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The Environmental Unsustainability of Public Debt: Non-Renewable Resources, Public Finances Stabilization and Growth

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Abstract

This paper introduces a public debt stabilization constraint in an overlapping generation model in which non-renewable resources constitute a necessary input in the production function and belong to agents. It shows that stabilization of public debt at high level (as share of capital) may prevent the existence of a sustainable development path. Public debt thus appears as a threat to sustainable development. It also shows that higher public debt-to-capital ratios (and public expenditures-to-capital ones) are associated with lower growth. Two transmission channels are identified. As usual, public debt crowds out capital accumulation. In addition, public debt tends to increase resource use which reduces the rate of growth. We also analyze the dynamics and we show that the economy is characterized by saddle path stability. Finally, we show that the public debt-to-capital ratio may be calibrated to implement the social planner optimal allocation.

Keywords: Non-renewable Resources; Growth; Public Finances; Overlapping Generations

JEL Codes: Q32; Q38; H63

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1. Introduction

In the 1970s, "The limits to growth" report alerted the general audience on the (in)feasibility of long-run growth. The report notably points out that economic development relies on natural resources, some of which being finite and non-renewable. The pessimistic Malthusian point of view exposed in the report has since been challenged by neoclassical economists who have highlighted the role of increasing returns to scale, technological progress and substitution between natural capital and man-made capital (Dasgupta and Heal, 1974, 1979; Solow, 1974; Stiglitz, 1974). Still, while they have shown that it is possible to experiment an infinite growth in a finite world, this is far from being guaranteed and the question of resource exhaustion is still a question of major interest.

Another major concern that has been developed during the last decades is linked with public indebtedness. Indeed, the last decades have been marked by an increase in public debt-to-GDP ratios. In the Eurozone, the Maastricht treaty imposes a public debt-to-GDP ratio lower than 60% of GDP. This stabilization level has been largely exceeded by many developed economies in the last decades, and the COVID19 crisis is generating a huge increase of public debt-to-GDP ratios. The impact of public debt on growth has been largely studied in the literature. There exist numerous theoretical papers that document a negative relationship between public debt and growth (Diamond, 1965; Blanchard, 1985; Barro, 1990; Saint-Paul, 1992). The famous paper by Reinhart and Rogoff (2010) documents a threshold effects and argues that a stabilization ratio of public debt larger than 90% of GDP is associated with significant decreases in the rate of growth.[1] To summarize, it is widely accepted that high levels of public indebtedness are associated with lower economic performances in the long run.

[1] While this paper is subject to controversy due to a shortcoming in the methodology (see Herndon et al., 2014), this type of results strongly influenced IMF policy recommendation in the last decades.

From these observations, it appears that the economic literature has identified two potential threats to long run economic development: resource exhaustion and public indebtedness. Despite the large literature on these topics, the two threats have never been studied conjointly. This is quite surprisingly since both issues are linked to agents' saving behaviors. In a nutshell, it is well known that public debt may crowd out investment in physical capital, and that physical capital accumulation is one major way to compensate for resource exhaustion.

In this paper, we propose an OLG model in which firms produce combining labor, capital, non-renewable resources, and public infrastructures. Public infrastructures are provided by the government and could be financed by taxes or debt. We consider however that the government faces public finances stabilization constraints in the spirit of the Maastricht Treaty. Such a framework allows us to study the impact of the public debt stabilization ratio on the sustainability of growth in the presence of a necessary finite input, namely the non-renewable resource.

The paper shows that high public debt stabilization ratios are incompatible with sustainable growth, because they prevent the existence of a positive balanced growth path. In addition, it shows that the rate of growth achieved by the market economy is negatively linked with the size of the stabilization ratio. The underlying mechanisms are quite simple. As usual, public debt crowds out savings from physical capital accumulation. More interestingly, public debt also increases the rate of resource exhaustion, because it crowds out households investment in the non-renewable resource stock. More resources are thus used in the production by firms, which threatens future growth feasibility.

The paper also analyzes the centralized economy where a benevolent social planner is free to choose the economic path on the ground of its time preference. Intuitively, the more the social planner cares about future generations, the lower will be the extraction rate and the larger will be the rate of growth. We then show, as a result of the monotonic and negative relationship occurring between public debt stabilization ratio and the economy rate of growth, that there exists one level of public debt stabilization ratio that allows to decentralize the optimal allocation.

This paper is related to the seminal papers on the feasibility of growth with non-renewable resources written in the 1970s who have highlighted the importance of (exogenous) technical progress, increasing returns to scale and substitution between natural and man-made production factor (Dasgupta and Heal, 1974, 1979; Solow, 1974; Stiglitz, 1974). This literature enjoyed a revival with the development of endogenous growth models. The interested reader can refer to Barbier (1999). One weakness of these papers is their use of the Infinitely Lived Agents (ILA) framework which implicitly assume dynastic altruism between agents, not supported by empirical results (Altonji et al., 1992). Such an assumption makes a sustainable management of resources more probable since each generation takes care of the following as of itself. This observation has lead to the work of Agnani et al. (2005) who have addressed this shortcoming using an OLG framework to analyze the sustainability of growth with a necessary non-renewable resource. They have shown that economic growth then requires that the labor share in production be sufficiently high to allow a level of savings (and then capital accumulation) sufficient to compensate for resource depletion. We add to this literature considering the existence of public productive infrastructures financed by debt, that could crowd out saving from capital accumulation.

This paper is also related to the literature on public debt. Using an OLG framework, Diamond (1965) shows that debt crowds out capital accumulation and reduces growth, a result confirmed by Blanchard (1985). The negative impact of public debt on growth also appears in the endogenous growth framework (Barro, 1990; Saint-Paul, 1992). Futagami et al. (2008) introduce public debt in an endogenous growth model with productive government spending. They show that financing productive public expenditures with debt might be growth reducing (enhancing) in developed (developing) countries. In a close set-up, Minea and Villieu (2013) show that increases in public debt reduce growth. As in our own, these papers introduce public debt stabilization targets in the size of the economy.[2] There exist some papers that introduce the environment in the discussion. Notably, Fodha and Seegmuller (2012) analyze the implications of an environmental tax under public debt stabilization constraint. This work has been extended in several ways (Fodha and Seegmuller, 2014; Clootens, 2017; Fodha et al., 2018).[3] However, these papers analyze the links between debt and environmental quality and don't take into account the resource exhaustion issue in their analysis. The present paper is an attempt to fill this gap. In this aspect, our paper does not focus on typical issues such as pollution and emissions that are detrimental to the quality of the environment but rather investigates the problems related to the exhaustion of non-renewable resources as fossils and minerals in an economy with debt.[4]

The remainder of the paper is organized as follows. Section 2 presents the model. The competitive equilibrium and the balanced growth path are characterized in Section 3. Section 4 deals with the stability of the balanced growth path. Section 5 analyses how movements in public debt and public stabilization ratios affect the balanced growth path. Section 6 is devoted to the presentation of the central planner problem and section 7 presents the decentralization of the optimal allocation. Section 8 concludes.

[2]They actually differ in the type of target. In Futagami et al. (2008), the size of the economy is captured with private capital while it is captured with GDP in Minea and Villieu (2013). This slight change explain the differences in their results.

