Interacting commons and collective action problems
(Very preliminary)

Nicolas Quérou*

Abstract

We consider a setting where a natural resource is jointly extracted in distinct local areas, which are owned by separate groups of individuals extracting the resource in their respective location. There are two types of collective action problems, any area user's individual extraction inducing an externality on others in the same area (intra-area problem), while aggregate extraction in one area induces an externality on each agent in other areas (inter-area problem). The interplay between both types of externality is shown to affect the results obtained in classical models of common-pool resources. We show how the fundamentals (magnitude of the externalities, size of user groups) affect the harvest strategies and welfare compared to the benchmark commons problems. Finally, different initiatives (local cooperation, inter-area agreements) are analyzed to assess whether they may alleviate the problems, and to understand the conditions under which they do so.

Key words: Common-pool resource; externalities; collective action.
JEL classification: C72, Q2, Q3, Q5, H7

1 Introduction

A substantial body of analysis and evidence highlights mismanagement of common-pool resources (CPRs) such as fisheries, pastures, forests, groundwater, pollution sinks, among other tragedies of the commons.[1] A large literature suggests economic instruments as solutions.[2] Under certain conditions, the proposed instruments may

*CNRS, LAMETA, F-34000 Montpellier, France. E-mail: querou@supagro.inra.fr

[1]Seminal contributions include, among others, Gordon (6), Hardin (8), Ostrom (17), and Maler (14). The reader is referred to Stavins (24) for empirical evidence.

[2]See Rubio and Escriche (21), or Montero (16), among many other examples.

even ”solve” the tragedy of the commons, and provide first-best outcomes.

Most CPR discussions, including the references quoted above, focus on how to secure reductions among agents across the resource. An important feature is that they rely on at least one of two different types of assumptions, which can be described as follows.[3] A first part of the literature considers that the spatial domain of the resource is entirely managed by one group of agents. As such, there is one type of collective action problem: an agent's extraction imposes negative externalities on others. Such a setting rules out interesting cases where different parts of the spatial domain are managed by different groups of agents. A second part of the literature acknowledges this issue, and stresses the importance of proposing spatially-explicit solutions to the commons problem (see Sanchirico and Wilen (22) or Kaffine and Costello (11)). In such contributions the resource spreads over distinct local areas, and each area is managed locally. Extraction in one area imposes negative externalities on others. Yet, this type of contributions relies on strong assumptions about the property rights structure: either each area is managed by a single owner, or no conflict of interest exists within a given area (the preferences of all owners within an area are perfectly aligned). This leaves aside interesting situations where there is a collective action problem within a given area.

The goal of this work is to reconcile the above two types of assumptions. This article theoretically analyzes how the interaction of collective action problems both within and between areas affect individual and group behavior. We thus consider a setting where the spatial domain of a natural resource is not managed by a single group. The resource is used jointly in local areas, and each local area is managed by a different user group. Extraction within one area is competitive and results in intra-area externality on the extractors in a given area. Moreover, extraction in one area imposes negative externalities on the other locations (inter-area externality). As illustrative examples, one might think of neighbouring fisheries, forest or oil concessions.

The main research questions of the present study can be described as follows. First, does this interaction between collective action problems matter? That is, are the conclusions of usual models of commons still valid? Second, if such interaction does matter, how does intra-group collective action impact inter-group collective action (and vice versa)? Finally, does this in turn impact the effect of cooperation and the emergence of intergroup agreements?

[3]Other practical problems with these instruments relate to wealth redistribution, heterogeneity, or political economy issues, see Besley (3), Karpoff (12), Johnson and Libecap (10), Libecap and Wiggins (13), or Gillet et al. (5).

Before describing our results, the relevance of accounting for interacting commons may be explained by describing the existing trade-off. Focusing on the effect of cooperation, one area group can locally cooperate and impose constraints on a remote other area by, say, threatening to revert to non-cooperative extraction patterns. The larger the group, the larger the potential cost of the threat to the other group, the more likely it will agree to cooperate. But the larger the cooperating group, the more severe the internal collective action problem might be, and the less likely this group will actually manage to cooperate. The relative severity of both types of problems may thus impede the success of cooperation initiatives and the effectiveness of inter-area agreements.

The present analysis shows that the conclusions of usual models of commons problem must be qualified. First, non-cooperative individual extraction levels may be inefficiently low compared to the efficient outcome. Other qualification may be obtained (under specific conditions), for instance an agent's extraction or payoff level may increase with the size of the user group in the same area. Second, the effect of potential remedies to the commons problem may differ significantly.[4] Specifically, it is proved that the effect of local cooperation, where there is cooperation within one area and non cooperation both between the areas and within the other area, is not always positive overall.[5] Furthermore, conditions are provided that characterize settings where inter-area contractual agreements may work. Some further insights are obtained on the features of model parameters (size of user groups, inter- and intra-area externality) that foster the success of inter-area agreements.

The conclusions are shown to hinge on the interaction between the two collective action problems. Regarding local cooperation, the inefficiency problem is shown to become more severe as the size of agents' population increases in the non-cooperating area, as the magnitude of inter-area externality increases, or as the size of agents' population in the cooperating area decreases. In such cases, local cooperation may actually result in an increase in total extraction, or in lower payoffs in certain areas (compared to the case of non cooperation).

Regarding the feasibility of inter-area contractual agreements, the analysis studies the factors that make such initiatives work when there are multiple separate

[4] Other ways to solve collective action problems have been analyzed in the literature, for instance the use of communication (see the experimental work by Ostrom et al. (18)).

[5] The main conceptual point is to assess whether the emergence of cooperation may correspond to an overall improvement of the situation. This differs from situations where cooperation is equivalent to such an improvement, as in empirical or field works by Rustagi et al. (19) and Stoop et al. (25).

groups. The focus will be put on self-consistent transfer schemes, more specifically an extreme case in which users in one area compensate the user group in the other area for not extracting the resource. There are two reasons for such a focus. First, the literature suggests that regulating activity (which is input-based) is more appropriate than output-based instruments (such as quotas) when the ability to monitor and to assess stocks is low, which is a valid assumption in many interesting cases (such as in many developing countries). Thus, focusing on a transfer scheme resulting in no extraction activity in one area will provide insights about the effectiveness of transfer schemes overall. Indeed, due to the significant increase in transaction costs involved by a transfer scheme specifying positive extraction levels in all areas, if the specific type of transfer schemes analyzed here fails to emerge, this will cast doubts about the feasibility of inter-area agreements overall.

The second reason for such a focus is that it will provide a very interesting byproduct of the analysis of interacting commons, that is, a rationale for the emergence of reserve-type initiatives relying on purely private incentives. Indeed, the use of reserves is the object of extensive discussions in the literature on natural resource management. This is actually part of a more general discussion in economics on the rationale of the private supply of public (or collective) goods. The related literature focuses on the use of such instrument as a conservation policy for either land-based conservation initiatives (see for instance Ando et al. (1), Church et al. (4), Armsworth et al. (2)) or marine reserve areas (see for instance White et al. (29)). With respect to studies on land-based initiatives, most of them consider exogenous costs of action or start from the assumption that they are the outcome of expensive public processes. Another part of the literature has developed, and consider contractual remedies to commons problems, where costs of action are endogenous (see Harstad and Mideksa (9) for a case of conservation contracts). Yet, they start from the assumption that the principal of the relationship has intrinsic preferences for conservation. By contrast, our analysis provides a rationale for the emergence of reserve areas based purely on strategic incentives of private agents focusing on economic profits. Regarding marine reserve areas, they have been shown to provide conservation benefits and even potential spillover benefits to adjacent fisheries (Roberts et al. (20)). Yet, they are almost always implemented as long and expensive public processes. The present results again stress the importance to acknowledge existing interactions between commons problems.

The remainder of the paper is organized as follows. The model is presented in Section 2, together with the characterizations of the non-cooperative and efficient outcomes. The effects of model parameters on harvest levels and payoffs are provided in section 3, together with the comparison between the two equilibrium outcomes. Section 4 presents results on the potential solutions to the collective action problems. Section 5 concludes. The proofs of all results are provided in an Appendix at the end of the article.

2 Model & benchmarks

2.1 The model

A natural resource is jointly extracted in distinct local areas A and B, which are owned by two separate groups of individuals that extract the resource in their respective location. This could model neighbouring fisheries, forest or oil concessions, among other examples. The size of the group $i = A, B$ having rights over the resource is denoted N_i. We consider a setting where a game is played simultaneously within and across areas. First, any area i owner's individual extraction induces a cost-type externality on others in the same area (intra-area externality modeled by parameter δ_{ii}). Second, aggregate extraction in area i induces a cost-type externality on each agent in area j (inter-area externality modeled by parameter δ_{ij}). Specifically, the payoff of a given agent $i \in \{1, ..., N_A\}$ is specified as follows:

$$\Pi_i^A = ax_{iA} - x_{iA}\left[\Gamma + \delta_{AA}X_A + \delta_{BA}X_B\right], \tag{1}$$

where x_{iA} denotes this agent's extraction level, $X_A = \sum_{j\in A} x_{jA}$ (respectively, X_B) the aggregate extraction level within group A (respectively, group B), and a and Γ are positive parameters. Parameter δ_{AA} captures the external effects of the other owners' extraction levels on agent i's payoff, while δ_{BA} captures the degree of spatial linkage between the two locations.

The present common-property resource model is similar to that developed by Walker et al. (28) and used in many other contributions[6], the main difference is that we consider a CPR which is exploited by different groups having access to different areas, and resulting extraction activities are characterized by potentially differing externality parameters.[7] Schnier (23) considers a related model of a spatially linked common property resource, but he does not account for the possibility that separate groups of users may have access to the resource in different locations. Moreover, his contribution relies mostly on experiments, and both contributions differ notably in terms of the underlying research questions.[8]

[6]See for instance Margreiter et al. (15).

[7]Some contributions incorporate heterogeneity in common property settings (see for instance Hackett (7)), but they focus on the case of a single common property resource.

[8]The reader is referred to Walker et al. (26) or Walker and Gardner (27) for other seminal experiments on commons problems.

We proceed with the analysis as follows. In order to keep the exposition of the results as simple as possible, we present some of them, namely Propositions 4, 7 and 10, for the case where $\delta_{AA} = \delta_{BB}$ and $\delta_{AB} = \delta_{BA}$ hold. They will be extended to allow for a more general degree of heterogeneity, and provided as supplementary material at the end of the article. The main differences between the results provided in the body of the article and their extended versions will be discussed following each extended result. Other findings will be provided in their most general form, as their exposition remains reasonably simple.

2.2 Nash equilibrium versus socially efficient outcome

We first derive the Nash equilibrium and the cooperative outcome that correspond to the initial model. In order to focus on the most interesting cases, we rule out situations where cost-externalities may be so strong that some agents would extract nothing at either the non cooperative equilibrium or the socially efficient outcome. For a given agent i in group A, the corresponding optimality condition is:

$$a - [\Gamma + \delta_{AA}(x_{iA} + X_{-iA}) + \delta_{BA} X_B] - \delta_{AA} x_{iA} = 0, \tag{2}$$

and a similar type of condition holds for any owner of resource B. Solving for the equilibrium extraction levels, we obtain the following results:

Proposition 1. *Assume that* $(N_A + 1)\,\delta_{AA} > N_A \delta_{AB}$ *and* $(N_B + 1)\,\delta_{BB} > N_B \delta_{BA}$ *hold. Then the unique Nash equilibrium of the game corresponds to vectors of extraction levels* $\left(x^N_{1A}, ..., x^N_{N_A A}\right)$ *and* $\left(x^N_{1B}, ..., x^N_{N_B B}\right)$ *characterized as follows:*

$$\forall i \in A \;\; x^N_{iA} = x^N_A = (a - \Gamma)\frac{(N_B + 1)\,\delta_{BB} - N_B \delta_{BA}}{(N_A + 1)(N_B + 1)\,\delta_{AA}\delta_{BB} - N_A N_B \delta_{AB}\delta_{BA}} \tag{3}$$

and

$$\forall i \in B \;\; x^N_{iB} = x^N_B = (a - \Gamma)\frac{(N_A + 1)\,\delta_{AA} - N_A \delta_{AB}}{(N_A + 1)(N_B + 1)\,\delta_{AA}\delta_{BB} - N_A N_B \delta_{AB}\delta_{BA}} \tag{4}$$

The first proposition provides a closed-form characterization of the equilibrium extraction strategies. It is easily checked that there is no equilibrium where no agent extracts a positive level of the resource in each area. Moreover, an equilibrium where agents in one area (say A) extracts some of the resource, while nothing is extracted in the other area, requires that the externality imposed on area B by area A is sufficiently strong, which is ruled out when condition $(N_A + 1)\,\delta_{AA} > N_A \delta_{AB}$ is satisfied. The characterization provided in Proposition 1 will be used extensively in the following analysis. The next result provides the characterization of the full cooperation outcome.

