# Policy Papers

## Evolution of EROIs of Electricity Until 2050 : Estimation and Implications on Prices

Adrien FABRE

PP 2018-09

Suggested citation:

A. Fabre *(2018*). Evolution of EROIs of Electricity Until 2050 : Estimation and Implications on Prices. *FAERE Policy Paper, 2018-09.*

# Evolution of EROIs of Electricity Until 2050: Estimation and Implications on Prices

Adrien Fabre[1]

**Abstract**

The EROI –for Energy Returned On Invested– of an energy technology measures its ability to provide energy efficiently. Previous studies draw a link between the affluence of a society and the EROI of its energy system, and show that EROIs of renewables are lower than those of fossil fuels. Logically, concerns have been expressed that system-wide EROI may decrease during a renewable energy transition. First, I explain theoretically that the EROIs of renewables themselves could then decrease as energy-efficient fossil fuels would be replaced by less energy-efficient renewables in the supply-chain. Then, using the multiregional input-output model THEMIS, I estimate the evolution of EROIs and prices of electric technologies from 2010 to 2050 for different scenarios. Global EROI of electricity is predicted to go from 12 in 2010 to 11 in 2050 in a business-as-usual scenario, but down to 6 in a 100% renewable one. Finally, I study the economic implication of a declining EROI. An inverse relation between EROI and price is suggested empirically, even though theory shows that both quantities may move in the same direction.

*Keywords:* EROI; input-output; THEMIS; MRIO; sustainability; energy transition

*Acknowledgments.* I am grateful to three anonymous reviewers for their insightful comments, and especially to one of them who spotted an important typo. I hail the work of Konstantin Stadler on pymrio, an invaluable open-source python library when dealing with Multi-Regional Input-Output Tables (MRIO). I am thankful to Thomas Pregger and his team at DLR for graciously providing me the tables of future energy consumption in the Greenpeace [R]evolution scenarios. I am indebted to Thomas Gibon, Anders Arvesen and the Norwegian University of Science and Technology (NTNU) for providing me the data of THEMIS and helping me using it. I am thankful to Arjan de Koning, Carey King and Adam Brandt for answering my questions. I am grateful to Cyril François for his thoughtful comments on this paper. I am thankful to the Isterre for providing me an office at Grenoble to conduct this research. I am grateful to Olivier Vidal and Mouez Fodha for their support.

*Code.* All the code is on-line, and can be accessed from a notebook at: bit.ly/future_eroi_code. A substantial share of this work has been to contribute to the python library *pymrio*: github.com/bixiou/pymrio. Using my fork of pymrio, one can now easily undertake EROIs and related computations on Exiobase and THEMIS.

## 1. Introduction

As the harmful impacts of climate change call for a prompt energy transition away from fossil fuels —not to mention their depletion that shall ultimately make this transition unavoidable, concerns have been expressed that, in a decarbonized energy system, the lower efficiency of renewable energy might not allow to sustain advanced standards of living (Lambert et al., 2014; Tverberg, 2017).[2] We measure the energy efficiency of a technology or energy system using the Energy Returned On Invested (EROI), which is the ratio between the energy it delivers throughout its lifetime and the energy required to build, operate and dismantle it. A minimal requirement for a technology or energy system to be energetically sustainable is to have an EROI above 1, meaning that it provides more energy than it requires.

One issue to assess future energy systems is that the future EROI of a given technology cannot be readily deduced from current estimates. Indeed, as King (2014) remarked, the EROI of a technology is not intrinsic, but depends on the whole technological structure of the economy. Indeed, suppose that solar panels have a lower EROI than thermal power plants, so they require more energy to supply the same amount of energy. Then a plant producing solar panels will require more energy if the electricity it uses is produced by solar panels rather than by thermal plants. Ultimately, solar panels built using electricity from solar panels rather fossils will require more energy, and have a lower EROI. Some have called to

[1]Paris School of Economics, Université Paris 1 Panthéon-Sorbonne. adrien.fabre@psemail.eu. 48 bd Jourdan 75014 Paris.

[2]The energy expert Jean-Marc Jancovici also expressed concerns over this subject during a presentation at the École Normale Supérieure in 2018: “What happens to the EROI when you have only wind and solar panels to build wind and solar panels? I think it crashes.”

compute the evolution of EROIs during a renewable energy transition (Brandt, 2017), and this study aims to do so while accounting for their system dependency. Indeed, provided that EROIs of renewables are lower than EROIs of fossils and that decreasing EROIs jeopardize prosperity, the evolution of EROIs during the energy transition is of critical importance: let us review these two hypotheses in turn.

Many estimations of EROIs have been made, and among the various different figures derived from diverse data sets and methodologies, none stands out as singularly authoritative. Dale (2010) reviews all EROI estimates until 2010, while Hall et al. (2014) aggregate the estimates of the literature in a meta-analysis. I choose to present the results of Weißbach et al. (2013) (see Figure 1), because they compute the EROIs of different technologies in a comparable manner. In addition, the *buffered* EROIs of Weißbach et al. (2013) take into account the supplementary capacity, grid and storage required for the deployment of renewable technologies, which yields lower but presumably more accurate estimates for their EROIs. As anticipated, the EROIs of renewable electricity sectors they find are significantly lower than those of electricity from fossil fuels, except for hydro.

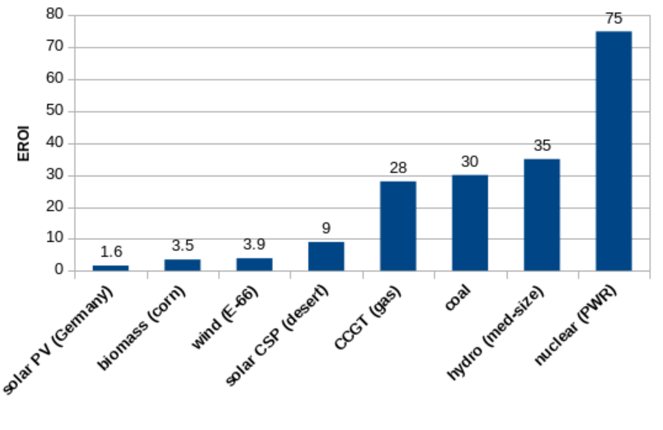


Figure 1: Estimates of EROIs of different electricity technologies, from Weißbach et al. (2013), where supplementary capacity and storage required for the deployment of these technologies is accounted for.

Some authors argue that the value of EROI is of primary concern, as they draw a link between the system-wide EROI and affluence of a society (Hall et al., 2009; Hall, 2011; Lambert & Lambert, 2011; Lambert et al., 2014; Fizaine & Court, 2016). Here is how Hall (2011) summarizes the argument:

> Think of a society dependent upon one resource: its domestic oil. If the EROI for this oil was 1.1:1 then one could pump the oil out of the ground and look at it. (...) Hall et al. (2009) examined the EROI required to actually run a truck and found that if the energy included was enough to build and maintain the truck and the roads and bridges required to use it (i.e., depreciation), one would need at least a 3:1 EROI at the wellhead. Now if you wanted to put something in the truck, say some grain, and deliver it that would require an EROI of, say, 5:1 to grow the grain. (...) 7 or 8:1 to support the families. If the children were to be educated you would need perhaps 9 or 10:1, have health care 12:1, have arts in their life maybe 14:1 and so on.

The reasoning of Hall relies on the observation that all sectors of the economy require energy, and that the more efficient is the energy production (i.e. the higher is the EROI), the more energy is available to the rest of the economy. In strict logic, Hall's argument relies on two questionable assumptions: that factors of production (and especially the labor force) are used at their full capacity, and that technical and organizational progress will not be sufficient to sustain current level of prosperity with significantly less labor (or other factors of production in limited supply). If one rejects these assumptions, one can imagine a sustained level of prosperity with a lower system-wide EROI, provided that a higher share of factors of production be devoted to the energy sector: for example, unemployed people could be mobilized to sustain the energy surplus available to the rest of society. In parallel to a shift in the labor force, Raugei (2019) explains that an increased efficiency of energy use may also counteract the decrease in energy services implied by a declining EROI. That being said, given that current system-wide EROI is already declining due to the decline in fossil fuels quality (Dale et al., 2011; Poisson et al., 2013; Court & Fizaine, 2017) and that technical progress is incremental, the aforementioned analyses should not be neglected. Under the current system of production, which will persist in the short term, EROI should not decrease too much for prosperous standards of living to be sustained.

In view of the potential implications of a declining EROI, this paper provides an assessment of the EROI of different electricity technologies in various prospective scenarios, which includes a 100% renewable electricity system. To this end, I employ input-output analysis and I rely on a prospective series of multi-regional Input-Output Tables (IOT): THEMIS (Gibon et al., 2015), which models two scenarios from the International Energy Agency (IEA, 2010): Baseline and Blue Map. In addition, I modify THEMIS' IOTs to embed two decarbonized scenarios of power generation: Greenpeace's Energy [R]evolution (ER) and Advanced Energy [R]evolution (ADV) (Teske et al., 2015). Although Pehl et al. (2017) and Arvesen et al. (2018) already computed energy requirements of electricity technologies for prospective scenarios, they focused on life-cycle assessment coefficients such as future $CO_2$ emissions, and did not provide results in terms of EROI, let alone system-wide EROI. Furthermore, they did not study a scenario with 100% renewable electricity. I intend to fill this gap.

Then, I analyze the economic implications of a declining EROI through its relation with price. Previous studies suggest an inverse relation between EROIs and energy prices, and such an average relation is retrieved empirically using prices observed and predicted from THEMIS. However, theoretical analysis tempers this finding. Indeed, while explaining to what extent EROI and price are related, I show that they do not necessarily move in opposite directions. This calls for taking prices predictions from input-output analysis with more caution than EROI estimates, because IOT is better suited to handle physical notions than economic ones. Finally, the economic analysis weakens the view that a decrease in EROI would necessarily lead to a surge in energy expenditures and hence to a contraction of GDP.

Section 2 explains theoretically why the EROI of a technology is not an intrinsic property; section 3 presents the methodology and the results; section 4 studies the implications of declining EROIs on prices and GDP; section 5 concludes.

