Contracting for The Management of a Non-Renewable Resource: Extracting Information or Extracting Resource?*

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Abstract. We characterize optimal contracts for resource extraction in a context where the concessionaire has private information on the initial stock. We show that the dynamics of extraction is characterized by a *Virtual Hotelling Rule* with costs of extraction being replaced with *virtual costs of extraction.* Asymmetric information introduces heterogeneity in the path of resource extraction with firms starting from high levels extracting more of the resource. We question the implementation of the optimal contract with simple menus of linear contracts specifying royalties per unit of output and license fees. Finally, we demonstrate that the price on markets where concessionaires operate under asymmetric information tend to converge faster to their long-run limits, exhibiting thus stable behavior.

Keywords. Non-Renewable Resource Management, Delegated Management, Optimal Contract, Asymmetric Information.

JEL codes. Q31; D82.

1. Introduction

Motivation. Throughout the world and over time, the oil and gas sectors have featured a large variety of contractual practices to run the relationships between, on the one hand, public authorities owning the land where exploration and production takes place, sometimes represented by *National Oil Companies* (or *NOC*) and, on the other hand, specialized firms in charge of managing the corresponding resource (*International Oil Companies* or *IOC*). These firms, because they are most often international ventures operating on a large scale, are key actors that can mobilize specific knowhow and gather

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sufficient resources to finance risky activities, such as exploration, but also the specific infrastructures that are needed for production and transportation.[1]

When it comes to assess the size of the stock of resources they are extracting, those firms involved in resource management are, because of their expertise, certainly at an informational advantage vis-à-vis public authorities This paper adresses the shape of optimal contracts that rule the principal-agent relationships between those public authorities and their concessionnaires in such asymmetric information contexts. We will bear a particular attention on how the quantity of resource extracted varies over time and on the price paths.[2]

Main Results. Of course, at a broad level one might *a priori* think that the delegated management of resource extraction shares some of the general features found with other forms of procurement and that the insights developed elsewhere carry over for resource as well.[3] In particular, the existing literature has forcefully stressed that the design of optimal procurement contracts under informational constraints result from a trade-off between efficiency and rent extraction.[4] Choosing to operate at efficient scale might also necessitate to leave excessive information rent to the contractor and those rents might be viewed as socially costly by public authorities. Reducing information rents instead calls for lower output, low-powered incentives and more generally procurement policies that are less sensitive to information. Yet, and although our analysis below will confirm that those insights have still some bite in the context of resource management, the phenomenon under scrutiny hereafter is inherently dynamic, requires specific analysis and unveils new effects.[5] This dynamic perspective is probably best illustrated by the importance taken by *Hotelling Rules* in resource economics. That rule shows how the resource price should evolve along an optimal path of extraction. Although much has been said on its validity,[6] it provides an important benchmark to assess how the trade-off between efficiency and the extraction of information rent must be revisited in a dynamic model of resource extraction.

[1]To illustrate and although similar patterns are observed for gas, three types of petroleum contracts are generally distinguished: *Concessions contracts* where the *IOC* owns the oil in the reservoir; *production sharing agreements* where the *IOC* owns only part of the produced oil; and *service contracts* which are such that the *IOC* receives a financial compensation for production. Historically, concessions have been the first type of contracts, since the industry took off in the U.S., where ownership of an estate includes subsurface mineral resources. Following the wave of nationalizations in the 1970's, the dominant type of contracts are now production sharing agreements and service contracts when the host country contracts with an *IOC* for the exploration and development of production.

[2]In practice, the contract concerns a concession area (a so called *"block"*), over which the *IOC* can decide at which speed to develop new wells, if any. This makes it possible for the firm to manage production overtime. As compared to the conventional oil and gas production, shale gas is characterized by greater flexibility. This is due to the following features. First, for a given potential productivity of the shale play, production increases proportionally to the number and length of horizontal fracking wells. Second, each single well produces most of the output over the first months. Third, reserves are embedded in shale plays that extend over very large areas. As a consequence, relative to conventional oil and gas, in shale gas and oil production can be managed in a much more flexible way over time.

[3]See Laffont and Tirole (1993) and Armstrong and Sappington (2007) for accounts of the *New Regulatory Economics.*

[4]Laffont and Martimort (2002).

[5]This view of the specifies of models of resource extraction from the perspective of Incentives Theory has also been pushed forward by Gaudet and Lasserre (2015).

[6]See Gaudet (2007) for a critical overview.

Before jumping to the asymmetric information scenario and to better assess the new insights brought by asymmetric information, it is useful to briefly remind the mechanism underlying the *Hotelling Rule* in an otherwise standard complete information setting. Suppose thus that the stock of resource is common knowledge and suppose also that, because of depletion, the marginal cost of extraction increases over time. The optimal dynamics of extraction under complete information should follow the celebrated *Hotelling Rule*. The resource price at any point in time must reflect the scarcity rent. Efficiency requires that, at any point in time, the marginal value of last unit extracted covers not only the current marginal cost (function of current reserves) but also the shadow cost of increasing the cost of extraction from that date on by depleting reserves. Resources decrease up to the point where the marginal cost of extraction goes beyond the choke price and extraction ceases at that point. At the same time , the resource price continuously increases over time to reflect the dynamic arbitrage between the benefits of consuming the resource today or letting more resource in the ground so as to facilitate extraction tomorrow. Under complete information, public authorities can simply capture all intertemporal profit that can be obtained by the concessionaire along this optimal path by imposing an entry fee whose value is perfectly known. An alternative would be to run a competitive auction for the franchise among potential concessionnaires.

Virtual cost of extraction and Virtual Hotelling Rule. The picture is radically different when the firm has private information on the initial stock of resource. Contracts must now induce firms with different initial stocks to select different paths of extraction as a means of revealing information. A firm which knows that its stocks are large and which thus faces low cost of extraction may just adopt the same extraction pattern as one with lower reserves and higher cost of extraction. By doing so, the high-stock firm produces the same quantity as the low-stock firm at any point in time, pays a lower license fee and pockets the corresponding cost saving. Slowing down extraction is thus a way to capture an information rent for those firms with the highest initial stocks. The temptation to underestimate stock is akin to the temptation to keep resource in the ground and move forward along the extraction phase.

An optimal contract must render such strategy less attractive so as to induce information revelation at the inception of the relationship. This is done by public authorities when committing to reduce extraction from those firms which claims the lower levels of reserves to start with. By reducing the production of those firms, the regulator makes less valuable for firms with higher stocks to strategically behave as such low-reserve types. In other words and more generally, everything happens as if the marginal cost of extraction was replaced by a greater *virtual cost of extraction* that accounts for the cost of extracting information.[7] Dynamic inefficiencies in resource extraction arise when information has also to be extracted. This points to an important dilemma in designing concession contracts for resource extraction: The cost of inducing information revelation is to leave resource in the ground, stopping extraction before what efficiency would command. An extreme case arises when the regulator chooses to eschew any production from fields with little stock to start with.

We characterize the path of the economy under asymmetric information by means of *Virtual Hotelling Rule* that governs how the resource price evolves over time. This

[7] To use the parlance of Myerson in numerous contributions.

Virtual Hotelling Rule has again a simple interpretation. The last unit extracted from a field whose reserves were announced to be of a given size at the start must be such the marginal benefit of consumption covers not only the current cost of extraction and the shadow cost of increasing future extraction as under complete information but also the cost of information rent captured by those firms with supramarginal reserves.

Comparative statics. The compounding of the rent for scarcity and the rent for information shapes the dynamics of resource extraction in significant ways. To illustrate, although under complete information, all firms whatever their initial stocks would evolve along the same trajectory for extraction and would extract up to a point where the cost of extraction at that level of stock just equals the choke price; asymmetric information introduces a significant heterogeneity across trajectories. Firms with lower stocks tend to extract less and leave thus more of the resource in the ground. The limiting level of stock is now obtained when the *virtual marginal cost of extraction* is equal to the choke price and that limiting stock varies negatively with the initial stock.

Interestingly, we provide closed-form expressions for the optimal paths and show that asymmetric information does not necessarily slow down resource extraction at least in the case of a single operator. In the case of linear demand and cost of extraction, the level of resource and the quantity extracted both converge towards their long-run limits at the same exponential rates as under complete information. Yet, the overall amount extracted is always lower under asymmetric information.

Implementation. From a practical point of view, it is important to ask whether the optimal contract can be implemented with simple instruments. Still in the case of linear demand and cost of extraction, we demonstrate that the choice of an optimal dynamic path of extraction can be reduced to the choice of the quantity extracted at the start; a reduction to a static problem. We show that a simple nonlinear payment links the firm's compensation to that quantity and that, under some technical condition that guarantees the convexity of this schedule, the optimal contract can be implemented by a menu of linear schemes that specify royalties per unit of output and a license fee. Firms with greater stocks to start with adopt a more attractive royalty but also pay a higher fee.

From one firm to a market of concessionaires. Finally, we extend our results to the case where the whole sector is run with concession contracts. An important take-away of our analysis is that, under asymmetric information, all production units that are active at a given date produce at the same virtual marginal cost of extraction. Since those virtual costs depend on the initial stocks of those units, all active firms at a given date produce different amounts and keep different stocks. This stands in sharp contrast with the case of complete information where firms may start to be active at different dates according to their initial stock and the current price but, from that point on, all active firms produce the same amount. The basic heterogeneity induced by asymmetric information remains although the price dynamics and the *Virtual Hotelling Rule* that now applies must account for the aggregation procedure among heterogeneous units along the price dynamics. Because some units, those with the lower stocks, never start to be used, the price remains quite high along the extraction path. Convergence towards its long-run limit is faster than under complete information. This shows that asymmetric information may in fact help justifying the stability of price around its long-run equilibrium value.

Literature Review. Our paper belongs to a burgeoning literature on resource extraction and incentives. Poudou and Thomas (2000) and Hung, Poudou and Thomas (2002) have analyzed the design of a mining concession contract in a (finite) multi-period adverse selection setting with a finite number of periods and a persistent shock on cost which is private information of the concessionaire. Reducing the firm's information rent requires downward distortions of production early in the relationship. Yet, with a finite number of periods, this also means that more resource can be extracted in a terminal phase of extraction. Focusing on a two-periods model, Gaudet, Lasserre and Van Long (1995) have studied optimal royalty contracts for a nonrenewable resource under asymmetric cost information with cost parameters which are intertemporally independent. A common finding of these papers is that asymmetric information shifts production to the future. We differ from these papers along several lines. First, our model has continuous time and infinite horizon in order not to artificially constrain the resource patterns. This means that all effects that are artifacts of the existence of a terminal date disappear in our context. Although output is reduced for rent extraction reasons, the consequence of such distortions with an infinite horizon are captured through both the long-run value of stocks and the intertemporal profile of extraction. Second, existing models suppose that the cost of extraction only depends on current production and not on the remaining stock; an assumption that has been often questioned in the literature (Solow and Wan (1979), Pindyck (1978, 1987), Swierzbinski and Mendelsohn (1989)).[8] Modeling a cost of extraction that depends on depletion allows us to see how asymmetric information interact with the dynamics of extraction. Third, we assume that the cost technology is common knowledge while the value of the initial stock is private information to the firm. This assumption is consistent with an earlier literature in resource economics that has stressed the significant uncertainty on the value of stocks at the inception of an exploitation phase.[9] While assuming private information on cost of extraction is akin to replace this cost by its *virtual value* which is greater, this transformation has only an indirect impact on the dynamics of extraction. To illustrate, assuming heterogeneity on the costs of extraction implies that the *Hotelling Rules* under complete information are already and *de facto* different across firms. Instead, assuming as we do below that private information is on initial stocks more deeply links private information to this dynamics. Although the trajectories of the economy remain the same under complete information and the price dynamics follow the same *Hotelling Rule* despite staring from different initial stocks, these trajectories now differ according to the value of the firm's *virtual cost of extraction* when firms have private information on the level of initial stock.

