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The effects of migration and pollution on cognitive skills in Caribbean economies: a theoretical analysis

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Abstract

This work analyses the interaction between demographic features and environmental constraints in the Caribbean Small Island Developing States. More specifically, it aims to clarify the impact of migration in the presence of pollution. To do so, an Intergenerational model is developed to reproduce the characteristics of these countries, which are highly dependent on migration gains such as brain gain or remittances. Moreover, production emits pollution that hinders the accumulation of human capital. Two cases emerge from the analysis, in the first an environmental policy is sufficient to correct the externality and in this case migration implies the same mechanisms as in the case without pollution. In the second case, if pollution emissions are high relative to the effectiveness of environmental policy, migration leads to an increase in per capita output and human capital. This only happens if the emigration rate is already high, because it leads to a reduction in demographic pressure on the environment.

Keywords: Pollution, Development, Migration, Caribbean Islands
JEL classification: Q01, Q56, F24, J24

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1. Introduction

Since the end of the twentieth century, the pursuit of sustainable development has occupied researchers, policy makers and more generally human societies. In particular, this has been consecrated by the Earth Summit held in Rio de Janeiro in 1992, where the international community has gathered to discuss this issue. On this occasion, Small Island Developing States (SIDS) were defined "as a special case both for environment and development" issues, because they share common economic, social and environmental vulnerabilities. To trigger a sustainable development of these territories, several interrelated demographic, social and economic determinants could be scrutinized. However, one topic is particularly important in the Caribbean SIDS' case: the human capital dynamics.[1]

[1]There are 16 countries in this group: Antigua and Barbuda, Bahamas, Barbados, Belize, Cuba, Dominica, Dominican Republic, Grenada, Guyana, Haiti, Jamaica, Saint-Kitts and Nevis, Saint-Lucia,

This is explained by two main characteristics. The first one is their structural lack of competitiveness, due to the small size of their economy, their remoteness and the scarcity of the natural resources. In this context, increasing the human capital level is the only way to increase the competitiveness (ECLAC, 2018). Secondly, these countries exhibit a high level of emigration, and more specifically of skilled emigration (ECLAC, 2017). According to the literature, one of the solutions for migration to improve economic growth is if it leads to improved human capital accumulation. In other terms, it depends on whether or not it is possible to have a *brain gain*–*i.e.* an increase in the average human capital in the sending economy–because migration possibility creates incentives to invest in education. However, *brain gain* occurs if the emigration rate is not too high and if the initial human capital is low (Stark et al., 1997, Beine et al. (2011), Docquier and Rapoport (2012), Docquier et al. (2012), Hatton (2014)). Considering the weight of migration in the Caribbean SIDS' demographic feature, it is crucial to determine if they are on a path where they benefit from migration, especially in terms of human capital. However, the possibility for an economy to drive the full potential from its population human capital is not solely defined by the investments in education. In fact, human capital can also be impacted by local pollution.

Several studies have highlighted the link between exposures to local pollutants–among others, metals, pesticides or Persistent Organic Pollutants (POPs)–and the reduction of the cognitive skills (Tzivian et al., 2015, Power et al. (2016), Pujol et al. (2016), Lett et al. (2017)). Moreover, small islands are characterized by the scarcity of land and the proximity between areas with different uses. This leads, on the one hand, to a high probability of contamination of the water sources or of the soil in the residential areas if pollutants are released without treatments. On the other hand, because of the high density of population in the inhabited areas, the share of the domestic population impacted by a local pollutant can be significant for these small countries.

In Caribbean islands, as in many developing countries, inefficient governance and informal dumping or recycling lead to inadequate environmental policies of waste management (Thonart Philippe et al., 2005, Wilson et al. (2006), Barton et al. (2008)) or wastewater. A review of this topic has been done by Mohee et al. (2015) for SIDS. Second, the close interaction between ecosystems implies that pollutants may easily travel through all the natural environment, ecosystems and vital stock of water. For example, the proximity of waste facilities and their lack of efficiency are directly responsible of soft water contamination and coasts degradation through anthropogenic seafloor debris (ECLAC, 2018). Finally, agricultural use of pesticides and fertilizers were considered as the main local pollution source between 1980 and 2000, knowing that many pesticides are very persistent

Saint Vincent and the Grenadines, Suriname as well as Trinidad and Tobago. Note, that Guyana and Suriname are not islands but continental countries, however they have the main characteristics of the SIDS.

There are seven non-independent territories: Anguilla, Aruba, British Virgin Islands, Montserrat, Netherlands Antilles, Puerto Rico and United States Virgin Islands.

in the ecosystems (Rawlins et al., 1998).

This work aims at examining the link between pollution, migration and economic growth, for Caribbean economies to draw qualitative conclusions regarding the impact of migration in a polluted area which is densely populated. An environmental tax is tested and a special focus is given to the evolution of the productive capital stocks–*i.e.* on the human capital level and on the physical capital–to assess the efficiency of this policy. In this context, what are the effects of an environmental policy? Is a *brain gain* from migration still possible when cognitive skills are undermined by local pollution; and if so, under which conditions?

This work arises from the observation that there is no model that includes migration and fertility, education choices while dealing with pollution effects on human capital accumulation. However, taking into account migration to study the link between human capital and pollution is necessary for the Caribbean SIDS, because they may suffer from both *brain drain* and environmental issues. To do so an Overlapping Generations (OLG) model is developed with a production responsible of the emissions of pollution, knowing that the human capital accumulation efficiency depends on the stock of pollution. Note that whether it is pollution in flux or in stock, any type of pollutant emitted by an economic activity could be analyzed with this model. This model, allows to incorporate in the analysis intergenerational choices and solidarity, as well as their impacts on production, the environment and the population dynamics.

In this paper, individuals care about their adult and old-age consumption which can be funded thanks to their savings or the intergenerational transfers from their children. The environment impacts the economy through an externality. Therefore, households' decisions in terms of savings, fertility and education depends on tradeoffs that do not include the pollution effect on human capital. Besides the usual intertemporal tradeoff between adult and old age consumptions there are two tradeoffs to fund consumption during the retirement period. The first is between savings and intergenerational transfers and the second one is between fertility and education of the children. For the former, migration increases the gain from human capital and thus leads to a substitution of the savings by the investments in children–in order to receive more transfers. This is not detrimental for the capital stock, if the increases in the household's income or in the number of people that can save, are large enough to compensate the reduction of the savings (in percentage of the income). This is more likely to happen, if the tradeoff between education and fertility leans in favor of education.

For this second tradeoff, the fertility choice depends on the migration incentives, but also on the opportunity cost of raising children in terms of time–a higher fertility reduces the participation to the labor market and then the wages. This cost includes an overcrowding effect linked to the population size (de la Croix and Gobbi, 2017, de la Croix and Gosseries (2012)). When the cost of the children increases, the parents have a higher incentive to invest in education. Therefore, the overcrowding effect decreases the fertility rate at the benefit of the education expenditures. At the aggregate level, migration has a positive effect on the population size if the increase in fertility induced by migration is higher than the loss due to the departures.

Until then, whether there is pollution or not, it is as the economy was not affected by the environment component, and the relationship between population size and migration is thus described by an inverted U-shaped curve. Without the environmental externality, at the aggregate level, the other variables are growing at the same rate on the long-run and the economy reaches a Balanced Growth Path (BGP). This also leads to inverted U-shaped relationship between the economic growth and migration–in terms of emigration rate or remittances–due to the combination of the increase (reduction) in human capital, population and capital stock when migration is low (high). These dynamics are described by the literature (Beine et al., 2006, Docquier et al. (2008)), and are presented in the case without externality.

In presence of the environmental externality two situations may emerge. In both cases, to cope with this degradation, an environmental policy is tested, consisting in a tax on pollution emissions and a publicly funded maintenance. However, in the first case, depending on the pollution intensity, the natural regeneration rate of the environment and the efficiency of cleaning expenses, it is possible to reach a green growth path. The pollution stock is thus maintained to zero, thanks to investments in maintenance, and the economy displays the same dynamics and characteristics than in the case without environmental degradation.

The second case–which is much more intricate–occurs when the environmental degradations are too large to be completely nullified by the environmental policy. To clarify some trends of this case, a numerical analysis is conducted for two Caribbean economies. This allows to study the model's dynamics, the optimal level of the environmental tax as well as the impacts of the different parameters linked to the emigration's scale or gain. First of all, the aggregate variables display different dynamics, despite the unchanged household choices. Instead of a balanced growth path, as in the previous cases, there is a steady state. This is explained by the pollution stock which hampers the human capital accumulation until the economy reaches an equilibrium. Second, the stability of the steady state values is linked mostly to the environmental damage function in the human capital dynamics. This function must be convex in order to observe a stable equilibrium. Third, in presence of the environmental externality, the relationships between on the one hand the emigration rate and on the other hand, production per capita, human capital and/or per capita utility are described by U-shaped curves. Indeed, if migration is low, an increasing rate of emigration leads to a rise in the population size and in the pollution stock. This decreases the average human capital, until a threshold, where the reduction of the population allows to have gains in human capital. The effects of intergenerational transfers–remittances–are more ambiguous because they do not reduce directly the population size. When this parameter is large, it leads to a decrease in the income which can result in a reduction in fertility. This is due to its negative effect on physical capital stock which is linked to the substitution between savings and investments in children.

Therefore the main contribution of this paper is to link *brain gain* analysis to environmental issues. Indeed, by considering and externality on human capital, it appears that in small open countries such as Caribbean demographic and environmental features can not be studied separately. While this result depends strongly on the environmental features considered, it appears that migration effects can be completely different from the usual analysis of the brain gain (Docquier and Rapoport (2012), Docquier et al. (2012)).

The rest of this paper is structured as follows. Section 2 gives some facts on the Caribbean region. Section 3 describes the model and the equilibrium. Section 5 is a discussion on the effects of migration and the environment on steady state values in a polluted area. Finally, the last section draws conclusions and defines a roadmap for future research.

2. Stylized Facts and Evidence

The first highlight of this article is the importance of migration in Caribbean SIDS. Using data from the World Bank's World Development Indicators (WDI), Figure 1 plots natural balance, migration balance, and population growth in percentage of population within 5 years, in Africa, Asia, the Caribbean, Latin America and Eastern Europe as well as countries of the Organisation for Economic Co-operation and Development (OECD).[2] United Nations Statistics Division (UNSD) releases migration flows data defined as the migrants stocks net change, between the years 1 and 5. The same 5-year interval is used in order to plot the other variables.

The extent of migration flows is quite significant for this regional group, the highest among emerging economies in our country sample.[3,4] As a result, population growth in the Caribbean is close to OECD levels, which is quite low compared to the other groups' levels.

Figure 1 shows that the emigration flows have decreased strongly since 1990 for Caribbean countries. However, the size of the diaspora compared to the domestic population is still significant, especially if compared to other regions. Figure 2 represents the average of the share of nationals living in a foreign country between 2000 and 2015, by region. It is defined as the ratio of the diaspora over the sum of the diaspora and the domestic population. On average, between 2000 and 2015, more than 20% of the persons born in the Caribbean were living in another country.

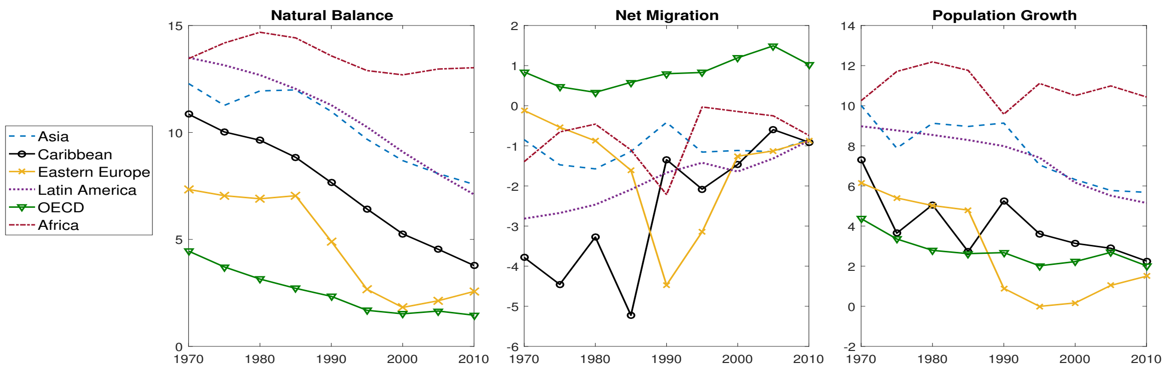
Two-thirds of Caribbean migrants live in the U.S. (ECLAC (2017)), with the rest living in European or other Caribbean countries. Although the majority of these migrants are

[2]Middle East and North African countries are not represented because their demographic features exhibit strong volatility due to conflicts.

[3]Eastern Europe exhibits also a strong migration features since the implementation of the Schengen Area.

[4]In the 1970s, migration was especially high. This is due to the immigration policies in the receiving countries, especially in the U.K. and the U.S.

Figure 1: Demographic features by region (% of population)



NB: Changes are given over a 5 year period.
Source: Author, based on the WDI

considered unskilled workers, the skill migration from the Caribbean is rising. Contrary to other sending regions, over half of the migrants heading to Northern America are skilled women. Moreover, the majority of migrants tend to be young and at a productive age.[5] Migration may have several negative effects on the Caribbean economy, the most obvious being the potential adverse effects from the *brain drain* of skilled people leaving the island. For example, over 80% of individuals with post-secondary education from Jamaica, Haiti, Guyana and Grenada live abroad (Arslan et al. (2016), Docquier and Marfouk (2006)).

However, the strong links between the diaspora and their family in the domestic area creates networks, especially through the *transnational family* (Thomas-Hope (2002)). These networks could trigger technological transfers between the receiving countries and the sending countries, as well as promote international integration (Alleyne and Solan (2019)). This connection between family members leads also to important remittances– *i.e.* transfers between the diaspora and their family in the domestic area (ECLAC (2017, 2018)). Figure 3 displays a comparison of the percentage of received personal remittances relative to GDP across regional groups for the period 2000-2015. Except for Eastern Europe and Central Asia at 4.96%, it shows that the median personal remittances received appear to be higher among the Caribbean countries (4.38% of GDP), followed by South Asia and Pacific (3.27%), Middle East and North Africa (2.25%), Latin America (1.77%), and Sub-Sahara Africa (1.81%). The EECA's context is very specific since the implementation of the Schengen area, which has led to a massive emigration starting at the end of the 1990s. Therefore, compared to other developing areas, Caribbean islands exhibit strong emigration pattern with high remittances.

Second, in this paper, the fertility includes a cost linked to the population density. This assumption is supported by the works of de la Croix and Gobbi (2017) or Sibly et al. (2002). Moreover, in a large country sample, Figure 4 plots logarithmic density per square kilometer against fertility, measured by the average number of children per woman

[5]This is not the case for those from Cuba, who are on average over 45 years old.

Figure 2: Average of nationals living abroad
(% of total population, 2000-2015)

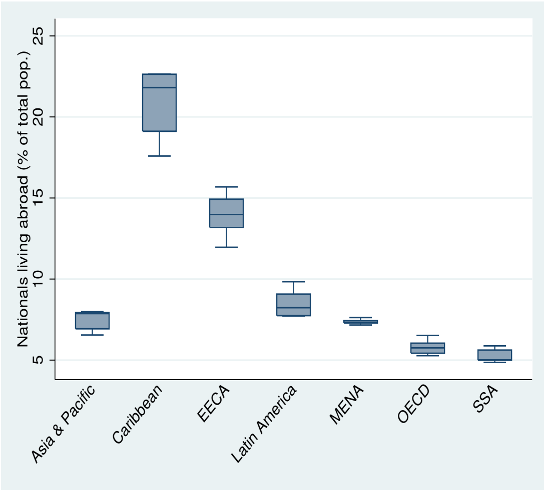
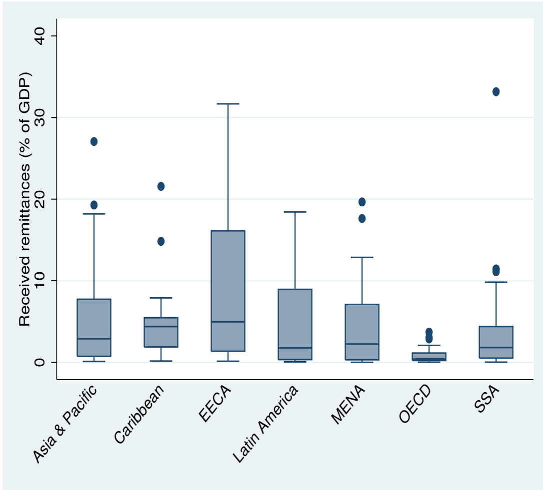


Figure 3: Received personal remittances
(% of GDP, 2000-2015)



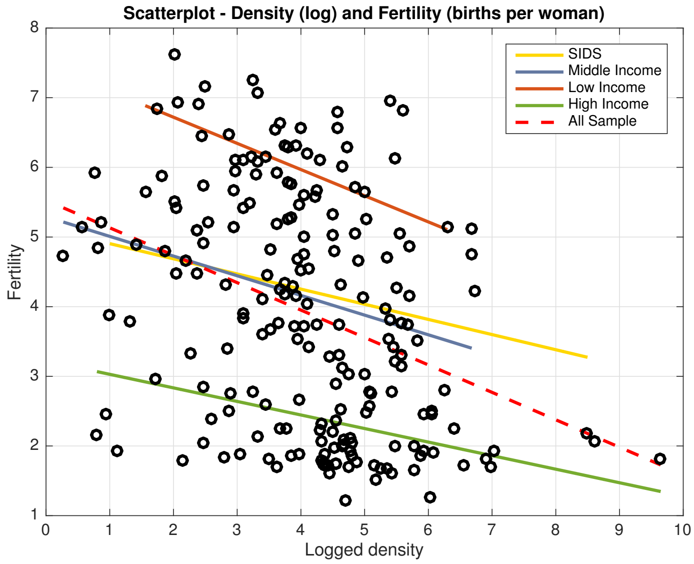
Legend: **EECA**: Eastern Europe and Central Asia, **Lat. Am.**: Latin America, **MENA**: Middle East and North Africa, **SSA**: Sub-Saharan Africa

Source: Author, based on the UNSD migration dataset and the WDI

in a sample set of 205 countries between 1960 and 2018. The sample set is then broken down into sub-categories according to the World Bank ATLAS income methodology. A separate SIDS group category is created using the UN methodology. The figure shows that the whole sample, as well as the different income and SIDS categories exhibit a downward sloping linear fit between logged density and fertility. There are however slight differences as to the sloping coefficients, suggesting that the relationship between the decline in fertility as density rises is not uniformly distributed across levels of income. Low income countries exhibit the steepest slope, which suggests that in these economies, a modest increase in logged density is associated with sharper decline in fertility–which may be accounted for by the fact that the initial levels of fertility are highest among the poorer countries in the sample and that the resource constraints are more important in the decision to have children. SIDS countries exhibit a flatter linear fit in comparison to the whole sample, and tend to mirror the sloping coefficient of High income countries instead. This may be accounted for by the already high population density that has led to a earlier and faster demographic transition in those countries in the 1960s. Indeed, their low level of fertility make them closer to fertility characteristics of developed countries.

