A dynamic model of recycling with endogenous technological breakthrough*

(Preliminary draft – Please do not quote)

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Abstract

We study an endogenous growth model in which the use of a non-renewable resource yields waste that can be recycled. The recycling activity only starts after the quality of recycled waste has reached a minimal threshold - which is not initially the case. The economy therefore has to invest in a specific R&D sector so as to improve this quality. We study the optimal trajectories of the economy; in particular, we analyze the discontinuity occurring at the date of the technological breakthrough, which is endogenous. We also discuss the environmental implications of the availability of the recycling technology. We show that the recycling option may have unexpected negative impacts on the environment, at least in the short run.

Keywords: Recycling; Non-renewable resource; Technical change; Growth.

JEL classifications: C61, O41, O44, Q32, Q53.

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1 Introduction

"Recycling is defined as any reprocessing of waste material in a production process that diverts it from the waste stream, except reuse as fuel. Both reprocessing as the same type of product, and for different purposes should be included. Recycling within industrial plants i.e. at the place of generation should be excluded." (United Nations[1]). A recycling activity presents two main benefits. First, by using waste as an input in the production process, it alleviates the scarcity of other resources. Second, it reduces the accumulated stock of waste, and, by doing so, reduces environmental damages.

Even if current levels of recycling activity greatly vary from sector to sector, recycling activity in the world is however low today. For instance, a UNEP report states that "many metal recycling rates are discouragingly low, and a "recycling society" appears no more than a distant hope" (UNEP, 2011). The main reasons behind this low-level activity are first that it remains comparatively expensive - i.e. non-recycled materials remain relatively cheap. Regarding municipal solid waste, for example, "in some cases the value of recyclables are less than the extra costs associated with collecting the disturbed waste" (World Bank, 2012). Second, the recycled materials are not always perfect substitutes to the virgin materials, which entails reduced marketing possibilities. If recycling pulp allows producing good-quality paper (ADEME, 2009), the recycling of carbon fiber reinforced polymer (CFRP) waste yields materials that cannot yet have the same industrial use as the virgin materials, particularly in advanced technology sectors such as aeronautics (Oliveux, 2015). This means that recycling technologies need to be improved.

The aim of the present paper is to understand how an economy will invest in research so that recycled waste can be used at a large scale. To do so, we consider an economy in which a recycling technology is available, but the current quality of recycled materials makes them non-usable by the production process. The only way to trigger the recycling activity is thus to improve the quality of these materials. This can be done by investing in a specific type of research and development (hereafter R&D). After a certain threshold quality level has been reached, a technological breakthrough occurs in the sense that the production of consumption goods starts using as inputs both the virgin (primary) resource and the recycled (secondary) waste. We characterize the optimal trajectories of the economy and their properties; in particular, we study the discontinuity occurring at the technological

[1] United Nations – Environmental indicators: http://unstats.un.org/unsd/environment/wastetreatment.htm

breakthrough. We also consider the impact of the recycling activity on the environment.

The economic literature has already considered the issue of recycling in dynamic contexts. Hoel (1978) analyzes the long-term path of an economy that consumes a non-renewable resource and a recycled resource. He shows how the impact of the use of these resources on the environment affects the optimal trajectories of the economy. André and Cerda (2006) study an economy that uses two types of natural resources, only one of which being recyclable. They show that if recycling may alleviate resource scarcity in the short-term, its ability to prevent negative long-run growth depends on how much the economy depends on non-renewable and renewable resources. Di Vita (2001) studies, in an endogenous growth growth context, an economy that uses both a non-renewable resource and recycled materials, the use of which harming the environment by producing waste. The model endogenizes the degree of recyclability of the accumulated waste: investing in a dedicated R&D sector allows improving recyclability. Di Vita (2007) focuses on the degree of substitutability between the non-renewable resource and recycled waste in the production process and analyzes its impact on the economy's growth path and the time profile of resource extraction. Amigues et al. (2010) also use an endogenous growth model with non-renewable and recycled resources; they consider that the waste flow resulting from the use of these resources depends on the level of economic activity. They show how a market for waste and subsidies to resource extraction and recycling allows restoring the social optimum. In all these studies, the recycling technology is immediately available and used by the economy. Conversely, in the present paper, we consider that the recycling technology initially produces materials of poor quality, that cannot be used in the production process (i.e. at a large scale). Therefore, as in Di Vita (2001), we consider a sector of R&D devoted to the recycled resource, but here technological improvements are needed so that the recycled resource reaches a minimal quality threshold. When this quality level is attained, the secondary material can be used and the recycling activity starts.

The endogenous growth model we develop can be sketched as follows. The production of a consumption good requires (general-purpose) knowledge, labor and raw material. Raw material corresponds to flows of a non-renewable resource, in its virgin or recycled form. The use of virgin (primary) resource yields waste flows that add to an overall stock. This waste can be recycled but the quality of the recycled waste is initially too low to allow its use within the production process. We consider that the economy can invest in a research sector dedicated to improving this quality; recycled waste starts being used as an input as soon as its quality meets a certain threshold. For simplicity, we assume that this threshold corresponds to the constant quality of the virgin material, taken as exogenous. However, the date at which the recycling activity starts is endogenous. The main trade-offs faced by the economy are the following: the intertemporal management of the stock of non-renewable resource, the intertemporal management of the stock of waste, the use of the virgin vs. recycled resource and the allocation of efforts between output production and R&D.

We characterize the socially optimal trajectories of the economy and we study their properties. In particular, we study the characteristics of the date of the technological breakthrough, that is, the discontinuity occurring at the date at which the quality of recycled waste meets the minimum standard and starts being used within the production process. We show for instance that resource extraction is less intensive and economic growth stronger once the technological breakthrough has occurred. We then analyze the impact of the recycling activity on the environment. If the availability of the recycling option is unambiguously beneficial for the environment in the long term, it is detrimental in the short term. Indeed, it increases waste flows over the period during which the recycled resource has not yet reached the minimal quality threshold. Furthermore, since we consider that the recycled resources are non-renewable resources, i.e. derived from fossil fuels, the impact of the recycling activity is bad for climate since it increases greenhouse gas emissions in the short (and possibly long) term.

The general model is exposed in Section 2, and Section 3 presents the optimal program of the economy. Then, we characterize the socially optimal trajectories and we study their properties in Section 4. In Section 5, we analyze the environmental impacts of the recycling activity. Section 6 concludes.

2 The model

We consider an economy where a final consumption good Y is produced from a raw material M and from labor L_Y according to the technology f. Denoting by A_Y the total factor productivity ("TFP" thereafter), the quantity produced at any time t is then given by $Y(t) = f(A_Y(t), M(t), L_Y(t))$, where the production function $f(\cdot)$ is increasing and concave in each argument. We also assume that labor and physical materials are essential in production: $f(A_Y, 0, L_Y) = f(A_Y, M, 0) = 0$.

For simplicity, we take the growth of the TFP as exogenous. Denoting by g_{A_Y} the growth rate (positive and constant) of this growth process, and by $A_{Y0} \equiv A_Y(0)$ the initial TFP index, we thus have $A_Y(t) = A_{Y0}e^{g_{A_Y}t}$.

The physical input M is made up of two types of materials: a non-renewable resource X – which will hereafter refer to as the virgin resource – and a recycled secondary material Z – which will refer to as the recycled resource. Each of these materials is associated with an index of quality. We denote by A_X the quality of the virgin resource, and by A_Z those of the recycled resource. $A_X(t)X(t)$ and $A_Z(t)Z(t)$ must then be viewed as the augmented material inputs that enter the production process at time t. In order to focus on the recycling-related activities, we assume that the quality index of the virgin resource is fixed and exogenous:[2] $\bar{A} \equiv A_X(t) > 0$, $\forall t$. The quality index of the recycled resource is subject to improvements resulting from specific R&D activities. We assume that, as long as this quality is lower than the quality of the virgin resource, the recycled material cannot be introduced into the production process. Once its quality index has reached the minimal threshold $\bar{A}$, then it can be used in combination with the virgin material. In this case, we assume that both types of resources are perfect substitutes (see Di Vita, 2001, or Pittel *et al.*, 2010). Consequently, the material input M can be expressed as follows:

$$M(t) = \begin{cases} \bar{A}X(t), & A_Z(t) < \bar{A} \\ \bar{A}X(t) + A_Z(t)Z(t), & A_Z(t) \geq \bar{A} \end{cases} . \tag{1}$$

The quality index of the recycled material can be improved through a specific endogenous R&D process. Starting from a given initial level $A_{Z0} \equiv A_Z(0)$, the quality $A_Z(t)$ of the recycled resource follows a standard law of motion:

$$\dot{A}_Z(t) = \delta L_A(t) A_Z(t), \tag{2}$$

where $\delta > 0$ is a parameter of productivity and $L_A(t)$ is the quantity of labor invested in this R&D activity at time t. For the problem to be meaningful, we clearly must assume that $0 < A_{Z0} < \bar{A}$.