Making similar informational assumptions to ours, Osmudsen (1998) shows that optimal contracts distort both the extent and the pace of depletion in a two-period (and more generally with a finite number of periods) framework. Having an infinite horizon, allows us to get a significantly different set of results. To illustrate, we provide an example (with linear inverse demand functions and marginal cost of extraction) which admits

[8]Once the pool of hydrocarbons in a field is precisely identified, the unit operating cost is typically decreasing in the size of the reserve. At the field level, this is due to the fact that the firm starts digging wells where oil is relatively abundant, and then moves on to areas within the field where oil is more difficult to extract and less abundant. At the regional level, where several exploration licenses can be handed, the same negative correlation between remaining reserves and unit operating cost may hold. In fact, the first applications for exploration licenses concern the most promising blocks, so that larger pools tend to be discovered first.

[9]See Hoel (1978) and Pindyck (1980) among others.

closed-form solutions for the optimal trajectory. This example shows that the pace of depletion may remain the same as under complete information at least in the case of a single operator, although the quantities extracted obviously differ. This again shows that the lessons of finite time models should be taken with a word of caution when asymmetric information is introduced. Following on this author's insights, we also study how royalty payments can be used to implement the optimal allocation. At this stage, we take advantage of the specific trajectory to reduce the infinite horizon dynamic implementation problem to a static one. In this static reduced-form problem, the regulator offers a menu of license contracts. The firm chooses within this menu a particular royalty that applies from date 0 on and pays upfront a license fee. This choice defines the whole trajectory of the economy with outputs and revenues following an exponential decay. Our analysis differs from that in Osmundsen (1998) because we qualify conditions on technologies under which such implementation through linear contracts is actually feasible. It is only so when the optimal nonlinear payment schedule is convex.[10]

Taking a broader perspective, this paper belongs also to a very active literature in dynamic mechanism design.[11] Our assumption that private information (here the initial stock of resource) is persistent throughout the whole relationship is reminiscent of the work of Baron and Besanko (1984) in a discrete-time dynamic regulation model although both the dynamics of resource extraction and the locus of informational asymmetries sharply differ from the stationary environment analyzed by these authors. The dynamics of resource extraction and the fact that the marginal cost of extraction depends on the remaining stock introduces an important linkage between the production technologies at different points in time. A technical insight of our analysis is that implementability conditions which are known to be technically quite challenging in general dynamic mechanism design environments[12] can be reduced to checking a simple monotonicity condition on the aggregate amount extracted. The dynamic linkage across periods is also reminiscent of other models like Gärtner, (2010), Auray, Martiotti and Moizeau (2011), and Lewis and Yildirim (2002) who provide discrete-time models to analyze how learning by doing or quality maintenance affect future cost structures.

Organization of the paper. Section 2 describes the model. Section 3 analyzes the optimal extraction of resource under complete information, and as benchmark for future comparison, the case of of an unregulated monopoly. Section 4 is the core of the analysis. We first derive incentive compatible allocations and then optimize within this class to find the second-best trajectory of the economy under informational constraints. We highlight there the role of the *virtual cost of extraction* as a driver of the dynamics. We also bear a particular attention to the shape of the concession contracts that implement this optimal allocation and argue that simple licensing contracts with fixed-fee and payment royalties may implement this optimal allocation. Finally, Section 5 shows how our modeling can be adapted to address the behavior of a market with heterogenous firms where multiple bilateral concession contracts are signed with each of those. All proofs are relegated to an Appendix.

[10] In a regulatory environment where cost-reimbursement rules are feasible and with both adverse selection and moral hazard elements, Laffont and Tirole (1993, Chapter 1) demonstrate that the optimal contract is necessarily convex and can be implemented with a menu of simple cost-reimbursement rules including fixed-price and cost-plus contracts. In our context, extraction costs remain nonobservable.

[11] See Bergemann and Pavan (2015) for an authoritative and up-to-date survey.

[12] Pavan, Segal and Toikka (2014).

2. The Model

• Resource Dynamics. We consider the dynamic management by a firm of a stock $S(\theta, t)$ of resources over time and how this management can be optimally regulated through a concession contract. The initial stock of resource at date $t = 0$ is

$$S(\theta, 0) = \theta. \tag{2.1}$$

The resource stock evolves according to the following state equation:

$$\frac{\partial S}{\partial t}(\theta, t) = -q(\theta, t). \tag{2.2}$$

where $q(\theta, t)$ denotes the non-negative amount of resources extracted at date t starting from an initial stock θ.

• Production technology and preferences. Following the existing literature,[13] we assume that extracting resources is all the more difficult that the remaining stock is low. Formally, we write the cost of extracting q units when the remaining stock is S as $C(S)q$. Therefore, we assume constant returns to scale in extraction but we also posit that the marginal cost of production $C(S)$ is decreasing and convex in the level of that stock ($C' < 0 \leq C''$). For technical reasons, we assume also that this marginal cost remains finite, $C(0) < +\infty$ and that $C''' \leq 0$.

The gross surplus from consuming q units of resource is denoted by $V(q)$. This function is increasing and strictly concave ($V' > 0 > V''$). We denote by $P(q) = V'(q)$ the inverse demand and assume that the choke price $P(0)$ is finite. We define S^* as the stock of resource such that the marginal cost of extraction at that level is equal to the choke price:

$$P(0) = C(S^*). \tag{2.3}$$

Any process of resource extraction should thus stop when it reaches that level.

• Asymmetric Information. The initial stock of resource θ is private information to the firm. We suppose that this parameter is drawn from the set $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$ according to the cumulative (atomless) distribution F with density $f = F'$. We assume that $\underline{\theta} \geq S^*$. In other words, the firm always starts with enough resources to find it valuable to begin extraction since $C(\underline{\theta}) \leq P(0)$.

Following the screening literature,[14] we also assume that the so called *Monotonicity of the Hazard Rate Property* holds:

$$\frac{d}{d\theta}\left(\frac{1 - F(\theta)}{f(\theta)}\right) \leq 0 \text{ for all } \theta \in \Theta. \tag{2.4}$$

That the firm has better information on the stock of resources that it exploits than regulators is in line with casual evidence that operators have more precise signals than

[13] Solow and Wan (1979), Pindyck (1978, 1987), Swierzbinski and Mendelsohn (1989) among others.
[14] Bagnoli and Bergstrom (2005).

outsiders (regulators, financiers, local communities) on the value of reserves.[15] Although in practice, operators may learn more about the exact reserves as they further exploit, we take the shortcut that the learning process is lumpy.[16]

• Running Example. We will sometimes provide closed-form solutions for some of the trajectories below by assuming linear inverse demand and linear cost, i.e., $P(q) = P(0) + qP'(0)$, $C(S) = C(S^*) + (S - S^*)C'(S^*)$. ■

3. The Dynamics of Resource Extraction: Complete Information

This section provides some important benchmarks in the simple scenario where the initial stock of resource is common knowledge. Although the dynamics of extraction in such environments is well-know, it is useful to remind its main feature to anchor comparisons with the case of regulation under asymmetric information.

3.1. Pareto-Optimal Trajectory

Let denote by r the discount rate, the optimal path of resource extraction maximizes the intertemporal discounted surplus subject to resource dynamics:

$$(\mathcal{P}^*(\theta)) : \quad \max_{q,S} \int_0^{+\infty} (V(q(\theta,t)) - C(S(\theta,t))q(\theta,t))exp(-rt)dt$$

subject to (2.1) and (2.2).

We denote respectively by $q^*(\theta,t)$ and $S^*(\theta,t)$, the quantity extracted at a time t and the remaining stock of resources that solve problem $(\mathcal{P}^*(\theta))$. Next proposition describes carefully the optimal dynamics of extraction.

Proposition 1. *Under complete information, the optimal path of resource extraction converges towards no extraction and a finite stock of resource in infinite time*

$$\lim_{t\to+\infty} q^*(\theta,t) = 0 \text{ and } \lim_{t\to+\infty} S^*(\theta,t) = S^*. \tag{3.1}$$

The price of resources $p^*(\theta,t) = V'(q^*(\theta,t))$ evolves according to the standard Hotelling Rule:

$$\frac{\partial p^*}{\partial t}(\theta,t) = r(p^*(\theta,t) - C(S^*(\theta,t))). \tag{3.2}$$

There are several important items in this proposition that need to be stressed in view of our analysis in the sequel.

Identical trajectories for heterogeneous fields. First, all trajectories of the economy follow the same path of extraction even though they may start at different

[15] To illustrate, much scrutiny is given in practice on the requirement that firms disclose the conditions of operations.

[16] Assuming that the concessionnaire and the public authority in charge of drafting the concession contracts have different beliefs on the size of the stock would lead to similar conclusions.

levels of resource at date $t = 0$. After some initial period of extraction moving an initial stock θ to a lower value θ', the continuation path is the same as if the stock had started from this lower value. This just follows from Bellmann Principle for optimization and the time-consistency of the planner's objective.

As a result, the initial stock of resource has no long-run impact neither on the rate of extraction nor on the final level of the stock. In the long-run, whatever the starting point, the resource is extracted up to the point that the marginal cost of extraction is equal to the choke price. In other words, all fields evolve the same way despite some *prior* heterogeneity.

HOTELLING RULE. Under complete information, the economy evolves according to the standard *Hotelling Rule*. At the optimum, the planner must be just indifferent at date t between consuming some extra amount dq today or delaying for an extra length of time that resource in the ground to facilitate extraction later on. Producing and consuming dq more units beyond $q^*(\theta, t)$ for a length of time dt around date t increases overall surplus at date t by approximatively:

$$(V'(q^*(\theta,t)) - C(S^*(\theta,t)))\, dqdt. \tag{3.3}$$

However, consuming those extra units dq reduces the stock of resources from date $t + dt$ onwards to $\approx S^*(\theta, \tau) - q^*(\theta, t)dt + dqdt$. Future costs are thus increased from date $t+dt$ onwards by amount which is worth, taking into account discounting, approximatively:

$$\approx -\left(\int_t^{+\infty} C'(S^*(\theta,\tau))q^*(\theta,\tau)exp(-r(\tau - t))d\tau\right) dqdt. \tag{3.4}$$

At the optimal path, (3.3) and (3.4) must be equal and thus:

$$V'(q^*(\theta,t)) - C(S^*(\theta,t) = -\left(\int_t^{+\infty} C'(S^*(\theta,\tau))q^*(\theta,\tau)exp(-r(\tau - t))d\tau\right).$$

Differentiating with respect to t immediately gives us the *Hotelling Rule* (3.2).

IMPLEMENTATION. Under complete information the planner could force the firm to extract the socially optimal quantity $q^*(\theta, t)$ at date t and compensate the firm for the cost $C(S^*(\theta, t))q^*(\theta, t)$ of producing this quantity. There is no real obstacle to implementing this forcing policy even though such forcing contracts are rarely seen in practice.[17] The optimal allocation can also be implemented if the firm adopts a competitive behavior and takes the price for resource $p^*(\theta, t)$ found above as given. As this resource rarefies, its price increases towards the choke price $P(0)$ and the amount extracted decreases over time. The overall profit is

$$\int_0^{+\infty} (p^*(\theta,t) - C(S^*(\theta,t)))q^*(\theta,t))exp(-rt)dt.$$

[17]Which in passing suggests that complete information might not be the right assumption to start with.

