Renewable resources and inequality aversion: what consequences for the future?*

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Abstract

This paper addresses intragenerational and intergenerational issues about a renewable natural resource exploitation. In particular, I analyze how different equity views, represented through different intragenerational inequality aversions, influence possible development paths for future generations. I suppose an agent has access to a renewable resource and works to exploit it, while another agent does not have access to it. A social planner implements a transfer mechanism from the former to the latter. I show that if the worker is originally better-off than the receiver, a higher inequality aversion implies a higher harvest with a lump-sum transfer, but potentially a lower one with a proportional tax. As reducing the global consumption may be desired in an intergenerational perspective, the mechanism used shall depend on the situation. These links strongly suggest to deal jointly with the two equity dimensions in order to design consistent environmental policies.

Keywords: renewable resource, intragenerational equity, intergenerational equity.
JEL Classification: D63, Q20, Q56

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> Even the narrow notion of physical sustainability implies a concern for social equity between generations, a concern that must logically be extended to equity within each generation.
>
> World Commission on Environment and Development (1987)

1 Introduction

Emphasis has been put on futurity in the sustainability debate. Economists translated this question on how weighting the future compared to the present in the evaluation of different development paths. Among these paths, some can be considered as fair if generations that arise at different dates count equally. This view is often summarized under the vocable *intergenerational equity*. But if society cares about the "difference of well-being" between two generations, it cares also about that of two individuals from the same generation. When they count equally, one talks about *intragenerational equity*[1]. The economics literature has generally separated these two equity dimensions, but there are several arguments for dealing with them together. First, it can be argued that an unjust society is likely to be unsustainable, either on the political side (revolution) or on the environmental side (degradation) (Haughton, 1999). Second, it seems curious to attach more importance to future generations, thus unborn, than to the current one (Solow, 1991; Anand and Sen, 2000). Finally, from a policy view, one may wonder if intragenerational and intergenerational concerns can be designed independently. On the contrary, one should be interested in how they interact to formulate consistent policies.

The interactions between these two dimensions are actually ever-present in economics. An example can be that of a fiscal policy that aims to reduce the public debt. But alongside that debt there is environmental debt, which can reduce the possibility of development for future generations. In this respect, three major dimensions can be taken into account: the climate change (with the question of burden-sharing between generations and into each one of them, between expected losers and winers), the exhaustion of non-renewable resources (but it requires an understanding of the industrial processes, especially to what extend these resources can be substitutes for manufactured capital) and the management of renewable resource stocks. I am interested in renewable resources in particular.

The linkages between the two equity dimensions seem not to have been extensively studied in the economics literature, and in the environmental and resources economics literature in particular. Nonetheless, some authors have highlighted this question in the climate change debate for some time. For example, Schelling (1992) argues that the best way for developing countries

[1] There is not a unique accepted definition of the word "equity". I take here the requirement of equal weights in decision making. Which is in line with a common definition: "Equity is the quality of being fair and reasonable in a way that gives equal treatment to everyone." (source: `https://www.collinsdictionary.com/dictionary/english/equity`, visited on May 2017).

to fight against the negative effects of climate change is to continue to develop. Heal (2009), on the opposite, explains that a preference for equality between generations and the preference for equality within each one of them may be opposed. If one expects consumption will grow, one can further discount the future, but that does not incite us to take preventive measures against the negative effects of climate change. Conversely, as developing countries are more vulnerable, more concerns about them would incite us to take actions more quickly. Kverndokk et al. (2014) proposed a model to analyze the burden-sharing between North and South in the reduction of greenhouse gas emissions through clean and dirty investments. These two dimensions are also present in the explanation of climate negotiations (e.g. Lecocq and Hourcade, 2012). Baumgärtner et al. (2012) proposed a framework to summarize the possible links between the two equity dimensions: independence, facilitation and/or rivalry. Those features are detailed in the context of ecosystem services by Glotzbach and Baumgärtner (2012).

As renewable resources do not have a market price, the choice of the welfare criterion is of all importance. More precisely this may allow for expressing the implicit values of stocks of natural resources. Associated *shadow prices* are essential to compute *Genuine savings* (Hamilton, 1994; Pearce et al., 1996; Asheim, 2007). Expenditures that enhance the environment are seen as savings and depletion of natural resources and environmental degradations as dissavings. It generalizes the traditional concept of savings, and its positivity indicates that welfare, however defined, is currently non-declining. Renewable resources have to be managed on the long run, but compared to non-renewable ones: they are generally directly consumed, they may have amenities and one can have "win-win" solutions. Further, some can be "essential" for life. For all of these reasons, the regulation can be justified. Here, renewable resources are viewed as a parable to link the two dimensions. Some current redistributions can be decided, but they will inevitably have consequences on redistribution between now and the future.

I am not aware of any work that builds analytically the welfare possibility between individuals that depends on a renewable resource and has consequences on future generations. The purpose of this study is to analyze the intragenerational and the intergenerational equity trade-off with a renewable resource. The question I am asking is what are the consequences of intragenerational concern on current and future welfares, when only some individuals have access to a renewable resource and a social planner orders a redistribution. And does the type of redistribution matters in such a context. The intergenerational equity may be represented in several ways (for a review, see Asheim, 2010). But to be consistent with the definition of equity, one may adopt the maximin criterion (Solow, 1974) as a benchmark. In terms of axioms, it satisfies weak Pareto (the criterion is increasing if the well-being of all generations increase) and finite anonymity (the outcome is indifferent to a finite permutation of a consumption stream) (Cairns, 2011). As it measures the highest sustainable welfare, it has appealing features in order to define sustainability (Cairns and Long, 2006; Cairns, 2011, 2013) and to measure it (Cairns and

Martinet, 2014; Fleurbaey, 2015). Here, when no conflict appears, a higher intergenerational welfare will be advocate.

The intragenerational equity is also expressed in different ways. The main ones are the utilitarianism (broad sense)[2], the liberal egalitarianism, the libertarianism and the marxism (Arnsperger and Van Parijs, 2003). I assume here a utilitarian view (in a strict sense). For that, society is assumed to have an inequality aversion. That is to say it restrains the substitution of the well-being of one agent for the well-being of another. This approach is not as restrictive as it could seem since it allows to deal with different theories of justice as special cases. Indeed, the utilitarianism (broad sense) assumes a nil inequality aversion, only the total of utility matters (Vickrey, 1945; Harsanyi, 1955, 1977). The liberal egalitarianism can take a "*maximin*" form, popularized by Rawls (1971), if utility are considered as "primary goods". Here, inequality aversion is infinite, no substitution is possible between the individual utility. "Right-libertarians" would promote no transfer and "left-libertarians" would promote a transfer such as to perfectly equalize utility. The marxist approach would determine thresholds of utility representing "needs". According to the intergenerational view, the intragenerational fulfillment may be constrained. Studying both together allows for determining all possible choices of policy and to estimate opportunity costs.

I use a social objective that allows to deal with the two main doctrines (utilitarianism and maximin) as well as all intermediary cases. Besides, I build sets of possibilities for the utilities. They indicate to what extend one can take from one agent to give to another agent. Then, efficient allocations will be represented by Pareto frontiers. From an allocation situated on those frontiers, one cannot increase any more the utility of one agent without decreasing that of another. Afterward, I will be able to choose between those optimal allocations using the social criterion. To respond to my issue I need, at least, two different agents. One agent has access to a resource and works to extract a part of it. The other agent, who does not have access to it, entirely devotes his/her time to leisure. I introduce a redistribution mechanism that takes a part of the harvest from the worker to give it to the receiver. The utility of the worker depends on his/her leisure time and on his/her available consumption. The utility of the receiver depends only on the amount s/he receives. I analyze the utility distribution possibilities offered by two redistribution mechanisms. The first one is a lump-sum transfer; whatever the effort of the worker, the receiver will get the same amount of the resource caught. The second one is a proportional tax. Whatever the effort of the first agent, the other will get the same proportion of the resource caught. I introduce a renewable resource which varies according to the catch. As the resource may affect the potential catch, it impacts the utility possibility sets. First, I will see how the optimal transfer (lump-sum and tax) evolve according to a change in the inequality

[2]The term utilitarianism in the broad sense refers to the sum of utilities. In the strict sense, it refers simply to the utilization of the concept of utility.

aversion. Second, I will analyze the consequences of the evolution of the resource on the well-being of the agents in a intergenerational perspective.

My contribution is having stated clearly the conditions underlying the construction of well-known utility possibility frontiers in the context of two heterogeneous agents. When the transfer is absolute (through a lump-sum), this does not pose any particular difficulties as long as leisure and consumption are normal goods. When the transfer is relative (proportional tax), it depends on the labor supply of the worker in reaction to the tax, which is given by his/her disposition to substitute leisure for consumption. In particular, if s/he works less when taxed, the utility possibility set is bounded by a "Laffer-like curve" (from Laffer, 2004); the amount received from a proportional tax is increasing, then decreasing, with respect to its rate. Knowing that, I was able to analyze the impact of a change in the inequality aversion on the transfers. If the worker is better-off than the receiver, one can expect that the higher the inequality aversion the higher the transfer (absolute or relative). Regarding the redistribution of a renewable resource over time, an increasing of the intragenerational inequality aversion favors the intergenerational dimension in two situations. If the transfer has to increase (i.e. the worker is better-off), it has to be through a tax: discouragement effect. While if it has to decrease (i.e. receiver is better-off), it has to be through a lump-sum: the worker gets more harvesting less.