Proposition 2. *Assume that* $\frac{\delta_{AB}+\delta_{BA}}{2} < \min_{k=A,B} \delta_{kk}$ *holds. Then the full co-operation outcome correspond to vectors of extraction levels* $\left(x^*_{1A},....,x^*_{N_A A}\right)$ *and* $\left(x^*_{1B},....,x^*_{N_B B}\right)$ *characterized as follows:*

$$\forall i \in A \;\; x^*_{iA} = x^*_A = \frac{a-\Gamma}{N_A}\frac{2\delta_{BB}-(\delta_{AB}+\delta_{BA})}{4\delta_{AA}\delta_{BB}-(\delta_{AB}+\delta_{BA})^2} \tag{5}$$

and

$$\forall i \in B \;\; x^*_{iB} = x^*_B = \frac{a-\Gamma}{N_B}\frac{2\delta_{AA}-(\delta_{AB}+\delta_{BA})}{4\delta_{AA}\delta_{BB}-(\delta_{AB}+\delta_{BA})^2} \tag{6}$$

The reader should notice that the above efficient outcome is not unique: the aggregate extraction levels in both areas are unique, but individual extraction levels within each area are not. Yet, since all agents are identical within a given area, it makes sense to focus on the outcome characterized by identical extraction levels for these agents. Moreover, it is easily checked that amending the part specifying the benefits in the expression of agents' payoffs (expression 1) as $ax_{iA} - \varepsilon\left(x_{iA}\right)^2$, with ε positive but small, would result in a unique efficient outcome that would get arbitrarily close to the outcome described in Proposition 2 as ε gets arbitrarily small.[9] In the next Section we will rely on these characterizations to compare the full cooperation outcome with the case of non-cooperation.

3 The effect of fundamentals

In this section we will analyze how model parameters affect the agents' harvest levels and payoffs. First, we will provide results of comparative statics on the agents' equilibrium harvest levels. Second, the effect of parameters on the comparison between non cooperative and cooperative outcomes will be analyzed. Finally, the same analysis will be performed on the equilibrium payoffs.

3.1 Comparative statics

The first Proposition allows for detailed comparative statics results on the effects of the various fundamentals (externality parameters, size of the populations) on the non cooperative equilibrium outcome. Specifically, we have:

Proposition 3. *Under the assumptions of Proposition 1, we have the following comparative statics results: for* $i, j = A, B$, $i \neq j$

[9]One can proceed with the same type of analysis in such a setting, and same results would follow. The main difference is that the form of the analytical characterizations would become more complex, and the exposition of the results would thus become more cumbersome. This is why we rely on the present simplification.

1. *The individual level of extraction in area i decreases (respectively, increases) with an increase in the intensity of within-area externality in this same area (respectively, the other area):*

$$\frac{\partial x_i^N}{\partial \delta_{ii}} < 0; \ \frac{\partial x_i^N}{\partial \delta_{jj}} > 0;$$

2. *The individual level of extraction in area i increases (respectively, decreases) with an increase in the intensity of inter-area externality from this same area to the other one (respectively, from area j to area i):*

$$\frac{\partial x_i^N}{\partial \delta_{ij}} > 0; \ \frac{\partial x_i^N}{\partial \delta_{ji}} < 0;$$

3. *The individual level of extraction in area i decreases as the number of agents in area j increases:*

$$\frac{\partial x_i^N}{\partial N_j} < 0$$

4. *The effect of an increase in the number of agents N_i in area i can be characterized as follows. We have:*

$$\frac{\partial x_i^N}{\partial N_i} > 0 \Longleftrightarrow N_j \delta_{ij} \delta_{ji} > (N_j + 1)\, \delta_{ii} \delta_{jj}$$

Cases $1, 2$ and 3 in Proposition 3 are fairly intuitive. For instance, regarding the effects of intra-area externality parameters, the main impact of an increase in δ_{ii} follows quite directly from the optimality condition, and tends to affect negatively the individual extraction level. By contrast, the main effect of a larger δ_{jj} is indirect, and follows from the resulting decrease in overall extraction in area j: each agent in area i then increases her own extraction.

The most interesting effect corresponds to the fourth case. Focusing on the case of area A, the expression (and the corresponding optimality condition) of x_A^N highlights the existing trade-off that relates to the effect of an increase in N_A. The first effect is direct, an increase in the number of agents in area A results in a more intense within-area competitive effect, and is driven by a marginal increase $(N_B + 1)\, \delta_{AA} \delta_{BB}$, which lowers the extraction level. The second effect is indirect, an increase in the number of agents results in higher external effects imposed on the other area: the corresponding marginal impact is given by $N_B \delta_{AB} \delta_{BA}$. This results in lower aggregate extraction in area B, which in turn tends to increase individual extractions in area A. The net change follows from the trade off between these two effects, and depends (as suggested by the previous reasoning) on an interplay between the number of agents in area B and the externality parameters. In order to illustrate this interplay, we can focus on two polar cases. First, when $\delta_{AB} = \delta_{BA} = 0$ we obtain immediately that $\frac{\partial x_i^N}{\partial N_i} < 0$ due to the second effect. By contrast, when $\delta_{AB} = \delta_{BA} = \delta'$, $\delta_{AA} = \delta_{BB} = \delta$ and $\delta' \approx \frac{N_i+1}{N_i}\delta$ then we deduce that $(N_i + 1)\,\delta^2 \approx \frac{N_i}{N_i+1}\,(\delta')^2 < N_i\,(\delta')^2$ and thus $\frac{\partial x_i^N}{\partial N_i} > 0$ holds due to the first effect.

A final feature of the fourth case deserves some discussion. A necessary condition for this case to hold is that $\delta_{ij}\delta_{ji} > \delta_{ii}\delta_{jj}$ be satisfied. This requires that, at least for one area, the magnitude of intra-area external effect be smaller than that of the inter-area externality. This might look like a non-intuitive assumption to make. Even though we do not intend to take a stance on this issue, we can make two points. A first point is that ruling out or supporting this condition is an entirely empirical question, and is outside the scope of this work. A second point is that this type of condition might be descriptive of real world cases, where members of a large group inflict individually low intra group costs, but the aggregate cross group effects on the other group are large. It might be consistent with very asymmetric situations among areas, where extraction in (say) area j might have small external effect both within area j and on area i, while the effect of individual extraction in area i might have small intra-area external effect and yet aggregate extraction in area i might have relatively strong effect on area j. This might be consistent with an asymmetry in relative geographical locations.

Before moving on to the next stage of the analysis, one can notice that the usual cases of commons problems are incompatible with the positive effect of larger sizes of agents' population on the effort level. Indeed, when $N_A = N_B = 1$ the problem is degenerate, and when $\delta_{AB} = \delta_{BA} = 0$ then the indirect effect disappears.

3.2 Comparison between full cooperation and Nash equilibrium

We now consider the cases where the full cooperation and the non-cooperative equilibrium outcomes are characterized by conditions (3)-(4) and (5)-(6). Thus, we assume that the following conditions are satisfied simultaneously:

$$(N_A + 1)\,\delta_{AA} > N_A\delta_{AB}, \;\; (N_B + 1)\,\delta_{BB} > N_B\delta_{BA}, \;\; \frac{\delta_{AB} + \delta_{BA}}{2} < \min\{\delta_{AA}, \delta_{BB}\}. \tag{7}$$

As the analysis will highlight it, the comparison between both outcomes is more complex than in the usual case of tragedy of the commons. Indeed, two problems interact in the present setting: a classical commons problem within each area (due to the common property regime), and a problem driven by aggregate interactions between areas (due to spatial externalities). Depending on which dimension matters most, the comparison requires qualifications, as the next result will show it. In order to keep the exposition as simple as possible, we present the result for the case where $\delta_{AA} = \delta_{BB}$ and $\delta_{AB} = \delta_{BA}$ are satisfied. Moreover, to save on notations, we focus on the case of area A, results for area B follow in an entirely similar manner. Specifically, we obtain:

Proposition 4. *Assume that conditions (7) are satisfied simultaneously, so that the full cooperation and Nash equilibrium outcomes are characterized by (3)-(4) and (5)-(6), and that $\delta_{AA} = \delta_{BB} = \delta$ while $\delta_{AB} = \delta_{BA} = \delta'$ are satisfied. We have the following comparisons:*

- *If $N_B \leq 3N_A - 1$ then $x_A^N \geq x_A^*$ holds generically;*
- *If $N_B > 3N_A - 1$ then $x_A^N \geq x_A^*$ when $\delta' \in [0, \underline{\delta}'(\delta, N_A, N_B)]$ and $x_A^N < x_A^*$ when $\delta' \in [\underline{\delta}'(\delta, N_A, N_B), \delta[$, where $\underline{\delta}'(\delta, N_A, N_B)$ increases as δ increases, as N_A increases, or as N_B decreases.*

This result can be explained as follows. Propositions 1 and 2 yield the following expressions of harvest levels:

$$x_A^N = (a - \Gamma) \frac{(N_B + 1)\delta - N_B \delta'}{(N_A + 1)(N_B + 1)\delta^2 - N_A N_B (\delta')^2}, \quad x_A^* = \frac{a - \Gamma}{2N_A(\delta + \delta')}$$

where the second expression follows from straightforward simplifications. Due to Proposition 3 we know that x_A^N decreases as N_B increases, while the efficient harvest level is not affected.[10] This suggests that sufficiently large values of N_B might be required for the non-cooperative harvest level to become smaller than the efficient outcome. Yet this necessary condition is not sufficient: indeed, for sufficiently small values of δ', we obtain that x_A^N gets close to $\frac{a-\Gamma}{(N_A+1)\delta}$ which is easily checked to be greater than x_A^*. By contrast, when δ' gets sufficiently close to δ, one obtains that x_A^N gets close to $\frac{a-\Gamma}{\delta(N_A+N_B+1)}$ and the comparison with the socially efficient outcome then depends on the value of N_B. Specifically, $x_A^N < x_A^*$ if and only if $N_B \geq 3N_A - 1$ is satisfied. Thus, this highlights the existing trade-off for sufficiently large values of N_B: the non-cooperative harvest level will be greater than the efficient one for low values of δ', while the opposite will hold otherwise.

[10] With benefits in the expression of agents' payoffs specified as $ax_{iA} - \varepsilon (x_{iA})^2$ with $\varepsilon > 0$ and small, the efficient harvest level would be affected by a change in N_B, but the qualitative conclusions obtained in Proposition 4 would remain valid.

Proposition 4 highlights a major difference with the classical commons problem: the conclusion that over extraction will prevail under non cooperative extraction does not always hold. The classical extraction problems studied in the literature correspond to cases where either $N_A = N_B = 1$ or $\delta' = 0$ holds, and $x_A^N < x_A^*$ cannot hold in such cases. This result is actually reinforced when intra- or inter-area externality parameters may differ: some trade-off emerges in (say) area A when the size of the user group in the other area is sufficiently large (compared to N_A), when the within-area externality problem is more (but not too) severe in area B, or when the inter-area externality problem is less severe for area B. Moreover, the effect of intra-area externality differs qualitatively from that of inter-area externality. The reader is referred to Proposition 11 (stated in the Appendix) for an extension of Proposition 4 that provides a precise statement and discussion of these last conclusions.

3.3 How does the number of users affect welfare?

We will conclude this section by discussing the effect of the number of users on the welfare of the different groups. This is done by simple comparative statics analysis. We have the following conclusion:

Proposition 5. *Assume that conditions $(N_A + 1)\,\delta_{AA} > N_A\delta_{AB}$ and $(N_B + 1)\,\delta_{BB} > N_B\delta_{BA}$ hold. Denote Π_A^N and Π_B^N the payoffs of any agent in, respectively, areas A and B corresponding to the Nash equilibrium characterized in Proposition 1. Then we have:*

1. *Regarding the effect on agents' payoffs in area A*

$$\frac{\partial \Pi_A^N}{\partial N_A} > 0 \Longleftrightarrow (N_B + 1)\,\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA} < 0 \qquad (8)$$

   The effect of changes in N_A depends on the spatial parameters and the size of agents' population in area B.

2. *Regarding the effect on agents' payoffs in area B, we have $\frac{\partial \Pi_B^N}{\partial N_A} \leq 0$: a decrease in the number of agents in one area yields an increase in agents' payoff in the other area.*

The effect of changes in the size of the agents' population in one area is unambiguous when looking at the agents' payoffs in the other area. In this case, a lower number of agents results in higher payoffs. The marginal effect of a change in the size of the agents' population in area A can be computed as follows:

$$\frac{\partial \Pi_B^N}{\partial N_A} = -x_B^N \left[\delta_{BB}\,(N_B - 1)\,\frac{\partial x_B^N}{\partial N_A} + \delta_{AB}\left[x_A^N + N_A\frac{\partial x_A^N}{\partial N_A}\right]\right]$$

The first part in the bracketed expression on the right-hand side of the equality reflects the effect due to changes in the aggregate extraction level in area B. Individual extraction level in area B decreases as N_A increases, which implies that the related effect on agents' payoffs in this area is positive. The second part in the bracketed expression characterizes the effect due to changes in the aggregate extraction level in area A. Aggregate extraction level increases as N_A increases, and the related effect on agents' payoffs in area B is thus negative. As long as x_A^N and x_B^N are positive, the second effect dominates the first one, and the overall effect on the agents' payoffs in area B is negative.