## 2. The EROI of a Technology Is Not Intrinsic

### *2.1. A Simple Model With A Unique Energy Technology*

The element $a_{i,j}$ of the technology matrix $A$ represents the quantity of input $i$ required to produce one unit of output $j$. Below is an illustrative technology matrix with three inputs (and the same three outputs): an energy technology, materials, and energy. $m_e$ denotes the quantity of materials ($m$) required to produce one unit of energy technology ($e$), and this notation extends naturally to all elements of $A$. The numerical values of the coefficients have a purely pedagogical purpose and have been arbitrarily chosen.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ m_e & m_m & 0 \\ E_e & E_m & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ m_e & 0.2 & 0 \\ 0.1 & 0.5 & 0 \end{pmatrix} \begin{array}{c} \text{energy techno.} \\ \text{materials} \\ \text{energy} \end{array}$$

The system-wide EROI, or Energy Returned On Invested, is the ratio between the energy delivered by the system, and the energy required to build, operate, maintain and dismantle it. In other words, it is the inverse of the amount of energy required to produce one unit of energy, when the series of all embodied inputs are taken into account.

The embodied inputs $x$ required for a final demand $y$ can be calculated using the Leontief inverse matrix (Leontief, 1986; Eurostat, 2008; Miller & Blair, 2009):

$$x(y) = (I - A)^{-1} \cdot y. \tag{1}$$

We denote by $\mathbb{1}_S$ the vector with 1 at the positions of the sectors $s \in S$, and zeros everywhere else. As energy $E$ is the last input of our list, $\mathbb{1}_E = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and the gross embodied energy required for a final demand $y$ is the last element of $x$: $\mathbb{1}_E^T \cdot (I_n - A)^{-1} \cdot y$. Thus, the EROI is

$$\begin{aligned} \text{EROI} &= \frac{\text{delivered energy}}{\text{net embodied energy}} \\ &= \frac{1}{\mathbb{1}_E^T \cdot \left((I-A)^{-1} \cdot \mathbb{1}_E - \mathbb{1}_E\right)}. \end{aligned} \tag{2}$$

After some calculations (available on-line), we find:

$$\begin{aligned} \text{EROI} &= \frac{(1-E_e)(m_m - 1) + E_m m_e}{E_e (m_m - 1) - E_m m_e} \\ &= \frac{0.72 - 0.5 m_e}{0.08 + 0.5 m_e} \end{aligned} \tag{3}$$

Unsurprisingly, one can see in Figure 2 that the EROI decreases with the material intensity of the energy technology, because extracting and processing material requires energy.

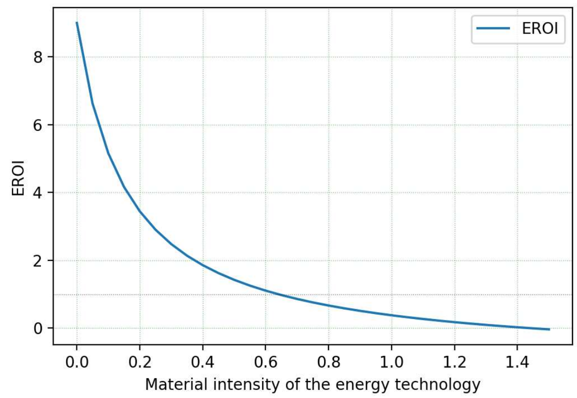


Figure 2: EROI in the simple model in function of the material intensity $m_e$ of the energy technology.

For an intensity above 0.6, the EROI is below 1. An EROI below 1 means that the energy technology is not worth developing, because (in net) it consumes energy rather than providing it. Such a system is not sustainable (and not realistic): for it to happen the society should have accumulated energy in the past from an energy source no more accessible, and would waste this energy in that absurd technology.

For even higher intensities, the EROI falls below 0, which means that the energy (recursively) required to produce one unit of energy is infinite. Here, free energy coming from the past would not suffice to build the energy technology: one would also need to have free materials (i.e. materials requiring no energy to access them). Such a world is physically impossible.

### *2.2. A Simple Model With A Mix of Two Energy Technologies*

Now, let us consider two energy technologies, with the same energy intensity, but different materials intensities.

Even if this example is purely illustrative, let us call them PV (for solar photovoltaic) and gas (for gas power-plant electricity) to grasp the motivation for this paper. The numbers are completely made up, but they respect the fact that PV is more material intensive than gas (Hertwich et al., 2015). Here is our new technology matrix, where $p$ represents the share of PV in the energy (or electricity) mix.

$$A = \begin{pmatrix} 0 & 0 & 0 & p \\ 0 & 0 & 0 & 1-p \\ m_{PV} & m_g & m_m & 0 \\ E_{PV} & E_g & E_m & 0 \end{pmatrix}$$

$$= \begin{array}{cc} \begin{pmatrix} 0 & 0 & 0 & p \\ 0 & 0 & 0 & 1-p \\ 0.7 & 0.1 & 0.2 & 0 \\ 0.1 & 0.1 & 0.5 & 0 \end{pmatrix} & \begin{array}{c} \text{PV} \\ \text{gas} \\ \text{materials} \\ \text{energy} \end{array} \end{array}$$

With some calculus (see on-line), we obtain:

$$\text{EROI} = \frac{0.67 - 0.3p}{0.13 + 0.3p} \tag{4}$$

This corresponds to the system-wide EROI. But now that we have two technologies, we can compute the EROI of each of them:[3]

$$\text{EROI}_{PV} = 1.558 - 0.698p$$
$$\text{EROI}_{gas} = 5.154 - 2.308p \tag{5}$$

Logically, the EROI of PV is lower as compared to gas because of its higher material intensity. But it is worth noticing that both EROIs depend on the energy mix $p$: the EROI of a technology is not an intrinsic property. Indeed, it depends on the whole economic system, or more precisely, of all technologies used in their chain of production.[4] Here, the higher the share of PV in the mix, the more the lower EROI of PV contaminates each technology, and the lower the EROI of both technologies.

One can see on Figure 3 that for highest penetration of PV, the EROI falls below unity. In other words, a renewable energy mix with 100% PV is not sustainable in this example. Even more worryingly, if one computes the EROI of PV in an energy mix relying mostly on gas, one would find a high-enough EROI for PV (meaning, above 1). Hence, one cannot conclude that a technology is sufficiently efficient (or sustainable) just by computing its EROI in the current energy mix. Yet, EROIs computations have always been done from actual data of our economy, and could falsely represent the efficiencies of energy technologies in another energy mix, say, a 100% renewable one. This uncertainty concerning the sustainability of a decarbonized energy system motivates the core of this paper: the estimation of EROIs after a global energy transition.

## 3. Estimation of Current and Future EROIs Using THEMIS

### *3.1. Definitions and Setting*

Different notions of EROIs have been used in the literature, and some papers clarify them all (e.g. Brandt & Dale, 2011; Murphy et al., 2011). The most relevant notion for this research is defined by Brandt & Dale (2011) as the Gross Energy Ratio (GER). The GER measures the ratio of energy delivered over energy embodied in inputs net of the energy of the fuels transformed in the process. Thus, for example, the denominator of the GER does not take into account the energy provided by gas in a gas powered plant. The term “gross” is used because all energy output is taken into account; on the contrary Net Energy Ratios subtract from the numerator all “self-use” output that is used in the pathway of production of the technology.[5] A related indicator that is sometimes used to compute EROI (as it is already included in many input-output databases) is the Cumulated Energy Demand (CED). I do not use it because Arvesen & Hertwich (2015) have shown that it is erroneous to use the CED directly for EROI computations, without making adjustments.

In most cases, EROIs (or energy ratios) are defined using quantities of primary energy. However, I adopt a different approach in this paper, and use only secondary energies in my computations. Indeed, as Arvesen & Hertwich (2015) put it, “EROI does not need to measure primary energy per se; the crucial point is to measure energy diverted from society in a unit of equivalence”. Also, the choice of secondary energy carriers is consistent with an energy system relying on renewable electricity, while for such systems the definition of primary energy is not harmonized and this can lead to inconsistencies: Frischknecht et al. (2015) spot for example a factor 6 between the cumulative (primary) energy demand for solar

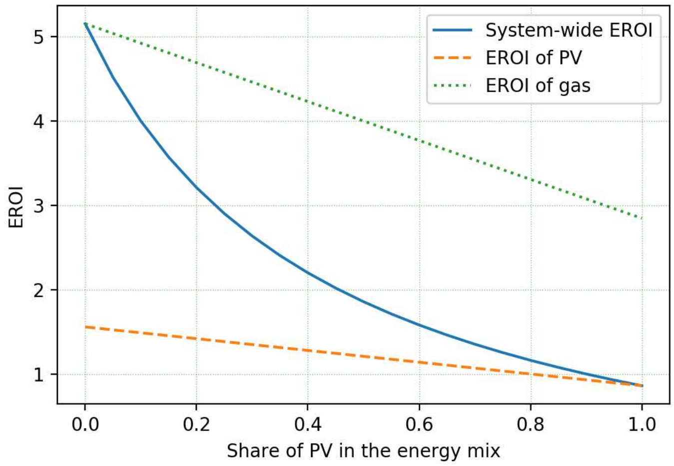


Figure 3: EROIs in the two-technology model in function of the share $p$ of PV in the energy mix.

[3] Similarly to the system-wide EROI, the EROI of a technology is the ratio between the energy delivered by one unit of this technology (over its lifetime), and the energy required to build, operate, maintain and dismantle it. Furthermore, one can show that $\frac{1}{\text{EROI}} = \frac{p}{\text{EROI}_{PV}} + \frac{1-p}{\text{EROI}_{gas}}$, and this formula generalizes to any number of technologies.

[4] *Chain of production*, *recursive* or *embodied inputs* are synonyms; their analysis is known as *structural path analysis* in the literature.

[5] It is worth noting that the Gross Energy Ratio is called by King (2014) the net external energy ratio. As the terminologies of these two papers are not compatible, I follow Brandt & Dale (2011), who aim at harmonising the terminology. For King, “gross” energy is the total energy diverted from Nature while “net” is the output of energy from the technology, what Brandt and Dale call “gross”. Furthermore, King would qualify “external” any notion that subtract the fuel transformed in production from the denominator, while Brandt and Dale always take this as a base case, and employ “external” when self-use output is also subtracted: it mirrors their notion of “net” for the denominator. As we study EROIs of electricity technologies, self-use output consists in electricity inputs in the pathway of production.

photovoltaic computed according to different methods. Although the sectors bringing energy are not the same in the two approaches (the primary approach uses crude oil when the secondary approaches uses gasoline, for example), both approaches are equally valid.