Finally, it is important to describe the potential epidemiological studies that justify the introduction of an externality on human capital. Pollution is found to be a key determinant of health. In particular, children are found to be much more vulnerable to pollution because of their developing systems and behaviors (Gordon et al. (2004)). Such health effects have important consequences on human capital accumulation through two main channels. First, there is a direct effect of pollution exposition on the ability to learn. Indeed many studies have showed that the cognitive skills could be hampered in a degraded environment (see for a meta-analysis of the subject Power et al., 2016,

Figure 4: Correlation between logged density and fertility. Average values 1960-2018.



Lett et al. (2017)). More pecularly, Pujol et al. (2016) finds that urban air pollution affects brain maturation of children under 12 years old. And Lett et al., 2017 finds that the more exposed children to industrial pollutant, have math scores that are 1.63 points lower than their less exposed peers. Second, the health effects have consequences on school attendance. Indeed, there is a strong consensus on the effect of air pollution on the occurrence of asthma or on health in general (Beasley et al., 2015, Arroyo et al. (2016), Marcotte and Marcotte (2017), Rosa et al. (2017), Liu et al. (2017)). Moreover, the effect of pesticides is clearly negative and many studies call for taking into account the effect of their presence on health, whether these particles are in water, soil or air (Lai, 2017, Lammoglia et al. (2017), Valcke et al. (2017)).

3. The Model

To analyze economic development in SIDS with pollution, migration and intergenerational transfers, an overlapping generations (OLG) model is used, with discrete time indexed by $t = 0, 1, 2, ..., +\infty$ [6]. OLG models are extensively described in de la Croix and Michel (2002) and are really convenient to study intergenerational transfers–as in Thibault (2008) or Del Rey and Lopez-Garcia (2016)–and human capital dynamics. The present model is kept as simple as possible in order to exacerbate the households' behavior toward savings, consumption, fertility and education.

[6] Each period is assumed to last twenty to thirty years

3.1. Production and the environment

The production of the composite good is carried out by a representative firm. As in Varvarigos (2013), the output is produced according to a constant returns to scale technology. The firm combines units of efficient work, $L_t h_t$–where h_t is the human capital per capita–and capital stock, K_t:

$$Y_t = AK_t^{\alpha} (L_t h_t)^{1-\alpha} \tag{1}$$

Here, $A > 0$ measures the technology level, and $\alpha \in (0, 1)$ is the share of physical capital in the production.

During the production process, the firm emits pollution which induces a negative impact on human capital accumulation. To correct this externality, the government implements a tax $\tau \in (0, 1)$ on production. The tax revenue is used to fund pollution emissions reduction, m_t, with $m_t = \xi\tau Y_t$ and ξ the efficiency of abatement expenditures. The firm profit is:

$$\Pi_t = A(1-\tau)K_t^{\alpha} (L_t h_t)^{1-\alpha} - w_t h_t L_t - R_t K_t \tag{2}$$

where w_t is the wage for one unit of efficient labor, $R_t \equiv 1 + r_t$ is the return factor of capital with r_t the interest rate.

Assuming that the capital fully depreciates in one period, factor prices are as follows:

$$w_t = A(1-\alpha)(1-\tau)K_t^{\alpha} (L_t h_t)^{-\alpha} \tag{3}$$
$$R_t = A\alpha(1-\tau)K_t^{\alpha-1} (L_t h_t)^{1-\alpha} \tag{4}$$

The production sector generates a pollution flow. The dynamics of the pollution stock is such that:

$$Z_{t+1} = \Omega Y_t - m_t + (1-a)Z_t = (\Omega - \xi\tau)AK_t^{\alpha} (L_t h_t)^{1-\alpha} + (1-a)Z_t \tag{5}$$

where, $\Omega \in]0; 1]$ is the pollution intensity of production and $a \in]0; 1]$ is the natural absorption rate of pollution.

Here, the abatement effort could include water treatments or waste processing for example.[7] The equation (5) describes the dynamics of any local pollution such as pesticides, metal or Persistent Organic Pollutant (POP) in water tables, soil, freshwaters, etc. This equation allows to take into account in a simple way, both the intensity of the pollution and the natural capacity of the environment to clean itself, a. If $a = 1$, the model describes a pollution which is not persistent in the environment and thus a pollution flow rather

[7]There are many alternative approaches in the modelling of the pollution dynamics. For instance, it is possible to consider an environmental quality which is degraded by the pollution and improved by a cleanup effort as in John and Pecchenino (1994). These approaches are interesting extensions of the current research and are left for future work.

than a stock. Here if $a = 0$, it implies that all emissions of pollution would be persistent in the environment forever. The only sustainable way of dealing with the pollution, is to abate completely the emissions of pollution. Otherwise, it is not possible to reach a steady state.

3.2. Family's behavior

The representative household lives through three periods: childhood, adulthood, and old age. At t, a new generation of $n_t N_t$ homogenous agents is born, where n_t is the number of children per household. As in de la Croix and Gosseries (2012), n_t is chosen by the adults of period t, knowing that raising n_t children takes a fraction of the parents income linked to the population size N_t.[8] The congestion component introduced in the cost of raising children, means that the higher the population density is, the more costly fertility is. This assumption is in line with works such as de la Croix and Gobbi (2017) or Sibly et al. (2002), which describe the negative correlation between fertility and population density in developing countries. This is also implemented with respect to the stylized facts described earlier.[9] Therefore, raising children takes a fraction $\sigma N_t^{\delta} n_t$ of the income, where $0 < \delta < 1$ captures the overcrowding effect.

The emigration rate is denoted by $\rho \in [0, 1[$. Migration implies that only $(1-\rho)n_t N_t$ children stay in the domestic country after childhood. The other $\rho n_t N_t$ children migrate to countries where wages are greater. The evolution of the size of the adult generation (or the labor force) is represented by the following equation:

$$N_{t+1} = n_t N_t (1 - \rho) \tag{6}$$

Adults born in $t-1$ care about their adult consumption level c_t and their old age consumption level d_{t+1}, according to the psychological discount factor β. Agents' preferences are represented by the following utility function:

$$U(c_t, d_{t+1}) = \ln(c_t) + \beta \ln(d_{t+1}) \tag{7}$$

During childhood, individuals are reared by their parents and do not make any decisions. If they remain in their home country as adults, they supply one inelastic unit of labor, remunerated at wage w_t, per unit of human capital h_t. Adults transfer a fraction γ of their revenue to their parents and use a share $\sigma n_t N_t^{\delta}$ for raising children. They allocate the rest of their income to consumption c_t, savings s_t and children's education $n_t e_t$. Adults who have migrated transfer the same share γ of their revenue to their parents. However, they can claim a higher wage in their country of residence, which is assumed to be proportional to the domestic wage, such that $w_t^F \equiv \varepsilon w_t$, where $\varepsilon > 1$ denotes the

[8]Note that in the paper of de la Croix and Gosseries (2012), the cost for rearing children σ was defined as a combination of parameters for available land, (T), fertility productivity factor, (λ), and weight of land in the children cost, (δ): $\sigma \equiv \frac{1}{\lambda T^{\delta}}$. Here this expression is simplified, by using directly σ.

[9]Moreover, this allows to have a constant population in the long run.

net gain from migration. This parameter can be interpreted as the increase in income that can be obtained in the receiving country, compared to the income in the domestic area, from which the costs associated with migration are subtracted. In this economy, incoming cash flows from migrants are remittances, while transfers from domestic workers are simply intergenerational transfers. Therefore, migrants are not economically active in the domestic country, except for the remittances sent to their parents. In this work, the focus is on the parents' tradeoff between savings, number of children and children's education, knowing that a fraction ρ of the new generation will leave the country and will transfer a larger cash amount. The budget constraint in the first period is given by the following:

$$c_t + s_t + n_t e_t = w_t h_t (1 - \gamma - \sigma n_t N_t^{\delta}) \tag{8}$$

Human capital per child h_{t+1} depends on education expenditures per child e_t and on the parents' human capital h_t:

$$h_{t+1} = \theta(Z_t) h_t^{1-\mu} e_t^{\mu} \tag{9}$$

where $\theta(Z_t) > 0$ is the efficiency of human capital accumulation and $0 < \mu < 1$ represents the efficiency of education. Note that, here, corner solutions are possible since there are two different forms of investments. However, they are not considered here, because $e_t = 0$ would bring the stock of human capital to 0; this leads to the following condition: $e_t > 0$.

When they are old, agents only consume their savings remunerated at the return factor R_{t+1} and the intergenerational transfers sent by their children, wherever they live. That said, there are two tradeoffs in this model; the first one concerns adult *versus* old age consumption. In addition, they must choose between savings or transfers–through human capital investments and the number of children–to finance their consumption when old. The budget constraint in the second period is written as follows:

$$d_{t+1} = s_t R_{t+1} + n_t \gamma (1 - \rho) w_{t+1} h_{t+1} + n_t \gamma \rho \varepsilon w_{t+1} h_{t+1} \tag{10}$$

Here one can note that $\gamma(1 - \rho + \rho\varepsilon) w_{t+1} h_{t+1}$ is the children's income received by the parents, whether there are in education or fertility. Therefore, the share of the children's income transferred is denoted by $\Lambda_h = \gamma(1 - \rho + \rho\varepsilon)$. It is positively correlated to ε, ρ and γ which are respectively, the net gain from migration, the emigration rate and the intergenerational transfers rate.

The consumer program is summarized by the following:

$$\begin{aligned} \max_{c_t, s_t, e_t, n_t} U(c_t, d_{t+1}) &= \ln(c_t) + \beta \ln(d_{t+1}) \\ s.t \qquad c_t + s_t + n_t e_t &= w_t h_t (1 - \gamma - \sigma n_t N_t^{\delta}) \\ d_{t+1} &= s_t R_{t+1} + n_t \Lambda_h w_{t+1} h_{t+1} \\ h_{t+1} &= \theta(Z_t) h_t^{1-\mu} e_t^{\mu} \end{aligned}$$

To solve this model, first, constraints are substituted in the utility:

$$U(s_t, e_t, n_t) = \ln\left[w_t h_t (1 - \gamma - \sigma n_t N_t^{\delta}) - s_t - e_t n_t\right] + \beta \ln\left[s_t R_{t+1} + n_t \Lambda_h w_{t+1} h_{t+1}\right] \tag{11}$$

The First Order Condition (FOC) of the household's problem with respect to s_t shows the consumption tradeoff over the life-cycle (equation (12)). It depends on the psychological discount factor, β, and the return factor on savings, R_{t+1}. The two others FOC of the household's problem with respect respectively to education and fertility (equations (13) and (14)) suggest that the remuneration from intergenerational transfers and savings should be equal on the equilibrium.

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{d_{t+1}} \tag{12}$$

$$\frac{1}{c_t} = \frac{\beta\mu\Lambda_h w_{t+1}h_{t+1}}{e_t d_{t+1}} \tag{13}$$

$$\frac{1}{c_t} = \frac{1}{\sigma N_t^\delta w_t h_t + e_t}\frac{\beta\Lambda_h w_{t+1}h_{t+1}}{d_{t+1}} \tag{14}$$

By combining (13) and (14), a first no-arbitrage condition appears. It ensures that the household is indifferent between education and fertility by equating the opportunity costs of education and fertility. It directly determines the level of education expenditures:

$$\frac{\mu\Lambda_h w_{t+1}h_{t+1}}{e_t} = \frac{\Lambda_h w_{t+1}h_{t+1}}{\sigma N_t^\delta w_t h_t + e_t} \tag{15}$$

$$e_t^\star = \frac{\mu\sigma N_t^\delta}{1-\mu}w_t h_t \tag{16}$$

Education choice depends solely on the adult income, on the cost of raising children and on the efficiency of education. Moreover, the adult population size has a positive effect on the education expenditures because of the congestion effect that increases the cost of fertility in a densely populated area. Thus, the larger the income and/or the adult population size, the higher the education expenditures are.

Second, combining (12) and (14), another no-arbitrage condition is obtained. This expression equates the returns of savings R_{t+1} and the future intergenerational transfers relative to the children cost. Thus, it ensures that the household is indifferent between investments in children and savings.

$$R_{t+1} = \frac{\Lambda_h w_{t+1}h_{t+1}}{\sigma N_t^\delta w_t h_t + e_t} \tag{17}$$

When the optimal choice of education is introduced into the second no-arbitrage condition a relationship essentially between equilibrium input prices at $t+1$ appears:

$$R_{t+1} = \frac{(1-\mu)\Lambda_h}{\sigma}\frac{w_{t+1}h_{t+1}}{w_t h_t N_t^\delta} \tag{18}$$

Rewriting the adult's budget constraint according to the optimal choice of education gives the investments in children in one term.[10]

$$w_t h_t(1-\gamma) = c_t + s_t + n_t \frac{\sigma w_t h_t N_t^\delta}{1-\mu} \tag{19}$$

The household's savings as defined in a global sense can be denoted as the present income which is not consumed at the first period by x_t. It is given on the left hand side of the following equation:

$$x_t = s_t + n_t \frac{\sigma w_t h_t N_t^\delta}{1-\mu} \tag{20}$$

The next step is to solve the inter-temporal optimization of the household. To do so, the expression (18) and the budget constraints are introduced in the FOC with respect to the savings (equation (12)) to get the following expression:

$$\beta R_{t+1} c_t = d_{t+1} \Leftrightarrow \frac{\beta(1-\gamma)}{1+\beta} w_t h_t = s_t + n_t \frac{\sigma w_t h_t N_t^\delta}{1-\mu} \tag{21}$$

Here, a new expression for $x_t \equiv \frac{\beta(1-\gamma)}{1+\beta} w_t h_t$ appears and leads directly to the consumption expression, given by the following expression:

$$c_t = \frac{1-\gamma}{1+\beta} w_t h_t \tag{22}$$

Equations (21) and (22) depict the tradeoff between adult and old age consumption through investment, *i.e.* between c_t and x_t. Both variables are negatively affected by parameter γ, the intergenerational transfer rate. A high γ generates a negative income effect on adults, thus reducing available resources for consumption, c_t, and investments, x_t. Nevertheless γ has an ambiguous effect on old age consumption, since high values for the parameter mean that the elderly get higher intergenerational transfers. This puts a lower burden on investing to fund future consumption d_{t+1}. Finally, the discount factor, β has a positive (negative) effect on x_t (c_t).

From the two expressions of x_t, it is possible to obtain a first relationship between the savings and the intergenerational transfers:

$$s_t = w_t h_t \left(\frac{\beta(1-\gamma)}{1+\beta} - n_t \frac{\sigma N_t^\delta}{1-\mu} \right) \tag{23}$$

At this point, the trade off between fertility and savings is not solved. To obtain an additional equation between the savings and the fertility, note that the representative household has perfect foresight as to future returns from investment (de la Croix and Michel (2002)). As a result, future remuneration of production inputs can be written

[10] The terms $n_t \frac{\sigma w_t h_t N_t^\delta}{1-\mu}$ is the combination of the two options in order to increase the intergenerational transfers (*i.e.* the fertility and the human capital).

as a function of present tradeoffs. Indeed, the optimal choices of the households solve simultaneously their problem of optimization of the inter-temporal income and the Market Clearing Conditions (MCC), given in the following equations:

$$K_{t+1} = s_t N_t \tag{24}$$

$$L_{t+1} = N_{t+1} = n_t N_t (1-\rho) \tag{25}$$

$$h_{t+1} = \theta(Z_t) e_t^{\mu} h_t^{1-\mu} \tag{26}$$

Combining the inputs prices, w_{t+1} and R_{t+1} (given by equations (3) and (4)) and the MCC in the equality (18) gives the following equation:

$$s_t = n_t \left[\frac{1-\alpha}{\alpha} \frac{(1-\mu)\Lambda_h}{(1-\rho)\sigma w_t h_t N_t^{\delta}} \right]^{-1} \tag{27}$$

Finally, solving the system defined by the equations (23) and (27) leads to the optimal choices of the consumer:

$$s_t^{\star} = \frac{\beta\alpha(1-\rho)(1-\gamma)}{(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} w_t h_t \tag{28}$$

$$n_t^{\star} = \frac{\beta\Lambda_h(1-\gamma)(1-\alpha)(1-\mu)}{\sigma(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} N_t^{-\delta} \tag{29}$$

A comparative static analysis on household choices–given by equations (16), (28) and (29)–is conducted with respect to migration related parameters.[11] On the one hand, there is a tradeoff between savings and investments in children to fund old age consumption–*i.e.* s_t versus $n_t \frac{\sigma w_t h_t N_t^{\delta}}{1-\mu}$. On the other hand, there is a tradeoff between quantity and quality of children to maximize intergenerational transfers–*i.e.,* n_t and e_t. First, according to equation (28), savings are negatively correlated to Λ_h, the share of children income received by the parents. Thus, increases in the net gain from migration, ε, the emigration rate ρ and/or the intergenerational transfer, γ, lead to increases in children investments at the expense of the savings. For γ this is aggravated by the negative income effect that comes from the term $(1-\gamma)$.