By contrast, the marginal effect on the agents' payoffs in area A depends on the spatial parameters and the size of agents' population in area B. Specifically, we have:

$$\frac{\partial \Pi_A^N}{\partial N_A} = x_A^N \left[\delta_{AA} \frac{\partial x_A^N}{\partial N_A} - \delta_{AA} \frac{\partial X_A^N}{\partial N_A} - \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A} \right]$$

where $X_A^N = N_A x_A^N$ which, after simplifications, yields:

$$\frac{\partial \Pi_A^N}{\partial N_A} = -x_A^N \left[\delta_{AA} x_A^N + \delta_{AA} \left(N_A - 1 \right) \frac{\partial x_A^N}{\partial N_A} + \delta_{BA} N_B \frac{\partial x_B^N}{\partial N_A} \right]$$

The first part in the bracketed expression on the right-hand side of the equality reflects the effect due to changes in the individual (first term) and the aggregate (second term) extraction levels in area A. Changes in the individual extraction level of the agent mitigate the effect of changes in the aggregate extraction level in the area. Overall, the sign of the related effect depends on spatial parameters and on N_B. The second part in the bracketed expression highlights the effect due to changes in the aggregate extraction level in area B. Aggregate extraction level decreases as N_A increases, and the related effect on agents' payoffs in area A is thus positive. As long as x_A^N and x_B^N are positive, the first effect dominates the second one, and the overall effect on the agents' payoffs in area A depends on the interplay between spatial parameters and N_B. In order to illustrate this interplay, we can again focus on two polar cases. First, when $\delta_{AB} = \delta_{BA} = 0$ the indirect effect disappears and $\frac{\partial \Pi_A^N}{\partial N_A} < 0$ obtains. By contrast, when $\delta_{AB} = \delta_{BA} = \delta'$, $\delta_{AA} = \delta_{BB} = \delta$ and $\delta' \approx \frac{N_B+1}{N_B} \delta$ then we deduce that $\left(N_B + 1 \right) \delta^2 \approx \frac{N_B}{N_B+1} \left(\delta' \right)^2 < N_B \left(\delta' \right)^2$ and thus $\frac{\partial x_i^N}{\partial N_i} > 0$ holds due to the first effect.

Proposition 5 yields several interesting implications. First, there are cases for which there is a specific conflict of interest between agents in different areas. This is so when agents' payoffs in area A are higher, while agents' payoffs in area B are lower, as the number of agents increases in area A. Second, when

$(N_B + 1)\,\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA} > 0$ is satisfied, agents in both areas benefit as the number of agents decreases in area A: agents might then have incentives to compensate some of them to give up extracting the resource. We will come back to the issue of self-consistent transfer schemes in a slightly different situation in the next section. Finally, since usual commons problems correspond to either $N_A = N_B = 1$ or $\delta_{AB} = \delta_{BA} = 0$, they are incompatible with a positive effect of a larger size of agents' population on profits (as depicted in the first case in Proposition 5). As in the fourth case in Proposition 4, this last conclusion requires a necessary condition on the comparison between the magnitude of inter-area externalities and that of intra-area externalities. We refer to the related discussion following the statement of Proposition 4.

4 Potential solutions to the collective action problem

The main point of this contribution is to show the importance of existing interactions between commons problems. As highlighted in the previous section, several classical results in commons problems can be reversed in such a setting: among others, non cooperative harvest levels may be inefficiently low.

The next section will highlight the importance of existing interactions by showing how it opens up new interesting research questions on the potential solutions to collective action problems. Indeed, compared to the usual commons problems, it will be shown that certain solutions may have significantly different effects. Specifically, we will now consider two potential solutions to the problem of collective action, namely (i) local cooperation and (ii) self-consistent transfer schemes, and analyze whether they might be effective and if so, we will characterize the conditions under which they alleviate the problem. Case (i) assumes that agents in a given area are able to solve their internal collective action problem: there is cooperation within one area, and non cooperation both between the areas and within the other area. Case (ii) will focus on one case, which is the object of extensive discussions: private reserve areas. Here, instead of assuming that public authorities select one of the two areas to become a reserve, we analyze whether there are cases for which there exists a self consistent transfer scheme such that agents in one area compensate the other agents for not extracting the resource in the other area.

4.1 Local cooperation

Again we consider the case where the non cooperative outcome is interior, that is, the situation where it is characterized by Proposition 1. In order to allow for the simplest comparison that is possible, we will characterize the conditions under which the partial cooperation outcome is interior too. This is provided in the next proposition:

Proposition 6. *Assume that agents in area A cooperate locally, while there is still non cooperation both within area B and between the two areas. Then, under conditions*

$$(N_B + 1)\,\delta_{BB} - N_B\delta_{BA} > 0,\ \ 2\delta_{AA} > \delta_{AB} \tag{9}$$

the partial cooperation outcome correspond to vectors of extraction levels $\left(x^{pc}_{1A}, ..., x^{pc}_{N_A A}\right)$ and $\left(x^{pc}_{1B}, ..., x^{pc}_{N_B B}\right)$ characterized as follows:

$$\forall i \in A\ \ x^{pc}_{iA} = x^{pc}_{A} = \frac{a-\Gamma}{N_A}\frac{(N_B+1)\,\delta_{BB} - N_B\delta_{BA}}{2\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA}} \tag{10}$$

and

$$\forall i \in B\ \ x^{pc}_{iB} = x^{pc}_{B} = (a-\Gamma)\,\frac{2\delta_{AA} - \delta_{AB}}{2\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA}} \tag{11}$$

The reader should notice that the above equilibrium is not unique: the aggregate extraction level in area A is unique, but individual extraction levels within this area are not. Yet, since all agents in area A are identical, it makes sense to focus on the outcome characterized by identical extraction levels for these agents. We refer the reader to the discussion in Section 2 supporting the use of such assumption following Proposition 2 (see footnote 9).

The main point is now to assess the effects of local cooperation on extraction levels and payoffs. This is done by comparing the outcome of Proposition 1 and that of Proposition 6. Specifically, we assume cooperation in area A, and we obtain:

Proposition 7. *Assume that conditions $(N_B+1)\,\delta_{BB} > N_B\delta_{BA}$ and $(N_A+1)\,\delta_{AA} > N_A\delta_{AB}$ are satisfied. Then, compared to non cooperation:*

1. *Local cooperation results in lower (respectively, higher) individual extraction levels in area A (respectively, B).*

2. *The effect on total extraction level depends on the interplay between spatial parameters and the size of the agents' population in area B. Specifically, when $\delta_{AB} \leq \delta_{BA}$ total extraction is higher under non cooperation. When*

$\delta_{AB} > \delta_{BA}$ total extraction is lower under non cooperation when $\frac{N_B}{N_B+1}\delta_{BA} < \delta_{BB} < \frac{N_B}{N_B+1}\delta_{AB}$ and higher under non cooperation when $\delta_{BB} \geq \frac{N_B}{N_B+1}\delta_{AB}$.

3. *The effect on payoffs depends on the interplay between spatial parameters and the size of the agents' populations in both areas. Specifically, we have $\Pi_A^{pc} \leq \Pi_A^N$ when*

$$\frac{N_A}{N_A+1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA} < \delta_{AA}\delta_{BB} \leq \frac{N_A+\sqrt{N_A}}{N_A-1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA}$$

and $\Pi_A^{pc} > \Pi_A^N$ for values of δ_{AA} and δ_{BB} such that the following condition is satisfied:

$$\delta_{AA}\delta_{BB} > \frac{N_A+\sqrt{N_A}}{N_A-1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA}.$$

Moreover, local cooperation in area A results in higher payoffs in area B (compared to the case of full non-cooperation).

Proposition 7 provides several interesting insights on the effect of local cooperation. First, the effect of local cooperation does not always mitigate the collective action problem in terms of the resulting extraction levels. Indeed, point 1 shows that local cooperation results in lower extraction levels in area A, while Proposition 4 highlights that whether the efficient extraction levels are lower or higher compared to the non-cooperative outcome depends on an interplay between spatial parameters and the size of agents' populations in both areas.

Secondly, the economic effect is not always positive overall. Specifically, while local cooperation in area A unambiguously results in higher payoffs in area B, the same conclusion does not always hold in area A. Looking at two polar cases helps to illustrate this feature. When $\delta_{AB} = \delta_{BA} = 0$ we obtain immediately that $\Pi_A^{pc} > \Pi_A^N$ holds. By contrast, when $\delta_{AA} = \delta_{BB} = \delta$, $\delta_{AB} = \delta_{BA} = \delta'$ and $\delta' \approx \min_{i=A,B} \frac{N_i+1}{N_i}\delta$ we obtain that $\frac{N_A+\sqrt{N_A}}{N_A-1}\frac{N_B}{N_B+1}(\delta')^2 \approx \frac{N_A+\sqrt{N_A}}{N_A-1}\frac{N_B+1}{N_B}\delta^2 > \delta^2$ and thus $\Pi_A^{pc} < \Pi_A^N$ holds.

Specifically, point 3 in Proposition 7 highlights that agents' payoffs in area A are lower under local cooperation for intermediate values of the intra-area spatial parameters:

$$\frac{N_A}{N_A+1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA} < \delta_{AA}\delta_{BB} \leq \frac{N_A+\sqrt{N_A}}{N_A-1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA}$$

It is easily checked that the size of the interval defined by the above inequalities increases as the size of agents' population in area B increases, as the values of inter-area parameters δ_{AB} and δ_{BA} increase, or as the size of agents' population in area A decreases. Thus, in any of such cases the inefficiency problem becomes more likely: the effect of partial cooperation is negative and becomes more severe on the specific area within which the collective action problem has been solved. This conclusion implies that, absent appropriate financial schemes which would target this area, the emergence of partial cooperation becomes less likely.

It is again interesting to contrast these results with the usual cases of commons problems. When $\delta_{AB} = \delta_{BA} = 0$ it is easily checked that partial cooperation always improves the situation overall, as it results in a strict Pareto improvement in area A and leaves area B unaffected. Obviously, when $N_A = N_B = 1$ local cooperation does not have any meaning.

4.2 What makes inter-area agreements work?

The second type of solution to the collective action problem that we will consider relies on self-consistent transfer schemes. The focus will be put on an extreme case in which users in one area compensate the user group in the other area for not extracting the resource. As explained in the introduction, there are two reasons for such a focus. Studying a transfer scheme resulting in no extraction activity in one area will provide insights about the effectiveness of transfer schemes overall: if this type of contractual agreements fails to emerge, this will cast doubts about the feasibility of arrangements specifying positive extraction levels in all areas (due to the significant increase in transaction costs). Moreover, this part will provide a very interesting result in terms of policy implications, as it will suggest a rationale for the emergence of reserve-type initiatives relying on purely private incentives. The support of reserves based on conservation motives or on public initiatives is being discussed extensively (see for instance Roberts et al. (20)).

We now analyze if inter-area agreements can emerge when based on private initiatives and, if so, the conditions under which it is more likely to emerge. In order to do so, we consider the situation where area A is sanctuarized, that is, the situation where no agent extracts the resource in this area. The sanctuarization of area A will emerge from private initiatives if and only if there exists a self-consistent transfer scheme, that is, a scheme that satisfies the following features:

- The transfer scheme induces a non-cooperative game that induces agents in area A to choose to not harvest the resource.

- The transfer compensates agents in area A for the absence of revenue derived from the extraction activity. They decide to comply with the arrangement as long as they obtain payoffs at least equal to those under non-cooperative extraction activities in both areas.

- The transfer makes the agents in area B at least as well off as compared to the case of non-cooperative extraction activities in both areas.

The above description highlights that an appropriate transfer scheme should possess both an incentive part, which induces agents in both areas to choose appropriate harvest levels, and a fixed part, which makes agents in both areas at least as well off compared to the case of non cooperative extraction activities. The first step thus consists in characterizing the non-cooperative game under the transfer scheme, and to prove that the unique outcome is that agents in area B extracts (some of) the resource, while there is no extraction activity within area A. More specifically, in order to assess the potential emergence of a reserve based on private initiatives, we will first characterize the incentive part of the scheme that enables to implement a given target level of harvest $\hat{x}_B$ by each agent in area B.[11] We obtain the following result:

Proposition 8. *Assume that conditions $(N_A+1)\,\delta_{AA} > N_A\delta_{AB}$ and $(N_B+1)\,\delta_{BB} > N_B\delta_{BA}$ hold, and consider the common-property setting induced by the transfer scheme defined by, for any $i \in B$ and $j \in A$:*

$$\tau_{iB}\left(x_{1A},...,x_{N_AA},x_{1B},..,x_{N_BB}\right) =$$

$$= x_{iB}\left[(N_B+1)\,\hat{x}_B - (a-\Gamma)\right] - \frac{\sum_{l\in A} x_{lA}}{N_B}\left[\delta_{BA}N_B\hat{x}_B - (a-\Gamma)\right] - T_{iB}$$

and

$$\tau_{jA}\left(x_{1A},...,x_{N_AA},x_{1B},..,x_{N_BB}\right) =$$

$$= x_{jA}\left[\delta_{BA}N_B\hat{x}_B - (a-\Gamma)\right] - \frac{\sum_{l\in B} x_{lB}}{N_A}\left[(N_B+1)\,\delta_{BB}N_B\hat{x}_B - (a-\Gamma)\right] + \frac{\sum_{l\in B} T_{lB}}{N_A}$$

where T_{iB} is the fixed part of the transfer corresponding to agent i in area B, and $\hat{x}_B$ is a given target level of harvest. Then the resulting equilibrium outcome corresponds to vectors of extraction levels $\left(x^s_{1A},...,x^s_{N_AA}\right)$ and $\left(x^s_{1B},...,x^s_{N_BB}\right)$ such that, for all $j \in A$ and $i \in B$:

$$x^s_{jA} = x^s_A = 0,\;\; x^s_{iB} = x^s_B = \hat{x}_B; \tag{12}$$

In other words, the transfer scheme induces the emergence of a reserve in area A.

[11] Since all agents in area B are identical, the case of an identical level of harvest makes sense.