The intragenerational dimension is built upon a framework proposed by Mas-Collel et al. (1995), borrowed itself from Atkinson (1973). I extend their example in two dimensions. First, I state clearly the conditions under their results, especially the construction of utility possibility frontiers. And second, I introduce a resource to take into account the intergenerational concern in a Gordon-Schaefer model (Clark, 1990). The welfare analysis is based upon social welfare functions (Bergson, 1938; Samuelson, 1966).

The next Section presents the model. I solve it and I present some welfare analyses. Section 3 exhibits the links between the two equity dimensions. Section 4 concludes.

2 A Model with Two Heterogeneous Agents

2.1 Framework and Notations

I consider an economy with two dynasties of heterogeneous agents. Virtually, each representative agent lives one instant in that continuous framework. But as I will not solve a dynamic program, I will only study the *potential* consequences of current decisions on the future. I assume one type of agent has access to a renewable natural resource and works to extract a part, while the other one does not have access to it. A social planner implements a mechanism of transfer at each period between the two agents. I consider two options: the first one is a lump-sum transfer and the second one a tax.

I note A the worker and B the receiver. I normalize their available time to unity. At a given date, the resource stock X is given. Its law of evolution is given by the gap between renewal and harvesting (the time index will be dropped since no confusion may arise): $\dot{X}(t) \equiv \frac{\mathrm{d}X_t}{\mathrm{d}t} = \phi(X_t) - H(l_A, X_t)$. The function ϕ describes a "bell curve" ($\phi(0) = 0 = \phi(X_{sup})$) and reaches a maximum for $\bar{X}$ (the Golden Rule stock). This formulation can represent resource issues as well as capital accumulation (Asheim and Ekeland, 2016). $H(l_A, X)$ is the production (catch-effort) function of the worker. I assume it depends linearly on labor (or leisure), and it is convex with respect to the resource: $H(l_A, X) = (1 - l_A)h(X)$, where $h(X)$ represents the catchability.[3] The production is bounded between zero (no work) and $h(X)$ (no leisure). Total consumption equal to the sum of individuals ones: $c \equiv c_A + c_B$. The utility of the worker depends on his/her leisure time and his/her effective consumption, $u_A(l_A, c_A)$, while the utility of the receiver depends only on his/her consumption, $u_B(1, c_B)$. The utility functions are assumed to be interpersonally comparable, strictly concave and homothetic (see the Appendix A for details). I assume no loss, so that the whole production is consumed: $c = H(l_A, X)$. By definition, the transfer amounts to $c_B \equiv c - c_A \geq 0$.

To represent the inequality aversion in a simple way, I use an ordinal social welfare function with a constant elasticity of substitution (CES) between individuals of the form $W(u_A, u_B) = \left(\frac{1}{2} \cdot u_A^{\eta} + \frac{1}{2} \cdot u_B^{\eta}\right)^{\frac{1}{\eta}}$. The elasticity of substitution is given by $\theta \equiv \frac{1}{1-\eta}$ (η inferior to unity but non-zero). A decrease of the elasticity of substitution represents an increase of the inequality aversion.[4] It is non-paternalist (only utility matters), paretian, symmetric[5] and concave.

The intergenerational dimension is dealt with through a value function of any maximal intertemporal welfare: $V(W(u_{A1}, u_{B1}), W(u_{A2}, u_{B2}), \dots)$. I do not solve such a problem, I simply assume the instantaneous welfare to be strictly increasing with respect to consumption and the value function to be non-declining with the stock.

2.2 Utility Possibility Frontier with Lump-Sum Transfer

An agent has access to a resource and another does not. If one wants to implement a mechanism of transfer, a first step can be to determine the constraint set of the social planner.[6] In this perspective, the social planner can virtually take an amount from what the worker harvests, to give it to the receiver. For each amount of what would be transfered, one can determine the optimal utility of each agent. By continuity, one can construct a frontier, which gives the maximum of utility of the worker, given that of the receiver (and vice versa). That frontier bounds

[3] I assume $\lim_{X \to 0} \phi'(X) > \lim_{X \to 0} \frac{\partial H(\cdot)}{\partial X}$. Otherwise, the stock is asymptotically exhausted.

[4] The cases of symmetric minimum and pure utilitarianism (mean) are obtained in the limit when, respectively, θ tends to zero or to infinity (see the Appendix B).

[5] I refrain myself to put different weights to individuals. Though, this might be interesting, in a political economics perspective for example, in order to measure "inequity". This is beyond the scope of the paper.

[6] Non-utilitarian societies might not need such sets, but as they necessary lies in such sets, there is no loss of generality.

the utility possibility set. From any state inside the set, one could make every agent better-off, but once one is on the boundary, one cannot increase the utility of an agent without decreasing the utility of the other one. This is so called "first-best Pareto frontier".

As leisure time of the worker is the only decision variable, for simplicity, I will express the problem and solve it with respect to leisure time. The transfer being absolute, the worker can consume what s/he harvests minus a constant transfer: $c_A = c - c_B$. The budget constraint can thus be rewritten as the consumption minus the transfer: $c_A = (1 - l_A)h(X) - c_B$.

For a given utility of the agent B (i.e. a given transfer), the agent A makes a labor-leisure trade-off so as to maximize his/her utility. As the utility function of the agent A is assumed to be homothetic, for a given consumption per leisure time, the marginal rate of substitution (MRS) is constant. It will be convenient to express this ratio as a function of the optimal MRS. Let Ω be such a function. It is an increasing function due to the strict convexity of indifference curves. The following proposition characterizes the First-Best frontier. All proofs are in the Appendix C.

Proposition 1 (First-Best Frontier)**.** *The maximal utility $u_A(l_A, c_A)$ subject to a constant transfer c_B represents a frontier in the (u_A, u_B) map, on which the following holds:*

- $u_A^\star = u_A\left(\frac{h(X)-c_B}{h(X)+\Omega(h(X))}, \frac{\Omega(h(X))(h(X)-c_B)}{h(X)+\Omega(h(X))}\right)$ *and* $u_B^\star = u_B(1, c_B)$
- $\frac{\mathrm{d}u_A^\star}{\mathrm{d}c_B} < 0$ *and* $\frac{\mathrm{d}u_B^\star}{\mathrm{d}c_B} > 0$

The frontier is parametrized by the amount transfered c_B. Not surprisingly, the higher the transfer the lower the utility of the worker and the higher the one of the receiver. At the limits, $u_A^\star$ tends to zero when the transfer tends to its highest level $h(X)$. And it is maximal if there is no transfer. Hence, the first-best possibility frontier is strictly downward-sloping in (u_A, u_B).

2.3 Utility Possibility Frontier with Tax

I study now the case where a social planner cannot transfer an absolute amount from the harvest of the agent A to the agent B. The transfer is decentralized through a proportional tax, at the rate τ. This problem can be related to a second-best approach. This case is comparable to the previous one, but relatively different in its implications since the (implicit) relative prices change. Here, for each tax rate, one can determine optimal utilities so as to construct the "second-best Pareto frontier". This frontier bounds the utility possibility set as before.

I will explicit every variable as a function of the tax to solve the problem in that variable. The transfer being proportional, the worker can consume what s/he harvests minus the taxed part: $c_A = (1 - \tau)c$. The receiver obtains $c_B = \tau c$. The budget constraint can thus be rewritten as the maximal possible consumption net of the tax: $c_A = (1 - \tau)(1 - l_A)h(X)$.

For a given tax rate, the agent A works so as to maximize his/her utility. Let $\sigma^{\star\star}_{l_A,c_A}$ be the elasticity of substitution of the utility of the worker evaluated at the optimum. And let $\varepsilon_{x,y}$ be the elasticity of x whit respect to y. I begin with the following proposition.

Proposition 2 (Second-Best Frontier)**.** *The maximal utility $u_A(l_A, c_A)$ subject to a constant tax rate τ represents a frontier in the (u_A, u_B) map, on which the following holds:*

- $u_A^{\star\star} = u_A\left(\frac{(1-\tau)h(X)}{(1-\tau)h(X)+\Omega((1-\tau)h(X)}, \frac{\Omega((1-\tau)h(X))(1-\tau)h(X)}{(1-\tau)h(X)+\Omega((1-\tau)h(X))}\right)$ *and*
  $u_B^{\star\star} = u_B\left(1, \frac{\Omega((1-\tau)h(X))\tau h(X)}{(1-\tau)h(X)+\Omega((1-\tau)h(X))}\right)$
- $\frac{\mathrm{d}u_A^{\star\star}}{\mathrm{d}\tau} < 0$ *and*

$$\begin{cases} - \text{ if } \sigma_{l_A,c_A} \leq 1, & \frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \text{always} \\ - \text{ if } \sigma_{l_A,c_A} > 1, & \frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \text{if} \quad \tau < \bar{\tau} \equiv \frac{1}{\sigma^{\star\star}_{l_A,c_A} - \varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}} \equiv \frac{1}{1+l_A^{\star\star}\left(\sigma^{\star\star}_{l_A,c_A}-1\right)} < 1 \end{cases}$$

Here, the frontier is parametrized by the tax rate. The tax is always negative for the worker. Not surprisingly, the tax reducing his/her budget set, the utility reached is lower. But for the receiver it depends. If the worker works more when s/he is taxed more, the catch received by the receiver increases: one taxes more a higher basis. If s/he works less (what should happen more generally), the effect is *a priori* ambiguous: one taxes more a lower basis. And if the worker is indifferent, the receiver is also better-off: one taxes more a constant basis.

