

Intergenerational equity under catastrophic climate change

Aurélie Méjean^{*1}, Antonin Pottier², Stéphane Zuber³ and Marc Fleurbaey⁴

¹CIREN - CNRS, Centre International de Recherche sur l'Environnement et le Développement (CNRS, Agro ParisTech, Ponts ParisTech, EHESS, CIRAD)

²CERNA, Centre d'Economie Industrielle, Mines ParisTech

³Paris School of Economics - CNRS

⁴Woodrow Wilson School of Public and International Affairs, Princeton University

Abstract

Climate change raises the issue of intergenerational equity, as catastrophes may unfairly affect some generations. As climate change threatens irreversible and dangerous impacts, possibly leading to extinction, one relevant trade-off may not just involve present and future consumption, but present consumption and the mere existence of future generations. This paper aims at identifying public policies that can reduce unfairness and strike a compromise between present and future generations when the potential impact of catastrophic climate change on the economy is accounted for.

Keywords: Climate change; Equity; Catastrophic risk; Climate-economy model

JEL: Q54; D63.

*Corresponding author: mejean@centre-cired.fr - CIREN, 45 bis avenue de la Belle Gabrielle, 94736 Nogent-sur-Marne, France

1 Introduction

Climate change raises the issue of intergenerational equity, as catastrophes may unfairly affect some generations. As climate change threatens irreversible and dangerous impacts (IPCC, 2014), possibly leading to extinction, the trade-off is not only between present and future consumption, but also between present consumption and the possible future extinction of civilization due to climate change (Weitzman, 2009). While the literature has mainly focused on the first trade-off, this paper aims at identifying public policies that can reduce unfairness and strike an acceptable compromise between present and future generations when the potential impact of catastrophic climate change on the economy is accounted for. We propose to evaluate policies aiming at limiting greenhouse gas emissions by considering different social welfare functions, while taking the risk of extinction into account. As we account for the risk of catastrophic climate change, total population over all generations can vary, which raises the issue of the weight given to total population size in the evaluation. We use an integrated assessment model, which provides a simple representation of the interaction between the economy and the climate, and allows to evaluate climate policies. We depart from the standard optimization framework, and instead consider various climate policies that are ordered according to their performance in terms of welfare. We explore the impact of inequality aversion, of attitudes towards population size and of the risk of extinction on the preferred climate policy. The paper is structured as follows. Section 2 presents the analytical framework, the model, and the numerical experiment. Section 3 presents and discusses the results. Section 4 concludes.

2 Analytical framework

2.1 Definitions

We introduce a model with a risk of extinction. Within a generation, consumption is equally distributed. The size of generation t is n_t . We denote $N_t = \sum_{\tau=0}^t n_\tau$ the total population up to generation t (or the total population when the world goes extinct after generation t). At each period t , aggregate consumption $C_t = c_t \cdot n_t$ is deterministic, provided extinction has not yet happened. The only risk is about extinction: in period t , there is a probability $(1 - p_t)$ of surviving to the next period, and thus a probability p_t of extinction. Hence the probability that there exists exactly t generations is: $P_t = p_t \prod_{\tau=0}^{t-1} (1 - p_\tau)$. We study a general class of social

welfare functions. The aggregate welfare W depends on the stream of consumption per capita c and on the stream of risk of extinction p . A given abatement policy i gives rise to a stream of consumption per capita over time, noted c_i , and to a stream of risk of extinction over time, noted p_i . The difference in welfare ΔW between abatement policies i and j thus depends on the pairs (c_i, p_i) and (c_j, p_j) . More generally, we note $W_{i,j} = W(c_i, p_j)$. We define below the concepts and methods used in the analysis.

Definition 1 (Equally Distributed Equivalent)

The equally distributed equivalent¹ level of income of a given distribution is “the level of income per head which if equally distributed would give the same level of social welfare as the present distribution” (Atkinson, 1970). With ϕ a continuous concave function, the equally distributed equivalent level of consumption, denoted EDE_t , is:

$$EDE_t = \phi^{-1}\left(\frac{1}{N_t} \sum_{\tau=0}^t n_\tau \cdot \phi(c_\tau)\right) \quad (1)$$

c_τ	consumption per capita at date τ
N_t	total population up to date t
ϕ	function that translates inequality aversion
n_τ	size of generation τ
EDE_t	equally distributed equivalent level of consumption at date t

Definition 2 (EPEDE social ordering) A social welfare function is an Expected Prioritarian Equally Distributed Equivalent (EPEDE) social ordering if there exist real numbers $(\alpha_{N_t})_{N_t \in \mathbb{N}} \in (\mathbb{R}_+)^{\mathbb{N}}$ and two concave continuous function ϕ and u such that:

$$\begin{aligned} W(c, p) &= \mathbb{E}\left(\alpha_{N_t} \left\{ u \circ \phi^{-1}\left(\sum_{\tau=0}^T \frac{n_\tau}{N_t} \phi(c_\tau)\right) - u(\bar{c}) \right\}\right) \\ &= \sum_{t=0}^{\infty} P_t \cdot \alpha_{N_t} \cdot (u(EDE_t) - u(\bar{c})) \end{aligned} \quad (2)$$

¹The principle was first introduced by Kolm (1969) and Atkinson (1970).

W	welfare
c_τ	consumption per capita at date τ
p	risk of extinction (per annum)
N_t	total population up to date t
α_{N_t}	a function of total population N_t
u	utility function, translates risk aversion from the social point of view
ϕ	a function that translates inequality aversion
n_τ	size of generation τ
\bar{c}	critical level of consumption per capita
P_t	probability that there exists exactly t generations
EDE_t	equally distributed equivalent level of consumption at date t

The EPEDE criterion (Fleurbaey and Zuber, 2015) is more flexible than discounted utilitarianism in terms of population ethics, as it includes a weight on population size. It disentangles risk aversion and inequality aversion. It accounts for the inequality aversion of the social planner and respects a weak form of Pareto. It is not separable, but we will nevertheless assume that past populations are not relevant to the analysis. This criterion includes a critical level of consumption (\bar{c}) which is the level of consumption enjoyed by an additional individual which leaves total welfare unchanged. The criterion therefore favours adding individuals to the population only if their consumption is above that critical level. The criterion admits utilitarianism as a special case. We choose isoelastic functions for ϕ and u , and an increasing function of cumulative population for α_{N_t} .

$$\phi(z) = \frac{z^{1-\eta}}{1-\eta} \quad ; \quad u(z) = \frac{z^{1-\gamma}}{1-\gamma} \quad ; \quad \alpha_{N_t} = N_t^\beta \quad (3)$$

η	inequality aversion parameter
γ	risk aversion parameter
β	population parameter

With these specifications, we demonstrate that the equally distributed equivalent level of consumption EDE_t is increasing with γ when $\gamma > 1$ and decreasing with γ when $\gamma < 1$). We also show that the utility of consumption $u(c)$ is decreasing with η when $c > \bar{c} > 1$ and increasing with η when $\bar{c} > c > 1$ (see proofs in appendix A). In the utilitarian case we have $\gamma = \eta$ (hence

$u = \phi$) and $u(ED E_t) = \sum_{\tau=0}^t \frac{n_\tau}{N_t} u(c_\tau)$. This enables us to rewrite welfare as:

$$\begin{aligned} W(c, p) &= \sum_{t=0}^{\infty} P_t \frac{\alpha N_t}{N_t} \sum_{\tau=0}^t n_\tau (u(c_\tau) - u(\bar{c})) \\ &= \sum_{\tau=0}^{\infty} n_\tau (u(c_\tau) - u(\bar{c})) \sum_{t=\tau}^{\infty} P_t \frac{\alpha N_t}{N_t} \end{aligned} \quad (4)$$

The weight given to consumption at period τ in total welfare does not depend only on what happens at period t but on the whole future history, and in particular on the probability of extinction P_t and on population size N_t .

2.2 Evolution of welfare differences

2.2.1 Analytical results

We assume two policies i and j , with paths of consumption c_i and c_j and paths of risks of extinction p_i and p_j . We suppose that c_i and c_j are increasing consumption streams. We assume that policy j has lower emissions than policy i and, assuming no climate damages, we have for each time step: $c_{i,t} \geq c_{j,t}$ (consumption is always higher in scenario i as less resources are devoted to mitigation), and $p_{i,t} \geq p_{j,t}$ (less mitigation in scenario i leads to a higher risk of extinction at each time step). The social welfare of each policy is computed from the consumption and extinction risk paths: $W(c_i, p_i)$ and $W(c_j, p_j)$, respectively. We examine the sign of the welfare difference between these policies: $\Delta W = W(c_j, p_j) - W(c_i, p_i)$: when this quantity is positive, policy j is preferred; when it is negative, policy i is preferred. We want to understand the evolution of ΔW with respect to η , γ and β . In the general case, we can write²:

$$\begin{aligned} \Delta W &= W(c_j, p_j) - W(c_i, p_i) \\ &= (W(c_j, p_j) - W(c_j, p_i)) - (W(c_i, p_i) - W(c_j, p_i)) \\ &= \Delta_p W - \Delta_c W \end{aligned} \quad (5)$$

The first term $\Delta_p W = W(c_j, p_j) - W(c_j, p_i)$ is the part of the welfare difference that is explained by the variation of *risk*, while the second term $\Delta_c W = W(c_i, p_i) - W(c_j, p_i)$ is the part of the

²We could also write $\Delta W = (W(c_j, p_j) - W(c_i, p_j)) - (W(c_i, p_i) - W(c_i, p_j))$ where the first term is the part of the welfare difference that is explained by the variation of *consumption*, while the second term is explained by the variation of *risk*. Using one decomposition or the other gives very similar results.

welfare difference that is explained by the variation of *consumption*. The first term $\Delta_p W$ reflects a situation where only the risk of extinction changes, which only affects P_t , the probability of extinction at time t exactly. We have $\Delta_p W = \sum_t \alpha_{N_t} \cdot (P_{j,t} - P_{i,t}) \cdot (u(EDE_t) - u(\bar{c}))$. As $P_t = p_t \cdot \prod_{s \leq t-1} (1 - p_s)$ and $W = \sum_{t=0}^{\infty} P_t \cdot \alpha(N_t) \cdot (u(EDE_t) - u(\bar{c}))$, the contribution of a lower risk of extinction at time t has two opposite effects: there are fewer generations living precisely t periods, which reduces total welfare, but there are more generations living strictly more than t periods, which increases total welfare³. Reducing the risk of extinction comes down to swapping extinction in early periods for extinction in later periods (where the welfare of the generation living exactly t periods, $\alpha_{N_t} \cdot (u(EDE_t) - u(\bar{c}))$, is higher). If EDE_t is above the subsistence level \bar{c} (which ensures that $u(EDE_t) - u(\bar{c}) > 0$) and if $\alpha_{N_t} \cdot (u(EDE_t))_u(\bar{c})$ is an increasing sequence⁴, a lower extinction rate at time t increases social welfare. In our example, $p_{i,t} \geq p_{j,t}$, hence $W(c_j, p_j) - W(c_j, p_i)$ is positive. The second term $\Delta_c W$ reflects a situation where only consumption changes. We have $\Delta_c W = \sum_t \alpha_{N_t} P_t (u(EDE_{j,t}) - u(EDE_{i,t}))$. In our example, $c_{i,t} \geq c_{j,t}$, hence $W(c_i, p_i) - W(c_j, p_i)$ is positive. We have thus shown that both $\Delta_p W$ and $\Delta_c W$ are positive.