The assumptions of Proposition 8 both imply the existence and uniqueness of the equilibrium outcome corresponding to non cooperation both within and between areas. Indeed, this is required to proceed with the full characterization of the scheme: the full non-cooperative outcome will constitute the status-quo and be used to define the participation constraint. Now, looking at the game induced by the transfer scheme, the resulting equilibrium payoffs are, for any $i \in B$ and $j \in A$

$$\Pi_{iB}^s = x_{iB}^s \left[a - \Gamma - \delta_{BB} X_B^s - \delta_{AB} X_A^s\right] + \tau_{iB}\left(x_{1A}^s, ..., x_{N_A A}^s, x_{1B}^s, ..., x_{N_B B}^s\right)$$

and

$$\Pi_{jA}^s = x_{jA}^s \left[a - \Gamma - \delta_{AA} X_A^s - \delta_{BA} X_B^s\right] + \tau_{jA}\left(x_{1A}^s, ..., x_{N_A A}^s, x_{1B}^s, ..., x_{N_B B}^s\right)$$

which, using Proposition 8, yield:

$$\Pi_{iB}^s = \delta_{BB} \left(\hat{x}_B\right)^2 - T_{iB}$$

and

$$\Pi_{jA}^s = \frac{N_B}{N_A} \hat{x}_B \left[(a - \Gamma) - (N_B + 1)\,\delta_{BB} \hat{x}_B\right] + \frac{\sum_{l \in B} T_{lB}}{N_A}$$

Proposition 8 highlights that the scheme is budget balanced (immediate from its characterization), and implements the desired outcome provided that the agents choose to comply with the terms of the scheme. Moreover, it highlights that such a transfer scheme does not have any meaning in the usual cases of commons problems: indeed, if there is no inter-area externality (in particular, if $\delta_{BA} = 0$) then the scheme characterized above is not well-defined. This again stresses the importance of accounting for existing interactions between commons problems.

The last step is thus to analyze the conditions under which all agents have incentives to participate. That is, we want to characterize the conditions under which there exist fixed payments T_{iB} for $i = 1, ..., N_B$ such that all agents are at least as well off as under full non-cooperation, which defines the status-quo scenario. This is the goal of the next result:

Proposition 9. *Assume that conditions $(N_A + 1)\,\delta_{AA} > N_A \delta_{AB}$ and $(N_B + 1)\,\delta_{BB} > N_B \delta_{BA}$ hold. Then there exist values of $\hat{x}_B > 0$ such that the transfer scheme defined in Proposition 8 is self-consistent if and only the following condition is satisfied:*

$$\left[(N_A + 1)(N_B + 1)\,\delta_{AA}\delta_{BB} - N_A N_B \delta_{AB} \delta_{BA}\right]^2 > \\ 4\left[N_B \left(\delta_{BB}\right)^2 \left[(N_A + 1)\,\delta_{AA} - N_A \delta_{AB}\right]^2 + N_A \delta_{AA} \delta_{BB} \left[(N_B + 1)\,\delta_{BB} - N_B \delta_{BA}\right]^2\right] \tag{13}$$

In such cases, there exist fixed parts of the transfer scheme $\{T_{iB}\}_{i \in B}$ such that all agents are at least as well off as under full non cooperation.

Proposition 9 provides several interesting conclusions. First, it allows to characterize the cases in which inter-area agreements may work and a reserve can emerge based on private initiatives. Moreover, it shows that the corresponding existence condition results from an interplay between the model parameters. This condition is easily checked to be non vacuous.[12] Secondly, even if its form is quite complex, this condition allows for further analysis of the features of area A that foster the emergence of such private initiatives. Indeed, we can analyze (at least partially) the situations where condition (13) is satisfied, and thus the effect of specific parameters on the potential rationality of private initiatives. In order to keep the exposition as simple as possible, we will focus on the case where $\delta_{AA} = \delta_{BB}$ and $\delta_{AB} = \delta_{BA}$ hold.[13] We obtain the following result:

Proposition 10. *Assume that conditions $(N_A + 1)\,\delta_{AA} > N_A\delta_{AB}$ and $(N_B + 1)\,\delta_{BB} > N_B\delta_{BA}$ hold. Moreover, assume that $\delta_{AA} = \delta_{BB} = \delta$ and $\delta_{AB} = \delta_{BA} = \delta'$ hold. Then we have:*

1. *When $\delta = \delta'$ a reserve based on private initiatives always Pareto dominates the full non-cooperative outcome.*

2. *When $N_A = N_B = N$ a reserve based on private initiatives Pareto dominates the full non-cooperative outcome*

    - *When $\delta' > \frac{-(N+1)+2\sqrt{2N}}{N}\delta$ provided $N \leq 5$;*
    - *for any value of $\delta' \in]0, \frac{N+1}{N}\delta[$ otherwise.*

This result provides several interesting implications. First, when the only source of potential heterogeneity is the size of the agents' populations in both areas, the transfer scheme is always consistent. Using the conclusions provided in Proposition 3, since a larger population in area B unambiguously decreases the non-cooperative extraction levels and the corresponding payoffs in area A, a larger population in area B corresponds to a less profitable status-quo scenario for agents in area A. This makes the emergence of self-consistent transfer schemes more likely. By contrast, the effect of a larger population in area A on extraction levels in the status-quo scenario (and on the resulting payoffs) depends on the interplay between N_B and the within and between-area externality parameters. As such, whether situations characterized by larger populations in area A are more or less desirable for the emergence of a reserve in this same area depends on this trade-off.

Secondly, when externality parameters are potentielly heterogeneous, the situation becomes more complex, and a trade-off emerges. When the size of agents'

[12] For instance, it is easily checked that $\delta_{AB} \approx \frac{N_A+1}{N_A}\delta_{AA}$ and $\delta_{BA} \approx \frac{N_B+1}{N_B}\delta_{BB}$ satisfy it.

[13] An extended version is provided as supplementary material at the end of the article.

populations in both areas remain small, looking at the polar cases where either $\delta' = 0$ or $\delta' \approx \frac{N+1}{N}\delta$ highlights the trade off. Indeed, in the first case, when δ' is sufficiently small we obtain immediately that condition (13) is not satisfied, and the transfer scheme is not self-consistent. By contrast, when δ' is sufficiently large ($\delta' \approx \frac{N+1}{N}\delta$) then condition (13) is always satisfied, and the transfer scheme is always self-consistent. Thus, depending on the size of agents' populations, a trade-off exists regarding the relative intensities of intra and inter-area externalities. This interplay between the sizes of agents' populations and the externality parameters is actually reinforced when one analyzes the problem allowing for more heterogeneity between parameters. This is shown in the supplementary material (in Proposition 12 in the Appendix).

5 Conclusion

The problem of CPR management has received a lot of attention from researchers in many disciplines, including economics. Yet, until recent years, most studies tend to focus on how to secure reductions among agents across the resource. By contrast, this article theoretically analyzes how the interaction of collective action problems both within and between areas affect individual and group behavior.

We focus on the interaction between these two types of collective action problem, and shows that the conclusions of usual models of commons problem should be qualified. Depending on the relative severity of both types of problem, non-cooperative management may result in inefficiently low individual harvest levels. Moreover, potential solutions may not have the same effect than in usual commons problems too. Indeed, it is shown that local cooperation does not always improve the situation overall. Specific conditions are provided that characterize the cases where it is actually effective. Moreover, inter-area agreements may be more or less effective depending on the specifics of the situation at hand. Finally, accounting for interactions between collective action problems may provide a rationale for the emergence of reserve-type initiatives based purely on strategic incentives of private agents focusing on economic profits. All together, these results highlight the importance of accounting for the existing interactions between commons problems.

The goal of this work was to assess the main differences that emerge when such interactions are acknowledged. As such, the corresponding model has been kept relatively simple, and we abstracted from several issues. For instance, allowing for asymmetric information and introducing dynamic considerations would constitute interesting and important extensions that deserve future research.

References

[1] Ando, A., Camm, J., Polasky, S., and Solow, A., (1998), Species distributions, land values, and efficient conservation, *Science* 279 (5359), 2126-2128.

[2] Armsworth, P.R., Daily, G.C., Kareiva, P., and Sanchirico, J., (2006), Land market feedbacks can undermine biodiversity conservation, *Proceedings of the National Academy of Sciences* 103 (14), 5403-5408.

[3] Besley, T., (1995), Property rights and investment incentives: Theory and evidence from Ghana, *Journal of Political Economy* 103, 903-937.

[4] Church, R.L., Stoms, D.M., and Davis, F.W., (1996), Reserve selection as a maximal covering location problem, *Biological Conservation* 76 (2), 105-112.

[5] Gillet, J., Schram, A., and Sonnemans, J., (2009), The tragedy of the commons revisited: The importance of group decision-making, *Journal of Public Economics* 93 (5), 785-797.

[6] Gordon, H.S., (1954), The Economic Theory of a Common-Property Resource: The Fishery, *Journal of Political Economy* 62 (2), 124-142.

[7] Hackett, S.C., (1992), Heterogeneity and the provision of governance for common-pool resources, *Journal of Theoretical Politics* 4 (3), 325-342.

[8] Hardin, G. J., (1968), The tragedy of the commons, *Science* 162, 1243-1248.

[9] Harstad, B., and Mideksa, T., (2017), Conservation contracts and political regimes, *Review of Economic Studies*, forthcoming.

[10] , Johnson, R.N., and Libecap, G.D., (1982), Contracting problems and regulation: The case of the fishery, *American Economic Review* 72, 1005-1022.

[11] Kaffine, D., and Costello, C., (2011), Unitization of spatially connected renewable resources, *BE Journal of Economic Analysis and Policy* 11 (1), 15.

[12] Karpoff, J.M., (1987), Suboptimal Controls in Common Resource Management: The case of the Fishery, *Journal of Political Economy* 95 (1), 179-194.

[13] Libecap, G.D., and Wiggins, S.N., (1985), The influence of private contractual failure on regulation: the case of oil field unitization, *Journal of Political Economy* 93 (4), 690-714.

[14] Maler, K.G., (1990), International environmental problems, *Oxford Review of Economic Policy* 6 (1), 80-108.

[15] Margreiter, M., Sutter, M., and Dittrich, D., (2005), Individual and collective choice and voting in common pool resource problem with heterogeneous actors, *Environmental and Resource Economics* 32, 241-271.

[16] Montero, J.P., (2008), A simple auction mechanism for the optimal allocation of the commons, *American Economic Review* 98, 496-518.

[17] Ostrom, E., (1990), *Governing the Commons: The Evolution of Institutions for Collective Action*, Cambridge University Press, Cambridge; New York and Melbourne.

[18] Ostrom, E., Gardner, R., and Walker, J., (1992), Covenants with and without the sword: self-governance is possible, *American Political Science Review* 86, 404-417.

[19] Rustagi, D., Engel, S., and Kosfeld, M., (2010), Conditional cooperation and costly monitoring explain success in forest commons management, *Science* 330, 961-965.

[20] Roberts, C.M., Bohnsack, J.A., Gell, F., Hawkins, J.P., and Goodridge, R., (2001), Effects of marine reserves on adjacent fisheries, *Science* 294 (5548), 1920-1923.

[21] Rubio, S., and Escriche, L., (2001), Strategic pigouvian taxation, stock externalities and polluting non-renewable resources, *Journal of Public Economics* 79, 297-313.

[22] Sanchirico, J.N., and Wilen, J.E., (2005), Optimal spatial management of renewable resources: matching policy scope to ecosystem scale, *Journal of Environmental Economics and Management* 50, 23-46.

[23] , Schnier, K.E., (2009), Spatial externalities and the common-pool resource mechanism, *Journal of Economic Behavior and Organization* 70, 402-415.

[24] Stavins, R., (2011), The problem of the commons: Still unsettled after 100 years, *American Economic Review* 101, 81-108.

[25] Stoop, J., Noussair, C. N., and van Soest, D., (2012), From the lab to the field: Cooperation among fishermen, *Journal of Political Economy* 120 (6), 1027-1056.

[26] Walker, J., Gardner, R., and Ostrom, E., (1990), Rent dissipation in a limited-access common-pool resource: experimental evidence, *Journal of Environmental Economics and Management* 19, 203-211.

[27] Walker, J., and Gardner, R., (1992), Probabilistic destruction of common-pool resources: experimental evidence, *Economic Journal* 120, 1149-1161.

[28] Walker, J., Gardner, R., Herr, A., and Ostrom, E., (2000), Collective choice in the commons: experimental results on proposed allocation rules and votes, *Economic Journal* 110, 212-234.

[29] White, C., Kendall, B.E., Gaines, S., Siegel, D.A., and Costello, C., (2008), Marine reserve effects on fishery profit, *Ecology Letters* 11 (4), 370-379.

Appendix

Proof of Proposition 1

The optimality conditions are necessary and sufficient, and given by, for any $i \in A$ and $j \in B$:

$$a - \Gamma - \delta_{AA} X_A - \delta_{BA} X_B - \delta_{AA} x_{iA} + \lambda_{iA} = 0$$

and

$$a - \Gamma - \delta_{BB} X_B - \delta_{AB} X_A - \delta_{BB} x_{jB} + \lambda_{jB} = 0,$$

where $\lambda_{iA} \geq 0$ and $\lambda_{jB} \geq 0$ are the lagrangian parameters associated to each optimality condition. First, it is easily checked that there is no equilibrium where no agent in each group extracts a positive level of the resource. Second, there cannot be an equilibrium outcome for which $\lambda_{iA} > 0$ and $x_{lA} > 0$ simultaneously (and the same holds for area B). Otherwise, we would have:

$$a - \Gamma - \delta_{AA} X_A - \delta_{BA} X_B < 0$$

while

$$a - \Gamma - \delta_{AA} X_A - \delta_{BA} X_B = \delta_{AA} x_{lA} > 0,$$

which is a contradiction. Now, if there is one equilibrium such that $x_{iA}^N > 0$ for any $i \in A$ while $\lambda_{jB} > 0$ for any $j \in B$, then one has:

$$a - \Gamma - \delta_{AB} X_A^N < 0$$

while

$$a - \Gamma - \delta_{AA} X_A^N = \delta_{AA} x_{iA}^N = \delta_{AA} x_{lA}^N,$$

which in turn implies that $x_{iA}^N = x_{lA}^N = x_A^N$ for any i and $l \in A$. Rewriting, we obtain:

$$x_A^N = \frac{a - \Gamma}{(N_A + 1)\, \delta_{AA}}$$

and

$$N_A \frac{a - \Gamma}{(N_A + 1)\, \delta_{AA}} > \frac{a - \Gamma}{\delta_{AB}}$$

or

$$N_A \delta_{AB} > (N_A + 1)\, \delta_{AA},$$

which is ruled out by assumption, a contradiction. The symmetric case for area B is ruled out in a similar way. All together, this implies that one must have $\lambda_{iA} = 0 = \lambda_{jB}$ for all $i \in A$ and $j \in B$, and more specifically that $x_{iA}^N = x_{lA}^N = x_A^N > 0$ for any i and $l \in A$, and the same property holds within area B. Now, coming back to the optimality conditions and solving for x_A^N and x_B^N, we obtain the desired expressions, which concludes the proof.

