Emissions Tax & Cap-and-Trade Schemes under Smooth Ambiguity Aversion

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Abstract: This article explores the comparative efficiency of tax and cap-and-trade (ETS) regimes under ambiguity when liable firms exhibit smooth ambiguity aversion. It thus extends the work of Baldursson & von der Fehr (2004,[3],B&vdF) carried out under risk aversion. We consider a two-period partial-equilibrium model with a continuum of homogeneous risk-neutral ambiguity-averse liable firms where ambiguity is exogenous and bears on the future allowance price or on future baseline emissions. In contrast to B&vdF, both tax and ETS are not conducive to intertemporal cost-efficiency. In the first period, relative to the ambiguity-neutral optimum, ambiguity-averse tax-liable firms always over-abate. In line with a precautionary effect, ambiguity-averse ETS-liable firms may over-abate or under-abate, which, as in B&vdF, relates to initial allocation: only when allocation is relatively low does ambiguity aversion raise date-1 abatement. In particular, firms always over-abate under symmetric allowance allocation of permits. As in Gierlinger & Gollier (2015,[28]) ambiguity aversion induces two effects: pessimism and ambiguity prudence. We provide a novel characterisation of these two effects and show with numerical simulations that pessimism generally dominates ambiguity prudence in terms of the magnitude of the induced abatement distortion. Tax-liable firms are only subject to ambiguity prudence while ETS-liable ones to both effects, so that optimal abatement decisions under ambiguity aversion are more distorted in an ETS than under a tax. When fairly-priced forwards are introduced, however, pessimism vanishes out and the two instruments are equivalent.

Keywords: Emissions Trading Schemes; Emissions Tax; Allowance Banking; Ambiguity; Ambiguity Aversion; Pessimism; Ambiguity Prudence.

JEL Classification codes: D81; D92; Q58.

1 Introduction

In the tradition following seminal work by Weitzman (1974,[68]) this article explores the comparative efficiency of price and quantity regimes under ambiguity when liable firms exhibit smooth ambiguity aversion. It extends the work of Baldursson & von der Fehr (2004,[3]), hereafter B&vdF, when ambiguity prevails and analyses ambiguity-averse firms' optimal abatement decisions in a two-period model. It also contributes to the burgeoning literature on decision-making under ambiguity aversion by providing a novel comparative characterisation of, and numerical simulations for two ambiguity aversion-induced effects: pessimism and ambiguity prudence (Gierlinger & Gollier, 2015,[28]). In essence, it thus brings together and advances two strands of literature, namely decision-making under ambiguity aversion and comparison of environmental economic instruments.

Motivation & related literature. According to Hasegawa & Salant (2014,[34]), "Permit markets may be subject to three kinds of uncertainty: (1) uncertainty about the aggregate demand for permits that will be resolved by an information disclosure at a fixed date in the future; (2) aggregate demand shocks in each period; and (3) regulatory uncertainty." Such pervasive uncertainty is notably at the source of ongoing design reforms in the form of *ex-post* allowance supply management in all existing Emissions Trading Schemes (ETS). Point (1) is illustrated by Phase I of the EUETS where EUAs suddenly lost two thirds of their value consecutive to the disclosure of verified emissions showing a low demand for permits, since Phase I vintage allowances were not bankable into Phase 2. Point (2) is reflected in the numerous price swings that have affected the EUETS since 2005. Koch et al. (2014,[40]) show that no more than 10% of the EUA price variation can be explained by abatement-related fundamentals, 40% of which stem from variations in expected economic conditions. For instance, EUAs have nearly lost 50% in early 2016, but the underlying causes were not known for sure[1]. Regarding point (3), Koch et al. (2015,[41]) find that the EUETS is highly responsive to political events and announcements. In particular, the backloading decision process is shown to have affected the EUA price negatively while the latter reacted positively to the announcement of the 2020 and 2030 policy packages, albeit with relatively smaller adjustments. Firms' anticipations hence fundamentally depend on the credibility of the regulator's announcements and environmental targets. Because there are continual interventions in cap-and-trade systems, ongoing concern about future regulatory action at an unknown time should affect allowance prices before uncertainty unveils[2]. That regula-

[1]Opinions were mixed as to which possible trigger among gloomy macroeconomic conditions, low crude oil prices, change in firms' hedging strategies, or a move of speculators effectively generated the price drop – see *e.g.* http://carbon-pulse.com/16460

[2]For instance, the recent and steady NZU price rise in the NZETS is attributable to the government's announcement that the 2:1 compliance rule should soon be abolished. Similarly, downward pressure on pilot prices in China results from regulatory uncertainty about the transition to a national market, especially regarding the carry-over provision for pilot allowances into the national market. RGA prices also increased when the 45% slash in the overall RGGI cap was discussed but before it was actually implemented.

tory uncertainty weighs on EUA prices is a point made by Salant (2015,[59]). Arguably, the perspective of linking up presently independent ETSs adds to this pervasive uncertainty, as it is a source of further regulatory uncertainty and shocks on permit demand.

Taken together, points (1-2-3) are well illustrated in a recent paper by Borenstein et al. (2015,[11]) applied to the cap-and-trade scheme in California[3]. They highlight two novel, previously underestimated, facts about cap-and-trade systems. First, there is significant uncertainty in baseline emissions, estimated to be "at least as large as uncertainty about the effect of abatement measures". This is attributable to uncertainty on economic growth, the reach of complementary policies that drive abatement otherwise made in response to the carbon price, the use of offset credits, electricity imports and reshuffling. Such factors have eroded the cap stringency in RGGI or in the EUETS, see *e.g.* de Perthuis & Trotignon (2014,[53]), who label this effect as a "tendency towards *ex-ante* overallocation"[4]. Second, there is little price elasticity of emissions between the price floor and price ceilings, even though banking provides additional permit demand elasticity. Together, these two facts suggest that the equilibrium price in a cap-and-trade scheme is likely to be very volatile and that very low or high prices are the most likely outcomes.

The independence property of initial allocation in a cap-and-trade scheme does not hold as soon as one of the requiring assumptions sustaining the results of the seminal paper of Montgomery (1972,[47]) is relaxed – see *e.g.* Hahn & Stavins (2011,[32]). In particular, B&vdF show that when risk neutrality is dropped and liable firms are risk averse, those which are handed out a relatively high amount of allowances (net sellers) tend to under-invest in abatement technology while those with relatively low allocation (net buyers) tend to over-invest, as compared with the risk-neutral optimum. This is because net sellers adopt a passive attitude and simply use the quotas allocated to them while net buyers invest more than what would be efficient so as to gain independence w.r.t. the risky price and hedge against high non-compliance penalties. Note that Ben-David et al. (2000,[5]) find similar results, which are also supported by laboratory experiments conducted by Betz & Gunnthorsdottir (2009,[10]). In practice, Ellerman et al. (2010,[22]) also note the asymmetry that always exists between net long and short liable entities. Because the former are under no compulsion to sell, they can adopt a passive 'wait-and-see' attitude towards market participation as long as price uncertainties are high and experience is being gained, as was the case in early Phase I of the EUETS[5]. B&vdF further show that risk aversion impedes allowance trading and that the introduction of forwards limits but does not eliminate risk. They also compare tax and cap-and-trade regimes and find that a tax regime does not deteriorate under risk aversion and remain intertemporally cost-efficient.

[3]Their analysis also applies to the EUETS or RGGI, as both systems also have significant complementary policies in place and have witnessed drops in prices to very low levels.

[4]Regarding policy overlap, Schmalensee & Stavins (2013,[62]) also underline the impact of railway deregulation in the US SO_2 trading program.

[5]The authors also mention that this was reinforced by two technical factors: registry set-up faced delays in eastern countries with potential sellers; industrial companies, acknowledged to be long, did not see a significant effect of the carbon price on their output cost, as did power companies, acknowledged to be short.

Two related articles also study the impacts of uncertainty in cap-and-trade systems. Colla et al. (2012,[16]) model an ETS where both risk-averse liable firms and speculators can trade permits and where the latter help the former hedging from risk originating from uncertain future demand. In a first trading round, firms may sell part of their initial allocation to speculators and buy allowances back in a second trading round where uncertainty has resolved. In compensation for carrying the risk from date 1 to date 2, speculators demand a premium. Because the presence of speculators augments the risk-bearing capacity of the market, it also tends to reduce price volatility. Zhao (2003,[74]) develops a general equilibrium model of permit trading with irreversible abatement investment under abatement cost uncertainty, assuming rational expectations and risk neutrality on the part of liable firms. He finds that investment incentives decrease in the level of cost uncertainty, but more so under a tax regime than under a cap and trade. This stands in sharp contrast with previous partial-equilibrium analyses that tend to favour taxes over permits on the grounds that: investments in abatement technology decrease future abatement costs, hence the future market price, and hence future benefits of investments; investment incentives in cap-and-trade is further reduced if one assumes exogenous uncertainty on the future allowance price (Xepapadeas, 2001,[72]) or aggregate demand (Chao & Wilson, 1993,[14]). The second effect occurs because meeting compliance by trading on the market for allowances provides more flexibility than investment in irreversible abatement technology does. In particular, intertemporal cost-efficiency does not obtain and the price for allowances may exceed the marginal cost of investment by an amount which can be seen as an option value.

It is apparent from the above that significant uncertainty prevails in existing ETSs. Moreover, uncertainty stems from different sources and can hardly be deemed objectively predictable. Arguably, this might be better captured in a setup accounting for ambiguity rather than mere objective risk[6]. In particular, liable firms need information in order to calibrate their expectations, which they can then base their investment and abatement decisions upon. So when they lack such relevant information, they can consider different possible scenarios. If we consider that these scenarios are objective, given and known to all liable firms, *e.g.* say they are provided by a set of experts[7], then this is situation that is particularly well represented by introducing ambiguity[8]. Ambiguity aversion is deemed quite pervasive in economic situations, and models explicitly accounting for ambiguity-consistent behaviours have been put forth since the 90's or so[9].

[6]Risk corresponds to a situation where decisions can be made based on a unique and perfectly known probability measure while under ambiguity there exists a set of such objective probability measures, and uncertainty lies upon which measure actually applies. More generally, ambiguity arises when one agent has too little relevant information to properly ascertain her beliefs

[7]One scenario corresponds to the prediction of one expert. Experts that may give allowance price evolution scenarios or allowance demand scenarios are, *inter alia*, BNEF, Energy Aspects, ICIS-Tschach, Point Carbon, diverse academic fora or think tanks, etc.

[8]Ambiguity aversion is reflected both in how an agent takes into account the extent of her knowing the odds relevant to her decision problem, and in her favouring acts whose odds are objectively known.

[9]For a recent review, see Etner et al. (2012,[24]) or Machina & Siniscalchi (2014,[43]). In simple ambiguous sit-

Ambiguity aversion has been applied to a variety of fields in economics, such as finance (Gollier, 2011,[30]), self-insurance and self-protection (Alary et al., 2013,[1]; Berger, 2015,[8]), formation of precautionary savings[10] (Gierlinger & Gollier, 2015,[28]; Berger, 2014,[7]) or health (Treich, 2010,[66]; Berger et al., 2013,[6]), and can explain otherwise unaccounted for empirical facts such as the equity premium puzzle (Collard et al., 2011,[17]) or the negative correlation between asset prices and returns (Ju & Miao, 2012,[35]). Of more relevance to our problem is the emerging theory of the competitive firm under ambiguity aversion à la Sandmo (1971,[58]) as in Wong (2015a,[69]) or the effects of ambiguity aversion on optimal abatement decisions under model uncertainty, applied to Integrated Assessment Models as in Millner et al. (2013,[46]) and Berger et al. (2016,[9]).

Our approach & results. In a two-period model, we consider a continuum of homogenous risk-neutral ambiguity-averse polluting firms liable under either a tax or an ETS. As in Slechten (2013,[63]) date-1 abatements have long-term effects, that is they are persistent[11] in that they both carry over to date 2 and affect date-2 abatement cost. If environmental regulation is effective at date 2 only, date-1 abatement can be interpreted as investment in abatement technology or 'early emission reduction' in the perspective of future regulation. If environmental regulation is effective at both dates, we assume that liable firms are already in compliance at date-1 and that they have exhausted all trades opportunities[12], so that date-1 abatement corresponds to additional abatement in expectation of a more stringent regulation at date-2, *i.e.* allowance banking. In both cases we control for the option-value issue by assuming that individual date-1 abatement or investment has no effect on the date-2 allowance price.

In our model, liable firms face one source of exogenous ambiguity: either (1) market-level extrinsic ambiguity, channelled via the allowance price; or, following Borenstein et al. (2015,[11]), (2) firm-level intrinsic ambiguity on baseline emissions, also engendering market-level ambiguity via the allowance price endogenously forming on the market. Agents display smooth ambiguity aversion (Klibanoff et al., 2005,[37];2009,[38]) in its recursive form – KMM for short[13]. This brings about nice comparative statics and tractability properties, thanks to which explicit computations of optimal date-1 abatement, otherwise

uations, Ellsberg (1961,[23]) highlight that individuals behave in ways incompatible with the Subjective Expected Utility (SEU) criterion (Savage, 1954,[60]) and, more precisely, with the sure-thing principle. In the standard urn experiment, it is the complementarity across events that brings about ambiguity, here characterized by a lack of relevant information. Roughly put, representation theorems accounting for ambiguity weaken the sure-thing principle.

[10] Note the similarity between saving and banking. Since banking also serves as a hedging device, also note the similarity between banking and both self-insurance and self-protection.

[11] One must acknowledge that not all abatements are irreversible in existing ETSs since many result from fuel switch. However, the US SO_2 program gives a notable example of early investment in abatement technology: significant investment in scrubbers occurred in Phase I in the perspective of a more stringent Phase II, see e.g. Schmalensee & Stavins (2013,[62]).

[12] In practice, environmental regulation generally kicks off with relatively slack objectives (trial periods) before moving to more stringent caps or higher tax rates.

[13] For the close topic of model uncertainty, the reader is referred to Marinacci (2015,[44]).

hard to come by, are bypassed. In the presence of one source of ambiguity, (1) or (2), we analyse how ambiguity aversion alters optimal date-1 abatement decisions under both instruments, where ambiguity neutrality serves as our natural benchmark. In this sense, our analysis differs from the Weitzman-like 'prices *vs* quantities' literature.

More specifically, we refer to the ambiguity-neutral case as the optimum, that is, we assume that it constitutes the first-best. This is because we reason at the firm level, and invoke the standard rationale that, were it not for market failures, ambiguity should not affect firms' behaviours. However, we note that from the regulator's standpoint, in a second-best setting accounting for distributional effects, the ambiguity-neutral case might not be optimal. We further control for the formation of precautionary date-1 abatement in the benchmark by assuming that abatement cost functions are quadratic, *i.e.* second-order Taylor expansions around baseline emissions. The optimum corresponds to the situation where the emission path is determined by the least-discounted abatement cost solution, *i.e.* where intertemporal cost-efficiency obtains, see *e.g.* Rubin (1996,[57]) or Schennach (2000,[61]) with stochastic shocks.

Ambiguity aversion induces two effects, pessimism and ambiguity prudence. Pessimism distorts ambiguity-averse agent's second-order subjective beliefs by granting relatively more weight to those *bad scenarios* with low profits. Ambiguity prudence is specific to the KMM representation theorem and we follow Gierlinger & Gollier (2015,[28]) and Berger (2014;[7]) in defining ambiguity prudence as Decreasing Absolute Ambiguity Aversion (DAAA) since absent pessimism, DAAA is always conducive to over-abatement at date 1 relative to ambiguity neutrality. In particular, as compared to ambiguity-neutral agents, ambiguity-prudent agents apply a higher discount factor and display a preference for an earlier resolution of ambiguity. In a tax regime with ambiguous baselines, because the future price of emissions is deterministic, liable firms do not pessimistically distort their intertemporal arbitrage but are subject to ambiguity prudence. In particular, ambiguity-averse tax-liable firms over-abate (under-abate) at date 1 relative to ambiguity neutrality when they exhibit DAAA (IAAA) and intertemporal cost-efficiency obtains only under CAAA. Numerical simulations show that a higher degree of ambiguity aversion does not necessarily imply higher date-1 abatement on the part of ambiguity-prudent tax-liable firms.

In an ETS with ambiguous allowance price, both pessimism and ambiguity prudence are present and can have aligned or opposite effects on date-1 abatement. Controlling for ambiguity prudence, pessimism can incent firms to over- or under-abate and intertemporal cost-efficiency does not obtain in general. For pessimism to lead to over-abatement, *bad scenarios* must coincide with those scenarios inducing high marginal date-2 profitability from date-1 abatement – hence corresponding to an anticomonotonicity criterion between date-2 profits and marginal profits across scenarios, which also obtains with other representation theorems, *e.g.* the pioneering MEU decision criterion of Gilboa & Schmeidler (1989,[29]). In line with a precautionary effect, this criterion translates into a threshold condition on initial allocation: Only when firms are allocated too few permits does anti-comonotonicity holds. The independence property of initial allocation does not hold under ambiguity aversion, just like under risk aversion in B&vdF. When ambiguity bears on firms' baselines, a more fine-grained threshold condition is derived and we show that, under symmetric allocation of allowances, anticomonotonicity always holds, *i.e.* pessimism always leads to over-abatement. In terms of comparative statics, we show that an increase in the degree of ambiguity aversion always leads to: An increase in pessimism in the sense of a monotone-likelihood deterioration; an increase in the ambiguity prudence coefficient provided that initial ambiguity prudence is not too high relative to ambiguity aversion. In general, we find it difficult to sign the effect of an increase in the initial allocation level on optimal date-1 abatement for a given ambiguity aversion degree.

With numerical simulations, we find that date-1 abatement unambiguously decreases with initial allocation and that there exists a unique threshold on initial allocation, below (above) which firms over-(under-)abate. While anticomonotonicity is a demanding criterion as it requires that a threshold condition be met in all scenarios, from our specific parametrical example, we conjecture that it might suffice that anticomonotonicity hold in expectations over the set of possible scenarios to sign pessimism in the direction of over-abatement. Accounting for pessimism only, in an ETS under CAAA, for a given allocation level, the higher the degree of ambiguity aversion, the higher the variation in date-1 abatement relative to neutrality and the higher the sensitivity of date-1 abatement around the threshold. Accounting for both pessimism and ambiguity prudence, in an ETS under DAAA, ambiguity prudence adjusts date-1 abatement upwards for all ambiguity aversion degrees and all initial allocation levels, but does so with different intensities. Ambiguity prudence is more pronounced when allocation is relatively small or high, albeit to a lower extent. In particular, when initial allocation is relatively small, a higher ambiguity aversion degree is not necessarily conducive to higher date-1 abatement. Overall, in terms of date-1 abatement adjustment relative to neutrality, it seems that pessimism dominates ambiguity prudence, especially around the threshold on initial allocation.

In terms of preferability of instruments, our paper mitigates B&vdF's findings that only ETS deteriorates in the presence of uncertainty: When ambiguity is prevalent and liable firms display ambiguity aversion, both price and quantity regimes are not conducive to intertemporal cost-efficiency. Because the tax regime is only subject to ambiguity prudence, hence less distorted than an ETS, one could conclude that a tax still performs better than an ETS under ambiguity[14]. However, we show that the ETS-specific pessimism effect vanishes out following on the introduction of fairly-priced forwards: An ETS with fairly-priced forwards performs like a tax[15]. Finally, we show that date-1 abatement decisions deviate

[14]In practice, note that the tax rate is also subject to uncertainty. For instance there might be uncertainty originating from different legislative texts that contradict with one another regarding the desired tax rate, or even uncertainty on the effective implementation of the tax regime in the future (credibility issues).

[15]Furthermore, when ambiguity bears on firms' abatement costs, we show in Appendix A that the two instruments are equivalent in terms of performance under ambiguity aversion. This also mitigates Zhao

further away from the optimum when the market for allowances is populated by both ambiguity averse and neutral firms. We also show that the presence of ambiguity averse firms in the market impedes trade. We also discuss the robustness of our results to the introduction of risk aversion, more inter-firm heterogeneity, two simultaneous sources of ambiguity and its extension to more periods.

OUTLINE. The remainder is organised as follows. Section 2 describes our modelling framework and underlying assumptions. Section 3 investigates the effects of ambiguity aversion in a tax regime. Sections 4.1 and 4.2 analyse the cap-and-trade regime properties under pure price ambiguity and (intrinsic) baseline ambiguity, respectively, while section 4.3 presents comparative statics results. Numerical simulations are then carried out in section 5. Section 6.1 presents three extensions of our model and section 6.2 discusses the robustness of our results in relation with the extant literature. Section 7 concludes.

2 The model

A set of polluting firms are alternatively liable under two type environmental policy regulations – price or quantity. There are two dates, 1 and 2. At date 2, regulation is effective and firms abate emissions so as to comply with the scheme they are liable under. At date 1, regulation is not effective yet but firms can already invest in abatement technology in the perspective of the future scheme coming into force at the beginning of date 2. Alternatively, at date 1, regulation is effective but firms are already in compliance so that they may wish to undertake additional abatement in anticipation of more stringent date-2 compliance requirements. In both cases, however, firms' *ex-ante* date-1 decision must be made in a context where ambiguity prevails at date 2, in a sense that will be defined below. The paper compares firms' optimal date-1 decisions under smooth ambiguity aversion relative to the ambiguity-neutral reference scenario, depending upon which type of regime they are covered under. That is, the two instruments are evaluated and compared in terms of ambiguity-induced date-1 investment/abatement distortions away from the optimum.

LIABLE FIRMS. Let there be a continuum $\mathcal{S} = [0; S]$ of infinitesimally small, identical, polluting, competitive firms indexed by $s \in \mathcal{S}$, where S is the mass of firms. When firm s does not make any abatement effort it emits $\xi(s)$, its baseline emissions level. Firm s' abatement decisions at both dates[16] are denoted $a_1(s)$ and $a_2(s)$. In a cap-and-trade scheme, let firm s be endowed with a volume $\omega(s)$ of allowances. We assume that a given level of abatement or investment cuts down emissions by a corresponding amount. We also assume that date-

(2003,[74])'s findings that an ETS better sustains investment incentives as does tax under cost uncertainty.

[16]Although a_1 can also be considered as investment in abatement technology, for simplicity, we retain the term abatement for the remainder.

1 abatement or investment reduces both date-1 and date-2 emissions by a corresponding amount. It means that: absent regulation at date 1, investment in abatement technology are persistent with a depreciation rate of capital equal to zero, or, liable firms are handed out early reduction permits as valid date-2 compliance instruments for abatement carried out at date 1; or, present regulation at date 1, and assuming date-1 compliance is effective and all trades opportunities are exhausted, additional date-1 abatements free up allowances that are banked into date 2. Therefore, after cleaning activities, firm s' date-2 emission are $\xi(s) - a_1(s) - a_2(s)$. Let capital letters denote aggregate quantities, that is

$$\Xi = \int_{\mathcal{S}} \xi(s)\mathrm{d}s,\ A_1 = \int_{\mathcal{S}} a_1(s)\mathrm{d}s,\ \Omega = \int_{\mathcal{S}} \omega(s)\mathrm{d}s.$$

Abatement cost functions are identical for all firms and given by twice continuously differentiable functions, C_1 and C_2. Following Slechten (2013,[63]), abatement is said to have long-term effect in the sense that C_2 also depends on the level of date-1 abatement, $C_2 \equiv C_2(a_1, a_2)$. Due to this effect the marginal cost of date-1 abatement is $\partial_{a_1}(C_1 + C_2)$. Abatement costs are assumed to be strictly increasing and strictly convex on $[0; \infty[$ with no fixed cost, that is $C_1' > 0$, $C_1'' > 0$, with $C_1(0) = 0$ and $C_1'(0) = 0$, and $\forall a_1 \geq 0$, $\partial_{a_2} C_2(a_1, \cdot) > 0$ and $\partial^2_{a_2 a_2} C_2(a_1, \cdot) > 0$ with $C_2(a_1, 0) = 0$. Firms are also assumed to face decreasing abatement opportunities as in Bréchet & Jouvet (2008,[13]), that is the date-2 marginal cost of abatement is increasing in the level of date-1 abatement, $\forall a_1, a_2 \geq 0$, $\partial^2_{a_1 a_2} C_2(a_1, a_2) \geq 0$. This effect is compensated by a learning-by-doing effect as firms gain experience in the abatement technology by abating at date 1. As in Slechten (2013,[63]), this is captured by assuming that $\forall a_1, a_2 \geq 0$, $\partial^2_{a_1 a_1}(C_1(a_1) + C_2(a_1, a_2)) \geq \partial^2_{a_1 a_2} C_2(a_1, a_2)$ and $\partial^2_{a_2 a_2} C_2(a_1, a_2) \geq \partial^2_{a_1 a_2} C_2(a_1, a_2)$. To derive analytical results, we will assume that abatement cost functions are equipped with the following quadratic specification[17]

$$\forall a_1, a_2 \geq 0,\ C_1(a_1) = \frac{c_1}{2} a_1^2 \text{ and } C_2(a_1, a_2) = \frac{c_2}{2} a_2^2 + \gamma a_1 a_2,\ \text{with } c_1, c_2 > 0,\ c_2 > \gamma, \tag{1}$$

where $\frac{1}{c_i}$ measures each firm's date-i flexibility in abatement and γ denotes the long-term abatement effect coefficient. Note that specification (1) satisfies the above assumptions. For tractability reasons, we will sometimes need to assume that there is no long-term effect of abatement, *i.e.*, $\partial_{a_1} C_2 \equiv 0$ or $\gamma = 0$.

