\right)^{\mu}\left(\frac{1-\xi x_t^p}{1-\xi}\right)^{\mu+\eta}\right]^{-1} \\
x_{t+1}^p &= (x_t^p)^{\mu+\eta}\left(\frac{1-\xi x_t^p}{1-\xi}\right)^{-\mu-\eta}\left(\frac{1-\xi x_{t+1}^p}{1-\xi}\right)\left(\frac{\pi_t^p}{1+\pi_t^p(\beta+\gamma\mu)}\right)^{\mu}\left(\frac{\pi_t^r}{1+\pi_t^r(\beta+\gamma\mu)}\right)^{-\mu}
\end{cases}
\tag{23}
$$

with π_t^i given by (21).

Relative human capital is a useful variable because it connects the two groups. A decrease in the relative human capital of poor individuals x^p corresponds to a proportional increase in the relative human capital of rich agents x^r. The lower x^p is, the lower the level of human capital of poor individuals is relative to rich agents, and hence, the wider the disparities are. Consequently, we can use the relative human capital of poor individuals x^p to approximate the level of human capital inequality in the economy. When there is no inequality, all individuals have the same human capital, which means that all relative human capital levels are equal ($x^p = x^r = 1$), while conversely, when inequality is at its maximum, x^p and x^r tend to 0 and $\frac{1}{1-\xi}$, respectively.[14]

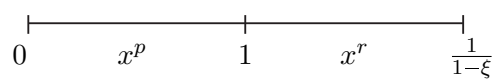

Figure 1: Representation of relative human capital.

In the remainder of this section, our aim is to analyze the long-term behavior of the economy.

Definition 2. *A balanced growth path (BGP) is an equilibrium satisfying Definition 1 and in which the stocks of physical and human capital grow at a single, constant rate ($g-1$).*

On a balanced growth path, the capital-labor ratio k_t, the growth in average human capital g_t, relative human capital x_t^i and the flow of pollution P_t are constant.

3.1 Balanced growth paths with and without inequality

From Definitions 1 and 2, we explore the properties of the dynamical system (23) and deduce the existence and the stability of balanced growth paths with and without inequality.

[14] From the definition of average human capital and the fact that we assume $h^p \leqslant h^r$ (as it is the only difference between the two types of individuals), it follows that $0 \leqslant x^p \leqslant 1$, while $1 \leqslant x^r \leqslant \frac{1}{1-\xi}$.

Proposition 1 *Under Assumptions 1 and 2 and the condition that $\alpha < \frac{1}{2}$ we have the following:*

- ***[Without inequality]*** *There exists a balanced growth path without inequality $(k_E, 1)$, with a positive growth rate ($g_E - 1 > 0$) for $\epsilon > \bar{\epsilon}$.*[15] *This BGP is locally stable when $\eta < \bar{\eta}(\tau)$ and corresponds to a saddle point otherwise. The thresholds $\bar{\epsilon}$ and $\bar{\eta}(\tau)$ correspond to*

$$\bar{\epsilon} \equiv \left[\frac{(a-b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta+\gamma\mu)}{\sigma\gamma\mu}\right]^{\mu} \quad \text{and} \quad \bar{\eta}(\tau) \equiv 1 - \mu\frac{2(a-b\tau)Ak_E^{\alpha} + (1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^{\alpha} + (1+\sigma(\beta+\gamma\mu))}$$

- ***[With inequality]*** *There exists at least one BGP with inequality when $2\mu + \eta > 1$ and $\eta < \tilde{\eta}(\tau)$, where the threshold $\tilde{\eta}(\tau)$ is the value of η such that*[16]

$$\frac{A^{\frac{1}{\alpha}}(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}(a-b\tau)^{\frac{1-\alpha}{\alpha}}}{\epsilon\gamma^{\mu}(1+\sigma(\beta+\gamma\mu))^{\frac{1-\alpha(1+\mu)}{\alpha}}} = \frac{(1-\mu-\eta)^{\frac{1-\alpha}{\alpha}}\mu\left[(1+\beta\sigma)+\sigma\gamma(1-\mu-\eta)\right]}{(2\mu+\eta-1)^{\frac{1-\mu}{\alpha}}}$$

Proof. See Appendix 6.1. ■

While there always exists a balanced growth path without inequality, one or several balanced growth path(s) characterized by inequality among households may also occur. Therefore, the economy may converge in the long run to an equilibrium without inequality among households[17], but this is not always the case. The economy may also be trapped in situations in which human capital inequality is persistant or increasing over time. For example, when the long-term equilibrium without inequality E is a saddle point ($\eta > \bar{\eta}(\tau)$), individuals' human capital and longevity will most likely diverge within the population.[18]

From Proposition 1, it emerges that the conditions determining the extent of inequality in the economy depend on η and μ, i.e., the weights in human capital accumulation. The reason is that human capital convergence or, conversely, the persistence of human capital inequality stems from the balance among several forces. Our model combines channels usually found in the literature on human capital and inequality (e.g., Tamura , 1991, Glomm and Ravikumar , 1992 or de la Croix

[15] The condition that $\epsilon > \bar{\epsilon}$ means that human capital accumulation has to be sufficiently efficient to have a positive growth rate in the long-term state without inequality. If it is not satisfied, the growth rate of human capital is always negative, causing the economy to collapse.

[16] The condition that $\eta < \tilde{\eta}(\tau)$ is sufficient but not necessary. The expression $\tilde{\eta}(\tau)$ is implicit but deduced from Appendix 6.1.2. The effects of τ on $\bar{\eta}(\tau)$ and $\tilde{\eta}(\tau)$ are detailed in Section 4.1.

[17] Such equilibrium is possible because agents differ only in the initial level of human capital. We do not assume in this model that poor individuals are less able to acquire skills.

[18] By definition, $0 \leqslant x^p \leqslant 1$. Thus, the economy can still achieve the BGP without inequality ($x^p = 1$) under few initial conditions where x_0^p is very high and k_0 is very low. We analyze the extent of inequality when $(k_E, 1)$ is unstable in the numerical illustration in Section 3.2.

and Doepke , 2003) and the uncommon longevity channel (see Castello-Climent and Domenech , 2008 for an exception).[19]

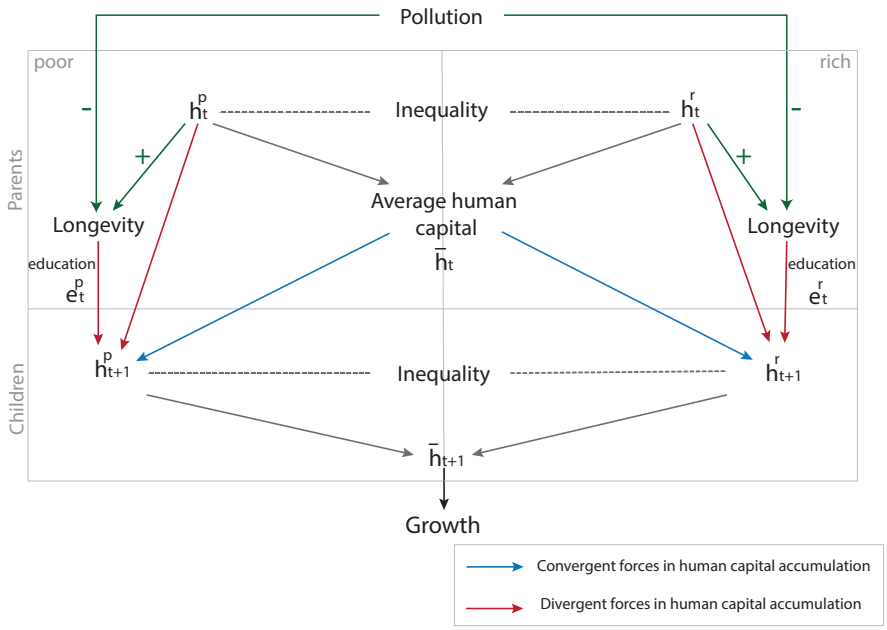

Figure 2: Structure of the model.

As represented in Figure 2, children's human capital is determined by their parents' human capital (intergenerational transmission), by parental investment in education and by the average human capital in the economy, which represents the quality of the educational system. The two first elements represent divergent forces in human capital accumulation, which perpetuate disparities among agents across generations, while the latter is a convergent force that reduces them over time. Human capital inequality is transmitted to the next generation directly through the intergenerational spillover and more indirectly through education choices because differences in human capital among parents also correspond to disparities in income and life expectancy. As a result, poor parents can less afford the cost of educating their children and die sooner, which discourages their investment in education, whereas rich parents are more able and willing to finance education. Conversely, the presence of average human capital in the production of human capital represents a convergent force, which is crucial to ensuring that human capital convergence is possible, as Tamura (1991) shows.

According to these effects, it emerges that the weights of the divergent forces - i.e., education and intergenerational transmission - in human capital accumulation must not be too high to

[19] Castello-Climent and Domenech (2008) isolate the life expectancy channel by assuming that human capital formation depends only on an investment in time. Thus, they do not consider the other aforementioned effects.

ensure that it is possible for an economy to avoid inequality in the long run. More precisely, we find that if the sum of these weights is at its maximum ($\mu + \eta \to 1$), there is only the long-term equilibrium without inequality and it is a saddle point, which means that inequality would grow steadily under most of the initial conditions of the economy. Conversely, when it is at its minimum ($\mu + \eta \to 0$), the equilibrium without inequality is the only equilibrium, but it is stable, meaning that inequality vanishes in the long run.[20]

Between these two extreme cases, the economy may or not exhibit long-term inequality. In this paper, the diminishing return of the divergent forces in human capital accumulation (i.e., $\mu + \eta < 1$) is not sufficient to ensure convergence, contrary to what is usually obtained in the literature on human capital and inequality (see, e.g., Tamura , 1991 or Glomm and Ravikumar , 1992). This is due to endogenous longevity. If longevity were exogenous in our model, the growth of individual human capital would always be higher for poor households, and thus, the economy would always converge to the long-term equilibrium without inequality, since $\mu + \eta < 1$ holds. Here, on the contrary, both human capital convergence and divergence are possible under this condition and the determinants of individuals' health play a crucial role in determining the long-term behavior of the economy. Pollution affects the returns on agents' investment in education through its impact on their life expectancy, and this effect is not the same for all agents. Indeed, some agents have less human capital than others and hence less knowledge, information or financial means to protect themselves from the harmful effect of pollution, making them more susceptible to it.[21] Therefore, when pollution and/or inequality are too high, the growth of individual human capital can be lower or equal for those who are poor.[22] In this case, the economy is stuck in an inequality trap, in which disparities among households are persistent and/or rising over time.