Proof of Proposition 2

The proof is omitted, as it follows mainly from the same type of calculations than in the proof of Proposition 1, except that the problem here is to maximize the sum of all agents' payoffs over the two areas.

Proof of Proposition 3

We prove the proposition for the case of a representative agent in area A, the conclusions will follow similarly for the case of agents in area B.
We use the expression of x_A^N provided in Proposition 1, and we denote $D := (N_A+1)(N_B+1)\delta_{AA}\delta_{BB} - N_AN_B\delta_{AB}\delta_{BA}$. First, we differentiate with respect to δ_{AA}, and we obtain:

$$\frac{\partial x_A^N}{\partial \delta_{AA}} = -\frac{a-\Gamma}{D^2}(N_A+1)(N_B+1)\delta_{BB}\left[(N_B+1)\delta_{BB} - N_B\delta_{BA}\right]$$

Since δ_{BB} and $(N_B+1)\delta_{BB} - N_B\delta_{BA}$ are positive by assumption, we conclude that the effect of δ_{AA} is negative. Now, differentiating with respect to δ_{BB} we obtain:

$$\frac{\partial x_A^N}{\partial \delta_{BB}} = \frac{a-\Gamma}{D^2}(N_B+1)N_B\delta_{BA}\left[(N_A+1)\delta_{AA} - N_A\delta_{AB}\right]$$

Since δ_{AA} and $(N_A+1)\delta_{AA} - N_A\delta_{AB}$ are positive by assumption, we conclude that the effect of δ_{BB} is positive. This concludes the proof of the first point in the proposition.
Differentiating with respect to δ_{AB} and δ_{BA} we obtain, respectively:

$$\frac{\partial x_A^N}{\partial \delta_{AB}} = \frac{a-\Gamma}{D^2}N_AN_B\delta_{BA}\left[(N_B+1)\delta_{BB} - N_B\delta_{BA}\right]; \quad \frac{\partial x_A^N}{\partial \delta_{BA}} = -\frac{a-\Gamma}{D^2}N_B(N_B+1)\delta_{BA}\left[(N_A+1)\delta_{AA} - N_A\delta_{AB}\right]$$

Arguments similar to those in the first point yield immediately that $\frac{\partial x_A^N}{\partial \delta_{AB}} > 0$ and $\frac{\partial x_A^N}{\partial \delta_{BA}} < 0$ are satisfied. This concludes the proof of the second point in the proposition.
Differentiating with respect to N_B we obtain:

$$\frac{\partial x_A^N}{\partial N_B} = -\frac{a-\Gamma}{D^2}\delta_{BB}\delta_{BA}\left[(N_B+1)\delta_{BB} - N_B\delta_{BA}\right]$$

We conclude immediately that the effect of an increase in N_B is negative. This concludes the proof of the third point in the proposition.
Finally, differentiating with respect to N_A we obtain:

$$\frac{\partial x_A^N}{\partial N_A} = \frac{a-\Gamma}{D^2}\left[(N_B+1)\delta_{BB} - N_B\delta_{BA}\right]\left[N_B\delta_{AB}\delta_{BA} - (N_B+1)\delta_{AA}\delta_{BB}\right]$$

Since $(N_B+1)\delta_{BB} - N_B\delta_{BA} > 0$ by assumption, we deduce that $\frac{\partial x_A^N}{\partial N_A} > 0$ if and only if $N_B\delta_{AB}\delta_{BA} - (N_B+1)\delta_{AA}\delta_{BB} > 0$ is satisfied. This concludes the proof.

Proof of Proposition 4

We have:

$$x_A^N - x_A^* = \frac{(N_B+1)\delta_{BB} - N_B\delta_{BA}}{(N_A+1)(N_B+1)\delta_{AA}\delta_{BB} - N_AN_B\delta_{AB}\delta_{BA}} - \frac{2\delta_{BB} - (\delta_{AB}+\delta_{BA})}{N_A\left[4\delta_{AA}\delta_{BB} - (\delta_{AB}+\delta_{BA})^2\right]}$$

Since the denominators of both terms are positive (by Propositions 1 and 2 respectively), rewriting the above expression as a single fraction its sign is given by that of the nominator, that is:

$$\begin{aligned} & N_A\left[(N_B+1)\,\delta_{BB}-N_B\delta_{BA}\right]\left[4\delta_{AA}\delta_{BB}-(\delta_{AB}+\delta_{BA})^2\right] \\ & -\left[2\delta_{BB}-(\delta_{AB}+\delta_{BA})\right]\left[(N_A+1)\,(N_B+1)\,\delta_{AA}\delta_{BB}-N_AN_B\delta_{AB}\delta_{BA}\right] \end{aligned} \quad (14)$$

We now provide the proof of the different cases in Proposition 4.

In case 1, using its assumptions to rewrite expression (14) allows to conclude that the sign of $x_A^N - x_A^*$ is given by that of:

$$N_A\left[(N_B+1)\,\delta-N_B\delta'\right]\left[4\delta^2-4\,(\delta')^2\right]-\left[2\delta-2\delta'\right]\left[(N_A+1)\,(N_B+1)\,\delta^2-N_AN_B\,(\delta')^2\right]$$

or, simplifying:

$$2N_A\left[(N_B+1)\,\delta-N_B\delta'\right](\delta+\delta')-\left[(N_A+1)\,(N_B+1)\,\delta^2-N_AN_B\,(\delta')^2\right].$$

This expression can be rewritten to deduce:

$$x_A^N \geq x_A^* \iff (N_B+1)\,(N_A-1)\,\delta^2+2N_A\delta\delta'-N_AN_B\,(\delta')^2 \geq 0,$$

where the second term is a polynomial function of δ' (keeping in mind that $0 \leq \delta' \leq \delta$ by conditions (7)). Solving it, we obtain:

$$x_A^N \geq x_A^* \geq 0 \iff \delta' \in [0, \min\{\delta, \underline{\delta}' = \delta\frac{N_A+\sqrt{(N_A)^2+N_AN_B\,(N_B+1)\,(N_A-1)}}{N_AN_B}\}].$$

To conclude the proof of this first case, it then remains to notice that $\frac{N_A+\sqrt{(N_A)^2+N_AN_B(N_B+1)(N_A-1)}}{N_AN_B} < 1$ if and only if $N_B > 3N_A - 1$ holds. Moreover, the conclusions on the effect of the parameters on $\underline{\delta}'$ follow from straightforward differentiations, which are thus omitted.

Proof of Proposition 5

We first differentiate the agents' payoffs in area A with respect to N_A, accounting for the optimality conditions characterizing x_A^N:

$$\frac{\partial \Pi_A^N}{\partial N_A} = x_A^N\left[-\delta_{AA}x_A^N-\delta_{AA}\,(N_A-1)\,\frac{\partial x_A^N}{\partial N_A}-\delta_{BA}N_B\frac{\partial x_B^N}{\partial N_A}\right]$$

Differentiating the expressions of x_A^N and x_B^N with respect to N_A and simplifying, we obtain:

$$\frac{\partial \Pi_A^N}{\partial N_A} = 2x_A^N\frac{a-\Gamma}{D^2}\delta_{AA}\left[(N_B+1)\,\delta_{AA}\delta_{BB}-N_B\delta_{AB}\delta_{BA}\right]\left[N_B\delta_{BA}-(N_B+1)\,\delta_{BB}\right]$$

with $D := (N_A+1)\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_AN_B\delta_{AB}\delta_{BA}$ and we can now conclude as follows. The second term between brackets on the right hand side of the equality is negative by assumption. Since $\delta_{AA} > 0$ by assumption, this implies that $\frac{\partial \Pi_A^N}{\partial N_A} < 0$ if and only if $(N_B+1)\,\delta_{AA}\delta_{BB} -$

$N_B\delta_{AB}\delta_{BA} > 0$holds. This concludes the proof of the first case of the proposition.

Similar calculations yield:

$$\frac{\partial \Pi_B^N}{\partial N_A} = -2x_B^N \frac{a-\Gamma}{D^2}\delta_{AA}\delta_{BB}\delta_{AB}\left[(N_B+1)\,\delta_{BB} - N_B\delta_{BA}\right]$$

The term between brackets on the right hand side of the equality is positive by assumption. This implies that $\frac{\partial \Pi_B^N}{\partial N_A} \leq 0$ is always satisfied, with strict inequality if and only if the externality parameter δ_{AB} is positive (since δ_{AA} and δ_{BB} are positive by assumption). This concludes the proof.

Proof of Proposition 6

It is immediately checked that the first order conditions are necessary and sufficient, and given by:

$$a - \Gamma - 2\delta_{AA}X_A^{pc} - \delta_{BA}X_B^{pc} + \lambda_{lA} = 0$$

for any agent $l \in A$ and

$$a - \Gamma - \delta_{AB}X_A^{pc} - \delta_{BB}X_B^{pc} - \delta_{BB}x_{iB}^{pc} + \lambda_{iB} = 0$$

for any agent $i \in B$. The reasoning used in the proof of Proposition 1 allows quickly to conclude that $x_{iB}^{pc} = x_{jB}^{pc} = x_B^{pc}$ for any i and $j \in B$, and that $(N_B+1)\,\delta_{BB} > N_B\delta_{BA}$ rules out the possibility of a corner solution $x_{lA} = 0$ while $2\delta_{AA} > \delta_{AB}$ rules out the possibility of a corner solution $x_{iB} = 0$. Solving the system of optimality conditions for X_A^{pc} and x_B^{pc} yields the desired expressions, keeping in mind that $x_{lA}^{pc} = \frac{X_A^{pc}}{N_A}$ in the statement of the proposition.

Proof of Proposition 7

Concerning individual extraction levels, using the expressions provided in Proposition1 and 6 we deduce that:

$$x_A^N \geq x_A^{pc} \Longleftrightarrow N_A \geq 1$$

which is always satisfied. Regarding extraction levels in area B, we obtain: and

$$x_B^N \geq x_B^{pc} \Longleftrightarrow \frac{(N_A+1)\,\delta_{AA} - N_A\delta_{AB}}{(N_A+1)\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_AN_B\delta_{AB}\delta_{BA}} \geq \frac{2\delta_{AA} - \delta_{AB}}{2\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA}}$$

which can be rewritten and simplified as

$$x_B^N \geq x_B^{pc} \Longleftrightarrow \delta_{AA}\delta_{AB}\,(N_A-1)\left[N_B\delta_{BA} - (N_B+1)\,\delta_{BB}\right] \geq 0,$$

which is only satisfied when $N_A = 1$ or $\delta_{AB} = 0$, since the term between brackets is negative by assumption. This concludes the proof of the first point.
Regarding the effect on total extraction levels, denoting $X^{pc} = N_Ax_A^{pc} + N_Bx_B^{pc}$ and $X^N = N_Ax_A^N + N_Bx_B^N$ we have:

$$X^{pc} \leq X^N$$

$$\Longleftrightarrow \frac{(N_B+1)\,\delta_{BB} - N_B\delta_{BA} + 2N_B\delta_{AA} - N_B\delta_{AB}}{2\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA}} \leq \frac{N_A\,(N_B+1)\,\delta_{BB} - N_AN_B\,(\delta_{AB}+\delta_{BA}) + N_B\,(N_A+1)\,\delta_{AA}}{(N_A+1)\,(N_B+1)\,\delta_{AA}\delta_{BB} - N_AN_B\delta_{AB}\delta_{BA}}$$

Rewritting and simplifying, we obtain:

$$X^{pc} \leq X^N \iff (N_A - 1)\left[-(N_B+1)^2(\delta_{BB})^2 + N_B(N_B+1)(\delta_{AB}+\delta_{BA})\delta_{BB} - (N_B)^2\delta_{AB}\delta_{BA}\right] \leq 0.$$

The term between brackets is a polynomial expression of δ_{BB}: solving for δ_{BB}, there are two cases:

- If $\delta_{AB} \leq \delta_{BA}$ then the term between brackets is non-positive if and only if $\delta_{BB} \geq \frac{N_B}{N_B+1}\delta_{BA}$ is satisfied, which is the case by assumption;
- If $\delta_{AB} > \delta_{BA}$ then the term between brackets is positive if δ_{BB} satisfies $\frac{N_B}{N_B+1}\delta_{BA} < \delta_{BB} < \frac{N_B}{N_B+1}\delta_{AB}$ and non-positive otherwise.