Second, the tradeoff between fertility and education depends on the one hand on the incentives to invest in education, and on the other hand, on migration. Indeed, as said earlier household chooses to invest in education expenditures if the efficiency of education and the child-rearing costs–measured, respectively, by parameters μ and σN_t^{δ}–are higher. On the other hand, migration has a strong impact on fertility and not directly on education. More specifically, ρ and ε are positively correlated to n_t, while the impact of γ depends on the interaction between the share of the children income received Λ_h and the negative income effect induced by $(1-\gamma)$. This interaction corresponds to the balance between two effects which are captured by the condition below:

$$\frac{\partial n_t}{\partial \gamma} > 0 \Leftrightarrow \frac{\gamma}{1-\gamma} < \frac{\alpha(1-\rho)}{\Lambda_h(1-\alpha)}$$

[11] Derivatives expressions are given in the Appendix A.

Third, the number of children per household depends also on the overcrowding effect. In a more densely populated area, fertility is decreased for the benefit of education expenditures. Therefore, migration can have an impact on education through its impact of fertility. To clarify this mechanism it is necessary to study first the population dynamics and the labor force market.

Finally, as households do not take into account the environmental externality their optimal choices cannot reflect the dynamic of human capital. Therefore, agents will invest in education, fertility or capital in the same way, than in a situation without externality.[12].

4. Equilibrium

To ease the discussion of the different effects induced by migration on the one hand and the environmental damages on the other, the equilibrium is computed in two steps: i) analysis of the intertemporal equilibrium in the absence of the externality; ii) analysis the intertemporal equilibrium with the environmental issues.

4.1. Case 1: The intertemporal equilibrium in the absence of the externality

In the absence of an environmental externality $\bar{\theta}$ denotes the efficiency of human capital accumulation, the MCC for human capital is given by:

$$\bar{\theta} h_t^{1-\mu} e_t^{\mu} \tag{30}$$

The MCC for capital and labor, are given respectively by the equations (24) and (25). The values of the household's optimal choices $s_t^\star$, $n_t^\star$ and $e_t^\star$ are given in equations (28), (29) and (16). The wage and the return factor on capital correspond, respectively, to (3) and (4). Using, all the previous findings the intertemporal equilibrium can be deduced.

Proposition 1. *Given the initial conditions $K_0 > 0$, $N_0 > 0$ and $h_0 > 0$, the intertemporal equilibrium is the sequence (K_t, N_t and h_t) that satisfies the following system $t \geq 0$:*

$$\begin{cases} K_{t+1} &= \Psi \alpha A (1-\rho) K_t^{\alpha} N_t^{1-\alpha} h_t^{1-\alpha} \\ N_{t+1} &= \Psi \Lambda_h (1-\rho) \dfrac{(1-\mu)}{\sigma} N_t^{1-\delta} \\ h_{t+1} &= \bar{\theta} \left[\dfrac{\sigma \mu A (1-\alpha)}{1-\mu} \right]^{\mu} K_t^{\alpha\mu} N_t^{\mu(\delta-\alpha)} h_t^{1-\alpha\mu} \end{cases} \tag{31}$$

where $\Psi = \frac{\beta(1-\alpha)(1-\gamma)}{(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}$.

[12] If the environment is introduced in the utility with private environmental maintenance (as in Mariani et al. (2010) for instance), the household's choices can change, even if parents are not aware of the environmental externality on their children's ability.

Therefore, the ratio of the capital to efficient units of labor k_t can be defined as follows:

$$k_{t+1} \equiv \frac{K_{t+1}}{N_{t+1}h_{t+1}} = \left(\frac{A\sigma}{1-\mu}\right)^{1-\mu} \frac{\alpha}{\bar{\theta}\Lambda_h[\mu(1-\alpha)]^{\mu}} k_t^{\alpha(1-\mu)} N_t^{\delta(1-\mu)} \tag{32}$$

The growth factors of the stocks of physical capital, the human capital per capita and the adult population are respectively denoted by g_t^K, g_t^h and g_t^N.

$$g_t^K = \frac{K_{t+1}}{K_t} = \Psi\alpha A(1-\rho)k_t^{\alpha-1} \tag{33}$$

$$g_t^h = \frac{h_{t+1}}{h_t} = \bar{\theta}\left[\frac{\mu A\sigma(1-\alpha)}{1-\mu}\right]^{\mu} k_t^{\alpha\mu} N_t^{\mu\delta} \tag{34}$$

$$g_t^N = \frac{N_{t+1}}{N_t} = \Psi\Lambda_h(1-\rho)\frac{(1-\mu)}{\sigma}N_t^{-\delta} \tag{35}$$

First of all, it appears that the evolution of the labor force, given by equation (35), depends solely on the structural parameters of the economy. Consequently, the population dynamics has an impact on the human capital as well as on the physical capital. However, the evolution of the production or the income does not change the fertility choices. Thus the steady state value is directly given by the following equation:[13]

$$N^{\star} = \left[\Psi(1-\rho)\Lambda_h\frac{(1-\mu)}{\sigma}\right]^{\frac{1}{\delta}} \tag{36}$$

The steady state size of the labor force is increased by all the parameters that are positively correlated to n_t, except for ρ, whose effect depends on the following condition.[14]

$$\frac{\partial N^{\star}}{\partial \rho} > 0 \Leftrightarrow \frac{1-\rho}{1-\rho+\rho\varepsilon} > \sqrt{\frac{\gamma(1-\alpha)}{\alpha(\varepsilon-1)}} \tag{37}$$

Due to the constant returns to scale for human and physical capital, there is no steady state in this economy but a balanced growth path (BGP).

Proposition 2. *On the BGP, the system satisfies the **Proposition 1** and the stock of physical and efficient units of labor grows at the same constant rate $g_{BGP} = g^K = g^h$; therefore, the long term ratio of capital per units of efficient labor is constant: $k_t \equiv \frac{K_t}{L_t h_t} = k_{BGP}$. There is a unique locally stable equilibrium for which the values of k and g are as follows:*

$$k_{BGP} = \left[\frac{\alpha[\Psi A(1-\rho)]^{1-\mu}}{\bar{\theta}[\mu\Lambda_h(1-\alpha)]^{\mu}}\right]^{\frac{1}{1-\alpha(1-\mu)}} \tag{38}$$

$$g_{BGP} = \left[[\bar{\theta}[\mu\Lambda_h(1-\alpha)]^{\mu}]^{1-\alpha}[\Psi\alpha^{\alpha}A(1-\rho)]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}} \tag{39}$$

[13]The growth rate of the population is directly given by $n_t^*(1-\rho)$, and on the steady state $n^* = 1/(1-\rho)$

[14]It is possible to have a decrease of the adult generation size until $N^{\star}$ is reached. This is the case, if the initial population size is larger than $N^{\star}$.

Proof of Proposition 2. see Appendix B.1 □

Proposition 3. *On the BGP, there is a negative correlation between k_{BGP} and the efficiency of human capital accumulation, $\bar{\theta}$, as well as the share of the children's income transfers to the parents, Λ_h–knowing that Λ_h is positively correlated to the emigration rate, ρ, the intergenerational transfers, γ and the net gain from migration, ε. The technology factor, A, and the cost of raising children, σ, have a positive effect on k_{BGP}.*

The positive effects of A and σ on long term ratio of capital per units of efficient labor, k_{BGP}, result respectively from the increase in the production and from the decrease in the number of children due to the extra cost–*i.e.* the decrease in the next generation size. The negative impact of the other parameters on this ratio is accounted for by the increase in the number of units of efficient labor in the economy–with respect to ε, $\bar{\theta}$, γ and ρ.

Proposition 4. *On the BGP, the economic growth, g_{BGP}, is positively impacted by: the technology factor, A, the psychological discount factor, β, the efficiency of human capital accumulation, $\bar{\theta}$ and the net gain from migration, ε. The effects of the intergenerational transfer rate, γ, and the emigration rate, ρ, depend on the condition below:*

$$\frac{\partial g_{BGP}}{\partial \rho} > 0 \quad \Leftrightarrow \quad \frac{1-\rho}{1-\rho+\rho\varepsilon} > \frac{[\varepsilon-(1-\alpha)(\varepsilon-1)(1-\rho)]}{\alpha(\varepsilon-1)(1-\rho)} \tag{40}$$

$$\frac{\partial g_{BGP}}{\partial \gamma} > 0 \quad \Leftrightarrow \quad \frac{1-\alpha}{\alpha}\frac{1-(1-\alpha)(1-\gamma)}{(1-\alpha)(1-\rho)-\gamma} > \frac{(1-\rho)}{\Lambda_h} \tag{41}$$

Firstly, it is important to note that the long-term growth factor gives directly the growth of the production per worker, and thus of the production per capita, because of the constant population size. In that case, the growth factor of the production per capita can be directly translated in the growth of utility, which depends strongly on the consumption (*cf.* Appendix B.3 for details).

For the growth rate of the economy, g_{BGP}, a rise in the technological factor, A, and in the efficiency of human capital accumulation, $\bar{\theta}$, lead to a more efficient economy. While, increases in the psychological discount factor, β, result in higher investments for the future through human capital or savings and subsequently to an increase in the economic growth. Moreover, an increase in the net gain from migration, ε enhances the income of the old age generation and the production per capita growth.

However the effects of the other features of migration–*i.e.* the emigration rate, ρ, and the intergenerational transfer rate, γ–are not clear. Intricate conditions are obtained for the sign of the derivatives of the growth factor on the BGP with respect to the emigration rate and the intergenerational transfer.[15] Difficulties to interpret the analytical results on ρ and γ, could be explained by the opposite effects that are observed on household choices and on aggregate variables.

[15]These conditions are given in Appendix B.2.

On the one hand, there is the effect of ρ, the emigration rate. This parameter creates an incentive to have more children through the increase in the net gain from migration. However, because there are more adults who leave the territory at the next period it can lead to a decrease in the number of units of efficient labor. Second, with the increase in population size, the education expenditures can increase because of the congestion effect, this is reinforced by the substitution effect between investments in children and savings that occurs when the emigration rate increases. In that context, a higher migration can lead to an increase of the human capital and thus to an increase of the income which is directly given by $w_t h_t$. Therefore, the emigration rate effect on the capital stock is three-fold. First, by increasing the number of children, there is a rise in the rearing expenditures and thus a decrease in the savings, this is the substitution effect, on the intensive-margin. Second, there is the role of the extensive-margin in capital stock. Migration can lead to a decrease (an increase) of the adult population size, which induces a reduction (a rise) of the capital stock because of the smaller (larger) number of contributors. Finally, there is the migration effect on the education expenditures and thus on the human capital dynamics. In a wealthier economy, even if the share of the income devoted to savings is reduced, the capital stock might be larger.

On the other hand, there is the effect of γ. As said earlier, γ reduces fertility because of a negative income effect, but only when it is very high. For low values, γ should lead to an increase in investment in children. Therefore, while the mechanisms are different, its impacts through the net gains from migration are the same than those of ρ. First it leads to a rise in the units of efficient labor, but if γ is too high, savings are low, and the population size may decline as well as the human capital.

In conclusion, two main intuitions can be driven from this first case. The first one is that there is a strong tradeoff between intergenerational transfers and savings, because migration enhances the net gain from the children's transfers. In that context, positive impacts from migration on the capital stock and the economy are possible, but only if there are a gain in human capital and in the labor force. Consequently, it is possible to have an emigration rate which has a negative impact on the economic growth of these countries because of the combined effects on capital stock and on units of efficient labor stock. However due to the complexity of the conditions obtained for the emigration rate and the intergenerational transfers, it is difficult to give clear insights of the migration effects. This could be clarified by a numerical analysis on two Caribbean islands: Barbados and Jamaica.

Note that a similar work has been conducted for a simpler model by Ait Benhamou and Cassin, 2018 for five Caribbean islands. They have then calibrated most of the parameters of the model. The objective here, is not to demonstrate the same conclusions but to give a first insight on the effect of the environmental issues on the migration effects. The table below gives the parameters retained in this first numerical analysis, and the method is detailed in Appendix C.

Finally, Figures 5 and 6 show economic growth according to the emigration rate and

Table 1: Calibrated values for structural parameters - SIDS Caribbean countries.

Parameters		**Barbados**	**Jamaica**
Preference factor for the future	β	0.940	0.944
Capital intensity in production	α	0.340	0.312
Technology level	A	1.034	1.014
Education efficiency	μ	0.130	0.162
Efficiency of human capital accumulation	θ	5.025	4.898
Cost of rearing a child	σ	0.171	0.063
Emigration rate	ρ	0.370	0.490
net gain from migration	ε	1.91	6.580
Share of income remitted	γ	0.121	0.200
Congestion parameter	δ	0.636	0.623
Capital stock	K_0	0.021	0.335
Human capital stock	h_0	1.367	1.083
Labor	N_0	0.006	0.051

Note: Calibrated values for individual countries use available data points for the period 1961-2014. Initial values for capital stock and labor are given with a factor of 10^6

the share of income transferred to the parents.[16] First, the relationship between economic growth and migration or remittances is described by inverted U-shaped curves. This result is in line with the literature, that defines that migration should not be tool large in order to prevent the depletion of productive capital stocks. Second, Barbados and Jamaica have been selected in this analysis because they present very different features. Indeed, Jamaica relies heavily on migration, this lead to a strong substitution effect which makes the optimal level of γ and ρ lower than in Barbados. The latter is not strongly impacted by migration–the curve on the left is relatively flat–because the net gain from migration is small.

Figure 5: Barbados

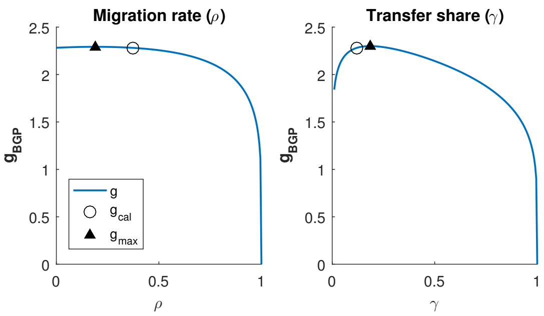
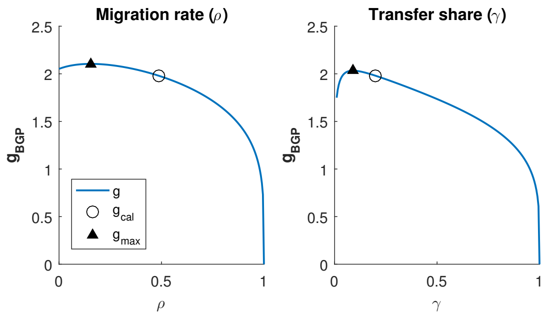


Figure 6: Jamaica



Consequently, what should be retained from this analysis is that it is possible to define

[16] The other parameters are not studied, because they are not strongly related to migration features.

an optimal level of migration and that whatever the characteristics of the country studied, it should not be too high. This optimal level of migration results in particular from the accumulation of stocks of human and physical capital. Migration should be at a level that does not lead to a depletion of investment in the country of origin.

4.2. Case 2: Equilibrium with the environmental externality

Two cases arise from the analysis of the environmental issues. That is if the pollution emissions can be completely abated and thus the pollution emissions maintained to 0 on the long run. This is possible only if the efficiency of the abatement effort is high enough and if the pollution intensity of the production is small enough, in order to have $\xi\tau = \Omega$. If it is not possible to completely overcome the pollution emissions, a steady-state equilibrium will be reached only if all stocks are constant over time.

In the presence of pollution emissions the intertemporal equilibrium becomes:

Proposition 5. *Given the initial conditions $K_0 > 0$, $N_0 > 0$, $h_0 > 0$, $Z \geq 0$ the intertemporal equilibrium is the sequence (K_t, N_t and h_t) that satisfies the following system $t \geq 0$:*

$$\begin{cases} K_{t+1} = \Psi\alpha A(1-\tau)(1-\rho)K_t^{\alpha}N_t^{1-\alpha}h_t^{1-\alpha} \\ N_{t+1} = \Psi\Lambda_h(1-\rho)\dfrac{(1-\mu)}{\sigma}N_t^{1-\delta} \\ h_{t+1} = \theta(Z_t)\left[\dfrac{\sigma\mu A(1-\alpha)}{1-\mu}\right]^{\mu} K_t^{\alpha\mu}N_t^{\mu(\delta-\alpha)}h_t^{1-\alpha\mu} \\ Z_{t+1} = (\Omega-\xi\tau)AK_t^{\alpha}(N_t h_t)^{1-\alpha} + (1-\alpha)Z_t \end{cases} \tag{42}$$

4.2.1. Case 2.1: Total abatement of emissions

If pollution is completely abated–*i.e.* $\Omega = \xi\tau$–the long term pollution stock is null. Thus the equilibrium obtained is the same that in the case without environmental issues, except for the tax rate, τ, that appears in K_{t+1} and h_{t+1}.