Not surprisingly, the reaction of the worker to the tax depends on his/her elasticity of substitution of leisure for consumption. In particular, if the worker considers his/her leisure and his/her consumption as quite complementary ($\sigma_{l_A,c_A} \leq 1$), the more s/he is taxed, the more s/he works (or does not react to), and then the agent B is getting better and better as the tax rate grows. If they are quite substitutable ($\sigma_{l_A,c_A} > 1$), the amount received by the agent B increases if and only if the tax rate is not too high. The limit being given by the inverse of the difference between the elasticity of substitution $\sigma^{\star\star}_{l_A,c_A}$ and the cross-price elasticity of leisure (with respect to the shadow price of consumption) $\varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}$. Notice that this is close to the concept of the Laffer curve (see Laffer, 2004), the receiver playing role of the State. If the threshold is strictly inferior to unity, the receiver takes advantage of a higher tax until a certain point ($\bar{\tau}$), from which on, a higher rate reduces the amount perceived.

As the cross-elasticity depends on the substitutability of leisure for consumption, the two parts of the denominator of the threshold (first expression) are linked. Indeed, the threshold is lower than one if and only if the elasticity of substitution is superior to unity. As this seems to be the more plausible case, I assume thereafter the worker has a high elasticity of substitution. This implies that the worker works less when the tax rate increases.

To sum up, under the assumption of an increasing labor supply, the utility of the worker is decreasing with respect to the tax rate while the utility of the receiver increases until a threshold and decreases afterward. Thus, one has a "bell curve" frontier. But actually only the decreasing part matters, since the increasing one represents states where both agents can be better-off. These states are Pareto-dominated, and thus not of interest from a welfare point of view.

2.4 Comparison of the Two Frontiers

I now compare the implications of the two instruments.

Proposition 3 (Comparison of the Frontiers)**.** *The Second-Best Utility Possibility Frontier lies below the First-Best Utility Possibility Frontier.*

That is to say, for a given utility of the worker, the tax is always less favorable for the receiver than the lump-sum transfer. Indeed, the tax on the production discourages the worker who works less and then harvests less. If the tax rate is higher than the threshold of the previous subsection, increasing its rate leads to worsen their both situations. As these states are not of interest in my framework, I do not consider high tax rates. Besides, from a social planner point of view, the lump-sum is always better than the tax since it allows more choices. I plotted them in the Fig 1, considering CES utility function for the worker and power function for the receiver.

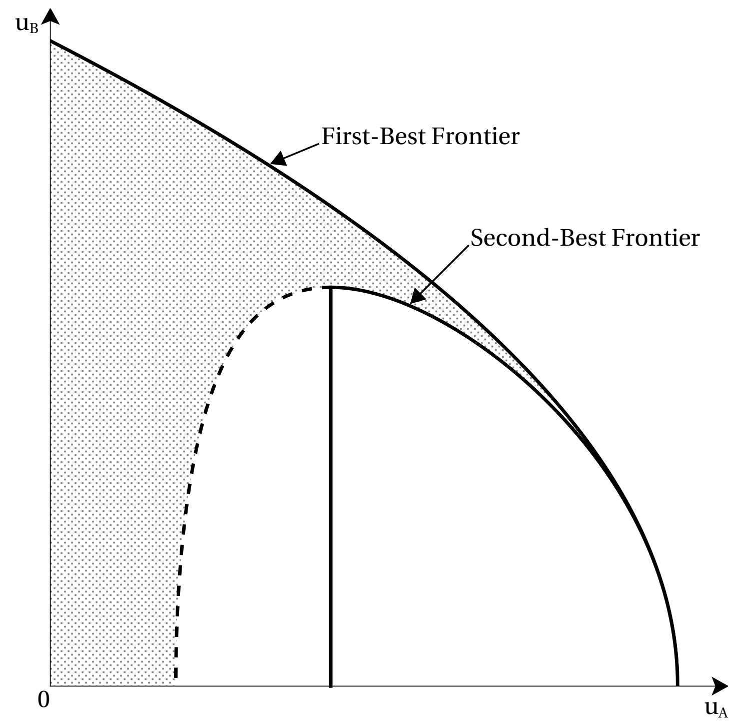


Figure 1: First and Second-Best Frontiers

Since the impact of the tax depends on the reaction of the worker, one may expect the difference between the two frontiers, i.e. the inefficiency of the tax (represented by the gray area in the Fig. 1), to be dependent on the worker preferences. Indeed, if they are characterized by a constant elasticity of substitution, it is shown in the Appendix C that the higher the elasticity, the more likely the gap between the two instruments is high[7]. At the limit, if leisure and consumption are perfectly complementary, the instrument used does no longer matters since the two frontiers merge together (Appendix C).[8] That latter result is not surprising since in this case the (implicit) relative prices do not matter for the worker's choice.

2.5 Welfare Analysis

I built utility possibility sets (bounded by the frontiers). I am now interested in finding the optimal allocations of utility, which indicate the optimal transfer (absolute or proportional). For that I want to compare two different societies with different inequality aversion. But for the sake of simplicity, I consider a single society that changes its inequality aversion, for whatever reasons. In this perspective, I will see the consequence of a change in the inequality aversion on the optimal allocation. Let us recall that a higher inequality aversion corresponds to a lower elasticity of substitution ($-\mathrm{d}\theta > 0$). The welfare analysis is qualitatively the same with both mechanisms. I present it only with the lump-sum.

In general terms, one has to seek the maximal welfare subject to the fact that utilities are elements of the utility possibility set. Here, as I solved already a maximization problem, I can maximize directly the welfare through the amount transfered. I will always be situated on the frontier.

$$\max_{c_B} \quad W^\star(c_B) = \left(\frac{1}{2}\cdot u_A^\star(c_B)^\eta + \frac{1}{2}\cdot u_B^\star(c_B)^\eta\right)^{\frac{1}{\eta}}. \tag{1}$$

Let us define the marginal social rate of substitution (MSRS) as the willingness of society to increase marginally the utility of the agent B taking from the utility of the agent A, keeping its global satisfaction equal. Let $\lambda := \frac{u_B}{u_A}$ be the utility ratio.

Proposition 4 (Optimal allocation)**.** *An optimal allocation of the problem* (1) *satisfies* $\lambda^\star = MSRS^{\star\theta}$. *Hence* $\frac{\mathrm{d}\lambda^\star}{-\mathrm{d}\theta} \geq 0 \Leftrightarrow \lambda^\star \leq 1$.

The optimal utility ratio is a function of the MSRS and the inequality aversion. In particular, the lower the slope of the frontier in absolute value, the lower the utility ratio. The more difficult is to get a high increasing of the utility of the agent B taking from the utility of the agent A, the more one moves toward a situation where the agent B has a low utility compared to that of the agent A. A MSRS* lower than one corresponds to a $\lambda^\star$ lower than one. Then, the optimal utility ratio depends negatively on the elasticity of substitution (positively on the inequality aversion) if it is originally lower than one, and vice versa. If the optimal MSRS equal to one, the utility

[7]Rigorously speaking, the more the worker is marginally productive (high $h(X)$) and weights consumption compared to leisure, the more likely a lower elasticity of substitution implies *always* a lower inefficiency of the tax. Otherwise, it is true only if the elasticity of substitution is under a threshold (see the Appendix C).

[8]I exclude the perfect substitutes case to avoid unrealistic corner solutions (i.e. no work or no leisure).

ratio equal to one, then the optimal ratio is independent to the inequality aversion in such a case. Therefore, the impact of the inequality aversion depends on the initial situation. Two types of frontier illustrate it in the Fig. 2.