Analytically, we are not able to conclude on the dynamics of the long-term equilibrium(equilibria) with inequality. Therefore, we numerically analyze the model in the following section to obtain a more comprehensive overview of the different scenarios in which the economy may end up.

[20] See Appendix 6.1.

[21] As in Blackburn and Cipriani (2002), we do not formalize health expenditures in this paper (neither private nor public), but we assume that individual human capital includes the capacity of agents to spend in healthcare. For models with explicit healthcare spending, see, e.g., Varvarigos (2010) or Raffin and Seegmuller (2014).

[22] The individual growth of human capital g^i is increasing and then decreasing in x^i. The maximum value of g^i is achieved in $x^i < 1$, while $g^i(0) = 0$ and $g^i(Max\{x^r\}) > 0$. Thus, there exists a level of x^p under (resp. above) which the individual growth of human capital is lower (resp. higher) for poor than for rich agents $g^p < g^r$ (resp. $g^p > g^r$).

3.2 Numerical illustration

In this section, we provide a numerical analysis of the model to provide further insights into the long-term behavior of the economy. To do so, we calibrate the model on the United States, which is particularly affected by health inequalities and officially intends to address this issue (U.S. Department of Health and Human Services , 2000). After motivating the choice of the parameter values summarized in Table 1, we study in detail the features of the different balanced growth paths.

3.2.1 Calibrations

Table 1: Description of the model parameters.

Parameter	Description	Calibrated value
ξ	Share of poor individuals in each cohort	0.5
β	Preference for old age consumption	0.3
γ	Preference for children's human capital	0.35
σ	Maximum share of old age that individuals can live	0.9
α	Share of physical capital in production	1/3
A	Total factor productivity	1
a	Emission rate of production	0.6
b	Efficiency of environmental maintenance	0.4
ϵ	Efficiency of human capital accumulation	6
μ	Weight of education in human capital accumulation	0.6
η	Weight of intergenerational transmission in human capital accumulation	[0,0.4)
τ	Pollution tax rate	[0,1)

To solve the model numerically, we assign values to the parameters of technology and preferences such that they fit empirical observations and projections for the U.S. economy. Assuming that a period represents thirty years, the parameter in the longevity function σ is set to 0.9 to obtain realistic values of individual life expectancy.[23] Consequently, an individual starts working at 30, retires at 60 and may live for up to 87 years, according to her/his longevity.[24] In the real business cycle literature, the quarterly psychological discount factor is estimated at 0.99 (see Cooley , 1995 or de la Croix and Michel , 2002). Thus, β is set to $0.99^{4\times 30} = 0.3$. Assuming that the two groups of workers have the same size ($\xi = 0.5$)[25], we set the scale parameter ϵ to 6 and

[23]While having only one parameter in the longevity function may appear restrictive, our results are robust to the introduction of another parameter, e.g. ν in $\pi_t^i = \frac{\sigma h_t^i/P_t}{\nu + h_t^i/P_t}$. Note that this parameter, driving the vulnerability of individuals to pollution, would only reinforce our mechanism by increasing the size of the trap when becoming higher. Proof available upon request.

[24]While the lower bound is equivalent to the U.S. life expectancy at birth in the 1930s (60), the upper bound is close to the value expected by the U.S. Census Bureau for 2060 (84 years).

[25]We provide a sensitivity analysis with respect to the distribution of the population ξ in Appendix 6.4.

the preference for children's human capital γ to 0.35 to match U.S. data on the annual long-term growth rate (i.e., approximately 1.7%) and the U.S. share of education expenditure in GDP on the balanced growth path (i.e., between 5 and 8%).[26] For the production technology, the share of physical capital in the production function α is set to 1/3 in accordance with empirical data, and total factor productivity A is a scale parameter set to 1. Regarding pollution, the weights of production and of environmental maintenance in the pollution flow are chosen to satisfy the condition that $a > b$, thus ensuring that there are pollution emissions in the presence of economic activity, i.e., $a = 0.6$ and $b = 0.4$.

The parameter μ represents the weight of education in human capital accumulation and also corresponds, in our model, to the elasticity of human capital with respect to education. In the literature, the return to schooling in developed countries is estimated to be between 8 and 16% (see Ashenfelter and Krueger , 1994; Psacharopoulos , 1994 or Krueger and Lindahl , 2001). These figures correspond to Mincerian returns, which means that they include only an opportunity cost (forgone earnings) and do not consider education expenditure. Following de la Croix and Doepke (2003), we assume that an additional year of schooling raises education expenditure by 20%. The resulting elasticity of schooling ranges from 0.4 to 0.8. Thus, we set μ to be 0.6.

The weight of intergenerational transmission of human capital η is a key parameter in our analysis, we will consider all values satisfying our assumption that human capital convergence is not impossible, i.e., $\eta \in [0, 1-\mu)$. But, these values are also consistent with the empirical literature which finds intergenerational education elasticities between 0.2 and 0.45 (see, e.g., Dearden et al , 1997 or Black et al , 2005). In the same way, we let the tax on pollution vary by considering all tax rates $\tau \in [0, 1)$.[27]

3.2.2 Long-term behavior of the economy

To obtain a clearer picture of the different scenarios, we analyze the existence and dynamics of the balanced growth paths in the calibrated economy. For the set of parameters considered, we obtain the following result and represent the dynamics of the economy in Figure 3.[28]

Numerical result 1 (i) *When $\eta > \bar{\eta}(\tau)$, there exists a unique BGP, which is that without*

[26] See the long-term projections for the U.S. economy of the OECD (2014a) on growth and the Digest of Education Statistics 2012 of the U.S. Department of Education for data on the education share.

[27] All the following numerical results are obtained by considering all η and τ with a pitch value of 0.000001 units.

[28] The second dynamical equation in (23), represented by the blue curve in Figure 3, is discontinuous at $x^p = 1$.

inequality $(k_E, 1)$*. It is a saddle point with a stable branch* SS_E*.*

(ii) When $\eta < \bar{\eta}(\tau)$*, there exist multiple balanced growth paths: the BGP without inequality* $(k_E, 1)$ *and an additional BGP with inequality* (k_I, x_I^p)*. The first is stable, while the latter is a saddle point with a stable branch* SS_I*.*

Consequently, as represented in Figure 3, if the initial conditions of the economy (k_0, x_0^p) *are*

- *to the left of* SS_j *(with* $j = E, I$*), the economy is caught in an inequality trap*
- *to the right of* SS_j *(with* $j = E, I$*), the economy will converge to the BGP without inequality.*

First, it is worth noting that in the numerical analysis, the threshold $\bar{\eta}(\tau)$ identified in Proposition 1 corresponds not only to the value under which the equilibrium without inequality E is stable but also to the value under which an equilibrium with inequality I appears, for all tax rates. This means that, for realistic calibrations of the model on the U.S. economy, there always exists an inequality trap in which disparities widen over time. Only the size of the trap, i.e., the size of the set of initial conditions for which an economy is caught in the trap, varies. The higher the weight of intergenerational transmission in human capital accumulation η is, the heavier the weight of the divergent forces, and therefore, the larger the size of the inequality trap.[29] In particular, when $\eta > \bar{\eta}(\tau)$ (represented in the left panel of Figure 3), the economy is stuck in the trap under most of its initial conditions.

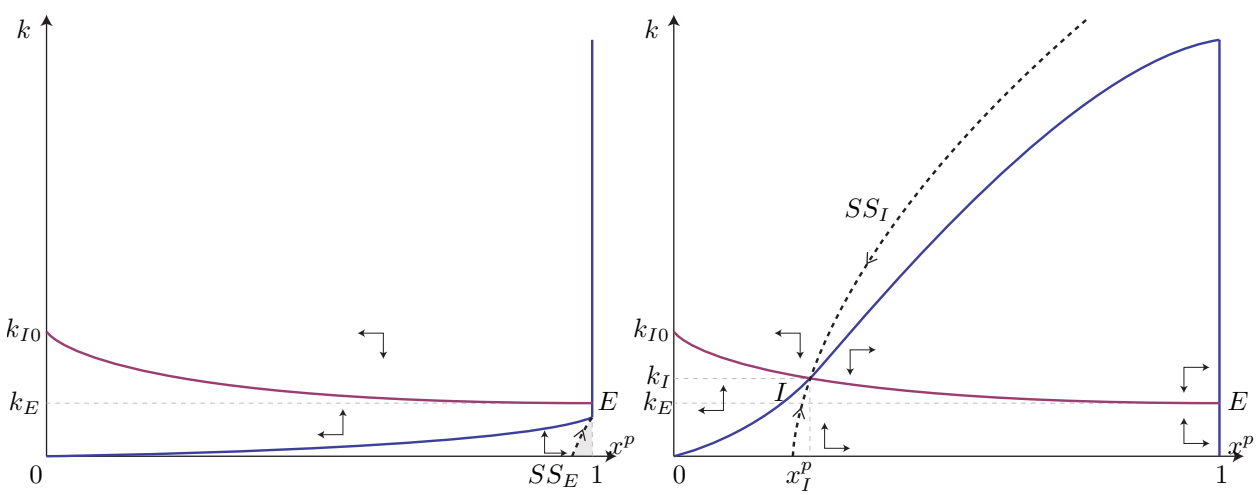

Figure 3: Phase diagrams when $\eta > \bar{\eta}(\tau)$ (left panel) and when $\eta < \bar{\eta}(\tau)$ (right panel).

The underlying mechanism for the coexistence of an inequality trap and a long-term equilibrium without inequality stems from the fact that the return on investment in education varies

[29] See the sensitivity analysis in Appendix 6.4 for further details on the effect of η on the results.

according to the levels of inequality and pollution intensity in the economy, as explained in Section 3.1. When the initial conditions of an economy (k_0, x_0^p) are to the right of curve SS_j (with $j = E, I$), inequality is sufficiently low and environmental quality is sufficiently high (i.e., high x_0^p and low k_0) that poor households have a higher return on education investment than rich households, which allows them to narrow existing disparities over the generations and to converge to an equal equilibrium in the long run.[30] However, when the initial conditions of an economy (k_0, x_0^p) are to the left of curve SS_j, inequality and/or pollution intensity are too high (i.e., low x_0^p and/or high k_0), and thus, health disparities are too wide for poor agents to be able to close the gap (the return on the investment in education is higher for rich households). Consequently, the economy is stuck in a trap in which inequality will steadily increase.