Ambiguity & Ambiguity aversion. Ambiguity is exogenous and originates from two sources: the date-2 allowance price or date-2 emission baselines[18]. However, our papers assumes that firms are subject to one source of ambiguity at a time. Under pure price ambiguity, ETS-liable firms are only subject to market-level ambiguity. When baselines are ambigu-

[17]This corresponds to a second-order Taylor expansion of abatement cost functions centred around baseline emissions. This assumption is standard, see *e.g.* Newell & Stavins (2003,[49]). The linear term is omitted for convenience – adding it would merely translate our results by a constant term.

[18]For ambiguity on abatement cost functions, see Appendix A.

ous, ETS-liable firms are subject to both intrinsic ambiguity and market-level ambiguity from the ambiguous allowance price endogenously emerging from the market. Note that firms covered under a tax regime can only be subject to intrinsic ambiguity. Consider for instance the case of pure price ambiguity and let the price risk $\tilde{\tau}$ be described by the objective cumulative distribution G^0, supported on $\mathrm{T} = [\underline{\tau}; \bar{\tau}]$, with $0 < \underline{\tau} < \bar{\tau} < \infty$. Liable firms exhibit smooth ambiguity aversion as in Klibanoff et al. (2005,[37]) and, as such, are uncertain about G^0. That is, they are confronted with a set of objective probability measures for $\tilde{\tau}$ and are uncertain about which of those truly govern the price risk. For each realisation θ of the random variable $\tilde{\theta}$ (called θ-scenario), let $G(\cdot;\theta)$ denote the objective probability measure for $\tilde{\tau}_\theta$, the θ-scenario price risk. Ambiguity is represented by a second-order subjective probability distribution of $\tilde{\theta}$, F, supported on $\Theta = [\underline{\theta}; \bar{\theta}]$[19], which captures the firms' beliefs about which scenario they feel will materialise. Attitudes toward ambiguity originate in the relaxation of the reduction of compound first and second order probabilities[20] so that firms' computations of their expected profits can be decomposed into three steps: first, in any given θ-scenario they compute their expected profits w.r.t. $G(\cdot;\theta)$; second, each θ-scenario first-order expected profits is transformed by an increasing function ϕ; third, their second-order expected profits obtain by taking the expectation of the ϕ-transformed first-order expected profits w.r.t. F. Ambiguity aversion is characterised by a concave ϕ so that an ambiguity-averse agent dislikes any mean-preserving spread in the space of second-order expected profits, see *e.g.* (4). To disentangle and separate out the effect of ambiguity aversion vis-à-vis that of risk aversion, liable firms are risk-neutral but ambiguity averse with $\phi' > 0, \phi'' < 0$[21]. Compared to other ambiguity aversion models, smooth ambiguity aversion comes with both advantages in terms of tractability and nice properties[22]. In particular, it disentangles ambiguity (beliefs) from ambiguity aversion (tastes), has nice comparative static properties for which the decision-making under risk machinery readily applies, nests other ambiguity aversion models as special cases[23] and, as exposed below, the KMM framework can be embedded in a dynamic framework. In essence, firms are identical but for their initial allocation $\omega(s)$ as this will be shown to be an essential driver of date-1 abatement adjustment under ambiguity aversion.

Intertemporal profits. Using the recursive smooth ambiguity model[24] of Klibanoff et al.

[19]Note that in Klibanoff et al. (2009,[38]) the scenario space Θ is finite but here we consider its continuous extension with a continuous subjective distribution F. Note also that the KMM axiomatisation is based on acts rather than probability distribution on T, the outcome space.

[20]When the reduction of compound lotteries applies the decision criterion reduces to the SEU criterion.

[21]This means that all firms are equally ambiguity averse. Indeed, different curvatures of the ambiguity function ϕ imply different degrees of ambiguity aversion. Section 6 slightly departs from the assumption of identical tastes toward ambiguity but this is a complicated task.

[22]A few other ambiguity aversion models share these fine properties, see *e.g.* Gajdos et al. (2008,[26]).

[23]When ϕ displays CAAA with $\phi(x) = \frac{e^{-\alpha x}}{-\alpha}$, Klibanoff et al. (2005,[37]) show that, under some conditions, when the ambiguity aversion coefficient α tends to infinity, the KMM model approaches the pioneering MEU decision criterion of Gilboa & Schmeidler (1989,[29]).

[24]Many papers in the literature use the KMM static formulation, *e.g.* Alary et al. (2013,[1]), Millner et al.

(2009,[38]), firms maximise their intertemporal profits w.r.t. date-1 abatement, that is

$$\max_{a_1 \geq 0} \pi_1(a_1) + \beta\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathbb{E}_G\left\{\pi_2\left(a_1, a_2^*; \tilde{\theta}\right)|\theta\right\}\right)\right\}\right) \tag{2}$$

where π_i is the date-i profits and the second term is the date-2 ϕ-certainty equivalent expected profit with $0 \leq \beta \leq 1$ the discount factor. $\mathbb{E}_F$ denotes the expectation parameter taken w.r.t. F conditional on all relevant information available to liable firms at date 1, and $\mathbb{E}_G\{\cdot|\theta\}$ denotes the expectation parameter taken w.r.t. $G(\cdot;\theta)$ conditional on the true scenario being $\theta \in \Theta$. Our model essentially focuses on abatement decisions and ignores firms' output decisions, as is the case in Zhao (2003,[74]) and Baldursson & von der Fehr (2004,[3]) so that the firms' net profits on the goods' markets are independent of the net emissions volume[25]. Let $\zeta_i > 0$ denote the firms' exogenous rate of profits at date i absent abatement. Assuming firms are liable under an ETS, for a given price for allowances τ, date-1 and date-2 profits write, $\forall a_1 \geq 0$,

$$\pi_1(a_1) = \zeta_1 - C_1(a_1) \text{ and } \pi_2(a_1, a_2; \tau) = \zeta_2 - C_2(a_1, a_2) - \tau(\xi - a_1 - a_2 - \omega). \tag{3}$$

Note that in the recursive formulation (2), a_2^* denotes the optimal date-2 abatement level for any given date-1 abatement level and observed allowance price. Note also that (2-3) assumes corner solutions away since firms always have an incentive to abate a positive volume at date-1 provided the expected future price is positive[26]. Finally, let the ambiguity function ϕ be three times differentiable, increasing and concave, so that under ambiguity aversion the Jensen's inequality gives

$$\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathbb{E}_G\left\{\pi_2\left(a_1, a_2^*; \tilde{\theta}\right)|\theta\right\}\right)\right\}\right) \leq \mathbb{E}_F\left\{\mathbb{E}_G\left\{\pi_2\left(a_1, a_2^*; \tilde{\theta}\right)|\theta\right\}\right\}, \tag{4}$$

where the right-hand side corresponds to the ambiguity-neutral case (ϕ is linear), which, by reduction of compound lotteries, also corresponds to a Savagian expectation operator taken w.r.t. $\bar{G} \equiv \mathbb{E}_F\left\{G(\cdot;\tilde{\theta})\right\}$. The second-order condition to problem (2) is satisfied provided that ambiguity tolerance $-\phi'/\phi''$ is concave – see Gierlinger & Gollier

(2013,[46]) or Wong (2015a,[69];2015b,[70];2015c,[71]), and hence consider an F-expected ϕ-valuation of G-expected date-2 profits. In this paper, the date-2 expected profit is taken as a ϕ-certainty equivalent, which will require stronger definition for ambiguity prudence. However, note that a significant advantage of considering a two-period ambiguity aversion model is that the abatement effort exerted at date-1 can effectively reduce ambiguity. Moreover, π_1 is deterministic and does not vary with ambiguity so that clearer results regarding the impact of the introduction of ambiguity aversion obtain.

[25]This is a restrictive assumption. For instance, it can justified if firms produce different goods and belong to different industrial sectors. If so, this assumption amounts to assuming that profits are always positive in the relevant range. Note that while an interaction between the goods' and environmental policy undoubtedly exists, there is uncertainty on its intensity: Martin et al. (2014,[45]) find that, in the UK, the carbon tax has reduced both energy use and intensity, but no evidence of impacts on employment or production; with regards to the labour market, see also the double dividend debate in Bovenberg & Goulder (1996,[12]). For an explicit treatment of the interaction of allowance trading with the output market, see *e.g.* Requate (1998,[55]) or Baldursson & von der Fehr (2012,[4]).

[26]In this sense, date-1 abatement can be interpreted as allowance banking from date-1 into date-2 since the Rubin-Schennach banking condition is always satisfied – see Rubin (1996,[57]) and Schennach (2000,[61]).

(2015,[28],Lemma2). This condition is satisfied for usual ϕ functions, as those we use in section 5, and we therefore assume that this holds throughout the paper.

3 The tax regime under ambiguity

We consider the situation where the regulator implements a proportional tax t on emissions[27] and analyse optimal date-1 abatement decisions for one representative ambiguity-averse tax-liable firm relative to ambiguity neutrality, when date-2 baseline emissions are ambiguous – the s index is hence dropped throughout the section. For any given θ-scenario, let the random variable $\tilde{\xi}_\theta$ denote the baseline risk, distributed according to the objective probability measure $G(\cdot;\theta)$, supported on $[\underline{\xi};\bar{\xi}]$ with $0 < \underline{\xi} < \bar{\xi} < \infty$, where θ is the realisation of the random variable $\tilde{\theta}$, distributed according to the subjective probability measure F defined over Θ. The resolution proceeds in two steps, using backward induction. At date 2, for a given level of date-1 abatement $a_1 \geq 0$ and after having observed its baseline emission level ξ, the liable firm maximizes its date-2 profit w.r.t. a_2, that is

$$\max_{a_2 \geq 0} \pi_2 \left(a_1, a_2; \xi\right) = \zeta_2 - C_2(a_1, a_2) - t \left(\xi - a_1 - a_2\right), \tag{5}$$

with the optimality condition $\partial_{a_2} C_2(a_1, a_2^*) = t$, so that cost-efficiency always obtains at date 2, where the optimal date-2 abatement is implicitly defined such that $a_2^* \equiv a_2^*(a_1; t)$. Note that this holds true irrespective of the firm's attitude towards ambiguity. Moving backward to date-1 abatement decisions, however, we need to distinguish between ambiguity neutrality and aversion. This is so because the optimal date-1 abatement varies with the ambiguity level, as seen from date 1, in conjunction with the degree of ambiguity aversion. Indeed, at date 1, the liable firm's program is recursively defined by

$$\max_{a_1 \geq 0} \pi_1(a_1) + \beta\phi^{-1} \left(\mathbb{E}_F \left\{\phi\left(\mathcal{V}(a_1;\tilde{\theta})\right)\right\}\right), \tag{6}$$

where $\forall\theta \in \Theta$, $\mathcal{V}(a_1;\theta) = \mathbb{E}_G \left\{\tilde{V}(a_1;\theta)\middle|\theta\right\}$ is the θ-scenario date-2 expected profit, with $\tilde{V}(a_1;\theta) = \max_{a_2} \pi_2(a_1, a_2; \tilde{\xi}_\theta) \equiv \pi_2(a_1, a_2^*(a_1;t); \tilde{\xi}_\theta)$, so that

$$\mathcal{V}(a_1;\theta) = \zeta_2 - C_2\left(a_1, a_2^*(a_1;t)\right) - t\left(\bar{\xi}_\theta - a_1 - a_2^*(a_1;t)\right), \quad \text{with} \quad \bar{\xi}_\theta = \mathbb{E}_G\left\{\tilde{\xi}_\theta\middle|\theta\right\}.$$

Using date-2 optimality, the θ-scenario expected marginal profitability from date-1 abatement satisfies $\mathcal{V}_{a_1}(a_1;\theta) = t - \partial_{a_1} C_2(a_1, a_2^*(a_1;t)) > 0$, $\forall a_1 \geq 0$. Since both t and $\partial_{a_1} C_2$ are deterministic, $\mathcal{V}_{a_1}$ is deterministic and does not depend on the θ-scenario considered.

Ambiguity neutrality. With ϕ linear, the necessary first-order condition of program (6)

[27] t is exogenously given and certain. Whether it is optimal or not is of no relevance here.

defines the optimal date-1 abatement under ambiguity neutrality, $\bar{a}_1^t$, by

$$-C_1'(\bar{a}_1^t) + \beta \mathbb{E}_F \left\{ \mathcal{V}_{a_1}(\bar{a}_1^t; \tilde{\theta}) \right\} = 0. \tag{7}$$

Since $\mathcal{V}_{a_1}$ is deterministic, (7) rewrites $C_1'(\bar{a}_1^t) + \partial_{a_1} C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t; t)) = \beta t$. In particular, combining optimality conditions at both dates yields

$$C_1'(\bar{a}_1^t) + \beta \partial_{a_1} C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t, t)) = \beta \partial_{a_2} C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t; t)), \tag{8}$$

so that intertemporal cost-efficiency obtains in that the overall marginal date-1 abatement cost is equated to the marginal date-2 abatement cost[28]. By reduction of compound lotteries, and as will be elaborated upon further in section 4.1, the ambiguity neutrality case corresponds to a decision under risk for a risk-neutral liable firm. B&vdF further show that the intertemporal cost-efficiency property of a tax regime remains valid under risk aversion. As exposed below, however, this property does not carry over to ambiguity aversion.

Ambiguity Aversion. With ϕ concave, the necessary first-order condition of program (6) defines the optimal date-1 abatement under ambiguity neutrality, $\hat{a}_1^t$, by

$$-C_1'(\hat{a}_1^t) + \beta \mathbb{E}_F \left\{ \frac{\phi'\left(\mathcal{V}(\hat{a}_1^t; \tilde{\theta})\right)}{\phi' \circ \phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(\hat{a}_1^t; \tilde{\theta})\right)\right\}\right)} \mathcal{V}_{a_1}(\hat{a}_1^t; \tilde{\theta}) \right\} = 0, \tag{9}$$

Since $\mathcal{V}_{a_1}$ is deterministic, (9) rewrites $-C_1'(\hat{a}_1^t) + \beta \mathcal{A}(\hat{a}_1^t)\left(t - \partial_{a_1} C_2(\hat{a}_1^t, a_2^*(\hat{a}_1^t; t))\right) = 0$ where, as explained below, $\mathcal{A}$ is a function characterising ambiguity prudence such that

$$\mathcal{A}(a_1) = \frac{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(a_1; \tilde{\theta})\right)\right\}}{\phi' \circ \phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(a_1; \tilde{\theta})\right)\right\}\right)}. \tag{10}$$

Proposition 3.1 describes the impact of ambiguity aversion on date-1 abatement decisions as compared with ambiguity neutrality.

Proposition 3.1. *Ambiguity aversion is conducive to higher (resp. lower) date-1 abatement than under ambiguity neutrality if, and only if, the liable firm displays Decreasing (resp. Increasing) Absolute Ambiguity Aversion. Moreover, under Constant Absolute Ambiguity Aversion, the introduction of ambiguity aversion has no effect on date-1 abatement decision.*

[28] In particular, with the quadratic abatement cost specification, it comes that

$$\bar{a}_1^t = \frac{c_2 - \gamma}{c_1 c_2 - \beta \gamma^2} \beta t \geq 0 \quad \text{and} \quad a_2^*(\bar{a}_1^t; t) = \frac{t - \gamma \bar{a}_1^t}{c_2} = \frac{c_1 - \beta \gamma}{c_1 c_2 - \beta \gamma^2} t \geq 0.$$

With no long-term dependency, *i.e.* $\gamma = 0$, a_2^* is independent of a_1 and the overall abatement under ambiguity neutrality is such that $\bar{a}_1^t + a_2^*(t) = \frac{\beta t}{c}$, where $\frac{1}{c} = \frac{1}{c_1} + \frac{1}{\beta c_2}$ is the liable firm's aggregate flexibility in abatement over the two dates. In particular, the overall abatement volume is optimally apportioned between the two dates, that is, in proportion to each date flexibility in abatement: $\bar{a}_1^t = \frac{c}{c_1}\left(\frac{\beta t}{c}\right)$ and $a_2^*(t) = \frac{c}{\beta c_2}\left(\frac{\beta t}{c}\right)$.

Proof. Let ϕ be thrice differentiable. We first prove the following claim: "ϕ is DAAA (IAAA) is equivalent to $\mathbb{E}\phi'(\cdot) \geq (\leq) \phi' \circ \phi^{-1}(\mathbb{E}\phi(\cdot))$". An agent is said to display Decreasing Absolute Ambiguity Aversion (DAAA) i.f.f. the Arrow-Pratt coefficient of absolute ambiguity aversion $-\frac{\phi''}{\phi'}$ is a decreasing function. This is true i.f.f. $-\phi'''\phi' + \phi''^2 < 0$ and, upon rearranging, i.f.f. $\frac{-\phi'''}{\phi''} > \frac{-\phi''}{\phi'}$. This is equivalent to $-\phi'$ being more concave than ϕ. In terms of certainty equivalent, this translates into $\phi^{-1}(\mathbb{E}\phi(\cdot)) > (-\phi')^{-1}(-\mathbb{E}\phi'(\cdot))$. Applying $-\phi'$ on both sides proves the claim.

In terms of comparative statics, it follows from the concavity of the objective function that $\hat{a}_1^t \geq \bar{a}_1^t$ i.f.f. $-C_1'(\bar{a}_1^t) + \beta\mathcal{A}(\bar{a}_1^t)\left(t - \partial_{a_1}C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t;t))\right) \geq 0$. Since $t - \partial_{a_1}C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t;t)) > 0$ and using the date-1 optimality condition under ambiguity neutrality, it comes that $\hat{a}_1^t \geq \bar{a}_1^t$ i.f.f. $\mathcal{A}(\bar{a}_1^t) \geq 1$. Finally, it follows from the claim that $\mathcal{A}$ is bigger, lower or equal to 1 i.f.f. ϕ is DAAA, IAAA or CAAA, respectively. □

First, Proposition 3.1 indicates that intertemporal cost-efficiency obtains only in the case where the firm displays CAAA. This suggests that the relative merits of an emissions tax *vs.* emissions trading as highlighted by B&vdF under risk aversion would tend to fade away under ambiguity aversion. Second, Proposition 3.1 is in line with the literature on the formation of precautionary saving under ambiguity aversion, *e.g.* Osaki & Schlesinger (2014,[51]), Gierlinger & Gollier (2015,[28]). Because the liable firm abates relatively more at date 1 than under ambiguity neutrality when it displays DAAA, we follow Gierlinger & Gollier (2015,[28]) and Berger (2014,[7]) in identifying ambiguity prudence with DAAA. Accordingly, we call $\mathcal{A}$ the ambiguity prudence coefficient, whose value is above one in case of ambiguity prudence. With this definition[29],

Corollary 3.2. *The liable firm forms precautionary date-1 abatement if, and only if, it displays prudence towards ambiguity.*

It is noteworthy that the effects of ambiguity prudence are clearer when there is no long-term abatement effects. In particular,

Corollary 3.3. *Assume there is no long-term dependency in abatement cost. One liable firm displaying ambiguity prudence applies a relatively higher discount factor, and hence abates more at date 1, than under ambiguity neutrality.*

[29] Presently, however, there is not a unique definition for ambiguity prudence in the literature. Baillon (2015,[2]) identifies ambiguity prudence with the less demanding condition that ϕ''' be positive (DAAA $\Rightarrow$ $\phi''' > 0$). He justly argues that his definition does not depend on the representation theorem and follows directly from that for risk prudence. With the certainty equivalent formulation we use, however, $\phi''' > 0$ is not sufficient to guarantee the formation of precautionary banking and only with ϕ DAAA is the ambiguity precautionary premium bigger than the ambiguity premium (Osaki & Schlesinger, 2014,[51]). Therefore, the DAAA property is the most natural definition in our analysis. Berger (2015,[8]) underlines the similarity between the KMM and Kreps-Porteus/Selden recursive formulations to pin down his definition of ambiguity prudence: just like the DARA property is required for precautionary saving under risk aversion in the K-P/S models, is DAAA required for precautionary saving under ambiguity aversion in the KMM framework. Finally, Gierlinger & Gollier (2015,[28]) also argue that DAAA should be a standard assumption, in parallel with the widely accepted DARA property in risk.

Proof. When $\partial_{a_1} C_2 \equiv 0$, then $\mathcal{V}_{a_1}(\hat{a}_1^t;\theta) = t$ so that the necessary first-order condition (9) reduces to $-C_1'(\hat{a}_1^t) + \beta\mathcal{A}(\hat{a}_1^t)t = 0$. Combining the necessary first-order conditions at the two dates yields $C_1'(\hat{a}_1^t) = \beta\mathcal{A}C_2'(a_2^*(t))$, where the optimal date-2 abatement is now independent of a_1. Under ambiguity prudence ($\mathcal{A} \geq 1$), one has the following chain $C_1'(\hat{a}_1^t) = \beta\mathcal{A}(\hat{a}_1^t)C_2'(a_2^*(t)) \geq \beta C_2'(a_2^*(t)) = C_1'(\bar{a}_1^t)$, so that $\hat{a}_1^t \geq \bar{a}_1^t$ since $C_1'' > 0$. □

Under ambiguity aversion, the effective discount factor becomes $\beta\mathcal{A}$ so that the liable firm exhibiting DAAA (resp. IAAA) applies a higher (resp. lower) discount factor than under ambiguity neutrality. In this light, ambiguity prudence puts relatively more weight on date-2 profits than under ambiguity neutrality – lowering impatience, as it were – thereby leading to higher date-1 abatement levels, since this reduces date-1 profits to the benefits of date-2 profits, all else equal. A similar interpretation is that the DAAA property worsens the importance of any date-2 profit risk so that it can be assimilated to a "preference for an earlier resolution of uncertainty", as described by Strzalecki (2013,[64],Theorem 4).
In short, our result that an emissions tax is generally not conducive to intertemporal cost-efficiency under ambiguity aversion mitigates B&vdF's findings that only emissions trading deteriorates under risk aversion. Let us now turn to the effects of ambiguity aversion in emissions trading systems.

4 The cap-and-trade regime under ambiguity

Section 4.1 analyses liable firms' abatement decisions in the situation where the date-2 allowance price is the only ambiguous variable, while other factors intrinsic to firms, such as abatement technology and emissions baselines, are assumed to be certain. That is, ambiguity is purely extrinsic to firms and transmitted via prices only[30]. In particular, because ξ is exogenously given, the allowance price and baseline emissions are treated as independent variables, *i.e.*, there is no connection between the prevailing market price and individual baselines[31]. This assumption will be relaxed to some extent in the section 4.2, where price ambiguity endogenously emerges from a (market-wide) ambiguity on baseline emissions. Finally, section 4.3 derives comparative statics results.

[30]As in B&vdF, this could result from uncertainty on the mass of liable firms S. In this sense, this setup captures uncertainty on market coverage, or linkage with other allowance markets, albeit in a crude manner. Although the underlying assumptions are quite restrictive, this setup is informative as it clearly separates out price and quantity systems (fixed price *vs* ambiguous price).

[31]A parallel assumption which is frequent in the literature on firms' decisions under uncertainty is that price and output be independent stochastic variables, see *e.g.* Viaene & Zilcha (1998,[67]) or Dalal & Alghalith (2009,[18]) and references therein.