We further show that both $\Delta_p W$ and $\Delta_c W$ are increasing with β and decreasing with η in the general case. We also find that the behaviour of $\Delta_p W$ and $\Delta_c W$ with respect to γ is ambiguous, but that both terms are decreasing with γ in the utilitarian case (i.e. when $\eta = \gamma$), cf. proofs in the appendix A.3. ΔW is therefore the difference between two positive quantities that behave exactly in the same way with respect to each ethical parameter. We can thus expect anything regarding the evolution of the preferred policy with respect to ethical parameters, as we will see in the results (section 3).

2.2.2 A two-period model

In order to get a better intuition on how the choice between policies i and j evolves according to β (weight on population) and η (inequality aversion) in the utilitarian case, we consider a simple two period-model. Here, we also examine the behaviour of the welfare difference with b , the marginal risk of extinction, defined as the additional risk of extinction per °C above the pre-industrial temperature⁵. Again, we consider two policies (i and j), each characterized by

³Provided that these generations are above the subsistence level \bar{c} , i.e. $u(EDE_t) > 0$

⁴This is the case if $u(EDE_t)$ is non decreasing, as α_{N_t} is increasing.

⁵A higher b therefore translates into a higher risk of extinction p (per annum) for a given level of temperature increase above the pre-industrial level.

a path of consumption c and a path of risk of extinction p . For simplicity, the paths of risk of extinction are here expressed directly in terms of P_t , the probability of extinction at t exactly⁶.

time step	policy i	policy j
$t = 1$	$c_{i,1}$	$c_{j,1}$
	$P_{i,1}$	$P_{j,1} = P_{i,1} - \Delta P$
$t = 2$	c_{i,t_2}	$c_{j,2}$
	$P_{i,2}$	$P_{j,2} = P_{i,2} + \Delta P$

In line with our specification, $\Delta P \geq 0$. We thus have:

$$\begin{aligned} \Delta_p W &= W(c_j, p_j) - W(c_j, p_i) \\ &= -\Delta P \cdot \alpha_{N_1} \cdot u(EDE_{j,1}) + \Delta P \cdot \alpha_{N_2} \cdot u(EDE_{j,2}) \end{aligned} \quad (6)$$

and

$$\begin{aligned} \Delta_c W &= W(c_i, p_i) - W(c_j, p_i) \\ &= P_{i,1} \cdot \alpha_{N_1} \cdot (u(EDE_{i,1}) - u(EDE_{j,1})) + P_{i,2} \cdot \alpha_{N_2} \cdot (u(EDE_{i,2}) - u(EDE_{j,2})) \end{aligned} \quad (7)$$

Table 1 summarises the behaviour of both terms of the welfare difference with b (marginal risk of extinction), β (weight on population) and η (inequality aversion). A higher mitigation effort translates into greater ΔP and smaller EDE .

- A higher b increases the reward of mitigation in terms of avoided probability of extinction (greater ΔP) but increases P_1 and reduces P_2 .
- A higher β increases the difference $\alpha_{N_2} \cdot u(EDE_{j,2}) - \alpha_{N_1} \cdot u(EDE_{j,1})$ of the first term. Also, $u(EDE_{i,1}) - u(EDE_{j,1})$ in the second term is then multiplied by a larger quantity.
- A higher η reduces the differences between EDE but also between the utility of EDE .

For each parameter, the terms of the difference evolve in the same direction. The resulting behavior of the difference is therefore not straightforward. The question is then which term dominates. There is a remarkable discrepancy between both terms: the main driver of $\Delta_p W$ is the difference of instantaneous welfare across time, whereas the main driver of $\Delta_c W$

⁶Note that $P_{i,1} + P_{i,2} = 1$ because P_t is the probability of extinction at time t exactly.

is the difference of instantaneous welfare across scenarios. If the difference between periods is larger than the difference between scenarios in absolute values (i.e if $u(EDE_{j,1}) - u(EDE_{j,2}) \geq u(EDE_{j,1}) - u(EDE_{i,1})$), $\Delta_p W$ may outweigh $\Delta_c W$ (although this also depends on the weight given to each sub-term in each case).

parameter	$\Delta_p W (\geq 0)$ $= \Delta P \cdot \alpha_{N_2} \cdot u(EDE_{j,2})$ $- \Delta P \cdot \alpha_{N_1} \cdot u(EDE_{j,1})$	Terms of the welfare difference $\Delta_c W (\geq 0)$ $= P_{i,1} \cdot \alpha_{N_1} \cdot (u(EDE_{i,1}) - u(EDE_{j,1}))$ $+ P_{i,2} \cdot \alpha_{N_2} \cdot (u(EDE_{i,2}) - u(EDE_{j,2}))$	ΔW $= \Delta_p W - \Delta_c W$
b	\nearrow (because $\Delta P \nearrow$)	if $u(EDE_{i,1}) - u(EDE_{j,1}) \geq u(EDE_{i,2}) - u(EDE_{j,2})$ (as $P_{i,1} \nearrow$ and $P_{i,2} \searrow$)	undetermined
β	\nearrow as $\alpha_N \nearrow$ and $u(EDE_{j,2}) \geq u(EDE_{j,1})$	\nearrow as $\alpha_N \nearrow$	undetermined
η	\searrow as $\partial_\eta u(EDE) \leq 0$ and $\partial_\eta^2 u(EDE) \leq 0$	\searrow as $\partial_\eta u(EDE) \leq 0$ and $\partial_\eta^2 u(EDE) \leq 0$	undetermined

Table 1: Behaviour of the contributions in social welfare with b , β and η

2.2.3 Contributions

In the numerical experiment, we propose a method to determine which of the stream of consumption c or the stream of risk of extinction p determines the sign of the welfare difference. This method will allow us to check the validity of our results and will prove particularly useful when we account for climate damages, as in this case one consumption path is not necessarily inferior or superior to the other over the whole period and we cannot use the previous analysis to understand the results. The difference in welfare $\Delta W = W(c_j, p_j) - W(c_i, p_i) = W_{j,j} - W_{i,i}$ can either be explained by a difference in consumption, a difference in risk, or both. If changing consumption for a given stream of extinction risk gives the same difference in welfare, the welfare variation can be attributed to consumption. The same logic holds if one changes the risk of extinction for a given stream of consumption. We examine the product of the variation of welfare $W_{j,j}$ compared to the variation of welfare when only the stream of consumption or the stream of risk of extinction changes ($W_{j,i}$ or $W_{i,j}$). Consumption and the risk of extinction change simultaneously in the model, so we vary consumption with a fixed stream of risk of extinction (p_j or p_i). Conversely, one can vary the risk of extinction with a fixed stream of consumption (c_j or c_i). The resulting variations of welfare may not be the same. If both variations have the same sign, one can attribute the initial welfare variation to consumption or to the risk of extinction. To explain $W_{j,j} - W_{i,i}$ by the variation of consumption, one can look

at either $W_{j,i} - W_{i,i}$ or $W_{j,j} - W_{i,j}$. Table 2 gives the possible cases as a function of the sign of the quantity at the top of each column⁷.

<i>product of welfare differences</i>				<i>diagnostic</i>
$(W_{j,j} - W_{i,i}) \cdot$ $(W_{j,j} - W_{i,j})$	$(W_{j,j} - W_{i,i}) \cdot$ $(W_{i,j} - W_{i,i})$	$(W_{j,j} - W_{i,i}) \cdot$ $(W_{j,i} - W_{i,i})$	$(W_{j,j} - W_{i,i}) \cdot$ $(W_{j,j} - W_{j,i})$	
+	+	+	+	Δc_t and Δp_t cause ΔW
+	+	+	-	Δc_t causes ΔW
+	-	+	+	Δc_t causes ΔW
+	-	+	-	Δc_t causes ΔW , Δp_t counteracts
+	+	-	+	Δp_t causes ΔW
-	+	+	+	Δp_t causes ΔW
-	+	-	+	Δp_t causes ΔW , Δc_t counteracts
+	-	-	+	inconclusive
-	+	+	-	inconclusive

Table 2: Which of the risk of extinction or consumption explains the difference in welfare?

2.3 The climate-economy model

The numerical exercise is performed using a climate-economy model. *Response* is a dynamic optimization model (Ambrosi et al., 2003), which belongs to the tradition of compact integrated assessment models such as DICE (Nordhaus, 1994), PAGE (Hope et al., 1993) or FUND (Tol, 1997). Response combines a simple representation of the economy and a climate module. The model can be used to determine the optimal climate objective by comparing mitigation costs and avoided climate damages. The economic module is a Ramsey-like growth model with capital accumulation and population growth. Population growth is considered to be exogenous and welfare is evaluated at the aggregate level in a given period in time. The model includes climate mitigation costs that account for the inertia of technical systems. The climate module describes the evolution of the global temperature and radiative forcing. The model includes a climate damage function. The inter-temporal social welfare is obtained by aggregating individual utilities over time. It assumes an infinite time horizon and a continuum of identical households who derive utility from the consumption of a composite good. Key parameters of the model include the pure time preference rate, the level of inertia of technical systems, technical progress⁸, climate sensitivity and the functional form of climate damages. A thorough description of the model and its equations can be found in (Dumas et al., 2012) and (Pottier et al., 2015).

⁷It is impossible to get two negative signs in columns 1 and 2 or in columns 3 and 4, leaving 9 possible cases from the original 16.

⁸In this paper, we assume technical progress decreases over time due to the very long time horizon considered. Indeed, assuming constant labour productivity growth would bring unrealistically high levels of consumption per capita at the time scales considered. This issue is usually not discussed in the literature due to the shorter time horizon used in models (typically 100 years).

