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Tax-subsidy schemes for recycling when quantity and quality of waste matter.

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Abstract

This paper seeks to theoretically understand the impact of a tax-subsidy system (as implemented in Extended Producer Responsibility) on packaging source reduction, waste generation, and recycling in the presence of economies of scale and quality concerns in the recycling industry. We use a static equilibrium and a non-homothetic technology function to study asymmetric substitution between the virgin and the recycled material. The model displays a trade-off between recycled content and material productivity, and between waste generation and the recycling industry's profitability. A tax-subsidy scheme in the form of an excise charge and a dual subsidy restores the social optimum, providing that the recycler reaches a positive profit. We find that the excise tax favors virgin material and packaging refinement, all else equal. At the same time, it decreases the use of recycled material, sales, and total waste generation. The subsidy granted to the producer has the opposite effect. The subsidy granted to the recycler increases its profit and the recycling rate.

Keywords: Waste; Recycling; Substitutability; Plastics; Tax-Subsidy

JEL codes: H23; Q53

1 Introduction

Plastic pollution and waste generation have become a priority target in the ecological agenda in recent years. [Persson et al. \(2022\)](#) have highlighted plastic pollution as a significant threat to Earth system processes. Only 14% of plastic packaging is collected for recycling, translating into an 80 to 120 billion loss for the global economy ([MacArthur, 2017](#)). Governments, businesses and the academic community seek solutions to reduce their use. For instance, Canada launched in 2018 a strategy for zero plastic waste.¹ The European Commission has adopted a Directive to restrict single-use plastics² and a Circular Action Plan ([Communication 2020/18](#); [CEAP, 2021](#)).

Many governments have introduced environmental incentive instruments to subsidize end-of-life markets and reduce waste generation, such as the packaging tax and subsidy of collection and recycling activities. In Extended Producer Responsibility policy, a widespread policy holding producers responsible for their products' end-of-life (e.g., [Fullerton and Wu, 1998](#)), the most used instrument is akin to a tax. The financial incentive takes the form of fees³ based on the weight of packaging placed on the market. The funds are then used to finance downstream waste treatment operations. An extensive theoretical literature on tax-subsidy schemes is reviewed in Section 2, demonstrating its optimality in several theoretical contexts ([Miedema, 1983](#); [Dinan, 1993](#); [Kohn, 1995](#); [Sigman, 1995](#); [Palmer and Walls, 1997](#); [Palmer et al., 1997](#); [Huhtala, 1999](#); [Fullerton and Wu, 1998](#); [Calcott and Walls, 2000a](#); [Eichner and Pethig, 2001](#); [De Beir et al., 2007](#); [Ba, 2019](#); [Arnaud, 2017](#); [Chang et al., 2019](#); [Yi et al., 2021](#)).

¹Approved in principle by the Council of Ministers on November 23, 2018.

²Directive (EU) 2019/904 of the European Parliament and of the Council of June 5, 2019, on the reduction of the impact of certain plastic products on the environment

³The term "fee" is usually preferred to "tax", as the fee revenues are not allocated to the State's general budget but to waste management operations.

However, theoretical approaches in the literature consider substitution symmetry between recycled and virgin materials (e.g., imperfect and perfect substitution as in the Cobb-Douglas), and also sometimes, perfect substitution between waste source reduction and disposal reduction. In the latter case, society can reduce damages through recycling or material use reduction alike.

This paper proposes a static partial equilibrium model that analyses the optimality and the effect of environmental taxes and subsidies on the use of virgin materials, recycled materials, packaging refinement, sales, total waste generation and the recycling rate *-when waste quality and quantity matter-*. More precisely, the paper aims to identify the trade-off between waste reduction and recycling, and the difficulties of integrating recycled materials of lesser quality. The scheme studied comprises an excise tax on packaging material and a downstream subsidy to recycling, which are standard features of packaging EPR schemes (OECD, 2016; Joltreau, 2022; Afif et al., 2022). Additionally, an upstream subsidy for using recycled materials is included, as some EPR schemes are moving in this direction (e.g., a discount is applied on the excise tax if the packaging integrates recycled content in France and Quebec; Citeo, 2019; Eco Entreprises Québec, 2021).

We consider that *(bad) quality matters* by modeling the recycled material as an inferior input, using a product-level non-homothetic technology function. Firstly, it is essential to study the substitution between virgin and recycled materials at product level, as the mix used can be decisive for product quality. Secondly, our modeling approach permits to establish asymmetrical demand functions between these two factors of production. We reject perfect substitution on grounds of quality and risk. We also reject symmetrical imperfect substitution (e.g., Cobb-Douglas), which favours the interior solution and prevents the null utilization of the recycled input. Products made

from 100% virgin inputs are perfectly feasible, while 100% recycled products remain a technological exception. The input relationship should be asymmetrical: Firstly, the producer can do without recycled materials. Secondly, from a technological point of view, it is more difficult to do without virgin materials than recycled materials, and increasingly difficult to do without virgin materials as the content of recycled materials increases. This modeling allows us to study production choices in a more realistic way than perfect homogeneity, or an approach that favors an inner solution, or symmetrical complementarity (i.e., where the virgin material must also be compensated for by the recycled one).

In addition, *quantity matters environmentally and economically*. *Environmentally*, because we consider waste reduction a superior environmental option to recycling. Indeed, waste reduction is listed as the top priority in the Waste Management Hierarchy, while recycling is listed third, after “reuse” and “reduce”. Recycling activities are energy-intensive and lead to secondary waste and pollution. For plastics, it does not prevent the fragmentation of plastics into micro and nanoparticles, posing potential long-term risks to the environment (Obbard et al., 2014; De-la Torre et al., 2020; Kwon et al., 2015). *Economically* because we assume scale economies in the recycling sector. Reducing the amount of waste for which the recycling process is already mature can diminish the profitability of the recycling industry, and works against recycling efficiency. For materials that are not “economically” recyclable, and thus not recycled, reducing their use only benefits society. However, this may reduce the probability of developing a profitable recycling industry (little economy of scale, little investment and innovation if little volume).

This theoretical framework should mostly fit plastic recycling issues. Plastic recycling presents economic challenges, such as the need for economies of scale and the

competitiveness of recycled resin with virgin resin (price and quality). For instance, in Quebec, the sorting centres explain that too little polystyrene (PS) waste is available, so most of the PS is thrown away (Recyc-Quebec, 2018); albeit the different existing recycling techniques (mechanical, chemical, and thermal recycling; Maharana et al., 2007). Finally, recycled plastics often suffer from a quality problem, which can deteriorate the final products' attributes or costs. For instance, *Plastiques GPR*, a Quebec injection moulding plant, states that processing recycled plastic⁴ is more expensive and complex than processing virgin material.⁵ More generally, depending on the initial sorting of the waste and the technology used, it may be difficult to decontaminate recycled plastics of additives or impurities previously contained, and the properties of the material may be degraded (BIR, 2020; Demets et al., 2021).

We make several modeling assumptions, among which open-loop recycling (exogenous international markets). The model highlights a trade-off between the integration of recycled material and investment in the product's material productivity, and between waste generation and the recycling industry's profitability. In the producer's program, the demand for recycled materials decreases with material productivity investment. In addition, the product's weight increases with the recycling content, as more materials must be added to maintain the product's functionality. In the recycler's program, economies of scale play an important role. Waste reduction (either by improving material productivity or decreasing the number of products) harms the profits of the recycler, who benefits from economies of scale. However, the recycling rate maximizing the recycler's profit solely depends on financial parameters.

All else equal, we find that the excise tax favors virgin material and packaging refinement. At the same time, it decreases the use of recycled material, sales, and

⁴They do not specify a plastic-type.

⁵Interview retrieved from <https://ici.radio-canada.ca/nouvelle/1418402/plastique-recyclage-quebec-cout-importation-materiau>

total waste generation. Hence, in our model, a classic EPR scheme (where the fee does not distinguish the recycled nature of the material) favors virgin materials. All else equal, the subsidy granted to the producer has the opposite effect. These results relies on the used technology function and on the price assumption (the virgin input is more expensive than the recycled one). The subsidy granted to the recycler increases its profit and the recycling rate but does not affect production because of open-loop recycling.

We find that the tax-subsidy system restores the social optimum if the recycler generates a positive profit despite the fixed costs. If not, the recycling industry is inactive and the social optimum cannot be reached. At the social optimum, the waste level is lower, decreasing the recycler's profit. Nevertheless, the recycler receives a compensation equal to the landfill externality, which positively impacts their profit. The optimum tax level depends on the optimum recycling rate in an U shape. The tax is minimized when the recycling rate reaches the social optimum.

Section 2 reviews the literature on tax-subsidy systems for recycling. It emphasizes the modeling choices of the relationship between virgin and recycled, and of waste-related externalities. Section 3 presents the theoretical model, with the assumptions, the private agents' programs and the decentralised equilibrium values. We also present the impact of the instrument system on agents' decisions. Section 4 presents the social optimum values and the optimal system of instruments. Section 5 provides a numerical example through a simulation. Section 6 concludes.

2 Related literature

Baumol (1977) justifies the need for public waste management policies because of the

presence of externalities. He says (p.84), “In the absence of externalities, competitive general equilibrium must produce an optimal quantity of recycling, as it does of every other economic activity”. Several authors theoretically demonstrate the social optimality of a combination of tax and subsidy in the presence of waste externalities (Dinan, 1993; Kohn, 1995; Palmer and Walls, 1997; Palmer et al., 1997; Huhtala, 1999; Fullerton and Wu, 1998; Eichner and Pethig, 2001).

In Europe and North America, such tax and subsidy schemes have often been introduced under Extended Producer Responsibility (EPR) (European Commission, 2014; OECD, 2016; Joltreau, 2022; Afif et al., 2022). With EPR, the producers have to pay for the end-of-life costs of their products. The funding is used to subsidize collection and recycling activities (or other treatments, such as reuse and incineration for energy recovery). Tax and subsidy schemes are also referred to as “deposit-refund” systems in the literature, a system that “combines a tax on product consumption with a rebate when the product or its packaging is returned for recycling” (Walls, 2011). Most of the time, the term “deposit-refund” is used when the tax and the subsidy concern the same agent (e.g., the consumer). In the presence of further market failures (incorrect price transmission), and when the model details the value chain, the question arises regarding which actor to target for this tax and subsidy, or refund.

For instance, the subsidy can be attributed to recyclers to compensate for lack of profitability (De Beir et al., 2007; Yi et al., 2021), or to producers using recycled materials to compete with the price of virgin material (Miedema, 1983; Dinan, 1993; Eichner and Pethig, 2001), or both. As for the tax, it can be in the form of a material charge or a waste charge. The tax can be differentiated according to the inputs’ nature (e.g., recycled or not). This system can be called “modulated tax” (or bonus/penalty system; Arnaud, 2017) or upstream tax-subsidy systems. The refund is generally attributed

to consumers to encourage the separate collection but could be granted to recyclable processors (Walls, 2011). Hence, many combinations of instruments are possible.

Value chain considerations in the presence of market failures were raised by Holtermann (1976). She considers the case when taxing the externality itself is not possible. For instance, producers' packaging use will result in waste, considered as the externality when disposed. However, taxing the externality may not be possible. Waste disposal taxation may give rise to illegal disposal, a typical market failure in the waste literature (Dinan, 1993; Palmer and Walls, 1997; Sigman, 1995). In organizing the end-of-life of products, the difficulty lies in the inability of the market to produce positive prices for waste (Palmer et al., 1997). A combination of financial instruments such as the deposit-refund is common in the waste literature to palliate such problems (Palmer and Walls, 1997). Alternatively, it is possible to go back up the value chain to apply the tax. In a static partial equilibrium, Holtermann (1976) shows that a tax on output and a subsidy on inputs can lead to the social optimum.⁶ In parallel, due to the risk of illegal dumping, consumers mostly hand over their waste for free or through a flat tax (Miedema, 1983). Calcott and Walls (2005) and Massarutto (2014) consider that flat pricing prevents consumers from passing their recyclability upstream. As a result, the authors again raise value chain consideration and advocate that the financial instrument should directly target the producer to influence its recyclability decisions.

We present the results of the literature in more detail. Table 1 summarizes the findings from the literature and the relevant parameters for our paper.

Miedema (1983) starts from the premise that there are two inefficiencies in the waste market. First, he argues that virgin materials are underpriced because they do not include waste management costs. In contrast, not only do recyclable materials

⁶In her model, the externality affects firms' output.

include collection and recycling costs, but they also avoid disposal costs. Second, he points to waste management flat rate pricing as in [Wertz \(1976\)](#). Flat rate pricing prevents incentivizing consumers towards the purchase of less waste-intensive goods. He compares the effect of the tax and the subsidy separately in a static general equilibrium, where he assumed diseconomies of scale in material (virgin and recycled) production. He finds that virgin material fees are preferred because they maximize social welfare. Recycling subsidies are the least preferred policy in terms of net income.

[Dinan \(1993\)](#) shows the limitations of solely using virgin material fees and contradicts [Miedema \(1983\)](#)'s results. She argues that flat pricing is the only market inefficiency and that underpriced virgin materials are only a consequence of the latter. Contrary to [Miedema \(1983\)](#), she considers that recycling does not necessarily replace virgin materials in production because of open-loop recycling and exports. In a two-period equilibrium, she shows that in addition to the tax, a recycling subsidy must be used to stimulate substitution and account for the net decrease in disposal costs permitted by using recycled materials.

[Kohn \(1995\)](#) studies the effects of a combination of a disposal tax, an excise tax (on goods) and a recycling subsidy in a general equilibrium model of [Pethig \(1977\)](#) with closed-loop recycling. Any convex combination of these three instruments is efficient in his modeling framework. The first tax is applied to companies and is equal to the sum of the marginal cost of recovering and recycling the good at the end of its life. The second tax is applied to companies and is equivalent to the cost of burying post-recyclable residues imposed on recyclers. The subsidy is redistributed to companies using recycled material as an input. At the social optimum, the subsidy is equal to the first tax. The two taxes are internalized in the sale and purchase prices of the good at its end-of-life.

[Sigman \(1995\)](#) compares the virgin material tax, the deposit refund, the recycling subsidy and the recycling content in a partial equilibrium. In her model, consumers choose between recycled and virgin lead for their batteries at the same price (perfect substitutes). Households choose to recycle the battery if the recycling effort is less than the battery's scrap value. [Sigman \(1995\)](#) advocates the superiority of virgin material fees and the deposit-refund because it has lower private costs than the recycling subsidy and content standards.