Furthermore, practitioners often use a factor of conversion (around 3) to account for the higher quality of electricity as compared to fossil fuels. I follow the recommendation of Murphy et al. (2011) by undertaking my computations without and with a quality-adjustment factor of 2.6. However, I prefer not to bring to the fore the quality-adjusted computations, provided in Appendix C, and I focus instead on non-quality adjusted EROIs. The reason for this is that the factor of conversion is not well established: it represents the inverse of the yield of a thermal power station (about 38%), but this yield depends on the technology and on the fuel used. Moreover, for certain usage like heating, the yield of fossil fuels is close to that of electricity, and fossil fuels are disproportionately used for these applications for which they have a higher yield, therefore the difference in quality between fossils and electricity may be smaller than usually assumed. Finally, Table 1 summarizes the choices that have been made to address common problems in Net Energy Analysis. These choices are consistent with the method of Brand-Correa et al. (2017) to compute national EROIs.

To avoid the possible ambiguity of sentences, I reproduce below the formulas used to compute the EROI for a technology (or an energy system) $t$, which I denote $GER_t^{2nd}$. Let us recall that $y$ is the vector of final demand, given by the scenario, and $A$ is the technology matrix (or input-output table). $E^S$ is the row vector of unitary energy supply per sector, meaning that $E_t^S$ is the energy supplied by one unit of sector $t$, hence $E^S \cdot y_t$ gives the energy supplied by the technology $t$:[6]

$$\text{supply}_t = E^S \cdot y_t \tag{7}$$

$\odot$ (resp. $\oslash$) denotes the Hadamard (or entrywise) product (resp. division), so that $E^S \odot \mathbb{1}_{\text{2nd}}$ is the vector of unitary *secondary* energy supply. The main term at the denominator of the *GER* is the secondary energy embodied in inputs, net of the energy supplied by the technology:

$$\text{net embodied}_t = E^S \odot \mathbb{1}_{\text{2nd}} \cdot \left( (I - A)^{-1} \cdot y_t - y_t \right) \tag{8}$$

To this term, we also need to subtract the energy supplied by secondary fuels which are direct inputs to thermal electricity somewhere in the supply-chain, including at the last stage. Indeed, such energy is not used to build or maintain the energy system; rather, it is an energy transformed and delivered by the electricity technology, so including it would amount to double-counting. This term is especially important when $t$ is some kind of thermal electricity.

[6] In practice, $y$ is obtained from the scenario of energy demand from the IEA:

$$y_t = \left( \text{demand} \oslash E^S \right) \odot \mathbb{1}_t. \tag{6}$$

$$\text{fuels inputs to elec}_t = E^S \odot \mathbb{1}_{\text{2nd fuel}} \cdot A \cdot \mathbb{1}_{\text{thermal elec}} \odot (I - A)^{-1} \cdot y_t \tag{9}$$

where $\mathbb{1}_{\text{thermal elec}} \odot (I - A)^{-1} \cdot y_t$ is the embodied thermal electricity.

Finally, we have:

$$GER_t^{2nd} = \frac{\text{supply}_t}{\text{net embodied}_t - \text{fuel input to elec}_t} \tag{10}$$

### 3.2. *Data, Sources and Method*

I apply these formulas to the IOTs (i.e. technology matrices $A$) and the vectors of unitary energy supply $E^S$ from THEMIS (Gibon et al., 2015). THEMIS contains hybrid input-output tables: precise data on electricity units (the *foreground*) is completed with data on other sectors that originates from life cycle inventories and national accounts (the *background*). Gibon et al. (2015) have compiled various life cycle inventories into the 609 sectors of the foreground, including original and up-to-date life cycle inventories for electricity sectors. Hertwich et al., 2015 and its Supplementary Information (SI) detail sources and values retained for the evolution of crucial parameters of electricity technologies, such as energy efficiency and market shares of different photovoltaic modules. The background contains data in physical units for 4,087 sectors from the life cycle inventory ecoinvent and data in monetary units for 203 sectors from the input-output database Exiobase (Wood et al., 2014). The 44 Exiobase regions are aggregated into 9 macro-regions that coincide with those of the International Energy Agency (IEA), so that the number of rows and columns in each IOT is 9 times the number of sectors: 44,046. Starting from data of the 2010 IOT, the 2030 and 2050 IOTs of THEMIS embed expected technological efficiency improvements of key background sectors, produced by the New Energy Externalities Development for Sustainability project (NEEDS, 2009). NEEDS' realistic-optimistic scenario was identified as the closest match to the Blue Map and Greenpeace's scenarios assumptions, namely the deployment of best available techniques and reasonable efficiency trends, while the realistic-pessimistic scenario matched the Baseline assumptions. Besides, improvements in foreground processes are modeled using (1) industry road maps, (2) technology learning curves, and (3) expert opinion (see SI of Hertwich et al. (2015) for more details). Furthermore, it is worth noting that THEMIS IOTs are constructed as if the whole economy were at a steady-state, contrarily to national accounts, which give the flows between sectors for a given year. This matches perfectly our purpose, because there is no need to adjust the EROI computations for the growth of some sector or for the lifetimes of some technologies. Finally, as THEMIS is multiregional, EROIs are given in total rather than internal terms, meaning that embodied energy contains energy embodied in imports.The two scenarios native in THEMIS are the baseline (BL) and the Blue Map (BM) scenarios of the IEA (IEA, 2010). While the former posits an almost constant electricity

Table 1: How this paper deals with classical problems of Net Energy Analysis

| Problem | Reference | Solution adopted |
|---|---|---|
| System boundary | Suh (2004) | Input-Output (exhaustive) approach |
| Dynamic vs. steady state | Müller et al. (2014) | Steady state with vintage capital |
| Predicting future coefficients | Gibon et al. (2015) | Use of THEMIS modeling |
| Meshing distinct energy types | Raugei (2019) | Compare only electricity technologies |
| Primary vs. secondary energy | Arvesen & Hertwich (2015) | Secondary energy |
| Quality adjustment | Murphy et al. (2011) | Emphasis on non-quality adjusted, both done |
| Definition of EROI | Brandt & Dale (2011) | Gross Energy Ratio |

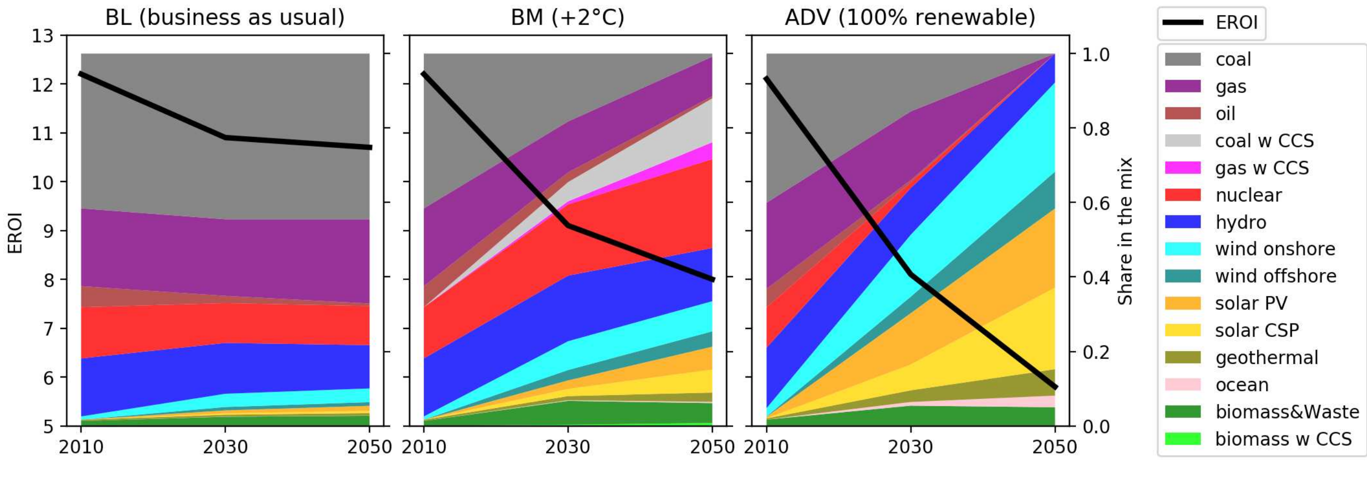


Figure 4: Evolution of global EROIs and mixes of electricity for different scenarios.

mix, the latter is compatible with a 50% probability to contain the global mean temperature anomaly to +2°C in 2100. As Blue Map still relies at 30% on fossil fuels based electricity in 2050 —including 17% with Carbon Capture and Storage (CCS); it does not allow to assess more decarbonized scenarios. Hence, I combine with THEMIS the scenarios from Greenpeace's Energy [R]evolution report (Teske et al., 2015). Greenpeace proposes a business as usual scenario (REF) close to baseline, as well as two scenarios compatible with the 2°C target. Both exclude CCS and phase out from nuclear between 2012 and 2050.[7] The first Greenpeace scenario, Energy [R]evolution (ER), comprises 93% of electricity from renewable sources in 2050, while the second one, Advanced Energy [R]evolution (ADV), attains 100% renewable. As the difference is small between these two scenarios, I focus on the 100% renewable one. I describe my methodology for embedding the regional electricity mixes of Greenpeace's scenarios into THEMIS in Appendix A.

In the literature, most EROIs estimations follow a bottom-up approach that use data from life cycle inventories. Bottom-up studies describe in details the power facilities and the most direct inputs to the energy technologies, but they do not cover the entire economy: indirect inputs such as clerical work or R&D are often beyond their system boundaries (Suh, 2004). On the contrary, the input-output method allows to encompass all embodied inputs exhaustively. As a consequence of this more comprehensive account of embodied energy than usual, we expect estimates of EROIs lower than the average of the literature. That being said, it is not a concern if our estimates are not directly comparable to those of the literature, as we are mainly interested in comparing them internally, among the different years and scenarios, and to scrutinize whether they vary substantially or not.

Because renewable sources are intermittent and dispersed, the capacity, grid extension and storage they require do not increase linearly with the electricity delivered. Hence, as Greenpeace scenarios are not native in THEMIS, they need further adjustments to account for these non-linearities. I explain in Appendix A how the need for overcapacity is addressed. Concerning transmission and storage, however, the requirements are not given by the Greenpeace report (Teske et al., 2015), so they have not been taken into account. Even if the report does not precise any plan relative to storage, hydrogen produced from renewables seems to play a substantial role in Greenpeace scenarios, as its share in the electricity mix is 5% in ADV 2050. However, as the sector 'Electricity from hydrogen' is absent from THEMIS, hydrogen has been excluded from this analysis. These limitations should be addressed in future

[7] The study funded by Greenpeace was in fact conducted by researchers at the Institute of Engineering Thermodynamics of the German Aerospace Center (DLR), who applied their model REMix. Using the same model, Berrill et al. (2016) minimize the cost of European electricity generation under different carbon prices. Interestingly, an outcome of the model was to phase nuclear out, but to select coal with CCS. This indicates that the choices of Greenpeace were not solely motivated by a minimization of costs, but also by expert judgment and ethical considerations.