The economy is endowed with a fixed labor amount L, which can be devoted either to production or to R&D. Any optimal path must then satisfies the following constraint:

$$L_A(t) + L_Y(t) = L. \tag{3}$$

[2]We will discuss about alternatives to this restriction in the last section.

The virgin resource is extracted from a non-renewable stock according to a one-to-one technology: one unit of extracted resource yields one unit of virgin material. We assume that the extraction cost is negligible. Denoting by $S(t)$ the stock of resource at time t, and by $S_0 \equiv S(0)$ the initial reserves, we have the following standard depletion process:

$$\dot{S}(t) = -X(t). \tag{4}$$

The consumption of $C(t)$ units of final good generates an instantaneous surplus $u\,(C(t))$ to consumers. The utility function $u(\cdot)$ satisfies the standard properties (increasing, concave, Inada conditions). Moreover, the utility flows are discounted by consumers at the social discount rate ρ, supposed to be positive and constant.

The final output production process generates waste that can be saved and reused. We assume that recycling is instantaneous, meaning that waste production and dismantling occur instantaneously and at the same time. Within the production process, only the primary physical inputs – virgin and recycled resources – yield waste. For simplicity, the waste content rates of the virgin and recycled materials, α and β respectively, with $\alpha, \beta \in (0,1)$, are taken as exogenous and constant. At any time, the incoming flow of wastes is then $\alpha X(t) + \beta Z(t)$. Let $W(t)$ be the cumulative amount of waste at time t, $W_0 \equiv W(0)$ be the initial stock inherited from the past. As a flow $Z(t)$ of waste is eventually used by the recycling sector and then removed from the stock of wastes, we can write:

$$\dot{W}(t) = \alpha X(t) - (1-\beta) Z(t). \tag{5}$$

In the remainder of the paper, we adopt the following conventional notations. We denote by φ_x the partial derivative of any function $\varphi(\cdot)$ with respect to variable x when this function contains more than one argument: $\varphi_x \equiv \partial\varphi(\cdot)/\partial x$. As usual, g_x characterizes the growth rate of variable x: $g_x(t) \equiv \dot{x}(t)/x(t)$. Last, for simplicity, we drop the time index when this causes no confusion.

3 The optimal program

The social planner program consists in determining the trajectories of resource extraction, waste recycling and efforts in R&D and production, that maximize the discounted sum of utility flows subject to the set of technical constraints. However, the problem turns out to be discontinuous since the final output has two different expressions depending on whether the quality index of recycled material is smaller or larger than the threshold $\bar{A}$. In this section, we consider separately these two successive phases.

Let T be the (endogenous) time at which A_Z is equal to $\bar{A}$, i.e. the date at which recycling becomes operational. As $g_{A_Z} = \delta L_A \geq 0$, the trajectory of A_Z is always non-decreasing. Henceforth, if such a time T exists then it is unique. We define respectively by $\mathcal{P}_1$ and $\mathcal{P}_2$ the social planner programs before and after time T. Following the standard optimal control methodology, we solve these two programs backward.

3.1 Recycling phase

Once the recycling option becomes available, i.e. after time T, the optimal program is:

$$(\mathcal{P}_2) : \max_{\{X,Z,L_A,L_Y\}} \int_T^{\infty} u(C)e^{-\rho(t-T)}dt,$$

subject to the technological condition $C = f(A_Y, \bar{A}X + A_Z Z, L_Y)$, to the labor use condition (3), to the dynamic constraints (2), (4) and (5), and to the initial condition $A_Z(T) = \bar{A}$. The following constraints on the control variables must also be satisfied:

$$X(t) \geq 0 \tag{6}$$

$$Z(t) \geq 0 \tag{7}$$

$$L_A(t), L_Y(t) \in [0, L]. \tag{8}$$

For the moment, we omit these non-negativity conditions which will be verified ex-post.

Denoting by λ_A, λ_S and λ_W the co-state variables associated with A_Z, S and W respectively, any optimal interior solution must satisfy the following first-order conditions:

$$u'(C)f_M\bar{A} = \lambda_S - \alpha\lambda_W \tag{9}$$

$$u'(C)f_M A_Z = (1-\beta)\lambda_W \tag{10}$$

$$u'(C)f_{L_Y} = \delta A_Z \lambda_A \tag{11}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{12}$$

$$\dot{\lambda}_W = \rho\lambda_W \tag{13}$$

$$\dot{\lambda}_A = (\rho - \delta L_A)\lambda_A - u'(C)f_M Z. \tag{14}$$

The transversality conditions are:

$$\lim_{t\to\infty} e^{-\rho(t-T)}\lambda_\kappa(t)\kappa(t) = 0, \quad \kappa = \{A_Z, S, W\}. \tag{15}$$

Conditions (9)-(11) state that the marginal social gain (in terms of utility) of one unit of input must be equal to its corresponding social marginal cost. More precisely, in (9), the marginal social gain of one unit of virgin resource equals the scarcity rent λ_S of the non-renewable resource stock, reduced by $\alpha\lambda_W$ to take into account that this unit generates wastes up to $\alpha\%$, which accumulate into the stock W whose shadow value is given by λ_W. Note that, as long as no negative externality is associated with the stock of wastes, λ_Z works as a scarcity rent and is unambiguously positive.[3] The same interpretation applies in (10) for the recycled resource, except that it does not involve the stock of natural resource but directly the stock of wastes. Last, equations (11) is a standard static arbitrage condition relative to the labor allocation between either production or R&D. The left-hand-side reads as the marginal social gain (in terms of utility) of increasing by one unit labor in production while the right-hand-side represents the marginal social cost (in terms of knowledge value) of these labor reallocation resulting from a diminution of the effort devoted to R&D.

Conditions (12) and (13) imply that $\lambda_S(t) = \lambda_S(T)e^{\rho(t-T)}$ and $\lambda_W(t) = \lambda_W(T)e^{\rho(t-T)}$. Consequently, and conditionally on the fact that both resource stocks have a positive value at time T (i.e. $\lambda_S(T) > 0$ and $\lambda_W(T) > 0$), the transversality conditions (15) associated with S and W reduce to $\lim_{t\to\infty} S(t) = \lim_{t\to\infty} W(t) = 0$. The stock of natural resource and the stock of waste must be asymptotically exhausted:

$$S(T) = \int_T^{\infty} X(t)dt \quad \text{and} \quad W(T) = \int_T^{\infty} [(1-\beta)Z(t) - \alpha X(t)]dt. \tag{16}$$

To get a general arbitrage condition on the resource use irrespective of its state – either virgin or recycled – we can replace λ_W into (9) by its expression coming from (10). We obtain the following equation:

$$u'(C)\left[\bar{A}f_M + \left(\frac{\alpha}{1-\beta}\right)A_Z f_M\right] = \lambda_S, \tag{17}$$

which reads as the equality between the marginal social gain of resource use that takes into account recycling possibilities and the marginal social value of the resource stock. In the right-hand-side of (17), the term in brackets is the total marginal productivity of one unit of extracted resource. This unit is used a first time, which increases the output by $\bar{A}f_M$. Next, this unit generates $\alpha\%$ of wastes from which $(1-\beta)\%$ can be valued through recycling. Then $\alpha/(1-\beta)$ can be interpreted as the recyclability rate of the virgin resource. Multiplying this rate by the marginal productivity $A_Z f_M$ of the recycled material yields the second increase in production induced by the virgin resource after recycling.

[3] The case of environmental externalities is discussed in the last section.