[3]There also exist some empirical works on the relationship between public debt and the environment. See for example Carratù et al. (2019).

[4]This is the reason why we decide to not consider an explicit demand for environmental quality. Nevertheless, we do not deny that emissions are often a byproduct of resource use that may affect the utility of households, but this is beyond the purpose of our analysis. Clootens (2021) analyses the effects of flow emissions in an OLG economy with non-renewable resources where emissions are a byproduct of resource use and extraction.

2. The Model

The model is in essence that of Diamond (1965) in which public expenditures in infrastructures *à la* Barro (1990) and non-renewable resources are introduced. For the sake of simplicity, the size of population has been normalized to one and a no demographical growth assumption is used. Lowercase letters represent per worker variables.

2.1. The Resource

A grandfathering economy is considered here. Thus, the economy is endowed with a finite quantity m_{-1} of a non-renewable resource which is held by the first generation of agents. At each date t, old agents sell their resource endowments m_{t-1} to the new generation of agents and to firms at a price p_t. A quantity x_t is used in the production. Thus the rate of resource use is given by

$$q_t = \frac{x_t}{m_{t-1}} \tag{1}$$

The law of motion of the resource stock is thus

$$m_t = (1 - q_t)m_{t-1} \tag{2}$$

Since the resource is finite and non-renewable, the initial stock imposes a limit on total quantities that can be extracted

$$1 \geq \sum_{t=0}^{+\infty} q_t \prod_{j=1}^{t} (1 - q_{j-1}) \tag{3}$$

This condition establishes that we cannot extract more resources than the quantity available at the beginning of the time horizon. This implies that the sequence of the extraction rates is subject to some restrictions.

2.2. The Consumers

In each period of time, the economy is composed of two generations of finite lived agents. An agent born in period t maximizes the following inter-temporal log-utility function

$$u(c_t; d_{t+1}) = \ln(c_t) + \frac{1}{1+\rho} \ln(d_{t+1}) \tag{4}$$

where c represents the consumption while young, d the consumption while old, and ρ is the individual rate of time preferences. In its first period of life, the agent works and earns a wage w. This wage is used to consume c, to save s in capital or in public bonds yielding a real interest rate r, to buy property rights on the resource stock m at a price p, and to pay lump-sum taxes τ. In his/her second period of life, the agent uses his/her savings (both in capital, public bonds and in resources) to consume. Thus, normalizing to one the price of the output, his/her budget constraints when young and old are respectively

$$w_t - \tau_t = c_t + s_t + p_t m_t \tag{5}$$

$$d_{t+1} = (1 + r_{t+1}) s_t + p_{t+1} m_t \tag{6}$$

The intertemporal budget constraint is then

$$w_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - \frac{p_{t+1} m_t}{1 + r_{t+1}} + p_t m_t + \tau_t \tag{7}$$

Maximization of (4) subject to (7) leads to the following first order conditions

$$\frac{d_{t+1}}{c_t} = \frac{1 + r_{t+1}}{1 + \rho} \tag{8}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1} \tag{9}$$

(8) is the standard Euler equation establishing that the marginal rate of substitution between the two consumptions has to be equal to their relative price. (9) represents the Hotelling rule. It is a non-arbitrary condition between the two types of savings which sets that the resource price has to increase at the interest rate. It means that our agents are indifferent between the three types of assets: resources, public bonds, and capital.

2.3. The Firms

The production sector of this economy is composed of an infinite set of competitive firms which produce the consumption and investment good Y by combining capital K, resources X, labor L and public infrastructures G. Capital and public infrastructures fully depreciate over a period. A represents the technical level and grows at an exogenous rate a. The production function of the representative firm is

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} X_t^{\nu} G_t^{\theta}$$

with $\alpha + \beta + \nu = 1$ and $\theta > 0$. Notice that our production function differs from Barro (1990) one where A_t is constant, $\nu = 0$ and $\theta = 1 - \alpha$. While in Barro (1990) the main growth engine rest upon constant returns in aggregate capital at the social level, in our case, by contrast, perpetual growth will be mainly the fruit of the exogenous technological progress and resource preservation.

It follows that real aggregate profits Π are given by

$$\Pi_t = A_t K_t^{\alpha} L_t^{\beta} X_t^{\nu} G_t^{\theta} - (r_t + 1) K_t - w_t L_t - p_t X_t \tag{10}$$

Since production in per worker terms is

$$y_t = A_t k_t^{\alpha} x_t^{\nu} g_t^{\theta} \tag{11}$$

where lower case letters denote per capita variables. We will assume through the paper that the contribution of public spending to production is lower than the share of labor in total income *i.e.* $\theta < \beta$. If this were not the case, the technology would display constant or increasing returns in capital and resources. The first order conditions of the maximization problem of firms can be written in the following way

$$r_t = \alpha A_t k_t^{\alpha-1} x_t^{\nu} g_t^{\theta} - 1 \tag{12}$$

$$p_t = \nu A_t k_t^{\alpha} x_t^{\nu-1} g_t^{\theta} \tag{13}$$

$$w_t = \beta A_t k_t^{\alpha} x_t^{\nu} g_t^{\theta} \tag{14}$$

where we assume full depreciation of capital. Notice that conditions (12)-(14) simply claim that production factor are remunerated at their marginal productivity. In addition, it is immediate to verify that public spending increases all the marginal productivities.

2.4. The Government

The government has the responsibility to provide a public good g (in the form of public infrastructures) using debt b or taxes τ. Its budgetary constraint is thus

$$b_{t+1} = (1 + r_{t+1})b_t - \tau_t + g_t \tag{15}$$

We assume that the government faces public finances stabilization rules in the spirit of the Maastricht Treaty which imposes to Eurozone countries a public debt no larger than 60% of GDP. We thus suppose that the public debt-to-capital ratio is a constant $\hat{B}$.[5] In the same spirit, we assume that public expenditures are kept constant as a share of national capital. We denote this constant $\hat{G}$. The choice of focusing simultaneously on two fiscal instruments implies that taxes are endogenously given in a way that balances the government budgetary constraint.

3. Intertemporal Equilibrium and Balanced Growth Path

Using the following market clearing condition

$$s_t = k_{t+1} + b_{t+1} \tag{16}$$

and equations (1), (2), (5), (6), (8), (9), (12), (13), (14), (15), an intertemporal equilibrium may be found. We denote $\mu_{h,t+1} = \frac{h_{t+1}}{h_t}$ the growth factor between t and $t+1$ of any variable h. The next proposition characterizes the intertemporal equilibrium of the decentralized economy.

[5]It will be demonstrated later (Proposition 2) that GDP and the capital stock increase at the same rate along the BGP. Thus, along the BGP, the public debt-to-GDP ratio is also constant.