When the economy is in the inequality trap, it converges asymptotically to a state in which inequality is at its maximum, i.e., the lower bound of the trap (k_{I0}, x_{I0}^p). Such an extreme case is not achieved, but it illustrates the constant deterioration of living conditions of disadvantaged households in the trap. Indeed, in this state, the human capital of poor agents tends to zero $(x_{I0}^p \to 0)$, as do their income and longevity π^p. Therefore, poor agents would die at the end of the second period of life (before retirement) and would no longer be able to consume, save or educate their children, meaning that the poor category would collapse. Conversely, rich households tend to a state in which they are richer, more educated and live longer than in the other long-term equilibria.

The numerical illustration provides a clearer picture of how endogenous longevity favors the intergenerational transmission of inequality. As in Castello-Climent and Domenech (2008), it plays a crucial role. However, we consider additional forces in human capital accumulation and additional determinants of longevity that contrast our results from theirs. While they find that the economy always converges to a long-term equilibrium with high but "sustainable" inequality, we find that there are two possible scenarios for an economy: human capital convergence or an inequality trap in which disparities among households widen until a part of the population collapses. Moreover, we expand the mechanism for the transmission of inequality through longevity to the environmental dimension and emphasize that pollution is a critical determinant of the long-term behavior of the economy.

Studying the features of the two long-term equilibria E and I and of the lower bound of the

[30] This result holds even if the trap is bounded by E (i.e., when $\eta > \bar{\eta}(\tau)$): As $x^p \in [0, 1]$, the economy will converge to E if its initial conditions are on the stable branch SS_E or to its right (shaded area in Figure 3).

trap $I0$, we obtain the following result.

Numerical result 2 *Wider human capital inequality in the long run ($1 > x_I^p > x_{I0}^p$) is associated with the following:*

- *a higher physical capital-labor ratio and hence a higher pollution intensity ($k_{I0} > k_I > k_E$),*
- *wider disparities in life expectancy ($\pi_E > \pi_I^p > \pi_{I0}^p$) and a lower average life expectancy in the economy ($\pi_E > \bar{\pi}_I > \bar{\pi}_{I0}$),*
- *a lower long-term growth rate ($g_E > g_I > g_{I0}$).*

The numerical analysis of the model indicates that the long-term physical capital-labor ratio is higher in long-term states with inequality. Given that the marginal propensity to save is higher for rich individuals, inequality consists in concentrating wealth in the hands of those who save more. It follows that aggregate savings is higher in such a state, which favors physical capital accumulation. Given that the capital-labor ratio drives the pollution intensity of production, the latter is also larger in equilibria with inequality.

A wider dispersion of human capital and a higher pollution intensity lead to wider inequality in health. While poor agents have lower relative human capital and suffer from the higher pollution intensity, rich agents benefit from an increase in their relative human capital, which more than offsets the increase in pollution intensity. The net outcome is that in the presence of inequality, rich individuals live longer whereas poor individuals die sooner. Moreover, the loss of longevity for poor individuals is greater than the benefit for rich individuals, and therefore, the average life expectancy in the economy declines with inequality in the long run (even if the population is equally distributed between the two categories).

In addition to this cost in terms of health, inequality also implies a cost in terms of growth. As an illustration, for an intermediate weight of intergenerational transmission ($\eta = 0.2$) and without an environmental policy ($\tau = 0$), the estimated losses generated by inequality equal approximately 10 years of average life expectancy in the long run and approximately 0.4 percentage points of long-term economic growth per year, which represents substantial damage to cumulative growth.[31] This latter result contributes to the substantial literature on the implications of inequality for growth, surveyed for example by Galor (2011). From these numerous analyses, it appears that inequality may have both positive and negative effects on economic performance and that the

[31] The exact figures are 83.2 and 72.4 years for average life expectancy at E and $I0$, respectively, and 1.68% and 1.31% for the compound annual economic growth rate at E and $I0$, respectively.

debate is still ongoing. Here, we conclude in favor of a net detrimental effect of disparities on economic growth through human capital accumulation. The explanation stems from the fact that human capital is embodied in individuals whose physiological constraints imply diminishing marginal returns in individual human capital accumulation. Therefore, greater inequality leads to a higher concentration of education spending in the economy, which reduces the aggregate stock of human capital.[32] In accordance with this result, the OECD (2015) finds "consistent evidence that the long-term rise in inequality of disposable incomes observed in most OECD countries has indeed put a significant brake on long-term growth" and that it is "in large part because it reduces the capacity of the poorer segments to invest in their skills and education".

From the model's illustration, it emerges that an economy is most likely to fall into a trap with rising inequality since the pollution intensity of its activities is high, even if inequality is initially weak. Examining the data, it appears that "inequality is today at its highest since data collection" in many OECD countries (including the U.S.) and follows an upward trend.[33] Given the adverse effects of such disparities on growth and health in the long run and the key role of pollution in this trap phenomenon as highlighted by our contribution, we wonder whether an environmental policy could be useful to break such a vicious circle. This is the purpose of the following section.

4 Environmental policy implications

In this section, we assess the effect of a higher pollution tax associated with increased public investment in pollution abatement activities on the dynamics and growth of the economy. In particular, we want to know whether such an environmental policy can have redistributive power, given the role of the pollution intensity of production in the persistence of inequality, and whether it can allow for enhanced long-term economic growth, which is driven by human capital. For each point, we first provide an analytical analysis of the policy implications and then illustrate them numerically.

[32] Theoretically, several channels have been illustrated to explain this relationship such as credit market imperfections (Galor and Zeira , 1993), fertility differentials (de la Croix and Doepke , 2003) or longevity differentials (Castello-Climent and Domenech , 2008), as in this paper.

[33] See OECD (2015) pages 5 and 25.

4.1 Environmental policy implications on the balanced growth paths

From Proposition 1, we know that the conditions under a stable long-term equilibrium without inequality and under long-term equilibria with inequality both depend on the environmental tax τ. Examining how an increase in the tax on pollution affects these thresholds, we make the following proposition.

Proposition 2 *Under Assumptions 1 and 2 and for $\alpha < 1/2$ and $2\mu + \eta > 1$, the thresholds $\bar{\eta}(\tau)$ and $\tilde{\eta}(\tau)$ depend positively on the tax rate τ. Moreover, there always exists a sufficient environmental tax rate τ such that*

- *the BGP without inequality is stable (i.e., $\eta < \bar{\eta}(\tau)$)*
- *there exists at least one BGP with inequality (i.e., $\eta < \tilde{\eta}(\tau)$).*

Proof. See Appendix 6.2. ■

A rise in the tax on pollution allows the associated investment in abatement to increase, which reduces the pollution flow. Consequently, the longevity of all agents increases, which in turn positively affects their returns from education. They accordingly choose a higher level of human capital. However, although the individual growth of human capital increases for all actors, the decrease in pollution has a relatively larger effect on poor households, which are more susceptible to pollution.[34] With all agents being proportionally taxed, it follows that an increase in the tax on pollution increases the likelihood of convergence toward the long-term equilibrium without inequality.

An increase in the tax rate also increases the likelihood of the existence of one or several balanced growth path(s) with inequality. This could mean that the tax on pollution favors the persistence of inequality in long run or that the tax restricts such inequality by stabilizing it (rather than allowing it to worsen) or even by reducing the size of the inequality trap. To be able to obtain more precise the implications of the environmental tax for the long-term behavior of the economy, we use the numerical illustration begun in Section 3.2.

For the parameters considered, we found that the threshold $\bar{\eta}(\tau)$ represents the value under which both the long-term equilibrium without inequality E becomes stable and the long-term equilibrium with inequality I appears. The result below thus follows:

[34] More precisely, $\frac{\partial g^i}{\partial \tau} > 0$, but $\frac{\partial^2 g^i}{\partial \tau \partial x^i}$ is > 0 when x^i is small and < 0 when x^i is high, with a threshold equal to $\frac{(a-b\tau)Ak^{\alpha}(2\mu+\eta-1)}{(1+\sigma(\beta+\gamma\mu))(2-\mu-\eta)}$. Thus, the increase in the tax reduces this threshold, meaning that there are more levels of x^i such that poor agents are more affected by the decrease in pollution than rich agents.

Numerical result 3 (i) *When $\eta > \bar{\eta}(0)$, there only exists the BGP without inequality E, which is a saddle point and defines the inequality trap for a low pollution tax τ. However, when the tax rate becomes sufficiently high, η becomes lower than $\bar{\eta}(\tau)$, which implies that the BGP without inequality E becomes stable while that with inequality I appears and is a saddle point, defining the new trap.*[35]

(ii) *When $\eta < \bar{\eta}(0)$, the condition such that the BGP without inequality E is stable and the BGP with inequality I exists as a saddle point is satisfied for all rates of the pollution tax.*

From this numerical result, we deduce that, when $\eta > \bar{\eta}(0)$, a sufficient tax on pollution may reduce the size of the inequality trap by allowing the long-term equilibrium I to exist and thus to be the upper bound of the trap instead of E, as SS_I defines a smaller set of initial conditions for which the economy is caught in the trap than SS_E. Furthermore, we obtain a more general result:

Numerical result 4 *An increase in the environmental tax always decreases the size of the inequality trap. Thus, a sufficient increase in the tax on pollution can allow an economy to escape from the trap and to converge toward the BGP without inequality.*

The net effect of the environmental policy on the level of inequality is hence negative. As described above, an increase in the tax rate enables a reduction in environmental damage, which improves the life expectancy of agents and hence increases their return on investment in education. This effect is even stronger for disadvantaged households, which are relatively more sensitive to pollution. As a result, a sufficient increase in the tax on pollution can allow unequal economies to escape from the trap and to reduce disparities along the convergence to a long-term equilibrium without inequality.

To illustrate the Numerical Results 3 and 4, we use Figure 4, corresponding to the case in which the trap is the largest, i.e., $\eta > \bar{\eta}(0)$, to represent all possible scenarios. As above, the inequality trap is bounded by the dotted curve SS_j. An economy with initial conditions to the left of this curve is in the trap in which inequality worsens at each generation, whereas an economy starting at the right of this curve will converge to the long-term equilibrium without inequality. For all tax rates, we observe that as τ increases, the inequality trap moves to the left, meaning that the size of the trap decreases. In other words, the set of initial conditions for which the economy achieves the equilibrium without inequality becomes larger.