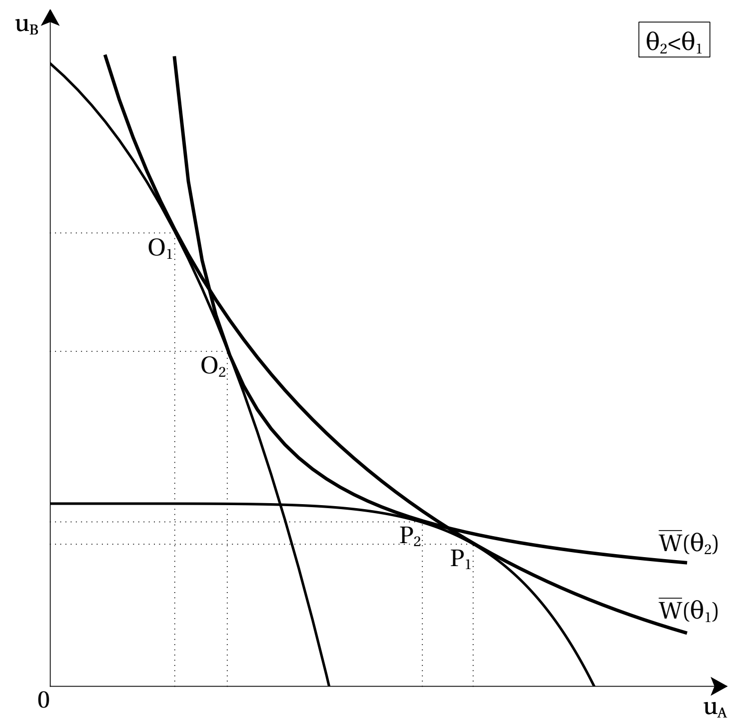


Figure 2: Different inequality aversions with two types of frontier

If the agent *B* is relatively better-off in the original situation, the higher the inequality aversion (the elasticity of substitution pass from θ_1 to θ_2) the lower the corresponding utility ratio (from O_1 to O_2). This implies to reduce the optimal transfer from the worker to the receiver. On the contrary, if the agent *A* is relatively better-off at the beginning, the higher the inequality aversion the higher the utility ratio (from P_1 to P_2). This implies to rise the optimal transfer. And finally, if there is a perfect equality between individuals (ratio equal to one), inequality aversion has no influence. In the end, a higher inequality aversion implies always a more egalitarian situation, but according to the shape of the frontier, a perfect equality may never be obtained.

The counter-intuitive case *O* arises because it is not a rich-poor metaphor. The social planner is assumed here to be able to judge the situation of individuals and to act according to "acceptable claims". And it may be the case that the receiver meets his/her needs with little, such that a higher inequality aversion implies to transfer less. The measure of utilities is therefore very important here.

3 Intragenerational and Intergenerational Considerations

Sustainability can be measured through net investment, once one adopts a value function (Asheim, 2007). Without a particular formulation of such a function, one can assume it to be non-declining with respect to the resource stock. Formally, the evolution of the value function over time is (for an autonomous problem)

$$\dot{V}(X) = \underbrace{\frac{\partial V(X,t)}{\partial X}}_{v} \dot{X} \; . \tag{2}$$

The value function is increasing over time if Genuine savings is positive (shadow price v times the evolution of the stock). But this is an indicator and cannot directly be used to measure intergenerational equity. I will thus simply focus on instantaneous welfare, but verifying that the value function does not decline along a path.

I will see first how the inequality aversion impacts the current consumption. Second, I will see how the evolution of the resource, in turn, transforms the welfare possibilities.

3.1 Inequality Aversion and the Current Consumption

The consequences of a variation of the inequality aversion, say an increase, on the current global consumption depends on the redistribution mechanism used.

Lump-sum Inequality aversion has ambiguous effects on the repartition. From the previous section, I know that one will converge toward a more egalitarian situation. That is, the evolution of the utility ratio depends on the initial situation. If the utility of the agent A is high compared to the utility of the agent B in the initial situation, the utility ratio grows with a higher inequality aversion. In this case, the agent A works more and gets a lower utility, while the agent B gets a higher utility.

As the global catch increases with work, global consumption rises.

Tax This case is more difficult to analyze since the reaction to the tax may be ambiguous (contrary to the transfer). Be that as it may, I still restrict myself to the case of an increasing labor supply (decreasing with respect to the tax). Inequality aversion has ambiguous effects here also. According to the initial situation, a higher inequality aversion may advantage one or the other agent. Let us analyze, as before, the case that benefits to the receiver. In that case, the tax rate rises. With my restriction, the agent A works less and gets a lower utility, while the agent B gets a higher utility.

As the global catch decreases with leisure, global consumption reduces here.

Whatever the mechanism used, if the utilities are equal, as the inequality aversion has no influence on the redistribution, it has no influence on the global consumption too. The table 1 summarizes the different situations.

$\text{sign}\left(\frac{\mathrm{d}c_{\text{opt}}}{-\mathrm{d}\theta}\right)$	$\frac{u_B}{u_A} < 1$	$\frac{u_B}{u_A} = 1$	$\frac{u_B}{u_A} > 1$
Lump-sum	+	0	-
Tax	-	0	+

Table 1: Evolution of the global consumption when inequality aversion rises

3.2 Evolution of the Resource and the Possibilities for Futures Welfares

The redistribution influences the current consumption which has consequences on the evolution of the resource stock. And reciprocally, the evolution of the resource stock has consequences on the possibilities of consumption and then on possibilities of redistributions. If the resource stock varies, I have to determine beforehand to whom will go that supplement (variations of X on $\lambda^\star$). This should be done according to the social welfare function. But, to begin, I am interested in the deformation of the frontier due to the variation of the resource stock.

In the first-best approach, let us consider that an agent keeps constant his/her utility, while the other one maximizes his/hers. For example, one may consider the receiver get a constant transfer. In the case of an increasing stock, the budget set of the worker is expansing. And therefore, the utility of the worker is increasing. That is to say, the agent A *can* take advantage of a higher resource for every level of the utility of the agent B (and vice versa). However, I cannot conclude that everybody will be better-off with more resource, but that they have the possibility to, since the first-best frontier is expansing.[9] In the tax case, as there is a (virtual) backward-bending relation between both utility with the tax, it is easier to fix the well-being of the worker. To make the worker indifferent when the stock evolves, one has to modify the tax rate. Obviously, it has to rise if the resource grows. Let us do it so as the net budget constraint remains the same, the receiver gets more while the worker is indifferent. The frontier is therefore always expansing with an increasing resource.

Proposition 5 (Variations of the frontiers)**.** *For $X' > X$, the Utility Possibility Frontiers associated with X lie strictly below the Utility Possibility Frontiers associated with X'.*

As the frontiers are shifting outwards with an increase of the resource stock, the welfare is unambiguously increasing too.[10] I can then link the evolution of the welfare with the evolution

[9] Strictly speaking, as individuals are supposed to live “one instant”, more resources allows more possibilities for their descendants.

[10] At this stage, I was not able to prove that the optimal utilities are effectively increasing. I suppose they are.

of the resource. Hence, a variation of the inequality aversion has a direct effect on the current generation through the transfer and an indirect one on future generations through the evolution of the stock.

3.3 Can Intragenerational and Intergenerational Equity be Reconciled?

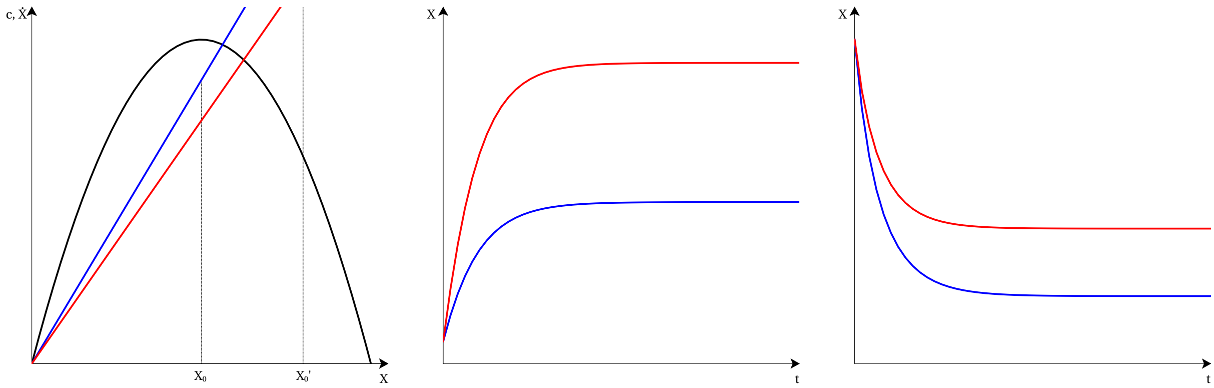
Let us recall that an increasing inequality aversion implies a rising of the global consumption in two situations. Either using a lump-sum when the worker is better-off or using a tax when the receiver is better-off. Is this good for the intergenerational concern? The answer shall be based on the evolution of welfare along time. More precisely, we should analyze the modification of the welfare path after a change in the inequality aversion. As we know that the welfare is positively linked with the resource (see Proposition 5), we will focus on the resources paths. Four cases can be distinguished depending on, on the one hand, whether the resource stock is rising or declining, and one the other hand, whether the steady state stock level is higher or lower than the Golden Rule one[11]. The steady state stock level depends on the productivity of the worker. I consider a productivity as being relatively low if the the steady state stock level if superior to $\bar{X}$, and vice versa.

The Fig. 3 plots the different situations, with a logistic growth and a linear catch-effort curve. The initial situations are in red, the news ones are in blue. I plotted a low productivity situation (a), and corresponding paths with a rising stock (b) and a decreasing one (c). Symmetrically, I plotted a high productivity situation (d), with a rising stock (e) and a decreasing one (f). In all the situations, a higher consumption impact negatively the resource: the growth is lower (resp. the decreasing is higher) and it converges to a lower steady state.