We develop a new simulation version of *Response* in python. This new version of the model is used as a simulation tool (and not as an optimisation tool) to evaluate and order various climate policies. The model is used to compute consumption and the extinction risk for a given policy. The associated welfare is then computed according to the EPEDE criterion. The model is not used to find an optimal policy. We account for the risk of extinction due to climate change by assuming that the risk of extinction p depends on the temperature increase compared to pre-industrial levels, noted T . We assume a linear risk of extinction p as a function of temperature increase T , below. Note that this risk of extinction will be later referred to as p_t , to associate this probability to a given time period.

$$p(T) = p_0 + \frac{\bar{p} - p_0}{\bar{T} - T_0} \cdot (T - T_0); \quad b = \frac{\bar{p} - p_0}{\bar{T} - T_0} \quad (8)$$

$p(T)$	risk of extinction (per annum)
p_0	minimum exogenous risk of extinction, set at 1E-3 per annum
T	temperature increase compared to pre-industrial levels ($^{\circ}C$)
T_0	temperature increase above which the risk of extinction starts rising with temperature, set at $1^{\circ}C$
\bar{p}	maximum risk of extinction per annum (set at 1)
\bar{T}	temperature increase at which \bar{p} is reached ($^{\circ}C$)
b	marginal risk of extinction (i.e. additonal risk of extinction per $^{\circ}C$ above T_0)

2.4 The numerical experiment

2.4.1 Parameter ranges

We consider the special case of utilitarianism, which is obtained from the welfare function described above by assuming $u = \phi$, i.e. risk aversion is equal to the inequality aversion of the social planner. In equation (3), this translates into $\eta = \gamma$. We choose $0 \leq \beta \leq 1$, hence the sequence α_{N_t} is increasing in N_t , which means there is a preference for large populations.

- For $\beta = 0$ (i.e. $\alpha_{N_t} = 1$), it is indifferent to add new individuals at the *EDE* level. If we assume $u = \phi$, this is average utilitarianism.
- For $\beta = 1$ (i.e. $\alpha_{N_t} = N_t$), and $u = \phi$, it is indifferent to add new individuals at the critical level \bar{c} . If we assume $u = \phi$, this is total utilitarianism.

Table 3 shows a summary of the parameter values considered.

<i>parameter</i>	<i>description</i>	<i>value</i>
η	inequality aversion parameter (= risk aversion γ)	from 0.5 to 5.0
β	population parameter	from 0 to 1
b	additional probability of extinction over per $^{\circ}C$ over T_0	0.0001 to 100% per annum

Table 3: Value ranges of scenario parameters

2.4.2 Climate policies

The model is used as a simulation tool to evaluate and order various climate policies: business-as-usual (*baa*), *550 ppm*, *450 ppm*. The policies considered here are defined in terms of emission reduction over time compared to a baseline. The social welfare criteria are used to evaluate pairs of emission reduction paths over time: the results show which policy is preferred according to a given social welfare function, i.e. for given values of the risk and inequality aversion parameters (γ and η , which are set to be equal), given values of the population parameter (β) and various forms of the probability of extinction as a function of temperature increase. Figure 1 illustrates the policies considered.

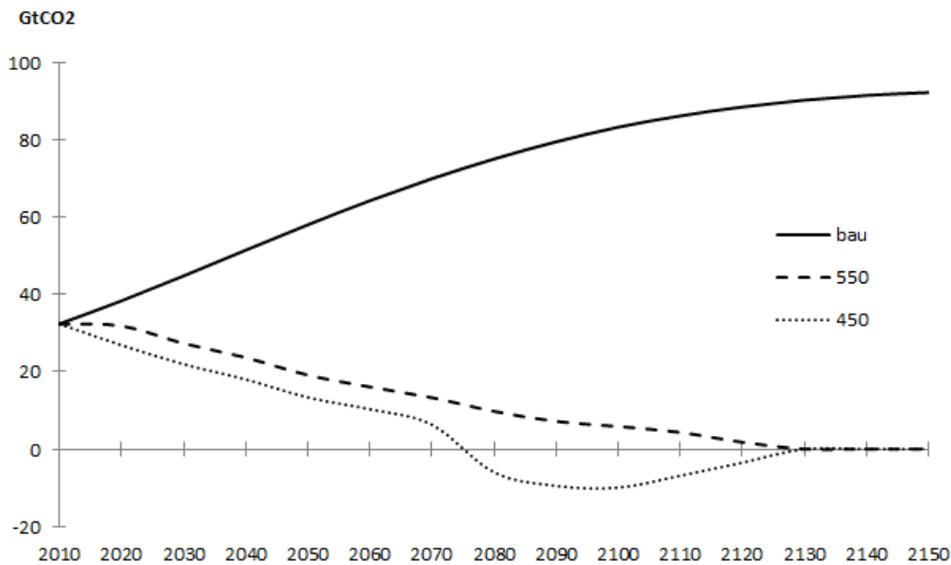


Figure 1: Emissions over time: business-as-usual (*baa*), *550 ppm*, *450 ppm* ($GtCO_2$)

2.4.3 The issue of the saving rate

In an optimization framework, integrated assessment models are used to determine the optimal policy in terms of greenhouse gas abatement and savings over time. Abatement and savings are the two decision variables of the model, and are both usually allowed to vary over time. Here, we use the integrated assessment model as a tool to order policies according to their performance given a social welfare function. The policies are thus defined exogenously in terms of both abatement and savings. The difficulty here is that, in principle, each parametrization of the social welfare function admits a different path for optimal savings over time. Two approaches can be adopted here. The first one is to adopt the socially optimal savings rate for each scenario that is examined with a given social welfare function. This means that it may be difficult to compare scenarios along the values of the ethical parameters that define the welfare function, as the underlying saving rate differs among scenarios. The second approach is to assume a certain path of savings over time, independently of the social welfare function considered. This way, scenarios with different ethical parameters are more readily comparable, as they assume the same exogenous evolution of the saving rate. However, the economy is necessarily in a state of under or over-investment (compared to the optimal accumulation path) in some scenarios, as the same path of savings is set for all social welfare functions. Therefore a new difficulty arises here, as climate policy (i.e. abatement) may be used by the model as a tool to correct for over-investment. This may lead to overestimate the value of optimal abatement, as abatement may then be used by the model to transfer wealth from present to future generations in place of savings. Here we choose to impose the same evolution of the saving rate in all scenarios, set at 15%.

2.5 Summary of the possible effects at play

Several effects may play a role in determining which policy is preferred in the model.

- The first effect is the intertemporal consumption trade-off. As future generations are assumed to be richer (as the model uses an exogenous rate of technical change), a high inequality aversion can give preference to present consumption. This could thus lead to favour no abatement in order to preserve the consumption of the present, poorer generation.
- The second effect is the trade-off between consumption today and the probability of future

generations existing. If the risk of extinction depends on the temperature increase compared to pre-industrial levels, climate policy can delay extinction due to climate change. In that case, short-term abatement could be favoured, which would then translate into lower consumption of the present generation, as abatement comes at a cost. Note that the risk on population size is implemented as 'all or nothing', i.e. there is no gradual decrease of population due to climate change. The risk is therefore on cumulative population. Also note that the model accounts for climate damages, but these are certain, i.e. they are not probabilistic. Therefore, strictly speaking, there is no risk on consumption in the model.

- The risk of extinction discounts future welfare and thus has an impact on the intertemporal consumption trade-off. Indeed, the contribution of the welfare of future generations can become negligible with a high probability of extinction (whether the probability of extinction is purely exogenous or depends on temperature).

3 Results

3.1 The role of the risk of extinction

3.1.1 Consumption vs. risk of extinction

We examine the preferred policy option (*bau* in black, *550 ppm* in grey) as a function of the probability of extinction for a given degree of inequality aversion ($\eta = 2.0$) and for a given value of the weight on population size ($\alpha_{N_t} = 1$, i.e. $\beta = 0.0$: the case of average utilitarianism). In a first approach, we do not account for climate damages, in order to primarily focus on the competition between consumption and the risk of extinction. In doing so, we do not consider the intertemporal consumption trade-off determined by the balance between abatement costs and climate damages. Emission reductions reduce both the consumption and the risk of extinction streams over the whole period⁹. The most ambitious policy will therefore be favoured when comparing the risk of extinction over time, while the least ambitious policy will be favoured when comparing consumption over time.

The results presented in table 4 show that very high and very low marginal risks of extinction (*b*) lead to favour the *bau* scenario (i.e. no abatement)¹⁰. If future generations are unlikely

⁹The stream of consumption is lower over the whole period due to the fact that we do not account for climate damages. This would most likely not be the case if climate damages were accounted for.

¹⁰Note that *b* is the slope of the linear function describing the risk of extinction as a function of temperature

to exist, it is not worth abating emissions today. The result that a rational social planner may voluntarily choose not to abate emissions in a case where it appears too difficult to avoid catastrophic climate damages has been shown in (Perrissin-Fabert et al., 2014). A high marginal risk of extinction (b) at low temperatures favours the *bau* scenario, as the state of the climate is then considered hopeless, and one might as well favour present consumption if future generations are unlikely to exist. As the marginal risk decreases, there is a chance that climate action may avoid extinction, and the *550 ppm* scenario is favoured over *bau*. Further tests show that for a given set of ethical parameters (η and β), the preferred policy is driven by the relative order of magnitude of the risk of extinction (p_0 , here set at 10^{-3} , following Stern), and of the marginal risk of extinction (b). Except in the previously described case of a doomed situation, the *bau* scenario is preferred only if b is inferior to or of the same order of magnitude as p_0 . In those cases, the difference of welfare between policies is very small, but still consistently in favour of the *bau* scenario. In the extreme case where $b = 0.0\%$ (i.e. the case of a purely exogenous risk of extinction), the *bau* scenario is preferred. Setting an exogenous risk of extinction is equivalent to introducing pure time discounting in the model, which favours present consumption. In that case, a policy that enhances present consumption is favoured as a way to improve the bad states of nature: if extinction occurs independently of climate policy, one way to improve the value of the social objective is to maximise the stream of consumption of early periods when extinction has not occurred yet. The way to do so is to maximise present consumption, hence delay climate policy, to improve the states of nature where extinction happens early.

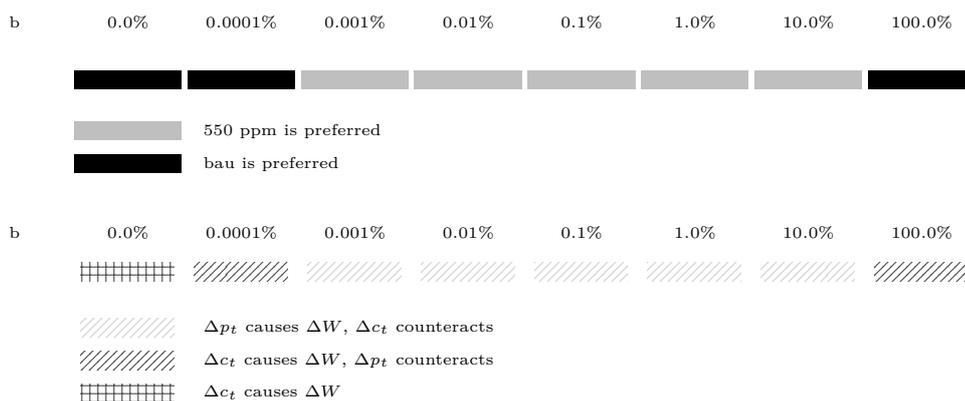


Table 4: Preferred policy and contributions as a function of the marginal risk of extinction (b) for $\beta = 0.0$ and $\eta = 2.0$

increase T . It can be described as the additional probability of extinction over p_0 per degree above $T_0 = 1^\circ C$. We refer to it as the marginal risk of extinction. A high b therefore translates a higher probability of extinction at a given temperature increase T .