This concludes the proof of the second point in the proposition.
Finally, we compute the difference between agents' payoffs in area A under the two outcomes. Regarding the fully non-cooperative case, accounting for the first order conditions satisfied by x_A^N and simplifying, we obtain:

$$\Pi_A^N = \delta_{AA}\left(x_A^N\right)^2.$$

Regarding the case of partial cooperation, accounting for the first order conditions satisfied by x_A^{pc} and simplifying, we obtain:

$$\Pi_A^{pc} = \delta_{AA}N_A\left(x_A^{pc}\right)^2.$$

Denoting $D = (N_A+1)(N_B+1)\delta_{AA}\delta_{BB} - N_AN_B\delta_{AB}\delta_{BA}$ and $M = 2(N_B+1)\delta_{AA}\delta_{BB} - N_B\delta_{AB}\delta_{BA}$ we can now compute the difference as follows:

$$\Pi_A^{pc} - \Pi_A^N = \frac{\delta_{AA}(N_A-1)\left[(N_B+1)\delta_{BB} - N_B\delta_{BA}\right]^2}{N_AD^2M^2}P(\delta_{AA}\delta_{BB})$$

where

$$P(\delta_{AA}\delta_{BB}) = (N_A-1)(N_B+1)^2(\delta_{AA}\delta_{BB})^2 - 2N_AN_B(N_B+1)\delta_{AB}\delta_{BA}\delta_{AA}\delta_{BB} + (N_B)^2N_A(\delta_{AB}\delta_{BA})^2$$

To derive the sign of $\Pi_A^{pc} - \Pi_A^N$ it suffices to analyze the sign of $P(\delta_{AA}\delta_{BB})$. Since this is a polynomial expression of $\delta_{AA}\delta_{BB}$ we obtain that $\Pi_A^{pc} \geq \Pi_A^N$ when either (i) $\delta_{AA}\delta_{BB} \leq \frac{N_B}{N_B+1}\frac{N_A-\sqrt{N_A}}{N_A-1}\delta_{AB}\delta_{BA}$ holds or (ii) $\delta_{AA}\delta_{BB} \geq \frac{N_B}{N_B+1}\frac{N_A+\sqrt{N_A}}{N_A-1}\delta_{AB}\delta_{BA}$ is satisfied. By assumption we know that $\delta_{AA}\delta_{BB} > \frac{N_A}{N_A+1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA}$ is satisfied, and it is easily checked that $\frac{N_A}{N_A+1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA} \geq \frac{N_B}{N_B+1}\frac{N_A-\sqrt{N_A}}{N_A-1}\delta_{AB}\delta_{BA}$ is satisfied whenever $N_A \geq 1$ holds. This rules out case (i) and implies that $\Pi_A^{pc} < \Pi_A^N$ when $\frac{N_A}{N_A+1}\frac{N_B}{N_B+1}\delta_{AB}\delta_{BA} < \delta_{AA}\delta_{BB} < \frac{N_B}{N_B+1}\frac{N_A+\sqrt{N_A}}{N_A-1}\delta_{AB}\delta_{BA}$ is satisfied, and that $\Pi_A^{pc} \geq \Pi_A^N$ otherwise. This concludes the comparison of agents' payoffs in area A.
Now regarding agents' payoffs in area B, accounting for the first order conditions satisfied by x_A^N and simplifying, we obtain in the full non-cooperative case:

$$\Pi_B^N = \delta_{BB}\left(x_B^N\right)^2.$$

Regarding the case of partial cooperation, accounting for the first order condition satisfied by x_B^{pc} and simplifying, we obtain:

$$\Pi_B^{pc} = \delta_{BB}\left(x_B^{pc}\right)^2.$$

This now implies that we have:

$$\Pi_B^{pc} - \Pi_B^N = \delta_{BB}\left[\left(x_B^{pc}\right)^2 - \left(x_B^N\right)^2\right] = \delta_{BB}\left(x_B^{pc} + x_B^N\right)\left(x_B^{pc} - x_B^N\right)$$

From the first point in Proposition 7 we know that $x_B^{pc} - x_B^N \geq 0$ is satisfied, which implies that $\Pi_B^{pc} - \Pi_B^N \geq 0$ holds. This concludes the proof.

Proof of Proposition 8

Considering the game induced by the transfer scheme and differentiating, we obtain the following optimality conditions, for $j \in A$ and $i \in B$:

$$a - \Gamma - \delta_{AA} X_A^s - \delta_{BA} X_B^s - \delta_{AA} x_{jA}^s + \frac{\partial \tau_{jA}}{\partial x_{jA}} + \lambda_{jA} = 0,$$

and

$$a - \Gamma - \delta_{BB} X_B^s - \delta_{AB} X_A^s - \delta_{BB} x_{iB}^s + \frac{\partial \tau_{iB}}{\partial x_{iB}} + \lambda_{iB} = 0,$$

where λ_{jA} and λ_{iB} denote the lagrangian parameters associated to the first order conditions of agents $j \in A$ and $i \in B$. Using the expressions of the transfers we obtain:

$$a - \Gamma - \delta_{AA} X_A^s - \delta_{BA} X_B^s - \delta_{AA} x_{jA}^s + \delta_{BA} N_B \hat{x}_B - (a - \Gamma) + \lambda_{jA} = 0,$$

and

$$a - \Gamma - \delta_{BB} X_B^s - \delta_{AB} X_A^s - \delta_{BB} x_{iB}^s + (N_B + 1)\, \delta_{BB} \hat{x}_B - (a - \Gamma) + \lambda_{iB} = 0,$$

We now proceed by contradiction and assume that $x_{jA} > 0$ for some $j \in A$. This implies that:

$$-\delta_{AA} X_A^s - \delta_{BA} X_B^s - \delta_{AA} x_{jA}^s + \delta_{BA} N_B \hat{x}_B = 0,$$

which in turn implies that the extraction levels of all other agents in A must be positive as well, and that $x_{lA}^s = x_{jA}^s = x_A^s$ for any agent $l \in A$ $(l \neq j)$. Now assume that $x_{iB}^s = 0$ for some $i \in B$, we obtain:

$$-\delta_{BB} X_B^s - \delta_{AB} X_A^s + (N_B + 1)\, \delta_{BB} \hat{x}_B \leq 0,$$

which in turn implies that necessarily $x_{kB}^s = 0$ for any $k \in B$ as well. All together, the two optimality conditions imply that one must have:

$$(N_B + 1)\, \delta_{BB} \hat{x}_B - \delta_{AB} N_A x_A^s \leq 0$$

while

$$\delta_{BA} N_B \hat{x}_B = \delta_{AA} (N_A + 1)\, x_A^s$$

and combining these two conditions allows to deduce that necessarily $(N_B + 1)\, \delta_{BB} \hat{x}_B \leq \frac{\delta_{AB} \delta_{BA} N_A N_B}{(N_A+1)\delta_{AA}} \hat{x}_B$ must hold which, since $\hat{x}_B$ is positive, implies that $(N_A + 1)(N_B + 1)\, \delta_{AA} \delta_{BB} - N_A N_B \delta_{AB} \delta_{BA} \leq 0$ must hold, which contradicts the assumptions of Proposition 8.

Now assume that $x_{iB}^s > 0$ for some $i \in B$: coming back to the corresponding optimality condition, we obtain:

$$-\delta_{BB} X_B^s - \delta_{AB} X_A^s - \delta_{BB} x_{iB}^s + (N_B + 1)\, \delta_{BB} \hat{x}_B + \lambda_{iB} = 0,$$

which implies that all other agents in area B finds it optimal to choose a positive extraction level, and that $x_{lB}^s = x_{iB}^s = x_B^s$ for any agent $j \in B$ $(l \neq i)$. The optimality condition can thus be rewritten as:

$$(N_B + 1)\, \delta_{BB} \hat{x}_B = \delta_{AB} X_A^s + \delta_{BB} (N_B + 1)\, x_B^s$$

or

$$\hat{x}_B - x_B^s = \frac{N_A \delta_{AB}}{(N_B + 1)\, \delta_{BB}} x_A^s,$$

while the optimality condition related to agents in area A imply that $\hat{x}_B - x_B^s = \frac{N_A \delta_{AB}}{(N_B+1)\delta_{BB}} x_A^s$ which, since $x_A^s > 0$ holds, implies:

$$\frac{N_A \delta_{AB}}{(N_B+1)\,\delta_{BB}} = \frac{N_A \delta_{AB}}{(N_B+1)\,\delta_{BB}},$$

which contradicts the assumptions in Proposition 8, as they imply that $(N_A+1)(N_B+1)\,\delta_{AA}\delta_{BB} - N_A N_B \delta_{AB}\delta_{BA} > 0$ is satisfied. We conclude by contradiction that $x_{jA}^s > 0$ for some $j \in A$ is impossible.

Thus the equilibrium outcome of the game results in $x_{jA}^s = x_A^s = 0$ for all agents j in area A. This implies that there is necessarily some agent, say i, in area B such that $x_{iB}^s > 0$ holds. Now the optimality condition related to agent $i \in B$ writes

$$-\delta_{BB} X_B^s - \delta_{BB} x_{iB}^s + (N_B+1)\,\delta_{BB}\hat{x}_B = 0.$$

This implies necessarily that $x_{lB}^s > 0$ for any other agent l in area B, and thus $x_{lB}^s = x_{iB}^s = x_B^s$. Rewriting the optimality condition for agent i we finally obtain:

$$(N_B+1)\,\delta_{BB}\hat{x}_B = N_B \delta_{BB} x_B^s + \delta_{BB} x_B^s,$$

thus $x_B^s = \hat{x}_B$ and this concludes the proof.

Proof of Proposition 9

Using Proposition 8 the transfer scheme induces the following equilibrium payoffs, for $j \in A$ and $i \in B$:

$$\Pi_{iB}^s = \delta_{BB}\left(\hat{x}_B\right)^2 - T_{iB}$$

and

$$\Pi_{jA}^s = \frac{N_B}{N_A}\hat{x}_B\left[(a-\Gamma) - (N_B+1)\,\delta_{BB}\hat{x}_B\right] + \frac{\sum_{l\in B} T_{lB}}{N_A}$$

Now, using Proposition 1 the payoffs corresponding to the full non-cooperative outcome are provided by the following expressions:

$$\Pi_A^N = \delta_{AA}\left(x_A^N\right)^2,\ \ \Pi_B^N = \delta_{BB}\left(x_B^N\right)^2,$$

with x_A^N and x_B^N characterized by expressions (3) and (4) respectively. Now, the transfer scheme will be self-consistent if and only if the fixed payments $\{T_{iB}\}_{i\in B}$ are defined such that the scheme makes all agents at least as well as under full non cooperation, that is, for any $j \in A$ and $i \in B$:

$$\Pi_{jA}^s \geq \Pi_A^N,\ \ \Pi_{iB}^s \geq \Pi_B^N$$

Using the expressions of equilibrium payoffs, and considering equal treatment of agents in area B, we obtain $T_{iB} = T_B$ and

$$\delta_{BB}\left[\left(\hat{x}_B\right)^2 - \left(x_B^N\right)^2\right] \geq T_B$$

and

$$T_B \geq \frac{N_A}{N_B}\delta_{AA}\left(x_A^N\right)^2 - \hat{x}_B\left[(a-\Gamma) - (N_B+1)\,\delta_{BB}\hat{x}_B\right].$$

Now the transfer scheme will be self-consistent if and only if there exists a non vacuous interval of values of T_B that satisfy these two inequalities. This is equivalent to showing

$$\delta_{BB}\left[(\hat{x}_B)^2 - \left(x_B^N\right)^2\right] > \frac{N_A}{N_B}\delta_{AA}\left(x_A^N\right)^2 - \hat{x}_B\left[(a-\Gamma) - (N_B+1)\,\delta_{BB}\hat{x}_B\right],$$

which can be rewritten as

$$-N_B\delta_{BB}\left(\hat{x}_B\right)^2 + (a-\Gamma)\,\hat{x}_B - \left[\delta_{BB}\left(x_B^N\right)^2 + \frac{N_A}{N_B}\delta_{AA}\left(x_A^N\right)^2\right] > 0.$$

This is a polynomial expression of $\hat{x}_B$, which can be solved explicitly. To get $\hat{x}_B > 0$ a necessary and sufficient condition is that

$$\Delta = (a-\Gamma)^2 - 4N_B\delta_{BB}\left[\delta_{BB}\left(x_B^N\right)^2 + \frac{N_A}{N_B}\delta_{AA}\left(x_A^N\right)^2\right] > 0, \tag{15}$$

because any $\hat{x}_B \in \left]\frac{(a-\Gamma)-\sqrt{\Delta}}{2N_B\delta_{BB}}, \frac{(a-\Gamma)+\sqrt{\Delta}}{2N_B\delta_{BB}}\right[$ will then satisfy the requirement. Using expressions (3) and (4) and rewritting condition (15) we obtain condition (13), which concludes the proof.

Proof of Proposition 10

First, when $\delta_{AA} = \delta_{BB} = \delta_{AB} = \delta_{BA}$ then condition (13) can be simplified as

$$(N_A + N_B - 1)^2 > 0,$$

which is always satisfied.

Secondly, when $N_A = N_B = N$ condition (13) can be simplified as follows:

$$\left[(N+1)\,\delta + N\delta'\right]^2 > 8N\delta^2$$

or

$$\left(N^2 - 6N + 1\right)\delta^2 + 2N\,(N+1)\,\delta\delta' + N^2\left(\delta'\right)^2 > 0,$$

which is a polynomial expression of parameter δ'. Solving for δ', we obtain that condition (13) is satisfied if and only if $\delta' \in \left]\max\{0, \frac{-(N+1)+2\sqrt{2N}}{N}\delta\}, \frac{N+1}{N}\delta\right[$ is satisfied. We can now conclude, since $\frac{-(N+1)+2\sqrt{2N}}{N}\delta \geq 0$ is easily checked to hold if and only if $N \leq 5$ is satisfied. This concludes the proof.