Integrating by parts, this intertemporal profit can be expressed as:

$$r\int_0^{+\infty}\left((\theta - S^*(\theta,t))C(S^*(\theta,t)) + \int_0^t C(S^*(\theta,\tau))d\tau\right) exp(-rt)dt$$

can then be fully extracted through a lump-sum tax on profit $T^*(\theta)$ which can be paid upfront if the firm has enough cash or which can be distributed over time to extract profit at any point in time.

An alternative implementation, much in the spirit of Loeb and Magat (1979), would be to organize an *ex ante* competitive tender for the right of running the field among several identical concessionaires (each of them already knowing the realization of θ). Through this process, the regulator selects arbitrarily a winner which has bid up the whole intertemporal profit of running operations. If it behaves *ex post* like a monopolist practicing first-degree discrimination, this firm may extract all consumer's surplus to cover his prior bid. The process clearly replicates the trajectory highlighted in Proposition 1.

RUNNING EXAMPLE (CONTINUED). Closed forms solutions for the optimal trajectory are obtained as:

$$S^*(\theta,t) - S^* = (\theta - S^*)exp\left(-\frac{t}{z^*}\right) \tag{3.5}$$

and

$$q^*(\theta,t) = \frac{\theta - S^*}{z^*}exp\left(-\frac{t}{z^*}\right) \tag{3.6}$$

where $z^* = \frac{1}{2\kappa^*}\left(1+\sqrt{1+\frac{4\kappa^*}{r}}\right) > \frac{1}{r}$ and $\kappa^* = \frac{C'(S^*)}{P'(0)} > 0$. The stock of resource and the quantity extracted decreases towards their long-run limits (S^* and 0 respectively) at the same exponential rate, the relaxation time being z^*. Intuitively, convergence is faster as κ^* increases and the cost function C sharply decreases as resource diminishes. ■

3.2. Unregulated Monopoly

Suppose instead that the firm behaves as an unregulated monopoly charging a unit price.[18] The solution under this scenario is very similar to what we derived above provided that the revenues $R(q) = qP(q)$ now replace the consumer's surplus in the objective function. Observe in particular that

$$R'(0) = P(0) = C(S^*).$$

Hence, the marginal revenue at zero is again equal to the choke price and it is thus the same for the monopolist and for the competitive firm. In the long run, there is as much resource extraction under monopoly provision as in a competitive market. Yet, the convergence towards this long-run outcome is slowed down because the monopolist wants to keep prices high enough along the path of extraction to exert market power. This

[18] The scenario where the monopolist can practice first-degree price discrimination is akin to Section 3.1 up to a redistribution of the overall surplus in its favor.

intuition which is developed formally in Appendix B is here presented for the functional forms of our RUNNING EXAMPLE.

RUNNING EXAMPLE (CONTINUED). Closed forms solutions for the trajectory under monopoly provision are obtained as:

$$S^m(\theta, t) - S^* = (\theta - S^*) exp\left(-\frac{t}{z^m}\right) \tag{3.7}$$

and

$$q^m(\theta, t) = \frac{\theta - S^*}{z^m} exp\left(-\frac{t}{z^m}\right) \tag{3.8}$$

where $z^m = \frac{1}{2\kappa^m}\left(1 + \sqrt{1 + \frac{4\kappa^m}{r}}\right) > \frac{1}{r}$ and $\kappa^m = \frac{C'(S^*)}{R''(0)} = \frac{C'(S^*)}{2P'(0)} < \kappa^*$.[19] The stock of resource and the quantity extracted decreases towards their long-run limit at an exponential rate with a relaxation time being now z^m. Observe that $z^m > z^*$ and hence, $S^m(\theta, t) \geq S^*(\theta, t)$. Indeed, the monopolist tends to maintain resources at a higher level than under perfect competition to keep prices high enough.[20] Turning to the quantity extracted, the comparison of (3.6) and (3.8) shows that it is lower under monopoly for t small enough whereas it is larger beyond a certain point. ■

4. THE DYNAMICS OF RESOURCE EXTRACTION: ASYMMETRIC INFORMATION

As pointed out by the literature in regulatory economics,[21] the true impediment to regulation is asymmetric information. We thus now assume that the initial stock of resource is privately known by the firm.

4.1. Incentive Compatibility

A long-term regulatory contract regulates the relationship between the public authority and the firm in charge of extracting the resource. This contract stipulates how much quantity should be extracted and possibly a payment at any point in time. From the *Revelation Principle*,[22] we can view without loss of generality such contract as a direct and truthful revelation mechanism that stipulates the output $q(\hat{\theta}, t)$[23] and the payment $\omega(\hat{\theta}, t)$ profiles over time as a function of the firm's announcement $\hat{\theta}$ on the resource stock. Allowing such communication in our theoretical model certainly echoes real world practices. Indeed, most contracts allow experts working for the host government to access data or use equipment from the *IOC* in order to check the likelihood of these

[19] The last inequality follows from $R''(0) = 2P'(0) < P'(0)$.

[20] General conditions on technologies for this property to hold are given in Proposition B.2 below.

[21] Baron and Myerson (1982), Laffont and Tirole (1993) and Armstrong and Sappington ()

[22] Myerson (1982).

[23] In practice, the quantity to be produced is not specified at a too early stage of contracting, i.e., before exploration has started. However, after this initial phase, it becomes feasible to negotiate on the level of production upon *"commercial discovery"* on the basis of the size of the field, i.e. the quantity of recoverable oil reserves in the area.

declared reserves. Because we assume that the regulator commits to the mechanism, this announcement takes place once for all at date 0^-.[24]

Henceforth, we may denote by $U(\theta)$ the firm's informational rent (or intertemporal payoff) when adopting the truthful strategy as:

$$U(\theta) = \int_0^{+\infty} (\omega(\theta, t) - C(S(\theta, t))q(\theta, t))exp(-rt)dt \tag{4.1}$$

where the trajectory $S(\theta, t)$ is defined through (2.1) and (2.2).

Incentive compatibility requires that this payoff is greater than what the firm with type θ may obtain by adopting the extraction patterns of a type $\hat{\theta}$ but starting from the initial stock θ. Let denote $\hat{S}(\theta, \hat{\theta}, t)$ the corresponding trajectory. From (2.1) and (2.2), it can be defined in integral form as:

$$\hat{S}(\theta, \hat{\theta}, t) = \theta - Q(\hat{\theta}, t) \tag{4.2}$$

where $Q(\hat{\theta}, t) = \int_0^t q(\hat{\theta}, \tau)d\tau$ is the cumulative amount extracted up to date t by a firm endowed with a stock of resource $\hat{\theta}$. Of course, $\hat{S}(\theta, \theta, t) = S(\theta, t)$ and the trajectory following the truthful strategy satisfies:

$$S(\theta, t) = \theta - Q(\theta, t). \tag{4.3}$$

Incentive compatibility can thus be written in a compact form as:

$$U(\theta) = \max_{\hat{\theta}\in\Theta} \int_0^{+\infty} (\omega(\hat{\theta}, t) - C(\hat{S}(\theta, \hat{\theta}, t))q(\hat{\theta}, t))exp(-rt)dt. \tag{4.4}$$

A contract $\{(\omega(\hat{\theta}, t), q(\hat{\theta}, t))_{t\geq 0}\}_{\hat{\theta}\in\Theta}$ induces an allocation $(U(\theta), (q(\theta, t))_{t\geq 0})_{\theta\in\Theta}$. Next Lemma provides a useful characterization of such allocations that are incentive compatible, i.e., that can be implemented by a particular contract. This dual approach turns out to be particularly useful to characterize the optimal trajectories under asymmetric information.

Lemma 1. *An allocation $(U(\theta), (q(\theta, t))_{t\geq 0})_{\theta\in\Theta}$ is incentive compatible if and only if:*

1. *$U(\theta)$ is absolutely continuous and thus a.e. differentiable with at any point of differentiability θ:*

$$\dot{U}(\theta) = -\int_0^{+\infty} C'(S(\theta, t))q(\theta, t))exp(-rt)dt; \tag{4.5}$$

2. *The amounts extracted satisfies the following conditions*

$$\int_0^{+\infty} \left(\int_\theta^{\hat{\theta}} \left(\int_{Q(\hat{\theta},t)}^{Q(\tilde{\theta},t)} C'(\tilde{\theta} - \tilde{Q})d\tilde{Q} \right) d\tilde{\theta} \right) exp(-rt)dt \geq 0 \quad \forall(\theta, \hat{\theta}) \in \Theta^2. \tag{4.6}$$

[24]Observe also that there is no loss of generality in making the convention an infinite planning horizon for the firm because the contract can just stipulates zero output and zero payment from a given date on if extraction was chosen over a finite period of time.

In particular, (4.6) holds if the following sufficient condition is satisfied

$$Q(\theta, t) \text{ non-decreasing in } \theta. \tag{4.7}$$

Condition (4.5) is a standard *Envelope Condition* that describes how incentive compatibility shapes the profile of the firm's information rent as its type varies. To understand this condition, it is useful to think of a firm knowing that it starts extraction from a stock of size θ. That type may mimic the behavior of a firm with type $\theta - d\theta$, thus endowed with $d\theta$ less units of resource and facing a greater cost of extraction. So doing means that, at any point in time, the type θ-firm can receive the same larger compensation destinated to this less-well endowed type $\theta - d\theta$ and extract the same amount, namely $q(\theta - d\theta, t)$ but it does so at a lower marginal cost of extraction $C(\theta - Q(\theta - d\theta, t))$. This mimicking strategy is thus akin to moving ahead in the extraction path. The intertemporal gains of adopting such mimicking strategy are thus:

$$\int_0^{+\infty} (C(\theta - Q(\theta - d\theta, t)) - C(\theta - d\theta - Q(\theta - d\theta, t)))q(\theta - d\theta, t))exp(-rt)dt$$

$$\approx -d\theta \int_0^{+\infty} C'(\theta - Q(\theta, t))q(\theta, t)exp(-rt)dt.$$

To be compensated for the opportunity cost of not adopting this mimicking strategy, a firm with type θ must gets an incremental rent above that of type $\theta - d\theta$, namely $U(\theta) - U(\theta - d\theta) \approx \dot{U}(\theta)d\theta$, which is just worth the cost saving above.