Denoting by $\sigma(C)$ the inverse of the elasticity of intertemporal substitution, i.e. $\sigma(C) \equiv -Cu''(C)/u'(C)$, the growth rate of the marginal utility can be simply expressed as $-\sigma(C)g_C$. Log-differentiating (17) with respect to time and using (12), we obtain the first following intertemporal arbitrage condition:

$$\rho + \sigma(C)g_C = \frac{d\left[\bar{A}f_M + \left(\frac{\alpha}{1-\beta}\right)A_Z f_M\right]/dt}{\left[\bar{A}f_M + \left(\frac{\alpha}{1-\beta}\right)A_Z f_M\right]}. \tag{18}$$

This condition is the Ramsey-Keynes condition in the specific context of our economy. The standard Ramsey-Keynes condition characterizes the socially optimal arbitrage made between consumption and capital accumulation. Here, the arbitrage is made between consumption and the use of the virgin resource. Assume, at date t, a marginal reduction of the production of consumption good through the diminution of resource use. At date $t + dt$, the economy uses this amount of resource whose total marginal productivity has increased while it was kept in situ. The extra amount of consumption good accordingly produced, represented by the right-hand-side of (18) must be the amount of consumption that compensates households from the loss of one unit of consumption at date t, represented by the left-hand-side of (18). What is new here is that, as previously mentioned, the total marginal productivity of the virgin resource features the term $\left(\frac{\alpha}{1-\beta}\right)A_Z f_M$, which accounts for the fact that the waste Z induced by the use of the virgin resource is recycled and used as an input for consumption good in production.

The second dynamic arbitrage condition is obtained by log-differentiating (11) and by using (14):

$$\rho + \sigma(C)g_C = \frac{\dot{f}_{L_Y}}{f_{L_Y}} + \frac{\delta A_Z Z f_M}{f_{L_Y}}. \tag{19}$$

The interpretation of this condition is similar to (18). Assume that, at time t, one unit of labor is transferred from output production to R&D activities, which implies an instantaneous marginal decrease in production and then in consumption. In order to maintain the same level of intertemporal welfare, the consumer must be compensated by an increase in consumption from t to $t + dt$ as given by the left-hand-side of (19). To be optimal, this compensation must be equal to the overall marginal increase in production during the same interval of time (right-hand-side), which is due to the increase in A_Z between t and $t + dt$ and to the reallocation of one unit of labor to production at time $t + dt$.

Finally remark that, from (9)-(10) and (12)-(13), the marginal productivity of the natural resource and of the recycled material, respectively $\bar{A}f_M$ and $A_Z f_M$, must grow at the same rate. This result is driven by the assumption of perfect substitution between the two types of resources. The additional assumption of a constant quality index of the virgin resource then hugely simplifies the analysis as it implies that the quality index of the recycled resource must be also constant. An immediate consequence is that no more effort in R&D is made once the quality of the secondary raw material has reached the one of the virgin resource:

$$\forall t \geq T: \ A_Z(t) = A_Z(T) = \bar{A}, \quad L_A(t) = 0 \quad \text{and} \quad L_Y(t) = L. \tag{20}$$

3.2 Pre-recycling phase

Before time T, as the secondary material cannot be used yet for production, we have $M = \bar{A}X$ from (1). Denoting by $V_2(A_Z(T), S(T), W(T))$ the value function of program $\mathcal{P}_2$ at time T, we can write the initial program $\mathcal{P}_1$ as follows:

$$(\mathcal{P}_1): \max_{\{X, L_A, L_Y\}} \int_0^T u(C)e^{-\rho t}dt + e^{-\rho T}V_2(A_Z(T), S(T), W(T)),$$

subject to the consumption condition $C = f(A_Y, \bar{A}X, L_Y)$, to the labor use constraint (3), and to the dynamic constraints (2), (4), (5). Note that, as the cumulative waste equation (5) is now reduced to $\dot{W} = \alpha X$, the trajectories of the resource reserves and of the waste stock are linked through the following relation:

$$W(t) = W_0 + \alpha(S_0 - S(t)), \ \forall t \in [0, T).$$

The first-order conditions of $\mathcal{P}_1$ are very similar to those of $\mathcal{P}_2$. Conditions (9), (11), (12) and (13) are the same. Condition (10) is no longer valid, whereas (14) becomes:

$$\dot{\lambda}_A = (\rho - \delta L_A)\lambda_A. \tag{21}$$

The transversality conditions at time T are:

$$\lambda_A(T^-) = \frac{\partial}{\partial A_Z(T^+)} V_2(A_Z(T^+), S(T^+), W(T^+)) \tag{22}$$

$$\lambda_S(T^-) = \frac{\partial}{\partial S(T^+)} V_2(A_Z(T^+), S(T^+), W(T^+)) \tag{23}$$

$$\lambda_W(T^-) = \frac{\partial}{\partial W(T^+)} V_2(A_Z(T^+), S(T^+), W(T^+)). \tag{24}$$

Last, the intertemporal trade-off condition writes:

$$\rho + \sigma(C)g_C = \frac{\dot{f}_M}{f_M} = \frac{\dot{f}_{L_Y}}{f_{L_Y}}, \tag{25}$$

which means that, under $\mathcal{P}_1$, the productivity of the resource and of labor must grow at the same rate.

4 Optimal trajectories of the economy

To illustrate the recycling problem with endogenous technical breakthrough and provide an example of optimal trajectories, we consider the following standard functional forms. Utility is characterized by a CES function of parameter $\sigma > 0$ and output production is described by a Cobb-Douglas function of parameter $\epsilon \in (0,1)$: $u(C) = C^{1-\sigma}/(1-\sigma)$ and $f(A_Y, M, L_Y) = A_Y M^{\epsilon} L_Y^{1-\epsilon}$.

Using these specified analytical forms, we analyze in this section the main qualitative properties of the particular optimal path we have obtained. All the details of trajectories computation are referred in Appendix.

4.1 Qualitative properties of the optimal trajectories

The solution of the planner program, as well as the existence conditions of this solution, are described in Appendix A.4. In this section, we describe the main qualitative properties of the optimal trajectories of the model. In particular, we explain their behavior at the time the economy switches from the pre-recycling to the recycling phases.

Resource use and waste accumulation

Resource extraction is always exponentially decreasing through time – at rate $\tilde{k} \equiv [\rho - (1-\sigma)g_{A_Y}]/\sigma$ during the pre-recycling phase and at rate $k \equiv [\rho - (1-\sigma)g_{A_Y}]/[1-\epsilon(1-\sigma)]$ during the recycling phase – and it tends asymptotically toward zero. In this sense, it follows a standard Hotelling depletion process. However, its trajectory is discontinuous here. Let $\Delta X(T) \equiv X(T^+) - X(T^-)$ denote the magnitude of the jump made by the resource extraction trajectory at time T. From (A.22), $\Delta X(T) = (k - \tilde{k})S_0 e^{-\tilde{k}T}$, which is negative as $\tilde{k} - k = (1-\sigma)(1-\epsilon)k/\sigma > 0$. This means that virgin resource use jumps down at time T and then follows a less sloping declining path, as illustrated in Figure 1-a. At that time indeed, recycling becomes operational and allows relaxing the constraint on the non-renewable resource consumption. Consequently the resource stock is continuously declining until its full exhaustion, but its trajectory is less sloping after T than before (not depicted).

As shown in Figure 1-b, the flow of recycled material is first nil. Then it jumps upwards at date T and follows a declining trajectory that asymptotically converges toward zero, similarly to the virgin resource trajectory (it particular, it decreases at the same rate k). The size of this jump is given by $\Delta Z(T) = [k(W_0 + \alpha S_0)]/(1-\beta)$. The stock of waste yielded by the use of the virgin and recycled resources is first increasing before T and next declining until exhaustion (see Figure 1-c). During the pre-recycling phase, virgin resource extraction thus has two purposes: first, the immediate production of output in order to meet consumption needs; second, the investment in a stock of waste which will be used as productive capital during the second phase, once recycling becomes operational. This also explains why resource extraction is more intensive before than after time T.

Effort in production and in R&D

The initial quality index of recycled material A_{Z0} is not high enough to allow using this input. During the first phase, the effort L_A devoted to the improvement of the recycled resource quality must continuously rise and stop once the required quality threshold is reached. This means that at time T where $A_Z(t) = \bar{A}$, the effort in R&D instantaneously falls to zero, as depicted in Figure 1-d. Consequently, since the total labor supply L is constant (see equation (3)), the effort in production L_Y declines through out the first phase, then jumps to level L at date T and remains at this level onwards.

Consumption

The translation of the previous results in term of consumption is more complex. On the one hand, the optimal consumption trajectory can be either increasing or decreasing. On the other hand, the nature of the jump at time T is not clear and depends on the relative magnitude of the respective jump made by each input.