Table 2: EROIs and share in electricity mix of electric technologies in the model THEMIS for different scenarios and years. The bottom line in columns *mix* gives the total secondary energy demand, in PWh/a.

| Scenario | Baseline (BL) | | | | | | Blue Map (BM, +2°) | | | | ADV (100% renewable) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | 2010 | | 2030 | | 2050 | | 2030 | | 2050 | | 2030 | | 2050 | |
| Variable | EROI | mix | EROI | mix | EROI | mix | EROI | mix | EROI | mix | EROI | mix | EROI | mix |
| **biomass w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | 4.6 | 0.00 | 4.0 | 0.01 | – | 0.00 | – | 0.00 |
| **biomass&Waste** | 11.4 | 0.01 | 6.3 | 0.02 | 5.9 | 0.03 | 5.5 | 0.06 | 5.2 | 0.05 | 5.2 | 0.05 | 4.6 | 0.05 |
| **ocean** | 5.5 | 0.00 | 2.4 | 0.00 | 2.9 | 0.00 | 3.7 | 0.00 | 5.8 | 0.00 | 4.8 | 0.01 | 4.9 | 0.03 |
| **geothermal** | 5.4 | 0.00 | 5.2 | 0.01 | 5.1 | 0.01 | 5.2 | 0.01 | 5.4 | 0.02 | 3.8 | 0.03 | 3.9 | 0.07 |
| **solar CSP** | 21.6 | 0.00 | 8.9 | 0.00 | 9.1 | 0.01 | 8.2 | 0.02 | 7.9 | 0.06 | 9.3 | 0.07 | 7.8 | 0.22 |
| **solar PV** | 9.3 | 0.00 | 7.4 | 0.01 | 7.2 | 0.01 | 6.4 | 0.02 | 6.0 | 0.06 | 5.4 | 0.14 | 4.7 | 0.21 |
| **wind offshore** | 9.4 | 0.00 | 11.0 | 0.01 | 10.5 | 0.01 | 7.7 | 0.03 | 6.3 | 0.04 | 6.5 | 0.04 | 6.4 | 0.10 |
| **wind onshore** | 9.5 | 0.01 | 9.3 | 0.04 | 8.1 | 0.04 | 7.1 | 0.08 | 7.3 | 0.08 | 7.2 | 0.17 | 5.8 | 0.24 |
| **hydro** | 13.2 | 0.16 | 11.9 | 0.14 | 11.9 | 0.12 | 12.8 | 0.18 | 13.1 | 0.14 | 11.0 | 0.13 | 10.9 | 0.08 |
| **nuclear** | 10.5 | 0.14 | 7.3 | 0.11 | 7.0 | 0.10 | 7.3 | 0.19 | 7.4 | 0.24 | 8.3 | 0.02 | – | 0.00 |
| **gas w CCS** | – | 0.00 | – | 0.00 | 7.5 | 0.00 | 7.9 | 0.01 | 9.1 | 0.05 | – | 0.00 | – | 0.00 |
| **coal w CCS** | – | 0.00 | – | 0.00 | 6.2 | 0.00 | 7.1 | 0.05 | 7.1 | 0.12 | – | 0.00 | – | 0.00 |
| **oil** | 8.4 | 0.06 | 9.8 | 0.02 | 9.9 | 0.01 | 9.5 | 0.03 | 7.3 | 0.01 | 10.0 | 0.01 | – | 0.00 |
| **gas** | 13.9 | 0.21 | 15.0 | 0.21 | 14.9 | 0.23 | 17.3 | 0.14 | 19.7 | 0.11 | 16.5 | 0.18 | – | 0.00 |
| **coal** | 12.9 | 0.42 | 11.5 | 0.45 | 11.5 | 0.45 | 11.6 | 0.18 | 12.4 | 0.01 | 10.4 | 0.16 | 11.5 | 0.00 |
| **Total** | 12.2 | 19.76 | 10.9 | 34.29 | 10.7 | 45.97 | 9.1 | 28.01 | 8.0 | 40.22 | 8.1 | 36.74 | 5.8 | 64.04 |

work, together with the study of an energy transition in the transportation sector (which also partly relies on hydrogen). Such extension will not be easy, as the transportation sectors are still not sufficiently disaggregated in THEMIS to study a change in their technology. Meanwhile, other references can provide information on orders of magnitude of storage and transmission (Berrill et al., 2016; Koskinen & Breyer, 2016; Scholz et al., 2017). Applying REMix, the same optimization model that is used in the Greenpeace report, Scholz et al. (2017) show that the cost of storage and transmission combined is 4.6% of total cost in a business-as-usual scenario and 10.6% in a 100% renewable one. The adjustment needed for the cost, around 6%, gives a rough estimate of the upward bias of unadjusted EROI estimates (see section 4.2 on the relation between price and EROI).

Finally, data for Concentrated Solar Panels (CSP) had to be adjusted, because the original data mistakenly contained an energy supplied by unit of solar CSP of 0 in some regions (leading to abnormally low EROIs, around 2). Backed by Thomas Gibon, core developer of THEMIS, I corrected this error by setting the unitary energy supplied for solar CSP in all regions to its value in *OECD North America* (still letting the value depend on the scenario and the year).

### 3.3. *Main Results*

Main results are shown in Figure 4 and in Table 2. Complementary results for quality-adjusted EROIs and all scenarios can be found in Appendix C. Complete results are provided in the Supplementary Information spreadsheet: they include e.g. regional estimates and a decomposition of EROIs' denominators between direct and indirect energy. Some EROIs are missing, because not all technologies already existed on an industrial scale in 2010, and some technologies are discarded in the future by some scenarios. Conversely, some EROIs are given for apparent shares of production of 0: this is the case when the share is rounded to 0, but not 0.

One can notice that, as expected, PV and wind panels have a lower EROI than electricity from fossil fuels. The EROIs of renewables decrease, as anticipated in the previous section. However, they remain largely above 1, suggesting that renewables are energetically sustainable. Recall that this was not evident as, *in theory*, nothing guarantees that EROIs stay above 1 when the energy mix changes (see section 2.2). Values for current EROIs range from 8 to 22. This range is in-line with that from Hall et al. (2014), but not with Weißbach et al. (2013), who find more contrasts between renewables and fossils. Such discrepancy is common in the EROI literature, may be due to differences in the methodology (Weißbach uses bottom-up data from specific locations) and does not affect this paper's results on the *evolution* of EROIs.

The system-wide EROI for the entire electricity sector is given at the bottom line of Table 2. It is estimated at 12.2 in 2010; it decreases slightly until $10.8 \pm 0.1$ in 2030 and 2050 in the Baseline scenario. An examination of regional estimates (see SI) reveals that this decrease is driven by a composition effect in the global mix. Indeed, the largest energy producer in 2010, North America, has higher EROIs and is replaced by China in 2030, which has lower ones; the EROIs in each world region remaining quite stable.[8] The decrease is more pronounced when the penetration of renewable is higher: down to 8.0 in 2050 in the Blue Map scenario and even 5.8 in the

[8] In Baseline, EROIs of *OECD Europe* and India are very close to global EROI, while those of Africa & Middle East and *Rest of developing Asia* are within ±3 to global ones.

100% renewable one. The magnitude of the decline is substantial: an expected halving of global EROI may prove to be a challenge for the success of an energy transition to renewables.

One may wonder whether our results are driven by conservative forecasts concerning the progress in renewable technologies, or any other hypothesis concerning the evolution of the technology matrix. Of course, the quality of input-output data is never perfect, and making predictions is notoriously difficult, as was recently proven by the unexpected fall in the price of photovoltaic (PV) modules. However, there are several reasons to be more confident into future EROIs estimates from THEMIS than into past predictions on prices from other sources. First, technical coefficients are more stable than prices. Second, THEMIS accounts for materials and energy efficiency gains for electricity technologies, and uses "fairly favorable assumptions regarding wind conditions, insolation and resulting load factors", which if anything would bias EROIs of renewables upward (see SI of Hertwich et al., 2015). Third, THEMIS already includes recent industry road maps in its prospective matrices (see section 3.1), e.g. concerning the shift of PV market shares from cristalline silicon modules towards more efficient cadmium telluride (CdTe) or CIGS modules. Overall, the data from THEMIS seems most accurate concerning materials, metallurgy and energy sectors, and further improvements should probably focus on other sectors, like transport or services.

## 4. Implications of a Decreasing EROI on Prices and GDP

The forecast of declining EROIs made in the previous section calls for an assessment of its economic implications. The main channel through which a decrease in EROI could affect the economy is arguably a rise in energy price (and correlatively, in energy expenditures). In this section, I review the literature on the relation between EROI and the price of energy, estimate it empirically, and extend a result from Herendeen (2015) to characterize this relation. As in previous work, an inverse relation is documented empirically. Yet, theoretical analysis shows that EROI and price might decrease together. This theoretical result tempers the view that a decreasing EROI necessarily leads to a contraction of GDP.

### *4.1. Inverse Relation Proposed in First Studies*

King & Hall (2011) point both theoretically and empirically that the price of a unit of energy $p_t$ and the EROI of a technology $t$ are inversely related. Defining the monetary return on investment MROI (i.e. the financial yield $\frac{\$_{\text{out}}}{\$_{\text{investment}}}$), they derive the formula:

$$p_t = \frac{\$_{\text{out}}}{E_{\text{out}}} = \frac{\text{MROI}_t}{\text{EROI}_t} \cdot \frac{\$_{investment}}{E_{\text{in}}} \tag{11}$$

Heun & de Wit (2012) find an equivalent formula. They designate MROI as the mark-up $m_t$, consider production costs per gross output $c_t = \frac{\$_{\text{investment}}}{E_{\text{out}}+E_{\text{in}}}$ and use their own notion of EROI:

Table 3: Predicted average global price of electricity (in €/MWh)

| year | 2010 | 2030 | | | 2050 | | |
|---|---|---|---|---|---|---|---|
| scenario | all | BL | BM | ADV | BL | BM | ADV |
| price | 27 | 28 | 30 | 30 | 28 | 30 | 32 |

$\text{EROI}_t^H = \frac{E_{\text{out}}+E_{\text{in}}}{E_{\text{in}}} = \text{EROI}_t + 1$, so that equation (11) rewrites

$$p_t = \frac{m_t}{\text{EROI}_t^H - 1} \cdot \frac{\$_{\text{investment}}}{E_{\text{out}} + E_{\text{in}}} \cdot \frac{E_{\text{out}} + E_{\text{in}}}{E_{\text{in}}} = \frac{m_t \cdot c_t}{1 - 1/\text{EROI}_t^H} \tag{12}$$

The problem with these formulas is that all variables move together: when EROI varies, so does the cost of production, so that we cannot predict the future price taking this cost as fixed. Heun & de Wit (2012) acknowledge this; and thus study the empirical link between EROI and price.