[35] Note that when τ tends to 1, $\bar{\eta}(\tau)$ is always greater than η.

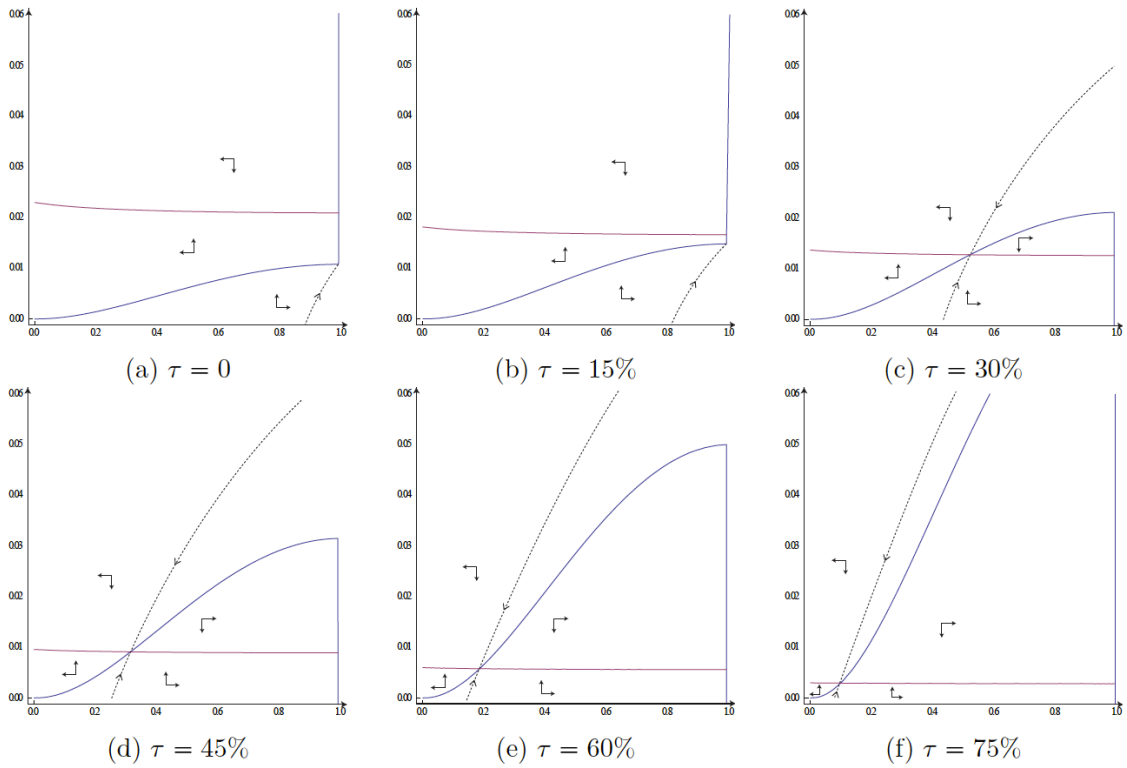

Figure 4: Phase diagrams when $\eta = 0.35$, i.e., $\eta > \bar{\eta}(0)$, for different tax rates τ, with x^p on the X-axis and k on the Y-axis.

More precisely, for a low pollution tax, we have that $\eta > \bar{\eta}(\tau)$. Thus, the only long-term equilibrium is that without inequality, and it defines a substantial inequality trap (phase diagram (a)). When the pollution tax increases, the size of the inequality trap decreases even if it is still bounded by E (phase diagram (b)). When the tax becomes sufficiently high (phase diagram (c)), the condition $\eta < \bar{\eta}(\tau)$ is now satisfied such that the BGP with inequality I appears and E becomes stable, which sharply diminishes the size of the trap. Finally, as the environmental tax continues to increase, BGP I continues to move to the left, thus the trap further decreases (phase diagrams (d) to (f)).

It is important to note that there are some limitations to the redistributive power of an environmental policy. First, even if the environmental policy always makes the trap smaller, it does not directly reduce inequality. Therefore, an economy may still be in the inequality trap after an increase in pollution taxation and hence still suffer from growing disparities. Our result that an environmental policy does not always succeed in reducing inequality contrasts, for example, with Aloi and Tournemaine (2013). This is because we consider differences in

susceptibility to pollution that are endogenously and continuously determined by an individual's human capital. When human capital inequality and/or pollution intensity are initially too high, a given improvement in environmental quality may be insufficient to overcome the existing gap in the returns on investment in education.

Second, even if, technically, there always exists a tax rate such that the economy can escape the trap, the more unequal a society is and the higher the pollution intensity is, the higher the tax rate necessary to escape the inequality trap. In extreme cases, the required tax can even be close to 100%. The problem is that when the tax rate is too high, it prevents households from consuming and therefore harms their welfare. An environmental policy composed of a reasonable tax rate on pollution may thus be insufficient to reduce inequality in the economy.

We conclude that an environmental policy, consisting of a public investment in environmental protection financed by a tax on pollution, can be a useful tool to reduce inequality through its positive effect on health but may also be insufficient for given social and environmental conditions. Moreover, as the longer a government waits, the higher the tax rate necessary to escape from the inequality trap, we emphasize that policy makers should implement an environmental policy as soon as possible to address inequalities.

When the environmental policy alone is not sufficient to remove the economy from the trap, other types of policies should be combined with it. In this respect, education and/or health policies targeting disadvantaged households would be much more efficient than an income transfer, as the lack of education is due - at least partly - to a low return on investment. However, these policies alone would be subject to the same kind of limitations than those mentioned above. In particular, for most of initial levels of inequality, the economy would not be able to escape from the inequality trap without an environmental policy if its pollution intensity is too high. An improvement in the environment therefore represents a key tool for addressing this issue.

4.2 Environmental policy implications for growth

The growth factor of human and physical capital on the balanced growth path without inequality is given by

$$g_E = \epsilon \left[\frac{\sigma\gamma\mu}{(a - b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta + \gamma\mu)} \right]^{\mu} \tag{24}$$

Analyzing the effect of the environmental policy on this long-term growth rate reveals the following:

Proposition 3 *Under Assumptions 1 and 2, an increase in the tax on pollution improves the growth rate on the BGP without inequality ($g_E - 1$).*

Proof. See Appendix 6.3 ■

Several effects occur. On the one hand, a rise in the tax on pollution implies a negative income effect, as firms pass it through to the wage rates (w_t) and returns on savings (R_t) of households. On the other hand, a higher tax rate leads to more maintenance activities, which improve environmental quality and hence health. Through this channel, individuals' longevity increases, which leads to greater returns on savings and children's education. For savings, the negative income effect outweighs the longevity effect, such that savings decrease as the tax rate increases. However, for education, a third effect operates. It is important to bear in mind that education is an investment in human capital that is made by parents and corresponds to an opportunity cost associated with the fact that they do not use this efficient labor to produce. Therefore, the negative income effect of the tax is neutralized by its positive effect through reduced opportunity costs. The net effect of the environmental policy on education is thus positive, meaning that the stock of human capital improves with the tax. Human capital being the engine of growth in the economy, the long-term growth rate is also enhanced under a stricter environmental policy.

Analyzing numerically the results in the other long-term states of the economy, i.e., the balanced growth path I and the lower bound of the trap (k_{I0}, x^p_{I0}) in which inequality is at its maximum, we observe the following:

Numerical result 5 *An increase in the tax on pollution $\tau \in [0, 1)$ decreases the long-term capital-labor ratios (k_I and k_{I0}) and improves the long-term growth rates (g_I and g_{I0}).*

Thus, a stricter environmental policy enables improving the long-term growth of an economy even when it does not enable the economy to escape from the inequality trap. In the same way that the policy favors economic growth on the balanced growth path without inequality E, it improves the life expectancy of all agents, which stimulates their investments in education and favors the growth rate of average human capital. However, in this case, the policy is not

sufficient to make the return to education of poor parents larger than that of rich parents, and thus, inequalities in human capital and life expectancy continue to worsen.

Note that the positive effect of an environmental policy on economic growth is bounded, just as its redistributive power is. Indeed, if the tax rate is too high, the policy becomes welfare-damaging through its negative effect on the ability of households to consume. All tax rates are therefore not economically desirable, despite their positive effect on human capital accumulation.

Finally, even if an increase in a tax on pollution combined with an investment in pollution abatement has a positive effect on the aggregate economy, it does not affect the two types of agents in the same way. The reason is that an increase in the tax rate improves environmental quality and thus everyone's health, but it may also modify the long-term state of the economy. When the policy is sufficient to rescue the economy from the trap, the economy will converge to a long-term state that is much better for poor agents but less favorable for rich agents. The tax rates that households desire may therefore differ, with a rate desired by rich lower than that desired by poor households.

5 Conclusion

In this paper, we are interested in how pollution can exacerbate inequality through health. In accordance with empirical evidence, we consider the role of pollution and individual human capital in determining life expectancy. By means of an analytical study and a numerical illustration of our model, we show that multiple long-term scenarios are possible. The economy may converge to a long-term equilibrium without inequality or be stuck in a trap with steadily increasing inequality. We show that pollution plays a crucial role in determining the long-term behavior of the economy. Even if inequality is initially low, the economy will be in the trap since the pollution intensity of production is sufficiently high. The underlying mechanism stems from the negative effect of pollution on longevity, which discourages investments such as education. Moreover, we show that inequality is costly for the economy in the long-run in terms of economic growth and average life expectancy in the economy.

Therefore, we assess the implications of an environmental policy, consisting of a tax on pollution and a public investment in pollution abatement activities. We find that such a policy can promote long-term economic growth and enable the economy to escape the inequality trap. This

is not only because a decrease in pollution enhances individuals' longevity, thereby encouraging them to invest in education but also because it is more favorable to individuals who are more susceptible to pollution as long as disparities among agents are not too great. We conclude that an environmental policy can be a useful tool to enhance growth and address inequality in addition to improving the environment, but to do so, the government should implement the policy before disparities among agents become too great. Thus, our analysis provides new insights into the relationship between pollution and inequality and the role that an environmental policy can play in determining economic conditions through the channel of life expectancy.