Proposition 6. *On the BGP, the system satisfies the* ***Proposition 5****. The economic growth factor $\hat{g}_{BGP}$ and the long term ratio of capital per units of efficient labor $\hat{k}_{BGP}$ are as follows:*

$$k_{BGP} = \left[\frac{\alpha[\Psi A(1-\rho)(1-\tau)]^{1-\mu}}{\bar{\theta}[\mu\Lambda_h(1-\alpha)]^{\mu}}\right]^{\frac{1}{1-\alpha(1-\mu)}} \tag{43}$$

$$g_{BGP} = \left[[\bar{\theta}[\mu\Lambda_h(1-\alpha)]^{\mu}]^{1-\alpha}[\Psi\alpha^{\alpha}A(1-\rho)(1-\tau)]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}} \tag{44}$$

The economic analysis of this equilibrium will be exactly the same than in the previous case. While the effect of pollution is completely canceled out, the economic cost of this operation leads to a reduction in growth and in the capital stock per efficient unit of labor.

4.2.2. Case 2.2: Partial abatement of pollution

The next step of the analysis is to compute the steady-state of the economy if pollution is not completely abated, thus if $Z^\star > 0$.

Proposition 7. *A Steady-State (SS) is an equilibrium satisfying* ***Proposition 1*** *and where N_t, h_t, K_t and Z_t are constant. There is an equilibrium, for which the values of $N^\star$, $K^\star$, $h^\star$ and $Z^\star$ are:*

$$N^\star = \left[\frac{(1-\mu)(1-\rho)\Lambda_h}{\sigma}\Psi\right]^{\frac{1}{\delta}} \tag{45}$$

$$K^\star = \alpha(1-\alpha)(1-\tau)\Psi(1-\rho)\frac{a\theta^{-1}(\chi)}{\Omega-\xi\tau} \tag{46}$$

$$h^\star = \frac{a\theta^{-1}(\chi)}{(\Omega-\xi\tau)A}\left[\alpha A(1-\tau)\right]^{-\frac{\alpha}{1-\alpha}}\left[\frac{\sigma}{\Lambda_h(1-\mu)}\right]^{\frac{1}{\delta}}(\Psi(1-\rho))^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}} \tag{47}$$

$$Z^\star = \theta^{-1}(\chi) \tag{48}$$

where $\chi = [\mu\Lambda_h(1-\alpha)]^{-\mu}[\Psi A\alpha^\alpha(1-\tau)(1-\rho)]^{-\frac{\mu}{1-\alpha}}$ is the efficiency of human capital accumulation in the steady state and $\theta^{-1}(.)$ is the inverse function of $\theta(Z_t)$.[17]

The model being a four-dimensional problem, it is quite difficult to study the stability of the equilibrium.[18] Therefore, here, insights of the mechanisms that might have an impact on the dynamics of the model are described precisely. There are two main determinants for the stability: the population dynamics and the pollution dynamics. Indeed, the stability analysis of the first case (cf. Appendix B.1) has shown that if the population size converges to its steady state value, both human capital and physical capital will grow at the same rate because of the accumulation of human capital. This has led to a balanced growth path in the case without pollution. Here, there is a steady state because of the pollution stock, which prevents human capital from increasing on the long run. In this chapter, with a growing human capital, production increases as well as the pollution stock. This generates an increase in the externality effect that reduces human capital at the next period, and thus the production too.

According to the population dynamics (given by equation (45)), population changes are totally independent of the other variables. Therefore, in this analysis the focus is on the determinants of the stability of the pollution stock. On the one hand, there is the damage function, $\theta(Z_t)$ that links the pollution dynamics and its economic effects. On the other hand, the focus is on the natural absorption rate, a, and the pollution intensity, Ω. In fact, robustness tests on the parameters have shown that these are the sole features that might have an impact on the stability. The parameters linked to the migration–such

[17] The stability of the equilibrium could not be proven analytically, however we conduct an analysis of the determinants of the stability in the next section.

[18] The population dynamics being independent of the other variables, the system can be reduced to a three-dimensional problem. However even in that case stability is not easy to prove analytically.

as ρ, ε or γ–change the level attained by the population size, the human capital and the capital stock, but they do not impact the stability.

To conduct the numerical analysis, parameters for Barbados (see Table 1) are used.[19] The pollution stock is set to 0 at the initial period. Besides, two functions for $\theta(Z_t)$ that respect the conditions given in the model (see Section 3) are tested. This means that those functions are defined for positive or null values of Z_t and their first derivatives with respect to Z_t are negative. The two functions tested are the following:

$$\theta_1(Z_t) = \frac{\bar{\theta}}{1+Z_t} \tag{49}$$

$$\theta_2(Z_t) = \frac{\bar{\theta}}{1+Z_t^2} \tag{50}$$

In Figure 7, dynamics of production (Y_t), capital stock (K_t), pollution stock (Z_t) and human capital (h_t) are displayed. The plain line represents the benchmark economy with $\theta_1(Z_t)$ while the dashed line is related to the situation with $\theta_2(Z_t)$. The first specification allows to have a stable equilibrium with damped-oscillations, while the second shows an unstable dynamics with regular oscillations around the steady state. In the figure 8, the second derivatives of the functions $\theta_1(Z_t)$ and $\theta_2(Z_t)$ are represented in order to define whether these functions are convex or concave. The signs of these second derivatives are not always the same in the interval represented here. It appears that the second function is concave for some values of Z_t, this leads to an unstable equilibrium when Z_t is in this interval. Therefore, in the rest of this analysis, the function $\theta(Z_t)$ is used. A sufficient assumption in the model to insure the stability of the equilibrium should be that: $\theta'(Z_t) < 0$ and $\theta''(Z_t) \geq 0$.[20]

Second, Ω and a effects are tested in the absence of any environmental policy. In Figures 9 to 11, production (Y_t), physical capital (K_t), pollution (Z_t) and human capital (h_t) of the benchmark economy, are represented according to different values of absorption rate–*i.e.* $a = \{0.2, 0.5, 0.8\}$–and of pollution intensity–*i.e.* $\Omega = \{0.2, 0.5, 0.8\}$.

To explain, these dynamics, recall that in the absence of the pollution externality, human capital increases monotonically on the steady state. This has a positive effect on production and on capital stock. In presence of pollution emissions, human capital cannot increase without leading to an increase in pollution. When the pollution is high, this leads to an abrupt decrease in human capital and in capital stock (because the of the income loss). With the resulting decrease in human capital, the production also is lessened and thus the pollution stock. At that moment another cycle begins, with human capital accumulation, growing production and physical capital. However, increases–for all variables–are slower because future human capital depends on past values of human capital. Consequently, it is possible to reach a steady state with damped oscillations. This

[19] As a robustness test, results for Jamaica are displayed in Appendix D.2.

[20] Depending on the function retained, the initial pollution stock might affect the stability of the equilibrium, however, whatever the value tested with the first function, the equilibrium was stable.

Figure 7: The effect of $\theta(Z_t)$ on the steady state stability

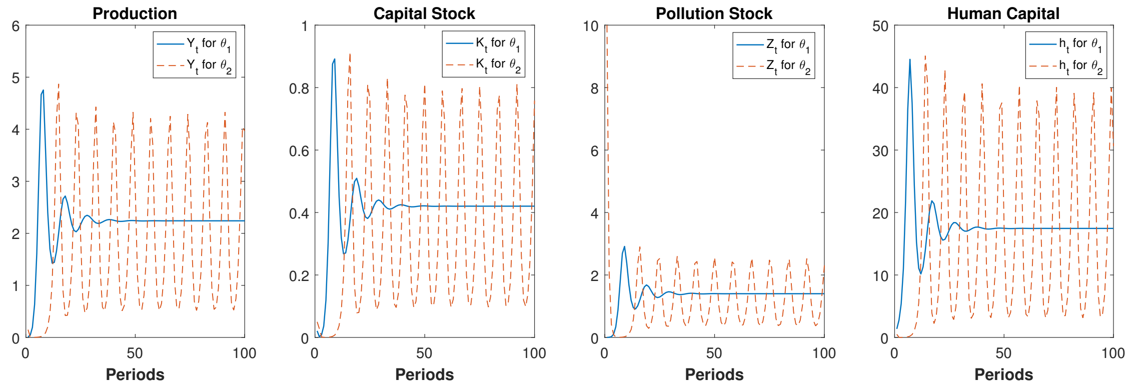
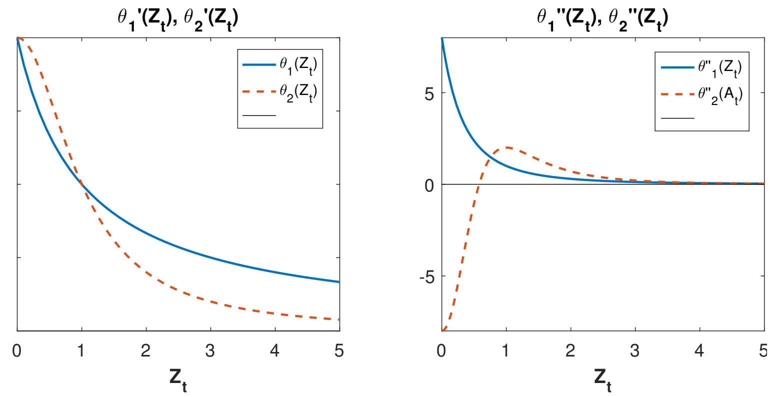


Figure 8: The functions $\theta_1(Zt)$ and $\theta_2(Z_t)$ and their second derivatives



cyclical convergence has also been found by Varvarigos (2013) with similar mechanisms.

Before the steady state is reached, a high pollution intensity, Ω, or a low natural absorption of pollution, a, accelerates the constitution of the pollution stock. This, results in a larger cycles' amplitude. On the steady state, the increase in human capital that would normally occur, is exactly compensated by the reduction in abilities due to pollution. The level where there is this equilibrium depends on χ.

What is surprising, is that the steady state pollution stock is the same whatever are the pollution intensity or the natural absorption of pollution by the ecosystems. This is due to the fact that the equilibrium is reached when the marginal increase in human capital is exactly compensated by the marginal loss in reduction in cognitive skills. This equalities does not depend on pollution intensity, but on parameters that changes the rate of human capital dynamics. Consequently, long-term pollution only depends on the level that will stop the accumulation of the human capital and thus on the economic tradeoffs, which appears in χ. When those tradeoffs are solved, all the economic variables are constant and thus the pollution linked to this level is also constant. Here, only the time necessary to stop the fluctuations, as well as their extent are impacted by those parameters, but not the steady-state value.

Therefore, effects of environmental features depend on the time scale considered. At short term, the pollution intensity has a strong impact on the pollution stock. This has an impact on the human capital obtained in the short-term, the income and thus on the amount saved. If a is small or Ω is high, the pollution stock is higher in the first periods. The level reached by the economic results is hampered by a higher pollution intensity or a smaller absorption rate, because, human capital accumulation depends on past values. If the earlier level of human capital are low, the later level of human capital are lower too. This leads to a decrease in the steady state values of production, $Y^\star$ and physical capital, $K^\star$. However, there will be convergence of the pollution stock value to the same steady state value–determined with χ–whatever the values of the environmental features.

Figure 9: The effect of Ω on the convergence, for $a = 0.2$

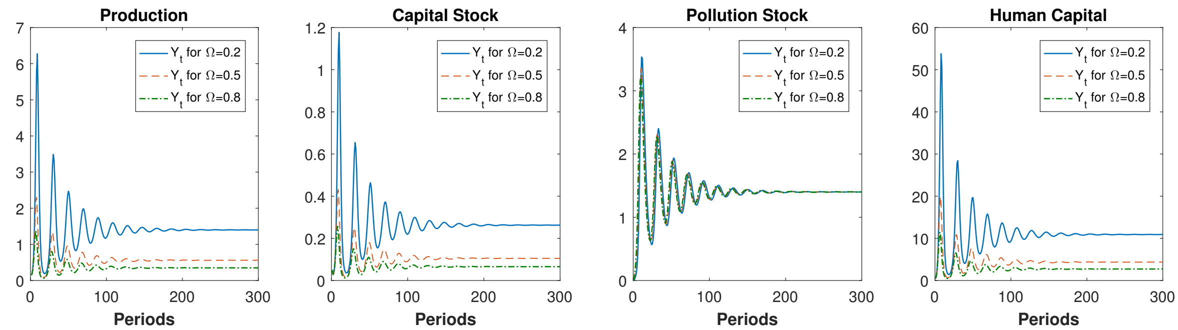


Figure 10: The effect of Ω on the convergence, for $a = 0.5$

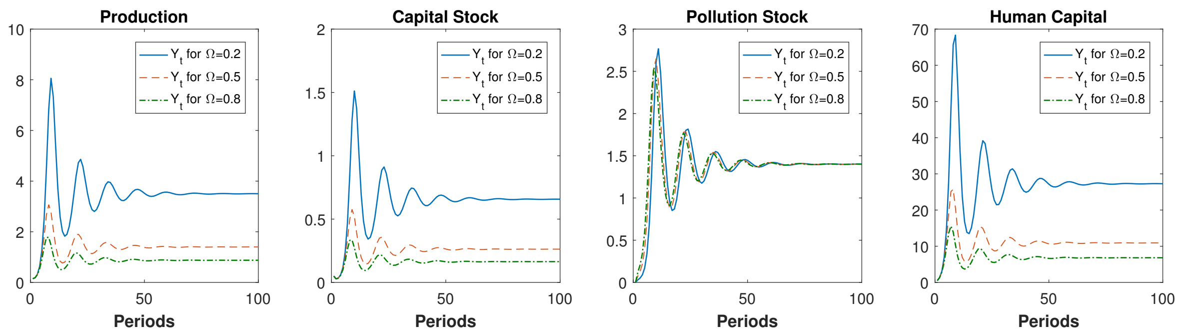
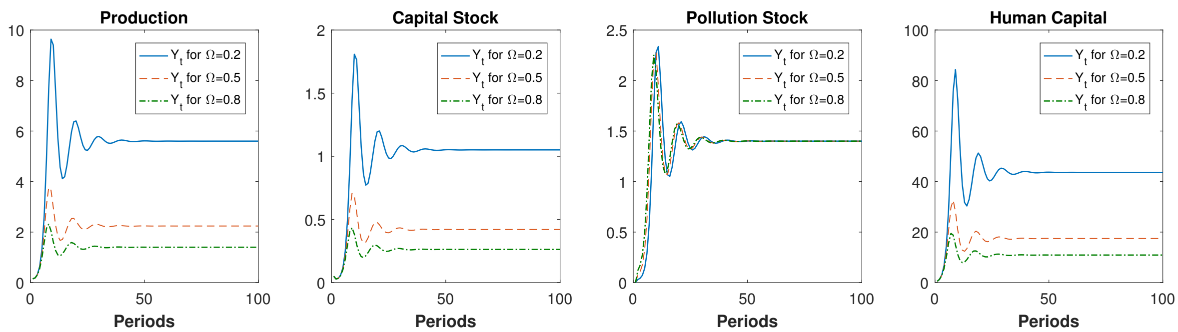


Figure 11: The effect of Ω on the convergence, for $a = 0.8$



Next, an analysis of the effects of the various parameters on the steady state values $N^\star$, $h^\star$, $K^\star$ and $Z^\star$ is conducted compared to the case without pollution. Note that in the steady state, the dynamics of the population size is considered to be equivalent to the adult generation size.[21] Therefore in the rest of this work, population size and adult generation size present exactly the same features and both terms can be used indifferently to describe the population.

5. Discussion on the migration and the environment

In this section, the focus is on the environmental features and on the parameters that control the migration impact. Static comparatives and numerical simulations are used in this discussion which is conducted in two steps: i) the environmental features, ii) the impact of the migration features.[22] Most of the discussions are derived from the numerical simulations because of the complexity of the expressions obtained from the analytical exercise. However, the conclusions from the simulations are very consistent to changes in the parameters. The numerical analysis is conducted for Barbados and Jamaica, with the parameters displayed in Table 1. The idea here was to illustrate that whatever the countries considered, qualitative results remain the same for the migration effects, especially if per capita variables are studied.

The aim of this work is not to scrutinize the values of the parameters, but rather to exhibit the impacts of their variations on the economies described by this model. Consequently, the variations induced by changes in the parameters will be described but not the levels reached by the different aggregates in the economies. As said earlier, computations use the following function $\theta_1(Z_t)$ of the previous section:[23]

$$\theta(Z_t) = \bar{\theta}\left[\frac{1}{1+Z_t}\right]$$

5.1. The environment and the public policy

Proposition 8. *On the steady state, the pollution intensity, Ω, has a negative effect on the stock of physical capital and the human capital, while it has no effect on the population size and the pollution stock. The productive capital stocks are positively correlated to the absorption rate, a. Finally, the tax reduces the pollution stock, while it has a positive effect on $K^\star$ and $h^\star$ under the following conditions:*

$$\frac{\partial K^\star}{\partial \tau} > 0 \Leftrightarrow \zeta_\tau < \frac{\tau}{1-\tau}\left[\frac{\xi-\Omega}{\Omega-\xi\tau}\right]$$

$$\frac{\partial h^\star}{\partial \tau} > 0 \Leftrightarrow \zeta_\tau < \frac{\tau}{1-\tau}\left[\frac{\xi(1-\tau)-\alpha(\xi-\Omega)}{(\Omega-\xi\tau)(1-\alpha)}\right]$$

where $\zeta_\tau = \frac{\partial\theta^{-1}(\chi)}{\partial\tau}\frac{\tau}{\theta^{-1}(\chi)}$ is the elasticity of steady state pollution with respect to the tax rate, τ

[21] The size of the adult and old age generation is the same, and before the migration there are $n^\star(1-\rho)$ children. In any case the total population size is directly proportional to the adult generation size, because the fertility is constant on the BGP: $n^\star = \frac{1}{1-\rho}$

[22] The detailed computations for the static comparatives are presented in the Appendix D.3

[23] For simplicity the subscript is removed.