### *4.2. Empirical Relation Between EROI and Price*

Using US data on oil and EROI from (Cleveland, 2005), Heun & de Wit (2012) regress $p_t$ on the EROI.[9] They obtain a good fit even in their simplest regression ($R^2 = 0.8$), and find $p_{\text{oil}} = \beta_0 \cdot \text{EROI}_{\text{oil}}^{-1.4}$.

This result is interesting, and documents a negative relationship between price and EROI, which is close to an inverse one. As the authors do not regress price on the inverse of EROI, one cannot compare whether an inverse specification would provide as good a fit as a log-log one. To undertake this comparison, I run these two regressions using all estimates of EROI computed using THEMIS, one for each combination of scenario, year, region and sector. To obtain the price corresponding to each EROI, which I take *before* taxes and subsidies on production; I assemble from the columns *compensation of employees* and *operating surplus* of the characterization matrix of THEMIS a row vector $v$ of value-added per unit of each sector. Indeed, the vector of prices excluding tax $p$ can be seen as emerging from value-added according to

$$p = v \cdot (I - A)^{-1} \oslash E^S \tag{13}$$

because the price of energy in sector $s$, $p_s$, is the sum of the value-added of inputs embodied in $s$: $v \cdot (I - A)^{-1} \cdot \mathbb{1}_s$, divided by the energy supplied by one unit of $s$: $E_s^S$. To the extent that the physical constituents and processes of a given technology will not change in an unexpected way, and as THEMIS models technical progress but not behaviors nor general equilibrium effects, prices forecast using the above formula seem less reliable than EROI estimates. For this reason, I report only the global average electricity prices of the main scenarios (see Table 3), but I do not detail the substantial variations between regions or sectors.[10]

[9] Although they claim that their explained and explanatory variables are respectively the cost of production and their notion of EROI, EROI$^H$, the former is indeed the producer price and the latter our notion of EROI, according to the source of their data: Cleveland (2005).

[10] The results are on-line and available on demand.

Table 4: Regressions of price on EROI (both estimated using THEMIS). All coefficients are significant at the 1‰ level.

| Obs. | N | Specification | Coefficients $a$ | Coefficients $b$ | $R^{2\,a}$ |
|---|---|---|---|---|---|
| All | 2079 | $p = \frac{a}{EROI} + b$ | 85 | 18 | 0.55 |
| 2010 | 104 | | 72 | 21 | 0.54 |
| All | 2079 | $\log(p) = a \cdot \log(\text{EROI}) + b$ | −0.57 | 2.0 | 0.58 |
| 2010 | 104 | | −0.46 | 1.9 | 0.62 |

[a] The $R^2$ given for log-log fits is not the original one, cf. text.

Table 4 reports the results of both the log-log and the inverse fits. I ran each model twice: first, on all 2079 positive observations available, and then on the 104 observations for year 2010. To make the $R^2$ of the log-log fit comparable to that of the inverse fit, I compute it as the sum of squared errors between "observed" prices and predicted prices (instead of their respective logarithms). As all $R^2$ are between 0.54 and 0.62, the inverse fit is almost as accurate than the log-log fit. Moreover, although the elasticity of price on EROI estimated here is different from that found by Heun & de Wit (2012) for oil (around −0.5 as compared to −1.4), both figures are close to 1. Empirical findings confirm an inverse relation between price and EROI. However, Figure 5 shows that a significant share of the variance in price remains unexplained by EROI, even more so for values of EROI around the global averages of 6-12, where the fit is almost flat and the errors substantial. In addition, theoretical analysis rejects the existence of a mapping between price and EROI.

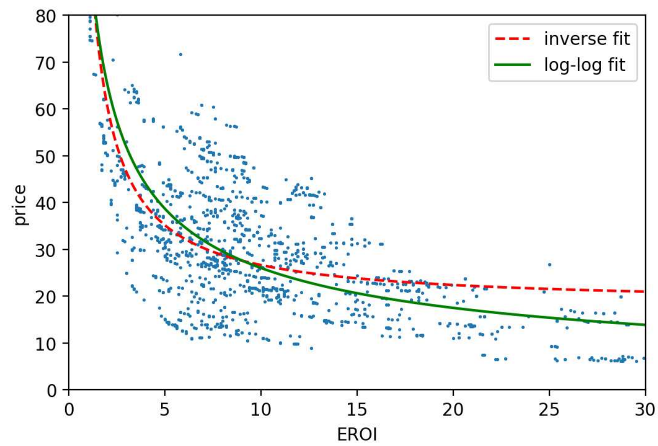


Figure 5: Regressions of price on EROI (all observations, from THEMIS).

### 4.3. A Case Against Any Simple Relation

Herendeen (2015) shed new light on the theoretical relation by treating the question from its matrix form and introducing the concept of value-added. Herendeen showed how to express rigorously the price in function of the EROI when the economy is constituted of two sectors (energy and materials), and explained the limits of such exercise. Hereafter, I extend the results of Herendeen to an arbitrary number of sectors, $n$. His approach relies on the concept of *energy intensity*.

To deliver one unit of energy technology $t$, the production mobilized is $(I-A)^{-1} \cdot \mathbb{1}_t$, while the energy mobilized, called the energy intensity of $t$, writes $\varepsilon_t = \mathbb{1}_E^T \cdot (I-A)^{-1} \cdot \mathbb{1}_t$, where $E$ is the set of all energies.[11] $\varepsilon_t$ is in fact the gross energy embodied in $t$, i.e. the sum of the delivered and the net embodied energy. Hence, the EROI of $t$ is a simple function of $\varepsilon_t$: $\text{EROI}_t = \frac{\mathbb{1}_E^T \cdot \mathbb{1}_t}{\mathbb{1}_E^T \cdot ((I-A)^{-1} - I) \cdot \mathbb{1}_t} = \frac{1}{\varepsilon_t - 1}$.

In Appendix D, I show that the price of a technology $t$ is a certain function of the coefficients of $A$,[12] and that each coefficient of $A$ can be expressed as a function of EROI. Composing two such functions, we obtain that the price is inversely related to EROI. However, the relation is not unique (as it depends on the coefficient of $A$ chosen to make the connection), and the other parameters in the relation are not constant. This leads to the following Proposition:

**Proposition 1.** *(Generalization of Herendeen, 2015) Assuming that all coefficients of the transformation matrix $A$ are constant except one, noted $x = a_{i_0,j_0}$, and that EROI varies with $x$; the price of $t$ can be expressed as a linear function of its energy intensity $\varepsilon_t = 1 + \frac{1}{EROI_t}$, so that:*

$$\exists!\,(\alpha,\beta) \in \mathbb{R}^2,\ p_t = \frac{\alpha}{EROI_t} + \beta \tag{14}$$

*Proof.* See Appendix D. □

*Remark.* With the terminology of Heun & de Wit (2012) or Herendeen (2015), the relation above would write: $p_t = \alpha \frac{\text{EROI}_t^H}{\text{EROI}_t^H - 1} + \gamma$, with $\gamma = \beta - \alpha$. This is because in their definition of EROI, the numerator is $\varepsilon_t$ instead of 1.

In the general case, we cannot obtain a *better* result, i.e. a formula that still holds when letting more than one coefficient vary. Indeed, denoting $\omega_{i,t}$ the coefficient $(i,t)$ of $(I-A)^{-1}$, the Laplace expansion of $I-A$ gives us $\omega_{i,t} = \frac{(-1)^{i+j}}{\det(I-A)} \det\left( \left((I-A)_{j,k}\right)_{\substack{j \in [\![1;n]\!] \setminus i \\ k \in [\![1;n]\!] \setminus t}} \right)$. Hence, we have $\varepsilon_t = \sum_{e \in E} \left((I-A)^{-1}\right)_{e,t} = \sum_{e \in E} \omega_{e,t}$ and $p_t = \sum_{i=1}^n v_i \left((I-A)^{-1}\right)_{i,t} = \sum_{i=1}^n v_i \omega_{i,t} = \sum_{e \in E} v_e \omega_{e,t} + \sum_{i \notin E} v_i \omega_{i,t}$. Denoting $\tilde{v} = \frac{\sum_{e \in E} v_e \omega_{e,t}}{\sum_{e \in E} \omega_{e,t}}$ and $r = \tilde{v} + \sum_{i \notin E} v_i \omega_{i,t}$, we obtain

$$p_t = \tilde{v} \varepsilon_t + \sum_{i \notin E} v_i \omega_{i,t} = \frac{\tilde{v}}{\text{EROI}_t} + r \tag{15}$$

However, one has to keep in mind that $r$, $\tilde{v}$ and $\text{EROI}_t$ all depend on the coefficients of $A$, and vary together when $A$ changes. If there is only one type of energy ($E = \{e\}$) or if value-added is equal for all types of energy ($\forall e \in E,\ v_e = \tilde{v}$), $\tilde{v}$

[11] I assume here that the unit of an output of an energy sector $e \in E$, hence of $\mathbb{1}_E^T$, is an energy unit, like TWh.

[12] More precisely, a function field of a certain algebraic variety.

does not depend on the coefficients of $A$ anymore, and we obtain a formula close to that of King & Hall (2011): $p_t = \frac{v_e}{\text{EROI}_t} + r$. Still, when the EROI varies because more than one coefficient of $A$ changes, $r$ varies concomitantly, and the EROI cannot be used as a sufficient statistic to infer the price. For this reason, one cannot identify empirically a linear relation between price and the inverse of EROI without strong assumption on the steadiness of $A$.