[Palmer et al. \(1997\)](#) study through theoretical and empirical modeling the effects of a trust fund, disposal fees and a subsidy for solid waste reduction. They use a simple theoretical supply and demand model and estimate elasticities from past empirical data. The deposit fund appears to be the least expensive instrument. Moreover, it encourages both recycling and reduction at source. On the other hand, the other two instruments take advantage of only source reduction or only recycling as opportunities for disposal reduction.

[Palmer and Walls \(1997\)](#), in a partial equilibrium, study the optimality of recycled content standards and compare it to the deposit-refund system. Contrary to previous studies, they assume a residual associated with the production process. The standards alone are not optimal and must be combined with taxes on the final product and virgin inputs. The deposit-refund (tax on production with a subsidy on recycled products, both equal to the marginal social cost of disposal) should be preferred: it is easier to implement because it does not require as much information (calculation).

[Huhtala \(1999\)](#) uses a dynamic general equilibrium with two sectors (conventional production and recycling) to study the optimality of a tax-subsidy scheme. In the model, only the output of the conventional production (virgin resources) includes a negative externality. She finds that the optimal policy consists in taxing conventional

products and subsidizing recycled products. The ultimate utility level is determined by how tax revenues and subsidies compensate each other in monetary terms.

De Beir et al. (2007) study the need to subsidize recycling activities using an overlapping generation model of Diamond (1965) with two periods. The utility of the representative agent is deteriorated by the production of virgin goods. The company representing the recycling sector produces a perfect substitute for virgin goods. The availability of input from the recycling sector depends on the virgin used good recovered (unique input). Studying the long term optimum, the authors find that recycling should be subsidized only in sectors where it is not profitable.

Ba (2019) theoretically studies the combined effect of a tax on the extracting activity, a subsidy on the recycling activity and a fixed entry cost. Using a two-period model (Cournot's oligopoly), he shows that combining these instruments makes it possible to restore the social optimum.

Eichner and Pethig (2001), Fullerton and Wu (1998), Calcott and Walls (2000b) and Arnaud (2017) consider *green design* of products. In this literature, a specific point is made to target the upstream section of the value chain. Indeed, design decisions are taken upstream at the production stage. Changing a product's design towards environmental considerations is referred to as "eco-design" and "design-for-environment" (Calcott and Walls, 2000b). Eichner and Pethig (2001) model green design as material contents. Their model illustrates a trade-off between material intensity and recycling productivity, as the environmental externality calls for reducing the material content per unit of output, while material content increases recycling productivity. They show that if all markets for embodied material per unit output fail (inactive) and recycling benefit exceeds environmental damage, the policy instruments needed for green design

are a tax on the good supply and a subsidy on the demand for material input. [Fullerton and Wu \(1998\)](#), [Calcott and Walls \(2000b\)](#) and [Arnaud \(2017\)](#) models recyclability as a parameter. In [Fullerton and Wu \(1998\)](#), the recyclability parameter is interpreted as the share of the product's weight that can be recycled (e.g., ease of disassembly). They show that when waste is collected for free, a social optimum can be reached if upstream industries are directly targeted by a tax on packaging use and a subsidy on recyclability. To be more realistic, [Calcott and Walls \(2000b\)](#) considers the infeasibility of taxing products according to their precise recyclability. They only accept a binary differentiation of "recyclable" or "not recyclable" according to a recyclability threshold. Their results show that the deposit-refund (the monetary transfer from producers to recyclers) must be supplemented by upstream instruments. In [Arnaud \(2017\)](#)'s model, recyclability is used as vertical differentiation. The market is a duopoly with Bertrand competition. The recyclability is modeled as a parameter between 0 and 1, representing the fraction of the recyclable material in the product. The choice of the materials (recyclable or not) affects the cost of each step (sorting, dumping/recycling). With the taxation of social costs, producers differentiate too much. The optimal policy couples the taxation of social disposal costs with upstream bonus/penalty systems on sorting and dumping costs.

Eco-design activities may imply innovation activities. In [Chang et al. \(2019\)](#)'s model, the recycler decides on the recycling rate and receives a reward subsidy accordingly. The tax targets manufacturers at the source. Their results show that such a joint tax-subsidy mechanism can motivate the manufacturer to pursue eco-innovation and to incorporate recycling considerations into its production and eco-innovation decisions. [Yi et al. \(2021\)](#) studies eco-innovation with an emphasis on resource conservation using game theory (Stackelberg and Cournot). They find that an optimal joint tax-subsidy policy can maximize social welfare and incentive eco-innovations. In their model, the

tax is on the manufacturer per unit of resources consumed, and the subsidy on the waste recycler for each unit of waste product recovered.

We contribute to the theoretical literature on tax and subsidy recycling schemes. The novelty of the paper is its emphasis on the effect of these systems on waste reduction and how it can impact recycling efficiency. Some papers in the literature studied considers source reduction and recycling as perfect environmental substitutes (Miedema, 1983; Kohn, 1995; Sigman, 1995; Palmer et al., 1997). In other words, the externality (or disutility, Miedema, 1983; Kohn, 1995) generally only applies to waste disposal (e.g., Palmer et al., 1997). In this context, public policy should reduce waste disposal through source reduction or recycling. However, Dinan (1993) consider that recycled newspaper still produce disposal costs in the future, and Palmer and Walls (1997) includes a production externality. Hence, production's size still matters. Huhtala (1999) and De Beir et al. (2007) consider a pollution externality from the production process of virgin goods. Here, public policy must favor the recycling activity to the detriment of the extraction sector. In this paper, in line with the Waste Management Hierarchy, we consider waste reduction as a superior environmental option. However, recycling has an economic value, and source reduction of recyclable materials can go against recycling profitability.

In addition, we contribute to this literature by adding quality concerns on recycled material. In the literature considered, Miedema (1983); Sigman (1995); Kohn (1995); Calcott and Walls (2000a); Eichner and Pethig (2001); De Beir et al. (2007); Ba (2019) consider virgin and recycled material as homogeneous goods. Dinan (1993), Fullerton and Wu (1998) and Palmer and Walls (1997) models production with a stylized production function that depends on recycled and virgin fibres.

The relationship (i.e., substitution, complementarity) between the virgin input and the recycled input in the technological production process (or as a behavioural component) is essential and an object of debate in the waste literature (Geyer et al., 2016; Zink and Geyer, 2017; Lorang et al., 2022). In the example of the pulp industry, Lorang et al. (2022) consider that virgin and recycled material can be complementary or substitutable by parameterizing different substitution elasticities. They find that this parameter is decisive in calculating the impact of recycling on climate mitigation.⁷ Indeed, this relation (substitution *vs* complementarity) allows determining a product's maximum potential recycled content, e.g., 100% in the case of perfect substitution, bounded to the complementary relation otherwise. The authors use a CES function, where this paper does not use CES as we wish to obtain explicit solutions and study substitution asymmetry.

⁷A positive effect is found in the case of strong substitution.

Table 1: Summary of relevant theoretical studies on tax-subsidy recycling schemes

Study	Methodology	Optimal instrument(s)	Relation between the two inputs in the technology function	Externality/Disutility
Miedema (1983)	static general eq.	virgin material fees	Homogeneous goods: containers are either produced from virgin or recycled material, with identical characteristics	Negative utility if littering Indifference between recycling and disposal
Dinan (1993)	2-period partial eq.	tax at point of sale & upstream subsidy	Stylized production function	Externality of disposal (present and future)
Kohn (1995)	static general eq.	any convex combination of a recycling subsidy, excise tax & disposal tax	Production function with perfect substitution	Disposal disutility
Sigman (1995)	partial eq.	virgin material fees or deposit-refund	No production function Homogeneous goods	Objective to reduce disposal
Palmer and Walls (1997)	partial eq.	deposit-refund	Stylized production function	Solid waste disposal price and firm's residual disposal price
Palmer et al. (1997)	partial eq.	deposit-refund	No production function	Disposal externality
Huhtala (1999)	dynamic (continuous) general eq.	Tax-subsidy	Two sectors: one uses virgin, the other recycled mat. Both consumptions increase utility without explicit formulation	Disposal damage function, Externality of virgin sector
Fullerton and Wu (1998)	static general eq.	Tax-subsidy	Stylized production function	Disposal externality
Calcott and Walls (2000b)	static general eq.	Deposit-refund + upstream instrument	Perfect substitution	Disposal externality
Eichner and Pethig (2001)	static general eq.	Tax-subsidy	Perfect substitution	Externality of disposal, but recycling generates some disposal
De Beir et al. (2007)	2-period overlapping generation model	Subsidy if recycling is not a profitable activity	Two sectors: one uses virgin, the other recycled mat. both are homogeneous goods	Externality of producing a virgin good
Ba (2019)	2-period Cournot eq.	tax, subsidy and fixed entry fee	Two sectors: one uses virgin, the other recycled mat. both are homogeneous goods	Environmental benefit of recycling, Environmental damage of extraction
Arnaud (2017)	Mussa-Rosen type of model	tariff by packaging type and bonus/penalty systems	Considers material recyclability; not the use of recycled materials	Social cost of disposal
Chang et al. (2019)	partial eq.	tax-subsidy	Considers product design; not the use of recycled materials	Externality of non-recycled products
Yi et al. (2021)	Stackelberg model	tax-subsidy	Considers eco-innovation (resource efficiency); not the use of recycled materials	Externality of disposal

In this paper, we model the recycled input as an inferior input. Hence, we reject perfect substitution because of quality and risk concerns. We also reject imperfect substitution (i.e., classic Cobb-Douglas), which favors the interior solution and prevents a null utilization of the recycled input. We argue that the relation should be asymmetric: Firstly, the producer can do without recycled material. Secondly, from a technological point of view, it is more difficult to do without virgin material than recycled material, and increasingly difficult to do without virgin material as the recycled content increases.

In addition, the inputs may be linked by a complementary relation, i.e., as a compensation ‘recipe’. For example, the use of recycled material can be accompanied by the use of virgin material to increase its quality. However, we argue that this complementary relationship is unidirectional. That is, the recycled material may need a certain amount of virgin material to improve its quality, but the opposite is not true. However, this modeling choice omits that, in reality, production processes can be calibrated for a certain mix of virgin/recycled, thus creating complementarity (e.g., [Lorang et al., 2022](#)).

In the literature, non-homothetic functions have been studied by [Sato \(2016\)](#) and [Sato and Ramachandran \(2014\)](#). They start from the observation that production functions are usually homogeneous, homothetic and symmetric. [Sato \(2016\)](#) defines a class of non-homothetic functions where marginal rate of substitution relation depends on output levels. However, in this paper, we model the input relationship at the product level using a non-homogeneous function, while [Sato \(2016\)](#)’s concerns are linked to the output level. However, in the case of recycling, the concerns of using recycled materials should not depend on the production size but on maintaining a product’s functionality (e.g., packaging functions).

The following section presents the model in detail. We study how a tax and subsidy recycling scheme works in this case.

3 The model

3.1 Assumptions and general framework

Figure 1 maps the flow of materials between the various agents. In this model, we consider that the production of goods in quantity n is achieved thanks to a total use of virgin materials V and recycled materials R , bought at prices respectively p_v and p_r . Consumers buy each good at a price p . These goods become waste W all at the same time. Some will be recycled locally for a recycling rate noted τ . An international market for homogeneous recycled materials balances supply and demand for recycled materials.

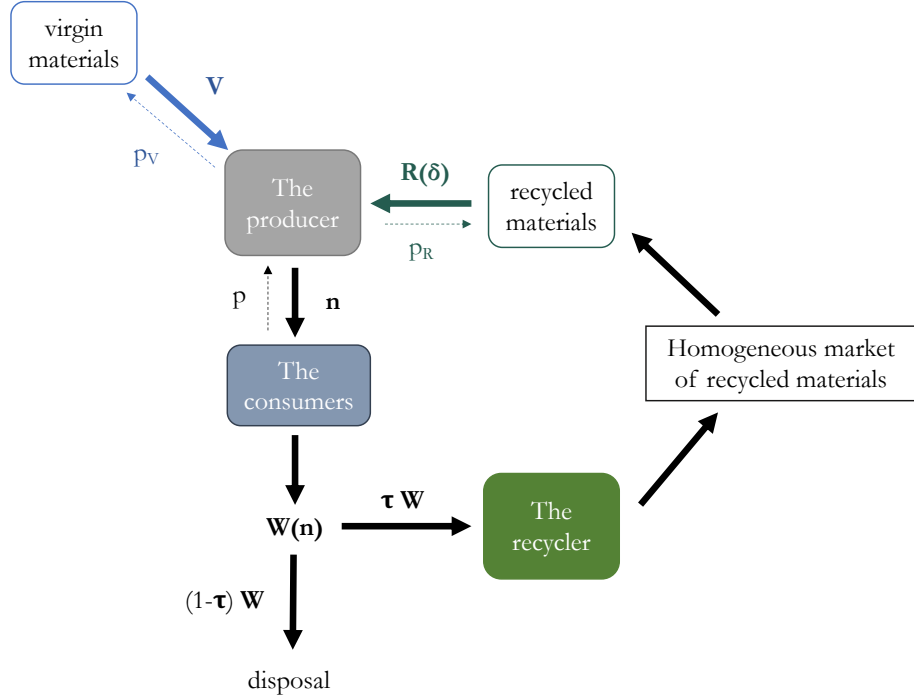


Figure 1: Conceptual framework

We detail the behavior of each agent below, including the various technical assumptions.

The producer arbitrates between using the virgin material v and the recycled material r , or a mix, to produce one unit of product q . Hence $q = 1$. The product is of constant, exogenous quality (e.g., quality standard). So the producer chooses his inputs in such a way as to optimize his profit and guarantee this quality. The total quantity of materials used for production is denoted with capital letters: V and R , whereas quantities for one unit is written in lowercase letters, v and r .

The use of recycled material incurs a default δ . The default limits the proportion of recycled material that can be incorporated into the product for a constant quality. The upstream market of virgin materials is assumed to be perfectly competitive and

exogenous.

We use a unit technology of production f . Notice that f is an implicit function such that:

$$1 = \alpha \cdot f(v, r, \delta)$$

with

$$\alpha \in [\alpha_0; \alpha_{max}]$$

$$v \in \mathbb{R}^*_+$$

$$r \in \mathbb{R}_+$$

$$\delta \in \mathbb{R}_+$$

where α is a material productivity parameter. The higher the α , the less material the company needs to produce a unit q . α_0 is the minimum material productivity of the firm, and α_{max} the maximal productivity achievable. Indeed, the company needs a minimum of material to pack its product, even to transport it to sell it in bulk (maximum productivity). We also add a minimum of productivity because over-packaging cannot be infinite.