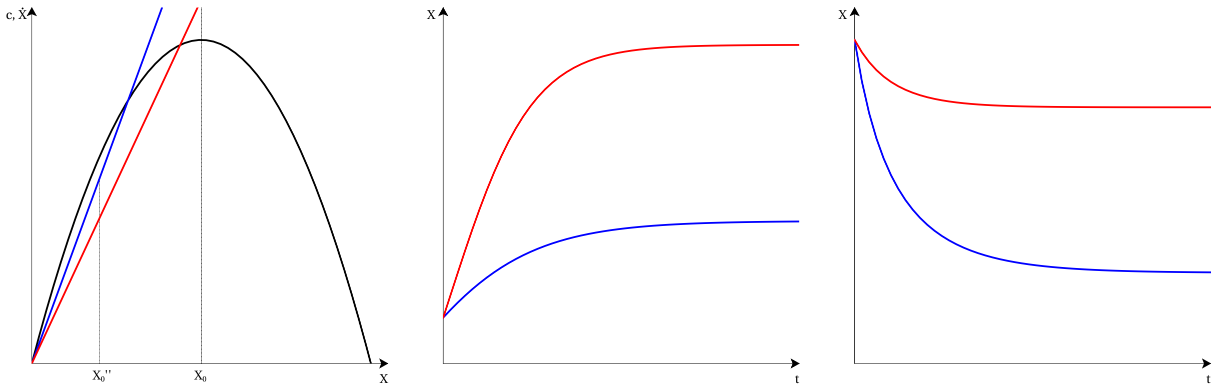
Therefore, if a society, for whatever reasons, decides to increase its inequality aversion, it has to be in such a way that the global consumption decreases. If people harvesting a resource are better-off than the receivers, the transfer has to be implemented by a proportional tax that discourage them. And, in the counter-intuitive case, if the receivers are better-off, the transfer has to be made through a lump-sum. In that way, decreasing the transfer makes the harvesters compensating less, and then harvesting less.

These results illustrate the facilitations and rivalries between the two dimensions guessed by Baumgärtner et al. (2012). Here, in an optimization framework, either the two dimensions facilitate each other (using the right mechanism) or trade-offs have to me made (when using the right mechanism is not possible).

[11]By definition, a steady state $\tilde{X} > 0$ satifies $\dot{X} = 0 \Leftrightarrow F(\tilde{X}) = (1 - l_A^\star)h(\tilde{X})$. And $\bar{X}$ is such that $F'(\bar{X}) = 0$.



(a) Low productivity... (b) ...with a rising stock (c) ...with a decreasing stock



(d) High productivity... (e) ...with a rising stock (f) ...with a decreseaing stock

Figure 3: Resource paths as indicators of intergenerational welfare

4 Conclusion

If one type of agent has access to a resource and another does not have access to it, society may want to implement a transfer of a part of the harvest from the first one to the second one. I studied the effects of an absolute transfer and of a relative one based on the resource caught. I constructed utility possibility frontiers in each case. They represent the necessary trade-offs a society has to make when it wants to enhance the well-being of one agent at the expense of another. Not surprisingly, the utility possibility set with a tax is lower than the one with a lump-sum transfer. The inefficiency of the tax depends on the preferences of the worker. Besides, the social criterion allowed for find the social optimum, and I was particularly interested in its variation due to a change in the inequality aversion. This depends on the initial situation and on the mechanism of transfers used. In particular, I found that society may afford a higher inequality aversion without worsening the future only with one mechanism. If the worker is better-off, a rising transfer shall be made through a tax. And if the receiver is better-off, a decreasing transfer shall be made through a lump-sum. In any cases, the global consumption decreases.

The heterogeneity was intentionally radical. One agent has access to a resource and another does not. Nonetheless it could be interesting to analyze the effects of a transfer between two agents having a different access. The redistribution could have influence on both of them.

A Assumptions on Utility

- The assumption of concavity needs no justification since its extensive use in the economic literature (notice that it implies cardinality[12]).
- The homothecy (only necessary for the agent A) is purely for mathematical convenience. Nonetheless many common utility functions have this feature.[13]
- The assumption of interpersonal comparison needs more comments. To explain it, I think that the utility function has to be differently understood. Here, it is not only an individualistic measure of well-being, but an objective evaluation of a 'legitimate request'. It can be obtained by revealed preferences for example. Further, I think that society is able to say whom is worse-off between to types of individuals. Here, it can be justifiable since natural resources may be inclusive of "primary goods". Besides, it should be recalled that avoiding any utility comparison lead to dictatorship according to the Arrow's classical theorem. And the assumption of comparison allows for comparing utilitarianism and leximin (lexicographic maximin) (d'Aspremont and Gevers, 1977).
- Besides, each good is assumed to be essential: $\lim_{c_k \to 0} \frac{\partial u_k(\cdot)}{\partial c_k}\Big|_{u_k} = \lim_{l_k \to 0} \frac{\partial u_k(\cdot)}{\partial l_k}\Big|_{u_k} = \infty,\ k = i, j.$

B Proof of Special Cases of CES Utility Function

It could seem to be a pointless exercise to demonstrate a very well-known result. But, to my knowledge, a rigorous demonstration of the minimum is not present in the literature.

Proof. Let us consider a continuously differentiable function with a constant elasticity of substitution $\theta \equiv \frac{1}{1-\rho}$: $f(x_1, \ldots, x_n) = \left(\alpha_1 x_1^\rho + \cdots + \alpha_n x_n^\rho\right)^{\frac{1}{\rho}}$. With $\sum_{i=1}^{i=n} \alpha_i = 1,\ \alpha_i > 0\ \forall i$ and $\rho < 1, \rho \neq 0$.

- Case 1: the elasticity tends to positive infinity. Trivially, if $\theta \to \infty$ ($\rho \to 1$), the CES function tends to the perfect substitutes function.

$$\lim_{\rho \to 1} f(x_1, \ldots, x_n) = \alpha_1 x_1 + \cdots + \alpha_n x_n \,. \tag{3}$$

[12] I thank Ludovic Julien for having pointed it to me.

[13] Formally, $-\frac{\mathrm{d}c_A}{\mathrm{d}l_A}\Big|_{U(c_A, l_A)} = -\frac{\mathrm{d}c_A}{\mathrm{d}l_A}\Big|_{U(\beta c_A, \beta l_A)}$, $(\beta > 0)$.

- Case 2: the elasticity tends to zero.[14] Let us rewrite the CES function as:

$$f(x_1,\ldots,x_n) = x_k\left(\alpha_1\left(\frac{x_1}{x_k}\right)^{\rho} + \cdots + \alpha_k + \cdots + \alpha_n\left(\frac{x_n}{x_k}\right)^{\rho}\right)^{\frac{1}{\rho}} . \tag{4}$$

  Let $\min\{x_1,\ldots,x_n\} = x_k$. Then

$$\lim_{\rho\to-\infty}\left(\frac{x_i}{x_k}\right)^{\rho} = 0,\ \forall i \neq k . \tag{5}$$

  Thus

$$\lim_{\rho\to-\infty} x_k\left(\alpha_1\left(\frac{x_1}{x_k}\right)^{\rho} + \cdots + \alpha_k + \cdots + \alpha_n\left(\frac{x_n}{x_k}\right)^{\rho}\right)^{\frac{1}{\rho}} = x_k . \tag{6}$$

  As x_k can be any good,

$$\lim_{\rho\to-\infty} f(x_1,\ldots,x_n) = \min\{x_1,\ldots,x_n\} . \tag{7}$$

- Case 3: for exhaustiveness, let the elasticity tend to one. Let us take the logarithm of $f(\cdot)$:

$$\ln\left(f(x_1,\ldots,x_n)\right) = \frac{\ln\left(\alpha_1 x_1^{\rho} + \cdots + \alpha_n x_n^{\rho}\right)}{\rho} . \tag{8}$$

  Let $g_1(\rho)$ and $g_2(\rho)$ be equivalent, respectively, to the numerator and to the denominator. As $\lim_{\rho\to0} g_1(\rho) = \lim_{\rho\to0} g_2(\rho) = 0$, by the Hospital's rule:

$$\lim_{\rho\to0}\frac{g_1(\rho)}{g_2(\rho)} = \lim_{\rho\to0}\frac{g_1'(\rho)}{g_2'(\rho)} , \tag{9}$$

  and as

$$\frac{g_1'(\rho)}{g_2'(\rho)} = \frac{\alpha_1 x_1^{\rho}\ln(x_1) + \cdots + \alpha_n x_n^{\rho}\ln(x_n)}{\alpha_1 x_1^{\rho} + \cdots + \alpha_n x_n^{\rho}} , \tag{10}$$

  one gets

$$\lim_{\rho\to0}\ln\left(f(x_1,\ldots,x_n)\right) = \alpha_1\ln(x_1) + \cdots + \alpha_n\ln(x_n) . \tag{11}$$

  Finally,

$$\lim_{\rho\to0} f(x_1,\ldots,x_n) = x_1^{\alpha_1}\ldots x_n^{\alpha_n} . \tag{12}$$

□

[14] I am grateful to Jean-Baptiste Michau for having provided to me this proof. I also thank Théo Benonnier for having informed me of its existence.