It is standard in the screening literature that implementability conditions be summarized by an *Envelope Condition* together with a monotonicity requirement on output.[25] The familiar approach consists in studying a so-called *relaxed* optimization problem where this monotonicity requirement is omitted and then check *ex post* that the solution of this relaxed problem is monotonic. Things are rather easy in the context of a single-dimensional screening variable even in the more involved settings where this monotonicity condition may be binding.[26] Here, our dynamic model allows the regulator to control output at all dates, making the sufficient conditions for implementability harder to summarize into a single condition. Fortunately, this reduction is possible in our structured model and the sufficiency condition (4.8) states that, for any incentive incentive compatible allocation, the total amount of extracted up to any date t must be non-decreasing in the initial stock. If the optimal path is such that the greater this stock, the greater extraction up to any date, then the solution to the relaxed problem is incentive compatible. This is thus a rather simple condition that needs to be checked *ex post* and that, in our context, turns out to be always satisfied.[27]

Finally, the *Envelope Condition* (4.5) also shows that the information rent of the firm is non-decreasing in its initial stock of resources. In other words, any type of the firm accepts the contract if that firm with the lowest reserve $\underline{\theta}$ already does so. We will thus impose on top of the incentive compatibility conditions (4.5) and (4.7) the following

[25] See Laffont and Martimort (2002, Chapter 3) for instance.
[26] Guesnerie and Laffont (1984).
[27] See Proposition 4 below.

participation constraint:

$$U(\underline{\theta}) \geq 0. \tag{4.8}$$

4.2. Virtual Cost of Extraction and Virtual Hotelling Rule

The regulator's objective is to maximize the expected intertemporal payoff[28]

$$\mathbb{E}_\theta \left(\int_0^{+\infty} (V(q(\theta,t)) - \omega(\theta,t))exp(-rt)dt \right)$$

subject to the equations (2.2) and (2.3) that describe how the stock of resource evolves on path and subject to the firm's incentive and participation constraints (4.5) and (4.6). We proceed as a large shrunk of the screening literature and, as suggested above, first focus on a *relaxed optimization problem* where (4.6) is omitted. We check *ex post* that the sufficient condition (4.7) holds which ensures that the solution to the relaxed problem satisfies the omitted constraints.

Expressing the intertemporal payment to the firm in terms of its information rent, we may rewrite the regulator's objective function as:

$$\mathbb{E}_\theta \left(\int_0^{+\infty} (V(q(\theta,t)) - C(S(\theta,t)q(\theta,t))exp(-rt)dt - U(\theta) \right).$$

This expression highlights the rent/efficiency trade-off faced by the regulator in this dynamic environment. On the one hand, the regulator would like to choose a path for resource extraction that maximizes the overall surplus. On the other hand, doing so might mean leaving excessive information rent to the firm and the corresponding extra payments are costly for the regulator. Observe also in passing that the regulator can always choose to extract nothing so that the value of this problem is always non-negative, and it is so for any possible realization of the initial stock of resource. In other words, the strategy consisting in *"shutting down"* the least productive units to reduce the rent of the most productive ones is already considered in the above formulation.

To get a more compact expression of the optimization problem, we observe that necessarily the participation constraint (4.8) is binding at the optimum. Otherwise, all payments could be reduced by a small amount, still ensuring participation, and keeping the same dynamics of resource extraction and such modification of the allocation would improve the regulator's expected payoff. Henceforth, $U(\underline{\theta}) = 0$ and, using this fact, a simple integration by parts together with (4.5) gives us:

$$\mathbb{E}_\theta (U(\theta)) = -\mathbb{E}_\theta \left(\int_0^{+\infty} \frac{1 - F(\theta)}{f(\theta)} C'(S(\theta,t))q(\theta,t)exp(-rt)dt \right).$$

This expression of the firm's expected information rent is an extra information cost incurred by the principal. Inserting this cost into the regulator's objective, we obtain a

[28]We choose to give no weight to the firm's profit in the regulator's objective to simplify notations. We already know from the work of Baron and Myerson (1982) that giving a weight less than one to the firm generates the same kind of rent/efficiency trade-off.

more compact expression of the maximand as:

$$\mathbb{E}_\theta\left(\int_0^{+\infty}\left(V(q(\theta,t))-\left(C(S(\theta,t))-\frac{1-F(\theta)}{f(\theta)}C'(S(\theta,t))\right)q(\theta,t)\right)exp(-rt)dt\right).$$

Maximizing this expression amounts to solving a collection of maximization problems for each possible realization of θ. We write this latter problem as:

$$(\mathcal{P}^{sb}(\theta)): \quad \max_{q,S}\int_0^{+\infty}(V(q(\theta,t))-\tilde{C}(\theta,S(\theta,t))q(\theta,t))exp(-rt)dt$$

subject to (2.1) and (2.2).

The maximand showcases that the marginal cost of extraction is now replaced by the *virtual marginal cost of extraction* that is defined as:

$$\tilde{C}(\theta,S)=C(S)-\frac{1-F(\theta)}{f(\theta)}C'(S)\quad\forall(S,\theta). \tag{4.9}$$

This expression accounts for the extra cost of information rent that has to be given to the firm to induce information revelation on the value of the stock. As a result, the *virtual marginal cost of extraction* is always above the true marginal cost of extraction:

$$\tilde{C}(\theta,S)\geq C(S)\quad\forall(S,\theta)$$

with an equality only for the highest possible type $\overline{\theta}$. Everything happens as if this increase in the cost of extraction which is induced by asymmetric information was akin to introducing a delay in extraction. This points at the core dilemma between extraction information rent and extracting resource.

The *virtual marginal cost of extraction* is increasing and convex in S under the assumptions made earlier on for C. Importantly, the marginal cost of extraction can also be ranked with respect to the initial stock θ since

$$\frac{\partial\tilde{C}}{\partial\theta}(\theta,S)\leq 0\quad\forall(S,\theta).$$

A type with a higher initial stock has thus a lower *virtual marginal cost of extraction* and is more willing to extract, at any level of stock along the trajectory.

Importantly for the dynamics of the economy, the cost of extraction at any given date now depends not only on the current stock but also on the initial stock. In other words, firms become heterogenous in terms of their costs of extraction while under symmetric information they all had the same technology and just differed by their initial stocks. As we will see below, the consequence of this heterogeneity induced by asymmetric information is that the dynamic paths of extraction now differ across types.

To stress those dynamics, we may for the time being just observe that firms differ in terms of the minimal level of resource beyond which extraction ceases to be valuable. Following on the steps of the complete information model, we define $\widetilde{S}(\theta)$ as the level of resource for a firm with type θ such that the marginal cost of extraction at this level is equal to the choke price:

$$P(0) = \tilde{C}(\theta, \tilde{S}(\theta)). \tag{4.10}$$

Thanks to the *Monotone Hazard Rate Property*, firms with different types are unambiguously ranked according to their limiting stock $\tilde{S}(\theta)$ as demonstrated in next Lemma.

Lemma 2. *$\tilde{S}(\theta)$ is a decreasing function of θ with $\tilde{S}(\overline{\theta}) = S^*$.*

We are now ready to state the characterization of the optimal trajectories under asymmetric information. Because virtual marginal costs of extraction now play the same role as true marginal costs of extraction under complete information, next proposition bears some strong resemblance with Proposition 1.

Proposition 2. *Under asymmetric information and when $\theta > \tilde{S}(\theta)$, the optimal path of resource extraction solving problem $(\mathcal{P}^{sb}(\theta))$ converges towards no extraction and a finite stock of resource in infinite time but this limiting level now depends on the initial stock:*

$$\lim_{t \to +\infty} q^{sb}(\theta, t) = 0 \text{ and } \lim_{t \to +\infty} S^{sb}(\theta, t) = \tilde{S}(\theta). \tag{4.11}$$

The price of resource $p^{sb}(\theta, t) = V'(q^{sb}(\theta, t))$ now evolves according to the Virtual Hotelling Rule:

$$\frac{\partial p^{sb}}{\partial t}(\theta, t) = r(p^{sb}(\theta, t) - \tilde{C}(\theta, S^{sb}(\theta, t))). \tag{4.12}$$

When $\theta \leq \tilde{S}(\theta)$, there is no exploitation of the resource at any point in time.

For a given initial stock of resource, the dynamics looks quite similar to the case of complete information with the *proviso* that the long-run limit of the stock remains at a higher level. To understand this effect, it is important to come back on (4.5). This formula shows that a type-θ firm enjoys some information rent by extracting the same amount as a firm with a lower initial stock but doing so at a lower marginal cost. This rent is reduced when the regulator calls for lower quantities. Asking for less extraction by a given type θ indeed reduces the information rent of all supramarginal types $\theta' \geq \theta$. In other words, doing as if more of the resource had already been exploited by a given type and pushing froward the extraction path for that type helps to extract rent from types endowed with more resource. The second-best path of extraction exhibits a trade-off between implementing ambitious extraction paths that come close to efficiency and, to do so, leaving excessive information rent to the firm. Extracting more of this information rent calls for extracting less of the resource.

Although similar to the complete information dynamics, the outcome under asymmetric information highlights a strong inefficiency that takes the form of insufficient extraction. To illustrate, remember that the long-run limit $\tilde{S}(\theta)$ remains above S^*. In other words, the regulator chooses to leave gains from trade *in the ground*. It would be optimal under complete information to keep on extracting but this limited extraction acts under asymmetric information as a commitment device to induce information revelation by making less attractive the pattern of extraction for concessions that claim a limited stock of resources, and thus high cost of extraction, to start with.

The most spectacular expression of the inefficient extraction are for those initial stocks, if they exist such that

$$\theta \leq \tilde{S}(\theta).$$

Because $\tilde{S}$ is decreasing, this corresponds to a (may be empty) lower tail of the types distribution. Those types are such that:

$$\tilde{C}(\theta,\theta) \geq \tilde{C}(\theta,\tilde{S}(\theta)) = P(0) = C(S^*) > C(\theta).$$

In other words, for those types, producing even the first unit is inefficient under asymmetric information while it is obviously efficient under complete information. Extraction does not even start for those fields. Everything happens as if the regulator was putting aside the whole field if it is of too small a size. Doing so helps to extract more rents if the firm was operating on bigger fields.

VIRTUAL HOTELLING RULE. To better understand the *Virtual Hotelling Rule*, it is again useful to see what are the main trade-offs involved when considering a slight modification of the production plan. These computations must now take into account that any such modification bears on a subset of types and has consequences on the information rent of supra-marginal types. Consider producing dq more units beyond $q^{sb}(\theta,t)$ for a length of time dt after date t and for a subset of types of mass $f(\theta)d\theta$ around θ. Such change increases overall surplus at date t roughly by

$$\left(V'(q^{sb}(\theta,t)) - C(S^{sb}(\theta,t))\right) f(\theta)d\theta dqdt. \tag{4.13}$$

where $S^{sb}(\theta,t)$ is the current stock of resources at a date t for a type θ. However, consuming those extra units dq reduces the stock of resources from date $t+dt$ onwards for all firms around θ to approximatively $S^{sb}(\theta,\tau)-q^{sb}(\theta,t)dt+dqdt$. Because of discounting, the increase in welfare from date $t+dt$ onwards is worth

$$\approx \left(\int_t^{+\infty} C'(S^{sb}(\theta,\tau))q^{sb}(\theta,\tau)exp(-r(\tau-t))d\tau\right) f(\theta)d\theta dqdt. \tag{4.14}$$

Under asymmetric information, such change in production plan for all types around type θ has also an impact on the information rent of all supramarginal types $\theta' \geq \theta$ whose mass is $1-F(\theta)$. Increasing by dq the production of all types around θ increases the slope of the information rent around that point. The overall incremental rent from date t for the mass of supramarginal types is roughly:

$$= (1-F(\theta))\left(-\int_t^{+\infty} C'(S^{sb}(\theta,t))exp(-r(\tau-t))d\tau\right) dqdtd\theta. \tag{4.15}$$

At the second-best optimum, the regulator should not find such modification of the production plan attractive which requires that (4.13) just compensate for the shadow costs of increased extraction costs (4.14) and increased information rents (4.15):

$$V'(q^{sb}(\theta,t))-C(S^{sb}(\theta,t)) = \underbrace{-\left(\int_t^{+\infty} C'(S^{sb}(\theta,\tau))q^{sb}(\theta,\tau)exp(-r(\tau-t))d\tau\right)}_{\text{Shadow cost associated to increased future cost of extraction}}$$

$$\underbrace{-\frac{1-F(\theta)}{f(\theta)}\int_t^{+\infty} C'(S^{sb}(\theta,t))exp(-r(\tau-t))d\tau}_{\text{Shadow cost of increased information rent}}$$

Differentiating with respect to t immediately gives us the *Virtual Hotelling Rule* (4.12).