It is convenient to write:

$$f(v, r; \delta) - \frac{1}{\alpha} = 0 \tag{1}$$

We choose a unitary technology function ($q = 1$), because this is where product quality must be guaranteed. The function uses a combination of (v, r) , which are substitutable according to an elasticity resulting from the functional form and the default parameter δ . The producer faces the purchase cost of materials p_v for the

virgin material (v) and p_r for the recycled material (r).

In total, the firm produces $n > 0$ units of composition $\alpha \cdot f(v, r, \delta)$.

Application: recycled input as an inferior input. We want to reflect that the recycled resin is an inferior input. Consequently, the substitution relationship between virgin and recycled must be asymmetric. We use a hybrid function, allowing us to have a product made of 100% virgin resin while preventing having 100% recycled resin. We add a defect parameter, which represents a linear need to use virgin to compensate for the defect of the recycled (uni-directional complementary relation).

We consider the following unit technology function (where q is defined at $q = 1$):

$$q = \begin{cases} \alpha \cdot (\ln(v - \delta) + (r + \delta)) & \text{if } r > 0 \text{ with } v > \delta \\ \alpha \cdot \ln(v) & \text{if } r = 0 \end{cases} \quad (2)$$

Considering $r > 0$, we can rewrite the technology function as an implicit function:

$$\ln(v - \delta) + (r + \delta) - \frac{1}{\alpha} = 0 \quad (3)$$

This function can be seen as a three-factor function: material productivity/refinement α , virgin v , recycled r . Figure 2 represents the isoquant curve for a given α . Figure 3 represents an isoquant curve for a given r and for a given v .

This function models a technology where, naturally, the company uses virgin resin, whose minimum use is essential and accentuated by the defect of recycled.⁸ Indeed, this technology requires (providing that r can tends to infinity) that:

⁸On the other hand, considering a negative δ (e.g., consumers' preferences for recycled materials) could improve the possibilities of substitution.

$$v > \delta \tag{4}$$

but allows to have $r = 0$. By construction here, the use of recycled materials (i.e., $r > 0$) requires that $\frac{1}{\alpha} - \delta - \ln(v - \delta) > 0$.

We prefer a hybrid function to a Cobb-Douglas function with the addition of a defect. Indeed, we show in Appendix A.2 that the Cobb-Douglas function produces symmetrical input demand functions, modulo a linear constant (the δ defect). We also prefer to avoid CES (constant elasticity of substitution) functions to be able to solve the model with explicit equations.

Figure 3 shows with the red isoquant, that there is a minimum of v to obtain $q = 1$, regarding the defect level δ and a constant r . However, the purple isoquant shows that the use of recycled can tend to zero. On Figure 3, we observe that r and α are more substitutable than v and α .

Figure 2: Isoquant curve, for a given α , and exogenous δ

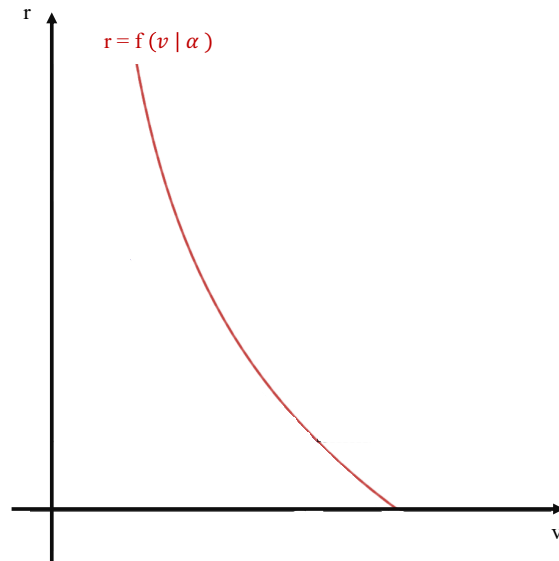
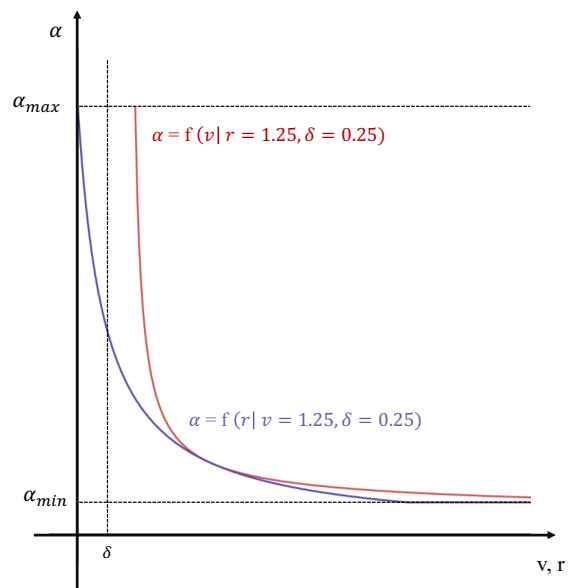


Figure 3: Isoquant curve, for a given r or v , and exogenous δ



Firms have the possibility to invest in material efficiency, i.e., to choose a high α at a cost:

$$g(\alpha) \tag{5}$$

We assume that the productivity cost function is:

$$g(\alpha) = \frac{\alpha^2}{2} \tag{6}$$

We use a square function to model that improving material productivity is less expensive when optimization is low. For instance, a standard method to refine the packaging is to use tailored materials in multi-layer films. This practice increases costs only moderately (Barlow and Morgan, 2013). With the square function, producing ever-thinner packages becomes then increasingly challenging.

The current model overlooks the threshold effect. For example, selling in bulk could be an alternative, but it would imply costly logistics changes. Modifying the packaging thickness may also require changing the machinery used (Nhamo, 2008). In this model, the reduction of packaging is mainly done on the thickness and size within a given threshold: $[\alpha_0; \alpha_{max}]$.

We consider that the cost of refining a package is unitary, particularly through all the precautions that must be taken for the good conservation of the product. However, the model omits research and logistics investment costs at the company level.

We introduce the function m : the conversion function of any entity (product unit or mass) to mass. By definition,

$$m(q) = r + v = m(r) + m(v) \tag{7}$$

The equation 7 means that the mass of a unit of product q is equal to the mass of the materials it contains, r and v . Since the variables r and v are already expressed in mass terms, their transformation by the function m is neutral.

The consumers display an aggregate demand of the form $n := A - p$, with A being the size of the market, p the product's price, and n the total quantity of products bought and produced $n = \sum_{i=1}^n q_i$. Consumers do not distinguish between different chemical compositions of the product: they are neutral towards the integration of recycled materials.

The consumption of n products generates the waste basket W . For simplicity, we assume that products are consumed within a single period. We consider the total conservation of the mass $m(n) = m(W)$.

As Dinan (1993), we assume open-loop recycling. We make this assumption for the tractability of the model.⁹ Moreover, this assumption illustrates a well-documented reality (Kellenberg, 2010; Bernard et al., 2012; BIR, 2016; OECD, 2018; Brooks et al., 2018) and allows us to analyze the recovery of the social optimum with our set of instruments in this specific situation. In our model, producers can buy recycled materials on an international exogenous market, and the recycler can sell the recycled materials on this market. The economy is assumed to be sufficiently small not to influence the global materials market. In addition, this means that the recycler may recycle more or less than the producer's demand. However, we assume that the recycler only faces a local waste supply.

The recycler faces the total waste production W . By definition, total waste production equals the total use of recycled materials (R) and virgin materials (V):

⁹Appendix E explores an endogenous recycling market. The equilibrium price is too complex to keep explicit equation forms.

$$W = R + V \tag{8}$$

We consider the same modeling for collection costs as [Fleckinger and Joltreau \(2020\)](#). The recycler collects the waste until a proportion τ over a standardized distance from 0 to 1. The transportation cost is equal to γ . The waste is uniformly distributed over $[0, 1]$ in a Hotelling fashion. The recycling facility is located at the address $x = 0$.

Hence collection costs are given by:

$$\int_0^\tau \gamma x dx = \frac{\gamma}{2} \tau^2. \tag{9}$$

The unit recycling costs are constant and equal to c . In addition, the infrastructure requires an investment of F , a fixed initial cost. The recycler produces τW of quality $(1 - \delta)$. Our modeling of an exogenous recycling market means that the quality produced by the recycler does not affect the producer's decisions. It is assumed that the world market trades a single quality of order $(1 - \delta)$. However, in reality, exports can consist of lower-quality processed waste without the receiving country being fully aware of traded qualities ([Velis, 2014](#)).

We consider the possibility for the State to implement a tax t on material use (R and V). A subsidy s_R is given to the recycler. This tax-subsidy scheme of t and s_R are the most common forms of EPR schemes ([OECD, 2016](#)). In addition, in our model, an upstream subsidy s_P is given to producers for each unit of recycled materials used in production.

Firstly, we use a dual subsidy because our model considers open-loop recycling on an exogenous international market. Thus, there is no reason for the subsidy given to

waste treatment operators to influence the world price of recycled material. So the incentive does not go back to the producer. Therefore, we suspect this dual subsidy will be necessary to restore the social optimum. Next, we do so to match the novel forms of EPR. For instance, in France a 50% discount applies to the contribution by weight of the plastic material of polyethylene units incorporating at least 50% recycled material (Citeo, 2019). In Quebec, a 20%-credit is granted if the percentage of post-consumer recycled content that meets or exceeds the established threshold (Eco Entreprises Québec, 2021).

In the model, we use the following assumptions:

A1: $p_r + t - s_P > 0$: producers are not paid to use recycled materials. Producers may have a discount on their fee but still not compensate 100% of the fee, nor price reimbursement

A2: $p_v > p_r - s_P$, the recycled resin is less costly. A sufficient condition is $p_v > p_r$. In practice, this corresponds to most cases, except in the context of high demand (e.g., green concerns, legislation¹⁰)

¹⁰Europe has mandated that clear PET plastic bottles include at least 25% recycled plastic by 2025. Historically, recycled PET was sold at a lower price than virgin PET. But recently, there has been a high demand for recycled PET in Europe due to the mandatory recycling inclusion. As a result, last year, recycled PET became more expensive than virgin PET (Chauvot, 2020).

3.2 The producer's program

The unit production cost (UPC) of the producer sums the purchasing costs of the virgin material v and the recycled material r , and the material efficiency cost function $g(\alpha)$

$$UPC = p_v \cdot v + p_r \cdot r + g(\alpha) \quad (10)$$

The producer's profit writes as:

$$\pi_{producer} = n \cdot (p - UPC) \quad (11)$$

To abstract from the effects of imperfect competition, we consider a competitive framework in which the producer's profit is equal to zero ($\pi_{producer} = 0$). This is justified by the constant returns to scale in production. Indeed, the unit production cost (UPC) is defined for each unit identically. Marginal cost is therefore equal to average cost. The producer can also be seen as a branch of a multitude of homogeneous producers, where companies can freely enter the market, until positive profits are dissipated.

The producer's program takes place in two stages:

1. The producer chooses its product design (the material productivity of a product unit α and the material mix in production (v, r)) by minimising unit production costs:

$$\min_{r \geq 0, v > \delta, \alpha_0 \leq \alpha \leq \alpha_{max}} UPC \quad (12)$$

$$s.t. \quad q = \alpha \cdot f(v, r, \alpha) \quad (13)$$

2. The producer chooses the number of units (n) by equalling its profit to zero:

$$\pi_{producer} := n(p - UPC) = 0 \quad (14)$$

with $p = A - n$.

3.2.1 First-stage

To decide on the material mix, the producer will minimize the production costs associated with purchasing materials and materials' possible association, according to the production technology f and the productivity parameter α . Since the consumer is neutral towards the integration of recycled materials, the demand for final goods does not affect this decision.

The program is:

$$\min_{v,r,\alpha} \text{UPC} = (p_v + t) \cdot v + (p_r + t - s_P) \cdot r + g(\alpha) \quad (15)$$

$$\text{s.t. } q = \alpha \cdot f(v, r, \delta) \quad (16)$$

The Lagrangian expression can be written as:

$$\mathcal{L} = (p_v + t)v + (p_r + t - s_P)r + g(\alpha) - \lambda(\alpha f(v, r, \delta) - q) \quad (17)$$

where λ is the coefficient associated to the technology constraint.

The first-order conditions with respect to v and r give:

$$\frac{p_r + t - s_P}{p_v + t} = \frac{\frac{df(v,r,\delta)}{dr}}{\frac{df(v,r,\delta)}{dv}} := e \quad (18)$$

where e denotes the technical rate of substitution of replacing v with r in q ($MRS_{v \rightarrow r}$).

We expect the marginal rate of substitution (e) to be less than 1. In the same product, if we want to add one unit of recycled resin, we will remove less than one unit of virgin resin; because some of the virgin resin will have to remain to improve the properties of the recycled material. At the optimum (and under the assumption of an interior solution), the productivity of the input factors is equal to the cost ratio when the producer minimizes production costs.

At the producer's optimum, the price of the inferior input should respect the following:

$$p_r = (e - 1) \cdot t + e \cdot p_v + s_P \quad (19)$$

Without economic instruments ($t = 0, s_P = 0$), the price of the recycled is equal to $e \cdot p_v$.¹¹ Under $e < 1$, the price of recycled must therefore be lower than the price of virgin resin to achieve the inner solution. Under $e < 1$ and to maintain equilibrium quantities, the material tax should induce a downward pressure on the price of recycled. At the same time, the subsidy should allow the recycler to offer a higher price. However, in our context of a globalized waste market, national policies are not likely to interfere with the global price p_r . Hence, adjustment is made by quantities.

The producer decides on the material productivity of the product for a given material mix (r^*, v^*) .

From the first-order condition with respect to α we obtain:

$$g'(\alpha) = \lambda \cdot f(v, r, \delta) \quad (20)$$

And hence:

¹¹Since a unique quality is traded, and if producers have the same technology, e is homogeneous across countries and is therefore reflected in the global price p_r .

$$\alpha^* = \min \left[\alpha_{max}, \frac{1}{g'(\alpha)} \cdot \frac{p_r + t - s_P}{df/dr} \cdot f(v, r, \delta) = \frac{1}{g'(\alpha)} \cdot \frac{p_v + t}{df/dv} \cdot f(v, r, \delta) \right] \geq \alpha_0 \quad (21)$$

The more expensive the materials (virgin or recycled) and the cheaper the packaging refinement $g'(\alpha)$, the more the company will invest in material productivity (high α^*).