C Proofs of the Propositions

C.1 First-Best Frontier

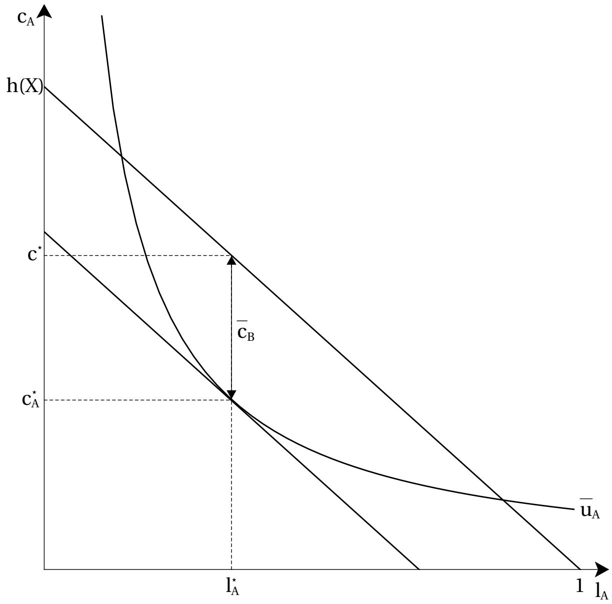


Figure 4: Construction of the utility possibility frontier with lump-sum transfers

Proof of Proposition 1. I have $u_A(l_A, c_A)$ and $c_A = c - c_B$. I substitute the budget constraint with the available consumption in the utility, so as to maximize it in l_A.

$$\max_{l_A} \quad u_A\left(l_A, (1-l_A)h(X) - c_B\right) \tag{13}$$

A necessary condition to maximize the utility is that the marginal productivity of labor equal to the MRS of leisure for consumption: $h(X) = \frac{\partial u_A(\cdot)/\partial l_A}{\partial u_A(\cdot)/\partial c_A}$.

Due to homothecy of the utility function, I can explicit the optimal consumption of the worker as a function of the leisure time, so as to obtain the expansion path.

$$\frac{c_A}{l_A} = \Omega\left(h(X)\right) \quad \Leftrightarrow \quad c_A = l_A \cdot \Omega\left(h(X)\right)\ . \tag{14}$$

Substituting it into the budget constraint to get $l_A^\star$, and substituting $l_A^\star$ into the expansion path to get $c_A^\star$.

$$l_A^\star = \frac{h(X) - c_B}{h(X) + \Omega(\cdot)} \quad \text{and} \quad c_A^\star = \frac{\Omega(\cdot)\left(h(X) - c_B\right)}{h(X) + \Omega(\cdot)}\ . \tag{15}$$

Trivially $c_B^\star = c_B$.

For the shape, I differentiate the utilities with respect to the transfer.

$$\frac{\mathrm{d}u_A^\star}{\mathrm{d}c_B} = \frac{\partial u_A^\star}{\partial l_A}\frac{\mathrm{d}l_A^\star}{\mathrm{d}c_B} + \frac{\partial u_A^\star}{\partial c_A}\frac{\mathrm{d}c_A^\star}{\mathrm{d}c_B} = \underbrace{\frac{\partial u_A^\star}{\partial l_A}}_{>0}\underbrace{\left(-\frac{1}{h(X)+\Omega(\cdot)}\right)}_{<0} + \underbrace{\frac{\partial u_A^\star}{\partial c_A}}_{>0}\underbrace{\left(-\frac{\Omega(\cdot)}{h(X)+\Omega(\cdot)}\right)}_{<0} < 0\,. \quad (16)$$

$$\frac{\mathrm{d}u_B^\star}{\mathrm{d}c_B} = \frac{\partial u_B^\star}{\partial c_B} = \frac{\partial u_B^\star}{\partial c_B} > 0\,. \quad (17)$$

Hence $\frac{\mathrm{d}u_B^\star}{\mathrm{d}u_A^\star} < 0$. □

C.2 Second-Best Frontier

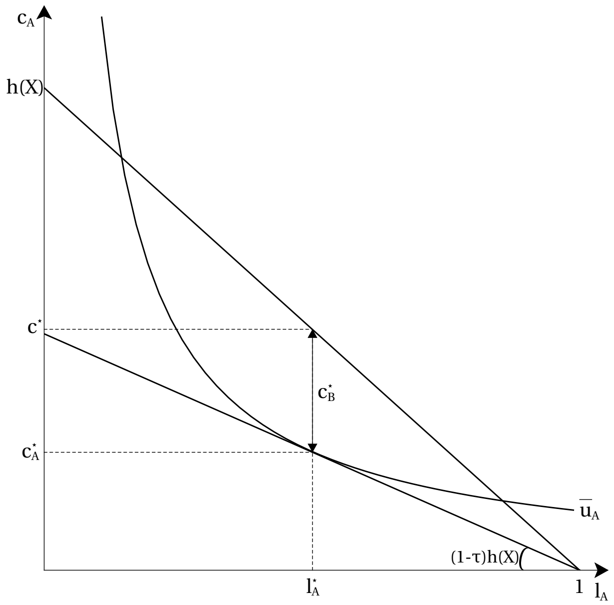


Figure 5: Construction of the utility possibility frontier with a proportional tax

Proof of Proposition 2. Let us consider a given tax rate. The agent A makes a labor-leisure trade-off so as to maximize his/her utility. I substitute, here also, the budget constraint with the consumption in the utility, so as to maximize it with respect to leisure time.

$$\max_{l_A} \quad u_A\left(l_A, (1-\tau)(1-l_A)h(X)\right)\,. \quad (18)$$

A necessary condition to maximize the utility is that the *net* marginal productivity of labor equal to the MRS of leisure for consumption: $(1-\tau)h(X) = \frac{\partial u_A/\partial l_A}{\partial u_A/\partial c_A}$. Due to homothecy of the utility function, I can express the optimal consumption as a function of leisure and of the tax rate.

$$\frac{c_A}{l_A} = \Omega\left((1-\tau)h(X)\right) \quad \Leftrightarrow \quad c_A = l_A \cdot \Omega\left((1-\tau)h(X)\right)\,. \quad (19)$$

Using the same methodology than with the absolute transfer:

$$l_A^{\star\star} = \frac{(1-\tau)h(X)}{(1-\tau)h(X)+\Omega(\cdot)} \quad \text{and} \quad c_A^{\star\star} = \frac{\Omega(\cdot)(1-\tau)h(X)}{(1-\tau)h(X)+\Omega(\cdot)} \,. \tag{20}$$

Recall that $c_B = \tau c$ and $c_A = (1-\tau)c$, hence $c_B = \frac{\tau}{1-\tau}c_A$. I have

$$c_B^{\star\star} = \frac{\Omega(\cdot)\tau h(X)}{(1-\tau)h(X)+\Omega(\cdot)} \,. \tag{21}$$

For the shape, as several stages are necessary, I put the proof in a separate subsection.

The shape of the Second-Best Frontier

Evolution of the Utility with Respect to the Tax Rate For the worker:

$$\frac{\mathrm{d}u_A^{\star\star}}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_A^{\star\star}}{\partial l_A}}_{>0}\underbrace{\frac{\mathrm{d}l_A^{\star\star}}{\mathrm{d}\tau}}_{?} + \underbrace{\frac{\partial u_A^{\star\star}}{\partial c_A}}_{>0}\underbrace{\frac{\mathrm{d}c_A^{\star\star}}{\mathrm{d}\tau}}_{?} \,. \tag{22}$$

Actually, since the constraint set is diminishing with the tax, the utility of the worker will reduce.

$$-\left.\frac{\mathrm{d}c_A}{\mathrm{d}l_A}\right|_{\tau_1} > -\left.\frac{\mathrm{d}c_A}{\mathrm{d}l_A}\right|_{\tau_2>\tau_1} \quad \Rightarrow \quad -\frac{\mathrm{d}c_A^{\star\star}}{\mathrm{d}l_A} > \frac{\frac{\partial u_A^{\star\star}}{\partial l_A}}{\frac{\partial u_A^{\star\star}}{\partial c_A}} \quad \Leftrightarrow \quad \frac{\mathrm{d}u_A^{\star\star}}{\mathrm{d}\tau} < 0\,. \tag{23}$$

For the receiver:

$$\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_B^{\star\star}}{\partial c_B}}_{>0}\underbrace{\frac{\mathrm{d}c_B^{\star\star}}{\mathrm{d}\tau}}_{?} \,. \tag{24}$$

Recall that $c_B = \tau c$.

$$\frac{\mathrm{d}u_B}{\mathrm{d}\tau} = \frac{\partial u_B}{\partial c_B}\frac{\mathrm{d}c_B}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_B}{\partial c_B}}_{>0}\left(\underbrace{(1-l_A)h(X)}_{>0} - \underbrace{\tau h(X)}_{>0}\underbrace{\frac{\mathrm{d}l_A^{\star\star}}{\mathrm{d}\tau}}_{?}\right) \,. \tag{25}$$

It is straightforward to see that if $\frac{\mathrm{d}l_A^{\star\star}}{\mathrm{d}\tau} \leq 0$ then $\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0$. But, otherwise, I cannot conclude.