Proposition 1. An intertemporal equilibrium is defined by the following equations:

$$\mu_{k,t+1}\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right]=\left[\frac{\beta q_t-\nu(1-q_t)(2+\rho)-\alpha\hat{B}q_t}{\alpha(2+\rho)q_t}\right](1+a)\mu_{k,t}^{\alpha+\theta}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1}-\frac{\hat{G}}{2+\rho} \tag{17}$$

$$\frac{\mu_{k,t+1}}{\frac{q_{t+1}}{q_t}(1-q_t)}=(1+a)\mu_{k,t}^{\alpha+\theta}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1} \tag{18}$$

together with the initial conditions q_{-1}, q_0, and $\mu_{k,0}$ and the usual transversality condition that rules out any explosive dynamics.

Proof. Proof is reported in Appendix A. □

(17) represents the dynamics of the accumulation of wealth by households. More precisely, it explains how households allocate their savings between resources, public bonds, and capital. (18) represents the dynamics of assets prices in the economy.

The present paper is interested in the balanced growth path because it constitutes the only case where a non-declining consumption path may be sustained in the presence of necessary non-renewable resources as pointed out by Agnani et al. (2005). Due to the presence of public debt and public expenditures, we will use the following assumption that ensures the existence of a balanced growth path.

Assumption 1. The debt-to-capital ratio satisfies $-\frac{(2+\rho)}{(1+\rho)}<\hat{B}<\beta/\alpha$.

This assumption is not restrictive. Indeed, standard parameter calibration implies that the debt-to-capital ratio should be lower than 2, which is a threshold rarely achieved. In addition, our paper is interested in positive levels of debt stabilization as it is the case for most developed countries. However, our results are fully consistent even for negative public debt stabilization ratios although bounded away from below.[6]

[6] Notice that the lower bound is a sufficient condition (not necessary) that ensures that all results given in the paper hold. In any case this bound is low enough to be empirically plausible for almost all countries.

In the present paper, we focus on the balanced growth path, *i.e.* paths characterized by constant growth factors. We can thus introduce the following proposition and show the existence of a unique balanced growth path.

Proposition 2. Under Assumption 1, the balanced growth path exists, is unique, and is defined by the following equations

$$
\begin{aligned}
&\mu = \mu_y = \mu_k = \mu_c = \mu_d = \mu_s = \mu_b = \mu_g = \mu_w = \mu_\tau = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}} \\
&\mu_x = \mu_m = 1-q \\
&\mu_p = \frac{\mu_y}{\mu_x} = 1+r = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}-1} \\
&\mu_r = 1 \\
&\mu_A = 1+a
\end{aligned}
$$

and q solving the following non-linear equation

$$
\begin{aligned}
F(q) \equiv \; & \frac{\alpha(1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}[(2+\rho)+(1+\rho)\hat{B}]q+\alpha\hat{G}q}{\beta q-(2+\rho)\nu(1-q)-\alpha\hat{B}q} \\
& = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{v}{1-\alpha-\theta}-1} \equiv H(q) \qquad (19)
\end{aligned}
$$

Proof. Proof is reported in Appendix B □

The balanced growth path could represents either a growing or a decreasing economy. In the present paper, we are interested in the sustainability of positive growth defined as follows:[7]

Definition 1. A balanced growth path is sustainable if $\mu \geq 1$.

Proposition 2 has important implications in terms of sustainability. Indeed, a balanced growth path is the only path compatible with a non-declining consumption

[7]It can be noticed that we use here a concept of weak sustainability which follows naturally from the use of the Cobb-Douglas production function that we consider.

(Agnani et al., 2005). High public debt-to-capital stabilization ratios, preventing the existence of a balanced growth path, thus prevent any sustainable growth possibility.

According to Definition 1 and taking into account Proposition 2, a sustainable balanced growth path requires

$$\frac{(2+\rho)\nu}{\beta - \alpha\hat{B} + (2+\rho)\nu} < q^* \leq 1 - (1+a)^{\frac{-1}{\nu}}$$

Thus, the economy is contracting if $1 - (1+a)^{\frac{-1}{\nu}} < \frac{(2+\rho)\nu}{\beta - \alpha\hat{B} + (2+\rho)\nu}$ which is less likely to hold for low level of public debt-to-capital stabilization ratios. Thus, a high level of public debt is associated with an unsustainable use of resources.

Since our economy can display long-run positive as well as negative balanced growth, some restrictions on the structural as well as policy parameters are necessary in order to guarantee a sustainable balanced growth path. The following proposition explains such a point.

Proposition 3. If $\frac{\beta}{\alpha} > \hat{B} > \frac{1}{\alpha}\left[\beta + (2+\rho)\nu - \frac{(2+\rho)\nu}{1-(1+a)^{-1/\nu}}\right]$, the economy is characterized by an unsustainable balanced growth path.

Proof. $1 - (1+a)^{\frac{-1}{\nu}} < \frac{(2+\rho)\nu}{\beta - \alpha\hat{B} + (2+\rho)\nu} \Leftrightarrow \hat{B} > \frac{1}{\alpha}\left[\beta + (2+\rho)\nu - \frac{(2+\rho)\nu}{1-(1+a)^{-1/\nu}}\right]$ □

Proposition 3 shows that there exists an intermediary level of debt-to-capital which is compatible with a balanced growth path, but which is (environmentally) unsustainable. Public debt thus appears as a threat to sustainable development.

Figure 1 summarizes the previous findings. Depending on the level of the public debt-to-capital ratio, the economy could experience no balanced growth, negative balanced growth, or positive balanced growth. Only the last case could be defined as sustainable.

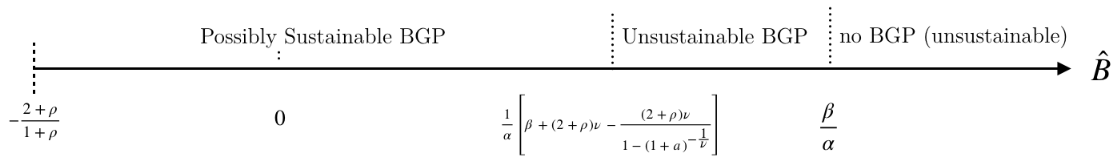

Figure 1: The growth experience is debt-to-capital dependent

For low level of public debt-to-capital, the economy experiences sustainable growth. Above a certain threshold , the growth becomes negative, but is still balanced. Then, if the debt-to-capital becomes very large, the existence of balanced growth is no longer possible, which disable the sustainability of development. Interestingly, no such conditions are found in the model once the resource dimension is removed. In the latter case, the rate of growth is simply $(1+a)^{\frac{1}{1-\alpha-\theta}}$. [8] In table 1, we propose to calibrate the model to obtain values for both thresholds. Two calibrations are thus performed. Calibration 1 captures standard developed economies features, while calibration 2 is more consistent for emerging or developing countries. For high income economies, the two thresholds are very close, so the likelihood to be trapped in the unsustainable BGP is very low. Interestingly, the spread between the two thresholds increases when the resource share increases, which is consistent for emerging and developing economies. It makes the occurrence of unsustainable balanced growth more likely. [9] This little exercise illustrate the importance of considering the resource dimension in studies that deal with public debt sustainability. It paves the way for future research analyzing the sustainability of a given country's debt.

[8] While this observation could be attributed to our modeling framework, it nevertheless proves the importance of the natural resource dimension in the debt-growth nexus.

[9] Anecdotally, depending on the labor share importance relative to the capital share, both thresholds may increase or decrease.