6 Appendix

6.1 Proof of Proposition 1

For technical reasons, the study of the existence of balanced growth path equilibria is done in two parts: when there is no inequality among households (i.e. $x^p = 1$) and when inequality exists among them (i.e. $x^p \neq 1$) at such long-term states.[36]

6.1.1 BGP without inequality $x^p = 1$

Existence and uniqueness of a BGP without inequality

The dynamics of the economy described in (23) when there is no inequality ($x^p = 1$) reduces to:

$$k_{t+1}\frac{1+\beta\pi_{t+1}}{1+\pi_{t+1}(\beta+\gamma\mu)} = \frac{A(1-\tau)(1-\alpha)\beta k_t^{\alpha}}{\epsilon(\gamma\mu)^{\mu}}\left[\frac{\pi_t}{1+\pi_t(\beta+\gamma\mu)}\right]^{1-\mu} \tag{25}$$

with $\pi_t = \frac{\sigma}{1+(a-b\tau)Ak_t^{\alpha}}$.

At this BGP, we have $k_{t+1} = k_t = k$. We rewrite equation (25) as $\Omega_1 = \Omega_2$ with:

$$\Omega_1 \equiv k\frac{(a-b\tau)Ak^{\alpha}+1+\beta\sigma}{(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)}$$

and

$$\Omega_2 \equiv \frac{A(1-\tau)(1-\alpha)\beta k^{\alpha}}{\epsilon(\gamma\mu)^{\mu}}\left[\frac{\sigma}{(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)}\right]^{1-\mu}$$

When $\alpha < \frac{1}{2}$ and under Assumptions 1 and 2, Ω_1 is increasing and convex in k and characterized by $\Omega_1(0) = 0$ and $\lim_{k\to+\infty}\Omega_1(k) = +\infty$, while Ω_2 is increasing and concave in k with $\Omega_2(0) = 0$ and $\lim_{k\to+\infty}\Omega_2(k) = +\infty$.

[36] When $x^p = 1$, rewrite the system (23) as two functions of k depending on x^p requires to divide by zero in the second dynamical equation.

Moreover, $\Omega_1'(0) < \Omega_2'(0)$. Thus, the two curves cross only once and there exists a unique positive BGP without inequality $(k_E, 1)$.

The growth factor on BGP E corresponds to:

$$g_E = \epsilon \left[\frac{\sigma\gamma\mu}{(a-b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta+\gamma\mu)} \right]^{\mu} \tag{26}$$

Thus, the growth rate is positive if $g_E > 1$, i.e.:

$$\epsilon > \left[\frac{(a-b\tau)Ak_E^{\alpha} + 1 + \sigma(\beta+\gamma\mu)}{\sigma\gamma\mu} \right]^{\mu} \equiv \bar{\epsilon} \tag{27}$$

Dynamics of the BGP without inequality

To analyze the stability of the BGP without inequality $(k_E, 1)$, we compute the Jacobian matrix associated to the system (23) in E:

$$J(k_E, 1) = \begin{pmatrix} \frac{\partial F_1}{\partial k_t}(k_E, 1) & \frac{\partial F_1}{\partial x_t^p}(k_E, 1) \\ \\ \frac{\partial F_2}{\partial k_t}(k_E, 1) & \frac{\partial F_2}{\partial x_t^p}(k_E, 1) \end{pmatrix} \tag{28}$$

where F_1 and F_2 are two implicit functions given by the dynamical system (23), such that: $k_{t+1} = F_1(k_t, x_t^p)$ and $x_{t+1}^p = F_2(k_t, x_t^p)$. Therefore, we use the implicit function theorem to obtain the elements of the Jacobian matrix. The partial derivatives of F_2 at a BGP (k, x^p) are given by:

$$\begin{aligned} &\frac{\partial F_2}{\partial k_t}(k, x^p) = \mu \left(\frac{1-\xi x^p}{1-\xi}\right)^2 \left(\frac{(1-\xi)x^p}{1-\xi x^p}\right)^{2\mu+\eta} \left(\frac{(a-b\tau)Ak^{\alpha} + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu))}\right)^{\mu-1} \\ &\left[\frac{(a-b\tau)A\alpha k^{\alpha-1}(1+\sigma(\beta+\gamma\mu))(x^p-1)}{((a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu)))^2}\right] \end{aligned} \tag{29}$$

$$\begin{aligned} &\frac{\partial F_2}{\partial x_t^p}(k, x^p) = \\ &\left(\frac{(1-\xi)x^p}{1-\xi x^p}\right)^{2\mu+\eta-1} \left(\frac{(a-b\tau)Ak^{\alpha} + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu))}\right)^{\mu-1} \left(\frac{(1-\xi x^p)^2}{1-\xi}\right) \\ &\left[(2\mu+\eta)\frac{1-\xi}{(1-\xi x^p)^2}\left(\frac{(a-b\tau)Ak^{\alpha} + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu))}\right) - \mu\,\frac{x^p}{1-\xi x^p}\,\frac{(1+\sigma(\beta+\gamma\mu))[(a-b\tau)Ak^{\alpha} + 1 + \sigma(\beta+\gamma\mu)]}{((a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu)))^2}\right] \end{aligned} \tag{30}$$

The partial derivatives of F_1 at a BGP (k, x^p) are given by:

$$\frac{\partial F_1}{\partial k_t}(k, x^p) = \frac{A(1-\tau)(1-\alpha)\beta}{\epsilon(\gamma\mu)^{\mu}(V_1V_2)^2}\;\frac{\alpha k^{\alpha-1}V_3V_1V_2 + k^{\alpha}\left[V_3'V_1V_2 - V_3\left(V_1V_2' + V_2\frac{\partial F_2}{\partial k_t}(k, x^p)W_1\right)\right]}{V_4} \tag{31}$$

with

$$V_1 \equiv \xi x^p \frac{(a-b\tau)Ak^{\alpha} + x^p(1+\sigma\beta)}{(a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu))} + (1-\xi x^p)\frac{(a-b\tau)Ak^{\alpha} + \frac{1-\xi x^p}{1-\xi}(1+\sigma\beta)}{(a-b\tau)Ak^{\alpha} + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))}$$

$$V_2 \equiv \xi \frac{\sigma^{\mu}(x^p)^{2\mu+\eta}}{((a-b\tau)Ak^{\alpha} + x^p(1+\sigma(\beta+\gamma\mu)))^{\mu}} + (1-\xi)\frac{\sigma^{\mu}(\frac{1-\xi x^p}{1-\xi})^{2\mu+\eta}}{\left((a-b\tau)Ak^{\alpha} + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{\mu}}$$

$$V_3 \equiv \xi \frac{\sigma (x^p)^2}{(a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu))} + \frac{\sigma \frac{(1-\xi x^p)^2}{1-\xi}}{(a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))}$$

$$V_4 \equiv \frac{A(1-\tau)(1-\alpha)\beta k^\alpha}{\epsilon(\gamma\mu)^\mu} \frac{V_3}{V_2(V_1)^2} \left(\xi x^p \frac{(a-b\tau)A\alpha k^{\alpha-1}\sigma\gamma\mu x^p}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^2} + (1-\xi x^p) \frac{(a-b\tau)A\alpha k^{\alpha-1}\sigma\gamma\mu \frac{1-\xi x^p}{1-\xi}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2} \right) + 1$$

$$W_1 = \xi \left[\frac{(a-b\tau)Ak^\alpha + x^p(1+\sigma\beta)}{(a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu))} - \frac{(a-b\tau)Ak^\alpha \sigma\gamma\mu x^p}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^2} \right.$$

$$\left. - \frac{(a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma\beta)}{(a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))} + \frac{(a-b\tau)Ak^\alpha \sigma\gamma\mu \frac{1-\xi x^p}{1-\xi}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2} \right]$$

$$V_2' = -\mu\sigma^\mu (a-b\tau)A\alpha k^{\alpha-1} \left[\frac{\xi (x^p)^{2\mu+\eta}}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^{1+\mu}} + \frac{(1-\xi)\left(\frac{1-\xi x^p}{1-\xi}\right)^{2\mu+\eta}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{1+\mu}} \right]$$

$$V_3' = -\sigma (a-b\tau)A\alpha k^{\alpha-1} \left[\frac{\xi (x^p)^2}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^2} + \frac{\frac{(1-\xi x^p)^2}{1-\xi}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2} \right]$$

and

$$\frac{\partial F_1}{\partial x_t^p}(k, x^p) = \frac{\frac{A(1-\tau)(1-\alpha)k^\alpha \beta}{\epsilon(\gamma\mu)^\mu (V_1V_2)^2} \left[W_3' V_1 V_2 - V_3 \left(V_1 W_2' + V_2 \frac{\partial F_2}{\partial x_t^p}(k, x^p) W_1 \right) \right]}{V_4} \tag{32}$$

with

$$W_2' = \xi\sigma^\mu \left[\frac{(2\mu+\eta)(x^p)^{2\mu+\eta-1}}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^\mu} - \frac{(x^p)^{2\mu+\eta}\mu(1+\sigma(\beta+\gamma\mu))}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^{1+\mu}} \right.$$

$$\left. - \frac{(2\mu+\eta)(\frac{1-\xi x^p}{1-\xi})^{2\mu+\eta-1}}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^\mu} + \frac{\left(\frac{1-\xi x^p}{1-\xi}\right)^{2\mu+\eta}\mu(1+\sigma(\beta+\gamma\mu))}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^{1+\mu}} \right]$$

$$W_3' = \xi\sigma \left[\frac{2x^p(a-b\tau)Ak^\alpha + (x^p)^2(1+\sigma(\beta+\gamma\mu))}{((a-b\tau)Ak^\alpha + x^p(1+\sigma(\beta+\gamma\mu)))^2} - \frac{2\frac{1-\xi x^p}{1-\xi}(a-b\tau)Ak^\alpha + \left(\frac{1-\xi x^p}{1-\xi}\right)^2(1+\sigma(\beta+\gamma\mu))}{\left((a-b\tau)Ak^\alpha + \frac{1-\xi x^p}{1-\xi}(1+\sigma(\beta+\gamma\mu))\right)^2} \right]$$