First, we have shown in the appendix (see Lemma 1 and 2) that the growth rates of consumption during the pre-recycling and recycling phases are different. These rates, respectively denoted by $\tilde{g}_C$ and g_C, are:

$$\tilde{g}_C = \frac{g_{A_Y} - \rho}{\sigma} \quad \text{and} \quad g_C = \frac{g_{A_Y} - \epsilon\rho}{1-\epsilon(1-\sigma)}\,. \tag{26}$$

From the existence condition (A.32), one can easily show that $g_C > \tilde{g}_C$. Then, consumption grows faster – or decreases more slowly in case of negative rates – after date T.

Second, we analyze the sign of these two growth rates. We know that consumption is a combination of three factors: TFP (A_Y), material input (M) and labor (L_Y). Its potential

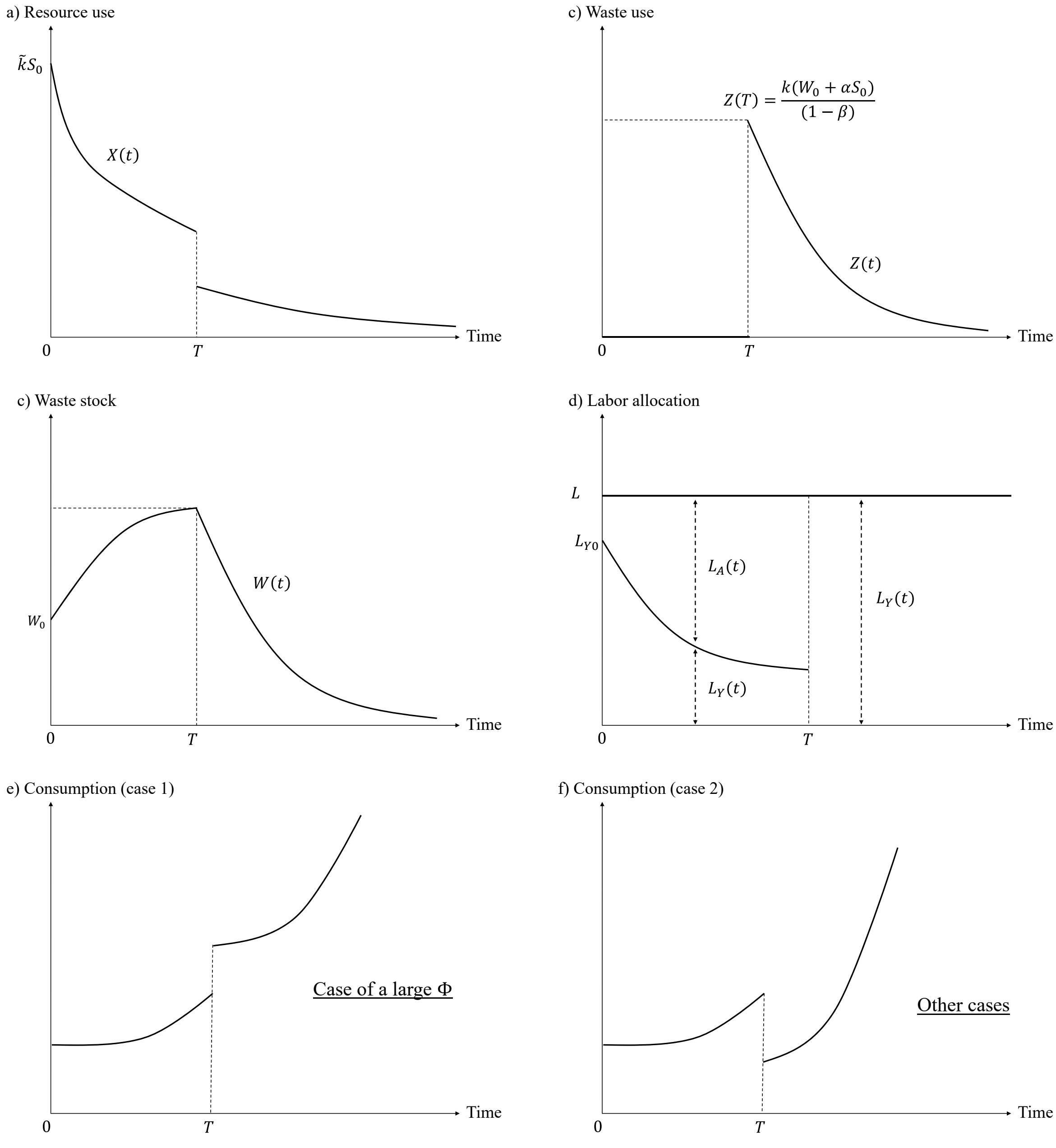

Figure 1: Optimal trajectories of the model

growth is only driven (exogenously) by A_Y, as L_Y is declining during the pre-recycling phase and constant afterward, and M is always decreasing. More precisely, we show that $\tilde{g}_C$ and g_C can be positive or negative depending upon the level of the TFP growth rate g_{A_Y}, relative to the social discount rate ρ and the input substitution parameter ϵ, as described by the three following cases:

$$\begin{cases} \rho < g_{A_Y} & \Rightarrow \ g_{\tilde{C}} > 0 \text{ and } g_C > 0 \\ g_{A_Y} \leq \rho < g_{A_Y}/\epsilon & \Rightarrow \ \tilde{g}_C \leq 0 \text{ and } g_C > 0 \\ g_{A_Y}/\epsilon \leq \rho & \Rightarrow \ g_{\tilde{C}} < 0 \text{ and } g_C \leq 0 \end{cases}$$

As usual, time impatience favors immediate consumption to the detriment of future consumption, and then speeds up input use. Consequently, the larger the social discount rate, the weaker the consumption growth rate, with negative values below a given threshold. For intermediate values of ρ, the optimal growth path may be U-shaped: decreasing through time during the first phase without recycling, and then increasing during the recycling phase. For simplicity, and to reduce the number of scenarios, we only illustrate the case where both g_C and $\tilde{g}_C$ are positive, that is the case where the social discount rate is not too high.

Next, we study the consumption jump at time T. Using the same notation as before, we can write that $\Delta C(T) \equiv C(T^+) - C(T^-)$, where $C(T^+) = A_{Y0}e^{g_{A_Y}T}M(T^+)^\epsilon L^{1-\epsilon}$ and $C(T^-) = A_{Y0}e^{g_{A_Y}T}M(T^-)^\epsilon L_Y(T^-)^{1-\epsilon}$. We already know that labor in production jumps up. A positive jump in the material input would thus be a sufficient condition for consumption to jump upwards. [4] From (1), the jump in M is given by $\Delta M(T) = \bar{A}[\Delta X(T) + \Delta Z(T)]$. As virgin and recycled resources uses vary in opposite directions at

[4]Formally, the jump in C is given by $\Delta C(T) = A_Y(T)\left[M(T^+)^\epsilon L^{1-\epsilon} - M(T^-)^\epsilon L_Y(T^-)^{1-\epsilon}\right]$. Then we can write:

$$\Delta C(T) \geq 0 \ \Leftrightarrow \ \left[\frac{M(T^+)}{M(T^-)}\right]^\epsilon \geq \left[\frac{L_Y(T^-)}{L}\right]^{1-\epsilon} .$$

Denoting by $\Delta M\%$ and $\Delta L_Y\%$ the percentage of instantaneous variation at time T of M and L_Y respectively, the previous expression becomes:

$$\Delta C(T) \geq 0 \quad \Leftrightarrow \quad (\Delta M\% + 1)^\epsilon \times (\Delta L_Y\% + 1)^{1-\epsilon} \geq 1\,,$$

which states that if the multiplier of M (first term in bracket) is larger than one, then consumption clearly jumps up. For the other cases, that is if this multiplier is smaller than one, the multiplier of L (second term in brackets, always larger than one) must be large enough and/or the substitution parameter ϵ must be small enough to compensate the instantaneous decrease in material inputs and to get a positive jump in C.

time T, the sign of $\Delta M(T)$ is unclear:

$$\begin{aligned} \Delta M(T) &= \bar{A}k\left[\left(\frac{W_0+\alpha S_0}{1-\beta}\right)-\frac{(1-\sigma)(1-\epsilon)S_0}{\sigma}e^{-\tilde{k}T}\right] \\ \Rightarrow \Delta M(T) &\geq 0 \quad \Leftrightarrow \quad \frac{W_0+\alpha S_0}{(1-\beta)S_0} \geq \frac{(1-\sigma)(1-\epsilon)}{\sigma}e^{-\tilde{k}T} \end{aligned} \tag{27}$$

From the expression (A.23) of Z given in the appendix, we can verify that the total use of the recycling resource $\int_T^\infty Z(t)dt$ amounts to $(W_0+\alpha S_0)/(1-\beta)$. Formally, this last expression reads as the maximal amount of wastes which can be generated along the planning horizon, divided by the net recycling rate of the secondary material. Then the left-hand-side in inequality (27) can be viewed as the maximal recycling rate of the initial reserves S_0 of the virgin resource. Let Φ denote this rate. Accordingly, the upward jump in Z overrides the downward jump in X and thus implies a positive jump in the material input M – and then in consumption – if Φ is large enough. This case is illustrated in Figure 1-e. The other case, that is the case of a downward jump in consumption, is represented in Figure 1-f.