It is important to note that consumers do not have an explicit preference (expressed in the demand function) towards over-packaging or ‘zero waste’ products. However, we can consider alpha’s lower and upper bounds as technical and cultural constraints.

We provide below the explicit results of the inner solution for the chosen functional forms:

$$v^* = \frac{p_r + t - s_P}{p_v + t} + \delta \quad (22)$$

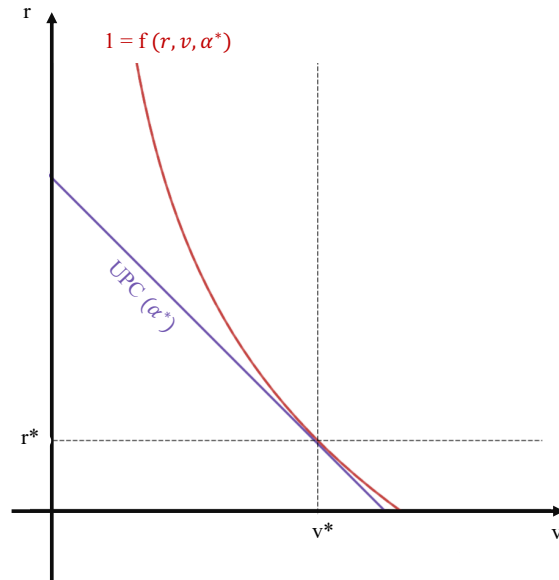
$$\lambda = (p_r + t - s_P)^{\frac{2}{3}} \quad (23)$$

$$r^* = \frac{1}{\alpha^*} - \ln \left(\frac{p_r + t - s_P}{p_v + t} \right) - \delta \quad (24)$$

$$\alpha^* = \lambda \cdot f(v, r, \delta) = (p_r + t - s_P)^{\frac{1}{3}} \quad (25)$$

The use of virgin materials increases with its price competitiveness and the technical difficulty to use recycled materials (i.e., the recycle’s default δ). The material productivity α increases with the price of the recycled resin. It is possible to see the problem as follows: The producer who uses virgin resin as the superior input, will choose to substitute, with recycled resin, or with material productivity (“void”). The use of the recycled resin is constrained by its technological defect, and is disadvantaged if its price is high, in particular, compared to the price of the virgin resin.

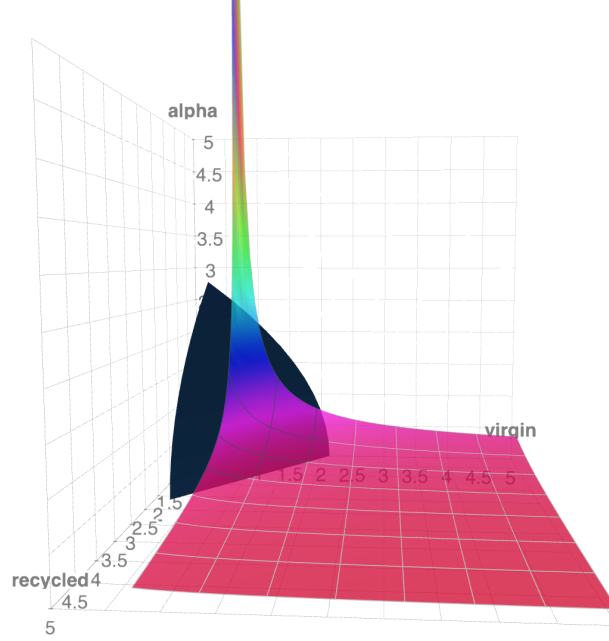
Figure 4: 2D-Illustration of the interior solution for one unit q (with optimal material productivity α^*)



Notes: *UPC* means 'unit production cost'. v (respectively, r) stands for the quantity of virgin material (resp. recycled material) used in one product.

Figure 4 illustrates the results of the interior solution ($r^* > 0$). It graphically represents the isoquant for a production unit in the (v, r) plane and the optimized isocost line for α^* (the productivity parameter) and the material mix (r^*, v^*) . Figure 5 represents the interior solution in three dimensions (v, r, α) . The coloured plane represents the isoquant for one production unit and the dark curved plane the isocost. These two planes are tangent and allow to find a unique equilibrium solution, optimizing the producer's program.

Figure 5: 3D-Illustration of the interior solution for one unit q



Parametrization used: $\delta = 0.2$, and for lisibility of the plot $r > 0.4$, $v > 0.75$ (math3D.org)

If the defect and price ratio are too unfavorable to the recycled product, $r^* = 0$. When no recycled is used in the product, there is no need for compensating the defect and hence $\delta = 0$. In this case, the relative quantity of inputs is equal to the inverse ratio of production factor costs at the producer's optimum, such as:

$$[r = 0] \quad \frac{v}{\alpha} = \frac{\alpha^2}{p_v + t} \quad (26)$$

The proof is presented in Appendix A. In the following, we only consider the interior solution for the three factors, i.e., where $v > 0$, $r > 0$ and $\alpha > 0$.

3.2.2 Second-stage

To define the total quantity n under perfect competition, we equalize the producer's profit to zero:

$$\pi_{producer} := n \cdot (p - UPC) \quad (27)$$

with $p = A - n$. We obtain:

$$n^* = A - UPC \quad (28)$$

The production cost of an additional unit is constant and equal to $UPC(v^*, r^*, \alpha^*)$.

Hence:

$$n^* = A - (p_v + t)v^* - (p_r + t - s_P)r^* - g(\alpha^*) \quad (29)$$

From the first-stage, we obtain for an interior solution:

$$n_{v^* >, r^* > 0}^* = A - \frac{3}{2}(p_r)^{\frac{2}{3}} - \delta(p_v - p_r) - p_r \left(1 - \ln \left(\frac{p_r}{p_v} \right) \right) \quad (30)$$

The waste generated by the production depends on the unit weight of the product ($r + v$) and the number of products sold (n). Thus the amount of waste W equals $n(r + v)$.

This number of units is what becomes waste for society and is available to the recycler. In the model, there is no accumulation of goods, and all are thrown away at the end of the first period (i.e., packaging). For an interior solution, we obtain:

$$W_{v^* >, r^* > 0} = \left[A - \frac{3}{2}(p_r)^{\frac{2}{3}} - \delta(p_v - p_r) - p_r \left(1 - \ln \left(\frac{p_r}{p_v} \right) \right) \right] \cdot \left[\frac{1}{p_r^{\frac{1}{3}}} - \ln \left(\frac{p_r}{p_v} \right) + \frac{p_r}{p_v} \right] \quad (31)$$

Lemma 1 (Interior solution with economic instruments). *In the presence of a tax and*

a subsidy, the producer uses v^* and r^* for each unit, with a material productivity parameter α^* , produced in n^* quantities.

$$v^* = \frac{p_r + t - s_p}{p_v + t} + \delta \quad (32)$$

$$r^* = \frac{1}{\alpha^*} - \delta - \ln\left(\frac{p_r + t - s_p}{p_v + t}\right) \quad (33)$$

$$\alpha^* = (p_r + t - s_p)^{\frac{1}{3}} \quad (34)$$

$$n^* = A - \frac{3}{2}(p_r + t - s_p)^{\frac{2}{3}} - \delta(p_v - p_r + s_p) - (p_r + t - s_p) \left(1 - \ln\left(\frac{p_r + t - s_p}{p_v + t}\right)\right) \quad (35)$$

The society produces waste in quantity $W = n^*(r^* + v^*)$.

Proof. The proof is presented in Appendix A. ■

Proposition 1. *With a production technology where the virgin resin is a superior input, the producer decides to substitute the virgin input with recycled resin and/or material productivity. Hence, the demand for recycled materials decreases with material productivity. A high price of recycled materials favors investments in material productivity. In parallel, material productivity is limited by the poor quality of recycled materials.*

3.3 The recycler's program

The profit of the recycler sums the sale revenues of recycled materials $p_r R$ and retrieves the recycling cost c , collection costs $\frac{\gamma}{2}\tau W$, and the fixed cost F .

Hence we can write:

$$\pi_{recycler} := n \left[\tau \left(p_r - c - \gamma \frac{\tau}{2} \right) (r + v) \right] - F \quad (36)$$

The recycler decides on the quantity of recycled R according to the amount of waste available W , the production costs, the market price for recycled materials p_r , and the recycling subsidy $s_R \geq 0$. We consider that there exists an international market of recycled materials, such that the price of recycled materials p_r is exogenous, and the recycler does not need to respond to local demand.

We assume that the recycler decides to recycle only if its profit is positive $\pi_{recycler} \geq 0$. With a recycling subsidy s_R , the recycler's profit is written:

$$\pi_{recycler} = (p_r + s_R - c) \tau W - \frac{\gamma}{2} (\tau)^2 W - F \quad (37)$$

The condition of profitability, noted P, of the recycling industry is

$$P : (p_r + s_R - c) \cdot \tau W - \frac{\gamma}{2} (\tau)^2 W \geq F \quad (38)$$

This condition is a prerequisite for the existence of a recycling channel for plastic packaging. The probability of meeting this condition increases with the price of the recycled material, the subsidy, and the amount of available waste.

If we were to replace the price of the recycled material p_r by the equalization of the

marginal rate of substitution of the implicit form with the price ratio (equation 19), we would obtain:

$$P : (ep_v - (1 - e)t + s_P + s_R - c) \cdot \tau \cdot n \cdot (r + v) \geq F \quad (39)$$

In this case, the recycling subsidy could be allocated to either the producer or the recycler to meet the profitability condition of the recycling industry. Several factors increase the probability of the existence of a recycling channel: a high virgin material price, high recycled material quality (e close to 1), subsidies, a large waste quantity.

However, in our applied modeling and chosen functional form, we consider p_r to be exogenous and the result of global supply and demand for recycled materials. Thus, the world price must include many parameters other than the local subsidy (the financial parameters of all the other countries). In this case, allocating the subsidy to the producer and/or recycler makes a difference.

To solve the recycler's program, we calculate:

$$\max_{\tau} \pi_{recycler} \quad (40)$$

$$\pi_{recycler} \geq 0 \quad (41)$$

We obtain :

$$\tau^* = \frac{p_r + s_R - c}{\gamma} \text{ if } \pi_{recycler} \geq 0 \quad (42)$$

$$\text{else } \tau^* = 0 \quad (43)$$

The recycling rate (τ) is strictly positive if recycling activities are profitable. If recycling operations are unprofitable, the rate equals zero, as no company is interested in pursuing recycling activities. This leads us to the following proposition:

Proposition 2. *The quantities of waste affect the industry's profitability and, therefore, its probability of existence. When the branch is profitable, the recycling rate is strictly positive. It increases with the recycled price and the subsidy and decreases with collection and processing costs. The recycling rate is not affected by the uniformly-distributed waste quantities.*

3.4 The consumers' surplus

Consumers are not concerned with the product composition, only with the quantity consumed (n). Thus, under constant marginal utility of income, the consumers' surplus is:

$$S_{cons} = \int_0^n D(n)dn - pn = \frac{n^2}{2} \quad (44)$$

where $D(n) = p$ is the equation describing the aggregate demand curve in the (n, p) plane, with $p = A - n$.

3.5 Externalities

On the one hand, we consider the externalities linked to the production of the product. We denote e_V and e_R , the externality for, respectively, extraction and recycling,

Recycling produces emissions and by-product waste. However, we assume that recycling is less detrimental to society than extraction since it avoids degrading ecosystems ($e_v > e_r$). The extraction branch is not modeled in our economy. It can be seen as an outside industry whose externality accounts for pollution from transportation

and the degradation of ecosystem services.

The unit environmental production cost $UEPC$ is, therefore:

$$UEPC = e_V v + e_R r \quad (45)$$

On the other hand, we consider the externalities linked to the end of the product's life. Like [Kohn \(1995\)](#), we assume that waste disposed in landfill degrades into a single period with a pollution e_D . The disposal causes pollution within this period, but we do not observe pollution accumulation.

The unit disposal costs are:

$$UDC = e_D(1 - \tau)(r + v) \quad (46)$$

We denote s_w the unit social cost of waste management:

$$s_w = e_D(1 - \tau) - \tau(p_r - c - \frac{\gamma\tau}{2}) \quad (47)$$

The unit social cost of end-of-life also adds the recycler's marginal profit. The externality of recycling is not counted in the end-of-life externalities to avoid double counting.

Externalities are considered in calculating the social optimum in [Section 4](#).

We summarize in [Table 2](#) the different notations used in the paper.

variable	description
q	One unit of product ($q = 1$)
α	Material productivity parameter
n	Number of products
A	Size of the market
p	Product's price
v	Quantity of virgin material used in q
r	Quantity of recycled material used in q
V	Total quantity of virgin material used in production
R	Total quantity of recycled material used in production
δ	Default associated to the recycled material
p_v	Price of the virgin material
p_r	Price of the recycled material
e_v	Externality of extraction
e_r	Externality of recycling
s_w	Unit social cost of waste management
W	Society's waste basket
τ	Recycling rate
c	Processing cost of recycling
γ	Marginal collection cost
F	Fixed cost of recycling
t	Material tax (applied to r and v)
s_P	Subsidy granted to the producer (for use of R)
s_R	Subsidy granted to the recycler (for producing R)
UPC	Unit production cost
$UEPC$	Unit environmental production cost
UDC	Unit disposal cost

Table 2: Notations used throughout the paper

3.6 Discussion on waste intensity and waste generation

The product will generate more or less waste depending on the material mix and the associated α parameter. The product's weight equals its material composition: $m(q) = v^*(\alpha^*) + r^*(\alpha^*)$. Figure 6 illustrates the product's waste intensity (the product's weight) when the price of the recycled material (p_r) varies (and therefore, with it, the quantities of materials and the productivity parameter α). Figure 7 illustrates the product's waste intensity when the price of the virgin material (p_v) varies (but remains superior to the recycled one).