Actually, the theoretical relation between EROI and price is so fragile that one cannot even conclude that it is a decreasing relation: I provide in Appendix B a numerical example showing that EROI and price can both increase at the same time when more than one coefficient varies. Such acknowledgment dissuades from predicting long run prices by simply looking at estimations of future EROIs.

Does this mean that EROI is unrelated to any economic concept? Fizaine & Court (2016) argue that there is a minimum EROI below which the US economy enters a recession. They first show that energy expenditure Granger causes growth in the US, then determine a threshold of energy expenditure above which the US enters in a recession (consistent with that of Bashmakov, 2007), and finally use a modified version of equation (11) to relate this to a minimum non-recessionary EROI. However, they misleadingly replace the inverse of the energy intensity of energy investment $\frac{\$_{investment}}{E_{\text{in}}}$ by that of the whole economy, $\frac{GDP}{E_{\text{out}}}$. This prevents them from noticing that cost reductions in energy production could compensate the effect of a decreasing EROI on energy prices and expenditures. As we have seen, EROI, price and energy expenditure may all decrease at the same time, which undermines the idea that a recession caused by a surge in energy expenditure is ineluctable as soon as EROI goes below some threshold. In addition, an energy price increase should have an expansionary effect on net exporters of energy, at odds with the mechanism extrapolated by Fizaine and Court from the case of the United States, which has been historically a net energy importer. Overall, the analysis of this section indicates that the economic consequences of a change in EROI are ambiguous, and that this physical notion cannot be used to predict future prices or GDP without empirical evidence.

## 5. Concluding Remarks

This work includes a first attempt at estimating future EROIs in a decarbonized electricity system. By examining a broad range of scenarios, it concludes that the system-wide EROI of the power sector should decrease until 2050, from 12.2 to 10.7 in a business-as-usual scenario, 8.0 in a partial transition away from fossil fuels, or 5.8 in a scenario with 100% renewable electricity. Even though the EROI of each technology is expected to remain well above 1, which was questioned theoretically, our results cast doubts on the energetic efficiency of renewable electricity.

As an inverse relationship between EROIs and energy prices is consistently found empirically, a declining EROI could mean higher energy prices. However, theoretical analysis of this relation showed that a declining EROI might also coincide with decreasing energy prices, and does not necessarily lead to a recession.

Finally, this paper assessed scenarios of transition in the electricity sector, but further research is still needed to estimate future EROIs in complete energy transitions, which include a mutation of the transportation system, agriculture and industry. Unfortunately, this could not been done using the current version of THEMIS, and the question remains open for future research.

## References


A. Arvesen & E. G. Hertwich. More caution is needed when using life cycle assessment to determine energy return on investment (EROI). *Energy Policy*, 2015.

A. Arvesen, G. Luderer, M. Pehl, B. L. Bodirsky, & E. G. Hertwich. Deriving life cycle assessment coefficients for application in integrated assessment modelling. *Environmental Modelling & Software*, 2018.

I. Bashmakov. Three laws of energy transitions. *Energy Policy*, 2007.

P. Berrill, A. Arvesen, Y. Scholz, H. C. Gils, & E. G. Hertwich. Environmental impacts of high penetration renewable energy scenarios for Europe. *Environmental Research Letters*, 2016.

L. I. Brand-Correa, P. E. Brockway, C. L. Copeland, T. J. Foxon, A. Owen, & P. G. Taylor. Developing an Input-Output Based Method to Estimate a National-Level Energy Return on Investment (EROI). *Energies*, 2017.

A. R. Brandt. How Does Energy Resource Depletion Affect Prosperity? Mathematics of a Minimum Energy Return on Investment (EROI). *BioPhysical Economics and Resource Quality*, 2017.

A. R. Brandt & M. Dale. A General Mathematical Framework for Calculating Systems-Scale Efficiency of Energy Extraction and Conversion: Energy Return on Investment (EROI) and Other Energy Return Ratios. *Energies*, 2011.

C. J. Cleveland. Net energy from the extraction of oil and gas in the United States. *Energy*, 2005.

V. Court & F. Fizaine. Long-Term Estimates of the Energy-Return-on-Investment (EROI) of Coal, Oil, and Gas Global Productions. *Ecological Economics*, 2017.

M. Dale, S. Krumdieck, & P. Bodger. Net energy yield from production of conventional oil. *Energy Policy*, 2011.

M. A. J. Dale. Global Energy Modelling : A Biophysical Approach (GEMBA). 2010.

Eurostat, editor. *Eurostat Manual of supply, use and input-output tables*. Amt für amtliche Veröffentlichungen der Europäischen Gemeinschaften, Luxembourg, 2008 edition edition, 2008. ISBN 978-92-79-04735-0.

F. Fizaine & V. Court. Energy expenditure, economic growth, and the minimum EROI of society. *Energy Policy*, 2016.

R. Frischknecht, F. Wyss, S. B. Knöpfel, T. Lützkendorf, & M. Balouktsi. Cumulative energy demand in LCA: the energy harvested approach. *The International Journal of Life Cycle Assessment*, 2015.

T. Gibon, R. Wood, A. Arvesen, J. D. Bergesen, S. Suh, & E. G. Hertwich. A Methodology for Integrated, Multiregional Life Cycle Assessment Scenarios under Large-Scale Technological Change. *Environmental Science & Technology*, 2015.

C. A. S. Hall. Introduction to Special Issue on New Studies in EROI (Energy Return on Investment). *Sustainability*, 2011.

C. A. S. Hall, S. Balogh, & D. J. R. Murphy. What is the Minimum EROI that a Sustainable Society Must Have? *Energies*, 2009.

C. A. S. Hall, J. G. Lambert, & S. B. Balogh. EROI of different fuels and the implications for society. *Energy Policy*, 2014.

R. A. Herendeen. Connecting net energy with the price of energy and other goods and services. *Ecological Economics*, 2015.

E. G. Hertwich, T. Gibon, E. A. Bouman, A. Arvesen, S. Suh, G. A. Heath, J. D. Bergesen, A. Ramirez, M. I. Vega, & L. Shi. Integrated life-cycle assessment of electricity-supply scenarios confirms global environmental benefit of low-carbon technologies. *Proceedings of the National Academy of Sciences*, 2015.

M. K. Heun & M. de Wit. Energy return on (energy) invested (EROI), oil prices, and energy transitions. *Energy Policy*, 2012.

IEA. Energy Technology Perspectives. 2010.

T. Junius & J. Oosterhaven. The Solution of Updating or Regionalizing a Matrix with both Positive and Negative Entries. *Economic Systems Research*, 2003.

C. W. King. Matrix method for comparing system and individual energy return ratios when considering an energy transition. *Energy*, 2014.

C. W. King & C. A. S. Hall. Relating Financial and Energy Return on Investment. *Sustainability*, 2011.

O. Koskinen & C. Breyer. Energy Storage in Global and Transcontinental Energy Scenarios: A Critical Review. *Energy Procedia*, 2016.

J. G. Lambert & G. P. Lambert. Predicting the Psychological Response of the American People to Oil Depletion and Declining Energy Return on Investment (EROI). *Sustainability*, 2011.

J. G. Lambert, C. A. Hall, S. Balogh, A. Gupta, & M. Arnold. Energy, EROI and quality of life. *Energy Policy*, 2014.

W. Leontief. *Input-Output Economics*. Oxford University Press, USA, 2 edition, 1986. ISBN 978-0-19-503527-8.

R. E. Miller & P. D. Blair. *Input-Output Analysis: Foundations and Extensions*. Cambridge University Press, 2009. ISBN 978-0-521-51713-3.

E. Müller, L. M. Hilty, R. Widmer, M. Schluep, & M. Faulstich. Modeling Metal Stocks and Flows: A Review of Dynamic Material Flow Analysis Methods. *Environmental Science & Technology*, 2014.

D. J. Murphy, C. A. S. Hall, M. Dale, & C. Cleveland. Order from Chaos: A Preliminary Protocol for Determining the EROI of Fuels. *Sustainability*, 2011.

NEEDS. LCA of Background Processes. Technical report, New Energy Externalities Developments for Sustainability, 2009.

M. Pehl, A. Arvesen, F. Humpenöder, A. Popp, E. G. Hertwich, & G. Luderer. Understanding future emissions from low-carbon power systems by integration of life-cycle assessment and integrated energy modelling. *Nature Energy*, 2017.

A. Poisson, C. Hall, A. Poisson, & C. A. S. Hall. Time Series EROI for Canadian Oil and Gas. *Energies*, 2013.

M. Raugei. Net energy analysis must not compare apples and oranges. *Nature Energy*, 2019.

Y. Scholz, H. C. Gils, & R. C. Pietzcker. Application of a high-detail energy system model to derive power sector characteristics at high wind and solar shares. *Energy Economics*, 2017.

S. Suh. Functions, commodities and environmental impacts in an ecological–economic model. *Ecological Economics*, 2004.

S. Teske, T. Pregger, S. Simon, & T. Naegler. Energy [R]evolution. Technical report, Greenpeace, 2015.

G. Tverberg. The "Wind and Solar Will Save Us" Delusion, 2017.

D. Weißbach, G. Ruprecht, A. Huke, K. Czerski, S. Gottlieb, & A. Hussein. Energy intensities, EROIs (energy returned on invested), and energy payback times of electricity generating power plants. *Energy*, 2013.

R. Wood, K. Stadler, T. Bulavskaya, S. Lutter, S. Giljum, A. de Koning, J. Kuenen, H. Schütz, J. Acosta-Fernández, A. Usubiaga, M. Simas, O. Ivanova, J. Weinzettel, J. H. Schmidt, S. Merciai, & A. Tukker. Global Sustainability Accounting—Developing EXIOBASE for Multi-Regional Footprint Analysis. *Sustainability*, 2014.

## Appendix A. Updating a Matrix *A* To a New Given Mix

The technology matrices $A$ for the IEA scenarios are readily available in THEMIS, but these matrices have to be updated to the new electricity mix for the Greenpeace scenarios. To do this, I exploit the fact that both THEMIS and Greenpeace use the world regions of the IEA, and I modify the electricity input of each sector by the regional mix given by Greenpeace. The most accurate algorithm to update an input-output matrix is known as GRAS (Junius & Oosterhaven, 2003). Although I implemented this algorithm in pymrio, I could not use it, because this algorithm uses the new sums of rows and columns to balance the matrix, and the vector of final demand $y$ or the vector of production $x$ is necessary to know them. As THEMIS does not include such vectors, I had to use a simpler method, which relies on the assumption that the electricity mix of inputs is the same across sectors for a given region. Given the perfect substitutability between electricity produced by different technologies and the uniqueness of electric grids, this assumption seems justified.