4.1 Cap-and-trade regime under pure price ambiguity

Since ambiguity is extrinsic, we consider the abatement decisions of one representative liable firm, hence dropping the s index. Let the allowance price risk $\tilde{\tau}$ be described by the objective cumulative distribution G^0, supported on $\mathrm{T} = [\underline{\tau}; \bar{\tau}]$, with $0 < \underline{\tau} < \bar{\tau} < \infty$. In any θ-scenario, let $\tilde{\tau}_\theta$ be the allowance price risk, described by the objective probability measure $G(\cdot;\theta)$, with continuously differentiable density $g(\cdot;\theta)$, defined over T. Again, subjective beliefs over Θ are represented by the prior F. When allowances are tradable[32] the liable firm solves the recursive program

$$\max_{a_1} \pi_1(a_1) + \beta\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(a_1;\tilde{\theta})\right)\right\}\right), \tag{11}$$

where $\forall\theta \in \Theta$, $\mathcal{V}(a_1;\theta) = \mathbb{E}_G\left\{\tilde{V}(a_1;\theta)\middle|\theta\right\}$ is the firm's θ-scenario date-2 expected profit, with $\tilde{V}(a_1;\theta) = \max_{a_2}\pi_2(a_1,a_2;\tilde{\tau}_\theta)$, where the uncertain θ-scenario date-2 profit writes $\pi_2(a_1,a_2;\tilde{\tau}_\theta) = \zeta_2 - C_2(a_1,a_2) - \tilde{\tau}_\theta(\xi - a_1 - a_2 - \omega)$. In particular, at date 2, for any given baseline level ξ, allocated volume of quotas ω and date-1 abatement a_1, the liable firm chooses the optimal abatement level a_2^* such that, $\partial_{a_2}C_2(a_1,a_2^*) = \tau$, the observed allowance price. The optimal date-2 abatement is thus implicitly defined by $a_2^* \equiv a_2^*(a_1;\tau)$. By virtue of the Envelop Theorem applied to $\tilde{V}$, one has that $\tilde{V}_{a_1}(a_1;\theta) = \tilde{\tau}_\theta - \partial_{a_1}C_2(a_1,a_2^*(a_1;\tilde{\tau}_\theta))$, $\forall a_1 \geq 0$, $\forall\theta\in\Theta$. Let us now characterize date-1 optimal abatement decisions under both ambiguity neutrality and aversion.

Ambiguity neutrality. With ϕ linear, the necessary first-order condition of program (11) defines the optimal date-1 abatement level under ambiguity neutrality, $\bar{a}_1$, by

$$-C_1'(\bar{a}_1) + \beta\mathbb{E}_F\left\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\} = 0. \tag{12}$$

Since the reduction of compound lotteries applies, this corresponds to a decision problem under uncertainty w.r.t. the F-averaged objective probability measure denoted $\bar{G}$, so that (12) combined with the date-2 optimality condition gives

$$C_1'(\bar{a}_1) + \beta\mathbb{E}_{\bar{G}}\left\{\partial_{a_1}C_2(\bar{a}_1,a_2^*(\bar{a}_1;\tilde{\tau}))\right\} = \beta\mathbb{E}_{\bar{G}}\left\{\tilde{\tau}\right\} = \beta\mathbb{E}_{\bar{G}}\left\{\partial_{a_2}C_2(\bar{a}_1,a_2^*(\bar{a}_1;\tilde{\tau}))\right\}, \tag{13}$$

with $\bar{G} \equiv \mathbb{E}_F\left\{G(\cdot;\tilde{\theta})\right\}$, implicitly taking the expected price risk under ambiguity neutrality to correspond to the objective price risk, *i.e.*, $\bar{G} \equiv G^0$. There is thus no bias in the ambiguity-neutral liable firm's beliefs, based on the grounds that an ambiguity-neutral decision-maker should not be affected by the introduction of, or a shift in, ambiguity. In particular, this means that under ambiguity neutrality, (i) intertemporal cost-efficiency obtains *in expectations*; (ii) $\bar{a}_1$ is independent of the initial allocation ω. Let us now state

[32] A non-tradable quota regime under extrinsic ambiguity corresponds to a problem of the repartition of a given abatement effort between two dates under certainty.

Proposition 4.1. *Let there be no long-term effect of abatement, $\partial_{a_1} C_2 \equiv 0$. Then, in the face of an increase in uncertainty in the sense of a mean-preserving spread, the ambiguity-neutral liable firm abates relatively more at date 1 if, and only if, $C_2''' > 0$.*

Proof. For a probability measure G^i, define the function O^i by

$$0 = -C_1'(\bar{a}_1^i) + \beta \mathbb{E}_{G^i} C_2'(a_2^*(\tilde{\tau})) \equiv O^i(\bar{a}_1^i),$$

where $\bar{a}_1^i$ is the date-1 optimal abatement when the price risk is distributed according to G^i and a_2^* does not depend on a_1 since we assume time separability. Let the measure G^j be a mean-preserving spread of G^i in the sense of Rothschild & Stiglitz (1971,[56]). Concavity of the objective function then yields

$$\bar{a}_1^j \geq \bar{a}_1^i \Leftrightarrow O^j\left(\bar{a}_1^i\right) \geq O^i\left(\bar{a}_1^i\right) = 0 \Leftrightarrow \mathbb{E}_{G^j} C_2'(a_2^*(\tilde{\tau})) \geq \mathbb{E}_{G^i} C_2'(a_2^*(\tilde{\tau})).$$

Using the Jensen's inequality, this holds true i.f.f. C_2' is convex. □

Under ambiguity neutrality, the formation of precautionary date-1 abatement is hence conditional on the positivity of the third derivative of the abatement cost function[33]. Note that this condition is reminiscent of the definition of risk prudence in the sense of Kimball (1990,[36])[34]. In addition of providing analytical results, considering quadratic abatement cost functions ($C''' \simeq 0$) therefore guarantees that no precautionary date-1 abatement is formed under ambiguity neutrality, so that we clearly separate out the sole effects due to the introduction of ambiguity aversion. In particular, with the abatement cost specification (1), it comes that for all $a_1 \geq 0$ and θ in Θ,

$$a_2^*(a_1; \tilde{\tau}_\theta) = \frac{\tilde{\tau}_\theta - \gamma a_1}{c_2}, \; \mathcal{V}_{a_1}(a_1; \theta) = \frac{c_2 - \gamma}{c_2} \tau_\theta + \frac{\gamma^2 a_1}{c_2}, \text{ and } \bar{a}_1 = \frac{c_2 - \gamma}{c_1 c_2 - \beta \gamma^2} \beta \langle \tilde{\tau} \rangle, \tag{14}$$

where $\tau_\theta = \mathbb{E}_G \{\tilde{\tau}_\theta | \theta\}$ is the θ-scenario average price and $\langle \tilde{\tau} \rangle = \mathbb{E}_{G^0} \{\tilde{\tau}\}$ is the expected price under ambiguity neutrality. Note, in particular, that $\bar{a}_1$ is invariant to any MPS in $\tilde{\tau}$.

Ambiguity aversion. When ϕ is concave, the necessary first-order condition of program (11) defines the optimal date-1 abatement under ambiguity aversion, $\hat{a}_1$, by

$$-C_1'(\hat{a}_1) + \beta \mathbb{E}_F \left\{ \frac{\phi'\left(\mathcal{V}(\hat{a}_1; \tilde{\theta})\right)}{\phi' \circ \phi^{-1}\left(\mathbb{E}_F \left\{\phi\left(\mathcal{V}(\hat{a}_1; \tilde{\theta})\right)\right\}\right)} \mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta}) \right\} = 0. \tag{15}$$

[33]Chevallier et al. (2011,[15]) obtain a similar result with a risk on the future allocation of permits, and argue that banking is an adequate tool for policy (here, allowance allocation) risk control. They also explore the role of a sector-level agency for optimally sharing risk across heterogeneous firms by pooling permits.

[34]Indeed, ignoring the linear trade term, firm's profits read, expressed in terms of emissions e_i, $\pi_i(e_i) = \zeta_i - C_i(\xi - e_i)$ with $\pi_i' > 0$, $\pi_i'' < 0$ and $\pi_i''' > 0$ following from assumptions on C_i and the convexity of C_i', such that π_i displays the same property than a risk-averse risk-prudent utility function.

Proposition 4.2 characterizes the comparative statics of $\hat{a}_1$ w.r.t. $\bar{a}_1$.

Proposition 4.2. *When ϕ displays DAAA (IAAA), ambiguity aversion is conducive to higher (lower) date-1 abatement than under ambiguity neutrality only if $(\mathcal{V}(\bar{a}_1;\theta))_\theta$ and $(\mathcal{V}_{a_1}(\bar{a}_1;\theta))_\theta$ are anti-comonotone (comonotone). When ϕ displays CAAA, ambiguity aversion raises date-1 abatement relative to ambiguity neutrality if, and only if, the anticomonotonicity criterion holds.*

Proof. By concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ is equivalent to

$$\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\} \geq \phi'\circ\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)\right\}\right)\mathbb{E}_F\left\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\}. \quad (16)$$

With ϕ DAAA, note that a sufficient condition for the above to hold is

$$\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\} \geq \mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)\right\}\mathbb{E}_F\left\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\}, \quad (17)$$

which is exactly $\text{Cov}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right);\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\} \geq 0$. Noting that ϕ' is non-increasing concludes. The above argument reverses when ϕ is IAAA. □

Note that under CAAA, the impact of ambiguity aversion on date-1 abatement relative to neutrality is solely dictated by the anticomonotonicity criterion (necessary and sufficient condition). Note also that when ϕ displays DAAA (IAAA), comonotonicity (anticomonotonicity) does not guarantee that date-1 abatement decreases (increases) relative to neutrality. As explained below, this is so because ambiguity aversion induces two effects that work in opposite direction in those two cases. It immediately follows from Proposition 4.2 that

Corollary 4.3. *Assuming ambiguity prudence on the part of the liable firm, it will form precautionary date-1 abatement only if the anticomonotonicity criterion holds.*

Contrary to a tax regime, Proposition 4.2 and Corollary 4.3 indicate that in an ETS, ambiguity prudence is not a necessary and sufficient condition for ambiguity aversion to be conducive to over-abatement at date 1. Rather, provided ambiguity prudence holds, over-abatement under ambiguity aversion is conditional on a covariance criterion, which requires that the marginal ambiguity $\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)$ be positively correlated with the date-2 expected marginal profitability from banking, $\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})$, both evaluated at the optimal date-1 abatement level under ambiguity neutrality. Note that the anticomonotonicity criterion is quite robust as it obtains with other ambiguity representation theorems – see Proposition B.1 for an application of the MEU representation theorem of Gilboa & Schmeidler (1989,[29]). Further inspection of (16) is informative on how smooth ambiguity aversion affects decision-making. Decomposing (16) into two terms[35] yields

$$\mathcal{A}(\bar{a}_1)\mathbb{E}_F\left\{\mathcal{D}(\bar{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\} \geq \mathbb{E}_F\left\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\}, \tag{18}$$

where, $\mathcal{A}$ is defined in (10) and again denotes the ambiguity prudence coefficient and where $\mathcal{D}$ is a distortion function satisfying, for all $\theta \in \Theta$,

$$\mathcal{D}(a_1;\theta) = \frac{\phi'\left(\mathcal{V}(\bar{a}_1;\theta)\right)}{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)\right\}}. \tag{19}$$

In addition to the ambiguity prudence effect ($\mathcal{A}$) and relative to ambiguity neutrality, smooth ambiguity aversion induces a second effect as $\mathcal{D}$ distorts the second-order subjective cumulative distribution of $\tilde{\theta}$, F. From the concavity of ϕ, the distortion function $\mathcal{D}$ overweights those scenarios inducing low $\mathcal{V}$-values[36]. In particular, the distorted second-order subjective probability measure, H, is given by

$$\forall\theta\in\Theta,\ H(\theta) = \int_{\underline{\theta}}^{\theta}\mathcal{D}(\bar{a}_1;X)\mathrm{d}F(X) = \frac{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{X})\right)|\tilde{X}\leq\theta\right\}}{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\bar{a}_1;\tilde{\theta})\right)\right\}}F(\theta), \tag{20}$$

so that $H(\underline{\theta}) = 0$, $H(\bar{\theta}) = 1$ and $H' > 0$ on Θ. Therefore,

$$\hat{a}_1 \geq \bar{a}_1 \ \Leftrightarrow\ \beta\mathcal{A}(\bar{a}_1)\mathbb{E}_H\left\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\} \geq C_1'(\bar{a}_1) = \beta\mathbb{E}_F\left\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\}. \tag{21}$$

In terms of comparative statics, up to the $\mathcal{A}$-effect, introducing ambiguity in the ambiguity-averse firm's decision is identical to a shift in the ambiguity-neutral firm's subjective beliefs from F to H. Due to ambiguity aversion, H (F) places relatively more weight on low-profit (high-profit) scenarios than F (H) does. This pessimism effect makes clear the intuition behind the anticomonotonicity criterion, as illustrated by Example 4.4.

Example 4.4. *Let $\Theta = \{\theta_1, \theta_2\}$ and $\mathcal{V}(a_1;\theta_i)$ be increasing in i, ϕ be* CAAA *so that $\mathcal{A}\equiv 1$, and $F = (q,\theta_1;1-q,\theta_2)$ with $0\leq q\leq 1$. The pessimism effect overweights scenario θ_1 relative to θ_2 so that $H = (\hat{q},\theta_1;1-\hat{q},\theta_2)$ with $q\leq\hat{q}\leq 1$[37]. Then, banking under ambiguity neutrality w.r.t. H, $a_{1,H}$, is bigger than than under neutrality w.r.t. F, $a_{1,F}$, i.f.f.*

$$\hat{q}\mathcal{V}_{a_1}(a_{1,F};\theta_1) + (1-\hat{q})\mathcal{V}_{a_1}(a_{1,F},\theta_2) \geq q\mathcal{V}_{a_1}(a_{1,F};\theta_1) + (1-q)\mathcal{V}_{a_1}(a_{1,F},\theta_2)$$

[35] This is the natural decomposition, *i.e.* normalisation of the additional factor in the future price estimation of (15) in comparison with (12), namely, $\frac{\phi'\left(\mathcal{V}(a_1;\tilde{\theta})\right)}{\phi'\circ\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(a_1;\tilde{\theta})\right)\right\}\right)}$, to obtain a probability distribution.

[36] The distortion function is a Radon-Nikodym derivative and is akin to the martingale distortion occurring in robust control theory – see Hansen & Sargent (2001,[33]).

[37] Under the MEU representation theorem of Gilboa & Schmeidler (1989,[29]), $\hat{q} = 1$. From this, it is clear that the MEU representation is equivalent to the KMM representation in the limiting case of infinite ambiguity aversion, as shown by Klibanoff et al. (2005,[37]).

which is true i.f.f. anticomonotonicity holds, since $\mathcal{V}_{a_1}(a_1;\theta_i)$ would be decreasing in i.

Assuming ambiguity prudence, only when low-$\mathcal{V}$ scenarios coincide with high-$\mathcal{V}_{a_1}$ scenarios does pessimism increase the H-averaged marginal benefit from abating at date-1 as compared to ambiguity neutrality, hence leading to over-abatement at date 1. Example 4.5 further illustrates the underlying effect of pessimism under anticomonotonicity.

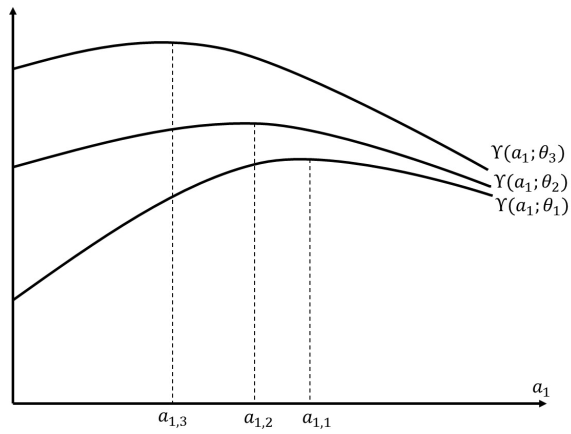


Figure 1: Ambiguity aversion and anticomonotonicity.

Example 4.5. *Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$ and $\Upsilon(a_1;\theta_i)$ denote the net intertemporal expected revenue from date-1 abatement in scenario θ_i, i.e. $\Upsilon(a_1;\theta_i) = \zeta_1 - C_1(a_1) + \beta\mathcal{V}(a_1;\theta_i)$. W.l.o.g. let Θ be ordered such that $\Upsilon(a_1;\theta_i)$ is increasing in i. Suppose that anticomonotonicity holds, i.e. $\Upsilon_{a_1}(a_1;\theta_i)$ is decreasing in i. This is depicted in Figure 1, where $a_{1,i}$ denotes the optimal banking level in scenario θ_i. Anticomonotonicity implies that $a_{1,i}$ is decreasing with i and that in moving towards higher date-1 abatement levels, the spread in $\Upsilon(a_1;\theta)$ across θ-scenarios is reduced*[38].

This is in line with the definition of ambiguity aversion that an ambiguity averse agent dislikes any MPS in the space of conditional second-order expected profit. Under CAAA where only pessimism applies, ambiguity aversion hence unconditionally adjusts date-1 abatement in the direction of reduced spread in Υ across scenarios: upwards if anticomonotonicity holds; downwards if comonotonicity holds. This also illustrates that anticomonotonicity is quite demanding a requirement since $\Upsilon(a_1;\theta_i)$-lines cannot cross between scenarios. It is a strong requirement because one needs to sign the covariance for sure. Roughly speaking, however, it might be sufficient that the level of discrepancy in expected profits across scenarios diminishes in a_1 for ambiguity aversion to raise date-1 abatement relative to neutrality, so that the anticomonotonicity could be relaxed somehow[39].

Let us now account for the two smooth ambiguity aversion-induced effects together. As-

[38]Symmetrically, under IAAA and when comonotonicity holds, reducing banking relative to ambiguity neutrality narrows the spread in $\Upsilon(a_1;\theta)$ across scenarios.

[39]We could not get there analytically but this intuition is illustrated with numerical simulations in section 5. *E.g.*, note that Berger et al. (2016,[9]) transform the anticomonotonicity criterion into a convergence effect between scenarios. They are able to do so because they use a binary structure, *i.e.*, good *vs* bas state, and ambiguity bears solely on the chances that these two states occur.

sume for clarity that there is no long-term effect of abatement, *i.e.* $\partial_{a_1} C_2 \equiv 0$. Concavity of the objective function yields

$$\hat{a}_1 \geq \bar{a}_1 \Leftrightarrow \mathcal{A}(\bar{a}_1) \left(\langle \tilde{\tau} \rangle + \mathcal{P}(\bar{a}_1) \right) \geq \langle \tilde{\tau} \rangle , \tag{22}$$

where $\langle \tilde{\tau} \rangle = \mathbb{E}_{\bar{G}} \{\tilde{\tau}\}$, and $\mathcal{P}(\bar{a}_1)$ can be interpreted as an ambiguity premium, evaluated at $a_1 = \bar{a}_1$, demanded by the liable firm to compensate for its exposure to ambiguity in abating one additional tonne at the margin at date 1, where

$$\mathcal{P}(a_1) = \frac{\text{Cov}_F \left\{ \phi' \left(\mathcal{V}(a_1; \tilde{\theta}) \right) ; \mathcal{V}_{a_1}(a_1; \tilde{\theta}) \right\}}{\mathbb{E}_F \left\{ \phi' \left(\mathcal{V}(a_1; \tilde{\theta}) \right) \right\}}. \tag{23}$$

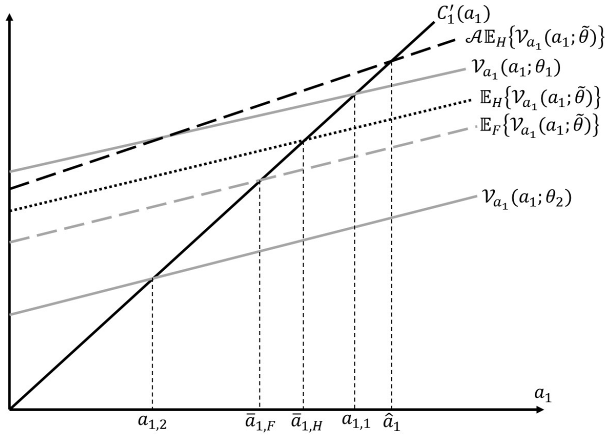
When ϕ is CAAA, *i.e.* $\mathcal{A} \equiv 1$, and the anticomonotonicity criterion holds, the ambiguity premium is positive so that the date-2 pessimistically-distorted allowance price is higher than the ambiguity-neutral one, which leads to over-abatement at date 1 under ambiguity aversion. One must also account for the ambiguity prudence effect under DAAA or IAAA, as illustrated in Proposition 4.6.

Proposition 4.6. *Let $\partial_{a_1} C_2 \equiv 0$. Then, the following equivalence conditions obtain*
(i) When ϕ displays CAAA, *$\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq 0$;*
(ii) When ϕ displays DAAA, *$\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq \frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)} \langle \tilde{\tau} \rangle < 0$;*
(iii) When ϕ displays IAAA, *$\hat{a}_1 \leq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \leq \frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)} \langle \tilde{\tau} \rangle > 0$.*

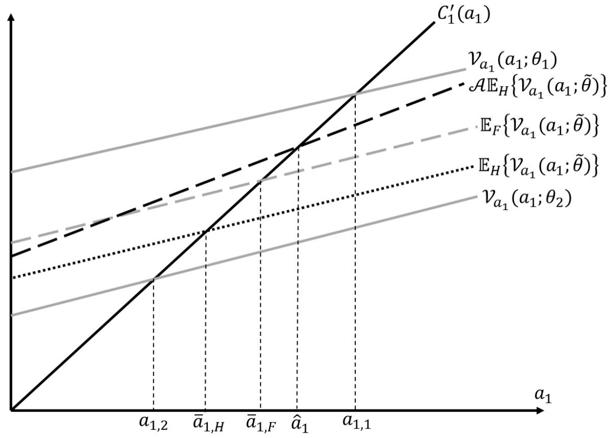
Proposition 4.6 both provides necessary and sufficient conditions for ambiguity aversion to lead to over-abatement and indicates the exact relation between the strengths of the pessimism and ambiguity prudence effects. In contrast, Proposition 4.2 gives sufficient conditions as it abstracts from the exact value of $\mathcal{A}$ by only considering whether it is bigger, lower or equal to unity. Proposition 4.6 thus refines the conditions under which shifts in date-1 abatement decisions due to ambiguity aversion can be determined, in particular when the two effects have opposite impacts on the date-1 abatement decision. For instance, under ambiguity prudence, when $\mathcal{P}(\bar{a}_1) \in \left[\frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)} \langle \tilde{\tau} \rangle ; 0 \right]$, the anticomonotonicity criterion does not hold but still, $\hat{a}_1 > \bar{a}_1$. That is, the ambiguity averse firm distorts the ambiguity-neutral subjective prior by overemphasising low-$\mathcal{V}_{a_1}$ scenarios, hence implying under-abatement. However, this is more than compensated by the decrease in impatience due to ambiguity prudence, and overall, precautionary date-1 abatement is formed.

Figure 2 graphically depicts the joint effects of pessimism and ambiguity prudence. Let $\Theta = \{\theta_1, \theta_2\}$, $\mathcal{V}_{a_1}(a_1; \theta_i)$ be decreasing in i and ϕ display DAAA. For clarity, consider that both H and $\mathcal{A}$ are constant[40]. In particular, Figure 2 separates the pessimism effect ($\bar{a}_1 = a_{1,F} \longrightarrow a_{1,H}$) from the ambiguity prudence effect ($a_{1,H} \longrightarrow \hat{a}_1$). Pessimism op-

[40]Assuming that H and $\mathcal{A}$ do not vary with a_1 does not alter the results and simplifies illustration – see *e.g.* Appendix C when this assumption is relaxed.



(a) Aligned effects.



(b) Opposite effects.

Figure 2: Joint effects of ambiguity prudence and pessimism.

erates a vertical translation of the F-averaged expected marginal profitability from date-1 abatement within the $\mathcal{V}_{a_1}(a_1;\theta_2)$-$\mathcal{V}_{a_1}(a_1;\theta_1)$ band, directed towards the lower $\mathcal{V}$-value scenario[41] and ambiguity prudence then increases the slope of the H-averaged expected marginal profitability from date-1 abatement. In Figure 2a, anticomonotonicity holds so that H overweights θ_1 relative to θ_2 as compared to F, and the two effects are aligned. In Figure 2b, comonotonicity holds so that H overweights θ_2 relative to θ_1 as compared to F, and the two effects are opposed. In this case, ambiguity prudence dominates pessimism since, overall, $\hat{a}_1 > \bar{a}_1$.