Running Example (continued). Observe first that

$$\tilde{S}(\theta) = S^* + \frac{1-F(\theta)}{f(\theta)}. \tag{4.16}$$

Closed forms solutions for the optimal trajectories under asymmetric information are now readily obtained as:

$$S^{sb}(\theta,t) - \tilde{S}(\theta) = (\theta - \tilde{S}(\theta))exp\left(-\frac{t}{z^*}\right) \tag{4.17}$$

and

$$q^{sb}(\theta,t) = \frac{\theta - \tilde{S}(\theta)}{z^*}exp\left(-\frac{t}{z^*}\right). \tag{4.18}$$

The stock of resource and the quantity extracted decreases towards their long-run limit at the same exponential rate than under complete information. Asymmetric information has an impact on total extraction but not on its pace.

To ensure that output remains positive under all circumstances, we shall assume that:

$$\underline{\theta} - S^* \geq \frac{1}{f(\underline{\theta})}. \tag{4.19}$$

When this condition does not all, the second-best production is zero at any point in time.

These formula are useful to immediately check the sufficient condition for implementability (4.7) since the total quantity extracted

$$Q^{sb}(\theta,t) = (\theta - \tilde{S}(\theta))\left(1 - exp\left(-\frac{t}{z^*}\right)\right) \tag{4.20}$$

is increasing in θ when $\tilde{S}(\theta)$ is decreasing as proved in Lemma 2. ■

4.3. Comparative Statics

The comparison of (3.5) and (4.17) highlights an important inefficiency. There is less long-run extraction of resource under asymmetric information. This conclusion is somewhat reminiscent of more standard screening models that show, over a broad range of contexts, that volumes of activities are lower to reduce information rent. The second and novel feature, more specific to a dynamic model like ours, which is highlighted by this running example is that, at any point in time, the quantity extracted is also lower under asymmetric information. Interestingly, those results hold over a broader ranges of functional forms.

PROPOSITION 3. *Under asymmetric information, the level of resource and the quantity extracted remains always smaller than under complete information at any point in time:*

$$S^{sb}(\theta,t) \leq S^*(\theta,t) \quad \forall t \geq 0 \text{ (with equality only at } t=0\text{)}; \tag{4.21}$$

$$q^{sb}(\theta,t) < q^*(\theta,t) \quad \forall t \geq 0. \tag{4.22}$$

The path of resource extraction under asymmetric information somewhat resembles that obtained with an unregulated monopoly. Here as well, resource extraction is limited and the stock remains also above its optimal value. Although similar, those patterns also exhibit striking differences. An unregulated monopoly drives prices above marginal cost of extraction at any point in time because it cares about marginal revenue and not marginal surplus, a wedge that keeps extraction low but depends only on the inverse demand function. Instead, the distortions found under asymmetric information are only driven by cost considerations. The virtual marginal cost of extraction being greater than the true value, prices are again above marginal cost of extraction; a first difference with complete information. As a consequence of this increase in cost, the long-run amount of resource remains above its complete information value while the unregulated monopolist was able to reach the same limit as a competitive market; a second significant difference.

Next proposition compares extraction paths across firms with different initial stocks. It shows in passing that the sufficient condition (4.7) is satisfied by the solution to the relaxed problem ($\mathcal{P}^{sb}(\theta)$). This validates our approach of focusing on this relaxed problem in the first place.

PROPOSITION 4. *The quantity extracted $Q^{sb}(\theta,t) = \theta - S^{sb}(\theta,t)$ under asymmetric information is increasing in θ:*

$$\frac{\partial Q^{sb}}{\partial \theta}(\theta,t) > 0. \tag{4.23}$$

4.4. *Implementation*

We have so far left apart the question of what sort of instruments can be used to achieve the second-best patterns highlighted in Proposition 3. We now turn to this issue. Before entering into the details of the analysis, it is worth stressing that the instruments that can be used for implementation have to be consistent with the information structure. For instance, profit-sharing arrangements or cost-reimbursement rules presuppose that costs are observable but, in our context this would mean that the initial resource stock of the firm would be, in sharp contrast with our assumptions. Hence, the only instruments that can be used in our setting can be contingent on production. Among the relevant class, we will bear particular attention to royalty payments.

The formulas (4.17) and (4.20) show that, for the case of our RUNNING EXAMPLE, the optimal extraction and resource levels are separable in time and type. Accordingly, let denote by $q_0^{sb}(\theta) = \frac{\theta - \tilde{S}(\theta)}{z^*}$ the optimal output at date $t=0$. Observe that this output contains all information on the firm's initial stock θ, and that $q_0^{sb}(\theta)$ is strictly increasing with θ because $\tilde{S}(\theta)$ is itself decreasing. Let denote by $\vartheta_0(q)$ the corresponding inverse function. Starting from this initial value of output, output declines over time at an exponential rate; $q^{sb}(\theta,t) = q_0^{sb}(\theta)exp\left(-\frac{t}{z^*}\right)$ and the total amount extracted also increases at such rate as $Q^{sb}(\theta,t) = z^*q_0^{sb}(\theta)\left(1-exp\left(-\frac{t}{z^*}\right)\right)$.

By picking a particular item within the menu of output and payment profiles available within the direct revelation mechanism $\{q(\hat{\theta},t);\omega(\hat{\theta},t)\}_{\hat{\theta}\in\Theta}$, the firm determines the whole trajectory of the economy once for all. The separability stressed above suggests that this choice is akin to picking only an initial output. In other words, the dynamic contracting model can be somehow reduced to a static problem. To illustrate, suppose that the firm is offered a nonlinear payment schedule $T(q_0)$ that depends on the starting level of extraction q_0 and that stipulates the firm's upfront payment for his services. Any departure from the trajectory $q_0exp\left(-\frac{t}{z^*}\right)$ could be heavily punished with penalties for non-provision of the right production at any point in time if the firm were to deviate from this extraction path. Formally, $q_0^{sb}(\theta) = \frac{\theta-\tilde{S}(\theta)}{z^*}$ should thus solve:

$$q_0^{sb}(\theta) = \arg\max_{q_0\geq 0} T(q_0) - \mathcal{C}(\theta,q_0) \tag{4.24}$$

where

$$\mathcal{C}(\theta,q_0) = \int_0^{+\infty} C\left(\theta - z^*q_0\left(1-exp\left(-\frac{t}{z^*}\right)\right)\right)q_0exp\left(-\frac{t}{z^*}\right)exp(-rt)dt.$$

denotes the intertemporal cost associated to a path of extraction that has starting value q_0 and initial stock θ.

Notice that the regulator's optimal problem $(\mathcal{P}^{sb}(\theta))$ is also akin to choosing that initial value $q_0^{sb}(\theta)$ so as to now maximize the virtual surplus of the allocation, taking again into account the exponential decay of output and stock that are specific to our functional forms. Formally, we know that, by construction of the optimal path under asymmetric information, $q_0^{sb}(\theta) = \frac{\theta-\tilde{S}(\theta)}{z^*}$ must also solve:

$$q_0^{sb}(\theta) = \arg\max_{q_0\geq 0} \mathcal{V}(q_0) - \mathcal{C}(\theta,q_0) + \frac{1-F(\theta)}{f(\theta)}\frac{\partial\mathcal{C}}{\partial\theta}(\theta,q_0) \tag{4.25}$$

where the intertemporal consumer's surplus associated to a path of extraction with starting value q_0 is defined as

$$\mathcal{V}(q_0) = \int_0^{+\infty} V\left(q_0exp\left(-\frac{t}{z^*}\right)\right)exp(-rt)dt.$$

The nonlinear payment schedule $T(q_0)$ implements the optimal choice $q_0^{sb}(\theta)$ when Problems (4.24) and (4.25) have the same solutions. Next proposition unveils the property of this schedule.

PROPOSITION 5. *Assume that functional forms are given as in our* RUNNING EXAMPLE. *The optimal choice $q_0^{sb}(\theta)$ is implemented by the following nonlinear schedule $T(q_0)$:*

$$T(q_0) = \mathcal{V}(q_0) - \mathcal{V}(q_0^{sb}(\underline{\theta})) + \mathcal{C}(\underline{\theta},q_0^{sb}(\underline{\theta})) + \frac{C'(S^*)}{r+\frac{1}{z^*}}\int_{q_0^{sb}(\underline{\theta})}^{q_0}\frac{1-F(\vartheta_0(\tilde{q}))}{f(\vartheta_0(\tilde{q}))}d\tilde{q}. \tag{4.26}$$

The marginal payment $T'(q_0)$ is non-negative,[29] always lower than the marginal social value $\mathcal{V}'(q_0)$ so as to induce the firm to choose low extraction paths which are attractive as a means to reduce information rent. The difference

$$\mathcal{V}'(q_0) - T'(q_0) = -\frac{C'(S^*)}{r + \frac{1}{z^*}} \frac{1 - F(\vartheta_0(q_0))}{f(\vartheta_0(q_0))} \geq 0$$

represents the corresponding discount. This discount is decreasing when the *Monotone Hazard Rate Property* holds, non-negative and worth zero only at $q_0(\overline{\theta})$ so as the firm's incentives to produce are aligned with the first-best at that point.

As the firm chooses a more ambitious extraction path and reveal by doing so that it has a larger stock, its marginal incentives to produce are more aligned with the socially optimal ones. The extraction paths for such firm come close to efficiency. The counterpart is that such a firm under high-powered incentives also gets a very large and costly information rent. A firm that instead chooses a modest extraction path and reveals by doing so that its initial stock is somewhat limited evolves under low-powered incentives, produces much below efficiency but gets by doing so very little information rent.

RUNNING EXAMPLE (CONTINUED). To get more precise results, we now specialize these findings in the framework of our linear functional forms but we now also assume that θ is uniformly distributed on $\left[\overline{\theta} - 1, \overline{\theta}\right]$ with $\overline{\theta} > S^* + 2$ to ensure that the second-best output is positive for all θ. We immediately derive that $\tilde{S}(\theta) = S^* + \overline{\theta} - \theta$, $q_0^{sb}(\theta) = \frac{2\theta - \overline{\theta} - S^*}{z^*}$ and $\vartheta_0(q_0) = \frac{1}{2}(\overline{\theta} - S^* + z^* q_0)$. Inserting into the above expression gives us:

$$T'(q_0) = \frac{P(0) + \frac{C'(S^*)}{2}(\overline{\theta} + S^*)}{r + \frac{1}{z^*}} + \frac{P'(0)}{2(2 + rz^*)}\left(z^* - \frac{2}{r}\right) q_0$$

and

$$T''(q_0) = \frac{P'(0)}{2(2 + rz^*)}\left(z^* - \frac{2}{r}\right).$$

Observe that the nonlinear payment schedule $T(q_0)$ is convex only when $z^* < \frac{2}{r}$ or $C'(S^*) < \frac{3}{4} r P'(0)$, i.e., when the marginal cost of extraction decreases sufficiently quickly with the level of remaining resource in comparison with inverse demand.