Supplementary material

Extension of Proposition 4

In order to assess the qualitative effect of each type of fundamentals (within-area externality parameter, between-area externality parameter, size of the populations), we successively consider two additional situations, where only one parameter is allowed to differ from one area to the other. Moreover, to save on notations, we focus on the case of area A, results for area B will follow in an entirely similar manner.

Proposition 11. *Assume that conditions (7) are satisfied simultaneously, so that the full cooperation and Nash equilibrium outcomes are characterized by (3)-(4) and (5)-(6). We have the following comparisons:*

1. *Assume* $\delta_{AB} = \delta_{BA} = \delta'$ *while* $N_A = N_B = N$*:*
   - *When* $\delta_{AA} \in]\frac{N^2-1}{N^2}\delta_{BB}, \delta_{BB}[$ *while* $(\delta')^2 < \delta_{AA}\delta_{BB} < \frac{N^2}{N^2-1}(\delta')^2$ *then there exists* $\underline{\delta}_{AA} \in]0, \delta_{AA}[$ *such that* $x_A^N \geq x_A^*$ *when* $\delta' \in [0, \underline{\delta}_{AA}]$ *and* $x_A^N < x_A^*$ *when* $\delta' \in]\underline{\delta}_{AA}, \delta_{AA}[$*;*
   - *Otherwise* $x_A^N \geq x_A^*$ *holds generically.*
2. *Finally, assume* $\delta_{AA} = \delta_{BB} = \delta$ *while* $N_A = N_B = N$*:*
   - *When* $\delta_{AB} < \delta_{BA}$ *then* $x_A^N \geq x_A^*$ *when* $\delta_{BA} \in [\delta_{AB}, \underline{\delta}_{BA}]$ *while* $x_A^N < x_A^*$ *when* $\delta_{BA} \in]\underline{\delta}_{BA}, \frac{N+1}{N}\delta[$*, where* $\underline{\delta}_{BA}$ *increases as* δ *increases, or as* δ_{AB} *increases. The effect of* N *on* $\underline{\delta}_{BA}$ *is ambiguous and depends on the interplay between* δ*,* δ_{AB} *and* N*.*
   - *When* $\delta_{AB} \geq \delta_{BA}$ *then* $x_A^N \geq x_A^*$ *holds generically.*

Proof. In case 1, using its assumptions (among others, $\delta_{AA} > \delta'$ and $\delta_{BB} > \delta'$ are satisfied) to rewrite expression (14) allows to conclude that the sign of $x_A^N - x_A^*$ is given by that of:

$$2N\left[(N+1)\delta_{BB} - N\delta'\right]\left[\delta_{AA}\delta_{BB} - (\delta')^2\right] - \left[\delta_{BB} - \delta'\right]\left[(N+1)^2\delta_{AA}\delta_{BB} - N^2(\delta')^2\right]$$

This expression can be rewritten to deduce:

$$x_A^N \geq x_A^* \Longleftrightarrow 2N\delta_{BB}\delta'(\delta_{AA} - \delta') + (\delta_{BB} - \delta')\left[(N^2-1)\delta_{AA}\delta_{BB} - N^2(\delta')^2\right] \geq 0, \qquad (16)$$

which is a polynomial function $\Phi(\delta')$ of δ' (keeping in mind that $0 \leq \delta' \leq \min\{\delta_{AA}, \delta_{BB}\}$ by conditions (7)). When $\delta_{AA} \geq \delta_{BB}$ we deduce immediately that

$$\Phi(\delta') \geq (\delta_{BB} - \delta')\left[2N\delta_{BB}\delta' + (N^2-1)(\delta_{BB})^2 - N^2(\delta')^2\right]$$

$$= (\delta_{BB} - \delta')\left[(N+1)\delta_{BB} - N\delta'\right]\left[(N-1)\delta_{BB} + N\delta'\right] > 0,$$

which allows to conclude that $x_A^N > x_A^*$ then. Now, when $\delta_{AA} < \delta_{BB}$ we deduce quickly from (16) that a necessary condition for $x_A^N < x_A^*$ to hold is that $\delta_{AA}\delta_{BB} < \frac{N^2}{N^2-1}(\delta')^2$ is satisfied. Now, differentiating $\Phi(\delta')$ with respect to δ', we obtain:

$$\Phi'(\delta') = 3N^2(\delta')^2 - 2N(N+2)\delta_{BB}\delta' + \left[2N - N^2 + 1\right]\delta_{AA}\delta_{BB}$$

The term on the right hand side of the equality is a polynomial function: solving it for δ' we obtain

$$\Phi'(\delta') \leq 0 \Longleftrightarrow \delta' \in [\delta'_1, \delta'_2],$$

where $\delta'_1 = \frac{(N+2)\delta_{BB} - \sqrt{\delta_{BB}\left[(N+2)^2\delta_{BB} - 3\delta_{AA}(2N-N^2+1)\right]}}{3N}$ and $\delta'_2 = \frac{(N+2)\delta_{BB} + \sqrt{\delta_{BB}\left[(N+2)^2\delta_{BB} - 3\delta_{AA}(2N-N^2+1)\right]}}{3N}$. The first conclusions are that, when $2N - N^2 + 1 \leq 0$ holds (which is the case if and only if $N \geq 3$) then $\delta'_1 \leq 0$ is satisfied, and that $\delta'_2 > \delta_{AA}$ as $\delta_{AA} < \delta_{BB}$ is satisfied. These two conclusions imply that $\Phi'(\delta') \leq 0$ on $[0, \delta_{AA}[$ for $N \geq 3$ and $\Phi'(\delta')$ is non-negative on $[0, \delta'_1]$ and negative on

$]\delta_1', \delta_{AA}[$ when $N \leq 2$ is satisfied. We obtain quickly that $Phi(0) > 0$ holds, and finally, looking at $\Phi(\delta' \to \delta_{AA})$ we obtain:

$$\Phi(\delta' \to \delta_{AA}) = \delta_{AA}\left[N^2 (\delta_{AA})^2 - \left(2N^2 - 1\right)\delta_{AA}\delta_{BB} + \left(N^2 - 1\right)(\delta_{BB})^2\right]$$

Solving for δ_{AA} we deduce that $\Phi(\delta' \to \delta_{AA}) < 0$ if and only if δ_{AA} lies in the interval $]\frac{N^2-1}{N^2}\delta_{BB}, \delta_{BB}[$. Together with the monotonicity of function Φ this implies that there exists $\underline{\delta}_{AA} \in]0, \delta_{AA}[$ such that $\Phi \geq 0$ for $\delta' \in [0, \underline{\delta}_{AA}]$ and $\Phi < 0$ for $\delta' \in]\underline{\delta}_{AA}, \delta_{AA}[$ if and only if $\delta_{AA} \in]\frac{N^2-1}{N^2}\delta_{BB}, \delta_{BB}[$ is satisfied. Otherwise Φ is non-negative. This concludes the proof of the first case.

In the second case, using its assumptions to rewrite expression (14) allows to conclude that the sign of $x_A^N - x_A^*$ is given by that of:

$$N\left[(N+1)\delta - N\delta_{BA}\right]\left[2\delta + (\delta_{AB} + \delta_{BA})\right] - \left[(N+1)^2\delta^2 - N^2\delta_{AB}\delta_{BA}\right]$$

This expression can be rewritten to deduce:

$$x_A^N \geq x_A^* \Longleftrightarrow (N+1)(N-1)\delta^2 - N(N-1)\delta\delta_{BA} + N(N+1)\delta\delta_{AB} - N^2(\delta_{BA})^2 \geq 0,$$

which is a polynomial function $P(\delta_{BA})$ of δ_{BA}. Solving for δ_{BA} we obtain:

$$P(\delta_{BA}) \geq 0 \Longleftrightarrow \delta_{BA} \in [0, \underline{\delta}_{BA} = \frac{-(N-1)\delta + \sqrt{(N-1)^2\delta^2 + 4(N+1)\delta\left[N\delta_{AB} + (N-1)\delta\right]}}{2N}]. \tag{17}$$

The above expression of $\underline{\delta}_{BA}$ yields the following implications:

$$\underline{\delta}_{BA} \leq \frac{N+1}{N}\delta \Longleftrightarrow \delta_{AB} \leq \frac{N+1}{N}\delta;\ \ \underline{\delta}_{BA} \leq \delta \Longleftrightarrow \delta_{AB} \leq \frac{N^2-N+1}{N(N+1)}\delta;\ \ \underline{\delta}_{BA} \leq \delta_{AB} \Longleftrightarrow \delta_{AB} \geq \frac{2+\sqrt{N^2+3}}{N}\delta. \tag{18}$$

In case 2, conditions (7)) are equivalent to either (i) $\delta_{AB} \leq \delta_{BA}$ and $\delta_{AB} < \delta$ while $\delta_{BA} < \frac{N+1}{N}\delta$, or (ii) $\delta_{AB} > \delta_{BA}$ and $\delta_{BA} < \delta$ while $\delta_{AB} < \frac{N+1}{N}\delta$. We prove the result in these two sub-cases. Regarding sub-case (i), the first inequality in (18) yields the conclusion that $\underline{\delta}_{BA} \leq \frac{N+1}{N}\delta$ is satisfied, while the last inequality in (18) yields the conclusion that $\underline{\delta}_{BA} \geq \delta_{AB}$ holds (since $\frac{2+\sqrt{N^2+3}}{N}\delta > \frac{N+1}{N}\delta > \delta_{AB}$ are satisfied). Using (17) then allows to conclude. The effect of δ and of δ_{AB} on $\underline{\delta}_{BA}$ follows from direct differentiations. Regarding the effect of N, differentiating and simplifying, we obtain that the sign of $\frac{\partial \underline{\delta}_{BA}}{\partial N}$ is given by that of the following expression:

$$-\sqrt{(N-1)^2\delta^2 + 4\delta(N+1)\left[N\delta_{AB} + (N-1)\delta\right]} - 2N\delta_{AB} + (N+3)\delta$$

A first conclusion is that $\frac{\partial \underline{\delta}_{BA}}{\partial N} < 0$ when $-2N\delta_{AB} + (N+3)\delta \leq 0$ is satisfied. Otherwise, $\frac{\partial \underline{\delta}_{BA}}{\partial N} \geq 0$ if and only if the following inequality is satisfied:

$$(N+1)(3-N)\delta^2 + N^2(\delta_{AB})^2 - 2N(N+2)\delta\delta_{AB} \geq 0.$$

Solving for δ_{AB} we conclude that $\frac{\partial \underline{\delta}_{BA}}{\partial N} \geq 0$ when $\delta_{AB} \leq \delta\frac{(N+2)-\sqrt{2N^2+2N+1}}{N}$ and $\frac{\partial \underline{\delta}_{BA}}{\partial N} < 0$ when $\delta_{AB} \in]\delta\frac{(N+2)-\sqrt{2N^2+2N+1}}{N}, \frac{N+3}{2N}\delta[$ is satisfied. It is easily checked that $\frac{(N+2)-\sqrt{2N^2+2N+1}}{N} > 0$

if and only if $N \leq 2$ holds. Thus, the sign of $\frac{\partial \delta_{BA}}{\partial N}$ depends on the interplay between δ, δ_{AB} and N. This concludes the proof of the first sub-case. Regarding the second sub-case, the last inequality in (18) allows again to conclude that $P(\delta_{BA}) \geq 0$ holds generically. This concludes the proof of the second case. □

Several remarks are worth noticing. Regarding the first case, a first point is that low values of δ' correspond to the classical conclusion: indeed, in such a situation this case is similar to the first one, and the conclusion follows similarly. When looking at large values of δ', if $\delta_{AA} \geq \delta_{BB}$ this amounts to looking at $\delta' \to \delta_{BB}$, but this implies that the efficient outcome gets close to zero, and obviously the classical conclusion obtains. By contrast, when $\delta_{AA} < \delta_{BB}$ we can look at situations where $\delta' \to \delta_{AA}$ is satisfied. Then x_A^N gets close to $\frac{a-\Gamma}{\delta_{AA}} \frac{(N+1)\delta_{BB} - N\delta_{AA}}{(N+1)^2\delta_{BB} - N^2\delta_{AA}}$ while x_A^* gets close to $\frac{a-\Gamma}{2N\delta_{AA}}$, and the comparison with the socially efficient outcome depends on the interplay between the within-area externality parameters and the size of the populations. Specifically, $x_A^N < x_A^*$ can be satisfied only if $\left(N^2 - 1\right)\delta_{BB} < N^2\delta_{AA}$ holds: there is some heterogeneity along this dimension, but the degree of heterogeneity is not too high.

The second case can be explained easily when $\delta_{AB} \geq \delta_{BA}$ is satisfied. Indeed, coming back to Proposition 3 we know that x_A^N increases as δ_{BA} increases, which implies that

$$x_A^N \geq (a - \Gamma)\frac{(N+1)\,\delta - N\delta_{BA}}{(N+1)^2\,\delta^2 - N^2\left(\delta_{BA}\right)^2} = \frac{(a-\Gamma)}{(N+1)\,\delta + N\delta_{BA}}$$

and the comparison with x_A^* follows easily then. By contrast, when $\delta_{AB} < \delta_{BA}$ holds, the conclusion depends on the value of δ_{BA}. When it is low, that is, close to δ_{AB}, the situation is similar to the case of homogeneous areas, and the classical conclusion follows. When it is high (close to $\frac{N+1}{N}\delta$ according to the assumptions), then x_A^N gets arbitrarily small (when the difference $\delta_{BA} - \delta_{AB}$ is sufficiently large) while $x_A^* = \frac{(a-\Gamma)}{N(2\delta + \delta_{AB} + \delta_{BA})}$ does not become so.