First of all, in the present work the households' choices are totally independent of the level of pollution or the environmental tax. Here, only the human and physical capital stocks are negatively impacted by the pollution stock–*i.e.* $\theta^{-1}(\chi)$–, while they increase with the value of χ.

As said earlier, the pollution stock in the steady state is directly given by the value of χ, and not by the environmental features of the pollution dynamics. However, through the study of the dynamics, we know that the pollution intensity of production impacts strongly the transitional dynamics of the pollution stock and thus the accumulation of the human and physical capital stocks in the first periods. If it is high, the efficiency of human capital accumulation is lessened in the first stages of the development. Thus human capital level that can be attained in the steady state is also lower, even if the value of $\theta(Z^{\star})$ is the same for all values of Ω. Similarly, the capital stock will be lessened even if the share of savings is independent of the pollution stock. This is due to the loss of income that occurs with the decline in the human capital when Ω is high. In that context, a change in the tax rate, τ, affects the steady state pollution through the long-run level of human capital accumulation, χ. This could entail an improvement in both human and physical capital, depending on conditions linked to the pollution intensity. To demonstrate this effect, two numerical illustrations are proposed.

In the first one, the pollution intensity, Ω, is set to 0.4, the absorption rate and the efficiency of the abatement effort are both set at 0.5. In that case, total abatement of the pollution emissions is possible. Figure 12 displays steady state values for production, production per worker, human capital, capital stock, population size, pollution stock, total utility of the local population and utility per capita of the resident according to the tax level, with $\tau \in (0, 1)$. Note that production per worker, will display exactly the same features than production per active individuals or production per capita, because those three populations definitions are proportional. Second, utility per capita is defined as the utility that can be obtained through adult and old consumptions. These definition are kept for all the numerical simulations presented in the rest of the papers. Moreover, values for Barbados and Jamaica are represented–on the same figure when it is possible–respectively by the plain line and the dotted line.

In this case, the highest levels of per capital utility or total utility (defined as the sum of the utility of adults) are obtained if $\tau\xi = \Omega$. Moreover, these figures depict asymptotes when $\xi\tau$ approaches Ω, this can be accounted for by the fact that the economy can reach a BGP in that case.

The second simulation depicts a context of high emissions with $\Omega = 0.6$ while the values of a and ξ are maintained at 0.5. Here again, production, population, pollution, total utility, production per capita, human capital, capital and utility per capita are displayed for Barbados–plan line–and Jamaica.

In that case, while it is possible to observe an increase in production, production per capita and human capital the tool fail to improve the utility per capita. One explanation is that the effects of the environmental policy on the environmental externality are too small

Figure 12: The effect of the environmental tax rate: τ for $\Omega = 0.4, \xi = 0.5, a = 0.5$

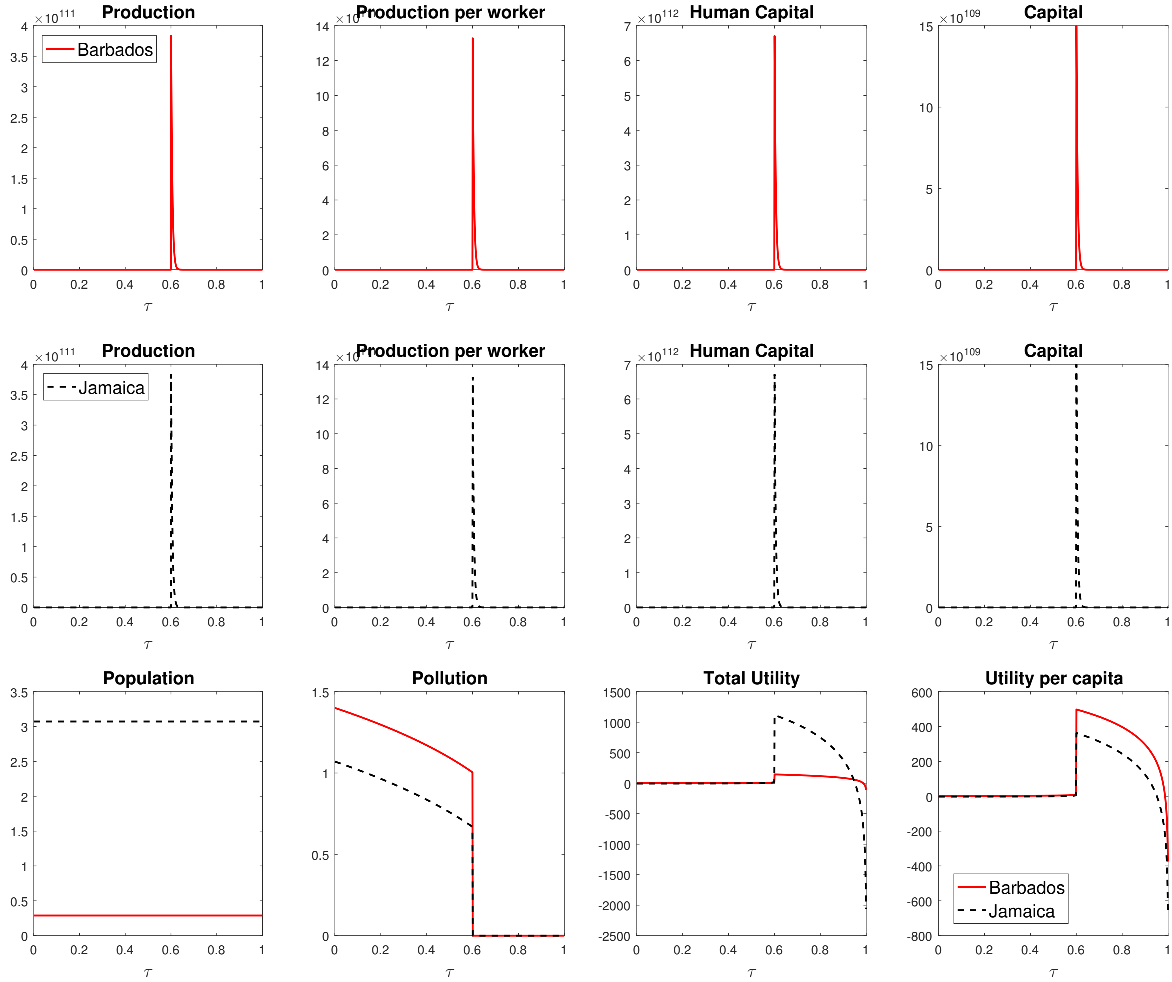
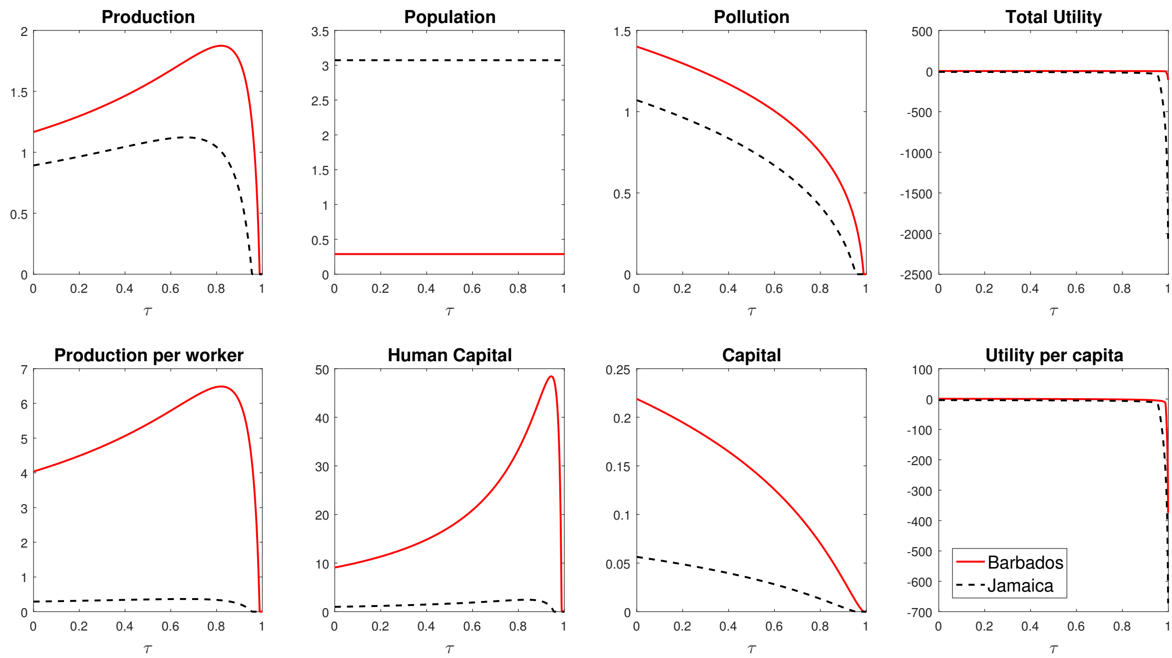


Figure 13: The effect of the environmental tax rate: τ for $\Omega = 0.6, \xi = 0.5, a = 0.5$



to overcome the loss due to the decreasing capital stock. Therefore, while the economic results are improved in terms of production, the important tax prevents an increase in utility. Note that, the criteria to define if emissions are high or not depend solely on the absorption rate and the efficiency of the maintenance effort. If those parameters are small, even a low level of emissions will be high and generates the same results.

Finally, note that these effects are very consistent to different values of parameters for migration–*i.e.* γ, ρ, ε–as well as for different values of environmental features or parameters that control the overcrowding effect. Indeed, ξ, a and δ introduce a scale effect but they do not change the shapes of the curves for the steady state values.

5.2. The impact of migration

The next step of the analysis is to study the impact of migration. Indeed, a situation with a non-optimal environmental policy can arise if the environmental degradations are large or if the cleaning expenses are inefficient. In the present section, the focus is thus on the interplay between the demographic characteristics of the Caribbean SIDS and the environmental degradation, in a context of high environmental vulnerabilities. As a reminder, in these small island states, whether due to natural or institutional characteristics, their vulnerabilities make them perfect examples where such situations could emerge. Therefore, here, the aim is to study the conditions under which a potential *brain gain* appears. To do so, the focus will be put on two parameters, the emigration rate, ρ, and the intergenerational transfer, γ.[24]

In the numerical simulations, the intensity of the emissions of pollution is as follows $\theta = 0.6$. The other environment related parameters–the absorption rate, a, the efficiency of the maintenance effort, ξ, and the tax rate, τ–are set to 0.5.

The effect of the emigration rate: ρ

Proposition 9. *The steady state values of the population size, the stock of pollution, the physical capital and the human capital are positively correlated to the emigration rate, ρ, under the following conditions:*

$$\frac{\partial N^\star}{\partial \rho} > 0 \Leftrightarrow \frac{1-\rho}{1-\rho+\rho\varepsilon} > \left[\frac{\gamma(1-\alpha)}{\alpha(\varepsilon-1)}\right]^{1/2}$$

$$\frac{\partial Z^\star}{\partial \rho} > 0 \Leftrightarrow \frac{(1-\rho)(\varepsilon-1)}{\Lambda_h} > \frac{1}{[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]}$$

$$\frac{\partial K^\star}{\partial \rho} > 0 \Leftrightarrow \zeta_\rho > \frac{\rho}{1-\rho}\left[\frac{\gamma\varepsilon(1-\alpha)}{\alpha(1-\rho)+\Lambda_h(1-\alpha)}\right]$$

$$\frac{\partial h^\star}{\partial \rho} > 0 \Leftrightarrow \zeta_\rho > \left[\frac{\rho(\varepsilon-1)}{\delta(1-\rho+\rho\varepsilon)} - \frac{\rho\gamma\varepsilon(1-\alpha(1-\delta))}{\delta(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]$$

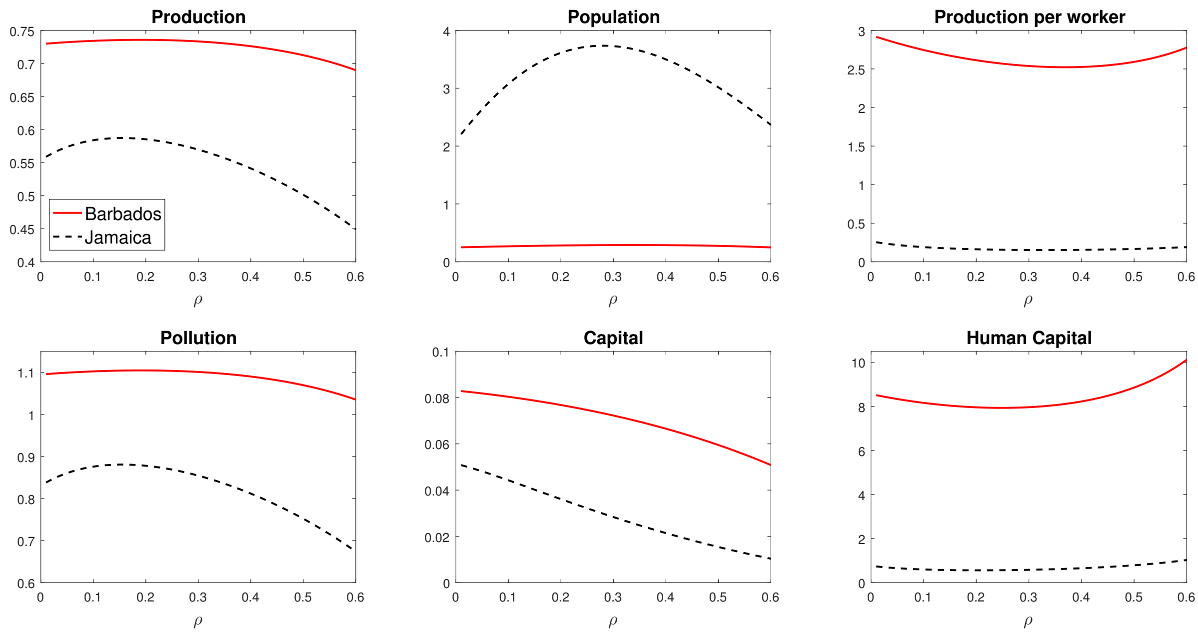
where $\zeta_\rho = \frac{\partial\theta^{-1}(\chi)}{\partial\rho}\frac{\rho}{\theta^{-1}(\chi)}$, *is the elasticity of steady state pollution with respect to the emigration rate,* ρ.

[24] ε gives the relative position in terms of GDP of the domestic economy compared to the migrant-receiving countries.

Pollution stock and population size are directly impacted by the value of the emigration rate. By contrast, the emigration rate effects on human capital and capital stock depend on the elasticity of pollution with respect to ρ. This is the main novelty of this case concerning migration with respect to the first case, without environmental issues. On the one hand, migration has a direct impact on these variables, as in the previous case. On the other hand, because it changes the population dynamics, it also changes the pollution level and thus can have adverse effect on human capital and capital.

To clarify, the mechanisms and the relationship involved, the evolution of the model's variables with respect to the value of ρ are depicted in Figures 14 and 15 for Barbados and Jamaica, for the domestic area. The former displays steady state values for production, population size, production per capita, pollution stock, capital stock and human capital according to the emigration rate, $\rho \in [0, 0.6]$. The latter depicts the effect of migration of total and per capita utility in the domestic area. Barbados simulations are depicted by the plain lines on this figures, while Jamaica results are displayed by the dashed lines. Note that the studied interval for ρ includes all the potential emigration rate observed in the Caribbean region.[25]

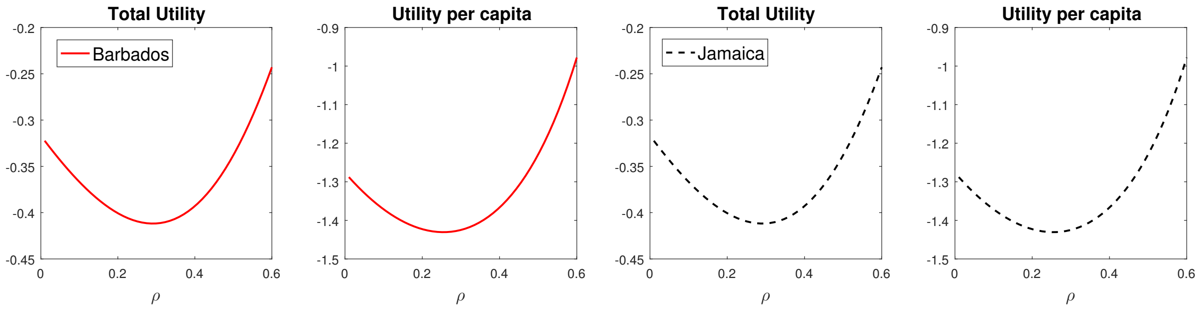
Figure 14: Effect of a variation of ρ on the economy



For both countries, values of the aggregate variables–production, and pollution–with respect to the emigration rate are still described by inverted U-shaped curves, except for capital. However, the emigration rate effects on human capital, production per capita and utility–total and per capita–are now described by U-shaped curves. While in the case without environmental issues these effects were described by inverted-U shaped curve. Indeed, under a certain level of ρ, the production stock increases thanks to the larger

[25] See Ait Benhamou and Cassin (2018) for details.