The table of contributions is produced using the method presented in section 2.2.3. The results show that the difference in the risk of extinction always explains the preference for the *550 ppm* scenario, while the difference in consumption counteracts (grey hatched cells). Conversely, the difference in consumption always explains the preference for the *baa* scenario while the difference in the risk of extinction counteracts (black hatched cells), except when the risk of extinction is purely exogenous (i.e. for a marginal risk $b = 0.0\%$), in which case probability streams are identical between the *550 ppm* scenario and the *baa* scenario. This means that when the *550 ppm* scenario is preferred to the *baa*, this is due to its effect on reducing the risk of extinction, while when the *baa* scenario is preferred to the *550 ppm* scenario, this is due to its effect on consumption streams.

3.1.2 Welfare and the role of the time horizon

We find that the choice of the time horizon of the model is crucial to correctly interpret the results. Indeed, as the risk of extinction p depends on the emissions path, the time horizon should be chosen so that the probability of survival at the end of the period is close to zero for all the emission paths considered, in order to ensure that long term welfare is not overlooked when calculating the aggregated welfare (used to determine which policy should be preferred). If the time horizon is too short, i.e. if the probability of survival at the end of the time horizon is still significant, the long term benefits of a given policy are cut out of the assessment, which for instance may lead to wrongly conclude that a *baa* scenario should be preferred to a *550 ppm* scenario. With an exogenous risk of extinction set at 10^{-3} per annum, the minimum time horizon is 10,000 years when the most aggressive climate policy considered is a *550 ppm* scenario. Examining the difference in cumulative welfare over time between two policies gives an idea of the appropriate time horizon: it is the time horizon at which the difference saturates, i.e. no significant share of welfare is left uncounted by truncating the horizon. In order to minimize the error that may derive from using a finite horizon, we add a final term to the sums, which is the welfare at the time horizon multiplied by the remaining probability of survival (this probability should be very small if the time horizon was already chosen appropriately).

Figure 2 shows the evolution of the difference in cumulative welfare (or more precisely the way it is calculated in the model) between the *550 ppm* scenario and the *baa* scenario. The total welfare difference used to order both policies is the last point on the curve (at period 1000 on the graph). We can see that for this particular set of parameters, the chosen time horizon is sufficient to

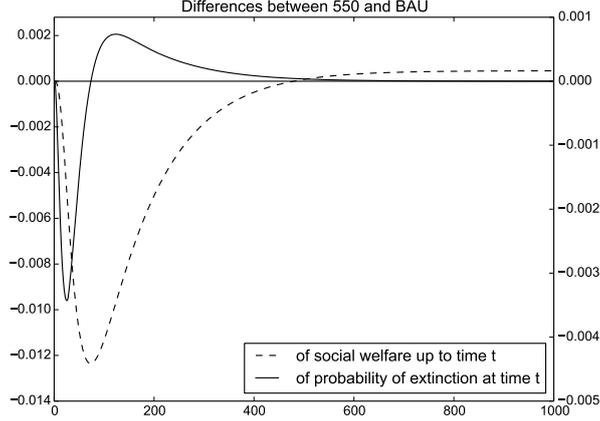


Figure 2: Differences in welfare and probability of extinction (P_t) over time between *550 ppm* and *bau* ($\beta = 0.0$, $\eta = 2.0$, $b = 0.01\%$)

compare both policies, as the difference in welfare saturates (and is positive) long before the time horizon. The welfare is calculated by adding the successive contributions of various generations to welfare. Indeed, as the welfare is a function of the equally distributed equivalent level of consumption over different generations, one can only calculate each component after extinction has been reached (with a given probability), i.e. when no more generations need to be accounted for. This is due to the fact that the equally distributed equivalent level of consumption at time t (EDE_t) depends on the consumption of all previous generations (cf. equation 1), so that consumption at time t affects all the EDE after time t . It means that the quantity shown on figure 2 is the instantaneous difference of welfare between policies, as a reserve of welfare is accounted for only after extinction. At the time horizon, the difference in welfare between the *550 ppm* scenario and the *bau* is positive, i.e. the *550 ppm* scenario should be preferred for ($\beta = 0.0$, $\eta = 2.0$, $b = 0.01\%$), which is consistent with the results shown on table 4. Figure 2 also shows the evolution of the difference of the probability of extinction at time t exactly, P_t , between the *550 ppm* scenario and the *bau* scenario. P_t is calculated as the probability of survival up to time $(t - 1)$ multiplied by the risk of extinction at time t (p_t). It is used in the calculation of the expected welfare according to equation 2. A negative value means that the probability of living exactly t periods is higher in the *bau* scenario. The evolution of this difference over time (first negative, then positive) stems from a pure composition effect of probabilities over time, as the sum of P_t over the whole time horizon must be equal to 1 for each scenario. The positive difference in probabilities after period 150 (i.e. after year 1500) simply reflects natural extinction (as modelled by the exogenous rate of extinction), and not extinction due to climate change. It reflects the fact that the probability of survival to that date has been

higher in the *550 ppm* scenario. However, note that it does not mean that the risk of extinction in the *550 ppm* would be higher than in a *baa* scenario. In fact, the risk of extinction is always lower in the *550 ppm* scenario. The difference in welfare starts increasing with a lag compared to the difference in the probability of extinction.

3.1.3 The role of damages

So far, the analysis has not accounted for the intertemporal consumption trade-off between abatement costs and climate damages. We now test the sensitivity of the results to the inclusion of climate damages in the model. Climate damages occur due to temperature increase, and are subtracted from production, thus reducing consumption. They can be mitigated thanks to abatement, which comes at a cost. The results show that the preferred policies remain largely unchanged whether or not climate damages are accounted for, except for low marginal risk b (with the exception of $b = 0.0\%$, i.e. for a purely exogenous risk of extinction). This result means that the risk of extinction is the main driver of the policy choice over the effect of climate damages on consumption. As expected, accounting for climate damages leads to prefer the *550 ppm* scenario over the *baa* in more cases, as emission reductions not only reduce the risk of extinction p but here also reduce climate damages (cf. table 5).

The table of contributions of consumption and risk in determining the preferred policy shows that the *550 ppm* scenario is preferred due to both the risk and the consumption effects for low values of b (in grid cells, and again with the exception of $b = 0.0\%$). This contrasts with the case without climate damages, where the *550 ppm* scenario was preferred due to the risk effect alone, while the consumption effect played in favour of the *baa*. Climate damages reduce the consumption of future generations more than that of present generations. The effect on consumption is indeed expected to favour the *550 ppm* scenario for low values of b , as the reduced consumption of future generations due to climate damages impacts total welfare in a significant way only if future generations are likely to exist (i.e. for relatively low values of b).

The results also show that when damages are accounted for and when there is no risk of extinction ($b = 0.0\%$), the *baa* scenario is preferred to the *550 ppm* scenario. This results differs from Stern, and is due to the choice $\beta = 0.0$ (average utilitarianism), which gives a relatively low weight to the population size. The following section will further explore the role of β .

It is interesting to note that, with damages, Δc_t no longer causes ΔW when b increases from

3.2 The role of ethical parameters

We examine the role of ethical parameters in determining the preferred policy in a case without climate damages. We consider in turn the role of the weight on population size (through β), and the role of the aversion towards inequality (η).

3.2.1 The role of population ethics

We examine the preferred policy option (*bau* in black, *550 ppm* in grey) as a function of the marginal risk of extinction b for various weights on population size (various α_{N_t} , i.e. various β) and for a given value of inequality aversion ($\eta = 2.0$), see table 6. For a given marginal risk b , a higher weight on population size favours the most aggressive climate policy (here the *550 ppm* scenario). The table of contributions shows that the difference in probability of extinction always explains the preference for the most aggressive scenario (here *550 ppm*), while the difference in consumption counteracts (grey hatched cells). Conversely, the difference in consumption always explains the preference for the least aggressive scenario (here *bau*) while the difference in the risk of extinction counteracts (black hatched cells), except when the risk of extinction is purely exogenous (i.e. for a marginal risk of extinction $b = 0.0\%$). This means that when the most aggressive climate policy scenario is preferred, this is due to its effect on reducing the risk of extinction, while when the least aggressive climate scenario is preferred, this is due to its effect on consumption streams.

We examine the preferred policy option between a *550 ppm* scenario (in grey) and a *450 ppm* scenario (in light grey) as a function of the same parameters, see table 7. Again, for a given b , a higher weight on population size favours the most aggressive climate policy (here the *450 ppm* scenario). Again, when the most aggressive climate policy scenario is preferred, this is due to its effect on reducing the risk of extinction, while when the least aggressive climate scenario is preferred, this is due to its effect on consumption streams.

3.2.2 The role of inequality aversion

We examine the preferred policy option (*bau* in black, *550 ppm* in grey) as a function of the marginal risk of extinction b for various degrees of inequality aversion (η) and for a given value of the weight on population size ($\alpha_{N_t} = 1$, i.e. $\beta = 0.0$: the case of average utilitarianism). Again, we examine this issue without accounting for climate damages, in order to primarily focus on

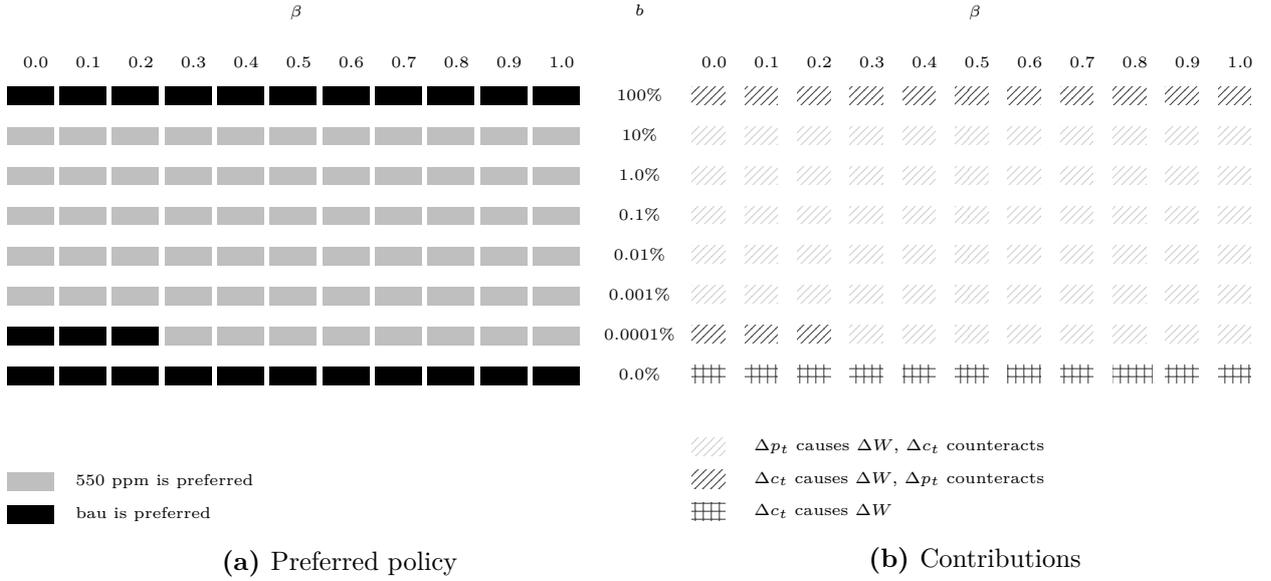


Table 6: Preferred policy and contributions as a function of β and the marginal risk of extinction (b) (for $\eta = 2.0$)