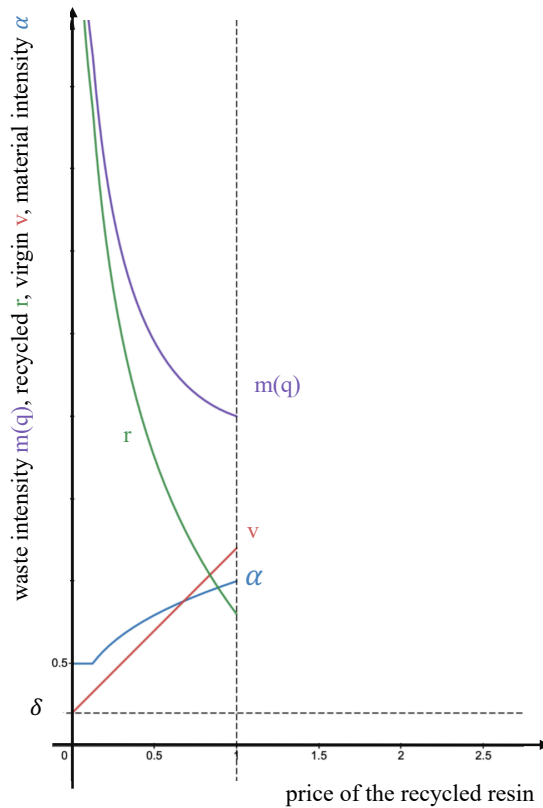


Figure 6: Illustration of the product's waste intensity as the price of the recycled material varies; with $v > \delta$, $\delta < 1$ and $p_r < p_v$

In Figure 6, we observe that the logarithm function causes an infinitely increasing amount of recycled material to compensate for the loss of virgin resin to maintain the product's quality. As the recycled price p_r decreases, the use of recycled material becomes very high compared to virgin material. The producer has to add more and more material to obtain the desirable quality of the product. Consequently, the product becomes more and more waste intensive (the weight increases). The material productivity parameter α increases with the recycled price, as r decreases. The recycled resin is substituted both with virgin resin and higher material productivity. Overall, the graphic highlights a trade-off between more recycled resin and the product's weight. A possible interpretation could be that the recycled resin is more fragile and cannot be refined as much as virgin plastic.

In Figure 6, we observe that when the virgin price augments, the virgin resin is substituted with the recycled resin. As the substitution is not symmetric, the product's weight increases because more recycled material is needed to replace the virgin resin. However, the material productivity parameter (α) does not react to the price increase. A possible interpretation is that it is impossible to associate the substitution (more recycled content) with higher material productivity, so material productivity only reacts to the recycled price when the substitution occurs in the opposite direction (more virgin content).

Does this effect correspond to reality, i.e., is recycled plastic more waste/material intensive and can this be attributed to its fragility?

The strength, flexibility, and lightness of polymers are attributed to the length of the chains of the atoms composing them and how they are arranged. When plastics are recycled, their polymer chain is shortened, which results in degraded properties (BIR,

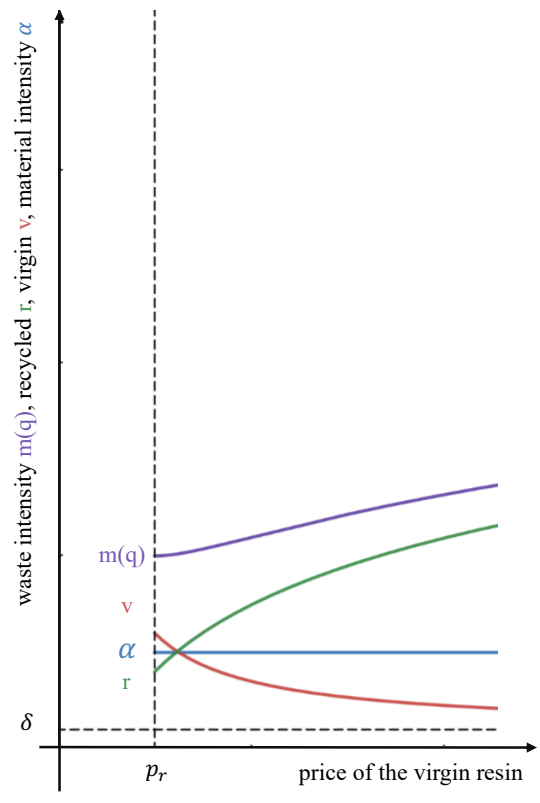


Figure 7: Illustration of the product's waste intensity as the price of the virgin material varies; with $v > \delta$, $\delta < 1$ and $p_r < p_v$

2020). Demets et al. (2021) reviews the technical properties¹² of three recycled resins and the substitution possibilities with virgin plastics. At the product level, they state that "mechanical properties of the recycled plastics appear to be reasonably close to that of the virgin plastic counterpart, mostly being a bit lower" (p.5). The presence of impurities can further negatively impacts the properties. Finally, the authors warn that the generalization of results is difficult due to product specificities.

Consequently, packaging refinement possibilities may be reduced when using recycled material, either because of the material's technical properties (however, relatively comparable; Demets et al., 2021), or because virgin plastic is often added to compensate for this quality degradation (limiting mass reduction possibilities or recycled content possibilities).

However, the situation on the far left of the graph, which represents an infinitely heavy product using almost exclusively recycled material, is not likely to happen. The model assumes constant product quality.¹³ In reality, high quality with close-to-100% recycled content is not possible for certain products. Manufacturers may reduce product quality rather than make the product a lot heavier (e.g., slightly grayer colors). Secondly, this situation corresponds to a meager relative price for recycled material, close to zero, which does not correspond to the observable economic situation.¹⁴

Finally, in reality, the waste intensity due to the use of recycled material can increase on a larger scale than that of the product. Demets et al. (2021) explain that recycled plastics can pose issues with the suitability of processing techniques. For instance, a Quebec industrialist states that it is easier to process virgin material and that the rejection rate is five times higher for recycled materials.¹⁵ Hence the use of

¹²elastic modulus, tensile strength, elongation at break and flow during processing

¹³In our setting, the fact that the product is heavy or light does not affect its quality

¹⁴Usually, the price of recycled material tries to keep pace with the price of virgin material, with a discount (with a few exceptions; e.g., Chauvot, 2020) to remain competitive (BIR, 2020).

¹⁵Interview retrieved from [https://ici.radio-canada.ca/nouvelle/1418402/plastique-recyclage-](https://ici.radio-canada.ca/nouvelle/1418402/plastique-recyclage)

recycled plastics could produce more waste at the firm level. According to [Castro et al. \(2022\)](#), increased recycled use, through legal recycled material content, increases material per product. The authors rely on the study of [Nhamo \(2008\)](#): In South Africa, the minimum thickness of plastic bags was legally required to increase to encourage recycling. In this case, recycling is associated with heavier products, but in anticipation that more oversized packaging will facilitate collection and recycling, and not due to manufacturing difficulties. In a similar vein, multi-layer packaging, used to reduce the volume of packaging material, comes at the expense of recyclability ([Barlow and Morgan, 2013](#)).

3.7 Impact of the tax and the subsidy

Table 3 summarizes the impact of the tax and subsidy on the different variables in the model.

Table 3: Impact of the tax and the subsidy, assuming $p_v > p_r - s_P$

variable	$\Delta t > 0 s_P$	$\Delta s_P > 0 t$	$\Delta s_R > 0 (t, s_P)$
virgin v	+	-	n.a
recycled r	-	+	n.a
refinement α	+	-	n.a
weight, waste/mat. intensity $m(q)$	-	+	n.a
sales n	-	+ if 48	n.a
Tot. waste W	-	+ if 48	n.a
Recycler's profit $\pi_{recycler}$	-	+ if 48	+
Recycling rate τ	none (only profitability condition)	none (only profitability condition)	+

Condition 48 writes as:

$$\frac{dn}{ds_P} > 0 \Leftrightarrow (p_r + t - s_P)^{\frac{-1}{3}} - \ln\left(\frac{p_r + t - s_P}{p_v + t}\right) > \delta \Leftrightarrow r^* > 0 \quad (48)$$

Implementing a material tax (t) increases the use of virgin materials in the product (v) and the productivity parameter (α) and reduces the use of recycled materials (r) and waste intensity ($m(q)$). Subsidizing the purchase of recycled resins increases the use of recycled materials and reduces the use of virgin resins, which implies a decrease in the material productivity parameter and an increase in the product's waste intensity.

The tax decreases sales volume while the subsidy increases sales via a reduction effect on the unit cost of production. Thus, the overall waste volume rises with the recycled material subsidy, while the tax reduces the waste volume through a downward pressure on sales and waste intensity per product.

Why does the material use tax favor the use of virgin plastics? Figure 8 illustrates

the impact of the tax on the material mix. An increase in t increases α and the relative price $\frac{p_r+t-s_P}{p_v+t}$ in favor of v , since $p_v > p_r - s_P$. When we pass from equilibrium E_1 to E_2 , the recycled resin is substituted with material efficiency and virgin resin.

All else equal, the excise charge decreases the recycler's profit because it reduces the total supply of waste through the number of sales and the product's weight. The subsidy granted to the producer has the opposite effect. The subsidy given to the recycler increases its profit and the recycling rate, all else equal, providing a positive profit. The upstream tax and subsidy do not influence the recycling rate, except through the profitability condition (threshold $\tau > 0$). The downstream subsidy do not influence producers' decision because international trade drowns out price transmission.

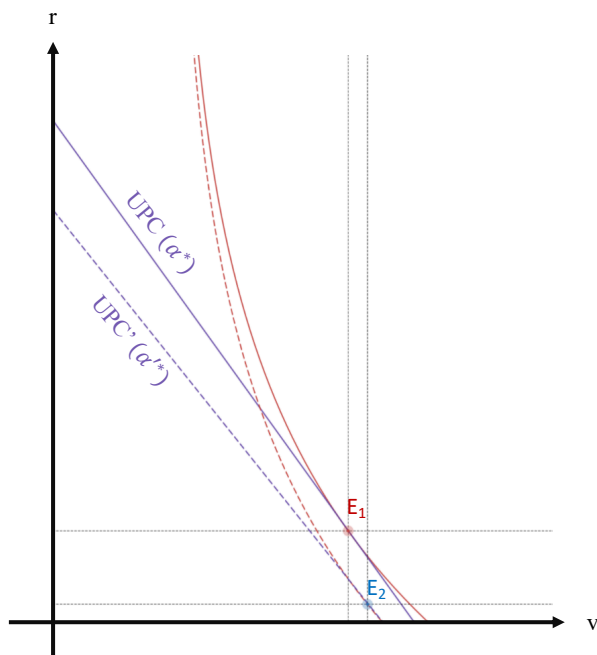


Figure 8: Interior solution ($v > 0, r > 0$) when t increases and s_P is fixed

Proposition 3 (Impact of economic instruments on waste generation). *The material use tax reduces society's waste generation and, thus, the recycler's profit, all else*

being equal. It encourages companies to refine their packaging and puts downward pressure on sales. The reduction in packaging use is accompanied by an increase in the proportion of virgin resin, due to a favorable price ratio and a quality allowing for refinement.

Subsidizing the use of recycled resin by the producer has the opposite effect overall. It encourages the use of recycled resin and therefore discourages the refining of packaging. The waste intensity per product increases. In addition, the subsidy induces a downward pressure on the product's unit cost and positively affects sales. Thus, the subsidy increases the amount of waste in society, all else equal.

Considering open-loop recycling, the subsidy granted to the recycler increases its profit and the recycling rate but has no impact on production.

Proof. The proof is presented in Appendix B. ■

4 The social optimum

In this section, we calculate the optimal values of the model variables, denoted by the index OP . The calculation of the optimum takes into account the externalities defined in Section 3.5. We then compare these solutions with the equilibrium values of private agents, calculated in Section 3 and denoted by the index $*$. By comparing these two outcomes, we identify a system of instruments able to restore the social optimum in the decentralized equilibrium.

The social optimum is solved in two stages.

4.1 First stage

The social planner minimizes the damages linked to the product's composition. The Lagrangian expression can be written as:

$$\mathcal{L} = \text{UPC} + \text{UEPC} + \text{UDC} - \lambda(\alpha f - q) \quad (49)$$

with λ , the coefficient associated with the technology constraint.

The interior solution of the first stage is:

$$v^{OP} = \frac{p_r + e_R + s_w}{p_v + e_V + s_w} + \delta \quad (50)$$

$$\alpha^{OP} = (p_r + e_R + s_w)^{\frac{1}{3}} \quad (51)$$

$$r^{OP} = \frac{1}{(p_r + e_R + s_w)^{\frac{1}{3}}} - \delta - \ln \left(\frac{p_r + e_R + s_w}{p_v + e_V + s_w} \right) \quad (52)$$

with $s_w = e_D(1 - \tau) - \tau(p_r - c - \frac{\gamma\tau}{2})$, the unit social cost of waste management.

At the social optimum, material quantities integrate their external costs at production and end-of-life.

4.2 Second stage

The social planner maximizes the total welfare \mathcal{S} given a specific product's composition, which sums up the consumers' surplus S_{cons} , the producer's profit $\pi_{producer}$, the recycler's profit $\pi_{recycler}$ and retrieves the environmental costs EC .

$$\max \mathcal{S} = S_{cons} + \pi_{producer} + \pi_{recycler} - n(\text{UEPC} + \text{UDC}) \quad (53)$$

The second stage gives us the following:

$$n^{OP} = A - \text{UPC}(r, v) - \text{UEPC}(r, v) - s_w(\tau)(r + v) \quad (54)$$

$$\tau^{OP} = \frac{p_r - c + e_D}{\gamma} \quad (55)$$

The number of products will be lower because external costs are deducted in addition to private production costs. The recycling rate is increased by the landfill externality that it avoids.

4.3 Optimal set of instruments

By comparing the solutions of social equilibrium and decentralized equilibrium, we can define the value of the instruments that restore the social optimum in decentralized equilibrium.

Lemma 2 (Social optimum with economic instruments). *Providing that the recycler makes a positive profit, it is possible to restore the social optimum with the following fiscal system:*

$$t = e_v + s_w(\tau) \quad (56)$$

$$s_P = e_v - e_r \quad (57)$$

$$s_R = e_D \quad (58)$$

The existence of the social optimum is not guaranteed because of the recycler's fixed cost. It is not reached if $\pi_{recycler} < 0$, i.e., if $\frac{(p_r + e_D - c)^2}{2\gamma} < \frac{F}{n^{OP}(r^{OP} + v^{OP})}$.

Proof. The proof is presented in Appendix D. ■

As mostly found in the literature review, a tax-subsidy scheme allows for restoring optimality. The material tax t sums up the extraction externality and the unit social cost of waste management (including recycling costs). The producer receives the externality differential between virgin and recycled material as a subsidy (s_P). As in [Dinan \(1993\)](#), the recycler receives the amount of the landfill externality as a subsidy (s_R), i.e., the cost he avoids for society.

In our model, the material tax (t) depends on the recycling rate (τ). The social cost of waste management (s_w) is linked to the recycling rate (τ) according to an U-shaped function. Before the optimal recycling rate, the cost function (s_w) decreases with the recycling rate. Once the optimum recycling rate has been exceeded, the function increases with the recycling rate. This is because the economic costs of recycling versus landfilling are becoming too high. The same variation applies to the optimal material tax, modulo a constant (e_v).