	Calibration 1	Calibration 2
α	0.3	0.2
β	0.65	0.5
ν	0.05	0.3
β/α	2.16667	2.5
$\frac{1}{\alpha}\left[\beta + (2+\rho)\nu - \frac{(2+\rho)\nu}{1-(1+a)^{-\frac{1}{\nu}}}\right]$	2.16664	1.55875

$\rho = 0.016$ and $a = 0.028$ (annual rates)

Table 1: Threshold Values

4. Local Stability

This section is devoted to the analysis of local stability of the dynamic system defined by equations (17) and (18). Defining $z_t = q_{t-1}$, the system may be re-written as follows

$$\begin{cases} \mu_{k,t+1}\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right] = \left[\frac{\beta q_t - \nu(1-q_t)(2+\rho) - \alpha\hat{B}q_t}{\alpha(2+\rho)q_t}\right](1+a)\mu_{k,t}^{\alpha+\theta}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1} - \frac{\hat{G}}{2+\rho} \\ \frac{\mu_{k,t+1}}{\frac{q_{t+1}}{q_t}(1-q_t)} = (1+a)\mu_{k,t}^{\alpha+\theta}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1} \\ z_{t+1} = q_t \end{cases}$$

Linearizing this system around the BGP, we get

$$\begin{pmatrix} d\mu_{k,t+1} \\ dq_{t+1} \\ dz_{t+1} \end{pmatrix} = J \begin{pmatrix} d\mu_{k,t} \\ dq_t \\ dz_t \end{pmatrix}$$

where

$$J = \begin{pmatrix} A & B & C \\ \frac{G-DA}{F} & \frac{H-E-DB}{F} & \frac{I-DC}{F} \\ 0 & 1 & 0 \end{pmatrix}$$

and

$$A = \frac{(\alpha+\theta)\left[\beta q - \alpha\hat{B}q - \nu(2+\rho)(1-q)\right]}{q\alpha(2+\rho)(1-q)\left[1+\frac{\hat{B}(1+\rho)}{2+\rho}\right]} \qquad F = -\frac{\mu}{(1-q)q}$$

$$B = \frac{\mu\left[(\nu-1)\left[\beta q - \alpha\hat{B}q - \nu(2+\rho)(1-q)\right]+\nu(2+\rho)\right]}{q^2\alpha(2+\rho)(1-q)\left[1+\frac{\hat{B}(1+\rho)}{2+\rho}\right]} \qquad G = \frac{\alpha+\theta}{1-q}$$

$$C = -\frac{\mu(\nu-1)\left[\beta q - \alpha\hat{B}q - \nu(2+\rho)(1-q)\right]}{q^2\alpha(2+\rho)(1-q)^2\left[1+\frac{\hat{B}(1+\rho)}{2+\rho}\right]} \qquad H = \frac{(\nu-1)\mu}{(1-q)q}$$

$$D = \frac{1}{1-q} \qquad I = -\frac{(\nu-1)\mu}{(1-q)^2 q}$$

$$E = \frac{\mu}{q(1-q)^2}$$

The stability features of the above Jacobian J depend on the associated eigenvalues. To this end, notice that our dynamic system includes two predeterminated variables, z and μ, and one forward looking, q. It follows that the equilibrium will be determinate if and only if the number of the stable roots is lower than three. The eigenvalues represent the solutions of the following third order characteristic polynomial

$$\lambda P(\lambda) = 0$$

where

$$P(\lambda) = \lambda^2 - \left[A - \left(\frac{DB+E-H}{F}\right)\right]\lambda - A\left(\frac{DB+E-H}{F}\right) + \left(\frac{DC-I}{F}\right) - B\left(\frac{G-DA}{F}\right)$$

It is immediate to verify that one root of the polynomial is always equal to 0. In order to characterize the remaining two roots, we may analyze the second order polynomial $P(\lambda)$.

Notice that $\lim_{\lambda\to+\infty} P(\lambda) = \lim_{\lambda\to-\infty} P(\lambda) = +\infty$, that $P(\lambda)$ is continuous, and that its domain of definition is a connected set. In order to fully characterize the third root, let us evaluate the polynomial $P(\lambda)$ at $\lambda = -1, 0$ and 1. We obtain the following expressions :

$$P(-1) = 1 + A - \left(\frac{DB+E-H}{F}\right) - A\left(\frac{E-H}{F}\right) + \left(\frac{DC-I}{F}\right) - B\left(\frac{G}{F}\right)$$

$$P(0) = -A\left(\frac{E-H}{F}\right) + \left(\frac{DC-I}{F}\right) - B\left(\frac{G}{F}\right)$$

$$P(1) = 1 - A + \left(\frac{DB+E-H}{F}\right) - A\left(\frac{E-H}{F}\right) + \left(\frac{DC-I}{F}\right) - B\left(\frac{G}{F}\right)$$

The first useful piece of information consists in evaluating $P(\lambda)$ at $\lambda = 1$. To this end, let us observe that the local unicity of the stationary solution implies that $P(1) \neq 0$. In addition, consider the following standard calibration for the model : $a = 0.028$ (annual rate), $\rho = 0.016$ (annual rate), $\alpha = 0.3$, $\beta = 0.65$, $\nu = 0.05$ and $\theta = 0.2$. We assume that $\hat{G}$ and $\hat{B}$ are respectively 0.01 and 0.6. It follows that $q \approx 0.61$ and $\mu \approx 3.61$. Under such calibration, we find $P(1) < 0$. This is sufficient to claim that one root is larger than one.

In order to characterize the third root, let us start by evaluating the sign of $P(0)$. Straightforward also tedious computations allow to verify that this sign is equal to the sign of

$$(\alpha + \theta + \nu - 1)\Gamma + (1 - \nu) + \frac{\nu(\alpha + \theta)}{q\alpha \left[1 + \hat{B}\frac{1+\rho}{2+\rho}\right]}$$

where

$$\Gamma \equiv \frac{\beta q - \alpha \hat{B} q - \nu(2+\rho)(1-q)}{q\alpha(2+\rho)(1-q)\left[1 + \hat{B}\frac{1+\rho}{2+\rho}\right]} > 0$$

By exploiting the dynamic equations (17) and (18) evaluated at the steady state, one obtains the following relationship

$$\Gamma = 1 + \frac{\hat{G}}{\mu(2+\rho)\left[1 + \hat{B}\frac{1+\rho}{2+\rho}\right]}$$

It follows that the sign of $P(0)$ is given by the sign of

$$(\alpha + \theta) + (\alpha + \theta + \nu - 1)\frac{\hat{G}}{\mu(2+\rho)\left[1 + \hat{B}\frac{1+\rho}{2+\rho}\right]} + \frac{\nu(\alpha + \theta)}{q\alpha\left[1 + \hat{B}\frac{1+\rho}{2+\rho}\right]}$$