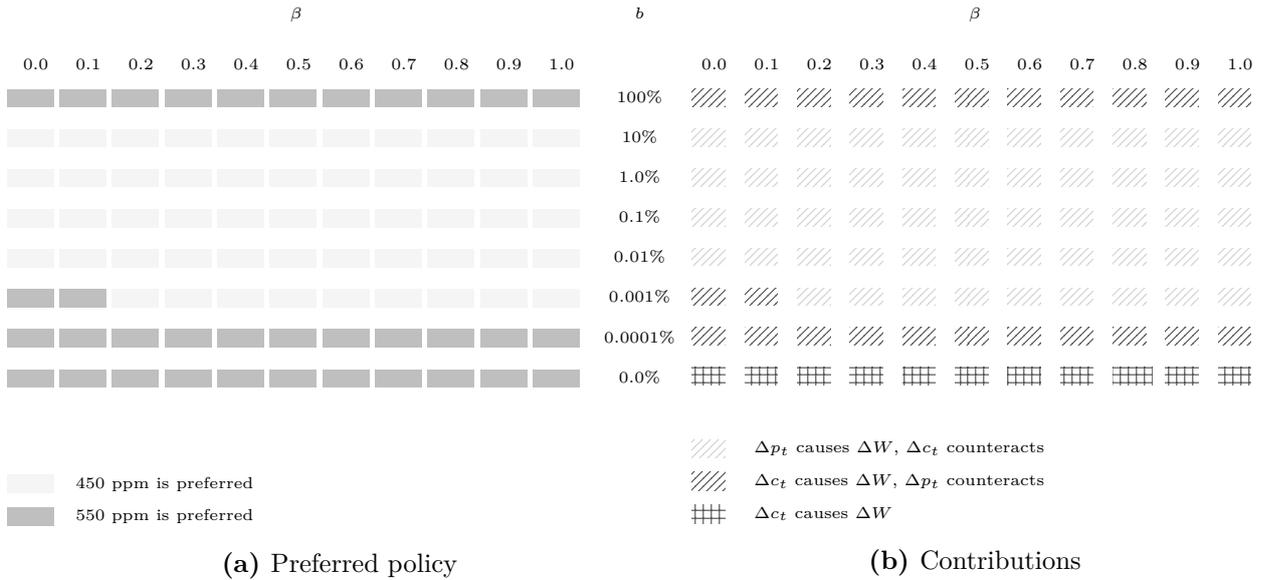


Table 7: Preferred policy and contributions as a function of β and the marginal risk of extinction (b) (for $\eta = 2.0$)

the competition between consumption and the risk of extinction. We therefore do not consider the intertemporal consumption trade-off determined by the balance between abatement costs and climate damages. In the case $\beta = 0.0$, at a given b , a higher inequality aversion favours the least aggressive policy (here the *bau* scenario), although the results are less straightforward for higher values of β (see for instance table 10 for $\beta = 0.01$ in section 3.2.3).

Also, a very low marginal risk of extinction favours the *bau* scenario for high levels of inequality aversion (a standard result in the literature). As the marginal risk of extinction decreases, the minimum level of inequality aversion that justifies a *bau* scenario is reduced: richer generations are added, which enhances inequalities between generations. At intermediate values of the marginal risk of extinction, there is a chance that climate action may avoid extinction, and the *550 ppm* scenario is favoured over *bau* for all values of inequality aversion. In the extreme case where $b = 0.0\%$ (i.e. the case of a purely exogenous risk of extinction), the *bau* scenario is preferred for all values of inequality aversion.

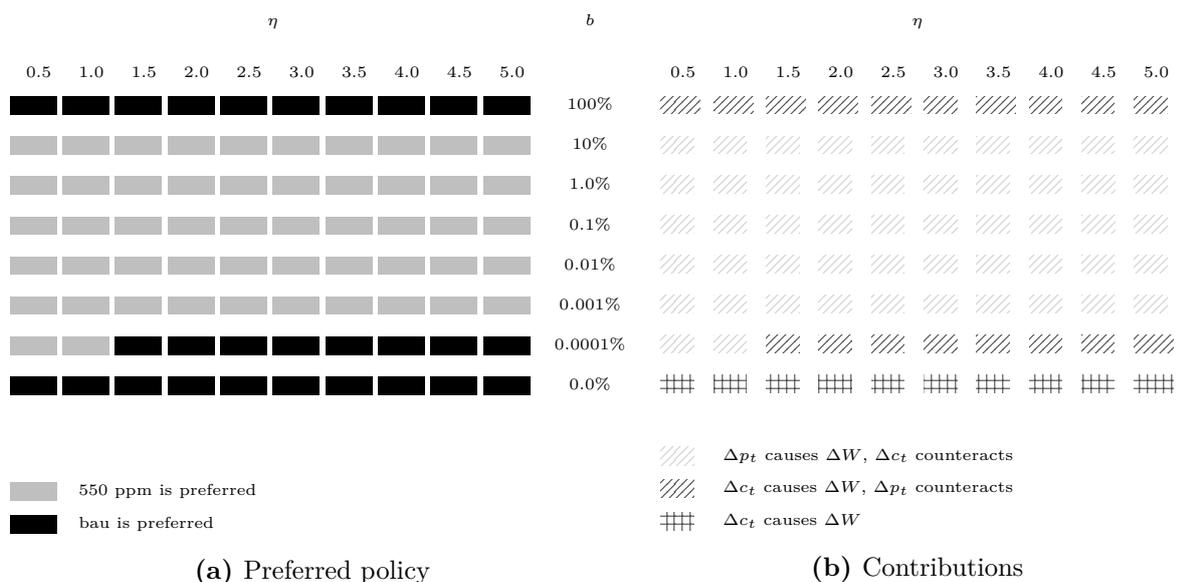


Table 8: Preferred policy and contributions as a function of inequality aversion (η) and the marginal risk of extinction (b) (for $\beta = 0.0$)

The comparison between *550 ppm* and *450 ppm* scenarios gives similar results as the comparison between *bau* and *550 ppm* scenarios. In the case $\beta = 0.0$, at a given b , a higher inequality aversion favours the least aggressive policy (here the *550 ppm* scenario), although, again, the results are less straightforward for higher values of β .

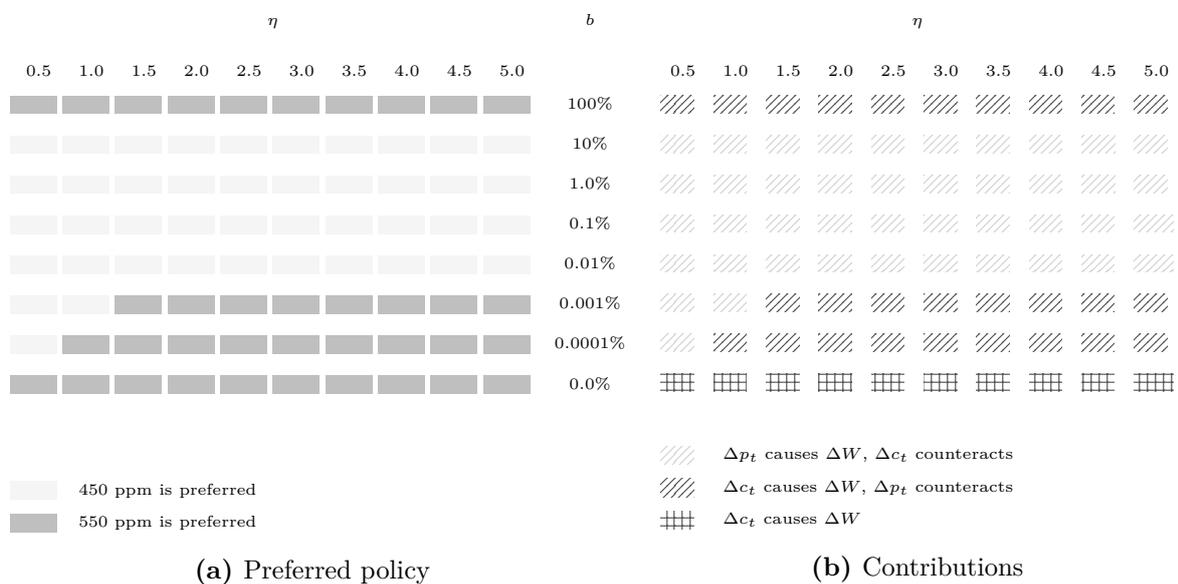


Table 9: Preferred policy and contributions as a function of inequality aversion (η) and the marginal risk of extinction (b) (for $\beta = 0.0$)

3.2.3 Evolution of the welfare difference with η

In order to understand the results, we examine the evolution of the terms of the welfare difference with η . Figure 3 shows the behaviour of $\Delta_c W$ and $\Delta_p W$ with η in the case ($\beta = 0.0$, $b = 0.0001\%$). This figure should be compared to the second to last row of table 8. The figure clearly shows that both terms decrease with η (as indicated in table 1), but at a different pace. The crossing of the green and red curves coincides with the value of η for which the preferred policy switches from *550 ppm* to *bau*. In this case, the first term decreases faster than the second term over the whole range of η considered. Keeping our two-period model in mind (cf. section 2.2.2), this suggests that the result is driven by the difference of instantaneous welfare across time, not by the difference of instantaneous welfare across scenarios. Figure 4 illustrates how both curves come closer and finally cross as the marginal risk of extinction decreases, i.e. as the risk of extinction is set to be less sensitive to the temperature increase.