Evolution of the Utility of the Receiver When the Tax Rate Varies The analyze of the sign of $\frac{\mathrm{d}c_B^{\star\star}}{\mathrm{d}\tau}$ is meaningless[15]. To obtain an explicable condition, I use the consumption of the worker

[15] $\frac{\mathrm{d}c_B^{\star\star}}{\mathrm{d}\tau} > 0 \Leftrightarrow \tau < \frac{h(X)+\Omega((1-\tau)h(X))}{\sigma_{l_A,c_A}^{\star\star}h(X)}$.

from the expansion path (eq. (19)). $c_B(l_A) = \frac{\tau}{1-\tau} c_A(l_A)$. Then;

$$\frac{\mathrm{d}c_B}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad \frac{c_A}{(1-\tau)^2} + \frac{\tau}{1-\tau}\frac{\mathrm{d}c_A}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad -\tau\frac{\mathrm{d}c_A}{\mathrm{d}\tau} < \frac{c_A}{1-\tau} \quad \Leftrightarrow \quad -\frac{\mathrm{d}c_A}{\mathrm{d}\tau}\frac{1-\tau}{c_A} < \frac{1}{\tau}\,;$$
$$\Leftrightarrow \quad -\left(\frac{\mathrm{d}l_A}{\mathrm{d}\tau}\Omega(\cdot) - l_A \cdot h(X)\Omega'(\cdot)\right) \times \frac{1-\tau}{l_A \cdot \Omega(\cdot)} < \frac{1}{\tau}\,;$$
$$\Leftrightarrow \quad \frac{\mathrm{d}l_A}{\mathrm{d}(1-\tau)}\frac{1-\tau}{l_A} + \Omega'(\cdot)\frac{(1-\tau)h(X)}{\Omega(\cdot)} < \frac{1}{\tau}\,. \tag{26}$$

The first term of the left-hand side is the elasticity of leisure with respect to $(1-\tau)$. Evaluated at the optimum, it is noted $\varepsilon^{\star\star}_{l_A,(1-\tau)}$. It can be easily shown[16] that $\varepsilon^{\star\star}_{l_A,(1-\tau)} = -\varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}$. So, the first term is the opposite of the cross-price elasticity of leisure.[17] The second term of the left-hand side equal to the elasticity of substitution of leisure for consumption, evaluated at the optimum, let it noted $\sigma^{\star\star}_{l_A,c_A} > 0$.[18] Thus (the positivity of the denominator is shown later):

$$\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad \tau < \bar{\tau} \equiv \frac{1}{\sigma^{\star\star}_{l_A,c_A} - \varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}}\,. \tag{27}$$

Links of the Two Parts of the Threshold As the cross-price elasticity depends on the reaction of the worker to the tax, the threshold can be linked with the elasticity of substitution. For that, let us compute the cross-price elasticity at the optimum:

$$\begin{aligned}
\varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}\bigg|_{l_A=l_A^{\star\star}} &= -\varepsilon^{\star\star}_{l_A,(1-\tau)}\bigg|_{l_A=l_A^{\star\star}}\,;\\
&= -\frac{1-\tau}{l_A^{\star\star}}\frac{\mathrm{d}l_A^{\star\star}}{\mathrm{d}(1-\tau)}\,;\\
&= -\frac{1-\tau}{\frac{(1-\tau)h(X)}{(1-\tau)h(X)+\Omega(\cdot)}}\times\\
&\quad \frac{h(X)((1-\tau)h(X)+\Omega(\cdot)) - (1-\tau)h(X)(h(X)+h(X)\Omega'(\cdot)))}{((1-\tau)h(X)+\Omega(\cdot))^2}\,;\\
&= -\frac{(1-\tau)h(X)+\Omega(\cdot)-(1-\tau)h(X)-(1-\tau)h(X)\Omega'(\cdot)}{(1-\tau)h(X)+\Omega(\cdot)}\,;\\
&= \frac{(1-\tau)h(X)\Omega'(\cdot)-\Omega(\cdot)}{(1-\tau)h(X)+\Omega(\cdot)}\,;\\
&= \left(\frac{(1-\tau)h(X)\Omega'(\cdot)}{\Omega(\cdot)} - 1\right)\frac{\Omega(\cdot)}{(1-\tau)h(X)+\Omega(\cdot)}\,;\\
&= \left(\sigma^{\star\star}_{l_A,c_A} - 1\right)\left(1 - l_A^{\star\star}\right)\,.
\end{aligned} \tag{28}$$

[16] $\frac{\mathrm{d}(1-\tau)}{1-\tau} = \frac{1}{1-\tau}\frac{1}{\frac{\mathrm{d}\left(\frac{1}{1-\tau}\right)}{\mathrm{d}(1-\tau)}}\mathrm{d}\left(\frac{1}{1-\tau}\right) = -\frac{\mathrm{d}\left(\frac{1}{1-\tau}\right)}{\frac{1}{1-\tau}}$.

[17] Notice that $c_A = (1-\tau)(1-l_A)h(X) \Leftrightarrow \frac{1}{1-\tau}c_A = (1-l_A)h(X)$, so $\frac{1}{1-\tau}$ is the shadow price of consumption.

[18] At the optimum: $\sigma^{\star\star}_{l_A,c_A} = \frac{\mathrm{d}\Omega(\cdot)}{\mathrm{dMRS}}\frac{\mathrm{MRS}}{\Omega(\cdot)}$.

What implies (for a non-zero work time)

$$\sigma^{\star\star}_{l_A,c_A} \leq 1 \quad \Leftrightarrow \quad \varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}\Big|_{l_A=l_A^{\star\star}} \leq 0\,. \tag{29}$$

Let us recall that $\frac{\mathrm{d}l_A^{\star\star}}{\mathrm{d}\tau} \leq 0 \Rightarrow \frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0$, and noticing that $\varepsilon^{\star\star}_{l_A,\tau}$ has the same sign as $\varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}$.[19] It comes

$$\sigma^{\star\star}_{l_A,c_A} \leq 1 \quad \Rightarrow \quad \frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0\,. \tag{30}$$

Besides, using the equation (28), the threshold of the inequality (27) becomes

$$\bar{\tau} \equiv \frac{1}{1+l_A^{\star\star}\left(\sigma^{\star\star}_{l_A,c_A}-1\right)}\,. \tag{31}$$

It is always positive. It can directly be seen that it is lower than one if and only if the elasticity of substitution is higher than one. To sum up:

$$\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \tag{32}$$

$$\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad \begin{cases} \text{if} \quad \sigma^{\star\star}_{l_A,c_A} \leq 1 & \text{always} \\ \text{if} \quad \sigma^{\star\star}_{l_A,c_A} > 1 & \text{if} \qquad \tau < \bar{\tau} \end{cases} \tag{33}$$

□

C.3 Comparison of Utility Possibility Sets

Proof of the Proposition 3. For a given utility of the agent A, let us seek the highest amounts one can transfer to the agent B, whatever the mechanism used. Let $\tilde{c}^i(l_A)$ be the image of an indifference curve of the agent A. And let us maximize the gap between the production and that curve.

$$\max_{l_A} \quad c_B = (1-l_A)h(X) - \tilde{c}^i(l_A)\,. \tag{34}$$

It is not hard to show that a necessary condition of this problem is the equalization of the MRS with the marginal productivity: $h(X) = \text{MRS}$. As this is done with the lump-sum transfer, this mechanism is then the most favorable for the agent B. For a given utility of the agent A, the first-best frontier corresponds to the highest utility for the agent B. Hence, the utility possibility set with the lump-sum transfer contains the set with the tax. The two frontiers coincide well when no transfer occurs, since net and gross productivity are equivalent. □

[19] $\text{sign}\left(\varepsilon^{\star\star}_{l_A,\tau}\right) = -\text{sign}\left(\varepsilon^{\star\star}_{l_A,(1-\tau)}\right) = -\text{sign}\left(-\varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}\right) = \text{sign}\left(\varepsilon^{\star\star}_{l_A,\frac{1}{1-\tau}}\right)$.

Assumption of Constant Elasticity of Substitution For a strictly convex indifference curve, the difference between the budget constraint (a straight line) and the indifference curve is strictly concave in (l_A, c_A). I know that the optimum is reached at the first-best optimal consumption-leisure ratio $\Omega(h(X))$. So, for a given utility of the worker, the higher is the difference between the first-best and the second-best optimal ratios, the higher is the distance between the two frontiers.

Let us assume here the worker has a constant elasticity of substitution function: $u_A(l_A, c_A) = \left(\gamma \cdot l_A^\rho + (1-\gamma) \cdot c_A^\rho\right)^{\frac{1}{\rho}}$. The function that links the MRS and the optimal consumption-leisure ratio is then of the form: $\Omega(z) = \left(\frac{1-\gamma}{\gamma} z\right)^\sigma$.

When the elasticity of substitution evolves, I want to determine the evolution of the difference between the two optimal ratios $D(\sigma) \equiv \Omega(h(X)) - \Omega((1-\tau)h(X))$. Let us note $r_1 \equiv \frac{1-\gamma}{\gamma} h(X)$ and $r_2 \equiv \frac{1-\gamma}{\gamma}(1-\tau)h(X)$, so that $D(\sigma) \equiv r_1^\sigma - r_2^\sigma$. Then $\frac{\mathrm{d}D(\sigma)}{\mathrm{d}\sigma} \geq 0 \Leftrightarrow \ln(r_1) r_1^\sigma - \ln(r_2) r_2^\sigma \geq 0$. There are three cases (with $\tau > 0$).