While Propositions 4.2 and 4.6 are intuitively appealing, the anticomonotonicity criterion is by no means informative in practice. In Proposition 4.7, we therefore derive more tangible conditions under which this criterion holds. In particular, abatement cost functions are equipped with the quadratic form (1), and we determine how the anticomotonicity criterion translates under this specification.

Proposition 4.7. *Under ambiguity prudence, over-abatement occur at date 1 only if*
(i) the liable firm expects to be in a net short position at date 2 under the abatement stream $(\bar{a}_1; a_2^*(\bar{a}_1;\hat{\tau}_\theta))$ *in all* θ*-scenarios with* $\hat{\tau}_\theta = (\mathbb{E}_{G_\theta}1)^{-1}\,\mathbb{E}_{G_\theta}X$*;*
(ii) for a given allowance allocation ω*, the liable firm abates too little at date 1 under ambiguity neutrality* $\bar{a}_1 < \min_{\theta\in\Theta} a_{1,\theta}$*, or reciprocally,*
(iii) the liable firm's allocation is relatively small $\omega < \omega^* \equiv \min_{\theta\in\Theta}\omega_\theta^* = \xi - \bar{a}_1 - a_2^*(\bar{a}_1;\hat{\tau}_\theta)$.

Proof. The proof consists in signing the covariance. For all $\theta \in \Theta$, $\mathcal{V}_{a_1}(\bar{a}_1;\theta)$, $\bar{a}_1$, and a_2^* are

[41] With the MEU preferences, the agent only considers the $\mathcal{V}_{a_1}(\bar{a}_1;\theta_2)$ line and ambiguity prudence is absent as it is specific to the KMM criterion.

given in (14). Differentiating $\mathcal{V}_{a_1}(\bar{a}_1;\theta)$ w.r.t. θ and then integrating by parts yields

$$\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) = \frac{c_2-\gamma}{c_2}\int_{\mathsf{T}} x\partial_\theta g(x;\theta)\mathrm{d}x = \frac{\gamma-c_2}{c_2}\int_{\mathsf{T}} G_\theta(x;\theta)\mathrm{d}x,$$

where $G_\theta(\cdot;\theta) = \partial_\theta G(\cdot;\theta)$. Similarly, by the Envelop Theorem and differentiation w.r.t. θ,

$$\begin{aligned}
\partial_\theta \mathcal{V}(\bar{a}_1;\theta) &= -\int_{\mathsf{T}} C_2(\bar{a}_1, a_2^*(\bar{a}_1;x)) + x\left(\xi - \bar{a}_1 - a_2^*(\bar{a}_1;x) - \omega\right)\partial_\theta g(x;\theta)\mathrm{d}x \\
&= -\int_{\mathsf{T}} x\left(\xi - \omega - \left(1-\frac{\gamma}{c_2}\right)\bar{a}_1 - \frac{x}{2c_2}\right) - \frac{\gamma^2\bar{a}_1^2}{2c_2}\partial_\theta g(x;\theta)\mathrm{d}x \\
&= \int_{\mathsf{T}}\left(\xi - \omega - \left(1-\frac{\gamma}{c_2}\right)\bar{a}_1 - \frac{x}{c_2}\right)G_\theta(x;\theta)\mathrm{d}x,
\end{aligned}$$

where the third equality obtains by integration by parts. For all $x \in \mathsf{T}$, let $k : x \mapsto \xi - \omega - \left(1-\frac{\gamma}{c_2}\right)\bar{a}_1 - \frac{x}{c_2}$. In all generality, k changes sign over T. By continuity of k, let $\tau_0 \in \mathsf{T}$ be such that $k(\tau_0) = 0$, *i.e.*, $\tau_0 = c_2(\xi-\omega) - (c_2-\gamma)\bar{a}_1$[42]. For all $\theta \in \Theta$, let

$$\Gamma_\theta(\tau_0) = \frac{1}{c_2}\int_{\mathsf{T}}(\tau_0 - x)\,G_\theta(x;\theta)\mathrm{d}x,$$

so that, differentiating w.r.t. τ_0 yields $\Gamma'_\theta(\tau_0) = \frac{1}{c_2}\int_{\mathsf{T}} G_\theta(x;\theta)\mathrm{d}x$.
When $G \nearrow^\theta$, $\Gamma'_\theta > 0$ so that by definition, $\Gamma_\theta(\underline{\tau}) < 0$ and $\Gamma_\theta(\bar{\tau}) > 0$. Symmetrically, when $G \searrow_\theta$, $\Gamma'_\theta < 0$ so that by definition, $\Gamma_\theta(\underline{\tau}) > 0$ and $\Gamma_\theta(\bar{\tau}) < 0$. In both cases, by continuity of Γ_θ, $\forall\theta \in \Theta$, there exists $(\hat{\tau}_\theta; a_{1,\theta})$ defined by $\hat{\tau}_\theta = c_2(\xi-\omega) - (c_2-\gamma)a_{1,\theta}$, such that $\Gamma_\theta(\hat{\tau}_\theta) = 0$. By definition,

$$\int_{\mathsf{T}}(\hat{\tau}_\theta - x)\,G_\theta(x;\theta)\mathrm{d}x = 0 \;\Rightarrow\; a_{1,\theta} = \frac{c_2}{c_2-\gamma}\left(\xi - \omega - \frac{1}{c_2}\frac{\int_{\mathsf{T}} xG_\theta(x;\theta)\mathrm{d}x}{\int_{\mathsf{T}} G_\theta(x;\theta)\mathrm{d}x}\right).$$

In each θ-scenario[43], for a given ω, $a_{1,\theta}$ hence corresponds to the required date-1 abatement effort when the allowance price prevailing at date 2 is $\hat{\tau}_\theta = \frac{\mathbb{E}_{G_\theta}X}{\mathbb{E}_{G_\theta}\mathbf{1}}$, *i.e.* when date-2 abatement is $a_2^*(\bar{a}_1;\tau^\theta)$. Two cases then arise depending on the monotonicity of G w.r.t. θ:
1. $G \nearrow^\theta$: $\forall\theta \in \Theta$, $\partial_\theta\mathcal{V}_{a_1}(\bar{a}_1;\theta) < 0$ and $\partial_\theta\mathcal{V}(\bar{a}_1;\theta) > 0$ i.f.f. $\frac{c_2}{2}\left(\xi - \omega - \left(1-\frac{\gamma}{c_2}\right)\bar{a}_1\right) > \hat{\tau}_\theta$, *i.e.* i.f.f., $\bar{a}_1 < a_{1,\theta}$;
2. $G \searrow_\theta$: $\forall\theta \in \Theta$, $\partial_\theta\mathcal{V}_{a_1}(\bar{a}_1;\theta) > 0$ and $\partial_\theta\mathcal{V}(\bar{a}_1;\theta) < 0$ i.f.f. $\frac{c_2}{2}\left(\xi - \omega - \left(1-\frac{\gamma}{c_2}\right)\bar{a}_1\right) > \hat{\tau}_\theta$, *i.e.* i.f.f., $\bar{a}_1 < a_{1,\theta}$.

In both cases, $\hat{a}_1 > \bar{a}_1$ i.f.f. $\bar{a}_1 < a_{1,\theta}$ $\forall\theta \in \Theta$, *i.e.* i.f.f. $\bar{a}_1 < \min_{\theta\in\Theta} a_{1,\theta}$, which proves (ii). Alternatively, transposing the above argument in terms of allowance allocation yields $\hat{a}_1 > \bar{a}_1$ i.f.f. $\omega < \omega^*_\theta = \xi - \bar{a}_1 - a_2^*(\bar{a}_1;\hat{\tau}_\theta)$, for all θ-scenarios, which proves (iii). Alternatively, $\bar{a}_1 < a_1^\theta$ is equivalent to $\xi - \omega - \bar{a}_1 - a_2^*(\bar{a}_1;\hat{\tau}_\theta) > 0$, which proves (i). Note finally that when ϕ display IAAA, $\hat{a}_1 < \bar{a}_1$ i.f.f. $\bar{a}_1 > \max_{\theta\in\Theta} a_{1,\theta}$. □

[42]This requires that $\underline{\tau} < c_2\left(\xi - \omega - \frac{c_2-\gamma}{c_1c_2-\beta\gamma^2}\beta\langle\bar{\tau}\rangle\right) < \bar{\tau}$, which we assume is the case.

[43]Consistently, note the absence of ϕ-weighting.

Proposition 4.7 shows that ETS-liable firms may over-abate or under-abate at date 1 and that the sign of the pessimism effect ultimately relates to the initial allocation. This result extends that of B&vdF under risk aversion that initial allocation actually matters, *i.e.* that the independence property of initial allocation does not hold. In particular, Proposition 4.7 indicates that ambiguity prudence leads to over-abatement at date-1 only in unfavourable situations where the liable firm expects to be net buyer of allowances under the abatement stream $(\bar{a}_1; a_2^*(\bar{a}_1; \hat{\tau}_\theta))$, for all θ-scenarios. In these situations, the marginal benefits of date-1 abatement (a lowering of both the likelihood of effectively being net short and the volume of allowance purchases) outweigh the marginal cost of date-1 abatement for sure. Otherwise, as soon the liable firm is net long when it abates $(\bar{a}_1; a_2^*(\bar{a}_1; \hat{\tau}_\theta))$ in at least one θ-scenario, one cannot conclude with certainty on the DAAA-related effects on date-1 abatement. Again, this suggests that anticomonotonicity might actually be too strong a criterion to sign pessimism. Because pessimism and ambiguity prudence have opposite effects, even in the extreme opposite favourable situations where the liable firm is net long in all θ-scenarios can one not sign the DAAA-related effects[44].

The anticomonotonicity criterion translates into a threshold criterion on initial conditions ($\bar{a}_1$), or, alternatively, on allocated volume (ω). Note that Berger (2015,[8]) also obtains threshold conditions in translating the anticomonotonicity criterion for optimal self-insurance and self-protection decisions under ambiguity aversion[45], in the specific case where ambiguity is concentrated on on state. In terms of initial conditions, ambiguity prudence is line with a one-sided precautionary principle[46]: Only when date-1 abatement is lower than a given threshold ($\min_{\theta \in \Theta} a_{1,\theta}$) will ambiguity aversion adjust date-1 abatement upwards. Since $\bar{a}_1$ is independent of the future abatement effort and only driven by the $\bar{G}$-expected allowance price, ambiguity aversion integrates considerations about the firm's expected future position on the allowance market into date-1 abatement decisions, and adjusts a_1 accordingly. Therefore, the key determinant in signing pessimism is initial allocation. When it is sufficiently low for the ambiguity-prudent liable firm to expect a net short position in all scenarios ($\omega < \omega^*$), will ambiguity aversion lead to over-abatement. Otherwise, no general results obtain[47].

Appendix D considers the situation where price ambiguity is binary. The interest of this

[44] This asymmetric effect of ambiguity prudence can be interpreted as being in line with a loss aversion rationale (if the firm increases date-1 abatement, it could potentially increase profits on the allowance market) or an endowment effect (it could reduce costly date-1 abatement and cover its emissions with received permits).

[45] There is also an noticeable parallel between banking and both self-insurance and self-protection: banking is costly, but (i) reduces the likelihood of being in a net short position at date 2 (role of self-protection); (ii) for a given date-2 net position, increases date-2 profits by either increasing sales or reducing purchases of allowances (role of self-insurance).

[46] Assuming CAAA, ambiguity aversion drives the liable firm to be more cautious in its date-1 abatement decisions than under neutrality as it displays a tendency to soothe 'extreme' ambiguity-neutral date-1 abatement levels, whether relatively high or low. This can be interpreted as a precautionary effect by which the ambiguity-averse firm rectifies its date-1 abatement decision, lest it deviate too far away from an optimal date-1 abatement path, which is more likely to occur for relatively high or low date-1 abatement levels.

[47] The cut-off allocation volume ω^* will be refined in section 4.2. Just like under CAAA, numerical simulations in section 5 will show that under DAAA, under-abatement occurs when ω is high enough − this is so because the $\mathcal{A}$-effect has almost no impact relative to the $\mathcal{P}$-effect around ω^*.

binary structure is twofold. First, the ambiguity aversion-induced effects can be more easily interpreted and the effects of an increase in the ambiguity level for a given degree of ambiguity aversion can be characterised. Second, with a similar binary structure, Alary et al. (2013, [1]) and Wong (2015a,[69]), *inter alia*, show that the anticomonotonicity criterion is always satisfied so that the impact of ambiguity aversion is clear. In our setup, however, Proposition D.1 indicates that initial continues to dictate how date-1 abatement is adjusted under ambiguity aversion. Yet, the condition for anticomonotonicity to hold is milder: The liable firm must be net short position under the sole abatement stream $(\bar{a}_1; a_2^*(\bar{a}_1; \langle\tau\rangle))$, with $\langle\tau\rangle = \frac{\underline{\tau}+\bar{\tau}}{2}$, and not across all θ-scenarios. This can be likened to a situation where the firm has no idea about the future allowance price at all and thus considers the equiprobable price scenario[48]. This translates into a uniquely defined initial allocation threshold $\omega^* = \xi - \frac{\beta\langle\tilde{\tau}\rangle}{c_1} - \frac{\langle\tau\rangle}{c_2}$.

Finally, Proposition 4.7 highlights that clear comparative statics results under ambiguity aversion are hard to come by. This is so because signing the covariance is a difficult exercise in general and hence requires restrictive threshold conditions, hence limiting the scope of applicability of Proposition 4.7. In particular, while the intuition behind the two effects of ambiguity aversion – pessimistic distortion of the subjective priors and prudence (or, reduced impatience) – is quite straightforward, how these practically transpose is not trivial. First, these two effects can work in opposite directions. Second, the distorted beliefs H are endogenous to the optimisation problem, which ultimately hinge upon initial conditions, $\bar{a}_1$. Third, it depends on both the underlying modelling assumptions and the functional form of the abatement cost functions.

4.2 Cap-and-trade regime under intrinsic ambiguity

In this section, ambiguity solely bears on firms' unregulated emission levels, denoted by $\tilde{\xi}_\theta(s)$ in scenario $\theta \in \Theta$. Because allowances are tradable, each liable firm will be subject to two types of ambiguity[49]: at the firm level, via $\tilde{\xi}_\theta(s)$; at the market level, via the ambiguous allowance price, $\tilde{\tau}_\theta$, endogenously formed on the market. In any θ-scenario, let $\tilde{\xi}_\theta(s)$ be described by the objective cumulative distribution $G(\cdot;\theta)$, supported on $\left[\underline{\xi};\bar{\xi}\right]$. Let F denote the subjective cumulative distribution of $\tilde{\theta}$ on Θ. To provide clear analytical results, assume that (i) abatement cost function are time separable[50]; (ii) $\tilde{\xi}_\theta(s)$ is equipped with a specific structure such that for all $\theta \in \Theta$ and for all $s \in \mathcal{S}$, $\tilde{\xi}_\theta(s) = \bar{\xi}_\theta + \tilde{\epsilon}_\theta(s)$. That is, individual

[48]Under ambiguity neutrality, the firm is not affected by ambiguity.

[49]When allowances cannot be exchanged between covered entities, hence with ambiguity at the individual level only, the liable firm's program reduces to a standard allocation problem under ambiguity. Indeed, each liable firm faces an ambiguous individual emissions cap it must comply with, such that in any θ-scenario, $\tilde{\xi}_\theta - a_1 - a_2 \leq \omega$. With this constraint binding, a_2 mechanically adjusts to a_1: $\tilde{a}_2(\theta) = \tilde{\xi}_\theta - \omega - a_1$. In particular, one tonne abated at date 1 necessarily decreases the abatement effort at date 2 by one unit.

[50]Assuming long-term dependency in abatement cost functions, our result carries over if we suppose symmetric allocation of permits. That is, all firms abate the same at date 2 provided they all abate the same at date one, which is feasible only if allocation is symmetric (since a_1 depends on ω under ambiguity aversion). However, this assumption does not alter our result.

baselines are decomposed into two parts: a first term $\bar{\xi}_\theta$ common to all firms, specific to any given θ-scenario and an idiosyncratic term $\tilde{\epsilon}_\theta(s)$, such that, for all $\theta \in \Theta$, $(\tilde{\epsilon}_\theta(s); s \in \mathcal{S})$ are i.i.d. with $\mathbb{E}_G\left\{\tilde{\epsilon}_\theta(s)|\theta\right\} = 0$ and finite variance. In any θ-scenario, the aggregate baseline emission level is hence given by

$$\int_{\mathcal{S}} \tilde{\xi}_\theta(s)\mathrm{d}s = \int_{\mathcal{S}} \bar{\xi}_\theta \mathrm{d}s + \int_{\mathcal{S}} \tilde{\epsilon}_\theta(s)\mathrm{d}s = S\bar{\xi}_\theta, \tag{24}$$

so that it is deterministic in any θ-scenario, where the last inequality obtains by the Law of Large Numbers for a continuum of i.i.d. variables. As before, firm s' recursive program under smooth ambiguity aversion reads

$$\max_{a_1} \pi_1\left(a_1(s)\right) + \beta\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(a_1(s);\tilde{\theta})\right)\right\}\right), \tag{25}$$

now where $\forall\theta \in \Theta$, $\pi_2\left(a_1(s), a_2; \tilde{\xi}_\theta(s)\right) = \zeta_2 - C_2(a_2) - \tau_\theta\left(\tilde{\xi}_\theta(s) - a_1(s) - a_2 - \omega\right)$ is firm s' date-2 profit in scenario θ. At date 2, for any given allocation plan $(\omega(s))_{s\in\mathcal{S}}$, firms choose how much to abate such that, $\forall s \in \mathcal{S}$, $C_2'\left(a_2(s)\right) = \tau$, the observed allowance price and upon observing their own baseline. In particular, all firms abate by the same amount at date 2, $a_2 \equiv a_2(s)$, $\forall s \in \mathcal{S}$, so that for any θ-scenario, market closure yields

$$\int_{\mathcal{S}} \tilde{\xi}_\theta(s) - a_1(s) - a_2 - \omega(s)\mathrm{d}s = 0 \Rightarrow a_2(\theta) = \bar{\xi}_\theta - \frac{A_1 + \Omega}{S}. \tag{26}$$

From this, the implied allowance price in scenario θ is $\tau_\theta = C_2'\left(\bar{\xi}_\theta - \frac{A_1+\Omega}{S}\right)$, noting that τ_θ is deterministic in any θ-scenario – but not across scenarios[51]. Therefore, by the Envelope Theorem and noting that individual date-1 abatement decisions have no influence on the date-2 allowance price (*i.e.*, $\partial_{a_1}\tau_\theta = 0$), one has that $\forall a_1 \geq 0$, $\forall\theta \in \Theta$, $\tilde{V}_{a_1}(a_1;\theta) = \mathcal{V}_{a_1}(a_1;\theta) = \tau_\theta$.

Ambiguity neutrality. With ϕ linear in (25), all ambiguity-neutral firms abate the same amount at date 1, $\bar{a}_1 = (C_1')^{-1}\left(\beta\left\langle\tau_\theta\right\rangle\right)$, with $\left\langle\tau_\theta\right\rangle = \mathbb{E}_F\left\{\tau_\theta\right\}$. With specification (1), it comes[52]

$$\bar{a}_1 = \frac{c}{Sc_1}\left(S\left\langle\xi_\theta\right\rangle - \Omega\right) \text{ and } a_2(\theta) = \bar{\xi}_\theta - \frac{c}{S}\left(\frac{S\left\langle\xi_\theta\right\rangle}{c_1} + \frac{\Omega}{\beta c_2}\right), \tag{28}$$

with $\left\langle\xi_\theta\right\rangle = \mathbb{E}_F\left\{\bar{\xi}_\theta\right\}$. In terms of comparative statics, *(i)* both $\bar{a}_1$ and $a_2(\theta)$ decrease with the overall cap Ω, with intensities depending on abatement flexibilities at the two dates and inversely proportional to the market size S – in particular, when the ratio of abate-

[51]This requires that, for all $\theta \in \Theta$, $S\bar{\xi}_\theta - A_1 - \Omega > 0$. When $A_1 = \bar{A}_1$ this is always the case provided that

$$\frac{\Omega}{S} > \frac{c_1 \max_{\theta\in\Theta}\bar{\xi}_\theta - c\left\langle\xi_\theta\right\rangle}{c_1 - c}. \tag{27}$$

[52]By definition, $\bar{a}_1 = \frac{\beta c_2}{Sc_1}\left(S\left\langle\xi_\theta\right\rangle - \bar{A}_1 - \Omega\right)$, so that $\bar{A}_1 = S\bar{a}_1 = \frac{c}{c_1}\left(S\left\langle\xi_\theta\right\rangle - \Omega\right)$, which then gives (28). Note that, for all $\theta \in \Theta$, the overall cap is met, *i.e.*, $\int_S \tilde{\xi}_\theta(s) - \bar{a}_1 - \bar{a}_2(\theta)\mathrm{d}s = \Omega$.

ment technology is one, *i.e.*, $c_1 = \beta c_2$, $\partial_\Omega \bar{a}_1 = \partial_\Omega a_2(\theta) = -\frac{1}{2S}$; *(ii)* $\bar{a}_1$ increases while $a_2(\theta)$ decreases with $\langle \xi_\theta \rangle$, with equal and opposite intensities, again depending on abatement flexibilities at the two dates, $\partial_{\langle \xi \rangle} \bar{a}_1 = -\partial_{\langle \xi \rangle} a_2(\theta) > 0$; *(iii)* holding the overall cap Ω constant, both $\bar{a}_1$ and $a_2(\theta)$ increase with the market size S, whose intensities depend on abatement flexibilities at the two dates and are inversely proportional to the square of S since the individual abatement effort increases (the constraint on emissions is unchanged while, on average, the individual allocation $\frac{\Omega}{S}$ diminishes).

Ambiguity Aversion. With ϕ concave, firm s' optimal date-1 abatement level under ambiguity aversion, $\hat{a}_1(s)$, is solution to

$$-C_1'\left(\hat{a}_1(s)\right) + \beta \frac{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\hat{a}_1(s);\tilde{\theta})\right)\mathcal{V}_{a_1}(\hat{a}_1(s);\tilde{\theta})\right\}}{\phi' \circ \phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{V}(\hat{a}_1(s);\tilde{\theta})\right)\right\}\right)} = 0, \tag{29}$$

which is identical to the necessary first-order condition for pure price ambiguity. The same criterion that $\mathcal{V}(\bar{a}_1;\tilde{\theta})$ and $\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})$ covary negatively thus provides a sufficient condition for the formation of precautionary date-1 abatement under ambiguity prudence. As Proposition 4.8 shows, however, the difference lies in the characterisation of when anticomonotonicity holds.

Proposition 4.8. *Let liable firms be ambiguity prudent and $\partial_{a_1} C_2 \equiv 0$. Then, $\forall s \in \mathcal{S}$, $\hat{a}_1(s) \geq \bar{a}_1$ only if $\omega(s) \leq \min_{\theta\in\Theta} \omega_*^\theta > \frac{\Omega}{S}$, which always holds under symmetric allowance allocation.*

Proof. Let us sign $\mathrm{Cov}_F\left\{\mathcal{V}(\bar{a}_1;\tilde{\theta});\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\right\}$. Because τ_θ and $a_2(\theta)$ are deterministic in any given θ-scenario, $\mathcal{V}(\bar{a}_1;\theta) = \zeta_2 - C_2(a_2(\theta)) - \tau_\theta\left(\bar{\xi}_\theta - \bar{a}_1 - a_2(\theta) - \omega(s)\right)$. It follows that

$$\partial_\theta \mathcal{V}_{a_1}\left(\bar{a}_1;\theta\right) = \partial_\theta \tau_\theta = C_2''(a_2(\theta))\partial_\theta a_2(\theta) = C_2''(a_2(\theta))\partial_\theta \bar{\xi}_\theta,$$

where the last equality follows from $\partial_\theta \bar{A}_1 = 0$ ($\bar{A}_1$ is decided ex-ante). Similarly, since $\partial_\theta \bar{a}_1 = 0$, it comes that

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) = \left(C_2''(a_2(\theta))\Psi(s;\theta) - C_2'(a_2(\theta))\right)\partial_\theta \bar{\xi}_\theta,$$

where $\Psi(s;\theta) = \bar{a}_1 + a_2(\theta) + \omega(s) - \bar{\xi}_\theta$ is firm s' expected net position on the allowance market in scenario θ under ambiguity neutrality. In general, anticomonotonicity holds provided that, for all $\theta \in \Theta$, $\Psi(s;\theta) < \frac{C_2'(a_2(\theta))}{C_2''(a_2(\theta))}$. Note that this allows a positive (*i.e.*, long) net position. In particular, with quadratic abatement cost functions, it comes from (28) that $\Psi(s;\theta) = \omega(s) - \frac{\Omega}{S}$, which is nil for a symmetric allocation allocation plan. Hence, when allowance allocation is symmetric, anticomonotonicity always holds, irrespective of the monotonicity of $\bar{\xi}_\theta$ w.r.t θ. Moreover, assuming for simplicity that the ratio of abatement technology between the two dates is unitary, $c_1 = \beta c_2$, one has that

$$\Psi(s;\theta) < \frac{C_2'(a_2(\theta))}{C_2''(a_2(\theta))} \quad \Leftrightarrow \quad \omega(s) < \min_{\theta\in\Theta} \omega_*^\theta, \quad \text{with} \quad \omega_*^\theta = \frac{\Omega}{2S} + \bar{\xi}_\theta - \frac{\langle \xi_\theta \rangle}{2},$$

where it follows from (27) that $\omega_*^\theta > \frac{\Omega}{S}$ for all $\theta \in \Theta$. □

As compared with pure price ambiguity, there is an additional term entering the determination of the covariance sign. The anticomonotonicity criterion is thus somewhat laxer since a net long position under the abatement stream $(\bar{a}_1; a_2(\theta))$ can be sufficient to increase date-1 abatement relative to neutrality, provided that the net positive position is not too big. Given that liable firms are identical, a grandfathered allocation corresponds to a symmetric allocation, under which ambiguity-prudent firms always over-abate at date 1 and intertemporal cost-efficiency does not obtain.