Figure 15: Effect of a variation of ρ on utility



population size. As described earlier, this can be accounted for by the increase in the gain from investments in education induced by migration, which thus creates incentives to have children. The problem is that a higher production leads to larger emissions of pollution, and thus to an increase in the environmental externalities on human capital accumulation. When the rise in the production is not accompanied by an increase in human capital, the production per capita, might be decreasing. This is amplified by the reduction in physical capital that might occur if savings are strongly reduced in order to increase fertility.

If the emigration rate exceeds a certain limit, the population size is reduced by migration, because, the larger number of children does not compensate the loss of adults with migration. Moreover, the substitution of savings in favor of investments in children is substantial if ρ is high. This is aggravated, by the reduction of the number of savers in the economy that comes from the reduction of the population. In that case, the physical capital decreases too much to sustain the production, this results in a production reduction which is larger than the reduction in population. Quite surprisingly, here, this is when the migration has a positive effect on utility per capita while it was exactly the opposite the case without environmental issues. Indeed, the reduction of the production decreases the pollution stock. This leads to an increase in human capital and thus to an increase in households income (defined as $w_t h_t$, where w_t is the wage). While the substitution effect between investments in children and savings is still an important mechanism in the economy, its impact on the physical capital stock is lessened by this positive income effect. Therefore, the decrease in physical capital is reduced. Combined with the increase in human capital, it results in gains in terms of utility per capita.

These results differ largely from those of usual analysis of migration. While there is still a debate on the overall effect from migration, it is quite well accepted, that an increase in migration will lead to a gain if the value of the emigration rate is low and to an economic decrease when ρ is high. Here, it is the opposite, the emigration rate reduces the damages from the externality when it is high and thus leads to economic gains.

The effect of intergenerational transfers: γ

Proposition 10. *The steady state values of the population size, the stocks of pollution and physical capital as well as the human capital level are positively correlated to γ under* *the following conditions:*

$$\frac{\partial N^\star}{\partial \gamma} > 0 \Leftrightarrow \gamma < \frac{\sqrt{\alpha(1-\rho)}\left[\sqrt{[\alpha(1-\rho)+(1-\alpha)(1\rho+\rho\varepsilon)]}-\sqrt{\alpha(1-\rho)}\right]}{(1-\alpha)(1-\rho+\rho\varepsilon)}$$

$$\frac{\partial Z^\star}{\partial \gamma} > 0 \Leftrightarrow \gamma < \frac{\sqrt{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)(2-\alpha)]^2+4\alpha(1-\rho)(1-\alpha)^3(1-\rho+\rho\varepsilon)}}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)} - \frac{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)(2-\alpha)]}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)}$$

$$\frac{\partial K^\star}{\partial \gamma} > 0 \Leftrightarrow \zeta_\gamma > \frac{\gamma(1-\rho+\rho\varepsilon)-\gamma\alpha\rho\varepsilon}{(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}$$

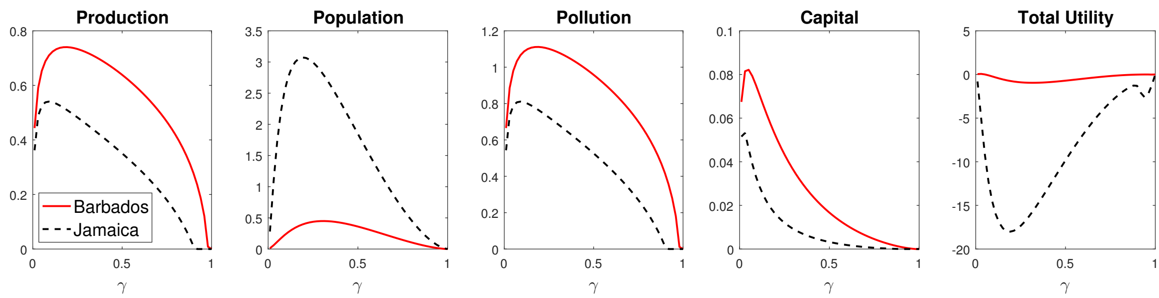
$$\frac{\partial h^\star}{\partial \gamma} > 0 \Leftrightarrow \zeta_\gamma > \left[\frac{1}{\delta} - \frac{\gamma(1-\alpha(1-\delta))[(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)]}{\delta(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]$$

where $\zeta_\gamma = \frac{\partial \theta^{-1}(\chi)}{\partial \gamma}\frac{\gamma}{\theta^{-1}(\chi)}$ *is the elasticity of steady state pollution with respect to the intergenerational tranfers,* ρ

Impacts of γ on the steady state variables are very intricate, because this parameter involves even more mechanisms than the migration rate. Indeed, without any environmental externality, on the one hand there is the negative effect from the reduction of the adult income. As said earlier, the negative income effect induced by γ leads to a decrease in adult consumption, in savings and in education expenditures. On the other hand, increase in intergenerational transfers changes incentives to invest in children through education and fertility. Therefore, here, in return of an increase in those variables there is a rise in production and in pollution. This pollution has a negative impact on human capital and thus on income, which can have an indirect effect on physical capital stock. This explains the conditions on the positive effect of γ. Here again, these conditions depend directly on the value of γ for $Z^\star$ and $N^\star$, while for $h^\star$ and $K^\star$, the effect of γ depends on the elasticity of pollution with respect to this parameter.

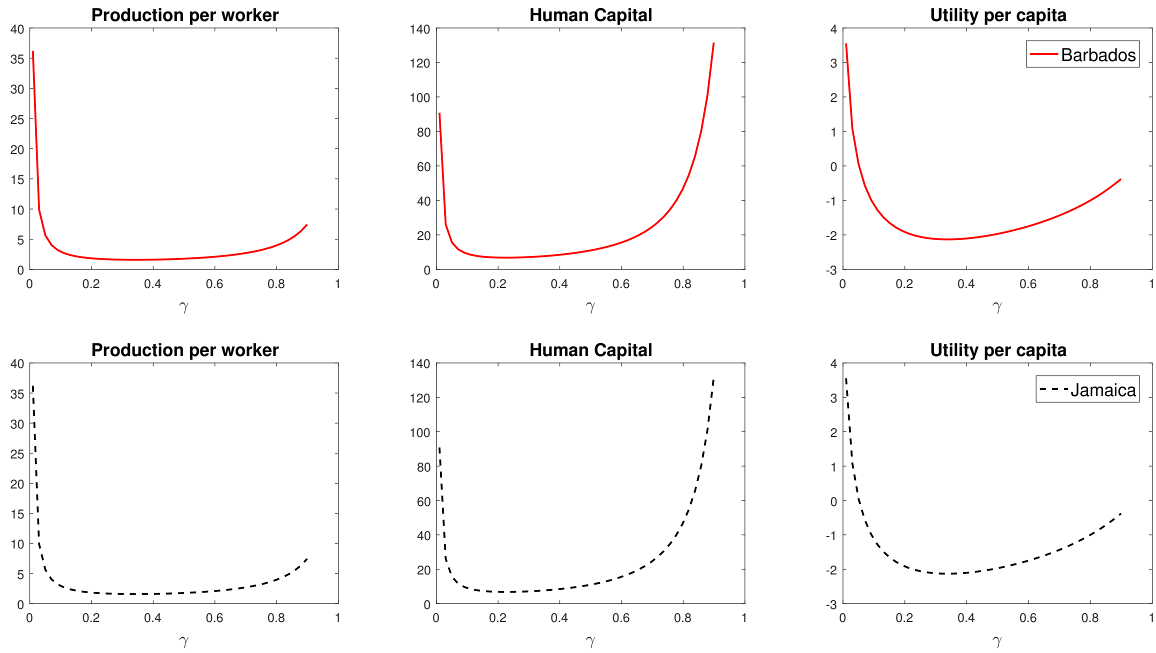
The numerical analysis is conducted for $\gamma \in (0;1)$. Figure 16 displays steady state values for production, population size, pollution stock, capital stock and total utility according to the intergenerational transfer share. Figure 17 displays steady state values for production per capita, human capital and utility per capita. As in the previous sections, Barbados and Jamaica are represented respectively by the plain and the dashed lines.

Figure 16: Effect of a variation of γ on the economy



Note that the net effect of γ is not trivial, because while the emigration rate might be null, a reduction of the intergenerational transfer to 0 leads to the collapse of the

Figure 17: Effect of a variation of γ on the economy



economy in the model (if $\gamma = 0$, there is no reason to have children). Therefore, on the graphics there are two asymptotes one at $\gamma = 0$ and the other one at $\gamma = 1$. Between these two values, inverted U-shaped curves are depicted, for population size, production and pollution stock with respect to the value of γ. With the strong decrease in the production and the pollution stock that occurs after a certain threshold, the human capital is enhanced. In that case, utility increases when the human capital is growing, until the collapse of the economy, because adult income is null when γ approaches 1. In fact except for the smallest (or the highest) value of γ, which are close to the asymptote, human capital and utility per capita are positively correlated to the intergenerational transfers rate. This is accounted for by the negative income effect, that induces a reduction of the fertility and thus of pollution stock. In that case, the intergenerational transfer is quite positive for utility per capita or human capital accumulation. However, if one consider the smallest values of γ, utility per capita might be very high, but with a quasi-null population size. This indicates, that pollution depends strongly on population features and demographic growth.

6. Conclusion

This work presented an overlapping generations model, to explain the interplay between economic activities, pollution emissions and investments in human and physical capital, according to the demographic structure of island economies. In this model, the demographic structure is strongly dependent on the pollution dynamics, which impacts directly human capital accumulation. Two cases are presented, in the first one, an environmental policy, consisting of a tax and public maintenance, is sufficient to overcome completely the pollution degradations. Migration could thus present the features described in the literature: it must not be too high and remittances must not exceed a certain volume. However, in the second case, depending on the pollution emissions, the environmental policy might not be sufficient to cope with the degradation. Migration and its interactions with pollution are thus a key determinants of the productive stocks accumulation.

Indeed, whether or not there is pollution, migration structure–the gain from migration and the scale of emigration–changes strongly the parents' choices in terms of education and savings. Knowing that children will help their parents during their retirement period, migration may lead to an increase in fertility, and thus in population size. However, in presence of an environmental externality, this change in population enhances aggregated production–but not necessarily production per capita–in the domestic area and thus the pollution stock. This last hampers the accumulation of human capital in the first periods and then leads to a reduction in long-term value of human capital. Therefore, even if the possibility to receive transfers–and especially remittances from migrants–creates a strong incentive to invest in their children education, migration can result in a reduction of human capital. In that case, while remittances were identified as a lever for economic growth in the literature, this work shows that they could also have a strong negative impact on development, and that a *brain gain* is not possible in most of the cases.

More specifically, on the one hand, migration could be a source of economic dynamism and thus a source of pollution. But on the other hand, more departures leads to a decrease in the population size and then to a decrease in the pollution stock. Therefore, in some cases, economic and (local) environmental gains can be obtained simultaneously in these economies thanks to migration. This occurs only if the emigration rate is already high, thanks to the reduction in the demographic pressure on the environment.

Moreover, the model reveals that the steady state level of human capital is not impacted by the steady state value of pollution, but by its dynamics during the transitional period. Therefore, the capital stock and the human capital generated in the early stages of the economic development are key features to trigger a strong economic growth in the next periods. In that context, the existence of remittances or intergenerational transfers could hamper the accumulation of human capital, because they boost quickly the emissions of pollution. To overcome this effect, other types of public policies could be tested, *e.g.* a tax on remittances or a policy on education to compensate the environmental damage on human capital accumulation.

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Appendix A Detailed comparative statics of household choices

Equations of the savings and the fertility are:

$$s_t^\star = \frac{\beta\alpha(1-\rho)(1-\gamma)}{(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} w_t h_t$$
$$n_t^\star = \frac{\beta\gamma(1-\rho+\rho\varepsilon)(1-\gamma)(1-\alpha)(1-\mu)}{\sigma(1+\beta)[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]} N_t^{-\delta}$$

The derivative of the fertility with respect to the emigration rate, ρ, the net migration gain, ε, and the transfer rate, γ are respectively:

$$\frac{\partial n_t^\star}{\partial \rho} = \frac{\beta\gamma\alpha\varepsilon(1-\alpha)(1-\gamma)(1-\mu)}{\sigma N_t^\delta(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} > 0$$
$$\frac{\partial n_t^\star}{\partial \varepsilon} = \frac{\beta\gamma\alpha\rho(1-\alpha)(1-\gamma)(1-\mu)}{\sigma N_t^\delta(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} > 0$$
$$\frac{\partial n_t^\star}{\partial \gamma} = \frac{\beta(1-\alpha)(1-\mu)(1-\rho+\rho\varepsilon)}{\sigma N_t^\delta(1+\beta)} \times \frac{\alpha(1-\rho)(1-2\gamma)-\gamma^2(1-\alpha)(1-\rho+\rho\varepsilon)}{[\alpha(1-\rho)+\Lambda_h(1-\alpha)]^2}$$

The sign of $\frac{\partial n_t^\star}{\partial \gamma}$ is given by the numerator of the second term of the derivative. Therefore, the next step is to solve the second degree equation given by:

$$\alpha(1-\rho)(1-2\gamma)-\gamma^2(1-\alpha)(1-\rho+\rho\varepsilon$$

The discriminant, denoted Δ, is positive. It is written as follows:

$$\Delta = 4\alpha(1-\rho)[\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)]$$

There is only one positive solution, which is given by the following equation. If γ is greater than this value, the derivative of the fertility with respect to the intergenerational transfer rate is negative.

$$\gamma = \frac{2\alpha(1-\rho)-\sqrt{4\alpha(1-\rho)[\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)]}}{-2(1-\alpha)(1-\rho+\rho\varepsilon)}$$
$$\gamma = \frac{\sqrt{\alpha(1-\rho)}\left[\sqrt{\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)}-\sqrt{\alpha(1-\rho)}\right]}{(1-\alpha)(1-\rho+\rho\varepsilon)}$$

It is worth noting that the population size is proportional to the optimal fertility with a factor $(1-\rho)$. Thus, the parameters' impact on fertility are the same than the impact on population size, except for the migration rate, which has also a negative effect on the number of adults staying in the domestic area. Here we give directly, the derivative of the steady state adult generation size with respect to ρ.

$$\frac{\partial N^\star}{\partial \rho} = \frac{1}{\delta}\left[\frac{\beta\gamma(1-\mu)(1-\gamma)(1-\alpha)}{\sigma(1+\beta)}\right]^{\frac{1}{\delta}}\left[\frac{(1-\rho)(1-\rho+\rho\varepsilon)}{\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)}\right]^{\frac{1-\delta}{\delta}}$$
$$\times\left[\frac{\alpha(1-\rho)(\varepsilon-1)(1-\rho)-\gamma(1-\rho+\rho\varepsilon)(1-\alpha)(1-\rho+\rho\varepsilon)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]$$

The sign of this derivative is given by the following condition:

$$\frac{\partial N^\star}{\partial \rho} > 0 \;\Leftrightarrow\; \alpha(1-\rho)(\varepsilon-1)(1-\rho)-\gamma(1-\rho+\rho\varepsilon)(1-\alpha)(1-\rho+\rho\varepsilon) > 0$$
$$\Leftrightarrow\; \frac{(1-\rho)}{(1-\rho+\rho\varepsilon)} > \sqrt{\frac{\gamma(1-\alpha)}{\alpha(\varepsilon-1)}}$$

Appendix B Case 1: Without the environmental externality

B.1 Proof of Proposition 2 and of the stability of the equilibrium

Proof of Proposition 2. The population size is independent of the evolution of the capital per unit of efficient labor. Therefore, when the labor force reaches its steady state level, the capital per efficient unit of labor is as follows:

$$k_{t+1} = k_t = k_{BGP} = \left(\frac{A\sigma}{1-\mu}\right)^{1-\mu}\frac{\alpha}{\theta\Lambda_h[\mu(1-\alpha)]^\mu}k_{BGP}^{\alpha(1-\mu)}(N^\star)^{\delta(1-\mu)}$$

Replacing $N^\star$ in this equation leads directly to the level of capital per unit of efficient labor on the BGP. □

Proof of the stability of the equilibrium. The population dynamics is completely independent on the evolution of the other variables of the economy. Defining $g(N_t) = N_{t+1}$.

$$\lim_{N_t\to 0} f'(N_t) = +\infty$$
$$\lim_{N_t\to +\infty} f'(N_t) = 0$$
$$\lim_{N_t\to +\infty} f(N_t) = +\infty$$

The function $g(N_t)$ is concave and there are two points such as $N_{t+1} = N_t$, which are $N_t = 0$ and $N_t = N^\star$ satisfying $0 < f'(N^\star) < 1$. Therefore, it exists a unique non-trivial equilibrium locally stable on the population dynamics, and N shows a regular convergence. When the population has reached its steady state value, the dynamics of the capital per unit of efficient labor writes as follows:

$$k_{t+1} = \left(\frac{A\sigma}{1-\mu}\right)^{1-\mu}\frac{\alpha}{\theta\Lambda_h[\mu(1-\alpha)]^\mu}k_t^{\alpha(1-\mu)}N^{\star\delta(1-\mu)}$$

Denote the function $f(k_t) = k_{t+1}$. It is characterized by the following:

$$\lim_{k_t \to 0} f'(k_t) = +\infty$$
$$\lim_{k_t \to +\infty} f'(k_t) = 0$$
$$\lim_{k_t \to +\infty} f(k_t) = +\infty$$

The function $f(k_t)$ is concave and there are two points such as $k_{t+1} = k_t$, which are $k_t = 0$ and $k_t = k_{BGP}$ satisfying $0 < f'(k_{BGP}) < 1$. Therefore, it exists a unique non-trivial equilibrium locally stable and the model shows a regular convergence. □