- $r_1, r_2 \geq 1$. $\frac{\mathrm{d}D}{\mathrm{d}\sigma} \geq 0 \Leftrightarrow \frac{r_1^\sigma}{r_2^\sigma} \geq \frac{\ln(r_2)}{\ln(r_1)}$. The inequality always holds.
- $r_1 \geq 1, r_2 < 1$. The inequality always holds.
- $r_1, r_2 < 1$. $\frac{\mathrm{d}D}{\mathrm{d}\sigma} \geq 0 \Leftrightarrow \frac{r_1^\sigma}{r_2^\sigma} \leq \frac{\ln(r_2)}{\ln(r_1)} \Rightarrow \sigma \leq \bar{\sigma} \equiv \frac{\ln\left(\frac{\ln(r_2)}{\ln(r_1)}\right)}{\ln\left(\frac{r_1}{r_2}\right)}$. The inequality holds as long as the elasticity of substitution is not too high.

Besides, $\lim_{\sigma \to 0} D(\sigma) = 0$.

C.4 Welfare Analysis

I study the variations of welfare with the lump-sum transfer, but it would be analytically equivalent to do it with the tax, substituting c_B for τ.

Proof of the Poposition 4.

$$\max_{c_B} \quad W^\star(c_B) = \left(\frac{1}{2} \cdot u_A^\star(c_B)^\eta + \frac{1}{2} \cdot u_B^\star(c_B)^\eta\right)^{\frac{1}{\eta}}. \tag{35}$$

First-order condition:

$$
\begin{aligned}
& \frac{\partial W^\star(c_B)}{\partial c_B} = 0\,; \\
\Leftrightarrow \quad & \frac{1}{\eta}\left(\frac{1}{2}\cdot u_A^{\star\eta} + \frac{1}{2}\cdot u_B^{\star\eta}\right)^{\frac{1-\eta}{\eta}} \left(\frac{\eta}{2} u_A^{\star\eta-1}\frac{\partial u_A^\star}{\partial c_B} + \frac{\eta}{2} u_B^{\star\eta-1}\frac{\partial u_B^\star}{\partial c_B}\right) = 0\,; \\
\Leftrightarrow \quad & u_A^{\star\eta-1}\frac{\partial u_A^\star}{\partial c_B} + u_B^{\star\eta-1}\frac{\partial u_B^\star}{\partial c_B} = 0\,; \\
\Leftrightarrow \quad & \left(\frac{u_B^\star}{u_A^\star}\right)^{1-\eta} = -\frac{\frac{\partial u_B^\star}{\partial c_B}}{\frac{\partial u_A^\star}{\partial c_B}}\,; \\
\Leftrightarrow \quad & \frac{u_B^\star}{u_A^\star} = \left(\frac{\frac{\partial u_B^\star}{\partial c_B}}{-\frac{\partial u_A^\star}{\partial c_B}}\right)^{\theta}\,. \qquad (36)
\end{aligned}
$$

The MSRS is by definition the marginal increase of the utility of the agent B when one decreases marginally the utility of the agent A, along a social indifference curve: MSRS := $\left|-\frac{\mathrm{d}u_B}{\mathrm{d}u_A}\Big|_W\right|$. It makes no difficulties to show that it equal to the ratio of social marginal welfare: MSRS $= \frac{\frac{\partial W}{\partial u_A}}{\frac{\partial W}{\partial u_B}}$.

Besides, let us differentiate the optimal welfare along a social indifference curve,

$$
\begin{aligned}
& \mathrm{d}W^\star(c_B) = 0\,; \\
\Leftrightarrow \quad & \frac{\partial W(\cdot)}{\partial u_A}\frac{\partial u_A}{\partial c_B}\mathrm{d}c_B + \frac{\partial W(\cdot)}{\partial u_B}\frac{\partial u_B}{\partial c_B}\mathrm{d}c_B = 0\,; \\
\Leftrightarrow \quad & \frac{\frac{\partial W}{\partial u_A}}{\frac{\partial W}{\partial u_B}} = \frac{\frac{\partial u_B^\star}{\partial c_B}}{-\frac{\partial u_A^\star}{\partial c_B}}\,. \qquad (37)
\end{aligned}
$$

Therefore, $\lambda^\star = \mathrm{MSRS}^{\star\theta}$.

Let us now analyze the sign of the differential of the optimal utility ratio with respect to the (inequality aversion.

$$
\frac{\mathrm{d}\lambda^\star}{-\mathrm{d}\theta} \geq 0 \quad \Leftrightarrow \quad \mathrm{MSRS}^{\star\theta}\ln\left(\mathrm{MSRS}^\star\right) \leq 0 \quad \Leftrightarrow \quad \mathrm{MSRS}^\star \leq 1 \quad \Leftrightarrow \quad \lambda^\star \leq 1\,. \qquad (38)
$$

□

C.5 Evolution of the Resource

Proof of the Proposition 5. The proof with taxes is trivial. Let us do it with lump-sums.

I am considering that the receiver keeps a fixed utility, and I study how evolves the optimal utility of the worker. But doing the converse would be equivalent. Here also, I use the consumption of the worker from the expansion path (eq. (19)).

$$\frac{\mathrm{d}u_B^\star}{\mathrm{d}X}=0 \quad \Leftrightarrow \quad \frac{\partial u_B^\star}{\partial c_B}\frac{\mathrm{d}c_B}{\mathrm{d}X}=0 \quad \Leftrightarrow \quad \frac{\partial u_B^\star}{\partial c_B}\left(\frac{\mathrm{d}c}{\mathrm{d}X}-\frac{\mathrm{d}c_A}{\mathrm{d}X}\right)=0 \quad \Leftrightarrow \quad \frac{\mathrm{d}c}{\mathrm{d}X}=\frac{\mathrm{d}c_A}{\mathrm{d}X}\,. \tag{39}$$

Recall that $c=(1-l_A)h(X)$ and $c_A=l_A\Omega(h(X))$. The previous equation becomes

$$\begin{aligned} -\frac{\mathrm{d}l_A}{\mathrm{d}X}h(X)+(1-l_A)h'(X) &= \frac{\mathrm{d}l_A}{\mathrm{d}X}\Omega(\cdot)+l_A\cdot\Omega'(\cdot)h'(X)\,; \\ \Leftrightarrow \qquad \frac{\mathrm{d}l_A}{\mathrm{d}X} &= \frac{h'(X)(1-l_A-l_A\cdot\Omega'(\cdot))}{h(X)+\Omega(\cdot)}\,. \end{aligned} \tag{40}$$

Given that, let us analyze how evolves the utility of the worker.

$$\begin{aligned} & \frac{\mathrm{d}u_A^\star}{\mathrm{d}X}>0\,; \\ \Leftrightarrow \quad & \frac{\partial u_A^\star}{\partial l_A}\frac{\mathrm{d}l_A}{\mathrm{d}X}+\frac{\partial u_A^\star}{\partial c_A}\frac{\mathrm{d}c_A}{\mathrm{d}X}>0\,; \\ \Leftrightarrow \quad & \frac{\partial u_A^\star}{\partial l_A}\frac{\mathrm{d}l_A}{\mathrm{d}X}+\frac{\partial u_A^\star}{\partial c_A}\left(\Omega(\cdot)\frac{\mathrm{d}l_A}{\mathrm{d}X}+l_A\cdot\Omega'(\cdot)h'(X)\right)>0\,; \\ \Leftrightarrow \quad & \frac{\mathrm{d}l_A}{\mathrm{d}X}>-\frac{\frac{\partial u_A^\star}{\partial c_A}\cdot l_A\cdot\Omega'(\cdot)h'(X)}{\frac{\partial u_A^\star}{\partial l_A}+\frac{\partial u_A^\star}{\partial c_A}\Omega(\cdot)}\,. \end{aligned} \tag{41}$$

Substituting the eq. (40) into the eq. (41)

$$\begin{aligned} \Rightarrow \quad & \frac{h'(X)(1-l_A-l_A\cdot\Omega'(\cdot))}{h(X)+\Omega(\cdot)}>-\frac{\frac{\partial u_A^\star}{\partial c_A}\cdot l_A\cdot\Omega'(\cdot)h'(X)}{\frac{\partial u_A^\star}{\partial l_A}+\frac{\partial u_A^\star}{\partial c_A}\Omega(\cdot)}\,; \\ \Leftrightarrow \quad & \frac{1-l_A}{l_A}>\Omega'(\cdot)\left(1-\frac{\frac{\partial u_A^\star}{\partial c_A}(h(X)+\Omega(\cdot))}{\frac{\partial u_A^\star}{\partial l_A}+\frac{\partial u_A^\star}{\partial c_A}\Omega(\cdot)}\right)\,. \end{aligned} \tag{42}$$

Notice that at the optimum $\frac{\frac{\partial u_A}{\partial l_A}}{\frac{\partial u_A}{\partial c_A}}=h(X)$. Thus, the previous term in the parenthesis is nil. In the end,

$$\left.\frac{\mathrm{d}u_A^\star}{\mathrm{d}X}\right|_{u_B^\star}>0 \quad \Leftrightarrow \quad 0<l_A<1\,. \tag{43}$$

□

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