4.3 Comparative statics

This section addresses the comparative statics of the above analysis, namely, the sensibility of optimal date-1 abatement decisions under ambiguity aversion to the degree of ambiguity aversion (tastes) and the volume of initial allocation of permits.

Increase in ambiguity aversion. In the sense of KMM, firm 2 is said to be more ambiguity averse than firm 1 if the ambiguity function of the former writes as an increasing and concave transformation of the latter's, *i.e.*, if there exists a function ψ such that $\phi_2 = \psi \circ \phi_1$ with $\psi' > 0$ and $\psi'' \leq 0$. Let $\hat{a}_i$ denote firm i's optimal date-1 abatement under ambiguity aversion with ϕ_i. Let us now state

Proposition 4.9. *Let there be two ambiguity-averse ambiguity-prudent firms 1 and 2, where firm 2 is more ambiguity averse than firm 1 such that $\phi_2 = \psi \circ \phi_1$, with ψ increasing, concave and almost quadratic, $\psi''' \simeq 0$. Assume $\mathcal{V}(a_1;\tilde{\theta})$ and $\mathcal{V}_{a_1}(a_1;\tilde{\theta})$ are anticomonotone, so that both firms form precautionary date-1 abatement. Then, firm 2 abates relatively more than firm 1 at date 1 provided that firm 1's ambiguity prudence is not too strong, i.e. $\frac{-\phi_1'''}{\phi_1''} \leq \frac{-3\phi_1''}{\phi_1'}$.*

Proof. By concavity of the objective function, $\hat{a}_2 \geq \hat{a}_1$ i.f.f.

$$\mathcal{A}_2(\hat{a}_1)\mathbb{E}_F\left\{\mathcal{D}_2(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\right\} \geq \mathcal{A}_1(\hat{a}_1)\mathbb{E}_F\left\{\mathcal{D}_1(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\right\},$$

with $\mathcal{A}_i$ and $\mathcal{D}_i$ denoting the ambiguity prudence coefficient and distortion function for firm i – replace ϕ by ϕ_i in (10) and (19). Note that, for all θ in Θ, one has that

$$\frac{\mathcal{D}_2(\hat{a}_1;\theta)}{\mathcal{D}_1(\hat{a}_1;\theta)} = \psi' \circ \phi_1 \left(\mathcal{V}(\hat{a}_1;\theta)\right) \frac{\mathbb{E}_F \left\{ \phi_1' \left(\mathcal{V}(\hat{a}_1;\tilde{\theta}) \right) \right\}}{\mathbb{E}_F \left\{ \phi_2' \left(\mathcal{V}(\hat{a}_1;\tilde{\theta}) \right) \right\}} \propto \psi' \circ \phi_1 \left(\mathcal{V}(\hat{a}_1;\theta)\right).$$

W.l.o.g., let $\mathcal{V}(\hat{a}_1;\theta)$ be non-decreasing in θ. By definition, $\psi' \circ \phi_1 \left(\mathcal{V}(\hat{a}_1;\theta)\right)$ is non-increasing in θ. Since for all θ, $\frac{\mathcal{D}_2}{\mathcal{D}_1}$ is non-increasing in θ, firm 2 displays a stronger pessimism effect than firm 1 in the sense that it overemphasises low-$\mathcal{V}$ states even further. By anticomonotonicity, $\mathcal{V}_{a_1}(\hat{a}_1;\theta)$ is non-increasing in θ so that it holds that

$$\mathbb{E}_F \left\{ \mathcal{D}_2(\hat{a}_1;\tilde{\theta}) \mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta}) \right\} \geq \mathbb{E}_F \left\{ \mathcal{D}_1(\hat{a}_1;\tilde{\theta}) \mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta}) \right\}.$$

Hence, provided that $\mathcal{A}_2(\hat{a}_1) \geq \mathcal{A}_1(\hat{a}_1)$, it is always true that $\hat{a}_2 \geq \hat{a}_1$.

We now investigate when $\mathcal{A}_2 \geq \mathcal{A}_1$ holds in general. This is equivalent to firm 2 being more ambiguity prudent than firm 1, *i.e.*, $\frac{-\phi_2'''}{\phi_2''} \geq \frac{-\phi_1'''}{\phi_1''}$. Assuming $\psi''' = 0$, it comes that

$$\phi_2'' = \left(\psi'' \circ \phi_1\right) \phi_1'^2 + \left(\psi' \circ \phi_1\right) \phi_1'', \text{ and } \phi_2''' = 3 \left(\psi'' \circ \phi_1\right) \phi_1' \phi_1'' + \left(\psi' \circ \phi_1\right) \phi_1'''.$$

That $\frac{-\phi_2'''}{\phi_2''} \geq \frac{-\phi_1'''}{\phi_1''}$ hence rewrites $\frac{-\phi_1'''}{\phi_1''} \leq \frac{-3\phi_1''}{\phi_1'}$. □

Proposition 4.9 separates out two effects consecutive to an increase in ambiguity aversion from ϕ_1 to ϕ_2. First, it leads to an unambiguous increase in pessimism in the sense of a monotone likelihood deterioration as in Gollier (2011,[30]): Being more concave, ϕ_2 places even more weight on those low-profit scenarios than ϕ_1. Second, in our two-period, ϕ-certainty-equivalent formulation, an increase in ambiguity aversion also induces a shift in ambiguity prudence. Controlling for this second effect by imposing that both firms display CAAA ($\mathcal{A}_i \equiv 1$, $i = 1, 2$) it is immediate from Proposition 4.9 that

Corollary 4.10. *Assuming* CAAA *on the part of liable firms and that anticomonotonicity holds, an increase in ambiguity aversion is always conducive to higher date-1 abatement.*

A similar result can also be found in Osaki & Schlesinger (2014,[51],Prop.3). For ambiguity prudence and pessimism to be aligned, *i.e.* for firm 2 to abate more than firm 1 at date 1 for sure, $\mathcal{A}_2$ must be at least as big as $\mathcal{A}_1$, that is, firm 2 must be more ambiguity prudent than firm 1. To be able to quantify the shift in ambiguity prudence, ψ must be equipped with an additional property and we impose the simplest one, namely $\psi''' = 0$. With this, an increase in ambiguity aversion via ψ increases ambiguity prudence provided that initial ambiguity prudence (for firm 1) is not too strong relative to ambiguity aversion, *i.e.* $\frac{-\phi_1''}{\phi_1'} \leq \frac{-\phi_1'''}{\phi_1''} \leq \frac{-3\phi_1''}{\phi_1'}$. In other words, when precautionary date-1 abatement for firm 1 is already substantial or when the ambiguity prudence effect for firm 1 is relatively strong, an increase in ambiguity aversion via ψ might not be conducive to an increase in date-1 abatement on the part of firm 2 for sure. A similar cut-off condition on the strength of ambiguity prudence is highlighted by Guerdjikova & Sciubba (2015,[31]). In a market populated by both ambiguity neutral (*i.e.* EU-maximisers) and ambiguity averse individuals (thus forming wrong beliefs as compared with EU-maximisers and, accordingly, having a tendency to disappear with time), they show that only those displaying *strong* ambiguity prudence, $\frac{-\phi'''}{\phi''} > \frac{-2\phi''}{\phi'}$, will survive[53].

Dependence to initial allocation volume. For clarity, let ϕ display CAAA and let there be no long-term effect of abatement. Under these assumptions, $\hat{a}_1$ is defined by

$$-C_1'(\hat{a}_1) + \beta \frac{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\hat{a}_1;\tilde{\theta})\right)\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\right\}}{\mathbb{E}_F\left\{\phi'\left(\mathcal{V}(\hat{a}_1;\tilde{\theta})\right)\right\}} = 0, \tag{30}$$

where $\mathcal{V}_{a_1}(\hat{a}_1;\theta) = \bar{\tau}_\theta = \mathbb{E}_G\{\tilde{\tau}_\theta|\theta\}$. Taking the total differential of (30) yields

$$\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}\omega} = \frac{\beta\Phi(\hat{a}_1)}{C_1''(\hat{a}_1) - \beta\Phi(\hat{a}_1)}, \tag{31}$$

where, since $\mathcal{V}_\omega = \mathcal{V}_{a_1} = \bar{\tau}_\theta$, and omitting arguments so as to avoid cluttering,

$$\Phi(a_1) = \frac{\mathbb{E}_F\left\{\mathcal{V}_{a_1}^2\phi''(\mathcal{V})\right\}\mathbb{E}_F\{\phi'(\mathcal{V})\} - \mathbb{E}_F\{\mathcal{V}_{a_1}\phi'(\mathcal{V})\}\mathbb{E}_F\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\}}{\mathbb{E}_F\{\phi'(\mathcal{V})\}^2}. \tag{32}$$

In particular, note that $\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}\omega} \in]-1;0[$ i.f.f. $\Phi(\hat{a}_1) < 0$. One can show that

$$\begin{aligned}\Phi(\hat{a}_1) &\propto \mathrm{Cov}_F\left\{\mathcal{V}_{a_1};\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\}\mathbb{E}_F\{\phi'(\mathcal{V})\} - \mathrm{Cov}_F\left\{\mathcal{V}_{a_1};\phi'(\mathcal{V})\right\}\mathrm{E}_F\left\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\} \\ &\propto \mathcal{P}(\hat{a}_1) - \mathcal{P}_2(\hat{a}_1) = \frac{\mathrm{Cov}_F\{\mathcal{V}_{a_1};\phi'(\mathcal{V})\}}{\mathbb{E}_F\{\phi'(\mathcal{V})\}} - \frac{\mathrm{Cov}_F\{\mathcal{V}_{a_1};\mathcal{V}_{a_1}\phi''(\mathcal{V})\}}{\mathbb{E}_F\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\}},\end{aligned} \tag{33}$$

where $\mathcal{P}(\hat{a}_1)$ is the ambiguity premium and $\mathcal{P}_2(\hat{a}_1)$ can be interpreted as a second-order ambiguity premium, both evaluated at $a_1 = \hat{a}_1$. Note that when the anticomonotonicity criterion holds, the two premia are positive and $\Phi(\hat{a}_1) \leq 0$ i.f.f. $\mathcal{P}_2(\hat{a}_1) \geq \mathcal{P}_1(\hat{a}_1)$. It is difficult to determine the variations of $\hat{a}_1$ w.r.t. ω because it is hard to sign $\mathcal{P}_2(\hat{a}_1) - \mathcal{P}_1(\hat{a}_1)$ in general. In section 5, numerical simulations show that, in line with intuition, the level of optimal date-1 abatement unambiguously decreases with the permit handout volume, with intensities depending on the degree of ambiguity aversion and the initial allocation volume itself[54].

[53]As pointed out by Baillon (2015,[2]), this motivates further work in the direction of extending the notion of ambiguity prudence to higher orders, which is beyond the scope of this paper.

[54]This would suggest that $\mathcal{P}_2$ is bigger than $\mathcal{P}_1$, and, again, this calls for studying higher orders for ambiguity prudence.

5 Numerical simulations

Given the difficulty to obtain general comparative static results, numerical simulations are presented in this section. This parametrical illustration also helps clarify and quantify both the pessimism and ambiguity prudence effects.

Quota regime under CAAA. Without loss of generality, let there be no long-term effect of abatement $\gamma = 0$ with $c_1 = c_2 = 1$ and $\beta = 1$. Let also $F \hookrightarrow \mathcal{U}(\Theta = [\![-\underline{\theta};\bar{\theta}]\!])$ and, for all scenario $\theta \in \Theta$, $G(\cdot;\theta) \hookrightarrow \mathcal{U}(\mathsf{T}_\theta = [\underline{\tau}+\theta;\bar{\tau}+\theta])$[55], where $0 < \bar{\theta} < \underline{\tau}$ and $\Delta\tau = \bar{\tau} - \underline{\tau} > 0$. With this, it follows that for all $a_1 \geq 0$ and $\theta \in \Theta$

$$\begin{aligned}\mathcal{V}(a_1;\theta) &= \zeta_2 - \frac{1}{\Delta\tau}\int_{\underline{\tau}+\theta}^{\bar{\tau}+\theta} x\left(\xi - a_1 - \omega - \frac{x}{2}\right)\mathrm{d}x \\ &= \zeta_2 - (\xi - a_1 - \omega)(\langle\tau\rangle + \theta) + \frac{1}{6}(3\theta(2\langle\tau\rangle + \theta) + 4\langle\tau\rangle^2 - \underline{\tau}\bar{\tau}),\end{aligned}$$

so that $\mathcal{V}_{a_1}(a_1;\theta) = \langle\tau\rangle + \theta$, with $\langle\tau\rangle = \frac{\underline{\tau}+\bar{\tau}}{2}$. It follows that anticomonotonicity holds provided that, for all $\theta \in \Theta$,

$$\partial_\theta \mathcal{V}(a_1;\theta) \leq 0 \;\Leftrightarrow\; \omega \leq \xi - a_1 - \langle\tau\rangle - \theta.$$

In particular, evaluated at $a_1 = \bar{a}_1 = \langle\tilde{\tau}\rangle = \mathbb{E}_F\left\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\right\} = \langle\tau\rangle$, anticomonotonicity holds i.f.f. $\omega \leq \xi - 2\langle\tau\rangle - \bar{\theta}$. Note that this corresponds to the threshold condition given in Proposition 4.7, *i.e.* $\omega^* = \xi - 2\langle\tau\rangle - \bar{\theta}$[56]. Finally, let $\underline{\tau} = 10$, $\bar{\tau} = 30$, $\bar{\theta} = 9$, $\xi = 100$ and $\omega \in [0;120]$. This gives $\langle\tau\rangle = 20$ and $\omega^* = 51$.

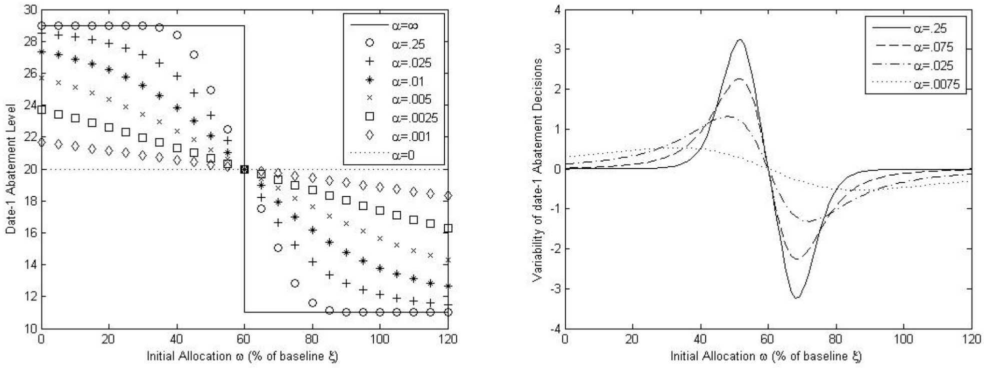
Let ϕ display CAAA such that $\phi(x) = \frac{e^{-\alpha x}}{-\alpha}$ with $\alpha > 0$ the coefficient of absolute ambiguity aversion. Let $\hat{a}_1^\alpha$ be the optimal date-1 abatement defined by

$$\hat{a}_1^\alpha = \langle\tau\rangle + \mathcal{P}(\hat{a}_1^\alpha), \text{ with } \mathcal{P}(a_1) = \left(\int_\Theta e^{-\alpha\mathcal{V}(a_1;\theta)}\mathrm{d}\theta\right)^{-1}\int_\Theta \theta e^{-\alpha\mathcal{V}(a_1;\theta)}\mathrm{d}\theta. \qquad (34)$$

where, by extension, $\hat{a}_1^\infty$ denotes the optimal date-1 abatement with the MEU representation theorem and $\hat{a}_1^0 = \bar{a}_1$. We can now characterize how $\hat{a}_1^\alpha$ evolves with ω and α. Figure 3a depicts the evolution of the optimal date-1 abatement level $\hat{a}_1^\alpha$ as a function of initial allocation for various ambiguity aversion degrees. Note that this also corresponds to the future price distortion due to ambiguity aversion. With our specification, there are implicit upper and lower constraints on $\hat{a}_1^\alpha$ (29 and 11, respectively) since the maximal future price variation, and hence date-1 abatement variation, is confined within a $-9;+9$ range around $\langle\tau\rangle = 20$. The dotted line represents the optimal date-1 abatement under ambiguity

[55] More generally, one could consider that G is uniform over $[\underline{\tau} - \varsigma\theta;\bar{\tau}+\theta]$ with ς a constant. This does not change the results and complicates computations.

[56] Indeed, $\omega_\theta^* = \xi - \bar{a}_1 - a_2^*(\bar{a}_1;\hat{\tau}_\theta)$, with $\bar{a}_1 = \langle\tilde{\tau}\rangle$ and $\hat{\tau}_\theta = \left(\int_{\mathsf{T}_\theta} G_\theta(x;\theta)\mathrm{d}x\right)^{-1}\int_{\mathsf{T}_\theta} xG_\theta(x;\theta)\mathrm{d}x = \langle\tau\rangle + \theta$, since $G_\theta(x;\theta) = -\frac{1}{\Delta\tau}$ for $\underline{\tau}+\theta \leq x \leq \bar{\tau}+\theta$ and 0 otherwise.



(a) $\hat{a}_1^\alpha = f(\omega)$ for different α's

(b) Optimal abatement variability

Figure 3: Cap-and-trade regime under CAAA.

neutrality $\bar{a}_1$, which is independent of the initial allocation volume. Other curves depict $\hat{a}_1^\alpha$ for various ambiguity aversion degrees α.

First, note that $\hat{a}_1^\alpha$ unambiguously decreases with ω, a fact we were unable not prove in general in section 4.3. Note also that there is a clear threshold at $\bar{\omega} = 60$ below (resp. above) which date-1 abatement increases (resp. decreases) under ambiguity aversion relative to $\bar{a}_1$, for all ambiguity aversion degree. This condition is laxer than that for anticomonotonicity to hold since $\omega^* < \bar{\omega}$. As mentioned earlier on, this suggests that anticomonotonicity might be too strong a requirement to sign pessimism. From the simulations, we infer that $\bar{\omega} = \mathbb{E}_F\{\omega_\theta^*\} = \xi - 2\langle\tau\rangle$ so that we can make the following claim: "The introduction of ambiguity aversion is conducive to an increase in date-1 abatement if, and only if, anticomonotonicity holds *in expectations* over Θ w.r.t. F".

Second, Figure 3a shows that, for any given initial allocation volume, the variation $|\hat{a}_1^\alpha - \bar{a}_1|$ increases with α. This was established in Corollary 4.10: When anticomonotonicity holds in expectations, $\hat{a}_1^\alpha$-lines are ordered by increasing α and never cross, so that an increase in ambiguity aversion always leads to higher date-1 abatement, for all $\omega < \bar{\omega}$. When comonotonicity holds in expectations ($\omega > \bar{\omega}$), the higher α the lower date-1 abatement. Moreover, Figure 3a indicates that the sensitivity of date-1 abatement w.r.t. ω also increases with α. That is, the bigger α, the bigger the variations in $\hat{a}_1^\alpha$ around $\bar{\omega}$, *i.e.* the quicker $\hat{a}_1^\alpha$ adjusts upwards/downwards. Note, in particular, that for $\alpha = .25$, $\hat{a}_1^\alpha$ has converged to its upper (resp. lower) limit when ω reaches 30 (resp. 90). This might be better explained with Figure 3b which plots $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha)$ as a function of ω. Using (22) and injecting the first-order condition for $\hat{a}_1^\alpha$, there is an incentive to increase date-1 abatement i.f.f. $\hat{a}_1^\alpha - \bar{a}_1 + \mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha) > 0$ so that $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha)$ can be interpreted as a proxy of the incentive to increase $\hat{a}_1^\alpha$ relative to $\bar{a}_1$, and hence roughly measures the sensitivity of how $\hat{a}_1^\alpha$ varies with ω. It is thus clear from Figure 3b that the bigger α, the quicker $\hat{a}_1^\alpha$ adjusts to ω around $\bar{\omega}$, *i.e.* the bigger the sensitivity of $\hat{a}_1^\alpha$ w.r.t. ω, within the $\bar{\omega}$-centred range, say, $[40; 80]$. Moving to lower α's, this sensitivity decreases around $\bar{\omega}$, but the incentive to adjust date-1 abatement relative to ambiguity neutrality is both smaller and more evenly spread over the entire allocation range $[0; 120]$.

Third, the solid line in Figure 3a represents the optimal date-1 abatement level with the MEU representation theorem. It is certainly too extreme in the sense that it is a step function ('bang-bang' date-1 abatement correction) since, provided that anticomonotonicity holds in expectations, $\hat{a}_1^{\infty}$ equals the upper limit; otherwise, it equals the lower limit. In this light, the KMM representation describes a continuum of optimal date-1 abatement levels for different α's between the two polar cases defined by ambiguity neutrality and the MEU criterion. In particular, in Figure 3b, $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^{\infty})$ corresponds to the $\bar{\omega}$-centred Dirac distribution. Next, we analyse the tax regime under DAAA as it clearly singles out the ambiguity prudence effect.

Tax regime under DAAA. For consistency with the above example and w.l.o.g., we take $\gamma = 0$, $c_1 = c_2 = 1$, $\beta = 1$ and $t = \langle \tilde{\tau} \rangle = 20$. Similarly, $F \hookrightarrow \mathcal{U}([\![-\bar{\theta}; \bar{\theta}]\!])$ with $\bar{\theta} = 9$ and, for all scenario $\theta \in \Theta$, $G(\cdot; \theta) \hookrightarrow \mathcal{U}(\Xi_\theta = [\underline{\xi} + \theta; \bar{\xi} + \theta])$ with $\Delta\xi = \bar{\xi} - \underline{\xi} > 0$. To enrich our results, we let the representative liable firm be endowed with ω permits, each permit exempting the firm to pay the tax for one tonne emitted, since (i) this provides further consistency with the cap-and-trade regime; (ii) such a scheme has certain policy relevance, see *e.g.* Pezzey & Jotzo (2013,[54]). That is, the tax is charged only on the difference between emissions and the threshold ω. In particular, the case $\omega = 0$ corresponds to the analysis presented in section 3. Therefore, for all $a_1 \geq 0$ and $\theta \in \Theta$

$$\mathcal{V}(a_1; \theta) = \zeta_2 - t\left(\xi_\theta - a_1 - \frac{t}{2} - \omega\right), \text{ with } \xi_\theta = \frac{1}{\Delta\xi}\int_{\underline{\xi}+\theta}^{\bar{\xi}+\theta} x \mathrm{d}x = \langle \xi \rangle + \theta,$$

so that $\mathcal{V}_{a_1}(a_1; \theta) = t$, where $\langle \xi \rangle = \frac{\underline{\xi}+\bar{\xi}}{2}$. W.l.o.g. we let $\underline{\xi} = 50$ and $\bar{\xi} = 150$ so that $\langle \xi \rangle = \left\langle \tilde{\xi} \right\rangle = \mathbb{E}_F \{\xi_\theta\} = 100$. Let ϕ display DAAA such that $\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}$, with $\alpha > 1$ the coefficient of absolute ambiguity aversion. Let also $\hat{a}_1^{t,\alpha}$ be defined such that

$$\hat{a}_1^{t,\alpha} = \mathcal{A}(\hat{a}_1^{t,\alpha})t, \text{ with } \mathcal{A}(a_1) = \left(\int_\Theta (\mathcal{V}(a_1;\theta))^{1-\alpha} \mathrm{d}F(\theta)\right)^{\frac{\alpha}{1-\alpha}} \int_\Theta (\mathcal{V}(a_1;\theta))^{-\alpha} \mathrm{d}F(\theta), \quad (35)$$

where, again, $\hat{a}_1^{t,\infty}$ and $\hat{a}_1^{t,1} = \bar{a}_1$ denote the optimal date-1 abatement level under the MEU criterion and ambiguity neutrality, respectively. We can now solve for $\hat{a}_1^{t,\alpha}$ as a function of ω for different α.