On the BGP without inequality, $\frac{\partial F_2}{\partial k_t}(k_E, 1) = \frac{\partial F_1}{\partial x_t^p}(k_E, 1) = 0$, while $\frac{\partial F_1}{\partial k_t}(k_E, 1)$ and $\frac{\partial F_2}{\partial x_t^p}(k_E, 1)$ are greater than 0. Thus, the two eigenvalues are given by: $\frac{\partial F_1}{\partial k_t}(k_E, 1)$ and $\frac{\partial F_2}{\partial x_t^p}(k_E, 1)$. Under Assumptions 1 and 2, under the condition $\alpha < 1/2$, and substituting the expression of k_E given in (25) in the BGP $(k_E, 1)$, we have $0 < \frac{\partial F_1}{\partial k_t}(k_E, 1) < 1$. Thus, BGP E is stable iif $\frac{\partial F_2}{\partial x_t^p}(k_E, 1) < 1$, which is equivalent to:

$$1 - \left(2\mu + \eta - \frac{\mu(1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^\alpha + (1+\sigma(\beta+\gamma\mu))}\right) > 0 \tag{33}$$

When the condition (33) is satisfied, the BGP without inequality is locally stable (a sink), otherwise it is a saddle point. This condition can be rewritten in terms of η as $\eta < \bar{\eta}(\tau)$ with

$$\bar{\eta}(\tau) \equiv 1 - \mu \frac{2(a-b\tau)Ak_E^\alpha + (1+\sigma(\beta+\gamma\mu))}{(a-b\tau)Ak_E^\alpha + (1+\sigma(\beta+\gamma\mu))} \tag{34}$$

The BGP without inequality E is stable when $\eta < \bar{\eta}(\tau)$ and corresponds to a saddle point when $\eta > \bar{\eta}(\tau)$. Note that when $\mu + \eta \to 0$, the condition (33) is always satisfied, i.e. BGP E is always stable, while when

$\mu+\eta \to 1$, (33) is never satisfied, i.e. BGP E is always a saddle point.

□

6.1.2 BGP with inequality $x^p \neq 1$

Now, we study the existence and uniqueness of a BGP with inequality ($x^p < 1 < x^r$). After computations, the dynamical system (23) at a BGP with inequality, when $x^p_{t+1} = x^p_t = x^p \neq 1$ and $k_{t+1} = k_t = k$, corresponds to:

$$\begin{cases} k^{1-\alpha}\frac{\epsilon(\sigma\gamma\mu)^\mu}{A(1-\tau)(1-\alpha)\beta}\left[\frac{\left[(a-b\tau)Ak^\alpha+\frac{1-\xi x^p}{1-\xi}(1+\sigma\beta)\right][(a-b\tau)Ak^\alpha+x^p(1+\sigma(\beta+\gamma\mu))]+\xi x^p(a-b\tau)Ak^\alpha\sigma\gamma\mu\left(\frac{1-\xi x^p}{1-\xi}-x^p\right)}{(a-b\tau)Ak^\alpha\sigma\left(\frac{(1-\xi x^p)^2}{1-\xi}+\xi(x^p)^2\right)+\frac{1-\xi x^p}{1-\xi}\sigma x^p(1+\sigma(\beta+\gamma\mu))}\right] \\ \left[\frac{\xi(x^p)^{2\mu+\eta}}{[(a-b\tau)Ak^\alpha+x^p(1+\sigma(\beta+\gamma\mu))]^\mu}+\frac{(1-\xi)\left(\frac{1-\xi x^p}{1-\xi}\right)^{2\mu+\eta}}{\left[(a-b\tau)Ak^\alpha+\left(\frac{1-\xi x^p}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^\mu}\right]-1=0\equiv\mathcal{A}(k,x^p) \\ k=\left[\frac{1+\sigma(\beta+\gamma\mu)}{(1-\xi)A(a-b\tau)}\frac{(1-\xi)x^p(1-\xi x^p)^{\frac{2\mu+\eta-1}{\mu}}-((1-\xi)x^p)^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^p)}{((1-\xi)x^p)^{\frac{2\mu+\eta-1}{\mu}}-(1-\xi x^p)^{\frac{2\mu+\eta-1}{\mu}}}\right]^{\frac{1}{\alpha}}\equiv\Psi_2(x^p) \end{cases} \tag{35}$$

Properties of the function Ψ_2

The second equation of (35) defines $k = \Psi_2(x^p)$. Under Assumptions 1 and 2 and the conditions $2\mu+\eta > 1$ and $\alpha < 1/2$, the properties of this function are:

- $Sign(\Psi_2') = u'v - uv'$ with:

$$u = (1-\xi)x^p(1-\xi x^p)^{\frac{2\mu+\eta-1}{\mu}} - ((1-\xi)x^p)^{\frac{2\mu+\eta-1}{\mu}}(1-\xi x^p) < 0$$

$$v = ((1-\xi)x^p)^{\frac{2\mu+\eta-1}{\mu}} - (1-\xi x^p)^{\frac{2\mu+\eta-1}{\mu}} < 0$$

$$v' = \frac{2\mu+\eta-1}{\mu}\left[((1-\xi)x^p)^{\frac{\mu+\eta-1}{\mu}}(1-\xi)+(1-\xi x^p)^{\frac{\mu+\eta-1}{\mu}}\xi\right] > 0$$

$$\begin{aligned} u' &= \xi((1-\xi)x^p)^{\frac{2\mu+\eta-1}{\mu}} + (1-\xi)(1-\xi x^p)^{\frac{2\mu+\eta-1}{\mu}} \\ &\quad -\frac{2\mu+\eta-1}{\mu}\left[\xi((1-\xi)x^p)(1-\xi x^p)^{\frac{\mu+\eta-1}{\mu}} + (1-\xi)(1-\xi x^p)((1-\xi)x^p)^{\frac{\mu+\eta-1}{\mu}}\right] \end{aligned}$$

  We rewrite this last equation as $u' = \mathcal{I}(x^p) - \mathcal{J}(x^p)$, where $\mathcal{I}(x^p)$ corresponds to the first part (first line) of the equation and $\mathcal{J}(x^p)$ corresponds to the second one.

  - $\mathcal{I}(0) = (1-\xi)$, $\mathcal{I}(1) = (1-\xi)^{\frac{2\mu+\eta-1}{\mu}} > \mathcal{I}(0)$ and $\mathcal{I}'(x^p) > 0$.
  - $\mathcal{J}(0) = +\infty$, $\mathcal{J}(1) = \frac{2\mu+\eta-1}{\mu}(1-\xi)^{\frac{2\mu+\eta-1}{\mu}} < \mathcal{I}(1)$ and

$$\begin{aligned} \mathcal{J}'(x^p) &= \frac{2\mu+\eta-1}{\mu}\left[\xi(1-\xi)\left((1-\xi x^p)^{\frac{\mu+\eta-1}{\mu}} - ((1-\xi)x^p)^{\frac{\mu+\eta-1}{\mu}}\right)\right. \\ &\quad \left. +\frac{\mu+\eta-1}{\mu}\left((1-\xi)^2(1-\xi x^p)((1-\xi)x^p)^{\frac{\eta-1}{\mu}} - \xi^2((1-\xi)x^p)(1-\xi x^p)^{\frac{\eta-1}{\mu}}\right)\right] \end{aligned} \tag{36}$$

$\mathcal{J}'(x^p)$ is an increasing function of x^p ($\mathcal{J}''(x^p) > 0$) which is always negative in $x^p = 0$ but may become positive for high x^p when $\xi > 1/2$ ($\mathcal{J}'(1) > 0$ when $\xi > 1/2$).

- u' is negative as long as $\mathcal{J}(x^p) > \mathcal{I}(x^p)$. Thus, we can define a threshold $\hat{x}^p \in (0,1)$ under which u' is negative and above which u' is positive for high level of ξ.

- The condition $u' < 0$ is sufficient to ensure that $\Psi_2' > 0$. Thus, we show that there exists a threshold $\hat{x}^p \in (0,1)$ under which Ψ_2 is an increasing function of x^p and above which Ψ_2 may become decreasing (for high level of ξ).
- Moreover, $\Psi_2 \geqslant 0\ \forall x^p$, $\Psi_2(0) = 0$ and

$$\lim_{x^p \to 1} \Psi_2(x^p) = \left[\frac{1+\sigma(\beta+\gamma\mu)}{A(a-b\tau)} \frac{1-\mu-\eta}{2\mu+\eta-1}\right]^{\frac{1}{\alpha}} > 0 \tag{37}$$

Properties of the function Ψ_1

The first equation of (35), $\mathcal{A}(k, x^p) = 0$, allows to define $k = \Psi_1(x^p)$, with $\Psi_1(x^p)$ an implicit function. Under Assumptions 1 and 2 and the conditions $2\mu + \eta > 1$ and $\alpha < 1/2$, we obtain that $\Psi_1(0)$ and $\Psi_1(1)$ are equal to two positive constants. More precisely, in $x^p = 0$ we have:

$$\begin{aligned} &\mathcal{A}(k,0) = 0 \Leftrightarrow k^{1-\alpha} \tfrac{\epsilon(\sigma\gamma\mu)^\mu (1-\xi)^{2-2\mu-\eta}}{A(1-\tau)(1-\alpha)\beta\sigma} \tfrac{(a-b\tau)Ak^\alpha + \frac{1}{1-\xi}(1+\sigma\beta)}{\left[(a-b\tau)Ak^\alpha + \left(\frac{1}{1-\xi}\right)(1+\sigma(\beta+\gamma\mu))\right]^\mu} = 1 \\ &\Leftrightarrow k^{1-\alpha}\left[(a-b\tau)Ak^\alpha + \tfrac{1+\sigma\beta}{1-\xi}\right] = \tfrac{A(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu(1-\xi)^{2-2\mu-\eta}}\left[(a-b\tau)Ak^\alpha + \tfrac{1+\sigma(\beta+\gamma\mu)}{1-\xi}\right]^\mu \end{aligned} \tag{38}$$

We analyze the properties of $\Psi_1(0)$ by studying the last equation. For that, we name the function on the left side $f_0(k)$ and the function on the right side $g_0(k)$. Their properties are:

- f_0 is increasing and concave in k, $f_0(0) = 0$ and $\lim_{k\to\infty} f_0(k) = +\infty$.
- g_0 is increasing and concave in k, $g_0(0)$ is equal to a positive constant and $\lim_{k\to\infty} g_0(k) = +\infty$.
- In $k = 0$, $g_0 > f_0$. The two curves have not cross yet, thus $\Psi_1(0) > 0$.
- When $k \to \infty$, we have $\lim_{k\to\infty} f_0 > \lim_{k\to\infty} g_0$. Thus, the two curves cross only once and for a positive and finite value of k.