4.2 Properties of the date of the technological breakthrough

We now perform some comparative dynamics so as to analyze the properties of the date of occurrence of the technological breakthrough. We do not intend to be exhaustive here, we rather focus on the main insights – a complete presentation of the comparative dynamics is provided in Appendix A.5.

The comparative dynamics show us that how the values of different types of parameters characterizing the technologies and preferences in this economy affect date T of the breakthrough.

Parameters characterizing the preferences

An increase in the psychological discount rate ρ means that the representative household obtains more utility from current consumption relative to future consumption. In order to increase consumption today, the social planner increases early extraction, which means that future extraction gets lower. The extraction of the virgin resource before T is thus accelerated: g_X decreases (recall that $g_X = \tilde{k}$). Simultaneously, a higher ρ entails stronger initial efforts in production: $L_Y(0)$ is increased. As a consequence, the initial effort in research gets lower, that is $L_A(0)$ decreases, and g_{A_Z}, the growth of the quality of the recycled resource is slower. Hence, for a given initial quality $A_Z(0)$, the date at which the recycled resource reaches the required quality $\bar{A}$ – and starts to get used in the production process – is postponed.

An increase in the elasticity of marginal utility σ means that the elasticity of marginal utility increases: the representative household derives more utility from a uniform consumption path, ceteris paribus. Here, in order to achieve a flatter consumption path, the social planner invests less in R&D (investing would imply a higher consumption tomorrow): $L_A(0)$ and both g_{A_Z} decrease. This too entails a later occurrence of the technological breakthrough: T increases.

Parameters characterizing the output production technology

In our framework, the total factor productivity (TFP) A_Y is taken as exogenous and growing at constant rate g_{A_Y}. A higher growth of TFP allows the planner to slow down resource extraction between 0 and T: less resource is used during the early periods and more in the future periods. In other words, g_X increases (that is, $\tilde{k}$) for all $t < T$. For a given level of $A_Y(0)$, the increase in g_{A_Y} entails higher A_Y for all t. Therefore, the planner devotes less labor to the production of output and more to research. Indeed, $L_Y(0)$ and g_{L_Y} getting larger with a higher g_{A_Y}, we can conclude that the effort in research is higher at each date $t < T$. Thus research is more intensive, the quality of the recycled resource grows faster, and the technological breakthrough is reached earlier.

An increase in the output elasticity of raw material ϵ means that the material input M gets pore productive (relative to labor). In this case, the economy relies more heavily on the flows of resource and hence the need for an additional resource is more pressing. Therefore the economy transfers labor from production to research and by this makes the technological breakthrough happen sooner: T gets lower.

Parameters characterizing the research sector

Parameter $\bar{A}$ characterizes the quality level of the virgin resource. With a higher $\bar{A}$, the economy requires that the recycled resource reaches a higher quality threshold before it starts to get used in the production process. Ceteris paribus and in particular for a given investment in R&D, this postpones the date of the breakthrough. The impact of $A_Z(0)$, the initial quality level of the recycled resource, is obviously opposite. A higher $A_Z(0)$

means that the distance to level $\bar{A}$ is shorter; the quality threshold is thus reached faster, and the technological breakthrough occurs at an earlier date.

A higher δ means that the R&D sector is more efficient. In such a case, $L_Y(0)$ is lower while g_{L_Y} is unchanged. This means that, at each date $t < T$, The effort in research L_A is higher and the quality of the recycled resource grows faster. The breakthrough occurs earlier here too.

Parameters characterizing the recycling sector

With a higher $S(0)$, the initial stock of virgin resource is larger and the need for a complementary resource is thus less urgent. Hence the social planner can use more virgin resource at each date between 0 and T, that is $X(t)$ gets higher, and devotes more labor to production and thus less to research: L_A is lower. Thus the quality of the recycled resource grows less fast and the technological breakthrough occurs later.

The initial stock of waste W_0 and the waste content rates of the two resources, α and β have similar effects on the economy's trajectories and henceforth on the date of the technological breakthrough. A larger initial stock of waste W_0 means that for a given path of resource use between 0 and T, the stock of recycled resource to be used from date T on is larger. This makes the technological breakthrough more significant. Hence the economy increases its effort in R&D, that is $L_A(t)$ gets higher for all $t < T$, and the recycled resource starts to get used sooner in the production process.

Similarly, if the waste content of the virgin resource, α, and the waste content of the recycled resource, β, are higher, the planner gets an incentive to put more efforts into research, and the technological breakthrough occurs earlier. We have introduced $\Phi \equiv \frac{W_0+\alpha S_0}{(1-\beta)S_0}$ as the maximal recycling rate; a higher Φ thus also brings forward the date of the breakthrough.

5 Discussion: environmental impact of the recycling option

We discuss here two negative environmental impacts[5] of the use of resources (recycled or not): first, the negative impact of the accumulated stock of waste $W(t)$; second, the impact

[5]These impacts are not explicit in the model as we do not consider environmental externality. In this section, we thus conduct a positive analysis of the environmental impact of recycling, not a normative one.

of resource use in terms of greenhouse gas (hereafter GHG) emissions and subsequent accumulated stock. In order to analyze the environmental impact of the recycling option, we first characterize the trajectory of the economy if this option is not available.

5.1 The non-recycling economy

Here we study the optimal trajectory of an economy in which no recycling technology is available at each date t, hereafter referred to as "the non-recycling economy". This is obviously a particular case of the economy studied in the last sections, in which date T never occurs. This corresponds formally to program $(\mathcal{P}_1)$ when T tends toward infinity. In this case, we can easily show that the marginal social value of R&D is nil and that it is never optimal to improve the quality of the recycled waste which will be never used. Consequently, the total amount of available labor is allocated at any point in time to production and the set of control variables reduces to resource extraction only. We thus obtain a simple "cake-eating" problem where the final consumption good is produced according to the following technological form: $C = A_Y(\bar{A}X)^{\epsilon}L^{1-\epsilon}$. [6]

5.2 Impact of recycling on the stock of waste

As shown in the preceding section, in an economy where recycling is not possible and will not be in the future, the stock of waste $\hat{W}(t)$ grows over time and asymptotically tends to its upper limit level $W(0) + \alpha S(0)$ (see Figue 2-d). In other words, as the economy uses the non-renewable resource, the associated waste adds to the existing stock until the whole resource stock is exhausted.

Consider now that the recycling option is available. We first assume that it is available at date 0. In other words, the quality threshold at which the recycled resource starts to be used is instantaneously met: $T = 0$, and thus $A_Z(0) = \bar{A}$. We will refer to this case as "the always-recycling economy". In this case, from (A.23), the expression of the stock of wastes would be $W(t) = W_0e^{-kt}$ for any t, implying an exponentially decline from the initial stock W_0 down to 0 (see Figure 1-c after T). This means that the economy tends to a

[6] The optimal trajectories of such a program are:

$$\hat{X}(t) = \hat{X}_0 e^{-kt}, \quad \hat{X}_0 = kS_0 \tag{28}$$
$$\hat{C}(t) = \hat{C}_0 e^{g_C t}, \quad \hat{C}_0 = A_{Y0}(\bar{A}kS_0)^{\epsilon}L^{1-\epsilon} \tag{29}$$
$$\hat{S}(t) = S_0 e^{-kt} \tag{30}$$
$$\hat{W}(t) = W_0 + \alpha S_0\left(1 - e^{-kt}\right), \tag{31}$$

long-run situation in which the amount of waste produced is instantaneously re-used by the production process: the waste stock progressively vanishes. Comparing this last expression with $\hat{W}(t)$ as given by (31), the environmental impact of the immediate availability of a recycling technology is therefore unambiguously positive: the stock of waste is lower at each date $t > 0$.