Figure 4 illustrates the specific effects of DAAA on date-1 abatement decisions in a tax regime. First, for all $\alpha > 1$, $\hat{a}_1^{t,\alpha}$ unambiguously decreases with ω but is always above $\bar{a}_1$ due to ambiguity prudence. Second, note that the $\mathcal{A}$-effect is more pronounced the smaller ω gets, especially for low α's with steep variations as soon as ω passes below 40. In particular, in the usual situation where $\omega = 0$, Figure 4 indicates that a higher degree of ambiguity

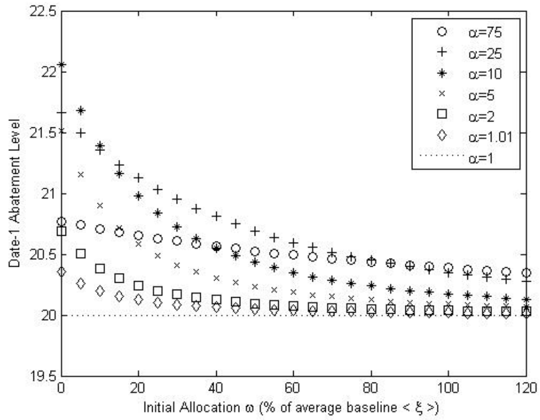


Figure 4: Tax regime under DAAA.

aversion is not necessarily conducive to higher date-1 abatement levels. One can show that there exists a threshold $\bar{\alpha}$ such that $\hat{a}_1^{t,\alpha}$ increases (resp. decreases) with α provided that α is below (resp. above) $\bar{\alpha}$ – numerically, we find $\bar{\alpha} \simeq 11.55$. One could also conjecture that for very high α, $\hat{a}_1^{t,\alpha} \to \bar{a}_1$. This should be the case since the $\mathcal{A}$-effect is specific to the KMM representation, hence absent with the MEU criterion[57]. On the other hand, if we consider the entire allocation continuum, we see that in moving towards higher ω levels, $\hat{a}_1^{t,\alpha}$ gets ranked by increasing ambiguity aversion degrees. Third, the ratio $\hat{a}_1^{t,\alpha}/\bar{a}_1 > 1$ is relatively smaller than under a cap-and-trade regime under CAAA as ω varies, even when α is close to $\bar{\alpha}$. This suggests that the magnitude of the $\mathcal{A}$-effect is relatively small, as compared with the $\mathcal{P}$-effect. Next we combine the two effects of ambiguity aversion in a quota regime under DAAA.

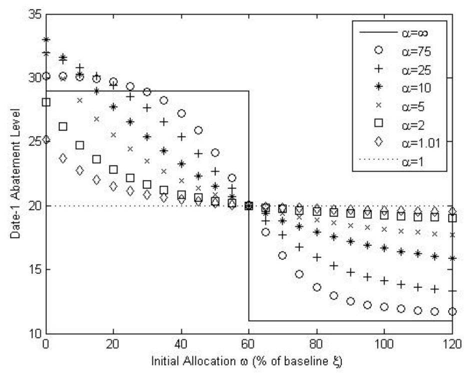
Quota regime under DAAA. We use the same modelling assumptions as for the quota regime under CAAA but now take ϕ to be DAAA instead, again with $\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}$, $\alpha > 1$. With this, $\hat{a}_1^\alpha$ satisfies the following necessary first-order condition

$$\hat{a}_1^\alpha = \mathcal{A}(\hat{a}_1^\alpha)\left(\langle\tau\rangle + \mathcal{P}(\hat{a}_1^\alpha)\right) \text{ with } \mathcal{P}(a_1) = \left(\int_\Theta \left(\mathcal{V}(a_1;\theta)\right)^{-\alpha} \mathrm{d}\theta\right)^{-1} \int_\Theta \theta \left(\mathcal{V}(a_1;\theta)\right)^{-\alpha} \mathrm{d}\theta, \quad (36)$$

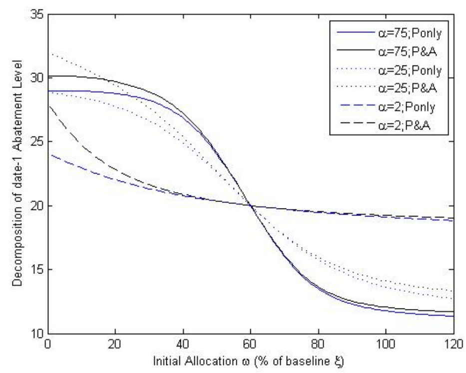
and with $\mathcal{A}(a_1)$ as in (35) for the tax regime.

Figure 5a depicts the variations of $\hat{a}_1^\alpha$ with ω for different ambiguity aversion degrees. It is similar to Figure 3a save for small disruptions due to the ambiguity prudence effect. In particular, when $\omega > \bar{\omega}$, the $\mathcal{A}$-effect pushes $\hat{a}_1^\alpha$ up towards the $\bar{a}_1$-line so that the lower limit $\hat{a}_1^\infty = 11$ is never reached. Note that the $\bar{a}_1$-line is never breached so that the $\mathcal{A}$-effect does not correct date-1 abatement above the ambiguity-neutral case. When $\omega < \bar{\omega}$, the $\mathcal{A}$-effect further adjusts banking upwards so that for relatively low allocation levels,

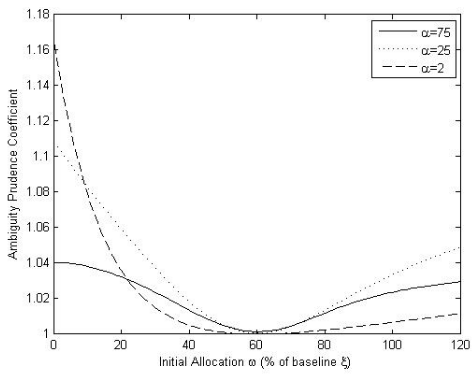
[57] More specifically, since $\mathcal{V}$ clearly is a decreasing function of θ, an MEU-agent only considers the scenario $\theta = \bar{\theta}$. Because a_1 is solely fixed based on the future tax rate, which is known and common to all scenarios, and because there is no $\mathcal{P}$-effect at play, $\hat{a}_1^{t,\infty} = \bar{a}_1$.



(a) $\hat{a}_1^\alpha = f(\omega)$ for different α's



(b) Decomposition of the $\mathcal{P}$ and $\mathcal{A}$ effects



(c) Magnitude of the $\mathcal{A}$ effect

Figure 5: Cap-and-trade regime under DAAA.

the upper limit $\hat{a}_1^\infty = 29$ can even be exceeded. Additionally and as in the tax regime, note that when ω tends to be very small, a higher ambiguity aversion degree is not necessarily conducive to higher date-1 abatement. More precisely, and as clear from Figure 5c, the magnitude of the $\mathcal{A}$-effect is relatively bigger for low α's when ω is really small. Note that within the $[40; 80]$ band, the $\mathcal{A}$-effect is really small and that it is ordered by increasing α around $\bar{\omega}$. Asymmetrically, when ω is big, it is relatively lower than when ω is small.
In particular, $\hat{a}_1^\alpha$-lines may cross for different α when ω is low enough. This fact was already underlined in Proposition 4.9: An increase in ambiguity aversion is not necessarily conducive to higher date-1 abatement under DAAA as the ambiguity prudence effect might disturb the pessimism-induced price distortion. More specifically, when ω is low enough and $\hat{a}_1^\alpha$ is relatively high, an increase in α might decrease date-1 abatement. In contrast, note that no such crossing exist when $\omega > \bar{\omega}$ so that the $\mathcal{A}$-effect asymmetrically disrupts the $\hat{a}_1^\alpha$-lines ordering. The joint $\mathcal{A}$ and $\mathcal{P}$ effect is further illustrated by the decomposition in Figure 5b where it is clear than the $\mathcal{A}$-correction is more substantial for small ω than for big ω while it is almost nil within the $[40; 80]$ band. Except for low allocation levels, this further suggests that the $\mathcal{P}$-effect is the main driving factor behind the date-1 abatement adjustment, while the $\mathcal{A}$-effect often has a residual impact in comparison.

6 Discussion

This section first develops three natural extensions to our previous analysis. Second, in relation to the literature, it briefly discusses how our results would carry over (or not) to more general assumptions than those used so far.

6.1 Three extensions

FORWARD TRADING. Given that ambiguity-averse firms demand an ambiguity premium ($\mathcal{P}$) to compensate for their exposure to ambiguity, it is natural to investigate whether the introduction of a forward market can diminish ambiguity and restore cost-efficiency. In practice, firms liable under cap-and-trade regimes have recourse to forward contracts for hedging purposes, *e.g.* power companies in the EUETS. Assume, therefore, that in addition to date-1 and date-2 abatement decisions, firms have the possibility to trade allowances in a forward market at date-1. Let a_f and p_f denote the volume of allowances contracted in the forward market and the forward price, respectively. Note that this does not change the optimal abatement decision at date-2. In particular, the firm's recursive program writes

$$\max_{a_1 \geq 0, a_f} \zeta_1 - C_1(a_1) - p_f a_f + \beta \phi^{-1} \left(\mathbb{E}_F \left\{ \phi \left(\mathcal{V}(a_1, a_f; \tilde{\theta}) \right) \right\} \right), \tag{37}$$

where $\mathcal{V}(a_1, a_f; \theta) = \mathbb{E}_G \{ \zeta_2 - C_2(a_1, a_2^*(a_1; \tilde{\tau}_\theta)) - \tilde{\tau}_\theta(\xi - a_1 - a_f - a_2^*(a_1; \tilde{\tau}_\theta) - \omega) | \theta \}$. The two necessary first-order conditions for the optimal date-1 abatement and contracted forward volumes under ambiguity aversion, $\hat{a}_1$ and $\hat{a}_f$, are given by

$$\begin{cases} -C_1'(\hat{a}_1) + \beta \dfrac{\mathbb{E}_F \left\{ \phi' \left(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \right) \mathcal{V}_{a_1}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \right\}}{\phi' \circ \phi^{-1} \left(\mathbb{E}_F \left\{ \phi \left(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \right) \right\} \right)} & = 0, \\ -p_f + \beta \dfrac{\mathbb{E}_F \left\{ \phi' \left(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \right) \mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \right\}}{\phi' \circ \phi^{-1} \left(\mathbb{E}_F \left\{ \phi \left(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \right) \right\} \right)} & = 0. \end{cases} \tag{38}$$

By the Envelop, $\mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) = \tau_\theta \geq \mathcal{V}_{a_1}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) = \tau_\theta - \mathbb{E}_G \{ \partial_{a_1} C_2(\hat{a}_1, a_2^*(\hat{a}_1; \tilde{\tau}_\theta)) | \theta \} > 0$, where $\tau_\theta = \mathbb{E}_G \{ \tilde{\tau}_\theta | \theta \}$. It follows that when there is no long-term effect of abatement, cost-efficiency in expectations is restored since $\beta \langle \tilde{\tau} \rangle = C_1'(\bar{a}_1) = p_f = C_1'(\hat{a}_1)$, as long as $p_f \in \mathrm{T}$ is predetermined but irrespective of how p_f is priced. Otherwise, in the general case, combining the two first-order conditions in (38) gives

$$-C_1'(\hat{a}_1) - \beta \mathcal{A}(\hat{a}_1, \hat{a}_f) \mathbb{E}_F \left\{ \mathcal{D}(\hat{a}_1, \hat{a}_f; \tilde{\theta}) \mathbb{E}_G \{ \partial_{a_1} C_2(\hat{a}_1, a_2^*(\hat{a}_1; \tilde{\tau}_\theta)) | \theta \} \right\} + p_f = 0.$$

Assume also that the forward price is unbiased such that $p_f = \beta \langle \tilde{\tau} \rangle$, that is, forward contracts are fairly priced. By (38), for any $a_1 \geq 0$, the firm chooses its optimal forward contracted volume $a_f^*(a_1)$ by equating $\langle\tilde{\tau}\rangle$ to $\mathcal{A}(a_1, a_f^*(a_1))\mathbb{E}_F\left\{\mathcal{D}(a_1, a_f^*(a_1); \tilde{\theta})\tau_\theta\right\}$[58]. Therefore, by concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ i.f.f.

$$\mathbb{E}_{\bar{G}}\left\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau}))\right\} \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1))\mathbb{E}_F\left\{\mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \tilde{\theta})\mathbb{E}_G\left\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau}_\theta))|\theta\right\}\right\}.$$

Using the quadratic specification, this is equivalent to

$$\langle\tilde{\tau}\rangle + \gamma(\mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) - 1)\bar{a}_1 \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1))\mathbb{E}_F\left\{\mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \tilde{\theta})\tau_\theta\right\},$$

which, under the fair price assumption, is equivalent to $\mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) \geq 1$. Let us now state

Proposition 6.1. *Consecutive to the introduction of forward contracts,*
(i) *assuming there is no long-term effect of abatement and irrespective of how forward contracts are priced, cost-e ciency in expectations is restored;*
(ii) *accounting for long-term effect of abatement and assuming forward contracts are fairly priced, cost-e ciency in expectations obtains only under* CAAA. *In particular, under* DAAA *(resp.* IAAA*), over-abatement (resp. under-abatement) at date-1 persists.*

Without long-term effect of abatement, Proposition 6.1 indicates that the introduction of a forward market restores intertemporal cost-efficiency, in expectations. That is, the optimal date-1 abatement level hinges neither upon the underlying ambiguity level nor upon the firm's attitude toward ambiguity. This contrasts with B&VDF who find that under risk aversion, with a separable abatement cost function, inefficient over-abatement at date-1 persists. However, our result is in line with recent extensions of the separation theorem under smooth ambiguity aversion, see *e.g.* Wong (2015b,[70]), Wong (2015c,[71]) or Osaki et al. (2015,[52])[59]. With long-term effect of abatement, Proposition 6.1 states that the introduction of a fairly-priced market for forward contracts only corrects for the pessimism effect, but the optimal date-1 abatement decision remains subject to the ambiguity prudence effect. In terms of date-1 abatement decisions, a cap-and-trade regime with forward contracts is hence akin to a tax regime, as in Proposition 3.1. This contrasts with Wong (2015b,[70]), Wong (2015c,[71]) and Osaki et al. (2015,[52]) in that they use the static KMM formulation, hence ignoring the $\mathcal{A}$-effect.

Traded volume. We now investigate whether ambiguity aversion diminishes the overall volume of trade as compared with ambiguity neutrality. In order to have clear variations of $\hat{a}_1$ w.r.t. $\bar{a}_1$ in both directions, let ϕ display CAAA. That is, under CAAA, when firm s (l) is allocated less (more) than $\min_{\theta\in\Theta}\omega^\theta$ ($\max_{\theta\in\Theta}$) it expects to be net short (long) in all

[58] In particular, when ϕ is CAAA, a_f^* solves $\text{Cov}_F\left\{\mathcal{V}(a_1, a_f^*(a_1); \tilde{\theta}); \mathcal{V}_{a_f}(a_1, a_f^*(a_1); \tilde{\theta})\right\} = 0.$

[59] In the presence of pure price ambiguity for a risk-averse ambiguity-averse competitive firm, see Wong (2015b,[70]). In the presence of price ambiguity and additive background risk for a risk-neutral and ambiguity-averse competitive firm, see Osaki et al. (2015,[52]). Finally, in the presence of price ambiguity and additive or multiplicative background risk for a risk-averse ambiguity-averse competitive firm, see Wong (2015c,[71]).

θ-scenarios under the abatement stream $(\bar{a}_1; a_2^\theta)$ so that $\hat{a}_1^s \geq \bar{a}_1 \geq \hat{a}_1^l$. At date 2, all firms equate their date-2 marginal abatement costs $\partial_{a_2} C_2(a_1; a_2^*)$ to the observed allowance price τ. In terms of total abatement for the three types of firms, one has that

$$a_2^*(\hat{a}_1^s; \tau) + \hat{a}_1^s = \frac{\tau}{c_2} + \left(1 - \frac{\gamma}{c_2}\right)\hat{a}_1^s \geq a_2^*(\bar{a}_1; \tau) + \bar{a}_1 \geq a_2^*(\hat{a}_1^l; \tau) + \hat{a}_1^l,$$

from which, since the net buying (selling) firm s (l) abates relatively more (less) and hence buys (sells) less allowances on the market, one can state the following

Proposition 6.2. *Let allowances be non-symmetrically distributed such that some liable firms are handed out $\omega \notin \left[\min_{\theta\in\Theta} \omega^\theta; \max_{\theta\in\Theta} \omega^\theta\right]$. Then, the equilibrium volume of trade is lower when liable firms are ambiguity averse than when they are ambiguity neutral.*

This is similar to B&vdF who find that risk aversion reduces the equilibrium volume of trade as compared with risk neutrality. Ambiguity and risk aversions might both reinforce and provide another explanation for what Ellerman (2000,[21]) calls 'autarkic compliance' in early phases of ETS. Thin traded volumes are indeed observed in nascent schemes, *e.g.* presently in the SKETS or the Chinese pilots. Because covered entities are waiting for increased price discovery and due to high regulatory uncertainty, they tend to hold on to their quota allocation so that trades are scarce. During Phase I of the EUETS, the volume of trades (both in EUAs and futures) increased steadily over time as uncertainty gradually vanished, see *e.g.* Ellerman et al. (2010,[22],Chap.5). Our result shows that the presence of ambiguity aversion might provide a theoretical underpinning for such a prudent behaviour.

Abatement decisions with a mix of AA and AN agents. We now consider the effect of having a mix of ambiguity averse and neutral firms in the market for allowances. Let $0 \leq \varepsilon \leq 1$ denote the share of ambiguity averse agents and assume that there is no long-term effect of abatement. For any $0 < \varepsilon < 1$, denote by $\hat{a}_1^\varepsilon$ and $\bar{a}_1^\varepsilon$ the optimal date-1 abatement levels for the ambiguity averse and neutral firms, respectively. Suppose also that ambiguity averse firms are allocated $\omega \leq \min_{\theta\in\Theta} \omega^\theta$ so that, in a market that contains either only ambiguity averse or ambiguity neutral firms, optimal date-1 abatement levels are defined by $\hat{a}_1^{\varepsilon=1} = \hat{a}_1 \geq \bar{a}_1^{\varepsilon=0} = \bar{a}_1$ and $\hat{A}_1 = S\hat{a}_1 \geq \bar{A}_1 = S\bar{a}_1$. For any mix ε, we assume[60] that market closure at date 2 gives the allowance price in each scenario $\theta \in \Theta$ by $\tau_\theta^\varepsilon = c_2\left(\bar{\xi}_\theta - \frac{\varepsilon\hat{A}_1 + (1-\varepsilon)\bar{A}_1 + \Omega}{S}\right)$. Let $\bar{\tau}_\theta$ and $\hat{\tau}_\theta$ denote the θ-scenario allowance price when $\varepsilon = 0$ and $\varepsilon = 1$, respectively, so that $\hat{\tau}_\theta \leq \tau_\theta^\varepsilon \leq \bar{\tau}_\theta$. Symmetrically, when ambiguity-averse firms receive a large allocation $\omega \geq \max_{\theta\in\Theta} \omega^\theta$, $\hat{a}_1 \leq \bar{a}_1$ so that $\bar{\tau}_\theta \leq \tau_\theta^\varepsilon \leq \hat{\tau}_\theta$. By comparing the necessary first-order conditions for $\bar{a}_1$ and $\bar{a}_1^\varepsilon$ on the one hand, and for $\hat{a}_1$ and $\hat{a}_1^\varepsilon$ on the other hand, one can thus state

[60]This is a conservative assumption. As will be clear from Proposition 6.3, defining τ_θ^ε with $\hat{A}_1^\varepsilon$ and $\bar{A}_1^\varepsilon$ instead of $\hat{A}_1$ and $\bar{A}_1$ would further amplify the deviation.

Proposition 6.3. *Let $\varepsilon \in]0;1[$ denote the share of ambiguity averse liable firms. Then,*
(i) when they are allocated $\omega < \min_{\theta\in\Theta} \omega^\theta$, $\bar{a}_1^\varepsilon < \bar{a}_1 < \hat{a}_1 < \hat{a}_1^\varepsilon$;
(ii) when they are allocated $\omega > \max_{\theta\in\Theta} \omega^\theta$, $\bar{a}_1^\varepsilon > \bar{a}_1 > \hat{a}_1 > \hat{a}_1^\varepsilon$.

Given that firms are symmetric in terms of marginal abatement costs, Proposition 6.3 shows that having a mix of ambiguity averse and neutral firms in the market where ambiguity averse firms are endowed with a relatively high or relatively low number of allowances brings the market further away from cost-efficiency since date-1 abatement levels are skewed even further away from the optimum. In particular, note that this also alters abatement decisions of ambiguity neutral agents.

6.2 Results robustness & relation to the literature

RISK AVERSION. Our paper assumes that liable firms are risk neutral to clearly identify the sole effects of ambiguity aversion[61]. However, one may wonder what would the combined effects of both risk and ambiguity aversions be. Taking our results together with B&VDF's, one could expect that risk and ambiguity aversions amplify the magnitude of date-1 abatement variations relative to the risk-neutral ambiguity-neutral reference case. However, ambiguity aversion is known to induce counter-intuitive effects so that this might not be this straightforward. Letting u denote the increasing concave vNM utility function, the liable firm's recursive program writes

$$\max_{a_1\geq 0} u(\pi_1(a_1)) + \beta\phi^{-1}\left(\mathbb{E}_F\left\{\phi\left(\mathcal{U}(a_1;\tilde{\theta})\right)\right\}\right), \tag{39}$$

with $\mathcal{U}(a_1;\theta) = \mathbb{E}_G\left\{u\circ\tilde{V}(a_1;\tilde{\theta})|\theta\right\}$ is the θ-scenario date-2 expected utility[62]. To account for the presence of risk aversion, the anticomonotonicity criterion must be restated. In particular, if we take the EU-maximiser as our reference scenario ($\bar{\bar{a}}_1$), one must determine under which conditions on u, ϕ and Θ the following holds

$$\forall\theta_1,\theta_2\in\Theta,\ \mathcal{U}(\bar{\bar{a}}_1;\tilde{\theta}_1)\geq\mathcal{U}(\bar{\bar{a}}_1;\tilde{\theta}_2)\ \Rightarrow\ \mathcal{U}_{a_1}(\bar{\bar{a}}_1;\tilde{\theta}_2)\geq\mathcal{U}_{a_1}(\bar{\bar{a}}_1;\tilde{\theta}_1). \tag{40}$$

This is not a simple task in general and leave this for future work[63]. The reader may refer to Gierlinger & Gollier (2015,[28],Prop3&4) for optimal saving decision under smooth ambiguity aversion. In a static setup one might also refer to Wong (2015a,[69],Prop2). For instance, in the case where u exhibits risk prudence, ϕ displays DAAA (IAAA) and (40) holds,

[61] Liable firms hence maximise their intertemporal profit. Note, however, that date-i profits are concave functions in a_i so that our formulation already *implicitly* comprises risk aversion. With quadratic abatement cost functions, we hence control for implicit risk prudence.

[62] Because u is increasing, V and a_2^* do not change as compared to our previous analysis.

[63] The effective utility function is $u\circ V$, which complicates the analysis. The problem becomes even more complicated if we take the risk-neutral ambiguity-neutral case as our reference scenario in lieu of the EU-maximising case.

one might expect that risk and ambiguity aversion reinforce one another (work in opposite direction). However, by assuming risk neutrality, our paper singles out the effects of ambiguity aversion; which allows us to both comparatively analyse and capture the ambiguity prudence and pessimism effects.