Therefore, $\Psi_1(0)$ is always a finite and positive constant.

In the same way, in $x^p = 1$ we have:

$$\begin{aligned} &\mathcal{A}(k,1) = 0 \Leftrightarrow k^{1-\alpha} \tfrac{\epsilon(\sigma\gamma\mu)^\mu}{A(1-\tau)(1-\alpha)\beta\sigma} \tfrac{(a-b\tau)Ak^\alpha + 1 + \sigma\beta}{[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)]^\mu} = 1 \\ &\Leftrightarrow k^{1-\alpha}\left[(a-b\tau)Ak^\alpha + 1 + \sigma\beta\right] = \tfrac{A(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}}{\epsilon(\gamma\mu)^\mu}\left[(a-b\tau)Ak^\alpha + 1 + \sigma(\beta+\gamma\mu)\right]^\mu \end{aligned} \tag{39}$$

As previously, we study the properties of $\Psi_1(1)$, by looking at the last equation. We name the function on the left side $f_1(k)$ and the function on the right side $g_1(k)$, whose properties are:

- f_1 is increasing and concave in k, $f_1(0) = 0$ and $\lim_{k\to\infty} f_1(k) = +\infty$.
- g_1 is increasing and concave in k, $g_1(0)$ is equal to a positive constant and $\lim_{k\to\infty} g_1(k) = +\infty$.
- In $k = 0$, $g_1 > f_1$, the two curves have not cross yet thus $\Psi_1(1) > 0$.
- When $k \to \infty$, we have $\lim_{k\to\infty} f_1 > \lim_{k\to\infty} g_1$. Thus, the two curves cross only once and for a positive and finite value of k.

Therefore, $\Psi_1(1)$ is equal to a finite and positive constant.

Comparison of Ψ_1 and Ψ_2

From the study of the properties of Ψ_1 and Ψ_2, we know that $\Psi_1(0) > 0$ and $\Psi_2(0) = 0$, it entails that $\Psi_1(0) > \Psi_2(0)$. It follows that if $\Psi_1(1) < \lim_{x^p\to 1} \Psi_2(x^p)$, there exists at least one BGP with inequality. From the study of Ψ_1, the condition $\Psi_1(1) < \lim_{x^p\to 1} \Psi_2(x^p)$ is equivalent to $f_1(k) > g_1(k)$ in $k = \lim_{x^p\to 1} \Psi_2(x^p)$ given in (37). We obtain that $\Psi_1(1) < \lim_{x^p\to 1} \Psi_2(x^p)$ if

$$\frac{A^{\frac{1}{\alpha}}(1-\tau)(1-\alpha)\beta\sigma^{1-\mu}(a-b\tau)^{\frac{1-\alpha}{\alpha}}}{\epsilon\gamma^{\mu}(1+\sigma(\beta+\gamma\mu))^{\frac{1-\alpha(1+\mu)}{\alpha}}} < \frac{(1-\mu-\eta)^{\frac{1-\alpha}{\alpha}}\mu\left[(1+\beta\sigma)+\sigma\gamma(1-\mu-\eta)\right]}{(2\mu+\eta-1)^{\frac{1-\mu}{\alpha}}} \tag{40}$$

where the right side of the equation corresponds to a function $\mathcal{R}(\eta)$ which is decreasing in η. It follows that this condition can be rewritten as $\eta < \tilde{\eta}(\tau)$ where $\tilde{\eta}(\tau)$ is implicitly given by (40). Note that this condition is never satisfied when $\eta \to 1-\mu$, but can be otherwise.

Thus, under Assumptions 1 and 2 and the conditions $2\mu + \eta > 1$ and $\alpha < 1/2$, the condition $\eta < \tilde{\eta}(\tau)$ is sufficient so that there exists at least one BGP with inequality. Note that when $\mu + \eta \to 0$ or $\mu + \eta \to 1$, Ψ_2 corresponds to strictly negative values of k $\forall x^p$, so that there is no BGP with inequality in these cases.

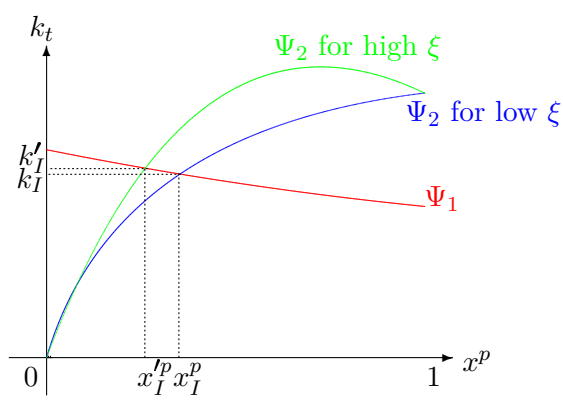

Figure 5: A representation of the dynamics when $x^p \neq 1$ (with Ψ_1 decreasing in x^p).

□

6.2 Proof of Proposition 2

Effect of τ on $\bar{\eta}(\tau)$: The threshold under which BGP E is stable, i.e. $\bar{\eta}(\tau)$, is given by (34) in Appendix 6.1.1. To analyze the effect of τ on the dynamics of E, we compute $\frac{\partial \bar{\eta}(\tau)}{\partial \tau}$:

$$\frac{\partial \bar{\eta}(\tau)}{\partial \tau} = \frac{\mu(1+\sigma(\beta+\gamma\mu))Ak_E^{\alpha-1}}{[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)]^2}\Big(bk_E-(a-b\tau)\alpha\frac{\partial k_E}{\partial \tau}\Big) \tag{41}$$

The effect of the pollution tax on the dynamics in BGP E depends on $\frac{\partial k_E}{\partial \tau}$. To compute this derivative, we use the dynamical equation (25) on the BGP:

$$\Phi(k,\tau) \equiv k\frac{(a-b\tau)Ak^{\alpha}+1+\beta\sigma}{(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)} - \frac{A(1-\tau)(1-\alpha)\beta k^{\alpha}}{\epsilon(\gamma\mu)^{\mu}}\left[\frac{\sigma}{(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)}\right]^{1-\mu} = 0$$

The effect of τ on k in E is given by the implicit function theorem:

$$\frac{\partial k}{\partial \tau}(k_E,1) = -\frac{\frac{\partial \Phi}{\partial \tau}}{\frac{\partial \Phi}{\partial k_E}}$$

After computations, we obtain the two partial derivatives:

$$\begin{aligned}\frac{\partial \Phi}{\partial \tau} \quad = \quad & \Big(-bAk^{1+\alpha}\sigma\gamma\mu + \tfrac{A(1-\alpha)\beta k^{\alpha}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^{\mu}}\left[Ak^{\alpha}(a-b(1-\mu(1-\tau)))+1+\sigma(\beta+\gamma\mu)\right] \\ & [(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)]^{\mu}\Big)\left[(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{-2}\end{aligned} \tag{42}$$

$$\begin{aligned}\frac{\partial \Phi}{\partial k} \quad = \quad & \Big[((a-b\tau)Ak^{\alpha})^2+(a-b\tau)Ak^{\alpha}[2(1+\beta\sigma)+\sigma\gamma\mu(1+\alpha)]+(1+\sigma(\beta+\gamma\mu))(1+\sigma\beta) \\ & -\tfrac{A(1-\alpha)(1-\tau)\beta\alpha k^{\alpha-1}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^{\mu}}\left[(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{\mu}\left[Ak^{\alpha}\mu(a-b\tau)+1+\sigma(\beta+\gamma\mu)\right]\Big] \\ & \left[(a-b\tau)Ak^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{-2}\end{aligned} \tag{43}$$

And, we have:

$$Sign\{\frac{\partial \bar{\eta}(\tau)}{\partial \tau}\} = Sign\{bk_E-(a-b\tau)\alpha\frac{\partial k_E}{\partial \tau}\}$$

Thus, $\frac{\partial \bar{\eta}(\tau)}{\partial \tau} > 0$ iif:

$$\begin{aligned}& bk_E\left[((a-b\tau)Ak_E^{\alpha})^2+(a-b\tau)Ak_E^{\alpha}[2(1+\beta\sigma)+\sigma\gamma\mu(1+\alpha)]+(1+\sigma(\beta+\gamma\mu))(1+\sigma\beta)\right] \\ & -\tfrac{A(1-\alpha)(1-\tau)\beta\alpha k_E^{\alpha}\sigma^{1-\mu}b}{\epsilon(\gamma\mu)^{\mu}}\left[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{\mu}\left[Ak_E^{\alpha}\mu(a-b\tau)+1+\sigma(\beta+\gamma\mu)\right] \\ & +(a-b\tau)\alpha\Big(-bAk_E^{1+\alpha}\sigma\gamma\mu+\tfrac{A(1-\alpha)\beta k_E^{\alpha}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^{\mu}}\left[Ak_E^{\alpha}(a-b(1-\mu(1-\tau)))+1+\sigma(\beta+\gamma\mu)\right] \\ & \left[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{\mu}\Big) > 0\end{aligned} \tag{44}$$

It can be rewritten as:

$$
\begin{aligned}
& bk_E\left[((a-b\tau)Ak_E^{\alpha})^2+(a-b\tau)Ak_E^{\alpha}[2(1+\beta\sigma)+\sigma\gamma\mu]+(1+\sigma(\beta+\gamma\mu))(1+\sigma\beta)\right] \\
& +(a-b\tau)\alpha\left(\tfrac{A(1-\alpha)\beta k_E^{\alpha}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^{\mu}}Ak_E^{\alpha}(a-b)\left[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{\mu}\right) \\
& +(1+\sigma(\beta+\gamma\mu))\tfrac{A(1-\alpha)\beta\alpha k_E^{\alpha}\sigma^{1-\mu}}{\epsilon(\gamma\mu)^{\mu}}\left[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)\right]^{\mu}(a-b)>0
\end{aligned}
\tag{45}
$$

Under Assumption 2, the condition $bk_E-(a-b\tau)\alpha\frac{\partial k_E}{\partial\tau}>0$ is always verified. Therefore, under Assumptions 1 and 2 and for $\alpha<1/2$, the threshold $\bar{\eta}(\tau)$ depends positively on the tax rate τ. When $\tau\to 1$, $k_E\to 0$ and $\bar{\eta}(1)\to 1-\mu$ which is always true since we assume that $\mu+\eta<1$. Thus, when τ tends to 1, the BGP without inequality is always stable.