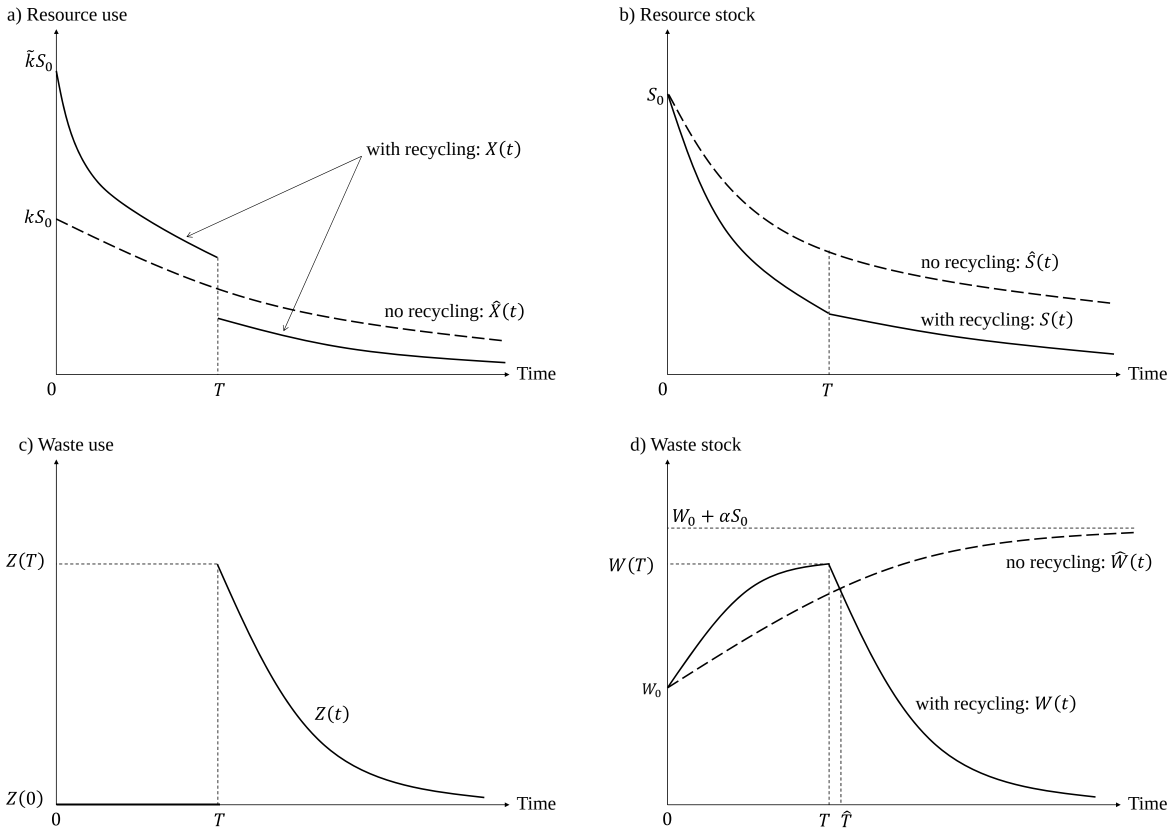

Figure 2: Optimal trajectories of the model

The impact of recycling on consumption when the technology is immediately available is the following. By comparing Equations (29) and (A.29)-post T, one can see that the growth rate of consumption is identical in the non-recycling and the always-recycling economies. The initial level of consumption in the non-recycling economy is $\hat{C}_0 = A_{Y0}(\bar{A}kS_0)^\epsilon L^{1-\epsilon}$, while this initial level in the always-recycling economy is $C(0) = A_{Y0}(\bar{A}k)^\epsilon[S_0 + (W_0 + \alpha S_0)/(1-\beta)]^\epsilon L^{1-\epsilon}$. It is straightforward that $C(0) > \hat{C}_0$. One can thus conclude that the immediate availability of a recycling technology has a positive impact on both the environment and consumption.

We assume now more generally that the recycled resource is used only when its quality level $A_Z(t)$ reaches the threshold level $\bar{A}$ at date T. This is the general case presented in Sections 2-4. We will simply refer to this case as "the recycling economy". Equation (A.25) presents the trajectory of the stock of waste in this case. This trajectory and the trajectory of the stock of waste $\hat{W}(t)$ in the non-recycling economy are depicted in Figure 2-c. Before date T, $W(t)$ increases and is higher than $\hat{W}(t)$ at each date. At date T, the stock reaches its maximum level $W(0) + \alpha S(0)(1 - e^{-\tilde{k}T})$ and then starts to steadily decline, to asymptotically converge toward 0 since, in the long-run, the recycling economy uses waste until its stock gets exhausted. Conversely, the non-recycling economy keeps accumulating waste after date T and $\hat{W}(t)$ gets higher than $W(t)$ at any date $\hat{T} > T$. In other words, the recycling option is bad for the environment until date $\hat{T}$. After this date, the environmental benefit becomes positive.

Regardless of its date of occurrence, the maximal level reached by a stock of pollutant is a serious concern for many - since irreversible damages may occur after certain thresholds. Here, this maximal level $W(T)$ is reached earlier by the recycling economy; however, it is lower than the maximal level reached by the non-recycling economy. Indeed, the difference between the two is equal to $\left[W(0) + \alpha S(0)(1 - e^{-\tilde{k}T})\right] - [W(0) + \alpha S(0)]$, which is negative since $e^{-\tilde{k}T} > 0$.

5.3 Impact of recycling on climate

As is well known, the use of many non-renewable resources in the production process yields greenhouse gas (GHG) emissions. If we consider that emissions at date t are proportional to resource use at the same date, and that there is no decay in the GHG stock (for simplicity), the accumulated stock of GHG is the sum of the flows of resource use over the interval $[0; +\infty)$ multiplied by a constant. For this reason, in what follows, we will approximate the stock of GHG at any date t by the sum of all resource flows from date 0 to t.

Using virgin (primary) material to produce output does not yield the same amount of GHG than using recycled materials. "Producing new products with secondary materials can save significant energy. For example, producing aluminum from recycled aluminum requires 95% less energy than producing it from virgin materials." (World Bank, 2012). Suppose thus for simplicity that only using the virgin resource $X(t)$ yields GHG: here, the recycled resource $Z(t)$ does not pollute. Figure 2-a shows that the availability of a recycling technology accelerates GHG emissions in the sense that emissions before date T

are higher and they are lower after this date. A well-known result of the climate-change literature is that emissions should be postponed (Withagen, 1994). Thus, while alleviating the burden of resource scarcity, the recycling technology has a negative impact on climate.

Before date T, the stock of GHG can be approximated by the stock of waste $W(t)$; Figure 2-c makes clear that this stock is unambiguously higher in the recycling economy. After date T, the stock of GHG increases in both the recycling and the non-recycling economies; in the non-recycling economy this stock may become higher at a certain date after T if virgin resource use is sufficiently higher than in the recycling economy. This means that recycling may have a positive impact on climate in the sense that it reduces the stock of GHG, but only in the long run.

Suppose now that both resource uses (X and Z) yield GHG emissions. Here, the negative impact of recycling on climate is reinforced. Before date T, the preceding conclusions apply. After this date, the emissions of the recycling economy are caused by the combined effect of both resource uses $(X(t) + Z(t))$, while in the non-recycling economy they stem from the non-renewable resource use $\hat{X}(t)$ only. This means that, depending on the level of $Z(t)$, emissions may be higher in the recycling economy. The possibility that recycling induces a lower stock of GHG in the long-run is thus reduced.