There are two different updates to make. First, I modify the vector of second energy demand (used to infer the final demand of technology $t$, $y_t$) so that it perfectly matches the demand of the scenario. Second, I modify the submatrix $D$ of $A$ containing the rows of electricity sectors. To convert $D$ in energy units, I multiply each row $t$ of $D$ by the corresponding energy supplied per unit of technology $t$, $E^S_t$. I call the result $E$: the coefficient $E_{is}$ of $E$ gives the electricity from sector $i$ required to make one unit of sector $s$' output, where $i = i(t,r)$ corresponds to technology $t$ in region $r$. Then, I premultiply $E$ by a block diagonal matrix with $R$ blocks of size $T * T$ containing only ones (where $R = 9$ and $T = 15$ are the number of THEMIS regions and electricity sectors, respectively) to obtain a matrix $B$. Each row of $B$ gives the total electricity from a given region $r$ required to produced each output, $E^{\text{tot}}_r$, and each row $E^{\text{tot}}_r$ is replicated $T$ times:

$$B = \begin{pmatrix} B_1 \\ \vdots \\ B_R \end{pmatrix}, \quad B_r = \begin{pmatrix} E^{\text{tot}}_r \\ \vdots \\ E^{\text{tot}}_r \end{pmatrix}$$

Next, each row of $B$ is multiplied by the share of a technology $t$ in the mix of the corresponding region, which defines a matrix $\tilde{E}$. Each coefficient $\tilde{E}_{i,s}$ of $\tilde{E}$ gives the electricity from sector $i$ required to make one unit of sector $s$' output, according to the new mix (by construction, for all electricity sector $j = i(t,r)$, the share of technology $j$ in the regional mix, $\frac{\tilde{E}_{j,s}}{\sum_t \tilde{E}_{i(t,r),s}}$, is the same across all sectors $s$). Eventually, I obtain the new submatrix $\tilde{D}$ by converting each row of $\tilde{E}$ to the original units of $A$ (by dividing each row by the appropriate unitary energy supplied $E^S_t$).

A last update is needed for Greenpeace scenarios, to account for the extra capacity needed when intermittent sources fail to deliver energy: the ratio of capacity (in GW) over production (in TWh) is somewhat higher in Greenpeace scenarios than in IEA/THEMIS ones. Thus, I multiply each column of an energy sector (representing all inputs required for one unit of output of this sector) by the ratio of the capacity-over-production ratios of Greenpeace and IEA/THEMIS. Doing so relies on the fact that the energy required to operate a power plant is negligible in front of the energy required to build it (see e.g. Arvesen et al. (2018)).

## Appendix B. Example of Non-Decreasing Relation Between EROI and Price

Herendeen (2015) proposes a calibration on US energy data of his toy model with 2 sectors (materials and energy), which yields as realistic results as a two-by-two model can yield. I start from a slightly modified version of his calibration (called *base*), in the sense that the figures are rounded, and I show how a deviation of two coefficients (in the *new* calibration) leads to an increase of both EROI and price of energy. This proves that in general, nothing can be said of the relation between EROI and price, not even that it is a decreasing relation.

For this, I use the formulas for EROI and price given by Herendeen (2015) (where I convert the price to \$/gal using the conversion factor 1 Btu = 114,000 gal):

$$\text{EROI} = \frac{1}{A_{ee} + \frac{A_{em}A_{me}}{1-A_{mm}}} \tag{B.1}$$

$$p = \frac{v_e(1-A_{mm}) + v_m A_{me}}{(1-A_{ee})(1-A_{mm}) - A_{em}A_{me}} \cdot 114{,}000$$

Table B.5: Example of sets of coefficients exhibiting a non-decreasing relation between EROI and price in a two sectors model (see desmos.com/calculator/ne4oqunhsm).

| | base | new |
|---|---|---|
| $v_m$ | 0.5 | |
| $v_e$ | $5 \cdot 10^{-6}$ | |
| $A_{em}$ | 1700 | |
| $A_{me}$ | $4 \cdot 10^{-6}$ | |
| $A_{mm}$ | 0.5 | 0.9 |
| $A_{ee}$ | 0.3 | 0.1 |
| **EROI** | 3.2 | 6.0 |
| **price** | 1.5 | 3.4 |

## Appendix C. Complementary Results

Results without quality adjustment for IEA/THEMIS scenarios are provided in section 3.3; those for Greenpeace's scenarios are in Table C.6. Quality-adjusted results follows in Table C.7 (IEA/THEMIS) and C.8 (Greenpeace). The quality adjustment consists in separating each energy in the formula of the EROI according to its origin (electric or thermal), and to weight electricity by a factor 2.6. For example, the quality-adjusted (gross) embodied energy for a unit of technology $t$ writes

$$\text{embodied}^{\text{qual. adj.}}_t = E^S \odot (2.6 \cdot \mathbb{1}_{\text{electric}} + \mathbb{1}_{\text{thermal}}) \cdot (I-A)^{-1} \cdot \mathbb{1}_{\text{t}} \tag{C.1}$$

Table C.6: EROIs and share in electricity mix of electric technologies in the model THEMIS for the Greenpeace scenarios.
The bottom line in columns *mix* gives the total secondary energy demand, in PWh/a.

| **Scenario** | **all** | | **REF** | | | | **ER** (+2°C, no CCS, no nuclear) | | | | **ADV** (100% renewable) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Year** | **2012** | | **2030** | | **2050** | | **2030** | | **2050** | | **2030** | | **2050** | |
| **Variable** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** |
| **biomass w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 |
| **biomass&Waste** | 8.5 | 0.02 | 6.4 | 0.03 | 5.0 | 0.03 | 5.3 | 0.06 | 4.7 | 0.06 | 5.2 | 0.05 | 4.6 | 0.05 |
| **ocean** | 4.7 | 0.00 | 2.0 | 0.00 | 2.5 | 0.00 | 4.3 | 0.01 | 4.5 | 0.03 | 4.8 | 0.01 | 4.9 | 0.03 |
| **geothermal** | 5.6 | 0.00 | 3.8 | 0.01 | 2.5 | 0.01 | 3.6 | 0.03 | 3.7 | 0.07 | 3.8 | 0.03 | 3.9 | 0.07 |
| **solar CSP** | 35.5 | 0.00 | 9.3 | 0.00 | 8.0 | 0.01 | 8.5 | 0.05 | 7.7 | 0.17 | 9.3 | 0.07 | 7.8 | 0.22 |
| **solar PV** | 13.7 | 0.00 | 7.0 | 0.02 | 5.3 | 0.02 | 5.6 | 0.11 | 4.4 | 0.20 | 5.4 | 0.14 | 4.7 | 0.21 |
| **wind offshore** | 9.1 | 0.00 | 8.6 | 0.01 | 7.8 | 0.01 | 5.6 | 0.03 | 5.9 | 0.08 | 6.5 | 0.04 | 6.4 | 0.10 |
| **wind onshore** | 9.7 | 0.02 | 9.1 | 0.05 | 7.2 | 0.05 | 7.2 | 0.15 | 6.0 | 0.22 | 7.2 | 0.17 | 5.8 | 0.24 |
| **hydro** | 12.2 | 0.16 | 11.4 | 0.14 | 11.2 | 0.13 | 11.0 | 0.14 | 11.1 | 0.10 | 11.0 | 0.13 | 10.9 | 0.08 |
| **nuclear** | 12.2 | 0.11 | 7.3 | 0.10 | 7.1 | 0.08 | 8.3 | 0.02 | – | 0.00 | 8.3 | 0.02 | – | 0.00 |
| **gas w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 |
| **coal w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 |
| **oil** | 8.4 | 0.05 | 11.1 | 0.02 | 11.4 | 0.01 | 10.0 | 0.01 | 9.2 | 0.00 | 10.0 | 0.01 | – | 0.00 |
| **gas** | 14.9 | 0.23 | 15.3 | 0.23 | 15.6 | 0.25 | 16.6 | 0.21 | 17.2 | 0.06 | 16.5 | 0.18 | – | 0.00 |
| **coal** | 11.8 | 0.40 | 11.3 | 0.40 | 11.3 | 0.39 | 10.7 | 0.19 | 10.8 | 0.01 | 10.4 | 0.16 | 11.5 | 0.00 |
| **Total** | 12.1 | 22.60 | 10.7 | 36.26 | 10.1 | 50.11 | 8.4 | 33.60 | 5.9 | 49.20 | 8.1 | 36.74 | 5.8 | 64.04 |

Table C.7: Quality-adjusted EROIs (with a factor of 2.6 for electricity) and share in electricity mix of electric technologies for IEA/THEMIS scenarios.
The bottom line in columns *mix* gives the total secondary energy demand, in PWh/a.

| **Scenario** | **Baseline (BL)** | | | | | | **Blue Map (BM, +2°)** | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Year** | **2010** | | **2030** | | **2050** | | **2030** | | **2050** | |
| **Variable** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** | **EROI** | **mix** |
| **biomass w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | 9.5 | 0.00 | 8.5 | 0.01 |
| **biomass&Waste** | 20.8 | 0.01 | 12.6 | 0.02 | 11.7 | 0.03 | 11.6 | 0.06 | 11.1 | 0.05 |
| **ocean** | 9.1 | 0.00 | 4.4 | 0.00 | 5.5 | 0.00 | 7.0 | 0.00 | 11.3 | 0.00 |
| **geothermal** | 11.5 | 0.00 | 10.3 | 0.01 | 10.2 | 0.01 | 10.6 | 0.01 | 11.4 | 0.02 |
| **solar CSP** | 44.6 | 0.00 | 17.7 | 0.00 | 18.2 | 0.01 | 17.3 | 0.02 | 16.9 | 0.06 |
| **solar PV** | 17.5 | 0.00 | 14.8 | 0.01 | 14.5 | 0.01 | 13.3 | 0.02 | 12.9 | 0.06 |
| **wind offshore** | 19.6 | 0.00 | 21.2 | 0.01 | 20.5 | 0.01 | 15.4 | 0.03 | 13.0 | 0.04 |
| **wind onshore** | 18.4 | 0.01 | 18.1 | 0.04 | 15.8 | 0.04 | 14.5 | 0.08 | 15.4 | 0.08 |
| **hydro** | 25.6 | 0.16 | 23.0 | 0.14 | 23.0 | 0.12 | 25.2 | 0.18 | 26.3 | 0.14 |
| **nuclear** | 19.2 | 0.14 | 13.4 | 0.11 | 13.0 | 0.10 | 13.6 | 0.19 | 13.9 | 0.24 |
| **gas w CCS** | – | 0.00 | – | 0.00 | 14.4 | 0.00 | 16.2 | 0.01 | 18.8 | 0.05 |
| **coal w CCS** | – | 0.00 | – | 0.00 | 11.7 | 0.00 | 13.7 | 0.05 | 14.3 | 0.12 |
| **oil** | 12.9 | 0.06 | 15.6 | 0.02 | 16.0 | 0.01 | 15.5 | 0.03 | 11.8 | 0.01 |
| **gas** | 22.5 | 0.21 | 24.6 | 0.21 | 24.4 | 0.23 | 29.0 | 0.14 | 33.5 | 0.11 |
| **coal** | 20.2 | 0.42 | 18.2 | 0.45 | 18.2 | 0.45 | 18.4 | 0.18 | 19.6 | 0.01 |
| **Total** | 20.4 | 19.76 | 18.7 | 34.29 | 18.4 | 45.97 | 17.0 | 28.01 | 16.0 | 40.22 |