Effect of τ on $\tilde{\eta}(\tau)$: Under Assumptions 1 and 2, for $2\mu+\eta>1$ and $\alpha<1/2$, we have obtained the condition (40) such that at least one BGP with inequality exists. This condition can be rewritten as $\mathcal{L}(\tau)<\mathcal{R}$, where τ only intervenes in $\mathcal{L}$ and $\partial\mathcal{L}(\tau)/\partial\tau<0$. Thus, an increase in the tax makes the condition (40) more easily satisfied. This condition corresponding also to $\eta<\tilde{\eta}(\tau)$, we deduced that the threshold $\tilde{\eta}(\tau)$ depends positively on τ. Moreover, (40) is always satisfied when τ tends to 1. Thus, a higher tax rate increases the range of parameters for which there exists at least one BGP with inequality. □

6.3 Proof of Proposition 3

We analyze the effect of the tax rate on the growth factor in the BGP without inequality g_E, given by (24). Its derivative with respect to τ is:

$$
\frac{\partial g_E}{\partial\tau}=\epsilon(\sigma\gamma\mu)^{\mu}\mu[(a-b\tau)Ak_E^{\alpha}+1+\sigma(\beta+\gamma\mu)]^{-\mu-1}k_E^{\alpha-1}\left\{bAk_E-(a-b\tau)A\alpha\frac{\partial k_E}{\partial\tau}\right\}
$$

The effect of the pollution tax on the growth rate in BGP E depends on $\frac{\partial k_E}{\partial\tau}$ and more precisely we have:

$$
Sign\left\{\frac{\partial g_E}{\partial\tau}\right\}=Sign\left\{bk_E-(a-b\tau)\alpha\frac{\partial k_E}{\partial\tau}\right\}
$$

From Appendix 6.2, we know that under Assumption 2, $bk_E-(a-b\tau)\alpha\frac{\partial k_E}{\partial\tau}>0$. Therefore, under Assumption 2, we have that $\frac{\partial g_E}{\partial\tau}>0$. The growth rate on the BGP without inequality g_E increases following an increase in the pollution tax.

6.4 Sensitivity Analysis

In this section, we analyze the robustness of our results with respect to two key parameters: the share of poor individuals in each cohort ξ and the weight of intergenerational transmission in human capital

accumulation η. Note that we have performed these analyses for a large number of tax rates, but as the results are similar, we only report here the results for $\tau = 0$ to save space.

Distribution of the two types of individuals in the population: ξ

The effect of ξ is illustrated in Figure 6 and Table 2. The share of poor people in each cohort does not affect the value of the BGP without inequality but modifies the BGP with inequality. On the latter BGP, the higher the share of poor individuals ξ is, the higher the capital-labor ratio k_I and the lower the relative human capital of poor agents x_I^p. This entails that, at I, human capital inequality will be wider and the growth rate will be lower. However, the dynamics of both BGPs remain the same. Thus, the threshold in terms of initial inequality under which the economy is in the inequality trap is lower. Other things being equal, a higher ξ implies that the relative disadvantage of poor agents with respect to the rest of the population is lower, which facilitates human capital convergence.

Table 2: Sensitivity Analysis with respect to ξ when $\tau = 0$ and $\eta = 0.25$.

ξ	k_I	x_I^p	g_I	$CAGR_I$	π_I^p	π_I^r	LE_I^p	LE_I^r	$AverageLE_I$
0.50	0.0241	0.0955	1.5412	1.452%	0.3197	0.8250	69.5919	84.7485	77.1702
0.70	0.0271	0.0738	1.4473	1.240%	0.2615	0.8515	67.8435	85.5435	73.1535
0.90	0.0355	0.0480	1.2439	0.730%	0.1762	0.8818	65.2848	86.4547	67.4018

ξ	k_E	g_E	$CAGR_E$	π_E	LE_E	$Eigenvalues_E$	$Eigenvalues_I$
0.50	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.17236; 0.321711\}$
0.70	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.20855; 0.324857\}$
0.90	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.27981; 0.328051\}$

Notes: $CAGR_j$ represents the compound annual growth rate on the balanced growth path $j = E, I$, while LE_j^i corresponds to the life expectancy in years of individual i on BGP j.

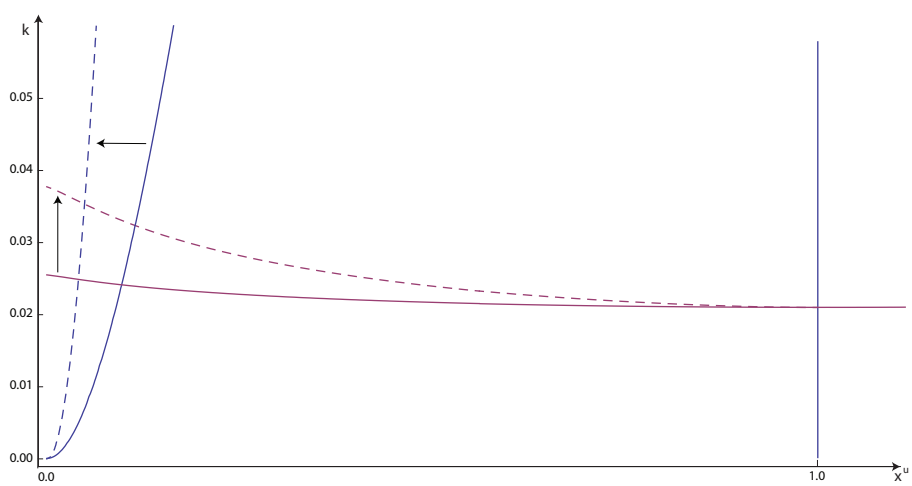

Figure 6: Sensitivity analysis with respect to ξ (when $\tau = 0$ and $\eta = 0.25$), in which the solid lines capture the case $\xi = 0.5$ and the dashed lines capture the case $\xi = 0.9$.

Weight of intergenerational transmission in human capital accumulation: η

Figure 7 and Table 3 illustrate the evolution of the two BGPs with respect to η. As for ξ, an increase in η has no effect on the variables on the BGP without inequality E. On the contrary, on the BGP with

inequality, it reduces the capital labor ratio k_I and increases the relative human capital of poor agents x_I^p. Thus, the growth rate on BGP I is higher while the level of inequality is lower. However, regarding the dynamics, I is a saddle point and defines the trap (when $\eta < \bar{\eta}(\tau)$). Therefore, up to the threshold $\bar{\eta}(\tau)$[37], an increase in η entails that the trap moves to the right and hence that its size increases. At this threshold, I disappears and E becomes a saddle point and defines the new trap of an even larger size. After $\bar{\eta}(\tau)$, a higher η continues to enlarge the trap, i.e., moves the blue curve (representing the second dynamical equation of (23)) to the right. Therefore, other things being equal, a higher η favors the transmission of inequality and hence makes it more likely that an economy is in a trap with rising inequality.

Table 3: Sensitivity Analysis with respect to η when $\tau = 0$ and $\xi = 0.5$.

η	k_I	x_I^p	g_I	$CAGR_I$	π_I^p	π_I^r	LE_I^p	LE_I^r	$AverageLE_I$
0.20	0.0259	0.0444	1.4847	1.326%	0.1801	0.8251	65.4019	84.7530	75.0775
0.25	0.0241	0.0955	1.5412	1.452%	0.3197	0.8250	69.5919	84.7485	77.1702
0.30	0.0223	0.2365	1.6010	1.581%	0.5251	0.8213	75.7518	84.6403	80.1960
0.35	$\varnothing$	$\varnothing$							
0.39	$\varnothing$	$\varnothing$							

η	k_E	g_E	$CAGR_E$	π_E	LE_E	$Eigenvalues_E$	$Eigenvalues_I$
0.20	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.861051; 0.318788\}$	$\{1.2362; 0.32274\}$
0.25	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.911051; 0.318788\}$	$\{1.17236; 0.321711\}$
0.30	0.0209	1.6506	1.684%	0.7724	83.1706	$\{0.961051; 0.318788\}$	$\{1.08036; 0.320204\}$
0.35	0.0209	1.6506	1.684%	0.7724	83.1706	$\{1.01105; 0.318788\}$	
0.39	0.0209	1.6506	1.684%	0.7724	83.1706	$\{1.05105; 0.318788\}$	

Notes: $CAGR_j$ represents the compound annual growth rate on the balanced growth path $j = E, I$, while LE_j^i corresponds to the life expectancy in years of individual i on BGP j.

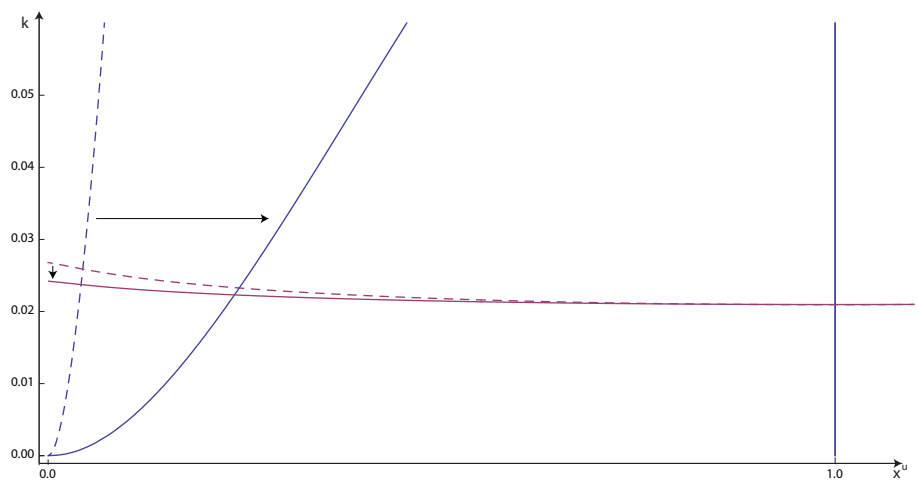

Figure 7: Sensitivity analysis with respect to η (when $\tau = 0$ and $\xi = 0.5$), in which the dashed lines capture the case $\eta = 0.2$ and the solid lines refer to the case $\eta = 0.3$.

[37] $\bar{\eta}(0) = 0.34$ and $\bar{\eta}(1) = 0.4$.

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