Since $\theta < \beta$ and in view of assumption 1, and considering that q is increasing in $\hat{G}$ and μ is decreasing in $\hat{G}$ as it will be demonstrated in the next section, we have that the above expression is monotonically decreasing in $\hat{G}$. In particular, it is positive up to a given $\hat{G}$ not too large, that we denote $\hat{G}^0$ and that is such that $P(0) = 0$. Then, for $\hat{G} > \hat{G}^0$, $P(0)$ becomes negative. It follows that for $\hat{G} < \hat{G}^0$, the third root will be included in $]0; 1[$ and the steady state will thus be locally determinate. One may wonder, by contrast, where the third root is located when $\hat{G} > \hat{G}^0$: in fact, if we know it is negative, we cannot still precise if it is included in $]-1; 0[$ or if it is lower than -1. In order to verify such a feature, let us evaluate the polynomial at $\lambda = -1$. As a matter of fact, if $P(-1) > 0$, then, the third root will be included in $]-1;0[$, otherwise, it will be lover than -1. Proceeding as we have done in the study of $P(0)$, it is possible to show that there exists a $\hat{G}^1 > \hat{G}^0$ such that $P(-1) = 0$, $P(-1) > 0$ for $\hat{G} < \hat{G}^1$ and $P(-1) < 0$ for $\hat{G} > \hat{G}^1$. By combining all these pictures of information, we obtain that the third root belongs to $[0;1[$ for $\hat{G} < \hat{G}^0$ and to $]-1;0[$ for $\hat{G} \in \left]\hat{G}^0;\hat{G}^1\right[$ and is lower than -1 for $\hat{G} > \hat{G}^1$. In addition, for $\hat{G} = \hat{G}^1$, the third root will be equal to -1. Accordingly, in a neighborhood of $\hat{G}^1$ there will emerge a two periods limit cycle bifurcating from the steady state. In the case of a supercritical flip bifurcation, there will appear a stable limit cycle near the unstable fixed point; were the bifurcation subcritical, conversely, the stable fixed point would be surrounded by an unstable limit cycle. As a consequence, along the bifurcation, the equilibrium will be always over-determinate. All these results are summarized in the following proposition.

Proposition 4. The Jacobian J of our dynamic system possesses an eigenvalue equal to zero and an eigenvalue larger than one. In addition, there exist $\hat{G}^0$ and $\hat{G}^1$, $\hat{G}^0 < \hat{G}^1$, such that:

i. when $\hat{G} < \hat{G}^0$ then the third root is included in $]0;1[$ and the equilibrium is determinate

ii. when $\hat{G} \in \left]\hat{G}^0;\hat{G}^1\right[$ then the third root is included in $]-1;0[$ and the equilibrium is determinate

iii. when $\hat{G} > \hat{G}^1$ then the third root is lower than -1 and the equilibrium is over-determinate

In addition, at $\hat{G} = \hat{G}^1$ the steady-state undergoes a flip bifurcation. Accordingly, in the neighborhood of $\hat{G}^1$, there will emerge a limit cycle near the steady state whose stability will depend upon the direction of the bifurcation.

In any case, several and robust simulations suggest that the occurrence of a negative third root requires a calibration for $\hat{G}$ very above all empirically plausible values. It follows that the standard configuration of the steady state corresponds to the saddle path one and therefore determinacy prevails.

5. The Impact of Public Finances Stabilization Ratios on Growth

This section analyses how movements of public debt-to-capital and public expenditures-to-capital ratio affects the rate of growth of the economy using comparative statics.[10] In the following propositions, we prove that an increase in the public expenditures stabilization ratio or in the public debt stabilization ratio is growth detrimental.

Proposition 5. An increase of the public expenditures stabilization ratio increases the extraction rate and decreases the rate of growth.

Proof. Proof is reported in Appendix C. □

This effect is quite intuitive. When the weight of public expenditures in the economy increases, it implies an increase in taxes and reduces the disposable income of households. They consume less but they also save less in both capital and resources. The rate of resource extraction increases which in turn affects negatively the rate of growth. More capital is needed in the long run to compensate for resource depletion but in the same time less capital is offered by households.

Proposition 6. An increase of the public debt stabilization ratio increases the extraction rate and decreases the rate of growth.

Proof. Proof is reported in Appendix D. □

In the long run, an increase in $\hat{B}$ imposes a higher level of taxes which depresses growth as explained before. In addition, an increase in the level of debt-to-capital stabilization ratio implies a larger crowding out effect of public debt on other assets (capital and resources) since a larger share of household savings is devoted to public bonds. Capital accumulation is reduced and resource use increases.

[10]We focus on the case where public debt (as a share of capital) is sufficiently low i.e. $\hat{B} < \tilde{B}$.

6. The Central Planner's Problem

This section is devoted to the study of the social planner program. We assume that the social planner faces the same expenditure stabilization rule than the one we imposed previously to the market economy. Formally, it means that the public expenditures-to-capital ratio should be stabilized at a level $\hat{G}$. This assumption has two advantages. *i)* It allows to see how the optimal rate of growth is affected by a change in the ratio of public expenditures-to-capital, *i.e.* in change in budgetary treaties; *ii)* It will also allow us to find an instrument (the public debt-to-capital stabilization ratio) that is able to decentralize the optimal equilibrium for each level of public expenditures stabilization ratio. Let's assume that the social planner discounts time at a rate ψ. As a consequence, it solves the following Ramsey problem:

$$\max_{\{c_t;d_t;m_t;k_t\}_{t=0}^{\infty}} = \frac{1}{1+\rho}\ln(d_0) + \sum_{t=0}^{\infty}\frac{1}{(1+\psi)^{t+1}}\left[\ln(c_t) + \frac{1}{1+\rho}\ln(d_{t+1})\right] \tag{20}$$

subject to:

$$c_t + d_t + k_{t+1} + (\hat{G}-1)k_t = A_t\hat{G}^{\theta}k_t^{\alpha+\theta}x_t^{\nu} \tag{21}$$

$$A_{t+1} = (1+a)A_t \tag{22}$$

$$m_t = m_{t-1} - x_t \tag{23}$$

$$m_{-1} = \sum_{t=0}^{\infty} q_t m_{t-1} \tag{24}$$

$$k_0,\ m_{-1},\ ,A_0 > 0 \tag{25}$$

where (21) is the resource constraint of the economy, (22) is the law of technical progress, (23) is the low of motion of the resource stock, (24) is a total exhaustibility condition for the resources while (25) represents initial endowments. The first order condition of the planner's program may be reduced to the following system:

$$\frac{1+\psi}{1+\rho} = \frac{d_t}{c_t} \tag{26}$$

$$(1+\rho)\frac{d_{t+1}}{c_t} = (\alpha+\theta)A_{t+1}\hat{G}^{\theta}k_{t+1}^{\alpha+\theta-1}x_{t+1}^{\nu} + 1 - \hat{G} \tag{27}$$

$$\frac{A_{t+1}\hat{k}_{t+1}^{\alpha+\theta}x_{t+1}^{\nu-1}}{A_t k_t^{\alpha+\theta}x_t^{\nu-1}} = (\alpha+\theta)A_{t+1}\hat{G}^{\theta}k_{t+1}^{\alpha+\theta-1}x_{t+1}^{\nu} + 1 - \hat{G} \tag{28}$$

$$\lim_{t\to\infty}\left(\frac{1}{1+\psi}\right)^t \frac{k_{t+1}}{c_t} = 0 \tag{29}$$

where (26) and (27) are, respectively, the intergenerational and intragenerational optimality conditions, (28) characterizes the optimal inter-temporal resources allocation and (29) is the transversality condition.