FULLY-FLEDGED DYNAMICS. Our paper develops a two-period model to analyse what is fundamentally a fully dynamic problem. However, extending our model to more periods would be technically difficult notably because this begs the question of the way preferences and beliefs are updated as new information comes in. The recursive KMM formulation is one of the few models there are in the literature which simultaneously satisfy dynamic consistency (DC) and learning under ambiguity operated via a prior-by-prior Bayesian updating of beliefs (BL)[64]. However, tensions between DC, BL and the positive value of information remain[65]. Furthermore, Strzalecki (2013,[64]) identifies a strong interdependence between ambiguity aversion and preference for early resolution of uncertainty in recursive models of ambiguity aversion. Papers in the literature generally try to circumvent these issues in different ways. Millner et al. (2013,[46]) opt for two simple but extreme exogenous learning scenarios: one where all ambiguity resolves after the first period, the other with persistent and unchanged ambiguity throughout. Guerdjikova & Sciubba (2015,[31]) consider two similar types of ambiguity structures: one where ambiguity vanishes at the first date, one where ambiguity changes but persists over time. In this vein, Ju & Miao (2012,[35]) consider Markov economies[66] that exhibit persistent ambiguity and propose a generalized recursive smooth ambiguity model. They show that ambiguity aversion, even with moderate degrees, may explain otherwise abnormal pricing phenomena with a representative EU-maximiser agent model such as the equity premium puzzle or the negative correlation between asset prices and returns. Using different specification for risk and ambiguity attitudes, Collard et al. (2011,[17]) obtain similar results. In particular, Collard et al. (2011,[17]) consider the case of CAAA to suppress ambiguity prudence and the induced preference for early resolution of uncertainty as it simplifies the Euler equations. An alternative is found in Gierlinger & Gollier (2015,[28]) and in Traeger (2014,[65]) who use a *one-step-ahead formulation*, which is composed of nested sets of identical ambiguity structure. In the end, however, even if we leave aside the technicality required by such dynamic simulations, one might face a lack the relevant data for calibration. Moreover, our two-period model already captures the essence of the effects of ambiguity aversion and is able to finely decompose the ambiguity prudence and pessimism effects in a special case.

[64]There are otherwise intrinsic tensions between DC and BL under ambiguity aversion.

[65]*E.g.*, since DC characterises ex-ante optimal plans that continue to be optimal in subsequent periods, irrespective of new information, the tension between DC and positive new information is conspicuous. For further references and proposals to weaken DC, see *e.g.* Galanis (2015,[27]).

[66]In particular, the ambiguity-averse agent has time-variant beliefs over consumption growth based on a binary hidden space (boom or recession) following a Markov-switching process.

Firms' heterogeneity. Our paper assumes that firms are symmetric but for their initial allocation. This single source of heterogeneity is relevant in that the allocation volume is shown to drive date-1 abatement adjustment under ambiguity aversion (Proposition 4.7). One may wonder to which extent this assumption could be relaxed. For instance, liable firms may also differ in their attitudes towards ambiguity or in their abatement cost functions. First, allowing for multiple tastes would tremendously complicate the analysis since aggregating different tastes or accounting for the interactions between differently ambiguity-averse agents remains a research topic to be investigated – on this see *e.g.* Danan et al. (2016,[19]). In a first step in this direction, (i) Proposition 6.3 shows that the presence of both (equally) ambiguity averse and neutral agents on the market for allowances would tend to skew date-1 abatement decisions further away from the two polar situations where agents either are all ambiguity averse or all ambiguity neutral; (ii) Proposition 4.9 characterizes the impact of an increase in ambiguity aversion on optimal date-1 abatement decisions. Second, assuming that firms have identical abatement cost functions allows us to clearly separate out the effects of ambiguity aversion relative to neutrality since marginal abatement costs parameters, c_1 and c_2, also determine the intertemporal arbitrage and hence the incentive to abate at date-1[67]. It is however of interest to consider that firms have ambiguous abatement cost functions at date 2 since this puts tax and ETS on an equal footing regarding date-1 abatement distortions – see Appendix A.

Background risk. Our paper considers that ambiguity stems from a unique source, either the market price or individual baselines. In particular, this means that the allowance price and emission baselines are treated as independent variables. In section 4.2, this assumption was relaxed to some extent because the price ambiguity results from ambiguity on aggregate emission baselines. We now further relax this assumption by allowing both price and individual baselines to be random variables at the same time. Let $G(\cdot;\theta)$ and $L(\cdot;\theta)$ be the objective probability distributions for the price risk $\tilde{\tau}_\theta$ and the baseline risk $\tilde{\xi}_\theta$ in scenario $\theta \in \Theta$, with densities $g(\cdot;\theta)$ and $l(\cdot;\theta)$, respectively. For simplicity, let G and L be first-order independent given a scenario θ, but second-order dependent across θ-scenarios, where F again denotes the subjective probability measure for $\tilde{\theta}$. The θ-scenario date-2 expected profit writes[68], for all $a_1 \geq 0$ and all $\theta \in \Theta$,

$$\mathcal{V}(a_1;\theta) = \zeta_2 - \iint_{\mathrm{T},\Xi} C_2(a_1, a_2^*(a_1;x)) + x(y - a_1 - a_2^*(a_1;x) - \omega)g(x;\theta)l(y;\theta)\mathrm{d}y\mathrm{d}x, \quad (41)$$

so that, with quadratic abatement costs, by first-order independence and denoting $\tau_\theta = \mathbb{E}_G\left\{\tilde{\tau}_\theta|\theta\right\}$ and $\xi_\theta = \mathbb{E}_L\left\{\tilde{\xi}_\theta|\theta\right\}$, it comes $\mathcal{V}_{a_1}(a_1;\theta) = \frac{\gamma^2 a_1 + (c_2-\gamma)\tau_\theta}{c_2}$ so that the optimal date-

[67] In particular, let firms be heterogeneous in their quadratic abatement costs functions and let $\gamma = 0$ for simplicity. The date-2 optimality condition gives $\forall s \in \mathcal{S}$, $c_2(s)a_2(s) = \tau$ so that $\tau = c\sum_{s\in\mathcal{S}} a_2(s)$ where $\frac{1}{c} = \sum_{s\in\mathcal{S}} \frac{1}{c_2(s)}$ is the date-2 market-wide flexibility in abatement. By market closure in all θ-scenarios, the allowance price is thus $\tau_\theta = c\left(S\bar{\xi}_\theta - A_1 - \Omega\right)$ so that $c_1(s)\hat{a}_1(s) = \beta\mathcal{A}(\hat{a}_1(s))\left(\langle\tau_\theta\rangle + \mathcal{P}(\hat{a}_1(s))\right)$.

[68] The date-2 optimality condition defining a_2^* is not altered by considering these two sources of ambiguity.

1 abatement decision under ambiguity neutrality is not affected by the introduction of ambiguity[69]. In particular, invoking the Envelop and differentiating w.r.t. θ gives

$$\begin{aligned}\partial_\theta \mathcal{V}_{a_1}(a_1;\theta) &= \frac{\gamma - c_2}{c_2}\int_{\mathsf{T}} G_\theta(x;\theta)\mathrm{d}x, \text{ and}\\ \partial_\theta \mathcal{V}(a_1;\theta) &= \int_{\mathsf{T}}\left(\xi_\theta - a_1\left(1-\frac{\gamma}{c_2}\right) - \omega - \frac{x}{c_2}\right)G_\theta(x;\theta)\mathrm{d}x + \tau_\theta\int_{\Xi} L_\theta(y;\theta)\mathrm{d}y.\end{aligned} \tag{42}$$

In the general case, it follows that anticomonotonicity is satisfied provided that $\omega \leq \min_{\theta\in\Theta}\xi_\theta - \bar{a}_1 - a_2^\theta$ and that G and L covary negatively with θ, *i.e.* $\mathrm{Cov}_F\{G;L\} < 0$. As clear from (41,42), this specification nests our previous analysis as special cases[70]: when ξ_θ is preserved across θ-scenarios ($\xi_\theta = \xi$, $\forall\theta \in \Theta$), the analysis is identical to that under pure price ambiguity; when τ_θ is preserved across θ-scenarios ($\tau_\theta = \tau$, $\forall\theta\in\Theta$), the analyses reduces to the case of a tax regime since $\mathcal{V}_{a_1}$ is independent of θ.

7 Conclusion

SUMMARY. Ambiguity aversion is shown to induce two effects, pessimism and ambiguity prudence. Ambiguity prudence can be assimilated to a decrease in impatience so that, controlling for pessimism, it always raises date-1 abatement. Pessimism leads to over-abatement provided that low expected date-2 profit scenarios coincide with high expected date-2 marginal profit scenarios – the anticomonotonicity criterion. This is because these two ambiguity aversion-induced effects can reinforce one another or work in opposite directions, and because it can be difficult to sign the pessimism effect, that it is complicated to have clear general results on the effects of the introduction of ambiguity aversion. In particular, the introduction of ambiguity aversion on the part of ETS-liable firms is not necessarily conducive to over-abatement at date 1, even under ambiguity prudence. Rather, it acts in line with a precautionary effect, which, as in B&vdF, ultimately relates to the initial allocation level and the independence property (Hahn & Stavins, 2011,[32]) does not hold: Only when liable firms are allocated too small a volume of allowances does ambiguity aversion raise date-1 abatement above the ambiguity-neutral optimal level. In contrast, ambiguity-prudent tax-liable firms always over-abate. Numerical simulations show that pessimism tends to be the dominant effect, while ambiguity prudence only has a residual role, except in extreme situations where initial allocation is very low, or very high, albeit to a lesser extent. In particular, when allocation is relatively small, the presence of ambiguity prudence implies that a higher degree of ambiguity aversion is not necessarily conducive to higher date-1 abatement. Simulations also highlight that anticomonotonicity in expecta-

[69] In a static setup, Wong (2015c,[71]) studies the competitive firm's optimal production and hedging decisions in the face of price ambiguity and (additive or multiplicative) background risk. In contrast to the case of a single source of ambiguity, the behaviour of an ambiguity-neutral firm is affected by the introduction of ambiguity when these two sources of ambiguity are first-order independent but second-order dependent.

[70] The situation where $L_\theta \equiv 0$ corresponds to section 4.1 with pure price ambiguity.

tions over the set of possible scenarios might in effect suffice to sign pessimism. In terms of preferability of instruments, our paper mitigates B&vdF's findings that only ETS deteriorates in the presence of uncertainty: When ambiguity is prevalent and liable firms display ambiguity aversion, both price and quantity regimes are not conducive to intertemporal cost-efficiency. Because the tax regime is only subject to ambiguity prudence, hence less distorted than an ETS, one could conclude that a tax still performs better than an ETS under ambiguity. However, the ETS-specific pessimism effect vanishes out following on the introduction of fairly-priced forwards.

Policy implications. First, the presence of ambiguity aversion in a context where ambiguity prevails, as *e.g.* in the EUETS, provides a theoretical justification for the formation of an allowance surplus, among others. For instance, extrapolating from Proposition 4.8 indicates that, when allowances are grandfathered, ambiguity-averse liable firms would tend to over-abate in early phases. Similarly, if firms receive no free allocation or a relatively small amount of allowances and if these firms exhibit ambiguity aversion, then this may also lead to excessive abatement in early phases of cap-and-trade systems, *i.e.* high banking. In practice, this might correspond to firms from the power sector liable under the EUETS or in RGGI, as they must acquire all allowances through auctions. Second, ambiguity-prudent liable firms apply a higher discount factor than ambiguity-neutral ones. Ambiguity prudence on the part of market participants, hence using relatively higher discount rates, could thus contribute to the drop in prices observed in the EUETS in the last few years. This contradicts the standard rationale that banking both adjusts so as to minimize the sum of discounted abatement costs and can be carried out at constant low rates (interest rates). Simulations show that this effect should be more pronounced under allowances auctioning. This result also complements the proposition of Neuhoff et al. (2012,[48]) that only speculators are willing to carry permits forward when the surplus exceeds the power sector's hedging demand (hence requiring higher appreciation rates) to explain the EUA price drop. Third, allocation is the key factor in determining how ambiguity aversion adjusts date-1 abatement, which further shows that the way allowances are distributed is not neutral. Ideally, if it were known which firms are ambiguity averse, the regulator should aim to allocate $\bar{\omega}$ allowances to the more ambiguity-averse firms since those firms are the most sensitive to allocation, so as to partially correct ambiguity-related inefficiencies. In practice, given a long-term emission objective, it is easier for the regulator to dynamically manage the supply of permits under full auctioning than free allocation because full auctioning of permits ensures that all firms over-abate. This can be seen as a severe decision by the regulator which can be compensated by him setting a cap on emissions higher than the objective. Then, this cap can be gradually revised down as over-abatement occurs, to meet the long-term target. This is relevant in the current context of market design revisions, notably in the form of *ex-post* allowance supply management (price-based cost-containment reserves in RGGI, California & Québec; quantity-based surplus-adjustment mechanism in the EUETS; *ex-post* allocation and cap adjustments in Chinese pilots).

ALLEYS FOR FUTURE RESEARCH. Besides extensions discussed in section 6, we identify two natural ways to build on this paper. First, output decisions could be endogenised, just like Baldursson & von der Fehr (2012,[4]) extended their 2004 paper. In their investment decision, firms also account for the investment-induced output price change. This alters, and sometimes reverts the 2004 results, in particular: endogenising output decisions mitigates (exacerbates) risk exposure for dirty firms (clean firms or highly-allocated dirty firms); with small allocation, risk-averse clean and dirty firms alike (both on average and at the margin) now reduce investment relative to the risk-neutral benchmark. If we distinguish between two types of firms, similar results could certainly obtain under ambiguity aversion. Second and as briefly touched upon above, one could address the normative question of the most socially desirable way to allocate allowances through time and across firms. One could test different dynamic cap-adjustment procedures so as to correct ambiguity aversion-induced inefficiencies through time, *e.g.* along the lines of Newell et al. (2005,[50]). In particular, one could define the optimal intertemporal trading ratio under ambiguity aversion, *e.g.* along the lines of Yates & Cronshaw (2001,[73]) or Feng & Zhao (2006,[25]). Another normative question is the effects of the introduction of ambiguity aversion on the part the regulator. That is, knowing how firms react to a given tax rate or emissions cap, one could compare the effects of ambiguity aversion on setting the optimal cap or tax rate for the regulator. To give a flavour of the results one may expect, we can show that with ambiguous baselines, linear environmental damages[71] and ambiguity-neutral liable firms[72], the introduction of ambiguity aversion has no effect on the socially optimal tax rate. In contrast, it is conducive to higher socially optimal emissions caps than under ambiguity neutrality, the higher the degree of ambiguity aversion on the part of the regulator.

[71]With quadratic environmental damages, the optimal tax rate under ambiguity aversion is lower than under ambiguity neutrality. Note also that, when emissions generate of flow of damages at each period, the least-discounted abatement-cost emission path does not minimise welfare, defined as the discounted sum of damages plus firms' abatement costs – see Kling & Rubin (1997,[39]) and Leiby & Rubin (2001,[42]).

[72]Relaxing this assumption might be complicated, but of great interest.

A Ambiguity on abatement cost functions

So far we considered an exogenously given random allowance price and analysed the price distortion induced by the introduction ambiguity aversion. However, it can be argued that, assuming the market for allowances is competitive, a major factor behind the allowance price formation, and hence randomness, stems for liable firms' abatement costs. This section therefore considers the situation where ambiguity bears on date-2 abatement cost function. This should affect the ambiguity-averse liable firm's optimal date-1 abatement decisions under a tax regime or quota regime alike. Hence the question we address is: Under which regime, tax or quota, does cost ambiguity under ambiguity aversion induce a larger departure from the reference scenario (ambiguity neutrality). Because cap-and-trade transmit individual uncertainties into allowance price uncertainty, a cap-and-trade regime can have a dampening effect which is absent in a tax regime where liable firms are subject to individual abatement cost shocks, so that it is possible that a quota regime outperforms the tax regime, as in Zhao (2003,[74]). Let abatement cost functions be quadratic at both dates with long-term effect of abatement such that, for all $s \in \mathcal{S}$ and $\theta \in \Theta$

$$C_1\left(a_1(s)\right) = \frac{c_1}{2}a_1(s)^2 \text{ and } \tilde{C}_2\left(a_2(s);\theta\right) = \frac{c_2}{2}a_2(s)^2 + \tilde{\delta}_\theta(s)a_2(s) + \gamma a_1(s)a_2(s),$$

where ambiguity enters abatement cost function as a linear term, *e.g.* as in Zhao (2003,[74]). The linear shock factor $\tilde{\delta}_\theta(s)$ is distributed according the objective measure $G(\cdot;\theta)$ defined over Δ in all θ-scenarios. Assume also that it is specified by $\tilde{\delta}_\theta(s) = \delta_\theta + \tilde{\epsilon}_\theta(s)$ where δ_θ is constant in each θ-scenario and common to all firms while $\tilde{\epsilon}_\theta(s)$ is firm-specific and satisfies $\mathbb{E}\tilde{\epsilon}_\theta(s) = 0$ with finite variance for all s and θ.

Quota regime. For a given allowance price τ, cost realisation δ and date-1 abatement a_1, date-2 optimality gives $a_2^* \equiv a_2^*(a_1;\delta) = \frac{\tau - \gamma a_1 - \delta}{c_2}$ so that market closure in each θ-scenario gives

$$\int_{\mathcal{S}} \xi(s) - \omega(s) - a_1(s) - a_2^*\left(a_1(s);\tilde{\delta}_\theta(s)\right) \mathrm{d}s = 0,$$

which in turn, by the Law of Large Numbers, gives the allowance price in scenario θ

$$\tau_\theta = \frac{c_2(\Xi - \Omega) + (c_2 - \gamma)A_1}{S} + \delta_\theta.$$

Two cases arise. (1) When $\mathbb{E}_G\left\{\tilde{\delta}_\theta|\theta\right\} = 0$ *i.e.* $\delta_\theta = 0$ for all s and θ, the date-2 allowance price is independent of θ. There is thus no pessimistic price distortion and the quota regime functions like the tax regime in section 3 with the rate t set equal to $\frac{c_2}{S}(\Xi - \Omega)$. In particular, only the ambiguity prudence effect dictates how firms adjust date-1 abatement relative to neutrality and, as characterised by Figure 4, DAAA-optimal date-1 abatement is a decreasing function of the initial allocation volume. (2) When $\mathbb{E}_G\left\{\tilde{\delta}_\theta|\theta\right\} \neq 0$ *i.e.* $\delta_\theta \neq 0$

for all s and θ, the date-2 allowance price is constant in ach θ-scenario so that both the pessimism and ambiguity prudence effects obtain, like in section 4.1.

TAX REGIME. For a given cost realisation δ and date-1 abatement a_1, date-2 optimality gives $a_2^* \equiv a_2^*(a_1, t; \delta) = \frac{t-\gamma a_1-\delta}{c_2}$, from which it follows that, for all $a_1 \geq 0$ and $\theta \in \Theta$

$$\mathcal{V}_{a_1}(a_1;\theta) = t\left(1 - \frac{\gamma}{c_2}\right) + \frac{\gamma}{c_2}(\gamma a_1 + \delta_\theta), \text{ and,}$$
$$\mathcal{V}(a_1;\theta) = \zeta_2 - t\left(\xi - a_1 - \omega - \frac{t}{2c_2}\right) - \int_\Delta \frac{\gamma a_1 + x}{2c_2}(2t - (\gamma a_1 + x))\mathrm{d}G(x;\theta),$$

and further differentiating w.r.t. θ and integrating by parts gives

$$\partial_\theta \mathcal{V}(a_1;\theta) = \frac{1}{c_2}\int_\Delta t - (\gamma a_1 + x)G_\theta(x;\theta)\mathrm{d}x \text{ and } \partial_\theta \mathcal{V}_{a_1}(a_1;\theta) = -\frac{\gamma}{c_2}\int_\Delta G_\theta(x;\theta)\mathrm{d}x.$$

Two cases also arise. (1) When $\mathbb{E}_G\left\{\tilde{\delta}_\theta|\theta\right\} = 0$ *i.e.* $\delta_\theta = 0$ for all s and θ, $\mathcal{V}_{a_1}$ is independent of θ so that it is similar to section 3 with the sole $\mathcal{A}$ effect. (2) When $\mathbb{E}_G\left\{\tilde{\delta}_\theta|\theta\right\} \neq 0$ *i.e.* $\delta_\theta \neq 0$ for all s and θ, there are both pessimism and ambiguity prudence effects at play. Like in Proposition 4.7, one can show that when $G_\theta \geq (\leq)0$,

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) \geq (\leq)0 \;\Leftrightarrow\; \bar{a}_1 \leq a_1^\theta = \frac{1}{\gamma}\left(t - \frac{\int_\Delta xG_\theta(x;\theta)\mathrm{d}x}{\int_\Delta G_\theta(x;\theta)\mathrm{d}x}\right),$$

so that anticomonotonicity holds provided that $\bar{a}_1 \leq \min_{\theta\in\Theta} a_1^\theta$.

Overall, in terms of date-1 abatement, liable firms behave similarly under the two instruments. That is, the advantage highlighted for the tax instrument as compared to the quota regime in the body of the paper vanishes when ambiguity bears on abatement cost functions as both instruments equally disrupt date-1 abatement away from the optimum in expectations (ambiguity neutrality).

B Alternative ambiguity representation theorems

The anticomonotonicity condition is robust in the sense that it obtains with other ambiguity aversion representation theorems. This is the purpose of Proposition B.1, where ambiguity aversion is characterized by the MEU decision criterion. Gilboa & Schmeidler (1989,[29]) put forth an axiomatic foundation of the maxmin EU decision rule, and with our interpretation that Θ represents the set of possible objective probability distributions, of the Wald's minimax decision criterion. In all generality, we consider the situation where the uses the α-maxmin decision criterion whereby it grants a weight $0 \leq \alpha \leq 1$ to the worst scenario in Θ, and the complementary weight to the best scenario. In particular, this reduces to the Wald's criterion for $\alpha = 1$.