In practice, we might expect this condition to hold given that it is easier to satisfy when r is small. When $T(q_0)$ is thus convex, we may replace this nonlinear schedule with the menu of its tangents. Those tangents are of the form:

$$\tilde{T}(q, q_0) = T(q_0) + T'(q_0)(q - q_0) \tag{4.27}$$

Indeed, convexity of T ensures that the maximization problem of a firm facing such menus of linear schemes, namely

$$\max_{q, q_0} \tilde{T}(q, q_0) - \mathcal{C}(\theta, q).$$

is globally concave.[30] The contract (4.27) can then be readily interpreted as a menu of license fees and royalties. The firm chooses an output target q_0 at date 0, pays a license

[29] See the Appendix.

[30] This argument is familiar from the incentive regulation literature (Laffont and Tirole (1993, Chapter 1) for instance) but it was developed there in a static framework. Osmudsen (1998) in his two-period

fee $q_0T'(q_0)-T(q_0)$ and receives an royalty $T'(q_0)$ (resp. a penalty) for any unit of output it produces. This implementation thus echoes real-world instruments observed in practice. ■

5. From a Single Firm to a Market of Concessionaires

So far, our analysis has supposed that the regulator contracts with a single firm whose initial stock of resource is private information. The implication of this is that the market price follows from matching demand with the particular supply of that firm according to its virtual cost of extraction. As a result, there was not a single price for the resource but a collection of such prices, one for each possible realization of the initial stock. However, in practice, we may expect that the regulator is involved into many simultaneous relationships with different concessionnaires producing on different fields and only their aggregate supply meets demand. We thus need to adapt our previous findings to the case of a market where a continuum of contractors operate.[31] Te paths of resource extraction and how production is optimally allocated among producers are somewhat different when one moves from the single-firm scenario to a market context, although much of our previous insights carry over.

5.1. Complete Information

With a continuum of firms, the regulator collects individual production from each of them according to their respective cost of extraction. Formally, let $Q^*(t) = \int_{\underline{\theta}}^{\overline{\theta}} q^*(\theta, t) f(\theta) d\theta$ be the overall production at date t from all active firms at this date ($q^*(\theta, t) > 0$ for such firms) and let $p^*(t)$ be the market price. Optimality of the regulator's plan of production certainly requires that aggregate supply meets demand at the market price so that the consumers' marginal benefit of an extra unit of resource consumed is equal to the shadow cost of the aggregate feasibility constraint above, i.e., the price of the resource:

$$p^*(t) = P(Q^*(t)) \quad \forall t \geq 0. \tag{5.1}$$

Following Herfindahl (1967)'s important insight, production units with the lowest costs are first mobilized. Given the fact that the marginal cost of extraction increases with depletion, the model generates an interesting dynamics for the last unit involved. For future reference, we thus define $\sigma^*(t)$ as the lowest level of resource currently extracted at date t. This characterizes the stock of the "marginal" firm , i.e. that firm with the lowest level of stock that starts being active at any date t. When all firms are already active, a case that arise when the price has sufficiently raised to justify that even the firm endowed with the minimal stock $\underline{\theta}$ has started extraction, we may use the convention that $\sigma^*(t) = S^*(\overline{\theta}, t)$.

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setting does not check for the global concavity of the firm's maximization problem when the optimal nonlinear schedule is replaced with the menu of its tangents. Global concavity requires convexity of T which is not always guaranteed.

[31] Screening models do not make much differences between those two scenarios. To illustrate, the optimal nonlinear pricing of a monopolist would be the same whether he faces a single consumer with private information or a continuum of consumers, each with his own preference parameters as long as the distributions of types remain the same in both scenarios and the monopolists offer an anonymous nonlinear price.

Equipped with these notations, we can define the global path of the economy under complete information as follows.

PROPOSITION 6. *Under complete information, the optimal path of resource extraction with a population of firms converges in infinite time towards no extraction and the same stock of resource S^* for all firms:*

$$\lim_{t\to+\infty} q^*(\theta,t) = 0 \text{ and } \lim_{t\to+\infty} S^*(\theta,t) = S^*. \tag{5.2}$$

All firms active at a given date t produces the same amount:

$$q^*(\theta,t) = -\dot{\sigma}^*(t) \quad \forall\theta \geq \sigma^*(t). \tag{5.3}$$

The price of resource $p^*(\theta,t)$ evolves according to the following Hotelling Rule:

$$\dot{p}^*(t) = r(p^*(t) - C(\sigma^*(t))) \tag{5.4}$$

where the common level of resource extracted at date t, $\sigma^*(t)$, satisfies $\sigma^*(0) = \overline{\theta}$, $\lim_{t\to+\infty}\sigma^*(t) = S^* \leq \underline{\theta}$ and

$$p^*(t) = P(-\dot{\sigma}^*(t)(1 - F(\sigma^*(t))). \tag{5.5}$$

The market price $p^*(t)$ is determined by applying *Hotelling rule* for a cost function which is that of the last production unit to start extraction at date t. In a market context, all the production units active at a given date face the same cost of extraction. Because those firms have the same cost of extraction, they thus also have the same remaining stock, extract the same amount at any future date and evolve along similar paths. In particular, the remaining stock of those firms converges towards S^* just as in the case of a single firm. This finding reiterates in a market context our previous observation that under complete information, firms follow the same path of extraction. The difference with the single-firm scenario is that the set of active units corresponds to an upper tail of the firms distribution, those such that $\theta \geq \sigma(t)$. This explains the term $1 - F(\sigma^*(t))$ in the market clearing condition (5.5).

Importantly in view of the results to come in Section 5.2 below, all firms end up being active at some point in time. Again, this result captures the idea that an efficient path of resource extraction should not leave any resource in the ground.

5.2. *Asymmetric Information*

Let us now turn to the scenario where the firms have private information on their stocks. The regulator offers a similar contract to all those firms and thus incentive compatibility and participation constraints are expressed as before. Of course, and as in Section 5.1 above, the fact that there is now a continuum of firms ready to supply has consequences on how the price is fixed on this market.

To see how, we may again define $\sigma^{sb}(t)$ as the lowest level of resource currently extracted at date t. Let also denote by $p^{sb}(t)$ the market price. Under asymmetric information, the allocation of aggregate production across firms does not depend on their respective cost of extraction but instead of their *virtual costs of extraction*. Constrained efficiency under asymmetric information now requires that the regulator asks firms to supply resources according to their virtual marginal cost of extraction. All those virtual marginal costs must be identical for all active firms, even though they started with different resources, and equal to that of the latest firm that becomes active. In other words, the resource in the production unit that starts from an initial level worth θ and is active at date t must be such that

$$\tilde{C}(\theta, S^{sb}(\theta,t)) = \tilde{C}(\sigma^{sb}(t), \sigma^{sb}(t)) \Leftrightarrow S^{sb}(\theta,t) = \Phi(\theta, \sigma^{sb}(t)) \tag{5.6}$$

where Φ is implicitly defined as the solution to

$$\tilde{C}(\theta, \Phi(\theta,\sigma)) = \tilde{C}(\sigma,\sigma).$$

Importantly, because firms with different initial stocks also differ in terms of virtual marginal costs of extraction, they keep different stocks at different points in time and extract different amounts. Various paths of extraction are thus compatible with optimal supply in this market context.

PROPOSITION 7. *Under asymmetric information, the optimal path of resource extraction with a population of firms converges in infinite time towards no extraction and a finite stock of resource $\tilde{S}(\theta)$ for a firm with initial stock θ:*

$$\lim_{t\to+\infty} q^{sb}(\theta,t) = 0 \text{ and } \lim_{t\to+\infty} S^{sb}(\theta,t) = \tilde{S}(\theta). \tag{5.7}$$

All firms active at a given date t, i.e., $\theta \geq \sigma^{sb}(t)$, produce different quantities:

$$q^{sb}(\theta,t) = -\dot{\sigma}^{sb}(t)\frac{\partial \Phi}{\partial \sigma}(\theta, \sigma^{sb}(t)) \quad \forall \theta \geq \sigma^{sb}(t). \tag{5.8}$$

The price of resource $p^{sb}(\theta,t)$ evolves according to the following Virtual Hotelling Rule:

$$\dot{p}^{sb}(t) = r(p^{sb}(t) - \tilde{C}(\sigma^{sb}(t), \sigma^{sb}(t))) \tag{5.9}$$

where the minimal level of resource extracted at date t, $\sigma^{sb}(t)$, satisfies $\sigma^{sb}(0) = \overline{\theta}$, $\lim_{t\to+\infty}\sigma^{sb}(t) = \tilde{S}^* > S^*$ such that

$$\tilde{C}(\tilde{S}^*, \tilde{S}^*) = P(0) \tag{5.10}$$

and

$$p^{sb}(t) = P\left(-\dot{\sigma}^{sb}(t)\int_{\sigma^{sb}(t)}^{\overline{\theta}} \frac{\partial \Phi}{\partial \sigma}(\theta, \sigma^{sb}(t)) f(\theta) d\theta\right). \tag{5.11}$$

At the aggregate level, the dynamics of resource extraction and prices resembles those found in Section 4 for the case of a single firm. The dynamics are also somewhat similar to that found in the complete information scenario of Section 5.1; it is again fully characterized by the minimal value of the stock that is extracted at that date. Contrary to the complete information scenario, that minimal surplus now converges over time towards a limit $\tilde{S}^*$ but this limit remains above S^*, meaning that inefficient extraction remains at the aggregate level.

As under complete information, the set of active firms at any given date remains an upper tail of the types distribution but all those firms produce different amounts at any point in time, in sharp contrast with complete information contexts. Indeed, because active firms produce according to their virtual marginal costs of production, the latest to become active as the price increases will keep on producing from a higher remaining stock and extract less than firms with lower stocks that start to become active from that date on.

RUNNING EXAMPLE (CONTINUED). The comparison of the price trajectories under complete and asymmetric information is significantly more complex than in the case of a single firm because it entails a detailed description of supply at any point in time and this aggregate supply depends itself on the price level. In order to nevertheless get some results in that direction, we specialize the analysis in the case of linear functional forms of our RUNNING EXAMPLE. On top of the linearity of inverse demand and marginal cost, we now also assume that θ is uniformly distributed on $[S^*, S^*+1]$. While $\sigma^*(t)$ converges as t goes to $+\infty$ towards $\tilde{S}^*$, $\sigma^{sb}(t)$ converges towards the greater value $\tilde{S}^* = S^* + \frac{1}{2}$. Half of the resource that could have been extracted under complete information thus remains in the ground under asymmetric information. Yet, both prices $p^*(t)$ and $p^{sb}(t)$ converge towards the choke price $P(0)$ with less and less extraction over time under both scenarios.

Next proposition nevertheless ranks those prices in terms of their asymptotic properties under the assumptions of our RUNNING EXAMPLE.

PROPOSITION 8. *The price $p^{sb}(t)$ under asymmetric information converges faster to $P(0)$ than the price $p^*(t)$ under complete information:*

$$p^{sb}(t) - P(0) \sim_{t\to+\infty} \frac{P'(0)}{z^{sb}} exp\left(-\frac{t}{z^{sb}} + t\eta^{sb}(t)\right)$$

while

$$p^*(t) - P(0) \sim_{t\to+\infty} \frac{P'(0)}{z^*} exp\left(-\frac{t}{z^*} + t\eta^*(t)\right)$$

where $z^{sb} = \frac{1}{4\kappa^*}\left(1+\sqrt{1+\frac{8\kappa^*}{r}}\right) < z^*$ *and* $\lim_{t\to+\infty}\eta^{sb}(t) = \lim_{t\to+\infty}\eta^*(t) = 0$.