An alternative optimal fiscal system could be written as follows:

$$t_v(:= t) = e_v + s_w(\tau) \tag{59}$$

$$t_r(:= t - s_P) = e_r + s_w(\tau) \tag{60}$$

$$s_R = e_D \tag{61}$$

with t_v a tax on virgin materials, and t_r on recycled ones.

In all cases, the model shows that the fiscal price of materials needs to be differentiated because of the different production externalities. Moreover, restoring the social optimum requires a dual subsidy, as the globalized recycling market makes it impossible to ensure that subsidies are passed on in the local value chain.

The results of our model have political implications in that they differ from the

financial transfers provided for in the EPR systems. Under EPR, producers pay a fee that is supposed to reflect the waste management costs incurred by their products. Hence producers pay a price corresponding to s_w (with $s_w = c_D(1 - \tau) - \tau(p_r - c - \frac{\gamma\tau}{2})$), where c_D correspond to the landfill costs (which may or may not be equal to the landfill externality e_D). The funding is then transferred to waste operators. If production externalities (e_v and e_r) are not reflected in the price of materials or other tax systems, the social optimum cannot be achieved through EPR. The use of materials and the production of waste will be under-priced and, therefore, too great. Furthermore, the recycler must receive a subsidy equal to the landfill externality (e_D) rather than the financial transfer from producers (s_w), as foreseen under EPR principles. Note that results may change in a dynamic model dealing with securing the supply of materials. In such a scenario, recycling would have advantage in ways beyond being greener and reducing landfill waste.

5 Simulation

We provide here a simulation to compare the social optimum with the business-as-usual scenario (BAU). The absence of tax, subsidies, and externalities internalization defines the BAU. A modified BAU scenario is calculated due to the negative profit of the recycler, which exits the market.

Table 4 provides the different parameter values. Table 5 distinguishes between values at the social optimum (restored by the optimal tax and subsidies) and the BAU situation.

In Table 5, we observe that the product's composition changes for the chosen parameters. At the social optimum, the packaging contains less virgin and more recycled

plastic (in absolute and relative terms). However, the recycled content is still relatively low (31%), although recycling is three times less pollutant than extraction. Material costs have increased due to the tax. Hence, the company invests more in material efficiency (higher α). Accordingly, the product's weight remains similar, despite the increase of recycled content. Unit production costs are higher, which leads to lower production and higher product prices. Thus, we observe a significant decrease in consumer surplus compared to the BAU scenario. This is because the consumer surplus here only depends on the number of units. It does not consider the usage value and the resulting circular economy strategies (e.g., sharing economy¹⁶).

The economy produces less waste at the optimum, and the recycling rate drastically improves (from 0% to 65%). At the social optimum, the local supply of recycled materials is larger than the local demand. This is possible because we have assumed the presence of international markets, and thus, recyclers can sell internationally. The disposal costs drastically decrease. We have allowed a positive profit for the recycler, which has increased significantly.

¹⁶For example, consumers sharing goods for a use comparable to the previous one could lead to a smaller drop in surplus.

Table 4: Parameters

parameter	description	value
p_v	Price of the virgin material	2
p_r	Price of the recycled material	1.8
δ	The recycled's default	0.5
e_v	Externality of extraction	1.5
e_r	Externality of recycling	0.5
e_D	Externality of disposal	1
c	Processing cost of recycling	1.5
γ	Marginal collection cost	2
F	Fixed cost of recycling	3
A	Market's size	20
outcome (social optimum)	description	value
$s_w(\tau^*)$	net social cost of end-of-life	0.6
t	optimal excise tax	2.1
s_P	optimal upstream subsidy	1
s_R	optimal downstream subsidy	1

Table 5: Simulation

	Social optimum	BAU	BAU modified
virgin material (v)	1.2	1.4	1.4
recycled material (r)	0.6	0.4	0.4
recycled content ($\frac{r}{r+v}$)	31%	23%	23%
material efficiency (α)	1.4	1.2	1.2
product's weight ($r + v$)	1.8	1.8	1.8
unit production cost (UPC)	7.5	4.3	4.3
unit environmental production cost (UEPC)	2.1	2.3	2.3
number of units (n)	12.5	15.7	15.7
demand of recycled material ($R = nr$)	6.9	6.7	6.7
consumers' surplus	77.9	123.1	123.1
product's price (p)	7.5	4.3	4.3
producer's profit	0.0	0.0	0.0
waste generation ($W = n(r + v)$)	21.9	28.7	28.7
recycling rate (τ)	65%	15%	0%
local supply of recycled material (τW)	14.3	4.3	0.0
disposal costs	7.7	24.4	28.7
recycler's profit	6.3	-2.4	0.0
social welfare	84.2	60.1	58.1

6 Conclusion

This paper considers a system of a material tax (in the form of an excise charge) with a dual subsidy (one granted to the producer, another to the recycler). We analyze the impact of such a system when the quality and quantity of waste matter. We study the quality problem by modeling the recycled material as an inferior input (using a product-level non-homothetic production function). We analyze the issue of quantities by modeling economies of scale in the recycling industry and considering pollution from recycling activities, on top of disposal and virgin production.

Our paper has implications for the theoretical literature on tax-subsidy schemes for recycling (Miedema, 1983; Dinan, 1993; Kohn, 1995; Sigman, 1995; Palmer and Walls, 1997; Palmer et al., 1997; Huhtala, 1999; Fullerton and Wu, 1998; Calcott and Walls, 2000a; Eichner and Pethig, 2001; De Beir et al., 2007; Ba, 2019; Arnaud, 2017; Chang et al., 2019; Yi et al., 2021). Our technology function displays a trade-off between increasing recycled content and material productivity. This effect cannot be identified with symmetric input relationships, as used in the literature. Indeed, in our modeling, recycled material must be compensated by virgin resin to improve its qualities, according to an additive linear factor and a non-linear dependency relationship (it is becoming increasingly difficult to reach the same quality with less and less virgin resin). So, when the producer replaces virgin resin with recycled resin, material productivity possibilities become more limited. The effects due to scale economies in the recycling industry are more standard. The reduction in waste production, through material productivity or the decrease in the number of products, harms the recycler's profit. However, the recycling rate that maximizes profit depends solely on financial parameters.

As it is standard in the literature, we show that a tax-subsidy system restores the

social optimum. However, the social optimum is only reached if the recycler makes a positive profit despite the fixed costs. If not, the recycling industry is not active. The waste level is lower at the social optimum, reducing the recycler's profit. But on the other hand, the recycler is compensated by a sum equal to the amount of the landfill externality, which positively affects the profit. The optimum tax level depends on the optimum recycling rate in an U shape. The tax is minimized when the recycling rate reaches the social optimum.

Finally, this paper has political implications regarding implementing a tax-subsidy scheme such as in Extended Producer Responsibility. Under our modeling assumptions, we study the impact of the tax and subsidy on the material intensity of the product, its recycled content, the generation of waste by society, and the recycling rate. All else equal, the excise tax favors using virgin material and packaging refinement. At the same time, it decreases the use of recycled material, sales, and total waste generation. All else equal, the subsidy granted to the producer has the opposite effect. The subsidy granted to the recycler increases its profit and the recycling rate but does not impact production because of open-loop recycling.

Hence, in our model, a classic EPR scheme (where no additional subsidy is given to the producer or where the fee does not distinguish the recycled nature of the material) favors virgin materials. This result relies on the used technology function and on the price assumption (the virgin input is more expensive than the recycled one).

Further research may explore the role of investment behavior in recycling quality, be it downstream (recycling technologies) or upstream (product design), as well as closed-loop recycling.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used Grammarly and DeepL in order to ensure a smooth and correct writing of the English language. After using this tool/service, the authors reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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APPENDIX

The appendices below contain all the calculation steps required to solve the model. Readers can refer to them if, after reading the paper, they wish to understand the proofs of the various propositions and lemmas. Each proof is referenced in the paper.

A Proofs - Producer's program

The Lagrangian is:

$$\mathcal{L} = (p_v + t)v + (p_r + t - s_P)r + g(\alpha) - \lambda(\alpha f(v, r, \delta) - q) \quad (62)$$

The first derivatives are:

$$\frac{d\mathcal{L}}{dv} = (p_v + t) - \lambda\alpha \frac{df}{dv} \quad (63)$$

$$\frac{d\mathcal{L}}{dr} = (p_r + t - s_P) - \lambda\alpha \frac{df}{dr} \quad (64)$$

$$\frac{d\mathcal{L}}{d\alpha} = \frac{dg}{d\alpha} - \lambda f(v, r, \delta) \quad (65)$$

Equalising to zero, we obtain:

$$\lambda = \frac{p_v + t}{\alpha \frac{df}{dv}} = \frac{p_r + t - s_P}{\alpha \frac{df}{dr}} = \frac{\frac{dg}{d\alpha}}{f(v, r, \delta)} \quad (66)$$

And

$$\alpha \cdot \frac{dg}{d\alpha} = \frac{p_r + t - s_P}{\frac{df}{dr}} \cdot f(v, r, \delta) = \frac{p_v + t}{\frac{df}{dv}} \cdot f(v, r, \delta) \quad (67)$$

Providing the existence of an interior solution, from the FOC we obtain :

$$\frac{p_v + t}{p_r + t - s_P} = \frac{\frac{df}{dv}}{\frac{df}{dr}} = MRS_{r \rightarrow v} \quad (68)$$

To define the total quantity n , the producer equals its profit to zero:

$$\pi = pn - TC(n) = (A - n)n - TC(n) \quad (69)$$

We obtain:

$$n = A - \frac{TC(n)}{n} = A - UPC \quad (70)$$

A.1 Application: hybrid technology function

We consider the following unit production function:

We consider the following unit technology function:

$$q = \begin{cases} \alpha \cdot (\ln(v - \delta) + (r + \delta)) & \text{if } r > 0 \text{ with } v > \delta \\ \alpha \cdot \ln(v) & \text{if } r = 0 \end{cases} \quad (71)$$

Let us first focus on the case where the producer considers the use of recycled materials, where he expects $r > 0$. This function requires $v > \delta$ to be defined, and $v > \delta + 1$ to be always positive even if $r \rightarrow 0$.

What is the condition for $r > 0$? We have:

$$1 = \alpha \cdot \ln(v - \delta) + \alpha \cdot r + \alpha \cdot \delta \quad (72)$$

Rearranging we obtain:

$$r = \frac{1}{\alpha} - \delta - \ln(v - \delta) \quad (73)$$

Hence $r > 0 \Leftrightarrow \frac{1}{\alpha} - \delta > \ln(v - \delta)$;

Because the exponential function is strictly increasing on \mathbb{R}_+ , we can write:

$$e^{\frac{1}{\alpha} - \delta} > v - \delta \quad (74)$$

Therefore, $r > 0$ requires:

$$\delta < v < e^{\frac{1}{\alpha} - \delta} + \delta \quad (75)$$

Hence, a necessary but not sufficient condition is $0 < e^{\frac{1}{\alpha} - \delta}$, which is respected.

We assume that the productivity cost function is: $g(\alpha) = \frac{\alpha^2}{2}$

The producer's program is:

$$\begin{cases} \min_{r,v,\alpha} UPC = (p_v + t)v + (p_r + t - s_p)r + \frac{\alpha^2}{2} \\ s.t. q = \alpha [\ln(v - \delta) + (r + \delta)] \end{cases} \quad (76)$$

Hence the Lagrangian is:

$$\mathcal{L} = (p_v + t)v + (p_r + t - s_p)r + \frac{\alpha^2}{2} - \lambda [\alpha (\ln(v - \delta) + (r + \delta)) - q] \quad (77)$$

The first-order conditions are:

$$\frac{d\mathcal{L}}{dv} = 0 \Leftrightarrow v^* = \frac{\lambda\alpha}{p_v + t} + \delta \quad (78)$$

$$\frac{d\mathcal{L}}{dr} = 0 \Leftrightarrow \lambda = \frac{p_r + t - s_p}{\alpha} \quad (79)$$

Replacing λ in v^* :

$$v^* = \frac{p_r + t - s_p}{p_v + t} + \delta \quad (80)$$

The use of virgin material is constrained by the default δ . To obtain r^* we replace v^* in the production function.

$$1 = \alpha \left[\ln \left(\frac{p_r + t - s_p}{p_v + t} \right) + r + \delta \right] \quad (81)$$

Hence

$$r^* = \frac{1}{\alpha} - \delta - \ln\left(\frac{p_r + t - s_p}{p_v + t}\right) \quad (82)$$

We derive the Lagrangian with respect to α

$$\frac{d\mathcal{L}}{d\alpha} = 0 \Leftrightarrow \alpha = \lambda [\ln(v - \delta) + (r + \delta)] \quad (83)$$

We replace by the value of λ , v^* and r^* . We obtain:

$$\alpha = \frac{p_r + t - s_p}{\alpha} \left[\ln\left(\frac{p_r + t - s_p}{p_v + t}\right) + \frac{1}{\alpha} - \delta - \ln\left(\frac{p_r + t - s_p}{p_v + t}\right) + \delta \right] \quad (84)$$

$$\alpha = \frac{p_r + t - s_p}{\alpha} \left[\frac{1}{\alpha} \right] \quad (85)$$

We obtain

$$\alpha^* = (p_r + t - s_p)^{\frac{1}{3}} \quad (86)$$

And therefore

$$r^* = \frac{1}{(p_r + t - s_p)^{\frac{1}{3}}} - \delta - \ln\left(\frac{p_r + t - s_p}{p_v + t}\right) \quad (87)$$

$$\lambda = (p_r + t - s_p)^{\frac{2}{3}} \quad (88)$$

Now, let us look at the corner solution, in the case where $r \leq 0$:

$$\frac{1}{(p_r + t - s_p)^{\frac{1}{3}}} - \delta - \ln\left(\frac{p_r + t - s_p}{p_v + t}\right) \leq 0 \quad (89)$$

The price of recycled material is too high compared to its need for compensation and

its natural inferiority. So the producer optimizes his program only in terms of material productivity and virgin resin.