Table C.8: Quality-adjusted EROIs (with a factor of 2.6 for electricity) and share in electricity mix of electric technologies for Greenpeace scenarios. The bottom line in columns *mix* gives the total secondary energy demand, in PWh/a.

| Scenario | all | | REF | | | | ER (+2°C, no CCS, no nuclear) | | | | ADV (100% renewable) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | 2012 | | 2030 | | 2050 | | 2030 | | 2050 | | 2030 | | 2050 | |
| Variable | EROI | mix | EROI | mix | EROI | mix | EROI | mix | EROI | mix | EROI | mix | EROI | mix |
| **biomass w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 |
| **biomass&Waste** | 15.7 | 0.02 | 12.6 | 0.03 | 9.8 | 0.03 | 10.4 | 0.06 | 9.2 | 0.06 | 10.3 | 0.05 | 9.1 | 0.05 |
| **ocean** | 7.8 | 0.00 | 3.6 | 0.00 | 4.6 | 0.00 | 8.4 | 0.01 | 9.1 | 0.03 | 9.3 | 0.01 | 9.7 | 0.03 |
| **geothermal** | 12.1 | 0.00 | 7.6 | 0.01 | 5.0 | 0.01 | 7.2 | 0.03 | 7.3 | 0.07 | 7.6 | 0.03 | 7.7 | 0.07 |
| **solar CSP** | 73.2 | 0.00 | 19.1 | 0.00 | 16.1 | 0.01 | 17.6 | 0.05 | 16.1 | 0.17 | 19.0 | 0.07 | 16.2 | 0.22 |
| **solar PV** | 25.8 | 0.00 | 13.7 | 0.02 | 10.5 | 0.02 | 11.5 | 0.11 | 9.2 | 0.20 | 11.3 | 0.14 | 9.9 | 0.21 |
| **wind offshore** | 17.3 | 0.00 | 16.0 | 0.01 | 15.1 | 0.01 | 11.6 | 0.03 | 12.2 | 0.08 | 13.3 | 0.04 | 13.2 | 0.10 |
| **wind onshore** | 18.6 | 0.02 | 17.6 | 0.05 | 14.1 | 0.05 | 14.9 | 0.15 | 12.4 | 0.22 | 14.8 | 0.17 | 12.1 | 0.24 |
| **hydro** | 23.5 | 0.16 | 22.1 | 0.14 | 21.9 | 0.13 | 21.4 | 0.14 | 21.3 | 0.10 | 21.4 | 0.13 | 21.1 | 0.08 |
| **nuclear** | 22.8 | 0.11 | 13.6 | 0.10 | 13.3 | 0.08 | 16.4 | 0.02 | – | 0.00 | 16.4 | 0.02 | – | 0.00 |
| **gas w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 |
| **coal w CCS** | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 | – | 0.00 |
| **oil** | 12.8 | 0.05 | 17.8 | 0.02 | 18.2 | 0.01 | 15.9 | 0.01 | 13.3 | 0.00 | 15.8 | 0.01 | – | 0.00 |
| **gas** | 23.8 | 0.23 | 25.0 | 0.23 | 25.8 | 0.25 | 27.1 | 0.21 | 28.2 | 0.06 | 27.2 | 0.18 | – | 0.00 |
| **coal** | 18.6 | 0.40 | 17.8 | 0.40 | 18.0 | 0.39 | 16.6 | 0.19 | 16.6 | 0.01 | 16.2 | 0.16 | 15.3 | 0.00 |
| **Total (PWh/a)** | 20.1 | 22.60 | 18.5 | 36.26 | 17.7 | 50.11 | 15.9 | 33.60 | 12.0 | 49.20 | 15.5 | 36.74 | 11.9 | 64.04 |

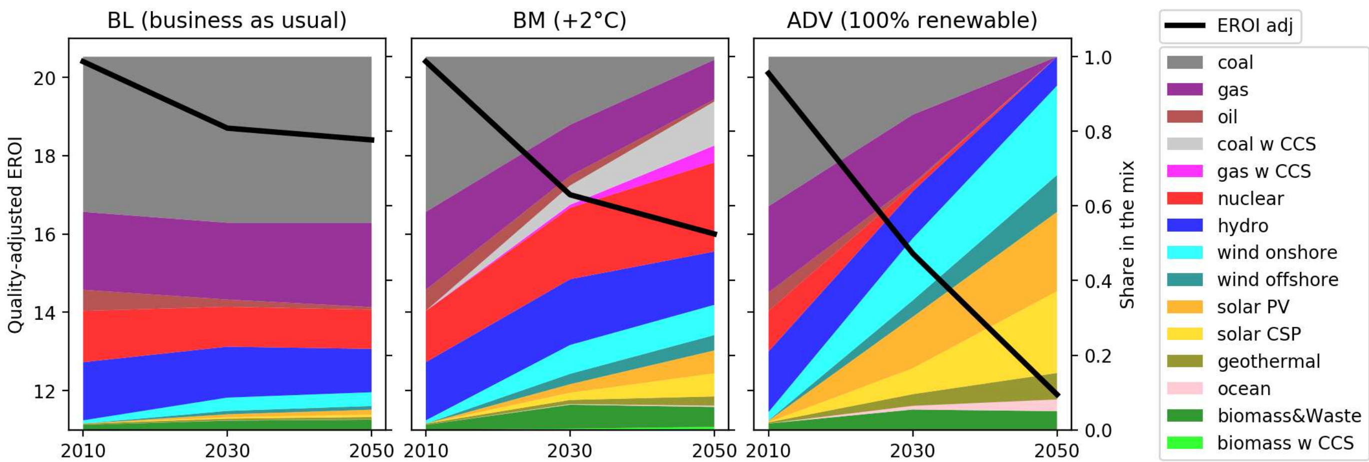


Figure C.6: Evolution of global quality-adjusted EROIs (with a factor 2.6 for electricity) and mixes of electricity for different scenarios.

## Appendix D. Proof of Proposition 1

The demonstration starts with a lemma:

**Lemma 1.** *Let $A$ be an invertible matrix and let $x$ be a coefficient of $A$. Then,*

*(i) the determinant of $A$ is a linear function of $x$, denoted $D^A$;*

*(ii) each coefficient (i,j) of the adjugate of $A$ is a linear function of $x$, denoted $P^A_{i,j}$;*

*(iii) each coefficient (i,j) of $A^{-1}$ is a rational function in $x$ of degree 1, which writes: $\left(A^{-1}\right)_{i,j} = \frac{P^A_{i,j}(x)}{D^A(x)}$.*

*Proof.* Let $A = \left(a_{i,j}\right)_{1\leq i,j\leq n} \in GL_n(\mathbb{R})$ an invertible matrix and let $\left(i_0, j_0\right) \in [\![1;n]\!]^2$ so that, without loss of generality, $x = a_{i_0,j_0}$. (i) From its definition by the Leibniz formula, the determinant of $A$ writes $\det(A) = \sum_{\sigma\in S_n} \mathrm{sgn}(\sigma) \prod_{i=1}^n a_{i,\sigma(i)}$, where $\mathrm{sgn}(\sigma)$ is the signature of permutation $\sigma$ and $S_n$ the set of all permutations of $n$ elements. In this linear combination, each term is a product containing $x$ at most once, it is thus a linear function of $x$. (ii) A minor being the determinant of a submatrix of $A$, we know from (i) that it is a linear function of $x$ (which reduces to a constant for submatrices that do not contain $x$). Each coefficient of the adjugate of A is (plus or minus) a minor of $A$, hence a linear function of $x$. (iii) Using (i) and (ii) and the Laplace expansion of $A$: $A^{-1} = \frac{\mathrm{adj}(A)}{\det(A)}$, we reckon $\left(A^{-1}\right)_{i,j} = \frac{P^A_{i,j}(x)}{D^A(x)}$. □

*Proof.* (Proposition 1) Defining $R(x) := D^{I-A}\left(\delta_{i_0,j_0} - x\right)$, lemma 1 yields that for all $(e,t) \in [\![1;n]\!]^2$, there is a unique linear function $P^{I-A}_{e,t}$ such that $\left((I-A)^{-1}\right)_{e,t} = \frac{P^{I-A}_{e,t}(\delta_{i_0,j_0}-x)}{R(x)}$, where $\delta_{i,j}$ is the Kronecker delta. As a linear combination of compositions of linear functions, the functions
$Q(x) := \sum_{e\in E} P^{I-A}_{e,t}\left(\delta_{i_0,j_0} - x\right)$ and $P(x) := \sum_{i=1}^n v_i P^{I-A}_{i,t}\left(\delta_{i_0,j_0} - x\right)$ are themselves linear. By definition, we have
$\varepsilon_t = \sum_{e\in E}\left((I-A)^{-1}\right)_{e,t}$, so that $Q(x) = \varepsilon_t R(x)$. As $P$, $Q$ and $R$ are linear, and as $\varepsilon_t$ varies with $x$, it is easy to show that there are unique real numbers $\alpha$ and $\gamma$ such that $P(x) = \alpha Q(x) + \gamma R(x)$. Finally, observing that $p_t = \sum_{i=1}^n v_i \left((I-A)^{-1}\right)_{i,t} = \frac{P(x)}{R(x)}$, we have:
$p_t = \frac{\alpha Q(x)+\gamma R(x)}{R(x)} = \alpha\varepsilon_t + \gamma$. □