Thus, the price on a market converges faster under asymmetric information than what one would expect under complete information. What is remarkable here is that, at the individual level, i.e., when a single firm is contracted on, the price dynamics under asymmetric information and under complete information exhibits similar patterns. Bringing back our previous findings, we got for the price dynamics under both scenario:

$$p^{sb}(\theta,t) = P(0) + P'(0)\max\left\{\frac{2\theta - 2S^* - 1}{z^*}; 0\right\} exp\left(-\frac{t}{z^*}\right)$$

while

$$p^*(\theta,t) = P(0) + P'(0)\frac{\theta - S^*}{z^*} exp\left(-\frac{t}{z^*}\right).$$

Yet, the subtlety is that those patterns are similar provided that $\theta \geq S^* + \frac{1}{2}$ so that extraction starts at all. Under asymmetric information, however, the smallest fields $\theta \in \left[S^*, S^* + \frac{1}{2}\right]$ do not even start extraction. At the aggregate level, this supply contracting which is induced by asymmetric information, raises the price very quickly towards its long run limit. Only firms with initial stocks $\theta \geq S^* + \frac{1}{2}$ start extraction and, even though they do extract over the whole time horizon, the price has in the mean time quickly converged towards its long-run limit.

This important finding somewhat reconciles the *Hotelling Rule* with empirical evidence that has repeatedly shown that prices tend to be more stable than what the *Rule* predicts.[32] Here, stability follows from the faster convergence of prices towards their long-run limit $P(0)$. From an empirical viewpoint, regressing $\log(p^{sb}(t) - P(0))$ on time should, according to the model, give a coefficient which is of greater magnitude than its complete information counterpart plus a residual that converges towards zero slowly other time. We leave to further research the empirical validation of such prediction. ■

6. CONCLUSION

We have analyzed optimal contracts for resource extraction in a context of asymmetric information. This analysis has stressed the fundamental dilemma between extracting information and extracting resource. Reducing the information rent of contractors who hold private information on the capacities of their fields requires diminishing extraction and leaving resource in the ground. We have derived a *Virtual Hotelling Rule* that characterizes the dynamics of resource extraction in such scenarios. Asymmetric information requires to replace the marginal cost of extraction by a *virtual marginal cost of extraction* that is greater, which explains the inefficient amount of extraction. Although fields of different initial values would evolve along the same extraction paths under complete information, these paths significantly differ under asymmetric information with fields not lesser magnitude being less extracted.

From an implementation viewpoint, we showed that the optimal contract for a concessionaire may be (but not always) implemented with a menu of license contracts. Contractors who operate on fields with larger stocks receive a higher royalty per unit of output but must also pay upfront a greater license fee to have the right to explore the resource.

Finally, we have generalized our model to the case where a whole market of concessionnaires is available. Under asymmetric information, constrained efficiency requires that all active fields at a given point in time operate at the same *virtual marginal cost of extraction*. Since those costs differ across firms with different initial stocks, all active firms are not producing the same amount even though their technologies for extraction are identical. We provide the *Virtual Hotelling Rule* in that environment and demonstrate that the market price converges faster to its long run limit under asymmetric information. This gives us an important empirical check to assess the validity of our approach in future research.

Beyond this important empirical aspects, our model also offers a building block that could certainly be extended into various directions that might impact on the magnitude of the rent/efficiency trade-off highlighted in our analysis.

[32]See Gaudet (2007) for a very nice account of the criticisms of this *Rule* but also of their own limits.

ON LIMITED COMMITMENT, RENEGOTIATION AND EXPROPRIATION. Of course, the optimal contract found above requires credibility on the side of the regulator; a standard blessing in the theory of dynamic contracts.[33] Once information is revealed, which arises immediately when the firm chooses among the various options offered by the incentive menu, the path of extraction $S^{sb}(\theta, t)$ is certainly no longer optimal and the regulator would like to switch to the complete information path $S^{*}(\theta, t)$. Anticipating such move, the firm may be reluctant to report the initial stock of resource in the first place. We leave the analysis of those complex dynamics under limited commitment for future research.[34]

ENFORCEMENT AND WEAK INSTITUTIONAL ENVIRONMENTS. Renegotiation and limited enforcement of contractual provisions may be particularly important in the context of developing countries which delegate to foreign company the management of resource. In weak institutional environments with insecure property rights, the threat of expropriation and hold-up is likely to induce contractors to extract resource early on.[35] This effect thus introduces countervailing incentives[36] that go counter the incentives to downplay the size of the stock. These countervailing incentives may thus limit the inefficient extraction stressed in our full scenario environment.

COMMON POOL. As rightly pointed out by Gaudet (2007), countervailing incentives are also likely in the context of common pool extraction. When various contractors draw from the same field, they certainly tend to over-harvest.[37] This effect is likely to be reinforced when contractors operating on the same fields have learned private information on the value of the pool. A phenomenon akin to the standard *"winner's curse"* of auction theory is likely to hold in this context. This effect would tend to push extraction forward.

EXPLORATION AND LEARNING. The discussion above suggests that a relevant information structure in practice is that contractors may have only better signals than public authorities on the value of the field. When it operates alone on a field, a contractor may learn on the stock as it is explored. The process of information gathering is dynamic and linked to the production process. Gathering information pushes to extract early and introduces again some kind of countervailing incentives.

We plan to investigate some of these issues in future research.

REFERENCES

Aghion, P., and L. Quesada (2010), "Petroleum Contracts: What Does Contract Theory Tell Us?" in in *The Natural Resources Trap*, eds. W. Hogan and F. Sturzenegger, MIT Press.

[33] See Dewatripont (1989) and Laffont and Tirole (1993, Chapter 9) among others.

[34] Gaudet, Lasserre and Van Long (1995) analyze a two-period model without commitment and private information on the extraction costs but the assumptions that those costs are drawn independently over time rules out the difficult issues of imperfect information revelation that would arise in our context where private information is persistent.

[35] On this issue, see Aghion and Quesada (2010) and Engel and Fischer (2010) although these authors adopt a reduced-form approach and take both the resource price and the probability of expropriation as given.

[36] Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995).

[37] See Libecap (1989) and Gaudet, Moreaux and Salant (2002) for some analysis.

Armstrong, M. and D. Sappington (2007), "Recent Developments in the Theory of Regulation," in *Handbook of Industrial Organization*, Vol. 3, eds. Mark Armstrong and Robert H. Porter, 1557-1700. Amsterdam: Elsevier, North-Holland.

Auray, S., T. Mariotti and F. Moizeau (2011), "Dynamic Regulation of Quality," *The RAND Journal of Economics,* 42: 246-265.

Bagnoli, M. and T. Bergstrom (2005), "Log-Concave Probability and its Applications," *Economic Theory*, 26: 445-469.

Baron, D. and D. Besanko (1984), "Regulation and Information in a Continuing Relationship," *Information Economics and Policy*, 1: 267-302.

Baron, D. and R. Myerson (1982), "Regulating a Monopolist with Unknown Costs," *Econometrica*, 50: 911-930.

Bergemann, D. and A. Pavan (2015), "Introduction to Symposium on Dynamic Contracts and Mechanism Design," *Journal of Economic Theory* 159: 679-701.

Dewatripont, M. (1989), "Renegotiation and Information Revelation Over Time: The Case of Optimal Labor Contracts," *The Quarterly Journal of Economics,* 104: 589-619.

Engel, E., and R. Fischer (2010), "Optimal Resource Extraction Contracts under Threat of Expropriation," in *The Natural Resources Trap*, eds. W. Hogan and F. Sturzenegger, MIT Press.

Gaudet, G. (2007), " Natural Resource Economics under the Rule of Hotelling," *Canadian Journal of Economics*, 40: 1033-1059.

Gaudet, G., and P. Lasserre (2015), "The Management of Natural Resources Under Asymmetry of Information," *Annual Review of Resource Economics,* 7: 291-308.

Gaudet, G., P. Lasserre and N. Van Long (1995), "Optimal Resource Royalties with Unknown and Temporally Independent Extraction Cost Structures," *International Economic Review,* 36: 715-749.

Gaudet, G., M. Moreaux, and S. Salant(2002), "Private Storage of Common Property," *Journal of Environmental Economics and Management,* 43: 280-302.

Gärtner, D. (2010), "Monopolistic Screening Under Learning By Doing," *The RAND Journal of Economics,* 41: 574-597.

Guesnerie, R. and J.J. Laffont (1984), "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm," *Journal of public Economics,* 25: 329-369.

Herfindahl, O. (1967), "Depletion and Economic Theory," in *Extractive Resources and Taxation,* ed. M. Gaffney (Madison WI: University of Wisconsin Press), 68?90.

Hoel, M. (1978), "Resource Extraction, Uncertainty, and Learning," *The Bell Journal of Economics,* 9: 642-645.

Hung, N., J.C. Poudou and L. Thomas (2006), "Optimal Resource Extraction Contract with Adverse Selection," *Resources Policy,* 31: 78-85.

Laffont, J.J. and D. Martimort (2002), *The Theory of Incentives: The Principal-Agent Model,* Princeton University Press.

Laffont, J.J. and J. Tirole (1993), *A Theory of Incentives in Procurement and Regulation,* MIT Press.

Lewis, T. and H. Yildirim (2002), "Learning By Doing and Dynamic Regulation," *The RAND Journal of Economics,* 33: 22-36.

Lewis, T. and D. Sappington (1989), "Countervailing Incentives in Agency Problems," *Journal of Economic Theory* , 49: 294-313.

Libecap, Gary D. (1989) Contracting for Property Rights (Cambridge: Cambridge University Press)

Loeb, M. and W. Magat (1979), "A Decentralized Method for Utility Regulation," *Journal of Law and Economics,* 22: 399-404.

Maggi, G. and A. Rodriguez-Clare (1995), "On Countervailing Incentives," *Journal of Economic Theory,* 66: 238-263.

Milgrom, P. and Segal I. (2002), "Envelope Theorems for Arbitrary Choice Sets," *Econometrica*, 70: 583-601.

Myerson, R. (1982), "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10: 67-81.

Osmundsen, P. (1998), "Dynamic Taxation of Non-Renewable Natural Resources under Asymmetric Information about Reserves," *Canadian Journal of Economics,* 31: 933-951.

Pavan, A., I. Segal and J. Toikka (2014), "Dynamic Mechanism Design: A Myersonian Approach," *Econometrica*, 82: 601-653.

Pindyck, R. (1978), "The Optimal Exploration and Production of Nonrenewable Resources," *The Journal of Political Economy,* 86: 841-861.

Pindyck, R. (1980), "Uncertainty and Exhaustible Resource Markets," *The Journal of Political Economy,* 88: 1203-1225.

Pindyck, R. (1987), "On Monopoly Power in Extractive Resource Markets," *Journal of Environmental Economics and Management,* 14: 128-142.

Poudou, J. C. and L. Thomas (2000), "Concession minière et asymétrie d'information," *Economie et Prévision,* 143: 129-138.

Rochet, J.-C. (1987), "A Necessary and Sufficient Condition for Rationalizability in a Quasi-Linear Context," *Journal of Mathematical Economics,* 16: 191-200.

Salant, S., M. Eswaran and T. Lewis, (1983), "The Length of Optimal Extraction Programs when Depletion Affects Extraction Costs," *Journal of Economic Theory,* 3: 364-374.

Seierstad, A. and K. Sydsaeter (1987), *Optimal Control Theory with Economic Applications*, North-Holland, Amsterdam.