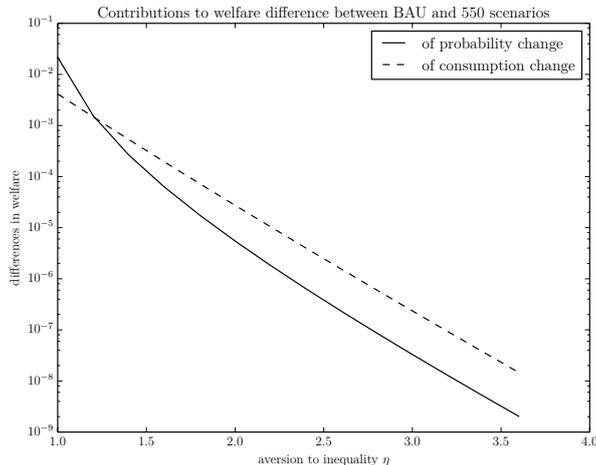
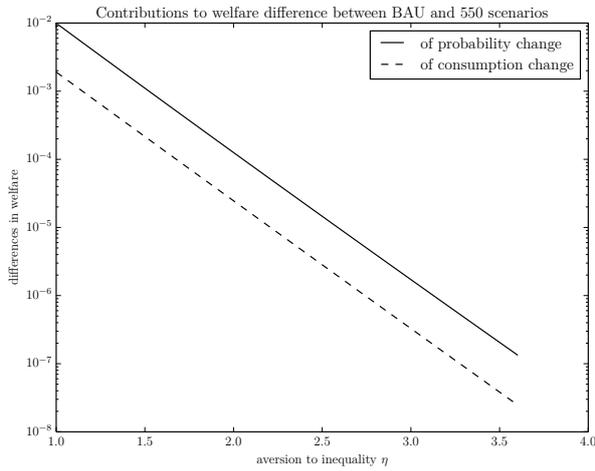
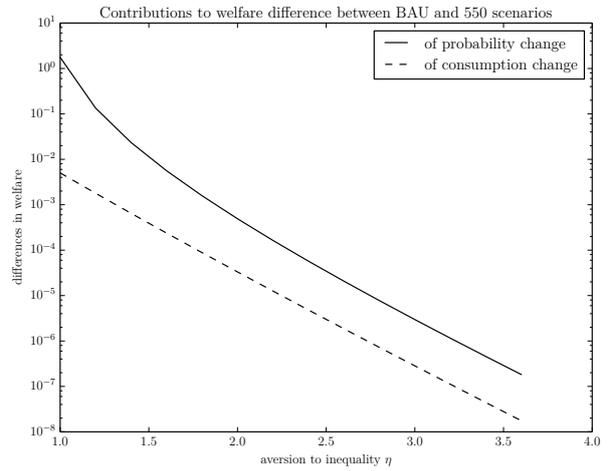


Figure 3: Evolution of both terms of the welfare difference between *550 ppm* and *bau* as a function of η ($\beta = 0.0$, $b = 0.0001\%$)

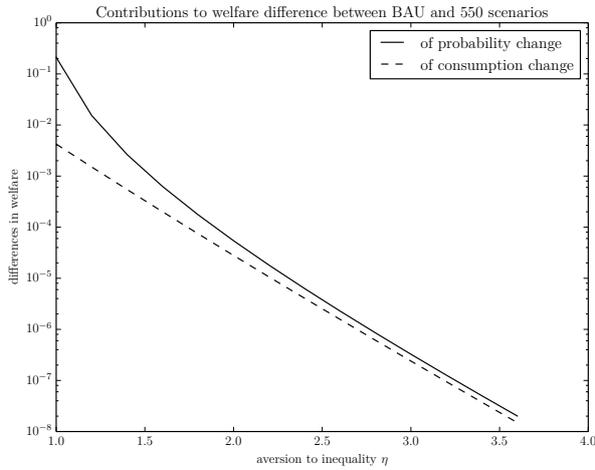
Further results demonstrate that the relative pace of evolution of both terms can change with η , for instance in the case $\beta = 0.01$ presented below (table 10 and figure 5). In that case, the terms of the welfare difference cross twice, which coincides with the result that the preferred policy switches twice, first from *550 ppm* to *bau* at low η , then from *bau* to *550 ppm* at high η (for $b = 0.0001\%$). These results are consistent with the analysis presented in section 2.2.1 which showed that we could expect anything regarding the evolution of the preferred policy with respect to ethical parameters, as the welfare difference ΔW is a difference between two positive quantities that behave exactly in the same way with respect to each ethical parameter.



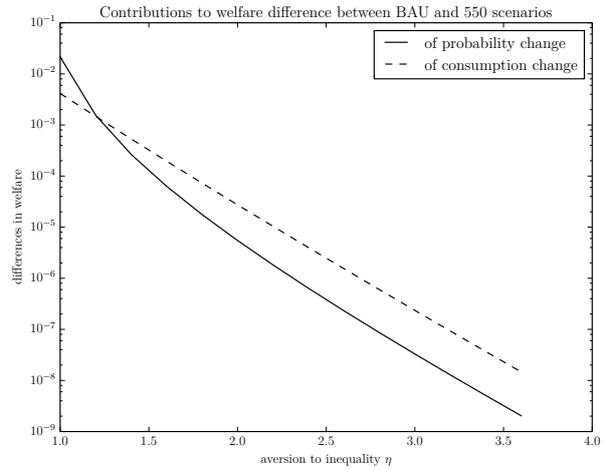
(a) $b = 10.0\%$



(b) $b = 0.01\%$



(c) $b = 0.001\%$



(d) $b = 0.0001\%$

Figure 4: The evolution of welfare components with η for various values of the marginal risk of extinction (b), for $\beta = 0.0$

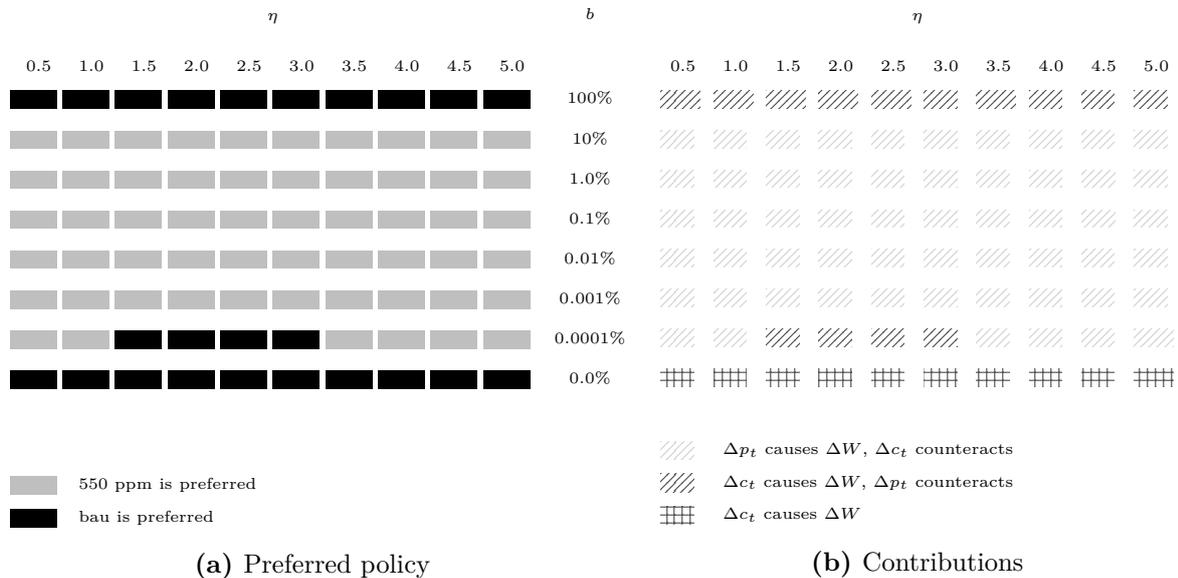


Table 10: Preferred policy and contributions as a function of inequality aversion (η) and the marginal risk of extinction (b) (for $\beta = 0.01$)

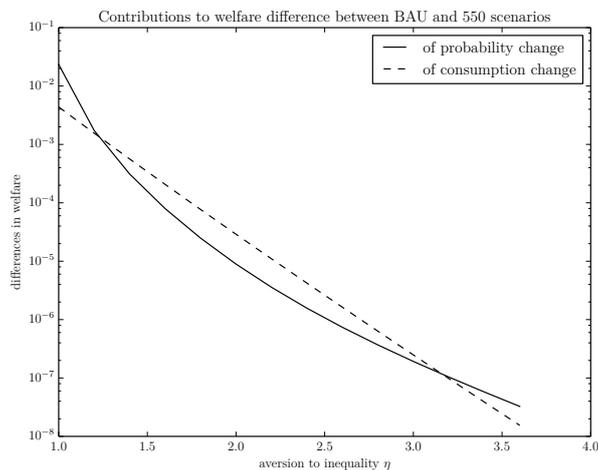


Figure 5: Evolution of both terms of the welfare difference between *550 ppm* and *bau* as a function of η ($\beta = 0.01$, $b = 0.0001\%$)

3.2.4 The role of damages

The table of contributions of consumption and risk in determining the preferred policy (table 11) shows that the *550 ppm* scenario is preferred due to both the differences in risk and the consumption streams for low values of inequality aversion ($\eta \leq 2.0$). This contrasts with the case without climate damages, where the *550 ppm* scenario was preferred due to the difference in risk of extinction alone, while the consumption effect played in favour of the *baa*. The difference in consumption streams is indeed expected to favour the *550 ppm* scenario for low inequality aversion values, and low inequality aversion values only, as climate damages reduce the consumption of future generations more than that of present ones. As future generations were originally richer than present ones due to technical change, this effect only occurs for low inequality aversion.

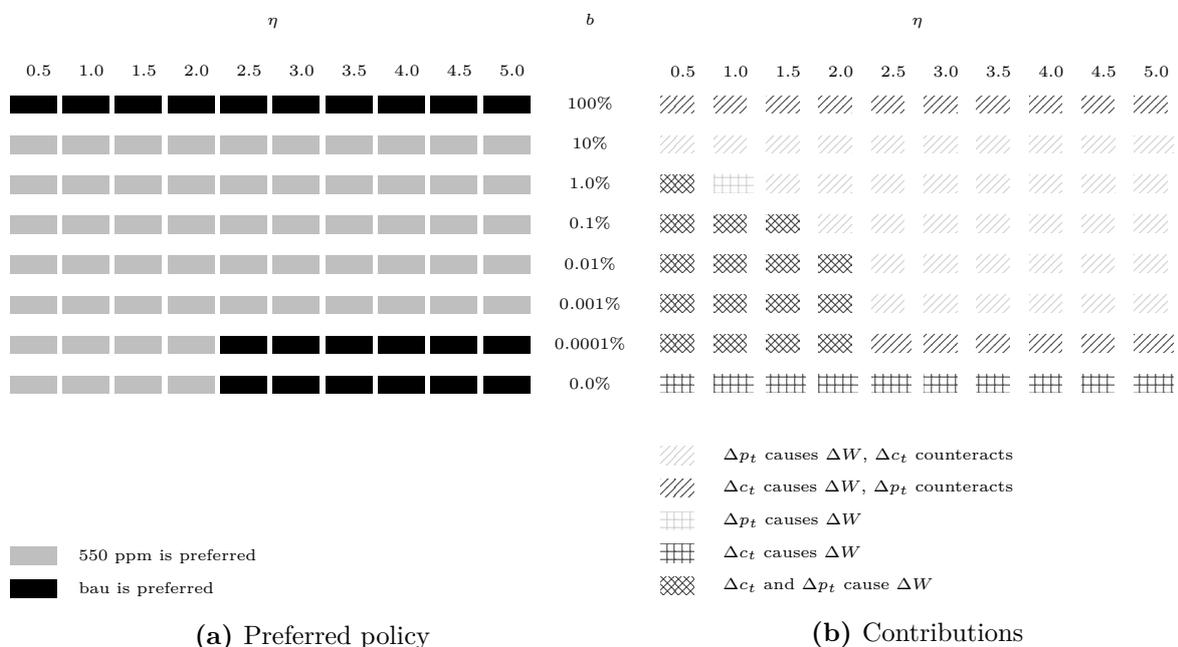


Table 11: Preferred policy and contributions as a function of inequality aversion (η) and the marginal risk of extinction (b), with damages (for $\beta = 0.0$)

We examine the preferred policy option between a *550 ppm* scenario (in grey) and a *450 ppm* scenario (in light grey) as a function of the same parameters, see table 12. This time we consider the case of total utilitarianism (i.e. $\beta = 1.0$), which is the case most commonly studied in the literature. We find that when climate damages are accounted for, the *550 ppm* scenario is preferred (except in the doomed situation for low η), even when the risk of extinction is purely exogenous. This last result is consistent with the results of Stern.