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Appendix

A.1 Analytical solution of program $\mathcal{P}_2$

We know that, for $t \geq T$, $A_Z(t) = \bar{A}$, $L_A(t) = 0$ and $L_Y(t) = L$, which implies $g_{A_Z} = g_{L_Y} = 0$. Given that $C = A_Y M^\epsilon L_Y^{1-\epsilon}$, equations (18) and (19) can be rewritten as:

$$\rho + \sigma g_C = \frac{\dot{f}_M}{f_M} \quad \Rightarrow \quad \rho - (1-\sigma) g_C = -g_M \tag{A.1}$$

$$\rho + \sigma g_C = g_C - g_{L_Y} + \frac{\delta \bar{A} Z f_M}{f_{L_Y}} \quad \Rightarrow \quad \rho - (1-\sigma) g_C = \frac{\delta \epsilon \bar{A} L Z}{(1-\epsilon) M}\,. \tag{A.2}$$

As $g_C = g_{A_Y} + \epsilon g_M + (1-\epsilon) g_{L_Y} = g_{A_Y} + \epsilon g_M$, equation (A.1) implies:

$$g_C = \frac{g_{A_Y} - \epsilon \rho}{1 - \epsilon(1-\sigma)}\,. \tag{A.3}$$

Combining (A.1) with (A.2), we obtain $g_M M = -(\delta \epsilon \bar{A} L Z)/(1-\epsilon)$, which imposes g_M to be negative. Replacing M by $\bar{A}(X+Z)$ in this last expression, we have:

$$\frac{X}{Z} = -\frac{\epsilon \delta L}{(1-\epsilon) g_M} - 1\,. \tag{A.4}$$

As g_C is constant (from (A.3)), g_M is constant too (from (A.1)). Log-differentiating (A.4) with respect to time, we obtain $g_X = g_Z$. Next, differentiating $M = \bar{A}(X+Z)$ with respect to time yields $\dot{M} = \bar{A}(g_X X + g_Z Z) = g_X M$, and then $g_M = g_X$. Finally, we obtain:

$$g_X = g_Z = g_M = -\left[\frac{\rho - (1-\sigma) g_{A_Y}}{1 - \epsilon(1-\sigma)}\right]\,. \tag{A.5}$$

Last, as X and Z grow at the same constant (negative) rate given by (A.5), we can easily solve the linear differential equation system (4)-(5) to get the optimal trajectories of resource extraction and of waste recycling. To conclude, the entire solution of $\mathcal{P}_2$ is characterized in the following lemma.

Lemma 1 *For $t \in [T, \infty)$, the optimal trajectories of the model are:*

$$\begin{aligned}
X(t) &= X(T) e^{-k(t-T)}; \quad X(T) = kS(T) \\
Z(t) &= Z(T) e^{-k(t-T)}; \quad Z(T) = \frac{k[W(T) + \alpha S(T)]}{(1-\beta)} \\
S(t) &= S(T) e^{-k(t-T)}; \quad W(t) = W(T) e^{-k(t-T)} \\
C(t) &= C(T) e^{g_C(t-T)}; \quad C(T) = A_Y(T) \bar{A}^\epsilon \left[X(T) + Z(T)\right]^\epsilon L^{1-\epsilon}
\end{aligned}$$

where $k \equiv [\rho - (1-\sigma) g_{A_Y}]/[1 - \epsilon(1-\sigma)$ and where g_C is given by (A.3).

From the expression of C as given in Lemma 1 and noting that $g_C = (\rho - k)/(1-\sigma)$, the optimal value of program $\mathcal{P}_2$ can be simply expressed as:

$$\begin{aligned} V_2(A_Z(T), S(T), W(T)) &= \int_T^\infty u(C) e^{-\rho(t-T)} dt \\ &= \frac{C(T)^{1-\sigma}}{(1-\sigma)} \int_T^\infty e^{-k(t-T)} dt = \frac{C(T)^{1-\sigma}}{(1-\sigma)k}, \end{aligned} \tag{A.6}$$

where $C(T) = A_Y(T)\left[\bar{A}X(T) + A_Z(T)Z(T)\right]^\epsilon L^{1-\epsilon}$ and $A_Y(T) = A_{Y0}e^{g_{A_Y}T}$. We can then compute the following derivatives:

$$\frac{\partial V_2}{\partial A_Z(T)} = \frac{\epsilon Z(T)}{kM(T)} C(T)^{1-\sigma} \tag{A.7}$$

$$\frac{\partial V_2}{\partial S(T)} - \alpha \frac{\partial V_2}{\partial W(T)} = \frac{\epsilon \bar{A}}{M(T)} C(T)^{1-\sigma}. \tag{A.8}$$

A.2 Analytical solution of program $\mathcal{P}_1$

In what follows, growth rates with an upper tilde refer to the optimal trajectories under program $\mathcal{P}_1$. As long as $t < T$, consumption amounts to $C = A_Y(\bar{A}X)^\epsilon L_Y^{1-\epsilon}$. The intertemporal trade-off condition (25) directly implies that $\tilde{g}_X = \tilde{g}_{L_Y} = (1-\sigma)\tilde{g}_C - \rho$. As $\tilde{g}_C = g_{A_Y} + \epsilon \tilde{g}_X + (1-\epsilon)\tilde{g}_{L_Y}$, we can deduce the optimal growth rate of consumption for $t \in [0, T)$:

$$\tilde{g}_C = \frac{g_{A_Y} - \rho}{\sigma}. \tag{A.9}$$

As we know the growth rates of all the control variables (note also that $L_A = L - L_Y$), we can then solve the state equations (2)-(5) and characterize the solution of program $\mathcal{P}_1$ as follows.

Lemma 2 *For $t \in [0, T)$, the optimal trajectories of the model are:*

$$\begin{aligned} X(t) &= \tilde{k} S_0 e^{-\tilde{k}t}; \quad S(t) = S_0 e^{-\tilde{k}t}; \quad W(t) = W_0 + \alpha S_0 (1 - e^{-\tilde{k}t}) \\ L_Y(t) &= L_Y(0) e^{-\tilde{k}t}; \quad L_A(t) = L - L_Y(t) \\ A_Z(t) &= A_{Z0} \exp\left[\delta L t - \frac{\delta L_Y(0)}{\tilde{k}}(1 - e^{-\tilde{k}t})\right] \\ C(t) &= C(0) e^{\tilde{g}_C t}; \quad C(0) = A_{Y0}\left(\bar{A}\tilde{k}S_0\right)^\epsilon L_Y(0)^{1-\epsilon}, \end{aligned}$$

where $\tilde{k} \equiv [\rho - (1-\sigma)g_{A_Y}]/\sigma$, where $\tilde{g}_C$ is given by (A.9), and where $L_Y(0)$ and T are endogenous variables that must be determined from the set of continuity and transversality conditions at time T.

A.3 Transversality conditions at time T

Given the expression of the state variables provided by Lemma 1 and 2, we can deduce the following continuity conditions at time T:

$$S(T^-) = S(T^+) \quad \Leftrightarrow \quad S(T) = S_0 e^{-\tilde{k}T} \tag{A.10}$$

$$W(T^-) = W(T^+) \quad \Leftrightarrow \quad W(T) = W_0 + \alpha S_0(1 - e^{-\tilde{k}T}) \tag{A.11}$$

$$A_Z(T^-) = \bar{A} \quad \Leftrightarrow \quad \delta LT = \frac{\delta L_Y(0)}{\tilde{k}}\left(1 - e^{-\tilde{k}T}\right) + \ln\left(\frac{\bar{A}}{A_{Z0}}\right). \tag{A.12}$$

Next, we analyze the transversality conditions as given by (22)-(24). We need first to characterize the trajectories of the co-state variables. Solving the differential equations (12), (13) and (21) for $t \in [0, T)$ results in:

$$\lambda_S(t) = \lambda_S(0)e^{\rho t} \tag{A.13}$$

$$\lambda_W(t) = \lambda_W(0)e^{\rho t} \tag{A.14}$$

$$\lambda_A(t) = \frac{\lambda_A(0)A_{Z0}}{A_Z(t)}e^{\rho t}, \tag{A.15}$$

where $\lambda_S(0)$, $\lambda_W(0)$ and $\lambda_A(0)$ are endogenous variables that must satisfy the first-order conditions (9) and (11) at time $t = 0$:

$$\lambda_S(0) - \alpha\lambda_W(0) = \frac{\epsilon}{\tilde{k}S_0}C(0)^{1-\sigma} \tag{A.16}$$

$$\lambda_A(0) = \frac{(1-\epsilon)}{\delta A_{Z0}L_Y(0)}C(0)^{1-\sigma}. \tag{A.17}$$

Combining (A.13)-(A.15) with (A.16)-(A.17), we have:

$$\lambda_S(T^-) - \alpha\lambda_W(T^-) = \frac{\epsilon}{\tilde{k}S_0}C(0)^{1-\sigma}e^{\rho T^-} \tag{A.18}$$

$$\lambda_A(T^-) = \frac{(1-\epsilon)}{\delta\bar{A}L_Y(0)}C(0)^{1-\sigma}e^{\rho T^-}. \tag{A.19}$$

Next, using the transversality conditions (22)-(24) together with (A.18)-(A.19) and rearranging the outcome, expressions (A.7)-(A.8) can be rewritten as follows:

$$M(T)\left[\frac{C(0)}{C(T)}\right]^{1-\sigma} e^{\rho T} = \frac{\epsilon\delta\bar{A}L_Y(0)Z(T)}{k(1-\epsilon)} \tag{A.20}$$

$$M(T)\left[\frac{C(0)}{C(T)}\right]^{1-\sigma} e^{\rho T} = \tilde{k}S_0\bar{A} \tag{A.21}$$

The two last equations allow for determining the optimal initial level of effort in production. Last, given this optimal value of $L_Y(0)$, the optimal switching time T is obtained as the solution of the continuity equation (A.12).