Combining equations (21)-(28), the dynamics of the economy is defined by the following system:

$$\begin{cases} (1+a)\mu_{k,t+1}^{\alpha+\theta-1}\mu_{x,t+1}^{\nu} = \dfrac{(1+a)\mu_{k,t+1}^{\alpha+\theta}\mu_{x,t+1}^{\nu-1} - 1 + \hat{G}}{(1+a)\mu_{k,t}^{\alpha+\theta}\mu_{x,t}^{\nu-1} - 1 + \hat{G}} & (30) \\ (1+a)\mu_{k,t+1}^{\alpha+\theta}\mu_{x,t+1}^{\nu-1} = (1+\psi)\mu_{k,t+1}\dfrac{\gamma_{c,t+1}}{\gamma_{c,t}} & (31) \\ \gamma_{c,t}\left[1 + \dfrac{1+\psi}{1+\rho}\right] + \mu_{k,t+1} + \hat{G} - 1 = \dfrac{(1+a)\mu_{k,t+1}^{\alpha+\theta}\mu_{x,t+1}^{\nu-1} - 1 + \hat{G}}{\alpha+\theta} & (32) \end{cases}$$

where $\gamma_{c,t} = c_t/k_t$. Evaluating the system at the BGP and using the definition of the extraction rate $q_t = x_t/m_{t-1}$, one can define the balanced growth path of this Ramsey economy.

Proposition 7. The optimal balanced growth path is defined by the following growth rates:

$$\tilde{\mu}_y = \tilde{\mu}_k = \tilde{\mu}_c = \tilde{\mu}_d = \tilde{\mu} = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}}$$

$$\tilde{\mu}_x = \tilde{\mu}_m = 1 - \tilde{q}$$

$$\tilde{\mu}_A = 1 + a$$

where

$$\tilde{q} = \tfrac{\psi}{1+\psi}.$$

Proof. Proof is reported in Appendix E. □

The optimal extraction rate is completely determined by the social rate of time preference. We can see here that the stabilization rule for public expenditures doesn't affect optimal extraction and growth, which are solely determined by the exogenous rate of technological progress, the social rate of time preference and factor elasticities in the production function. The Ramsey economy is thus a kind of cake-eating problem where the speed of resource exhaustion depends on a trade-off between different generations' welfare. It thus appears that the optimal rate of growth depends also on such a trade-off. When ψ is low, the rate of resource use is low so that the resource stock is preserved for future generations and long-run growth is enhanced. In opposition, when ψ is large, the rate of resource use is large so that the resource stock exhaustion goes faster depressing future growth.

7. Decentralization of the Optimal Allocation

Comparing the growth rates that appear in Propositions 2 and 7, it is immediate that the decentralization of the optimal allocation requires to put the market equilibrium extraction rate at the optimal level. This can be done using the public debt-to-capital stabilization level. Indeed, this ratio affects the rate of resource extraction as highlighted in Proposition 5. We thus propose to find the level of public debt-to-capital ratio such that $q = \tilde{q}$. Using equation (19), it can be inferred that the optimal level of debt-to-capital ratio $\tilde{\hat{B}}$ should satisfy

$$\frac{\alpha(1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}}[(2+\rho)+(1+\rho)\tilde{\hat{B}}]\tilde{q}+\alpha\hat{G}\tilde{q}}{\beta\tilde{q}-(2+\rho)\nu(1-\tilde{q})-\alpha\tilde{\hat{B}}\tilde{q}} = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{v}{1-\alpha-\theta}-1}$$

where $\tilde{q} = \psi/(1+\psi)$. It is straightforward to demonstrate that this condition imposes

$$\tilde{\hat{B}} = \frac{\beta\tilde{q}-(2+\rho)\nu(1-\tilde{q})-(2+\rho)\alpha\tilde{q}-\hat{G}\alpha\tilde{q}}{\alpha\tilde{q}\left[1+(1+\rho)(1-\tilde{q})\right]} \tag{33}$$

Therefore, we have the following proposition.

Proposition 8. The optimal allocation may be decentralized with a public debt-to-capital ratio $\tilde{\hat{B}}$.

From equation (33), it immediately appears that a higher level of public expenditures-to-capital ratio imposes a lower level of debt. The reason is quite simple. The optimal extraction rate is exclusively defined by the social rate of time preference. However, the market equilibrium extraction rate is endogenously determined and $\hat{G}$ and $\hat{B}$ are key parameters in its determination as highlighted by Propositions 5 and 6. Since increases in each of these two parameters increase the extraction rate, it is not surprising that an increase of the (exogenous) level of public expenditures-to-capital ratio reduces the optimal level of public debt-to-capital ratio.

8. Conclusion

Environmental issues are ones of the main cores of economic agenda. The faith in a lasting capital accumulation process and in a never-ending increase in GDP, consumption and investment is more and more questioned on the ground of an environment characterized by its limited amount of resources not all of them perfectly renewable. This seems apparently to put an upper bound on the increase of what Adam Smith referred as to the "wealth of the nations". Another highly questioned issue in recent literature is the impact of public debt on growth. Indeed, it is often thought that savings devoted to finance public debt is crowded out from productive capital accumulation. However, public expenditures allowed by the emission of public debt may be growth enhancing since they can endow the economy with productive infrastructures (Barro, 1990). In addition, public debt can be seen as a speculative bubble able to restore dynamic efficiency in economies characterized by capital over-accumulation (Tirole, 1985).

In this paper we have focused on such issues by studying an OLG model with stationary population and Cobb-Douglas preferences in which individuals accumulate physical capital, a non-renewable resource, and government liabilities. The government fiscal policy consists in targeting the public debt-to-capital ratio and the public expenditures-to-capital ratio, where public spending is assumed to contribute to production although in an unintended way. As a consequence, in order to respect public budget constraint, taxation is endogenously adjusted in response to the evolution of aggregate variables. Within such a framework, we have studied the impact of the fiscal rules on the balanced growth path.

More in details, we have proved that a higher public debt stabilization ratio and/or a larger public spending stabilization ratio is growth detrimental, since it compels agents to reallocate their savings from physical capital and the stock of natural resources to debt and this in turn accelerates the extraction rate at the cost of reducing the long run sustainability of the system.

In a further section of the paper we have carried out a detailed analysis of the centralized economy where a benevolent social planner is free to choose the economic path on the ground of its time preference and subject uniquely on the public expenditures stabilization target. The main result we obtain is that the stationary growth rate increases as soon as the social planner cares more and more about future generations and therefore tries to avoid a too much fast exploitation of the non-renewable resource which may be detrimental for the future economy size. In addition, we show that the optimal balanced growth path can be opportunely decentralized by calibrating the fiscal instruments, as the public debt ratio; this is the immediate consequence of the monotonic and negative relationship occurring between public debt stabilization ratio and the economy rate of growth.