The Lagrangian writes as:

$$\mathcal{L} = (p_v + t)v + \frac{\alpha^2}{2} - \lambda(\alpha \ln(v) - 1) \quad (90)$$

From the first conditions we obtain:

$$\frac{d\mathcal{L}}{dv} = 0 \Leftrightarrow p_v + t - \frac{\lambda\alpha}{v} = 0 \quad (91)$$

Which gives $\lambda = \frac{v(p_v+t)}{\alpha}$.

$$\frac{d\mathcal{L}}{d\alpha} = 0 \Leftrightarrow \lambda = \frac{\alpha}{\ln(v)} - 1 \quad (92)$$

We obtain:

$$\frac{v(p_v + t)}{\alpha} = \frac{\alpha}{\ln(v)} - 1 \quad (93)$$

$$\frac{d\mathcal{L}}{d\lambda} = 0 \Leftrightarrow \ln(v) = \frac{1}{\alpha} \quad (94)$$

So:

$$\frac{v}{\alpha} = \frac{\alpha^2}{p_v + t} \quad (95)$$

At the producer's optimum, the relative quantity of inputs is equal to the inverse ratio of production factor costs.

In the second stage, the producer decides on the number of units that it wants to sell. In the case of the inner solution:

$$TC'(n) = UPC = (p_r + t - s_p)r^* + (p_v + t)v^* + \frac{\alpha^{*2}}{2} \quad (96)$$

We obtain that:

$$UPC(v > 0, r > 0) = \frac{3}{2}(p_r + t - s_p)^{\frac{2}{3}} + \delta(p_v - p_r - s_p) + (p_r + t - s_p) \left(1 - \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) \right) \quad (97)$$

Hence:

$$n^* = A - \frac{3}{2}(p_r + t - s_p)^{\frac{2}{3}} - \delta(p_v - p_r + s_p) - (p_r + t - s_p) \left(1 - \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) \right) \quad (98)$$

We verify under which conditions the producer produces $n > 0$:

$$A > \frac{3}{2}(p_r + t - s_p)^{\frac{2}{3}} + \delta(p_v - p_r + s_p) + (p_r + t - s_p) - (p_r + t - s_p) \cdot \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) \quad (99)$$

Simply $A > UPC$

A.2 Application: Cobb-Douglas with a linear transfer

In this section we explore an another technology function. We consider the functional form:

$$1 = \alpha [(v - d)(r + d)] \quad (100)$$

The Lagrangian of our program writes as:

$$\mathcal{L} = p_v v + p_r r + g(\alpha) - \lambda [\alpha(v - d)(r + d) - 1] \quad (101)$$

The first-order conditions gives us:

$$MRS_{v-r} = \frac{v - d}{r + d} = \frac{p_r}{p_v} \quad (102)$$

We obtain

$$r = \left(\frac{1}{\alpha} \frac{p_v}{p_r} \right)^{\frac{1}{2}} - \delta \quad (103)$$

$$v = \left(\frac{1}{\alpha} \frac{p_r}{p_v} \right)^{\frac{1}{2}} + \delta \quad (104)$$

$$\alpha = (p_v p_r)^{\frac{1}{3}} \quad (105)$$

Here the demand for virgin and recycled is symmetric, within one linear parameter (δ). Material productivity is affected symmetrically by prices of materials.

B Proofs - Impact of the tax and subsidy

We study the impact of the tax and the subsidy on the different variables of the model. We begin by studying the impact of the tax in isolation and in the context of the interior solution.

Figure 9 illustrates the impact of the tax (t) and the subsidy (s_P) on the product's waste intensity, the material productivity parameter and the material mix (v, r). Im-

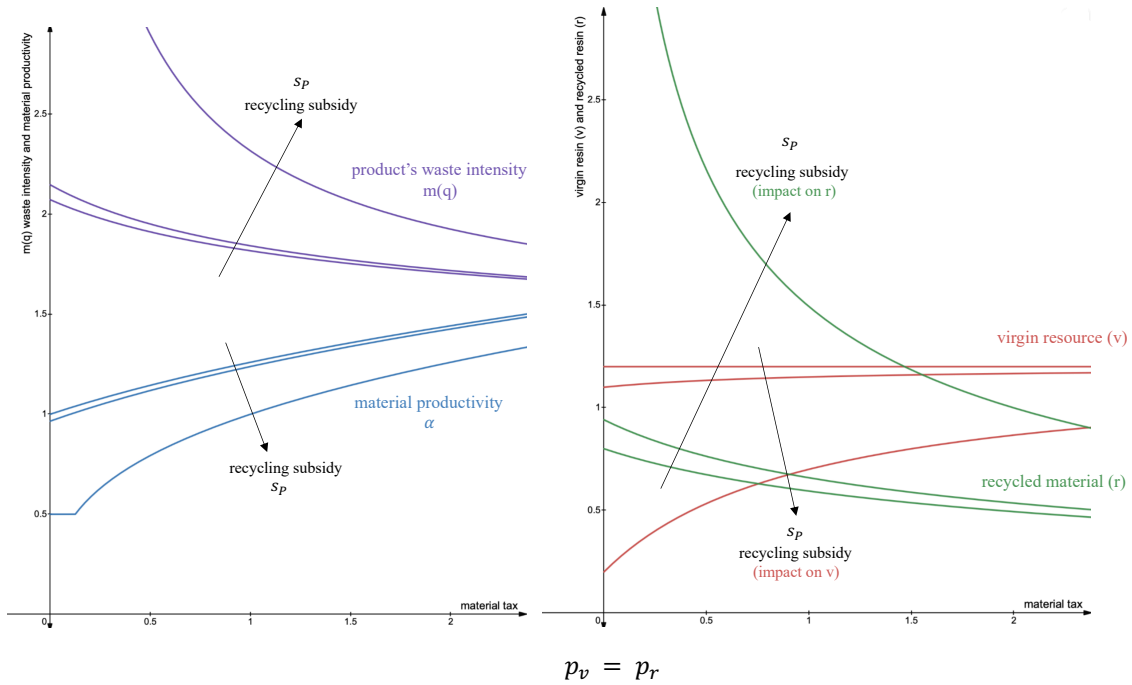


Figure 9: Impact of the tax and subsidy on the product's waste intensity, the material productivity parameter and the material mix (v, r)

plementing a material tax (t) increases the use of virgin materials in the product (v) and the productivity parameter (α), and reduces the use of recycled materials (r) and waste intensity ($m(q)$). Subsidizing the purchase of recycled resins increases the use of recycled materials and reduces the use of virgin resins, which implies a decrease in the material productivity parameter and an increase in the product's waste intensity.

Figure 10 illustrates the impact of the tax (t) on products' sales (n) and total waste generation (W). The tax decreases the volume of sales while the subsidy increases sales, via a reduction effect on the unit cost of production. Thus, the overall waste volume increases with the recycled material subsidy, while the tax reduces the waste volume through a downward pressure on sales and waste intensity per product.

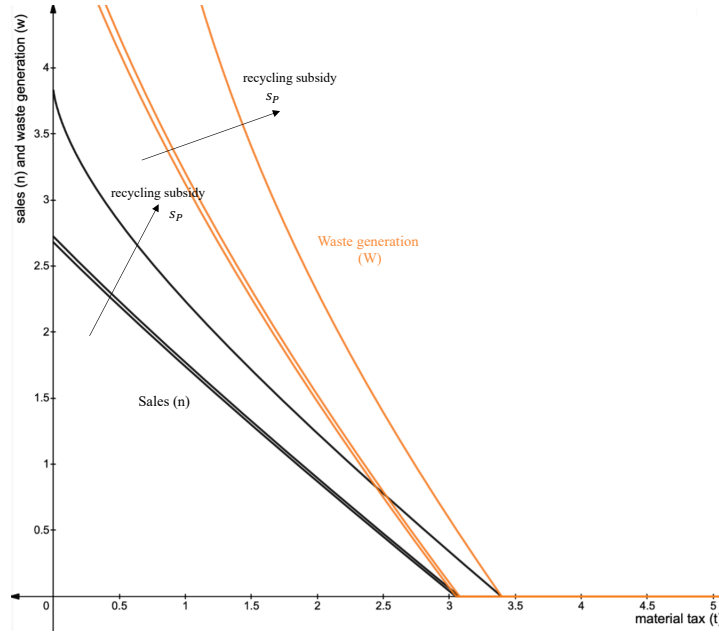


Figure 10: Impact of the tax and subsidy on products' sales (n) and total waste generation (W)

B.1 Use of the virgin resin

$$\frac{dv}{dt} = \frac{p_v - p_r + s_p}{(p_v + t)^2} \quad (106)$$

The tax increases the use of virgin resin in the product if:

$$\text{[necessary condition for } \frac{dv}{dt} > 0 \text{]} : \quad p_v > p_r - s_p \quad (107)$$

Increasing the tax increases the use of virgin material if it is initially relatively more expensive than recycled material. Indeed, a uniform tax on both materials increases the price of the initially cheaper material relatively more.

$$\frac{dv}{ds_p} = \frac{-1}{p_v + t} < 0 \quad (108)$$

Subsidizing the use of recycled material reduces the use of virgin resin in the product.

B.2 Use of the recycled resin

Replacing α^* we have:

$$r^* = \frac{1}{p_r + t - s_P} - \delta - \ln \left(\frac{p_r + t - s_P}{p_v + t} \right) \quad (109)$$

We obtain

$$\frac{dr}{dt} = -\frac{1}{3}(p_r + t - s_P)^{\frac{4}{3}} - \frac{p_v - p_r + s_P}{(p_v + t)(p_r + t - s_P)} \quad (110)$$

Here $\frac{dr}{dt} < 0$ is always verified if $p_r + t - s_P > 0$ (producers are not paid to use recycled materials) and $p_v > p_r - s_P$ (the recycled resin has a discount on its price compared to the virgin resin, when including the subvention).

Simplifying the expression, we obtain:

$$\frac{dr}{dt} < 0 \Leftrightarrow -1 < \frac{3(p_r + t - s_P)^{\frac{1}{3}}(p_v - p_r + s_P)}{p_v + t} \quad (111)$$

The impact of the subsidy is given by:

$$\frac{dr}{ds_P} = \frac{1}{(p_r + t - s_P)^2} + \frac{1}{p_r + t - s_P} > 0 \quad \text{if } p_r + t - s_P > 0 \quad (112)$$

The subsidy has a positive impact on the use of recycled material via the impact on material productivity which decreases (need for more material) and via the price ratio between the two inputs (recycled material becomes relatively cheaper).

B.3 Material productivity

The tax has a positive effect on material productivity and the subsidy a negative effect.

$$\frac{d\alpha}{dt} = \frac{1}{3(p_r + t - s_P)^{\frac{2}{3}}} > 0 \quad \text{if } p_r + t - s_P > 0 \quad (113)$$

$$\frac{d\alpha}{dt} = \frac{-1}{3(p_r + t - s_P)^{\frac{2}{3}}} < 0 \quad \text{if } p_r + t - s_P > 0 \quad (114)$$

B.4 Waste intensity

We have:

$$m(q) = r + v = \frac{p_r + t - s_P}{p_v + t} + \frac{1}{(p_r + t - s_P)^{\frac{1}{3}}} - \ln\left(\frac{p_r + t - s_P}{p_v + t}\right) \quad (115)$$

Deriving with respect to t , we obtain:

$$\frac{dm(q)}{dt} = -\frac{1}{3}(p_r + t - s_P)^{-\frac{4}{3}} - \frac{(p_v - p_r + s_P)^2}{(p_v + t)^2(p_r + t - s_P)} \quad (116)$$

An increase in the material use tax reduces the product's waste intensity.

Deriving with respect to s_P , we obtain:

$$\frac{dm(q)}{ds_P} = \underbrace{\frac{-1}{p_v + t}}_{\downarrow v} + \underbrace{\frac{1}{3}(p_r + t - s_P)^{-\frac{4}{3}}}_{\uparrow \alpha} + \underbrace{\frac{1}{p_r + t - s_P}}_{\uparrow r} \quad (117)$$

We obtain

$$\frac{dm(q)}{ds_P} = \frac{p_v - p_r + s_P}{(p_v + t)(p_r + t - s_P)} + \frac{1}{3}(p_r + t - s_P)^{\frac{-4}{3}} \quad (118)$$

The subsidy systematically increases the waste intensity of the product if $p_v > p_r - s_P$, i.e., if the substitution effect works against the material volume (if more recycled material replaces the virgin material saved: this is consistent with the model framework).

B.5 Sales

The sales are given by:

$$n_{v^* >, r^* > 0}^* = \left[A - \frac{3}{2}(p_r + t - s_P)^{\frac{2}{3}} - \delta(p_v - p_r + s_P) - (p_r + t - s_P) \left(1 - \ln \left(\frac{p_r + t - s_P}{p_v + t} \right) \right) \right] \quad (119)$$

Deriving with respect to t we obtain:

$$\frac{dn}{dt} = -(p_r + t - s_P)^{\frac{-1}{3}} - 1 + \ln \left(\frac{p_r + t - s_P}{p_v + t} \right) + \frac{p_v - p_r + s_P}{p_v + t} \quad (120)$$

The tax decreases sales if :

$$-(p_r + t - s_P)^{\frac{-1}{3}} - 1 + \ln \left(\frac{p_r + t - s_P}{p_v + t} \right) + \frac{p_v - p_r + s_P}{p_v + t} < 0 \quad (121)$$

Because $p_r - s_P < p_v$, we have $p_r + t - s_P < p_v + t$ and thus $\frac{p_r + t - s_P}{p_v + t} < 1$. Hence

$$\ln \left(\frac{p_r + t - s_P}{p_v + t} \right) < 0 \quad (122)$$

In addition, we have assumed that $p_v > p_r - s_P$.

Therefore

$$\frac{dn}{dt} < 0 \quad (123)$$

Deriving with respect to s_P we obtain:

$$\frac{dn}{ds_P} = (p_r + t - s_P)^{\frac{-1}{3}} - \delta - \ln\left(\frac{p_r + t - s_P}{p_v + t}\right) \quad (124)$$

$$\frac{dn}{ds_P} > 0 \Leftrightarrow (p_r + t - s_P)^{\frac{-1}{3}} - \ln\left(\frac{p_r + t - s_P}{p_v + t}\right) > \delta \quad (125)$$

The subsidy increases sales if the defect in the recycled material is small enough, i.e., a lower cost substitution is feasible.

B.6 Total waste

The total volume of waste is given by the number of products sold multiplied by the waste intensity of the product.

The tax reduces the product's waste intensity, as well as the sales. Hence, the tax on material use decreases total waste.

The subsidy increases the product's waste intensity, systematically if $p_v > p_r - s_P$, and the sales if the default δ is small enough (condition 125). Hence, under these conditions, the subsidy increases total waste.