B.2 Proof of Proposition 3 and 4

Proof of Proposition 3. Replacing Ψ and Λ_h by their values in the equation (43), the following is obtained:

$$k_{BGP} = \left[\frac{\alpha}{\theta[\mu\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^{\mu}}\right]^{\frac{1}{1-\alpha(1-\mu)}} \times \left[\frac{\beta A(1-\rho)(1-\alpha)(1-\gamma)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{1-\mu}{1-\alpha(1-\mu)}}$$

Without calculations, it appears that $\frac{\partial k_{BGP}}{\partial \theta} < 0$, $\frac{\partial k_{BGP}}{\partial \gamma} < 0$, $\frac{\partial k_{BGP}}{\partial \varepsilon} < 0$, $\frac{\partial k_{BGP}}{\partial A} > 0$, $\frac{\partial k_{BGP}}{\partial \beta} > 0$

For the emigration rate, the following is obtained:

$$\frac{\partial k_{BGP}}{\partial \rho} = -\frac{1}{1-\alpha(1-\mu)}\left[\left[\frac{\alpha}{\theta[\mu\gamma(1-\alpha)]^{\mu}}\right]\left[\frac{\beta A(1-\alpha)(1-\gamma)}{(1+\beta)}\right]^{1-\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}} \times \left[(1-\rho+\rho\varepsilon)^{-\mu}\left[\frac{(1-\rho)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{1-\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}} \times \left[\frac{\mu(\varepsilon-1)}{1-\rho+\rho\varepsilon} + \frac{\gamma\varepsilon(1-\alpha)(1-\mu)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right] < 0$$

□

Proof of Proposition 4. Replacing Ψ and Λ_h by their values in the equation (43), the growth factor writes as follows:

$$g_{BGP} = \left[[\theta(\mu\gamma(1-\rho+\rho\varepsilon)^{\mu}]^{1-\alpha}\left[\frac{\beta A\alpha^{\alpha}(1-\alpha)^2(1-\gamma)(1-\rho)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$

Without calculations, it appears that $\frac{\partial g_{BGP}}{\partial \theta} > 0$, $\frac{\partial g_{BGP}}{\partial \varepsilon} > 0$, $\frac{\partial g_{BGP}}{\partial A} > 0$, $\frac{\partial k_{BGP}}{\partial \beta} > 0$

The derivative with respect to the net gain from migration writes as follows:

$$\frac{\partial g_{BGP}}{\partial \varepsilon} = \left[\theta^{1-\alpha}\left[\frac{\beta A\alpha^{\alpha}(1-\gamma)(1-\alpha)[\mu\gamma(1-\alpha)]^{1-\alpha}}{(1+\beta)}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times \frac{\mu\alpha\rho(1-\alpha)}{1-\alpha(1-\mu)}\left[\frac{(1-\rho+\rho\varepsilon)^{1-\alpha}}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{\mu}{1-\alpha(1-\mu)}-1}(1-\rho+\rho\varepsilon)^{-\alpha}$$
$$\times\left[\frac{(1-\rho)+\Lambda_h}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right] > 0$$

For the emigration rate, the following is obtained:

$$\frac{\partial g_{BGP}}{\partial \rho} = \left[\theta^{1-\alpha}\left[\frac{\beta A\alpha^{\alpha}(1-\gamma)(1-\alpha)[\mu\gamma(1-\alpha)]^{1-\alpha}}{(1+\beta)}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times \frac{\mu(1-\alpha)}{1-\alpha(1-\mu)}\left[\frac{(1-\rho+\rho\varepsilon)^{1-\alpha}(1-\rho)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{\mu}{1-\alpha(1-\mu)}-1}(1-\rho+\rho\varepsilon)^{-\alpha}$$
$$\times\left[\frac{\alpha(1-\rho)(\varepsilon-1)(1-\rho)-\Lambda_h[\varepsilon-(1-\alpha)(\varepsilon-1)(1-\rho)]}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]$$

Thus, this leads to the following condition:

$$\frac{\partial g_{BGP}}{\partial \rho} > 0 \Leftrightarrow \frac{(1-\rho)}{\Lambda_h} > \frac{[\varepsilon-(1-\alpha)(\varepsilon-1)(1-\rho)]}{\alpha(\varepsilon-1)(1-\rho)}$$

The derivative of the growth factor on the BGP with respect to γ is:

$$\frac{\partial g_{BGP}}{\partial \gamma} = \left[\theta^{1-\alpha}\left[\frac{\beta A\alpha^{\alpha}(1-\rho)(1-\alpha)[\mu(1-\rho+\rho\varepsilon)(1-\alpha)]^{1-\alpha}}{(1+\beta)}\right]^{\mu}\right]^{\frac{1}{1-\alpha(1-\mu)}}$$
$$\times \frac{\mu}{1-\alpha(1-\mu)}\left[\frac{\gamma^{1-\alpha}(1-\gamma)}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]}\right]^{\frac{\mu}{1-\alpha(1-\mu)}-1}\gamma^{-\alpha}$$
$$\times\left[\frac{\alpha(1-\rho)[(1-\alpha)(1-\gamma)-\gamma]-(1-\alpha)\Lambda_h[1-(1-\alpha)(1-\gamma)]}{[\alpha(1-\rho)+\gamma(1-\rho+\rho\varepsilon)(1-\alpha)]^2}\right]$$

Hence, the following condition can be defined:

$$\frac{\partial g_{BGP}}{\partial \gamma} > 0 \Leftrightarrow \frac{\Lambda_h}{(1-\rho)} < \frac{\alpha}{1-\alpha}\frac{(1-\alpha)(1-\gamma)-\gamma}{1-(1-\alpha)(1-\gamma)}$$

However it is important to note that if the intergenerational transfer rate is high, $(1-\alpha)(1-\gamma)-\gamma$ can be negative. □

B.3 BGP values of consumptions

On the steady state, the adult and old age consumptions are given by the following equations:

$$\begin{aligned} c_t &= \frac{1-\gamma}{1+\beta}w_t h_t \\ d_{t+1} &= s_t R_{t+1} + n_t \Lambda_h w_{t+1} h_{t+1} \end{aligned}$$

Introducing in the household choices in terms of savings and fertility (s_t and n_t), the factor prices (w_{BGP} and R_{BGP}) and the labor force on the BGP, ($N^\star$), the following is obtained:

$$\begin{aligned} c_t &= \frac{1-\gamma}{1+\beta}(1-\alpha)A{k_{BGP}}^{\alpha}h_t \\ d_{t+1} &= \frac{(1-\alpha)A{k_{BGP}}^{\alpha}}{\alpha\Psi A\alpha}h_t\left[{k_{BGP}}^{1-\alpha} + \frac{(1-\alpha)\Lambda_h}{(1-\rho)\alpha\Psi A\alpha}\frac{h_{t+1}}{h_t}\right] \end{aligned}$$

On the BGP, $\frac{h_{t+1}}{h_t}$ is given by the growth rate of the economy, g_{BGP}, which leads to:

$$\begin{aligned} c_t &= \frac{1-\gamma}{1+\beta}(1-\alpha)A{k_{BGP}}^{\alpha}h_t \\ d_{t+1} &= \frac{(1-\alpha)A{k_{BGP}}^{\alpha}}{\alpha\Psi A\alpha}h_t\left[{k_{BGP}}^{1-\alpha} + \frac{(1-\alpha)\Lambda_h}{(1-\rho)\alpha\Psi A\alpha}g_{BGP}\right] \end{aligned}$$

On the BGP, when the growth factors $\frac{c_{t+1}}{c_t}$ and $\frac{d_{t+2}}{d_{t+1}}$ are written, everything simplifies except for the human capital dynamics. This means than on the BGP, it is equivalent to study the growth rate of the economy or of the inter-temporal utility per capita linked to consumption.

Appendix C Structural parameter values estimation and calibration

Table 2 below reports the model's structural economic parameters, their respective economic interpretations, the support range for credible values as well as the calculation methods used in Ait Benhamou and Cassin (2018). Data was extracted from the World Bank (2018) World Development Indicators (WDI) as well as the University of Pennsylvania World Table (PWT). In this section, only the parameters obtained with a method different from Ait Benhamou and Cassin (2018)' work are described in detail. Indeed, in their paper, there was no congestion and fertility was exogenous, but parameters $\beta, \alpha, \rho, \gamma, \mu, \bar{\theta}, \varepsilon$ can be used directly. Parameters δ and σ are defined with the method described below.

The purpose of the calibration exercise is twofold. First, calibration ensures that the model performs credibly well for each parameter value with respect to the features of economies we seek to replicate. Second, when the model proves to be able to match defining moments for the benchmark economy, it provides an adequate analytical framework, and thus predicts a set of relevant outcomes with respect to policy changes and instruments. As such, proper calibration can yield useful results for policymaking. Nonetheless, credible values for structural parameters are contingent upon available data. This is particularly the case for small emerging economies, such as the Caribbean islands.

Kydland and Prescott (1991) provide a comprehensive framework for discussing calibration in general equilibrium models. While they insist on the method to choose the benchmark values for structural parameters, in the absence of panel studies on households

Table 2: Model structural parameters

Economic Parameters	**Range**	Method	Data source
Preference factor for the future	$\beta \in [0,1[$	Calibration	WDI
Capital intensity in production	$\alpha \in [0,1]$	*idem*	WDI & PWT
Technology level	$A > 0$	*idem*	PWT
Emigration rate	$\rho \in [0,1]$	*idem*	WDI
Net gain from migration	$\varepsilon > 1$	*idem*	*idem*
Share of income remitted	$\gamma \in [0,1]$	*idem*	*idem*
Efficiency - education	$\mu \in [0,1]$	Estimation	WDI & PWT
Efficiency - human capital accumulation	$\theta > 0$	*idem*	*idem*
Cost of child-rearing	$\sigma \in [0,1]$	Calibration	*idem*
Congestion parameter	$\delta \in [0,1]$	Calibration	*UN Data*

and firms–which are optimal to compute agent behavior parameters–one should focus as much as possible on standard calibration. It relies on steady state expressions of the model (without environmental externalities), and use long-run averages of variables in the dataset built for the sample of SIDS countries.

Most available data can be traced back to the 1970s, and a dataset for the time period 1970-2014 is built. Numerical simulations will be then computed with initial values corresponding to the year 1970. Due to their differences two countries are scrutinized in order to illustrate the model: Barbados and Jamaica.

- $N^\star$ is the steady-state population value. Using data from the World Bank, the forecast of population levels (initially until 2050) are extended until demographic growth is close or equal to zero. This allows to extrapolate steady-state population $N^\star$ at the corresponding dates.

- δ is the congestion parameter, and captures the speed of convergence to the steady-state population level. The value is estimated thanks to the following expression:

  $$N_{t+1} = \Lambda_n N_t^{1-\delta}$$

  Where Λ_n is a collection of structural parameters of the model. The equation is re-written in log terms and differentiated, so that δ is estimated by regressing future demographic growth on logged present population, namely:

  $$\Delta\% N_{t+1} = \ln \Lambda_n - \delta \ln N_t$$

  A logistical transformation is introduced in order to make sure that estimated values for parameter δ always belong to the interval $(0, 1)$.

- σ denotes the child-rearing cost per individual. Its value is calibrated in order to match the parameters of the model, as well as the estimated values of μ and θ.

The congestion component of the model is incorporated such that σ matches the following expression:

$$\sigma = \frac{\gamma\beta(1-\rho+\rho\varepsilon)(1-\gamma)(1-\alpha)(1-\rho)(1-\mu)}{N^{\star\delta}(1+\beta)\left[\alpha(1-\rho)+(1-\alpha)\gamma(1-\rho+\rho\varepsilon)\right]}$$

For the initial values, output is normalized to unity in 1970, and the capital stock is computed using capital-to-output ratio for the same year. The figures in Table 1 report harmonized initial values for physical capital for comparison purposes. The same calibration is computed for efficient units of labor, which are derived from normalized output and capital. Using the Cobb-Douglas equation (1), it is possible to deduce $N_0 h_0$ for a given K_0 and $y_0 = 1$. Finally, given that human capital is reported as an index in PWT, we retain the 1970 value for all countries.

Appendix D Case 2: Equilibrium in a polluted area

D.1 Proof of proposition 7

Proof of Proposition 7. The population size dynamics remains the same than in the case without pollution emissions. Indeed, the population dynamics is independent of the other variables, and depends exclusively on the structural parameters of the economy as well as the population size. Using the equation (45) in the dynamics of the capital, given by the equation K_{t+1} in the system (31), a relationship is obtained between the steady state values of the capital stock $K^\star$ and the human capital level $h^\star$.

$$K^\star = \left[\frac{\alpha\beta A(1-\tau)(1-\alpha)(1-\gamma)(1-\rho)}{(1+\beta)\left[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]}\right]^{\frac{1}{1-\alpha}} N^\star h^\star \tag{51}$$

When introduced in the equation of the human capital dynamics, given by the equation (9), equations (51) and (45) ($N^\star$) leads to the following:

$$h^\star = \theta(Z^\star)\left[\frac{\mu A\sigma(1-\alpha)(1-\tau)}{1-\mu}\right]^{\mu} (K^\star)^{\alpha\mu}(N^\star)^{\mu(\delta-\alpha)}(h^\star)^{1-\alpha\mu} \tag{52}$$

After some computations, the steady state value of $\theta(Z^\star) \equiv \chi$ is:

$$\chi = \left[\gamma\mu(1-\rho+\rho\varepsilon)(1-\alpha)\right]^{-\mu}\left[\frac{\beta A\alpha^{\alpha}(1-\tau)(1-\rho)(1-\gamma)(1-\alpha)}{(1+\beta)\left[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]}\right]^{-\frac{\mu}{1-\alpha}} \tag{53}$$

It is worth noting that the steady state value of the efficiency of human capital accumulation is not linked to the level of emissions. The stock of pollution can be defined as the value of the inverse function of $\theta(Z_t)$ written $\theta^{-1}(.)$. Thus, on the steady state, $Z^\star$ is defined as $Z^\star = \theta^{-1}(\chi)$.[26] Finally, using the dynamics of the pollution stock, given by

[26] The properties of the function $\theta(.)$ are studied in the numerical analysis.

the last equation in the sytem (31), another expression of the steady state human capital appears:

$$h^{\star} = \left[\frac{aZ^{\star}}{(\Omega - \tau\xi)A} K^{\star -\alpha} N^{\star \alpha - 1}\right]^{\frac{1}{1-\alpha}} \tag{54}$$

Replacing the steady state values of population and pollution stock (given respectively by (45) and (48)), in the system of equations defined by (51) and (54), the values of $h^{\star}$ and $K^{\star}$ can be determined as:

$$\begin{aligned} h^{\star} &= \frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi} [\alpha A(1-\tau)]^{-\frac{\alpha}{1-\alpha}} \left[\frac{\sigma}{\gamma(1-\mu)(1-\rho+\rho\varepsilon)}\right]^{\frac{1}{\delta}} [\Psi(1-\rho)]^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}} \\ K^{\star} &= \Psi\alpha(1-\rho)(1-\tau)\frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi} \end{aligned}$$

where $\Psi \equiv \frac{\beta(1-\gamma)(1-\alpha)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]}$ □

D.2 Stability of the Steady-State: further results

D.3 Static Comparatives

D.3.1 The efficiency of human capital accumulation, χ, and the pollution, Z_t

The first step to study the variations of pollution, human capital or capital with respect to the parameters, is to define the effect of the different parameters on the long term efficiency of human capital accumulation $\chi \equiv \theta(Z^{\star})$. On the SS, derivatives of χ with respect to the parameters are given by the following equations:

$$\frac{\partial\chi}{\partial\beta} = -\frac{\mu}{1-\alpha}\frac{\chi}{\beta(1+\beta)} < 0 \tag{55}$$

$$\frac{\partial\chi}{\partial A} = -\frac{\mu}{1-\alpha}\frac{\chi}{A} < 0 \tag{56}$$

$$\frac{\partial\chi}{\partial\tau} = \frac{\mu}{1-\alpha}\frac{\chi}{\tau} > 0 \tag{57}$$

Moreover, conditions for the parameters linked to the impact of migration features can be computed. The derivative of χ with respect to ρ is given by this equation:

$$\frac{\partial\chi}{\partial\rho} = -\mu\chi\left[\frac{\gamma\varepsilon(1-\alpha)}{(1-\alpha)(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{\varepsilon-1}{1-\rho+\rho\varepsilon}\right] \tag{58}$$

Therefore the sign of this derivative, depends on the following condition:

$$\begin{aligned} \frac{\partial\chi}{\partial\rho} > 0 \quad &\Leftrightarrow \quad \left[\frac{\gamma\varepsilon(1-\alpha)}{(1-\alpha)(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{\varepsilon-1}{1-\rho+\rho\varepsilon}\right] > 0 \\ \frac{\partial\chi}{\partial\rho} > 0 \quad &\Leftrightarrow \quad \frac{\varepsilon}{(\varepsilon-1)} > \frac{(1-\rho)[\alpha(1-\rho)+\lambda_h(1-\alpha)]}{\Lambda_h} \end{aligned}$$