Since the rate of growth chosen by the social planner is increasing in his degree of patience, we find that the optimal public debt stabilization ratio will be lower in economies run by less shortsighted rulers. Therefore, in some circumstances, larger public debts could be welfare-improving. Of course, such results hold within our economy characterized by short-lived agents, Cobb-Douglas preferences and the absence of increasing returns to scale sufficient to generate unbounded growth (with the necessary requirement of exogenous technological progress). By removing each of these hypotheses, one could improve the analysis in terms of the role of the fiscal policy on growth, of the equilibrium (in)determinacy and of the extraction rate of non-renewable resources. In addition, the polluting features of non-renewable resources could also be considered by introducing environmental quality in the utility function. We leave such purposes for future research.

Appendix A. Proof of Proposition 1

Combining (9) and (12), we get

$$\frac{p_{t+1}}{p_t} = \alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^{\nu} g_{t+1}^{\theta}$$

Taking the ratio of this equation evaluated in $t+1$ and in t, and using equation (13), we obtain equation (18).

Combining equations (5) and (6) together with the market clearing condition for capital and bonds (16) and the Euler equation (8), it is immediate that

$$(2+\rho)(k_{t+1}+b_{t+1}) = w_t - \tau_t - (2+\rho)p_t m_t$$

Using the government budget constraint (15), it gives

$$(2+\rho)k_{t+1} + (1+\rho)b_{t+1} = w_t - (1+r_{t+1})b_t - g_t - (2+\rho)p_t m_t$$

Since the government stabilizes its debt-to-capital at $\hat{B}$ and its expenditures to GDP ratio at $\hat{G}$ and substituting w_t, p_t and r_{t+1} by their expressions, it follows that we obtain

$$(2+\rho)k_{t+1} + (1+\rho)\hat{B}k_{t+1} = \left[\beta - \frac{\nu(2+\rho)(1-q_t)}{q_t} - \alpha\hat{B}\right] A_t k_t^{\alpha} x_t^{\nu} \hat{G}^{\theta} k_t^{\theta} - \hat{G}k_t$$

Dividing both sides by k_t, and noticing than $A_t k_t^{\alpha} x_t^{\nu} \hat{G}^{\theta} k_t^{\theta} = \frac{p_t/p_{t-1}}{\alpha}$, we obtain equation (17).

Appendix B. Proof of Proposition 2

Along the BGP, all variables grow at a constant rate.

- The extraction rate should be constant at the BGP. Thus, equation (2) implies $\mu_m = 1-q$
- From equation (1), we obtain $\mu_x = \mu_m$
- $\mu_a = 1+a$
- The Hotelling rule (9) implies $\mu_p = 1 + r_{t+1}$
- A BGP requires a constant interest rate because the resource price's rate of growth is constant so $\mu_r = 1$
- We assume constant GDP ratios of public expenditures and public indebtness. Thus $\mu_g = \mu_b = \mu_y$

- The production function at the BGP gives $\mu_y = (1+a)\mu_k^\alpha \mu_x^\nu \mu_g^\theta$
- Equation (14) gives $\mu_w = \mu_y$
- The Euler Equation (8) along the BGP gives $\mu_c = \mu_d$
- The government budget constraint (15) implies $\mu_\tau = \mu_b = \mu_y$
- Evaluating the budgetary constraint at the BGP leads to $\mu_d = \mu_y$
- Evaluating (16) at the BGP gives $\mu_s s_t = \mu_k(k_{t+1} + b_{t+1})$. Since $s_t = k_{t+1} + b_{t+1}$ we have $\mu_s = \mu_k$
- Evaluating (18) along the BGP, we obtain $\frac{\mu_k}{1-q} = (1+a)\mu_k^{\alpha+\theta}(1-q)^{\nu-1}$. Thus we have

$$
\begin{aligned}
\mu_k^{1-\alpha-\theta} &= (1+a)(1-q)^\nu \\
\mu_k &= (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}
\end{aligned}
$$

- Evaluating (17) at the BGP, we obtain

$$
\mu_k\left(1+\frac{1+\rho}{2+\rho}\hat{B}\right) + \left(\frac{\nu(1-q)(2+\rho) - \beta q + \alpha q\hat{B}}{\alpha q(2+\rho)}\right)(1+a)\mu_k^{\alpha+\theta}(1-q)^{\nu-1} = -\frac{\hat{G}}{2+\rho}
$$

$$
\frac{\mu_k[(2+\rho)+(1+\rho)\hat{B}] + \hat{G}}{2+\rho} = \frac{\beta q - \nu(1-q)(2+\rho) - \alpha q\hat{B}}{\alpha q(2+\rho)}\frac{\mu_k}{1-q}
$$

$$
\frac{\alpha q\mu_k[(2+\rho)+(1+\rho)\hat{B}] + \alpha q\hat{G}}{\beta q - \nu(1-q)(2+\rho) - \alpha q\hat{B}} = \frac{\mu_k}{1-q}
$$

Replacing μ_k by its value, we get:

$$
\begin{aligned}
F(q) &\equiv \frac{\alpha q(1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}[(2+\rho)+(1+\rho)\hat{B}] + \alpha q\hat{G}}{\beta q - \nu(1-q)(2+\rho) - \alpha q\hat{B}} \\
&= (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}-1} \equiv H(q)
\end{aligned}
$$

and q is the solution of this non-linear equation.

$H(q)$ is a positive, increasing and convex function defined on $[0;1[$ admitting a vertical asymptote for $q=1$. Depending on the size of the public debt stabilization ratio, $F(q)$ may be positive or negative.

Since the numerator of $F(q)$ is always positive under assumption 1, the sign of the denominator determines the sign of the function.

For $(2+\rho)\nu < (\beta - \alpha\hat{B} + (2+\rho)\nu)q$, $F(q)$ is positive and admits a vertical asymptote in $q = \hat{q} = \frac{(2+\rho)\nu}{\beta-\alpha\hat{B}+(2+\rho)\nu}$. Since $\lim_{q\to\hat{q}^+} F(q) = +\infty$ and $\lim_{q\to 1} F(q) = \frac{\alpha\hat{G}}{\beta-\alpha\hat{B}} > 0$, it exists a unique q^* such that equation (19) is satisfied. Thus, if $\hat{B} < \tilde{B}$, there exists a unique balanced growth path. Moreover, if the last condition holds, $F'(q) < 0$ and $F''(q) > 0$.