Proposition B.1. *With the general* MEU *representation theorem, the introduction of ambiguity aversion is conducive to higher date-1 abatement levels than under ambiguity neutrality if, and only if, the sequences* $(\mathcal{V}(\bar{a}_1;\theta))_\theta$ *and* $(\mathcal{V}_{a_1}(\bar{a}_1;\theta))_\theta$ *are anticomonotone, where* $\bar{a}_1$ *denotes the optimal date 1-abatement under ambiguity neutrality.*

Proof. For the purpose of the proof, let Θ be a discrete finite set of cardinality $k = |\Theta|$, ordered such that $\theta_1 \leq \cdots \leq \theta_k$. Let $(q_i)_{i=1,\dots,k}$ be the subjective prior such that q_i denotes the agent's subjective probability that the θ_i-scenario will materialize and $\sum_i q_i = 1$. Without loss of generality, let the sequence $(\mathcal{V}(\bar{a}_1;\theta_i))_i$ be non-decreasing in i. Recall that $\bar{a}_1$ is defined by $-C_1'(\bar{a}_1) + \beta \sum_i q_i \mathcal{V}_{a_1}(\bar{a}_1;\theta_i) = 0$. The α-maxmin objective function, evaluated at $a_1 = \bar{a}_1$, reads

$$
\begin{aligned}
\Upsilon_\alpha(\bar{a}_1) &= \pi_1(\bar{a}_1) + \beta\left(\alpha \min_{\theta\in\Theta} \mathcal{V}(\bar{a}_1;\theta) + (1-\alpha)\max_{\theta\in\Theta}\mathcal{V}(\bar{a}_1;\theta)\right) \\
&= \pi_1(\bar{a}_1) + \beta\left(\alpha\mathcal{V}(\bar{a}_1;\theta_1) + (1-\alpha)\mathcal{V}(\bar{a}_1;\theta_k)\right),
\end{aligned}
$$

and let $\hat{a}_1^\alpha$ be the maximizer of Υ_α. By concavity of Υ_α, $\hat{a}_1^\alpha \geq \bar{a}_1$, i.f.f.

$$
\alpha\mathcal{V}_{a_1}(\bar{a}_1;\theta_1) + (1-\alpha)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k) \geq \sum_{i=1}^{k} q_i \mathcal{V}_{a_1}(\bar{a}_1;\theta_i).
$$

Note also that, for all $a_1 \geq 0$, $\Upsilon_\alpha(a_1) \leq \Upsilon_{\mathsf{SEU}}(a_1)$ by virtue of ambiguity aversion. That is,

$$
\alpha\mathcal{V}(a_1;\theta_1) + (1-\alpha)\mathcal{V}(a_1;\theta_k) \leq \sum_{i=1}^{k} q_i \mathcal{V}(a_1;\theta_i),
$$

which, upon rearranging, yields

$$
\begin{aligned}
(\alpha - q_1)\mathcal{V}(a_1;\theta_1) &\leq \sum_{i=2}^{k-1} q_i\mathcal{V}(a_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}(a_1;\theta_k) \\
&\leq \left(\alpha + \sum_{i=2}^{k} q_i - 1\right)\mathcal{V}(a_1;\theta_k) = (\alpha - q_1)\mathcal{V}(a_1;\theta_k)
\end{aligned}
$$

since $(\mathcal{V}(a_1;\theta_i))_i$ is non-decreasing and $\sum_i q_i = 1$. Since $\mathcal{V}(a_1;\theta_k) \geq \mathcal{V}(a_1;\theta_1) > 0$, that $\alpha \geq q_1$ is a sufficient condition for $\Upsilon_\alpha(a_1) \leq \Upsilon_{\mathsf{SEU}}(a_1)$ to hold. With this, $\hat{a}_1^\alpha \geq \bar{a}_1$, i.f.f.

$$
(\alpha - q_1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_1) \geq \sum_{i=2}^{k-1} q_i\mathcal{V}_{a_1}(\bar{a}_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k).
$$

Note finally that it is sufficient for this to hold that $(\mathcal{V}_{a_1}(\bar{a}_1;\theta_i))_i$ be non-increasing in i

since one would get

$$\sum_{i=2}^{k-1} q_i \mathcal{V}_{a_1}(\bar{a}_1;\theta_i)+(\alpha+q_k-1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k) \geq \left(\alpha+\sum_{i=2}^{k} q_i - 1\right)\mathcal{V}_{a_1}(\bar{a}_1;\theta_2) = (\alpha-q_1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_2),$$

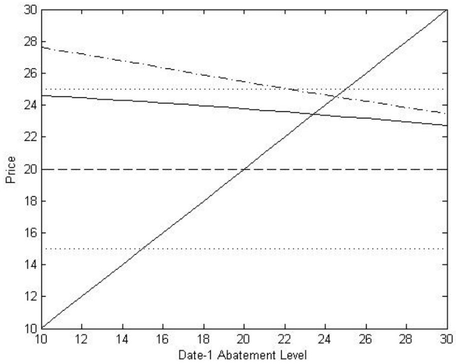
which concludes the proof. □

Note also that when $\alpha = 1$, an increase in the cardinality of Θ (say, from $|\Theta|$ to $|\Theta'|$), *i.e.* an increase in the ambiguity level, also corresponds to an increase in the degree of ambiguity aversion ($\min_{\theta\in\Theta'} \mathcal{V}(\cdot;\theta) \leq \min_{\theta\in\Theta} \mathcal{V}(\cdot;\theta)$ provided that $|\Theta'| > |\Theta|$), so that beliefs and tastes are not disentangled (here, due to the min operator). Another downside of these representation theorems is that they might be considered as to extreme, as illustrated by the numerical example in this paper – see section 5. By linearity of the objective function, the above proposition straightforwardly applies to the ϵ-contamination model (Eichberger and Kelsey, 1999[20]), which corresponds to a convex combination of the SEU model, with weight $0 \leq \epsilon \leq 1$, and the Wald's criterion, with weight $1-\epsilon$. The agent therefore grants a degree of confidence ϵ to the SEU model being the correct one. With probability $1-\epsilon$, the agent recognizes that other criteria are possible, and takes the worst-case scenario to account for this. The reader is also referred to Gierlinger & Gollier (2015,[28]) for the case of multiplier preferences using robust control theory.

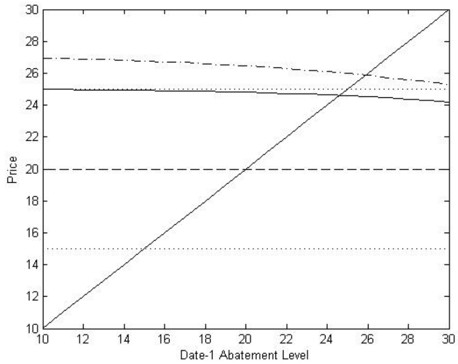
C Accounting for the two effects of ambiguity aversion

With numerical simulations, this section clarifies the decomposition of the two effects of ambiguity aversion, namely, the subjective prior pessimistic distortion $F \longrightarrow H$ and the ambiguity prudence effect $\mathcal{A}$, as clear from the necessary first-order condition for $\hat{a}_1$ that $-C_1'(\hat{a}_1)+\beta\mathcal{A}(\hat{a}_1)\mathbb{E}_H\left\{\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\right\} = 0$. It relies on the same set of assumptions as in section 5 where, in particular, $\beta = 1$, F is uniform over Θ, and for all $\theta \in \Theta$, $G(\cdot,\theta)$ is also uniform over $\mathrm{T}_\theta = [\underline{\tau}+\theta;\overline{\tau}+\theta]$. Because there is no long-term effect of abatement decision, it follows that for all $\theta \in \Theta$ and admissible $a_1 \geq 0$, $\mathcal{V}_{a_1}(a_1;\theta) = \langle\tau\rangle+\theta$, where $\langle\tau\rangle = 20$. As compared with Figure 2, this results in having flat $\mathcal{V}_{a_1}$ curves. As in Figure 2, let there be only two scenarios $\Theta = \{\theta_1 = +5, \theta_2 = -5\}$. ϕ displays DAAA such that $\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}$, with $\alpha > 1$ the degree of ambiguity aversion. Let (q_1, q_2) denote the subjective relevance of the two possible scenarios according to F, *i.e.*, $q_1 = q_2 = \frac{1}{2}$. Then, the two effects of ambiguity aversion are functions of the date-1 abatement level such that, for all $a_1 \geq 0$,

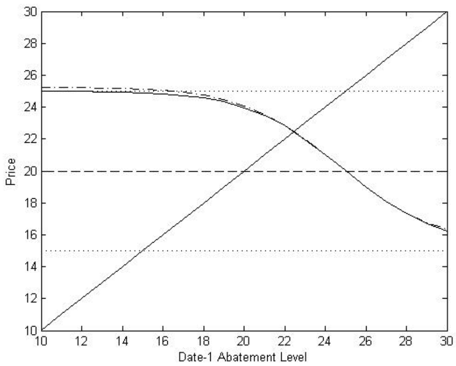
$$H(a_1) = \begin{cases} \hat{q}_1(a_1) = & q_1 \dfrac{\phi'(\mathcal{V}(a_1;\theta_1))}{q_1\phi'(\mathcal{V}(a_1;\theta_1)) + q_2\phi'(\mathcal{V}(a_1;\theta_2))} \\ \hat{q}_2(a_1) = & q_2 \dfrac{\phi'(\mathcal{V}(a_1;\theta_2))}{q_1\phi'(\mathcal{V}(a_1;\theta_1)) + q_2\phi'(\mathcal{V}(a_1;\theta_2))}, \end{cases}$$



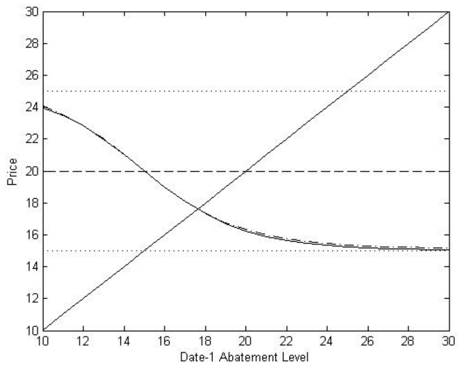
(a) $\alpha = 5$ and $\omega = 20$



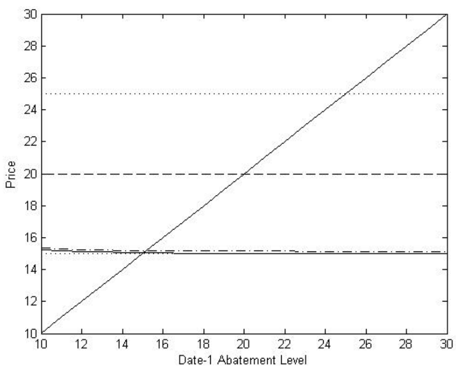
(b) $\alpha = 10$ and $\omega = 20$



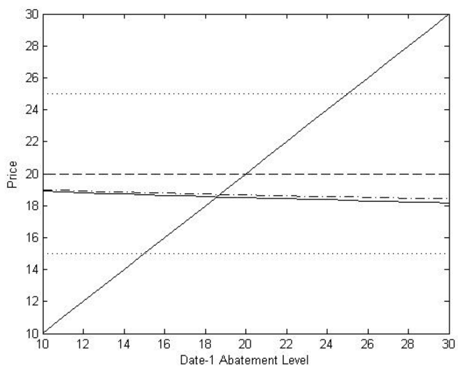
(c) $\alpha = 75$ and $\omega = 55$



(d) $\alpha = 75$ and $\omega = 65$



(e) $\alpha = 75$ and $\omega = 80$



(f) $\alpha = 5$ and $\omega = 90$

Figure 6: The two effects of ambiguity aversion

and

$$\mathcal{A}(a_1) = \frac{q_1\phi'\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi'\left(\mathcal{V}(a_1;\theta_2)\right)}{\phi' \circ \phi^{-1}\left(q_1\phi\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi\left(\mathcal{V}(a_1;\theta_2)\right)\right)}.$$

With this, the necessary-first order condition for $\hat{a}_1$ rewrites

$$-C_1'(\hat{a}_1) + \beta\mathcal{A}(\hat{a}_1)\left(\langle\tau\rangle + \hat{q}_1(\hat{a}_1)\theta_1 + \hat{q}_2(\hat{a}_1)\theta_2\right) = 0,$$

and is graphically depicted in Figure 6 for different combinations of α's and ω's. The slanted solid line is C_1', the two dotted lines represent $\mathcal{V}_{a_1}(a_1;\theta_i)$, the dashed line is $\mathbb{E}_F\left\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\right\}$, the other solid line is $\mathbb{E}_F\left\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\right\}$ and the dash-dotted line is $\mathcal{A}(a_1)\mathbb{E}_F\left\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\right\}$. As compared with the assumption behind Figure 2, Figure 6 shows that both the H distortion and the $\mathcal{A}$ effect are not constant with a_1. Figure 6 illustrates that the bulk of the variation in date-1 abatement level under ambiguity aversion is driven by the pessimism effect and that the relatively weaker ambiguity prudence effect has more influence for lower α's (see Figures 6a and 6b). Figures 6c and 6d highlight the high sensibility of date-1 abatement around the threshold $\omega = 60$ for relatively high α's. Figures 6e and 6f compare the prior distortion for different level of ambiguity aversion, indicating that the distortion is more pronounced for higher α's. Finally, Figures 6b and 6e underline that when ω is outside of the 40-80 band, and for relatively high α's, the pessimism effect redistributes almost all the weight to the worst scenario ('all-or-nothing switch').

D Cap-and-trade regime under binary ambiguity

This section considers the case of binary price ambiguity, that is, in all θ-scenarios, $\tilde{\tau}_\theta$ either takes the value $\underline{\tau} > 0$ with probability $0 \leq p(\theta) \leq 1$ or $\overline{\tau}$ with complementary probability, and $\Delta\tau = \overline{\tau} - \underline{\tau} > 0$. W.l.o.g. assume for clarity that abatement cost functions are time separable. Let the underlying objective allowance price lottery be $(p, \underline{\tau}; 1-p, \overline{\tau})$. Then, the no-ambiguity bias requires that $p = \mathbb{E}_F p(\tilde{\theta})$, so that, for the ambiguity-neutral firm, $\langle\tilde{\tau}\rangle = p\underline{\tau} + (1-p)\overline{\tau}$. Interestingly, with a similar binary structure, Alary et al. (2013, [1]) and Wong (2015a,[69]), *inter alia*, show that the anticomonotonicity criterion is always satisfied so that the impact of ambiguity aversion is clear. In our setup, as Proposition D.1 makes clear, the expected net position under ambiguity neutrality continues to dictate how the optimal date-1 abatement decision under ambiguity aversion compares with that under ambiguity neutrality – but the condition for anticomonotonicity to hold is milder. In particular, letting $\Upsilon(a_1;\theta)$ denote the θ-scenario expected date-2 net revenue from date-1 abatement, one has that, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$\begin{aligned}\Upsilon(a_1;\theta) &= \zeta_1 - C_1(a_1) + \beta\mathcal{V}(a_1;\theta)\\ &= \zeta - C_1(a_1) - \beta p(\theta)\left(C_2(a_2^*(\underline{\tau}) + \underline{\tau}\left(\xi - a_1 - a_2^*(\underline{\tau}) - \omega\right)\right)\\ &\quad - \beta(1-p(\theta))\left(C_2(a_2^*(\overline{\tau}) + \overline{\tau}\left(\xi - a_1 - a_2^*(\overline{\tau}) - \omega\right)\right),\end{aligned}$$

where $\zeta = \zeta_1 + \beta\zeta_2$. With quadratic abatement cost functions, it follows that,

$$\Upsilon(a_1;\theta) = \zeta - C_1(a_1) + \beta\left(p(\theta)\Delta\tau\left(\xi - a_1 - \omega - \frac{\langle\tau\rangle}{c_2}\right) - \overline{\tau}\left(\xi - a_1 - \omega - \frac{\overline{\tau}}{2c_2}\right)\right), \quad (43)$$

where $\langle\tau\rangle$ denotes the date-2 average price when $p = \frac{1}{2}$, *i.e.*, $\langle\tau\rangle = \frac{\overline{\tau}+\underline{\tau}}{2}$. Similarly, the θ-scenario expected net marginal revenue from date-1 abatement, evaluated at $a_1 = \bar{a}_1$, writes

$$\Upsilon_{a_1}(\bar{a}_1;\theta) = -C_1'(\bar{a}_1) + \beta\mathcal{V}_{a_1}(\bar{a}_1;\theta) = -C_1'(\bar{a}_1) + \beta(\overline{\tau} - p(\theta)\Delta\tau),$$

which is decreasing in θ i.f.f. $p(\theta)$ is increasing and, by optimality under ambiguity neutrality, nil when $p(\theta) = p$. Intuitively, it follows that $\Upsilon_{a_1}(\bar{a}_1;\theta)$ changes sign from positive to negative at $p(\theta) = p$. Also, from (43), one sees that when the liable firm expects to be net buyer of allowances under the abatement stream $\left(\bar{a}_1; \frac{\langle\tau\rangle}{c_2}\right)$, $\Upsilon(\bar{a}_1;\theta)$ is relatively high (resp. low) when $p(\theta)$ is relatively big (resp. small). Hence, for those θ-scenarios satisfying $p(\theta) < p$ where $\Upsilon(\bar{a}_1;\theta)$ is relatively low, $\Upsilon_{a_1}(\bar{a}_1;\theta) > 0$ so that increasing a_1 will increase $\Upsilon(a_1;\theta)$. Conversely, for those θ-scenarios satisfying $p(\theta) > p$ where $\Upsilon(\bar{a}_1;\theta)$ is relatively high, $\Upsilon_{a_1}(\bar{a}_1;\theta) < 0$ so that increasing a_1 will decrease $\Upsilon(a_1;\theta)$. Combining the two cases results in a reduced spread of expected profits across θ-scenarios. This describes the intuition behind the pessimism effect and the anticomonotonicity criterion. More formally, let us now state

Proposition D.1. *Let allowance price ambiguity be binary and abatement cost functions be quadratic and time separable. Assuming ambiguity prudence, the prevalence of ambiguity aversion increases date-1 abatement relative to ambiguity neutrality*
(i) *only if the liable firm expects to be net buyer of allowances under the abatement stream* $(\bar{a}_1;\bar{a}_2)$, *with* $\bar{a}_1 = \frac{\beta\langle\tilde{\tau}\rangle}{c_1}$ *and* $\bar{a}_2 = \frac{\langle\tau\rangle}{c_2}$, *or equivalently,*
(ii) *only if* ω *is below the threshold* $\bar{\omega} = \xi - \frac{\beta\langle\tilde{\tau}\rangle}{c_1} - \frac{\langle\tau\rangle}{c_2}$;
(iii) *only if* p *is above the threshold* $\bar{p} = \frac{1}{\beta c_2\Delta\tau}\left(\beta c_2\overline{\tau} + c_1\langle\tau\rangle - c_1c_2(\xi-\omega)\right) \in [0;1]$.

Proof. Again, the proof consists in signing the covariance. By differentiation w.r.t. θ, one has that, $\partial_\theta\mathcal{V}_{a_1}(\bar{a}_1;\theta) = -p'(\theta)\Delta\tau$, $\forall\theta\in\Theta$. Similarly, using (43),

$$\partial_\theta\mathcal{V}(\bar{a}_1;\theta) = p'(\theta)\Delta\tau\left(\xi - \bar{a}_1 - \omega - \frac{\langle\tau\rangle}{c_2}\right) = p'(\theta)\Delta\tau\left(\xi - \omega - \frac{\beta\langle\tilde{\tau}\rangle}{c_1} - \frac{\langle\tau\rangle}{c_2}\right).$$

Therefore, anticomonotonicity holds i.f.f. $\xi - \omega - \frac{\beta\langle\tilde{\tau}\rangle}{c_1} - \frac{\langle\tau\rangle}{c_2} > 0$, *i.e.*, the firm is a net buyer of allowances when it abates $(\bar{a}_1;\bar{a}_2)$. Note that by definition, $\langle\tilde{\tau}\rangle = \overline{\tau} - p\Delta\tau$, which is decreasing with p. Anticomonotonicity holds i.f.f.

$$2\beta c_2\left(\overline{\tau} - p\Delta\tau\right) + c_1\left(\overline{\tau} + \underline{\tau}\right) < 2c_1c_2\left(\xi - \omega\right), \tag{44}$$

that is, i.f.f., $p > \bar{p}$. For $\bar{p}$ to be admissible, one needs[73] $\beta\underline{\tau} \leq c\left(\xi - \omega\right) \leq \beta\overline{\tau}$. □

Proposition D.1 shows that the expected net position under ambiguity neutrality remains the key criterion in determining the effects of ambiguity aversion on date-1 abatement. As compared with Proposition 4.7, however, this condition is weaker since the ambiguity-

[73] When the date-2 allowance price is $c\left(\xi - \omega\right)$, the overall abatement effort $\xi - \omega$ has been optimally apportioned between the two dates, *i.e.*, in proportion to the flexibility in abatement at the two dates. With this in mind, it makes sense to have a possible price range such that $\beta\underline{\tau} < c\left(\xi - \omega\right) < \beta\overline{\tau}$.

averse liable firm must expect to be in a net short position for only one given abatement stream – not across all θ-scenarios. This abatement stream corresponds to the situation where the firm abates its ambiguity-neutral volume at date 1, $\bar{a}_1$, and a date-2 volume, $\bar{a}_2$, for which the underlying allowance price is $\langle\tau\rangle$. This scenario can be likened to a situation where the firm has no idea about the future allowance price at all and thus considers the equiprobable price scenario – under ambiguity neutrality, the liable firm is not affected by ambiguity.
Like before, this translates into an upper-threshold condition for the initial allocation level, but here also into a lower-threshold condition for p. That is, when p is relatively high, $\bar{a}_1$ is relatively low, so that the likelihood of being net short at date 2 is relatively high, all else equal. Interestingly, this transposition to a precise $\bar{p}$ threshold allows us to characterize the effects of an increase in the ambiguity level, here proxied by the size of the possible price range $\Delta\tau$, for given degree of ambiguity aversion. To do so, we analyse how $\bar{p}$ varies consecutive to an increase in $\Delta\tau$, and distinguish the cases of symmetric shifts in upper and lower price bounds, $\underline{\tau}$ and $\overline{\tau}$. In response to an infinitesimal positive shift in $\Delta\tau$ from $\Delta\tau$ to $\Delta\tau + \delta\tau$, where $\delta\tau > 0$ is small enough, we analyse the reaction of $\bar{p}$, denoted δp, around the equilibrium (44) in the two polar cases where $\overline{\tau}$ increases by $\delta\tau$ with $\underline{\tau}$ fixed, or symmetrically, where $\underline{\tau}$ decreases by $\delta\tau$ with $\overline{\tau}$ fixed. For an upward nudge in $\Delta\tau$, $\bar{p}$ reacts such that

$$2\beta c_2\left(\delta\tau - \bar{p}\delta\tau - \overline{\tau}\delta p - \delta p\delta\tau + \underline{\tau}\delta p\right) + c_1\delta\tau = 0, \quad \text{i.e.,} \quad R^{\uparrow} = \frac{\delta p}{\delta\tau} = \frac{2\beta c_2(1-\bar{p}) + c_1}{2\beta c_2\Delta\tau} > 0,$$

where $\delta p\delta\tau \simeq 0$ in the first order and $R^{\uparrow}$ denotes the rate of increase in $\bar{p}$ consecutive to an increase in $\overline{\tau}$ by $\delta\tau$. Similarly, for a downward nudge in $\Delta\tau$, $\bar{p}$ reacts such that

$$2\beta c_2\left(\bar{p}\delta\tau + \overline{\tau}\delta p + \delta p\delta\tau - \underline{\tau}\delta p\right) + c_1\delta\tau = 0, \quad \text{i.e.,} \quad R^{\downarrow} = -\frac{\delta p}{\delta\tau} = \frac{2\beta c_2\bar{p} + c_1}{2\beta c_2\Delta\tau} > 0,$$

where $\delta p\delta\tau \simeq 0$ again and $R^{\downarrow}$ denotes the rate of decrease in $\bar{p}$ consecutive to a decrease in $\underline{\tau}$ by $\delta\tau$, in absolute terms. It follows that

$$R^{\uparrow} - R^{\downarrow} = \frac{1-2\bar{p}}{\Delta\tau} > 0 \ \text{ i.f.f. } \ \bar{p} < \frac{1}{2}. \tag{45}$$

First notice that for a symmetric increase in $\Delta\tau$, so that $\langle\tau\rangle$ is unchanged, from $\Delta\tau$ to $\Delta\tau + 2\delta\tau$, $\langle\tilde{\tau}\rangle_{\Delta\tau+2\delta\tau} = \langle\tilde{\tau}\rangle_{\Delta\tau} + \delta\tau(1-2p)$ so that

$$\langle\tilde{\tau}\rangle_{\Delta\tau+2\delta\tau} \geq \langle\tilde{\tau}\rangle_{\Delta\tau} \ \Leftrightarrow \ p \leq \frac{1}{2} \ \Leftrightarrow \ \langle\tilde{\tau}\rangle_{\Delta\tau} \leq \langle\tau\rangle\,.$$

An increase in the ambiguity range hence always brings $\langle\tilde{\tau}\rangle$ closer to $\langle\tau\rangle$. That is, $\langle\tau\rangle$ is the central price scenario in determining how ambiguity aversion adjusts date-1 abatement relative to neutrality. This is also clear from (45) where an increase in $\Delta\tau$ always brings

$\bar{p}$ closer to $\frac{1}{2}$, which reflects the precautionary principle behind ambiguity aversion. More precisely,

- when $\bar{p} > \frac{1}{2}$, the firm abates more at date 1 under ambiguity aversion than under neutrality only if $p \geq \bar{p} > \frac{1}{2}$, that is, only if $\langle\tilde{\tau}\rangle$ is below $\langle\tau\rangle$: it foresees a price below that under the $\langle\tau\rangle$-scenario (and does not abate enough relative to this scenario) and ambiguity aversion corrects this by increasing a_1. In increasing $\Delta\tau$ symmetrically, both $\bar{p}$ and $\langle\tilde{\tau}\rangle$ decrease overall, which renders the criterion for ambiguity aversion to increase date-1 abatement relative to neutrality laxer;
- when $\bar{p} < \frac{1}{2}$ in particular, the firm abates more at date 1 under ambiguity aversion than under neutrality even in the case where $p \in \left[\bar{p}; \frac{1}{2}\right]$ so that $\langle\tilde{\tau}\rangle > \langle\tau\rangle$ and the ambiguity-neutral firm abates more at date 1 than under the $\langle\tau\rangle$-scenario. In increasing $\Delta\tau$ symmetrically, both $\bar{p}$ and $\langle\tilde{\tau}\rangle$ increase overall, which renders the criterion for ambiguity aversion to increase date-1 abatement relative to neutrality stricter.

That is, when the condition for ambiguity aversion to raise date-1 abatement relative to neutrality is relatively demanding (lax), an increase in the ambiguity range $\Delta\tau$ makes it laxer (more demanding), which is in line with a precautionary principle.

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