Figure 18: The effect of $\theta(Z_t)$ on the steady state stability

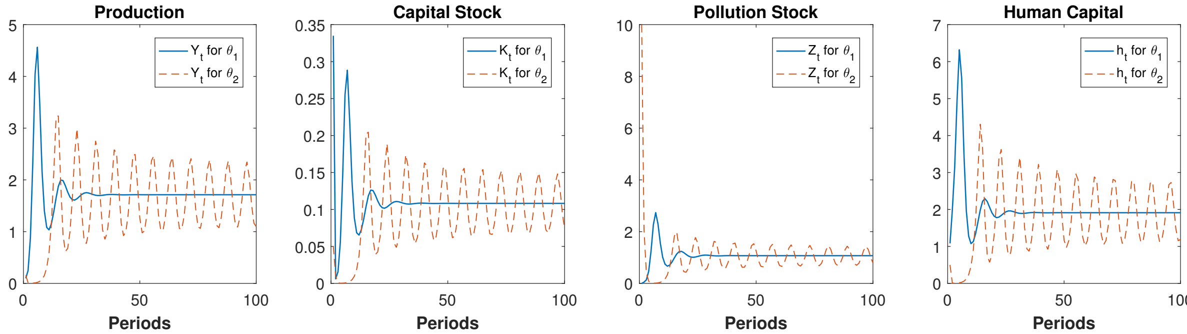


Figure 19: The effect of Ω on the convergence, for $a = 0.2$

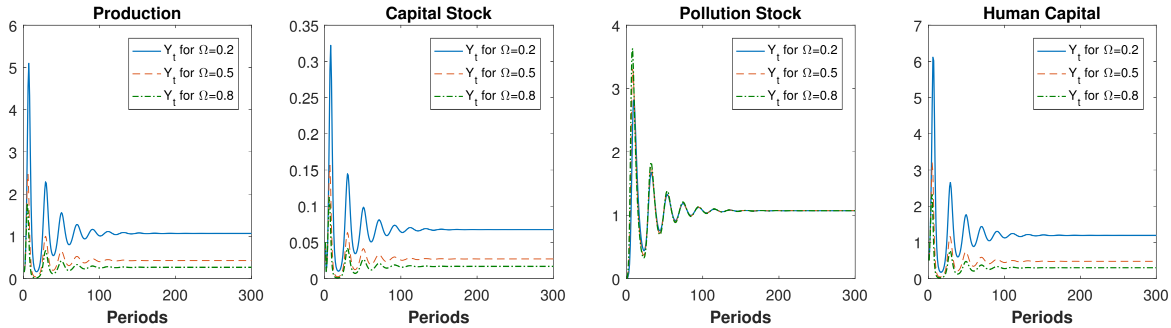


Figure 20: The effect of Ω on the convergence, for $a = 0.5$

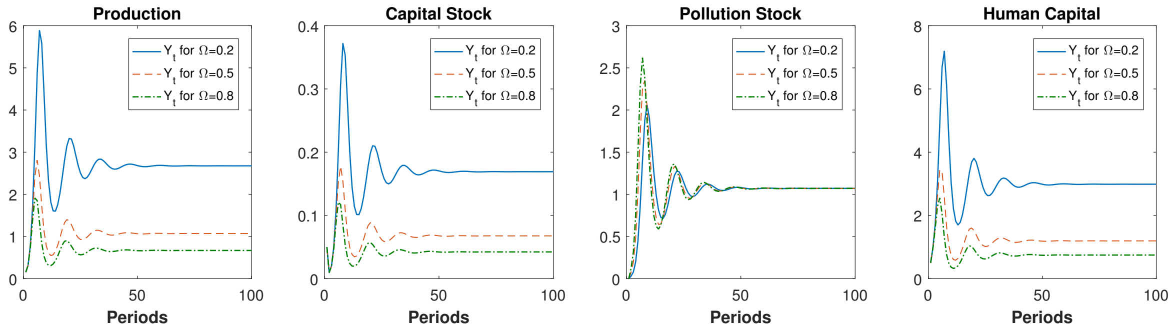
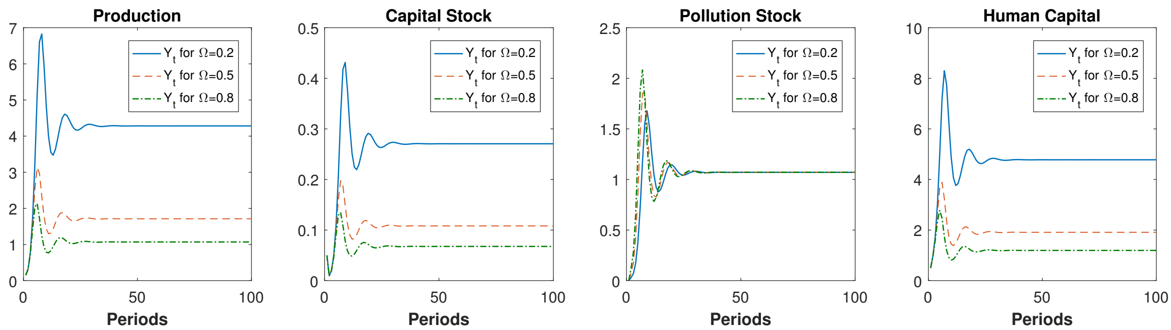


Figure 21: The effect of Ω on the convergence, for $a = 0.8$



The derivative of χ with respect to the net gain from migration is as follows:

$$\begin{aligned}\frac{\partial \chi}{\partial \varepsilon} &= \mu\rho \left[\frac{\beta A \alpha^{\alpha}(1-\tau)(1-\rho)(1-\gamma)(1-\alpha)^{2-\alpha}}{(1+\beta)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]^{\frac{-\mu}{(1-\alpha)}} (\gamma\Lambda_h)^{-\mu} \\ &\times \left[\frac{\gamma}{\alpha(1-\rho)+\Lambda_h(1-\alpha)} - \frac{1}{1-\rho+\rho\varepsilon}\right]\end{aligned} \tag{59}$$

The sign of this derivative depends on the second term of this product:

$$\frac{\partial \chi}{\partial \varepsilon} \Leftrightarrow \frac{\gamma}{\alpha(1-\rho)+\Lambda_h(1-\alpha)} - \frac{1}{1-\rho+\rho\varepsilon} > 0$$
$$\Leftrightarrow \Lambda_h > (1-\rho)$$

This can be rewritten in order to keep only ε on the left side. The larger γ and ρ are, the lower the threshold to have a positive impact from an increase in ε is.

$$\varepsilon > \frac{(1-\rho)(1-\gamma)}{\gamma\rho}$$

The derivative of χ with respect to the intergenerational transfer rate is as follows:

$$\frac{\partial \chi}{\partial \gamma} = \mu\chi \left[\frac{\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)}{(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{1}{\gamma}\right] \tag{60}$$

The sign of this derivative depends on the following condition which is quite difficult to interpret:

$$\frac{\partial \chi}{\partial \gamma} > 0 \Leftrightarrow \left[\frac{\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)}{(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{1}{\gamma}\right] > 0$$
$$\frac{\partial \chi}{\partial \gamma} > 0 \Leftrightarrow \frac{(1-\alpha)(1-\gamma)}{\gamma} < \frac{\alpha(1-\rho)+(1-\alpha)(1-\rho+\rho\varepsilon)}{\alpha(1-\rho)+\Lambda_h(1-\alpha)}$$

Knowing the effects of the parameters on the steady state value of the efficiency of human capital accumulation, it is now possible to study the other variables on the steady state. First $Z^\star$ depends directly on the level attained by the efficiency of human capital acucmulation, χ, knowing that $\theta(Z_t)$ and its inverse are decreasing and monotonic functions. The stock of pollution is positively correlated to the parameters A, β and is negatively correlated to τ. The effect of the other parameters depends on the opposite of the conditions given for χ.

D.3.2 The capital stock: $K^\star$

The equation of $K^\star$ is:

$$K^\star = \frac{\beta\alpha(1-\rho)(1-\tau)(1-\gamma)(1-\alpha)}{(1+\beta)\left[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]} \frac{a\theta^{-1}(\chi)}{\Omega-\tau\xi}$$

Without calculations it appears that $K^\star$ is negatively correlated to the emissions of pollutions Ω and positively correlated to the absorption rate a and the efficiency of the abatement effort, ξ. The derivatives of this equation with respect to β and A, are given by equation (61) and (62) respectively.

$$\frac{\partial K^\star}{\partial \beta} = K^\star \left[\frac{1}{\beta(1+\beta)} + \frac{\partial \theta^{-1}(\chi)}{\partial \beta}\frac{1}{\theta^{-1}(\chi)}\right] \tag{61}$$
$$\frac{\partial K^\star}{\partial A} = K^\star \frac{\partial \theta^{-1}(\chi)}{\partial A} \tag{62}$$

Knowing that the derivatives of $\theta^{-1}(\chi)$ with respect to β and A are positive, both derivatives (61) and (62) are positive and $K^\star$ also positively correlated to these variables.

The effects of the other parameters according on the natural capital stock depend on conditions. First let's give, the derivative of $K^\star$ with respect to the tax on production, τ.

$$\frac{\partial K^\star}{\partial \tau} = K^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \tau} \frac{\Omega - \tau\xi}{\theta^{-1}(\chi)} + \frac{\xi - \Omega}{1-\tau} \right] \tag{63}$$

We denote the elasticity of pollution with to the tax rate as:

$$\zeta_\tau = -\frac{\partial \theta^{-1}(\chi)}{\partial \tau} \frac{\tau}{\theta^{-1}(\chi)} > 0$$

Therefore the condition to observe an increase in the long term capital stock is:

$$\frac{\partial K^\star}{\partial \tau} > 0 \Leftrightarrow \zeta_\tau < \frac{\tau(\xi - \Omega)}{(1-\tau)(\Omega - \tau\xi)}$$

Second, the derivatives of $K^\star$ with respect to the emigration rate, ρ, the net gain from migration, ε, and the intergenerational transfer, γ, are given by the following equations:

$$\frac{\partial K^\star}{\partial \rho} = K^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \rho} \frac{1}{\theta^{-1}(\chi)} - \frac{\gamma\varepsilon(1-\alpha)}{(1-\rho)[\alpha(1-\rho) + \Lambda_h(1-\alpha)]} \right] \tag{64}$$

$$\frac{\partial K^\star}{\partial \varepsilon} = K^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \varepsilon} \frac{1}{\theta^{-1}(\chi)} - \frac{\gamma\rho(1-\alpha)}{[\alpha(1-\rho) + \Lambda_h(1-\alpha)]} \right] \tag{65}$$

$$\frac{\partial K^\star}{\partial \gamma} = K^\star \left[\frac{\partial \theta^{-1}(\chi)}{\partial \gamma} \frac{1}{\theta^{-1}(\chi)} - \frac{(1-\rho+\rho\varepsilon) - \alpha\rho\varepsilon)}{(1-\gamma)[\alpha(1-\rho) + \Lambda_h(1-\alpha)]} \right] \tag{66}$$

The sign of these derivatives depend on the term between brackets. We denote respectively by ζ_ρ, ζ_ε and ζ_γ the elasticities of the pollution with respect to ρ, ε and γ.

$$\zeta_\rho = \frac{\partial \theta^{-1}(\chi)}{\partial \rho} \frac{\rho}{\theta^{-1}(\chi)}, \qquad \zeta_\varepsilon = \frac{\partial \theta^{-1}(\chi)}{\partial \varepsilon} \frac{\varepsilon}{\theta^{-1}(\chi)}, \qquad \zeta_\gamma = \frac{\partial \theta^{-1}(\chi)}{\partial \gamma} \frac{\gamma}{\theta^{-1}(\chi)}$$

However, it is worth noting that the signs of ζ_ρ, ζ_ε and ζ_γ are not always positive because, the sign of the derivatives of θ^{-1} with respect to these parameters depends on their level. Therefore, their impacts depend on the following conditions:

$$\frac{\partial K^\star}{\partial \rho} > 0 \quad \Leftrightarrow \quad \zeta_\rho > \frac{\rho}{1-\rho} \frac{\gamma\varepsilon(1-\alpha)}{[\alpha(1-\rho) + \Lambda_h(1-\alpha)]}$$

$$\frac{\partial K^\star}{\partial \varepsilon} > 0 \quad \Leftrightarrow \quad \zeta_\varepsilon > \frac{\varepsilon\gamma\rho(1-\alpha)}{\alpha(1-\rho) + \Lambda_h(1-\alpha)}$$

$$\frac{\partial K^\star}{\partial \gamma} > 0 \quad \Leftrightarrow \quad \zeta_\gamma > \frac{\gamma(1-\rho+\rho\varepsilon) - \gamma\alpha\rho\varepsilon}{(1-\gamma)[\alpha(1-\rho) + \Lambda_h(1-\alpha)]}$$

D.3.3 The human capital: $h^\star$

Finally, we study the impact of the parameters on steady state human capital. Steady state value of $h^\star$ writes as follows:

$$h^\star = \frac{a\theta^{-1}(\chi)}{\Omega - \tau\xi}\left[\alpha A(1-\tau)\right]^{-\frac{\alpha}{1-\alpha}}\left[\frac{\sigma}{\gamma(1-\mu)(1-\rho+\rho\varepsilon)}\right]^{\frac{1}{\delta}} \times \left[\frac{\beta(1-\gamma)(1-\alpha)(1-\rho)}{(1+\beta)\left[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)\right]}\right]^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}}$$

Without calculations it appears that $h^\star$ is negatively correlated to the emissions of pollution Ω and positively correlated to the absorption rate a and the cost for rearing children. Following the same method as for the capital stock, we give the derivatives of $h^\star$ with respect to A (equation (67)), β (equation (68)), τ (equation (69)), γ (equation (72)), ε (equation (71)) and γ (equation (72)).

$$\frac{\partial h^\star}{\partial A} = h^\star\left[\frac{\partial\theta^{-1}(\chi)}{\partial A}\right]\frac{1}{\theta^{-1}(\chi)} - \frac{1}{A(1-\alpha)}] \quad (67)$$

$$\frac{\partial h^\star}{\partial \beta} = h^\star\left[\frac{\partial\theta^{-1}(\chi)}{\partial \beta}\frac{1}{\theta^{-1}(\chi)} - \frac{1-\alpha(1-\delta)}{\delta\beta(1-\alpha)(1+\beta)}\right] \quad (68)$$

$$\frac{\partial h^\star}{\partial \tau} = h^\star\left[\frac{\partial\theta^{-1}(\chi)}{\partial \tau}\frac{1}{\theta^{-1}(\chi)} + \frac{\xi(1-\tau)-\alpha(\xi-\Omega)}{(1-\alpha)(1-\tau)(\omega-\xi\tau)}\right] \quad (69)$$

$$\frac{\partial h^\star}{\partial \rho} = h^\star\left[\frac{\partial\theta^{-1}(\chi)}{\partial \rho}\frac{1}{\theta^{-1}(\chi)} - \left[\frac{\varepsilon-1}{\delta(1-\rho+\rho\varepsilon)} - \frac{\gamma\varepsilon(1-\alpha(1-\delta))}{\delta(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]\right] \quad (70)$$

$$\frac{\partial h^\star}{\partial \varepsilon} = h^\star\left[\frac{\partial\theta^{-1}(\chi)}{\partial \varepsilon}\frac{1}{\theta^{-1}(\chi)} - \left[\frac{\alpha\rho[(1-\rho)+\Lambda_h\delta]}{\delta(1-\rho+\rho\varepsilon)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]}\right]\right] \quad (71)$$

$$\frac{\partial h^\star}{\partial \gamma} = h^\star\left[\frac{\partial\theta^{-1}(\chi)}{\partial \gamma}\frac{1}{\theta^{-1}(\chi)} + \left[\frac{(1-\alpha(1-\delta))[(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)]}{\delta(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} - \frac{1}{\delta\gamma}\right]\right] \quad (72)$$

The sign of these derivatives depend on the terms between brackets. We denote respectively by ζ_A, and ζ_β the elasticities of the pollution with respect to ρ, ε and γ.

$$\zeta_A = \frac{\partial\theta^{-1}(\chi)}{\partial A}\frac{A}{\theta^{-1}(\chi)}, \qquad \zeta_\beta = \frac{\partial\theta^{-1}(\chi)}{\partial \beta}\frac{\beta}{\theta^{-1}(\chi)}$$

On the SS, the human capital level $h^\star$ is negatively correlated to the emissions of pollutions Ω. It is positively impacted by a, A. Under the following conditions $h^\star$ is positively correlated to the other parameters:

$$
\begin{aligned}
\frac{\partial h^\star}{\partial A} > 0 \quad &\Leftrightarrow \quad \zeta_A > \frac{1}{1-\alpha} \\
\frac{\partial h^\star}{\partial \beta} > 0 \quad &\Leftrightarrow \quad \zeta_\beta > \frac{1-\alpha(1-\delta)}{\delta(1-\alpha)(1+\beta)} \\
\frac{\partial h^\star}{\partial \tau} > 0 \quad &\Leftrightarrow \quad \zeta_\tau < \tau \left[\frac{\xi(1-\tau)-\alpha(1-\Omega)}{(1-\tau)(\xi-\alpha)(\Omega-\tau\xi)} \right] \\
\frac{\partial h^\star}{\partial \rho} > 0 \quad &\Leftrightarrow \quad \zeta_\rho > \left[\frac{\rho(\varepsilon-1)}{\delta(1-\rho+\rho\varepsilon)} - \frac{\rho\gamma\varepsilon(1-\alpha(1-\delta))}{\delta(1-\rho)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} \right] \\
\frac{\partial h^\star}{\partial \varepsilon} > 0 \quad &\Leftrightarrow \quad \zeta_\varepsilon > \left[\frac{\alpha\rho[(1-\rho)+\Lambda_h\delta]}{\delta(1-\rho+\rho\varepsilon)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} \right] \\
\frac{\partial h^\star}{\partial \gamma} > 0 \quad &\Leftrightarrow \quad \zeta_\gamma > \left[\frac{1}{\delta} - \frac{\gamma(1-\alpha(1-\delta))[(1-\alpha)(1-\rho+\rho\varepsilon)+\alpha(1-\rho)]}{\delta(1-\alpha)(1-\gamma)[\alpha(1-\rho)+\Lambda_h(1-\alpha)]} \right]
\end{aligned}
$$