Extension of Proposition 10

Proposition 12. *Assume that conditions* $(N_A + 1)\,\delta_{AA} > N_A\delta_{AB}$ *and* $(N_B + 1)\,\delta_{BB} > N_B\delta_{BA}$ *hold. Then we have:*

1. *When* $N_A = N_B = N$ *and* $\delta_{AA} = \delta_{BB} = \delta$ *a reserve based on private initiatives Pareto dominates the full non-cooperative outcome:*
   - *When* $\min\{\frac{4}{N+1}\delta, \frac{N+1}{N}\left(1 - \frac{\sqrt{N}}{2N}\right)\delta\} \leq \delta_{BA} < \frac{N+1}{N}\delta$ *is satisfied;*
   - *When* $\delta_{BA} < \min\{\frac{4}{N+1}\delta, \frac{N+1}{N}\left(1 - \frac{\sqrt{N}}{2N}\right)\delta\}$ *there exists* $\underline{\delta}_{AB} \in]0, \frac{N+1}{N}\delta[$ *such that Pareto dominance holds when* $\delta_{AB} \in [\underline{\delta}_{AB}, \frac{N+1}{N}\delta[$ *is satisfied;*
2. *When* $N_A = N_B = N$ *and* $\delta_{AB} = \delta_{BA} = \delta'$ *a reserve based on private initiatives Pareto dominates the full non cooperative outcome:*
   - *Provided that both* $\delta_{BB} \leq \frac{(N+1)^2+2N}{N+1}\delta'$ *holds and*
     - *Either conditions* $N \geq 3$ *and* $\delta' \geq \frac{4}{N+1}\delta_{BB}$ *are satisfied,*
     - *Or there exists* $\underline{\delta}_{AA} > \frac{N}{N+1}\delta'$ *such that Pareto dominance holds for* $\delta_{AA} > \underline{\delta}_{AA}$*. When* $N \geq 6$ *then* $\underline{\delta}_{AA}$ *belongs to* $]\frac{N}{N+1}\delta', \delta_{BB}]$*.*

- *Provided that* $\delta_{BB} > \frac{(N+1)^2+2N}{N+1}\delta'$ *is satisfied, there exists* $\underline{\delta}_{AA} > \frac{N}{N+1}\delta'$ *such that Pareto dominance holds for* $\delta_{AA} > \underline{\delta}_{AA}$. *When* $N \geq 6$ *then* $\underline{\delta}_{AA}$ *belongs to* $]\frac{N}{N+1}\delta', \delta_{BB}[$.

Proof. Now, when $N_A = N_B = N$ and $\delta_{AA} = \delta_{BB} = \delta$ we can rewrite condition 13 as $\Phi(\delta_{AB}) > 0$, where

$$\Phi(\delta_{AB}) = \left[(N+1)^2\delta^2 - N^2\delta_{AB}\delta_{BA}\right]^2$$

$$-4\left[N\delta^2\left[(N+1)\delta - N\delta_{AB}\right]^2 + N\delta^2\left[(N+1)\delta - N\delta_{BA}\right]^2\right]$$

Simple differentiations yield:

$$\Phi'(\delta_{AB}) = 2N^2\left[4\delta^2\left[(N+1)\delta - N\delta_{AB}\right] - \delta_{BA}\left[(N+1)^2\delta^2 - N^2\delta_{AB}\delta_{BA}\right]\right]$$

and

$$\Phi''(\delta_{AB}) = 2N^3\left[-4\delta^2 + N(\delta_{BA})^2\right]$$

Regarding the first sub-case, when $\delta_{BA} \geq \frac{2}{\sqrt{N}}\delta$ then $\Phi'' \geq 0$, which implies that Φ' is non decreasing which, together with $\Phi'(\delta_{BA} \to \frac{N+1}{N}\delta) \leq 0$, implies that Φ' is non increasing. Finally, since $\Phi(\delta_{BA} \to \frac{N+1}{N}\delta)$ is easily checked to be non negative, this implies that $\Phi \geq 0$. Now, when $\frac{4}{N+1}\delta \leq \delta_{BA} < \frac{2}{\sqrt{N}}\delta$ then $\Phi'' < 0$ and $\Phi' \leq 0$ (since $\Phi'(0) \leq 0$), which finally implies that $\Phi \geq 0$ by using the same argument than when $\delta_{BA} \geq \frac{2}{\sqrt{N}}\delta$ is satisfied, and concludes the proof of the first sub-case.

When $\delta_{BA} < \frac{4}{N+1}\delta$ we know that $\Phi'' \leq 0$ and $\Phi'(0) > 0$ which, combined with $\Phi'(\delta_{BA} \to \frac{N+1}{N}\delta) \leq 0$, implies that there exists $\delta'_{AB} \in]0, \frac{N+1}{N}\delta[$ such that Φ increases on $]0, \delta'_{AB}]$ and decreases on $]\delta'_{AB}, \frac{N+1}{N}\delta[$. Since $\Phi(\delta_{AB} \to \frac{N+1}{N}\delta) \geq 0$ this in turn implies that there exists $\underline{\delta}_{AB} \in [0, \delta'_{AB}[$ such that $\Phi \leq 0$ on $[0, \underline{\delta}_{AB}]$ and $\Phi > 0$ on $]\underline{\delta}_{AB}, \frac{N+1}{N}\delta]$ if and only if $\Phi(0)$ is negative; otherwise we conclude that $\Phi \geq 0$. We obtain:

$$\Phi(0) = \delta^2\left[(N+1)^2\left[N^2 - 6N + 1\right] + 8N^2(N+1)\delta\delta_{BA} - 4N^3(\delta_{BA})^2\right],$$

which is a polynomial expression of δ_{BA}. Solving it, we obtain the following conclusion: $\Phi(0) < 0$ when either $\delta_{BA} < \delta\frac{N+1}{N}\left(1 - \frac{\sqrt{N}}{2N}\right)$ or $\delta_{BA} > \delta\frac{N+1}{N}\left(1 + \frac{\sqrt{N}}{2N}\right)$ is satisfied, and $\Phi(0) \geq 0$ when $\delta_{BA} \in [\delta\frac{N+1}{N}\left(1 - \frac{\sqrt{N}}{2N}\right), \delta\frac{N+1}{N}\left(1 + \frac{\sqrt{N}}{2N}\right)]$ holds. All together, noting that $\frac{N+1}{N}\left(1 + \frac{\sqrt{N}}{2N}\right)\delta > \frac{N+1}{N}\delta$ is always satisfied, this concludes the proofs of the second and third sub-cases.

Finally, when $N_A = N_B = N$ and $\delta_{AB} = \delta_{BA} = \delta'$, again rewriting condition 13 as $\Phi(\delta_{AA}) > 0$ and differentiating, we obtain:

$$\Phi'(\delta_{AA}) = 2(N+1)^2\delta_{BB}\left[(N+1)^2\delta_{AA}\delta_{BB} - N^2(\delta')^2\right] - 8N(N+1)(\delta_{BB})^2\left[(N+1)\delta_{AA} - N\delta'\right]$$

$$-4N\delta_{BB}\left[(N+1)\delta_{BB} - N\delta'\right]^2$$

and

$$\Phi''(\delta_{AA}) = 2(N+1)^2(\delta_{BB})^2(N-1)^2,$$

which is non negative. This implies that Φ' increases. We obtain quickly:

$$\Phi'(\delta_{AA} \to \frac{N}{N+1}\delta') = 2N\delta_{BB}\left[(N+1)\,\delta_{BB} - N\delta'\right]\left[\left\{(N+1)^2 + 2N\right\}\delta' - 2\,(N+1)\,\delta_{BB}\right]$$

This implies that $\Phi'(\delta_{AA} \to \frac{N}{N+1}\delta') \geq 0$ if and only if $\delta_{BB} \leq \frac{(N+1)^2+2N}{2(N+1)}$ is satisfied. We first prove the Proposition in this case. Thus we deduce that $\Phi' \geq 0$, which implies that Φ increases as δ_{AA} increases. We obtain:

$$\Phi(\delta_{AA} \to \frac{N}{N+1}\delta') = N^2\delta'\left[(N+1)\,\delta_{BB} - N\delta'\right]^2\left(\delta' - \frac{4}{N+1}\delta_{BB}\right)$$

Since $\delta_{BB} > \frac{N}{N+1}\delta'$ by assumption, we deduce that $\delta' - \frac{4}{N+1}\delta_{BB}$ is negative if and only if either $N \leq 2$ or $N \geq 3$ and $\delta' < \frac{4}{N+1}\delta_{BB}$. Thus, when $N \geq 3$ and $\delta' \geq \frac{4}{N+1}\delta_{BB}$ we deduce that $\Phi(\delta_{AA} \to \frac{N}{N+1}\delta') \geq 0$ and in turn that $\Phi \geq 0$ in this case. Otherwise, we know that, when $\delta_{AA} \geq \delta_{BB}$ and large enough, we deduce easily that $\Phi(\delta_{AA}) \approx (N+1)^2\,(N-1)^2\,(\delta_{AA})^2\,(\delta_{BB})^2 \geq 0$, which implies that there exists $\underline{\delta}_{AA} > \frac{N}{N+1}\delta'$ such that $\Phi \geq 0$ for $\delta_{AA} \geq \underline{\delta}_{AA}$. More specifically, computing $\Phi(\delta_{AA} = \delta_{BB})$ we obtain:

$$\Phi(\delta_{BB}) = \left[(N+1)\,\delta_{BB} - N\delta'\right]^2\left[\left(N^2 - 6N + 1\right)(\delta_{BB})^2 + 2N\,(N+1)\,\delta_B B\delta' + N^2\,(\delta')^2\right]$$

Solving for δ' we obtain that $\Phi(\delta_{BB}) \geq 0$ when $\delta' \geq -\frac{N+1}{N} + 2\frac{\sqrt{2N}}{N}\delta_{BB}$, and $-\frac{N+1}{N} + 2\frac{\sqrt{2N}}{N}\delta_{BB} \leq 0$ when $N > 5$. This implies that $\underline{\delta}_{AA} \leq \delta_{BB}$ when $N > 5$ is satisfied. This concludes the proof of the first sub-case.

Finally, when $\delta_{BB} > \frac{(N+1)^2+2N}{2(N+1)}$ then $\Phi'(\delta_{AA} \to \frac{N}{N+1}\delta') < 0$ and, when $\delta_{AA} \geq \delta_{BB}$ and large enough we have $\Phi'(\delta_{AA}) \approx 2\,(N+1)^2\,(\delta_{BB})^2\,\delta_{AA}\,(N-1)^2 \geq 0$, which implies that there exists $\underline{\delta}_A A > \frac{N}{N+1}\delta'$ such that Φ is non increasing on $]\frac{N}{N+1}\delta', \underline{\delta}_A A]$ and then increasing. Since we have

$$\Phi(\delta_{AA} \to \frac{N}{N+1}\delta') = N^2\delta'\left[(N+1)\,\delta_{BB} - N\delta'\right]\left[\delta' - \frac{4}{N+1}\delta_{BB}\right] < 0$$

and still

$$\Phi(\delta) \approx (N+1)^2\,(N-1)^2\,(\delta_{AA})^2\,(\delta_{BB})^2 \geq 0$$

There exists $\overline{\delta}_{AA} > \frac{N}{N+1}\delta'$ such that $\Phi \geq 0$ for $\delta_{AA} \geq \underline{\delta}_{AA}$. Similarly than in the first sub-case, we conclude this time that $\Phi(\delta_{AA} = \delta_{BB}) > 0$ when $N > 5$ is satisfied, which implies that $\overline{\delta}_{AA} < \delta_{BB}$ and concludes the proof of the second sub-case. □

This extended result allows for several interesting comments. First, the case of potentially differing inter-area externality parameters exhibits qualitatively different features. When the intensity of the externality imposed by area B on area A is sufficiently strong, the transfer scheme defined in Proposition 8 will ensure a Pareto improvement over the case of non-cooperation. In such situations, the higher the value of δ_{BA}, the lower the level of non-cooperative extractions in area A in the status-quo scenario (according to Proposition 3). Thus, this corresponds to situations where the status-quo results in low extraction levels and, as such, to fairly low payoffs within area A. This makes it fairly simple to compensate agents within this area. By contrast, when the value of δ_{BA} is sufficiently low, there is a trade-off and a self-consistent transfer scheme is not always feasible. Whether such a scheme exists will depend on the intensity of the externality imposed by area A on area B. Due to the effect of δ_{AB} on extraction levels in both areas, a sufficiently high value of δ_{AB} is required to balance the low value of δ_{BA}.

Secondly, the case of potentially differing within-area externality parameters is more complicated. Compared to the second case, low intensities of within-area externality for at least one area is not a sufficient property to ensure the feasibility of an appropriate transfer scheme. When the intensity of within-area externality is not too high (compared to the inter-area externality), a sufficient condition is then that the intensity of the other types of externality be sufficiently large. This is equivalent to sufficiently large number of agents and intensity of inter-area externality: yet, if both conditions are not met simultaneously, the feasibility of a transfer scheme is not ensured. It then requires that the within-area externality is sufficiently strong in area A, so that the effects driven by low values of δ_{BB} be balanced. Finally, when the value of δ_{BB} is sufficiently high (still compared to δ'), the feasibility of setting area A as a reserve becomes relatively more difficult to achieve: the externality within area A must then be sufficiently strong.