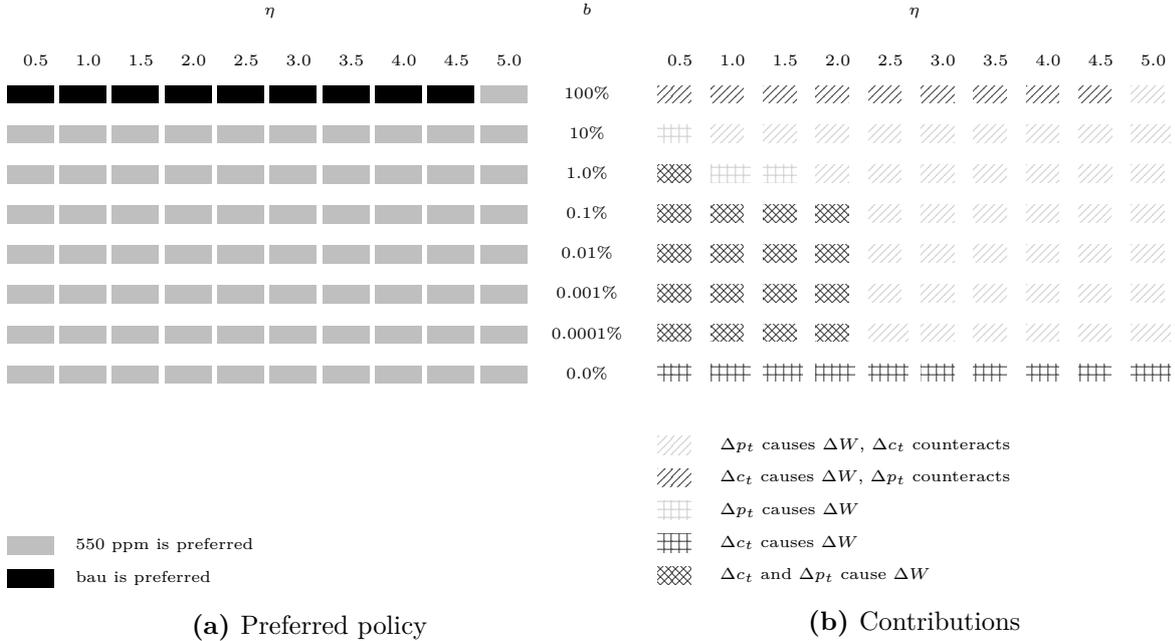


Table 12: Preferred policy and contributions as a function of inequality aversion (η) and the marginal risk of extinction (b), with damages (for $\beta = 1.0$)

4 Conclusion

With a probability of extinction that depends on temperature increase compared to pre-industrial levels, two effects are competing. On the one hand, future generations are assumed to be richer, and a high inequality aversion thus gives preference to present consumption. This plays in favour of the least aggressive climate policy as a way to preserve the consumption of the present, poorer generation. On the other hand, emission reductions can prevent extinction, which can favour aggressive climate policies. The main results follow:

- The risk of extinction is the main driver of the preferred policy over climate damages, as the preferred scenario is largely unchanged whether or not climate damages are accounted

for.

- If the risk of extinction is very sensitive to the temperature increase (high marginal risk of extinction b), the least aggressive climate scenario is preferred: if the state of the climate is considered hopeless (doomed state), one might as well favour present consumption if future generations are unlikely to exist.
- Except in the doomed case, the preferred policy is driven by the relative order of magnitude of the exogenous risk of extinction and of the marginal risk of extinction. In our specification, the most aggressive climate scenario is preferred for intermediate probabilities of extinction (where $b > p_0$), as there is then a chance that climate action may avoid early extinction.
- A higher weight on population size (i.e. high values of β) favours the most aggressive climate scenario, as a higher weight is then given to the welfare of future generations. Reducing the extinction risk for future generations has thus a higher impact on welfare.
- If the risk of extinction is relatively not sensitive to the temperature increase (i.e. for a low marginal risk of extinction b), a higher inequality aversion can favour the most aggressive policy for $\beta \geq 0$. This result contrasts with the standard result obtained when no risk of extinction is accounted for.

Acknowledgements

This research has been supported by the Chair on Welfare Economics and Social Justice at the Institute for Global Studies (FMSH - Paris), the Franco-Swedish Program on Economics and Philosophy (FMSH - Risksbankens Jubileumsfond), the ANR project EquiRisk, Grant # ANR-12-INEG-0006-01, and Princeton University's Program in Science, Technology and Environmental Policy (STEP).

References

- AMBROSI, P., HOURCADE, J.-C., HALLEGATTE, S., LECOCQ, F., DUMAS, P., AND DUONG, M. H. 2003. Optimal Control Models and Elicitation of Attitudes towards Climate Damages. Environmental Modeling & Assessment 8:133–147.
- ATKINSON, A. B. 1970. On the measurement of inequality. Journal of Economic Theory 2:244–263.
- DUMAS, P., ESPAGNE, E., PERRISSIN-FABERT, B., AND POTTIER, A. 2012. Comprehensive Description of RESPONSE. CIRED Working Paper 41-2012, CIRED.
- FLEURBAEY, M. AND ZUBER, S. 2015. Discounting, beyond Utilitarianism. Economics: The Open-Access, Open-Assessment E-Journal 9:1–52.
- HOPE, C., ANDERSON, J., AND WENMAN, P. 1993. Policy analysis of the greenhouse effect - An application of the PAGE model. Energy Policy 21:327–338.
- IPCC 2014. Climate Change 2014: Synthesis Report. Summary for Policymakers. Technical report.
- KOLM, S.-C. 1969. The optimal production of social justice. Public Economics pp. 145–200.
- NORDHAUS, W. D. 1994. Managing the global commons: the economics of climate change. MIT Press, Cambridge, Mass.
- PERRISSIN-FABERT, B., POTTIER, A., ESPAGNE, E., DUMAS, P., AND NADAUD, F. 2014. Why are climate policies of the present decade so crucial for keeping the 2 °C target credible? Climatic Change 126:337–349.

POTTIER, A., ESPAGNE, E., FABERT, B. P., AND DUMAS, P. 2015. The Comparative Impact of Integrated Assessment Models' Structures on Optimal Mitigation Policies. Environmental Modeling & Assessment 20:453–473.

TOL, R. S. J. 1997. On the optimal control of carbon dioxide emissions: an application of FUND. Environmental Modeling & Assessment 2:151–163.

WEITZMAN, M. L. 2009. On Modeling and Interpreting the Economics of Catastrophic Climate Change. Review of Economics and Statistics 91:1–19.

Appendices

A Proofs

A.1 Variation of EDE with respect to γ

Given a consumption sequence (c_t) , a population sequence (n_t) , the equally distributed equivalent level of consumption EDE is defined by:

$$\phi(EDE_T) = \sum_{t=0}^T \frac{n_t}{N_T} \phi(c_t) \quad (\text{A-1})$$

with $N_T = \sum_{t=0}^T n_t$ (so that $\sum_{t=0}^T \frac{n_t}{N_T} = 1$) and $\phi(x) = x^{1-\gamma}$. Differentiating (A-1) with respect to γ , we obtain:

$$\partial_\gamma \phi(EDE_T) + \phi'(EDE_T) \partial_\gamma EDE_T = \sum_{t=0}^T \frac{n_t}{N_T} \partial_\gamma \phi(c_t) \quad (\text{A-2})$$

Which means that:

$$\phi'(EDE_T) \partial_\gamma EDE_T = \left(\sum_{t=0}^T \frac{n_t}{N_T} \partial_\gamma \phi(c_t) \right) - \partial_\gamma \phi(EDE_T) \quad (\text{A-3})$$

Because $\phi' > 0$, the sign of $\partial_\gamma EDE_T$ is equal to the sign of the right hand-side term. The right hand-side term can be rewritten as:

$$\left(\sum_{t=0}^T \frac{n_t}{N_T} (\partial_\gamma \phi) \circ \phi^{-1}(\phi(c_t)) \right) - (\partial_\gamma \phi) \circ \phi^{-1}(\phi(EDE_T))$$

If $(\partial_\gamma \phi) \circ \phi^{-1}$ is strictly convex, the right hand-side is positive. Indeed:

$$\sum_{t=0}^T \frac{n_t}{N_T} (\partial_\gamma \phi) \circ \phi^{-1}(\phi(c_t)) > (\partial_\gamma \phi) \circ \phi^{-1} \left(\sum_{t=0}^T \frac{n_t}{N_T} \phi(c_t) \right) = (\partial_\gamma \phi) \circ \phi^{-1}(\phi(EDE_T))$$

Now $\partial_\gamma \phi(x) = -\log(x)x^{1-\gamma}$ and $\phi^{-1}(x) = x^{\frac{1}{1-\gamma}}$, so that $(\partial_\gamma \phi) \circ \phi^{-1}(x) = \frac{1}{\gamma-1} \log(x)x$. This is convex when $\gamma > 1$ and concave when $\gamma < 1$. We therefore have the following result:

$$\partial_\gamma EDE_T \begin{cases} > 0, & \text{when } \gamma > 1 \\ < 0, & \text{when } \gamma < 1 \end{cases} \quad (\text{A-4})$$

A.2 Variation of u with respect to η

The utility is defined as $u(c) = G(c) - G(\bar{c})$ where $G(x) = \frac{x^{1-\eta}}{1-\eta}$. We are interested in the sign of $\partial_\eta u$. We have:

$$\partial_\eta u(c) = \partial_\eta G(c) - \partial_\eta G(\bar{c}) \quad (\text{A-5})$$

The sign of $\partial_\eta u$ depends on whether $\partial_\eta G$ is increasing or decreasing, but $(\partial_\eta G)'(x) = \partial_\eta G'(x)$ (cross derivative). Now $G'(x) = x^{-\eta}$, so that $\partial_\eta G'(x) = -\log(x)x^{-\eta}$, which is negative as long as $x > 1$, so that $\partial_\eta G$ is decreasing when $x > 1$. We therefore have the following result:

$$\partial_\eta u(c) \begin{cases} < 0, & \text{when } c > \bar{c} > 1 \\ > 0, & \text{when } \bar{c} > c > 1 \end{cases} \quad (\text{A-6})$$

Because $\partial_\eta G$ is decreasing, this also proves that $\partial_\eta u$ is also decreasing.