C Proofs - Recycler's program

We consider the same modeling as Fleckinger and Joltreau (2020) for collection costs. The recycler collects the waste until a proportion τ over a standardized distance from 0 to 1. The transportation cost is equal to 1. The waste is uniformly distributed over $[0, 1]$ in a Hotelling fashion. The recycling facility is located at the address $x = 0$.

Hence collection costs are given by:

$$\int_0^\tau \gamma x dx = \frac{\gamma}{2} \tau^2. \quad (126)$$

The unit recycling costs are constant and equal to c . In addition, the infrastructure requires an investment F , a fixed initial cost.

Hence the profit is

$$\pi_{recycler} = (p_r + s_R - c) \tau W - \frac{\gamma \tau^2}{2} W - F \quad (127)$$

The condition of profitability, noted P, of the recycling industry is

$$P : (p_r + s_R - c) \cdot \tau W - \frac{\gamma \tau^2}{2} W \geq F \quad (128)$$

C.1 Application

The recycler's profit can be written as:

$$\pi_{recycler} = \tau \left(p_r + s_R - c - \frac{\gamma \tau}{2} \right) \cdot [n] \cdot [r + v] - F \quad (129)$$

Deriving with respect to t we obtain:

$$\frac{d\pi_{recycler}}{dt} = \tau \left(p_r + s_R - c - \frac{\gamma\tau}{2} \right) \cdot \left[(r + v) \left(\frac{dn}{dt} \right) + n \left(\frac{dv}{dt} + \frac{dr}{dt} \right) \right] \quad (130)$$

We have already demonstrated that $\frac{dn}{dt} < 0$, $\frac{dv}{dt} > 0$ and $\frac{dr}{dt} < 0$.

$$\frac{dv}{dt} + \frac{dr}{dt} = \frac{p_v - p_r + s_p}{(p_v + t)^2} - \frac{1}{3}(p_r + t - s_p)^{\frac{4}{3}} - \frac{p_v - p_r + s_p}{(p_v + t)(p_r + t - s_p)} \quad (131)$$

We obtain

$$\frac{dv}{dt} + \frac{dr}{dt} = -s_p \frac{p_v - p_r + s_p}{(p_r + t - s_p)(p_v + t)^2} - \frac{1}{3}(p_r + t - s_p)^{\frac{4}{3}} < 0 \quad (132)$$

Hence

$$\frac{d\pi_{recycler}}{dt} < 0 \quad (133)$$

The tax decreases the recycler's profit. Let us verify this result:

We can rewrite the recycler's profit as:

$$\begin{aligned} \pi_{recycler} = & \tau \left(p_r + s_R - c - \frac{\gamma\tau}{2} \right) \cdot \\ & \left[A - \frac{3}{2}(p_r + t - s_p)^{\frac{2}{3}} - \delta(p_v - p_r + s_p) - (p_r + t - s_p) \left(1 - \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) \right) \right] \\ & \cdot \left[\frac{1}{(p_r + t - s_p)^{\frac{1}{3}}} - \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) + \frac{p_r + t - s_p}{p_v + t} \right] - F \quad (134) \end{aligned}$$

$$\begin{aligned} \frac{d\pi_{recycler}}{dt} = & \tau \left(p_r + s_R - c - \frac{\gamma\tau}{2} \right) \cdot ((r+v) \left(-(p_r + t - s_p)^{\frac{-1}{3}} + \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) - 1 + \frac{p_r + t - s_p}{p_v + t} \right) \\ & + n \left(-\frac{1}{3}(p_r + t - s_p)^{\frac{4}{3}} - \frac{p_v - p_r + s_p}{(p_v + t)(p_r + t - s_p)} + \frac{p_v - p_r + s_p}{(p_v + t)^2} \right) \end{aligned} \quad (135)$$

Rewriting, we obtain:

$$\begin{aligned} \frac{d\pi_{recycler}}{dt} = & \tau \left(p_r + s_R - c - \frac{\gamma\tau}{2} \right) \cdot ((r+v) \left(-(p_r + t - s_p)^{\frac{-1}{3}} \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) + \frac{p_r + t - s_p}{p_v + t} - 1 \right) \\ & + n \left(-\frac{1}{3}(p_r + t - s_p)^{\frac{4}{3}} - s_p \frac{p_v - p_r + s_p}{(p_r + t - s_p)(p_v + t)^2} \right) \end{aligned} \quad (136)$$

Deriving with respect to s_p we obtain:

$$\frac{d\pi_{recycler}}{ds_p} = \tau \left(p_r + s_R - c - \frac{\gamma\tau}{2} \right) \cdot \left[(r+v) \left(\frac{dn}{ds_p} \right) + n \left(\frac{dv}{ds_p} + \frac{dr}{ds_p} \right) \right] \quad (137)$$

We know that $\frac{dn}{ds_p} > 0$ under the following condition:

$$\frac{dn}{ds_p} > 0 \Leftrightarrow (p_r + t - s_p)^{\frac{-1}{3}} - \ln \left(\frac{p_r + t - s_p}{p_v + t} \right) > \delta \quad (138)$$

At the same time,

$$\frac{dv}{ds_p} = \frac{-1}{p_v + t} < 0 \quad (139)$$

And

$$\frac{dr}{ds_p} = \frac{1}{(p_r + t - s_p)^2} + \frac{1}{p_r + t - s_p} > 0 \quad (140)$$

$$\frac{dv}{ds_P} + \frac{dr}{ds_P} = \frac{-1}{p_v + t} + \frac{1}{(p_r + t - s_P)^2} + \frac{1}{p_r + t - s_P} \quad (141)$$

We obtain

$$\frac{dv}{ds_P} + \frac{dr}{ds_P} = \frac{(p_r + t - s_P)(p_v - p_r + s_P) + (p_v + t)}{(p_v + t)(p_r + t - s_P)^2} > 0 \quad (142)$$

The subsidy (systematically) increases the recycling profit when $\frac{dn}{ds_P} > 0 \Leftrightarrow (p_r + t - s_P)^{\frac{-1}{3}} - \ln\left(\frac{p_r + t - s_P}{p_v + t}\right) > \delta$.

D Proofs - the social optimum

Under constant marginal utility of income, the consumers' surplus is:

$$S_{cons} = \int_0^n D(n)dn - pn = \frac{n^2}{2} \quad (143)$$

The unit production cost (UPC) of the producer sums the purchasing costs of the virgin material v and the recycled material r , and the material efficiency cost function $g(\alpha)$

$$UPC = p_v \cdot v + p_r \cdot r + g(\alpha) \quad (144)$$

The producer's profit writes as:

$$\pi_{producer} = n \cdot (p - UPC) \quad (145)$$

Replacing $p = A - n$, we obtain:

$$\pi_{producer} = An - n^2 - nUPC \quad (146)$$

The profit of the recycler sums the sale revenues of recycled materials $p_r\tau W$ and retrieves the recycling cost c , collection costs τW and the fixed cost F .

Hence we can write:

$$\pi_{recycler} := n \left[\tau \left(p_r - c - \frac{\gamma\tau}{2} \right) (r + v) \right] - F \quad (147)$$

We denote e_V and e_R , the externality for, respectively, extraction and recycling. We assume that recycling is less detrimental to the society than extraction, since it avoids degrading eco-systems ($e_v > e_r$).

The unit environmental production cost $UEPC$ is therefore:

$$UEPC = e_V v + e_R r \quad (148)$$

On the other hand, we consider the externalities linked to the end of the product's life. Like [Kohn \(1995\)](#), we assume that waste disposed in landfill degrades into a single period with a pollution e_D . The disposal causes pollution within this period, but we do not observe pollution accumulation. Unit disposal costs are:

$$UDC = e_D(1 - \tau)(r + v) \quad (149)$$

We denote s_w the social cost of recycling per unit of product:

$$s_w = e_D(1 - \tau) - \tau \left(p_r - c - \gamma \frac{\tau}{2} \right) \quad (150)$$

The social cost of end-of-life also adds the recycler's marginal profit. The externality of recycling is not counted in the end-of-life externalities, to avoid double counting.

D.1 The social welfare optimization

The social optimum of the first period is solved in two stages.

The social welfare function can be written as:

$$\mathcal{S} = -\frac{1}{2}n^2 + An - n(UPC + UEPC) + n \left[\tau \left(p_r - c - \frac{\gamma\tau}{2} \right) (r + v) \right] - F - ne_D(1-\tau)(r+v) \quad (151)$$

D.1.1 first stage

The social planner minimizes the costs and the damages linked to the product's composition.

$$\min UPC + UEPC + s_w(r + v) \quad (152)$$

$$s.t. \quad q - \alpha f = 0 \quad (153)$$

The Lagrangian expression can be written as:

$$\mathcal{L} = UPC + UEPC + s_w(r + v) - \lambda(q - \alpha f) \quad (154)$$

with λ , the coefficient associated to the technology constraint.

The Lagrangian can be rewritten as:

$$\mathcal{L} = (p_v + e_v)v + (p_r + e_r)r + \frac{\alpha^2}{2} + e_D(1 - \tau)(r + v) - \tau(p_r - c - \frac{\gamma\tau}{2})(r + v) + \lambda(1 - \alpha [\ln(v - \delta) + r + \delta]) \quad (155)$$

Taking the first derivatives:

$$\frac{d\mathcal{L}}{dv} = p_v + e_v + e_D(1 - \tau) - \tau(p_r - c - \frac{\gamma\tau}{2}) - \frac{\alpha\lambda}{v - \delta} \quad (156)$$

$$\frac{d\mathcal{L}}{dv} = 0 \Leftrightarrow v = \frac{\alpha\lambda}{p_v + e_v + s_w} + \delta \quad (157)$$

$$\frac{d\mathcal{L}}{dr} = p_r + e_r + s_w - \alpha\lambda \quad (158)$$

$$\frac{d\mathcal{L}}{dr} = 0 \Leftrightarrow \lambda = \frac{p_r + e_r + s_w}{\alpha} \quad (159)$$

We obtain

$$v^{OP} = \frac{p_r + e_r + s_w}{p_v + e_v + s_w} + \delta \quad (160)$$

To obtain r^{OP} , we use the implicit function:

$$\frac{1}{\alpha} = \ln\left(\frac{p_r + e_r + s_w}{p_v + e_v + s_w}\right) + r + \delta \quad (161)$$

Thus

$$r^{OP} = \frac{1}{\alpha} - \ln\left(\frac{p_r + e_r + s_w}{p_v + e_v + s_w}\right) - \delta \quad (162)$$

Deriving with respect to α :

$$\frac{d\mathcal{L}}{d\alpha} = \alpha - \lambda [\ln(v - \delta) + r + \delta] \quad (163)$$

Replacing λ , v and r :

$$\frac{d\mathcal{L}}{d\alpha} = 0 \Leftrightarrow \alpha = \frac{p_r + e_r + s_w}{\alpha} \left[\frac{1}{\alpha} \right] \quad (164)$$

Thus,

$$\alpha^3 = (p_r + e_r + s_w) \quad (165)$$

$$\alpha^{OP} = (p_r + e_r + s_w)^{\frac{1}{3}} \quad (166)$$

The interior solution of the first stage is:

$$v^{OP} = \frac{p_r + e_R + s_w}{p_v + e_V + s_w} + \delta \quad (167)$$

$$\alpha^{OP} = (p_r + e_R + s_w)^{\frac{1}{3}} \quad (168)$$

$$r^{OP} = \frac{1}{\alpha^{OP}} - \delta - \ln(v^{OP} - \delta) \quad (169)$$

with $s_w = e_D(1 - \tau) - \tau(p_r - c - \frac{\gamma\tau}{2})$

D.1.2 second stage

The social planner maximizes the total welfare \mathcal{S} given a certain product's composition, which sums up the consumers' surplus S_{cons} , the producer's profit $\pi_{producer}$ and retrieves the environmental costs.

$$\mathcal{S} = -\frac{1}{2}n^2 + An - n(UPC + UEPC) + n \left[\tau \left(p_r - c - \frac{\gamma\tau}{2} \right) (r + v) \right] - F - ne_D(1-\tau)(r+v) \quad (170)$$

The second-stage gives us:

$$n^{OP} = A - UPC(r^{OP}, v^{OP}) - UEPC(r^{OP}, v^{OP}) - s_w(r^{OP} + v^{OP}) \quad (171)$$

$$\tau^{OP} = \frac{p_r - c + e_D}{\gamma} \quad (172)$$

It is possible to restore the social optimum with

$$t = e_v + s_w \quad (173)$$

$$s_P = e_v - e_r \quad (174)$$

$$s_R = e_D \quad (175)$$

Unless it does not allow a positive profit for the recycler. Indeed, the social planner does not take into account the fixed cost in its calculation.

Let us analyse under which conditions the social optimum is compatible with a positive profit of the recycler.

Let us rewrite the recycler's profit with the optimum values.

$$\pi_{recycler} = W^{OP} \cdot \left(\left(\frac{p_r + e_D - c}{\gamma} \right) \left(\frac{p_r - e_D - c}{2} \right) \right) - F \quad (176)$$

$$\pi_{recycler} = W^{OP} \cdot \left(\frac{(p_r + e_D - c)^2}{2\gamma} \right) - F \quad (177)$$

The profit of the recycler is negative if :

$$\frac{(p_r + e_D - c)^2}{2\gamma} < \frac{F}{WOP} \quad (178)$$

E Time consistency and endogenous p_r

In this section, we explore the possibility of having an endogenous and local recycling market.

E.1 Time consistency

In an unchanged world, the equilibrium in t must be repeated in $t + 1$. For this, the equilibrium in t must be compatible in $t + 1$. That means if $R_{t+1} = R_t$, we have:

$$\tau W = R \quad (179)$$

For $n > 0$ this means:

$$\frac{r}{v} = \frac{\tau}{1 - \tau} \quad (180)$$

E.2 Endogeneous p_r

The demand of the producer is $R(p^*)$, the supply of the producer is $\tau W(p^*)$. We look for a time consistent p_r , i.e., unchanged over time.

$$\tau W = R \quad (181)$$

Hence,

$$\tau = \frac{R}{W} = \frac{r}{r+v} \quad (182)$$

$$\frac{p_r + s_r - c}{\gamma} = \frac{r}{r+v} \quad (183)$$

Hence

$$p_r = \frac{r}{r+v} + c - s_r \quad (184)$$

$$p_r = \gamma \frac{\frac{1}{p_r} - \delta - \ln\left(\frac{p_r}{p_v}\right)}{\frac{1}{p_r} - \ln\left(\frac{p_r}{p_v}\right) + \frac{p_r}{p_v}} + c - s_R \quad (185)$$

The equilibrium price allows little tractability of the model to solve in general equilibrium.