A.4 Optimal trajectories: Summary

The optimal solution is characterized by the following trajectories:

- Virgin and recycled resource use:

$$X(t) = \begin{cases} \tilde{k}S_0 e^{-\tilde{k}t} & , \quad t < T \\ kS_0 e^{(k-\tilde{k})T-kt} & , \quad t \geq T \end{cases} \tag{A.22}$$

$$Z(t) = \begin{cases} 0 & , \quad t < T \\ \frac{k(W_0+\alpha S_0)}{(1-\beta)} e^{-k(t-T)} & , \quad t \geq T \end{cases} \tag{A.23}$$

- Virgin resource and waste stocks:

$$S(t) = \begin{cases} S_0 e^{-\tilde{k}t} & , \quad t < T \\ S_0 e^{(k-\tilde{k})T-kt} & , \quad t \geq T \end{cases} \tag{A.24}$$

$$W(t) = \begin{cases} W_0 + \alpha S_0(1-e^{-\tilde{k}t}) & , \quad t < T \\ \left[W_0 + \alpha S_0(1-e^{-\tilde{k}T})\right] e^{-k(t-T)} & , \quad t \geq T \end{cases} \tag{A.25}$$

- Effort intensities in production and R&D:

$$L_Y(t) = \begin{cases} L_{Y0} e^{-\tilde{k}t} & , \quad t < T \\ L & , \quad t \geq T \end{cases} \tag{A.26}$$

$$L_A(t) = \begin{cases} L - L_{Y0} e^{-\tilde{k}t} & , \quad t < T \\ 0 & , \quad t \geq T \end{cases} \tag{A.27}$$

  where the initial level of productive labor is given by:

$$L_{Y0} \equiv L_Y(0) = \frac{\tilde{k}(1-\epsilon)(1-\beta)S_0}{\delta\epsilon(W_0+\alpha S_0)}; \tag{A.28}$$

- Consumption:

$$C(t) = \begin{cases} C(0)e^{\tilde{g}_C t} & , \quad t < T \\ C(T)e^{g_C(t-T)} & , \quad t \geq T \end{cases} \tag{A.29}$$

- R&D level in recycling:

$$A_Z(t) = \begin{cases} A_{Z0} \exp\left[\delta L t - \frac{\delta L_{Y0}}{\tilde{k}}(1-e^{-\tilde{k}t})\right] & , \quad t < T \\ \bar{A} & , \quad t \geq T \end{cases} \tag{A.30}$$

- Optimal switching time T, solution of the following equation:

$$\delta L T = \ln\left(\frac{\bar{A}}{A_{Z0}}\right) + \frac{S_0(1-\epsilon)(1-\beta)}{\epsilon(W_0+\alpha S_0)}\left(1-e^{-\tilde{k}T}\right). \tag{A.31}$$

We can easily verify that that such an interior solution exists, i.e. that the non-negativity constraints (6)-(8) hold, if and only if $L_{Y0} \in (0, L)$, $k > 0$ and $\tilde{k} > 0$. This corresponds to a set of parameters that must satisfy the following conditions:

$$(1-\sigma)g_{A_Y} \leq \rho \tag{A.32}$$

$$L_{Y0} = \frac{\tilde{k}(1-\epsilon)(1-\beta)S_0}{\delta\epsilon(W_0+\alpha S_0)} \leq L\,. \tag{A.33}$$

Condition (A.32) states that, to justify resource extraction and recycling, the social discount rate must be large enough as compared with the exogenous trend parameter of technical progress (this condition guarantees that both k and $\tilde{k}$ are positive). Condition (A.33) says that the total amount of effort (i.e. labor) must be large enough.

A.5 Dynamic comparative analysis

We conduct a sensitivity analysis of the key variables of the model, with respect to the set of parameters. The results are described in Table 1. Each box indicates the sign of the partial derivative of the variable mentioned in line with respect to the parameter given in column. This sign can be positive ("+") or negative ("−"). An empty box means that there is no relation between the variable and the parameter whereas "?" indicates that the sign is ambiguous.

Partial differentiation of the growth rates k, $\tilde{k}$, g_C and $\tilde{g}_C$, and of the initial values $X(0)$, $Z(T)$, $W(T)$, L_{Y0} and $C(0)$ are trivial so that their computations are not detailed here. However, the sensitivity analysis of the switching date T is less obvious as we do not have its analytical expression. We simply know that T is characterized by the implicit function (A.31). Let us define the following functions h and j:

$$\begin{aligned} h(t) &= \delta L t - \ln\left(\frac{\bar{A}}{A_{Z0}}\right) \\ j(t) &= \frac{S_0(1-\epsilon)(1-\beta)}{\epsilon(W_0+\alpha S_0)}\left(1-e^{-\tilde{k}t}\right) . \end{aligned}$$

These functions are depicted in Figure 2. Given their analytical properties, we can observe graphically that the solution T to the equation $h(T) = j(T)$ is unique. Moreover, for this solution to exist, we must have $h'(T) > j'(T)$.

We apply now the implicit function theorem to h and j. For any parameter x, we obtain:

$$\frac{dT}{dx} = \frac{\partial j/\partial x - \partial h/\partial x}{h'(T) - j'(T)} \quad \Rightarrow \quad \text{sign}\left(\frac{dT}{dx}\right) = \text{sign}\left(\frac{\partial j}{\partial x} - \frac{\partial h}{\partial x}\right) . \tag{A.34}$$

This equation, together with the computations of $\partial h/\partial x$ and $\partial j/\partial x$, thus allows to identify the sign of the derivatives of T with respect to any parameter x.

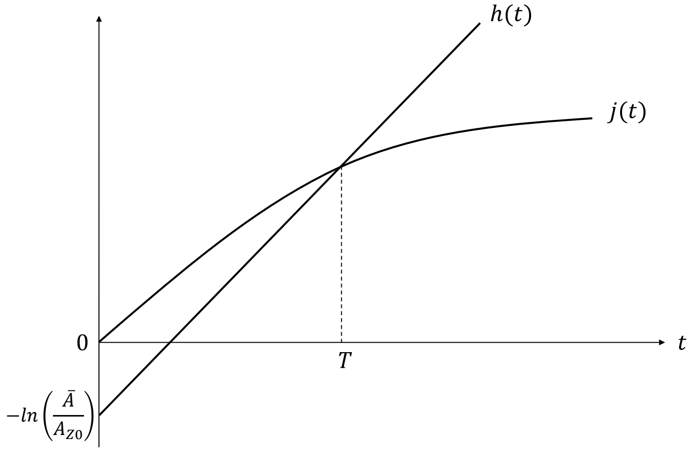

Figure 3: Graphical identification of T

Table 1: Comparative dynamics analysis

	ρ	σ	g_{A_Y}	ϵ	$\bar{A}$	δ	S_0	W_0	α	β
k	$+$	$+$	$-$	$+$	$\tilde{k}$	$+$	$+$	$-$	g_C	$-$
$\tilde{k}$	$+$	$+$	$-$							
g_C	$-$	$-$	$+$	$-$	$\tilde{g}_C$	$-$	$-$	$+$	T	$+$
$\tilde{g}_C$	$-$	$-$	$+$							
T	$+$	$+$	$-$	$-$	$+$	$-$	$+$	$-$	$-$	$-$
$X(0)$	$+$	$+$	$-$				$+$			
$Z(T)$	$+$	$+$	$-$	$+$			$+$	$+$	$+$	$+$
$W(T)$	$+$	$+$	$-$	$-$	$+$	$-$	$+$	?	?	$-$
L_{Y0}	$+$	$+$	$-$	$-$		$-$	$+$	$-$	$-$	$-$
$C(0)$	$+$	$+$	$-$	?	$+$	$-$	$+$	$-$	$-$	$-$