A.3 Evolution of welfare differences

A.3.1 Lemmas

We present three lemmas that will be useful for our demonstration.

Lemma A.1 *Suppose that c' is a consumption stream that is always lower than the consumption stream c , that is $\forall t, c'_t \leq c_t$. The EDE' stream is always lower than the EDE stream, i.e. $\forall t, EDE'_t \leq EDE_t$.*

Proof. We have $N_T \phi(EDE'_T) = \sum_{t=0}^T n_t \phi(c'_t) \leq \sum_{t=0}^T n_t \phi(c_t) = N_T \phi(EDE_T)$. Hence the

result. \square

Lemma A.2 *Suppose that the consumption stream c is an increasing sequence, that is $\forall t, c_t \leq c_{t+1}$. Then the sequence of EDE is increasing, i.e. $\forall t, EDE_t \leq EDE_{t+1}$.*

Proof. As $N_{T+1} = N_T + n_{T+1}$, we have $N_{T+1} (\phi'(EDE_{T+1}) - \phi'(EDE_T)) = n_{T+1} (\phi'(c_{T+1}) - \phi'(EDE_T))$. Now ϕ' is increasing (in its argument), so that $\phi'(EDE_T) = \sum_{t=0}^T \frac{n_t}{N_T} \phi'(c_t) \leq \sum_{t=0}^T \frac{n_t}{N_T} \phi'(c_T) = \phi'(c_T) \leq \phi'(c_{T+1})$ since c is an increasing sequence. Hence $\phi'(EDE_{T+1}) > \phi'(EDE_T)$ and the result since ϕ'^{-1} is increasing. \square

Lemma A.3 *Let u be a non null sequence so that $\sum_{s \geq t} u_s \geq 0$ for all t and $\sum_{s \geq 0} u_s = 0$. If a is an increasing (resp. decreasing) sequence then $\sum_s a_s u_s$ is positive (resp. negative).*

Proof. The proof relies on the transformation of the sequence u . If we introduce $U_t = \sum_{s \geq t} u_s$, the conditions on u become that U is positive and $U_0 = 0$. We have $u_t = U_t - U_{t+1}$. So $\sum_{s \geq 0} a_s u_s = \sum_{s \geq 0} a_s (U_s - U_{s+1}) = \sum_{s \geq 1} U_s \cdot (a_s - a_{s-1})$ since $U_0 = 0$. This rewriting of the sum proves the lemma. \square

A.3.2 Evolution of welfare differences when consumption differs

We have:

$$\Delta_c W = \sum_t \alpha_{N_t} P_t (u(EDE_t) - u(EDE'_t)) \quad (\text{A-7})$$

We suppose that c' is an increasing consumption stream that is always lower than the increasing consumption stream c , that is $\forall t, c'_t \leq c_t$. By lemma A.1, EDE' is always lower than EDE . $\Delta_c W$ is positive because each term of the sum is positive since $EDE'_t \leq EDE_t$ and u is increasing.

- **variation of $\Delta_c W$ with respect to β**

The quantity $\Delta_c W$ increases in β , as each $\partial_\beta \alpha_{N_t}$ is positive as long as $N_t > 1$.

- **variation of $\Delta_c W$ with respect to η**

We have:

$$\partial_\eta \Delta_c W = \sum_t \alpha_{N_t} P_t (\partial_\eta u(EDE_t) - \partial_\eta u(EDE'_t)) \quad (\text{A-8})$$

The quantity $\Delta_c W$ decreases in η because $\partial_\eta \Delta_c W$ is a sum of negative terms. This is because $\partial_\eta u$ is decreasing (as long as $EDE > 1$ and $EDE' > 1$) and $EDE > EDE'$.

- **variation of $\Delta_c W$ with respect to γ**

We have:

$$\partial_\eta \Delta_c W = \sum_t \alpha_{N_t} P_t (u'(EDE_t) \partial_\gamma EDE_t - u'(EDE'_t) \partial_\gamma EDE'_t) \quad (\text{A-9})$$

The variation of $\Delta_c W$ with respect to γ is ambiguous. With the only hypothesis that the consumption stream c' is inferior the consumption stream c , we cannot infer anything on the ranking of $\partial_\gamma EDE_t$ and $\partial_\gamma EDE'_t$. Indeed, $\partial_\gamma EDE_t$ is related to the magnitude of the EDE and to the dispersion of consumptions as a closer look at equation (A-3) reveals. To see this more clearly, let us take two examples for $\gamma > 1$.

1. take a sequence c' that is constant and a sequence c that is constant up to a certain rank, and constant but higher afterwards. $\partial_\gamma EDE'$ will always be zero, whereas $\partial_\gamma EDE$ will be positive, so that in the end $\partial_\gamma \Delta_c W$ will be positive.
2. take a sequence c that is constant and a sequence c' that is constant up to a certain rank, and constant but higher afterwards. $\partial_\gamma EDE$ will always be zero, whereas $\partial_\gamma EDE'$ will be positive, so that in the end $\partial_\gamma \Delta_c W$ will be negative.

If we suppose we are in the utilitarian case (i.e. $\gamma = \eta$), the variation of γ and η should be considered together, as it may be that one variation dominates the other, which could remove the ambiguity. We then have:

$$\Delta_c W = \sum_t n_t \cdot Q_t (u(c_t) - u(c'_t)) \quad (\text{A-10})$$

with $Q_t = \sum_{\tau=t}^{\infty} P_t \frac{\alpha_{N_\tau}}{N_\tau}$. The partial derivative with respect to η is a sum of negative terms since $\partial_\eta u$ is decreasing (as long as $c > 1$ and $c' > 1$) and $c > c'$. Here, the variation with respect to η is sufficient to remove the ambiguity of the variations with respect to γ .

A.3.3 Evolution of welfare differences when the risk of extinction differs

We have:

$$\Delta_p W = \sum_t \alpha_{N_t} (P'_t - P_t) u(EDE_t) \quad (\text{A-11})$$

We suppose that c is an increasing consumption stream and that p' is an extinction risk stream that is always lower than the extinction risk stream p , that is $\forall t, p'_t \leq p_t$. Recall that the

probability of extinction stream P_t is built from the extinction risk stream p_t according to the following formula: $P_t = p_t \cdot \prod_{s < t} (1 - p_s)$. P_t is the probability of dying exactly at period t . Note that because $p'_t < p_t$ at the beginning, P_t will be higher than P'_t , whereas the situation will likely be reverse at further dates. The probability of dying at period t or after is always higher with extinction risk p'_t than with extinction risk p_t . Indeed, if we note $P_{\geq t}$ this probability, we have $P_{\geq t} = \sum_{s \geq t} P_s = \prod_{s < t} (1 - p_s)$. Because $p'_t \leq p_t$, we have obviously that $P'_{\geq t} \geq P_{\geq t}$. Note that $P_{\geq 0} = 1$. The sequence $P' - P$ has the properties required for the sequence u of lemma A.3. Since $\alpha_{N_t} u(EDE_t)$ is increasing when, by assumption, EDE is increasing, this proves that $\Delta_p W$ is positive.

- **variation of $\Delta_p W$ with respect to β**

$\Delta_p W$ increases in β because $\partial_\beta \alpha_{N_t}$ is increasing and positive as long as $N_t > 1$, which means that $\partial_\beta \alpha_{N_t} u(EDE_t)$ is increasing, so that lemma A.3 can be applied to obtain $\partial_\beta \Delta_p W > 0$.

- **variation of $\Delta_p W$ with respect to η**

$\Delta_p W$ decreases in η , as $\partial_\eta u(EDE_t)$ is negative and decreasing (providing that $EDE > \bar{c}$, whereas α_{N_t} is increasing and positive, so that $\alpha_{N_t} \partial_\eta u(EDE_t)$ is decreasing. Lemma A.3 can thus be applied to obtain $\partial_\eta \Delta_p W < 0$.

- **variation of $\Delta_p W$ with respect to γ**

The variation of $\Delta_p W$ with respect to γ is ambiguous. Indeed $P'_t - P_t$ is multiplied by $\alpha_{N_t} u'(EDE_t) \cdot \partial_\gamma u(EDE_t)$. When $\gamma > 1$, this is the product of three positive terms, where the first is increasing, the second is decreasing, and the variation of the third is unknown (but likely to be increasing, as the dispersion between consumption increases with t). Therefore we cannot assert the direction of variation of this sequence to apply lemma A.3. In the utilitarian case however (i.e. when $\gamma = \eta$), the variation with respect to η is sufficient to remove the ambiguity of the variation with respect to γ . Indeed, we have:

$$\Delta_p W = \sum_{t=0}^{\infty} (P'_t - P_t) \alpha_{N_t} \sum_{s=0}^{\infty} \frac{n_s}{N_t} u(c_s) \quad (\text{A-12})$$

so that:

$$\partial_\eta \Delta_p W = \sum_{t=0}^{\infty} (P'_t - P_t) \alpha_{N_t} \sum_{s=0}^{\infty} \frac{n_s}{N_t} \partial_\eta u(c_s) \quad (\text{A-13})$$

Since α_{N_t} is increasing and $\sum_{s=0}^{\infty} \frac{n_s}{N_t} \partial_{\eta} u(c_s)$ is negative, we only need to show that the sequence $a_t = \sum_{s=0}^{\infty} \frac{n_s}{N_t} \partial_{\eta} u(c_s)$ is decreasing to apply lemma A.3 and get that $\partial_{\eta} \Delta_p W$ is negative, so that $\Delta_p W$ is decreasing in η in the utilitarian case. Now consider that $N_{t+1}(a_{t+1} - a_t) = n_{t+1}(\partial_{\eta} u(c_{t+1}) - a_t)$. Remember that $\partial_{\eta} u$ is decreasing and that c_t is increasing. Thus $a_t \geq \partial_{\eta} u(c_{t+1})$. Thus a_t is decreasing, which proves our claim and that $\partial_{\eta} \Delta_p W \leq 0$.