Appendix C. Proof of Proposition 5

The rate of growth of the economy is defined as

$$\mu = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}$$

The rate of technological progress is considered as exogenous. In opposition, the rate of resource extraction is affected by changes in $\hat{G}$. Indeed, while $H(q)$ is not affected by changes in $\hat{G}$, $\frac{\partial F(q)}{\partial \hat{G}} > 0$. Graphically:

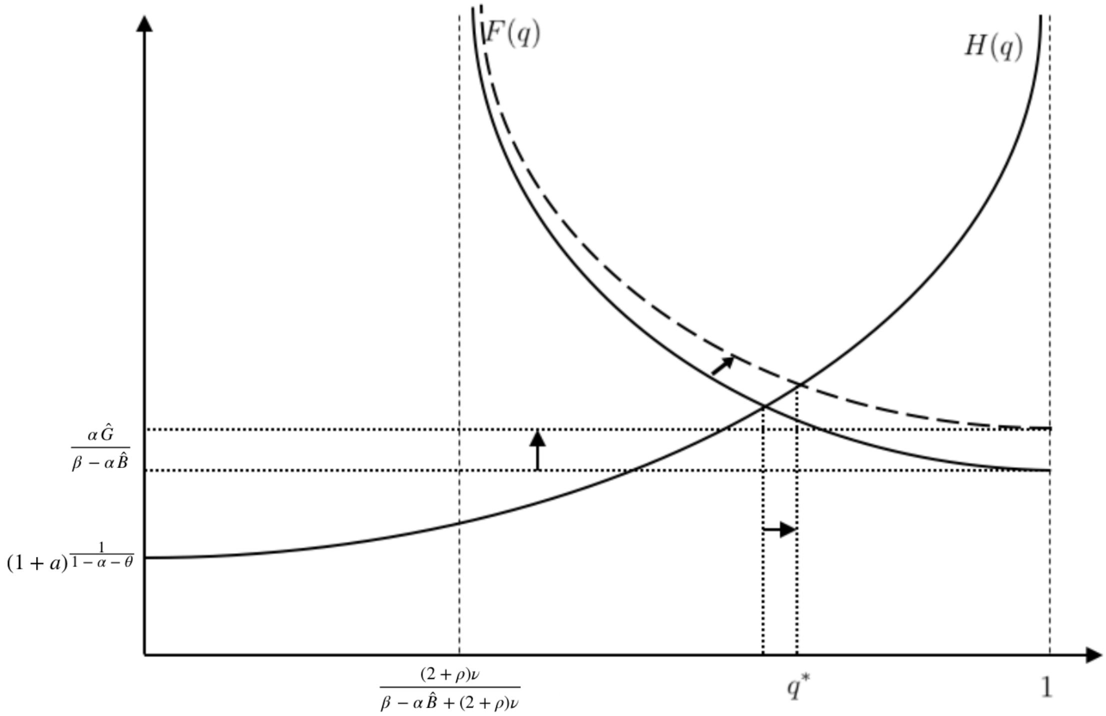

Figure C.2: Effects of an increase in G on the extraction rate

Appendix D. Proof of Proposition 6

The rate of growth of the economy is defined as

$$\mu = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}$$

The rate of technological progress is considered as exogenous. By contrast, the rate of resource extraction is affected by changes in $\hat{B}$. Indeed, while $H(q)$ is not affected by changes in $\hat{B}$, $\frac{\partial F(q)}{\partial \hat{B}} > 0$. Graphically:

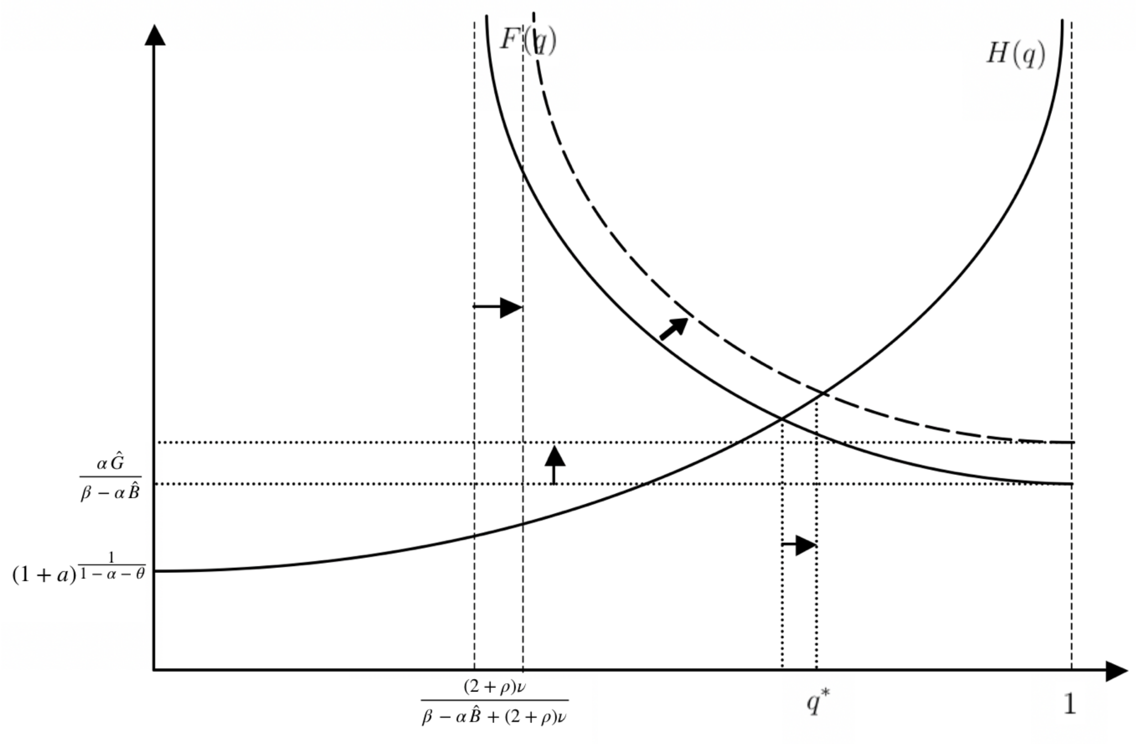

Figure D.3: Effects of an increase in G on the extraction rate

Appendix E. Proof of Proposition 7

- Evaluating equation (30) at the BGP, we obtain $\tilde{\mu}_k = (1+a)\tilde{\mu}_k^{\alpha+\theta}\tilde{\mu}_x^{\nu}$. Then, it follows that $\tilde{\mu}_k = (1+a)^{\frac{1}{1-\alpha-\theta}}\tilde{\mu}_x^{\frac{\nu}{1-\alpha-\theta}}$

- Taking the ratio of (32) in t and $t-1$ we get

$$\frac{\gamma_{c,t}\left[1+\frac{1+\psi}{1+\rho}\right]+\mu_{k,t+1}+\hat{G}-1}{\gamma_{c,t-1}\left[1+\frac{1+\psi}{1+\rho}\right]+\mu_{k,t}+\hat{G}-1} = \frac{(1+a)\mu_{k,t+1}^{\alpha+\theta}\mu_{x,t+1}^{\nu-1}-1+\hat{G}}{(1+a)\mu_{k,t}^{\alpha+\theta}\mu_{x,t}^{\nu-1}-1+\hat{G}}$$

  Evaluating this equation at the BGP, it follows that $\gamma_{c,t} = \gamma_{c,t-1}$ which implies $\tilde{\mu}_k = \tilde{\mu}_c$.

- Evaluating (31) at the BGP, one obtains $\frac{\tilde{\mu}_k}{\tilde{\mu}_x} = (1+\psi)\tilde{\mu}_c$. It follows that $\tilde{\mu}_x = \frac{1}{1+\psi}$. Since $\tilde{\mu}_x = 1-\tilde{q}$, we then have $\tilde{q} = \frac{\psi}{1+\psi}$

- Equation (26) at the BGP implies $\tilde{\mu}_d = \tilde{\mu}_c$.

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