Solow, R. and F. Wan (1976), "Extraction Costs in the Theory of Exhaustible Resources," *The Bell Journal of Economics,* 7: 359-370.

Stiglitz, J. (1976), "Monopoly and the Rate of Extraction of Exhaustible Resources," *The American Economic Review,* 64: 655-661.

Swierzbinski, J. and R. Mendelsohn (1989), "Exploration and Exhaustible Resources: The Microfoundations of Aggregate Models," *International Economic Review,* 30: 175-186.

Appendix A

Proof of Proposition 1. We slightly modify the statement of problem $(\mathcal{P}^*(\theta))$ by introducing a free-end point T. This device allows us to determine most rigorously that convergence takes place in infinite time. Formally, Problem $(\mathcal{P}^*(\theta))$ can be written as a maximization problem with free-end point to which we apply Pontryagin Principle.[38]

$$(\mathcal{P}^*(\theta)) : \quad \max_{T,q,S} \int_0^T (V(q(\theta,t)) - C(S(\theta,t))q(\theta,t))exp(-rt)dt$$

subject to (2.1) and (2.2).

We write the Hamiltonian for problem $(\mathcal{P}^*(\theta))$ as

$$\mathcal{H}(S,q,t,\lambda) = (V(q) - C(S)q(t))exp(-rt) - \lambda q$$

where λ is the costate variable for (2.2). This Hamiltonian is strictly concave in (q,S). The necessary conditions for optimality are thus also sufficient conditions of the Mangasarian type.[39] We write those conditions as follows.

- *Costate variable:*

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial S}(S^*(\theta,t), q^*(\theta,t), t, \lambda(t))$$

which amounts to

$$\dot{\lambda}(t) = exp(-rt)q^*(\theta,t)C'(S^*(\theta,t)) \tag{A.1}$$

- *Control variable:*

$$q^*(\theta,t) \in \arg\max_q \mathcal{H}(S^*(\theta,t), q, t, \lambda(t))$$

which amounts to

$$exp(-rt)(V'(q^*(\theta,t)) - C(S^*(\theta,t))) - \lambda(t) \leq 0 \text{ with } = 0 \text{ if } q^*(\theta,t) > 0. \tag{A.2}$$

- *Free-end point and transversality condition:*

1. If $T^*(\theta) < +\infty$, we should have:

$$\lambda(T^*(\theta)) = 0 \text{ and } V(q^*(\theta,T^*(\theta))) - C(S^*(\theta,T^*(\theta)))q^*(\theta,T^*(\theta)) = 0. \tag{A.3}$$

2. If $T^*(\theta) = +\infty$, we should have:

$$\lim_{t\to+\infty} \lambda(t) = 0 \text{ and } \lim_{t\to+\infty} exp(-rt)(V(q^*(\theta,t)) - C(S^*(\theta,t)))q^*(\theta,t)) = 0. \tag{A.4}$$

Several facts follow from those optimality conditions.

1. First, observe that differentiating (A.2) with respect to t when $q^*(\theta,t) > 0$ yields:

$$\dot{\lambda}(t) + r\lambda(t) = exp(-rt)\left(V''(q^*(\theta,t))\frac{\partial q^*}{\partial t}(\theta,t) + C'(S^*(\theta,t))q^*(\theta,t)\right). \tag{A.5}$$

[38] For a similar analysis although mostly developed in the case of discrete time, we refer to Salant, Eswaran and Lewis (1983).

[39] Seierstad and Sydsaeter (1987).

Using (A.1) and (A.2) and simplifying yields:

$$V''(q^*(\theta,t))\frac{\partial q^*}{\partial t}(\theta,t) = r(V'(q^*(\theta,t)) - C(S^*(\theta,t))) \tag{A.6}$$

which can be written as (3.2).

2. If $T^*(\theta) < +\infty$ and $q^*(\theta, T^*(\theta)) > 0$, then (A.2) implies:

$$V(q^*(\theta,T^*(\theta))) = q^*(\theta,T^*(\theta))V'(q^*(\theta,T^*(\theta))$$

which requires $q^*(\theta,T^*(\theta)) = 0$; a contradiction. Thus, if $T^*(\theta) < +\infty$, we must have $q^*(\theta,T^*(\theta)) = 0$. Then, (A.2) together with $\lambda(T^*(\theta)) = 0$ (from (A.3) also imply $V'(0) - C(S^*(\theta,T^*(\theta)) \le 0$ and thus $S^*(\theta,T^*(\theta)) \le S^*$. But $S^*(\theta,0) = \theta \ge S^*$, by assumption. Because $S^*(\theta,t)$ is non-increasing, there exists $t_0 \le T^*(\theta)$ such that $S^*(\theta,t_0) = S^*$. Hence, from (A.2) and the fact that $\lambda(t)$ (being non-increasing and zero at $T^*(\theta)$) remains non-negative, we deduce that $q^*(\theta,T^*(\theta)) = 0$ over $[t_0,T^*(\theta)]$. Thus, there is no loss of generality in considering that $T^*(\theta)$ is the lowest time at which $q^*(\theta,t) = 0$. Taking left-side limit of (A.2) at $T^*(\theta)$ immediately gives that $S^*(\theta,T^*(\theta)) = S^*$.

3. For the optimal path of resource extraction, we rewrite (2.2) as

$$\frac{\partial S^*}{\partial t}(\theta,t) = -q^*(\theta,t). \tag{A.7}$$

We also rewrite (A.6) as:

$$\frac{\partial q^*}{\partial t}(\theta,t) = \frac{r}{V''(q^*(\theta,t))}(V'(q^*(\theta,t)) - C(S^*(\theta,t))). \tag{A.8}$$

Observe that the system made of (A.7) and (A.8) admits a unique stationary point $(S^*,0)$. We want to prove that this system together with the initial condition $(S^*(\theta,0),q^*(\theta,0)) = (\theta,q^*(\theta,0))$ converges towards this stationary point in infinite time. If $T^*(\theta) < +\infty$, Item 3. shows that, the stationary point is achieved at $T^*(\theta)$. Suppose that, for $t < T^*(\theta)$ but close enough, $q^*(\theta,t) > 0$ and thus $S^*(\theta,t) > S^*$, then Item 2. shows that (A.6) and thus (A.8) hold. But by Cauchy-Lipschitz Theorem, there is a unique solution to the system made of (A.7) and (A.8) with the initial condition $(S^*(\theta,T^*(\theta)),q^*(\theta,T^*(\theta))) = (S^*,0)$ and this is $(S^*(\theta,t),q^*(\theta,t)) = (S^*,0)$ itself. But this contradicts the initial condition $S^*(\theta,0) = \theta > \underline{\theta} > S^*$. Hence, the system cannot converge in finite time towards $(S^*,0)$ and $q^*(\theta,t)$ remains positive.

We deduce from this last fact that $S^*(\theta,t)$ is decreasing over time. It is bounded below, so it converges. At $t = 0$, we must have from (A.1), $\lambda(t)$ is decreasing and from (A.4), it remains non-negative. From the fact that $q^*(\theta,t)$ remains positive, we also deduce from (A.2) that $V'(q^*(\theta,t)) - C(S^*(\theta,t)) \ge 0$. From (A.8), $q^*(\theta,t)$ is thus also decreasing, and since it is positive, it must converge. Hence, the system made of (A.7) and (A.8) admits a limit, which is necessarily the unique stationary point $(S^*,0)$. Finally, gathering everything the system (A.7)-(A.8) converges towards $(S^*,0)$ in infinite time. Hence, (3.1).

4. For future reference, we propose an alternative description of the solution to the system (A.7)-(A.8). The first step is to notice that (A.8) can be rewritten as:

$$V''(q^*(\theta,t))\frac{\partial q^*}{\partial t}(\theta,t) - rV'(q^*(\theta,t)) = exp(rt)\frac{\partial}{\partial t}\left(V'(q^*(\theta,t))exp(-rt)\right) = -rC(S^*(\theta,t))). \tag{A.9}$$

Integrating once more yields:

$$V'(q^*(\theta,t)) = rexp(rt)\int_t^{+\infty} C(S^*(\theta,\tau))exp(-r\tau)d\tau. \tag{A.10}$$

Now using (A.7), we find that $S^*(\theta,t)$ must solve the following differential equation together with the initial condition $S^*(\theta,0)=\theta$ and the limit behavior (3.1):

$$\frac{\partial S^*}{\partial t}(\theta,t) = -V'^{-1}\left(rexp(rt)\int_t^{+\infty} C(S^*(\theta,\tau))exp(-r\tau)d\tau\right). \tag{A.11}$$

5. Finally, and also for future reference, we also observe that the system (A.7)-(A.8) is autonomous in time. This means that one can express q in terms of S along the optimal trajectory as a simple differential equation. More precisely, taking the ratio between (A.7) and (A.8) (and slightly abusing notations), we obtain that the function $\Omega^*(S)$ so obtained solves the first-order differential equation:

$$\Omega^*(S)V''(\Omega^*(S))\frac{d\Omega^*}{dS}(S) + rV'(\Omega^*(S)) = rC(S) \tag{A.12}$$

where $S\in[S^*,\theta]$.

Defining the elasticity of the inverse demand function as $\varepsilon(q) = -\frac{qP'(q)}{P(q)} = -\frac{qV''(q)}{V'(q)}$, and for φ the inverse function $\int_0^q \varepsilon(\tilde{q})d\tilde{q}$, the solution to (A.12) is of the form:

$$\Omega^*(S) = \varphi(r(S-k^*(S))) \tag{A.13}$$

where $k^*(S)$ solves

$$\frac{dk^*}{dS}(S) = \frac{C(S)}{V'(\varphi(r(S-k^*(S))))} \tag{A.14}$$

with the initial condition $k^*(S^*) = S^*$. (Observe that $\frac{dk^*}{dS}(S^*) = 1$ so that the Cauchy-Lipschitz Theorem applies and such solution is unique.)

Finally, inserting the solution of (A.14) into (A.12) gives that $S^*(\theta,t)$ solves

$$\frac{\partial S^*}{\partial t}(\theta,t) = -\varphi(r(S^*(\theta,t)-k^*(S^*(\theta,t)))) \tag{A.15}$$

with the initial condition $S^*(\theta,0)=\theta$.

6. RUNNING EXAMPLE. Observe that we may rewrite (A.12) as a differential equation for the inverse function $S^*(\theta,q)$, namely:

$$\frac{\partial S^*}{\partial q}(\theta,q) = -\frac{qV''(q)}{r(V'(q)-C(S^*(\theta,q)))} \tag{A.16}$$

For the specific functional forms, this becomes

$$\frac{\partial S^*}{\partial q}(\theta,q) = -\frac{1}{r\left(1-\kappa^*\frac{S^*(\theta,q)-S^*}{q}\right)} \tag{A.17}$$

where $\kappa^* = \frac{C'(S^*)}{P'(0)} > 0$. This is a first-order differential equation of the Clairaut kind. It admits a singular solution (and this is the only one satisfying the terminal condition) such that:

$$S^*(\theta, q) - S^* = z^* q \tag{A.18}$$

for some constant $z^* > 0$. Inserting into (A.17), z^* must satisfy:

$$z^* = -\frac{1}{r\,(1 - \kappa^* z^*)} \tag{A.19}$$

and, selecting the relevant solution to this second-degree equation, we have:

$$z^* = \frac{1}{2\kappa^*}\left(1 + \sqrt{1 + \frac{4\kappa^*}{r}}\right). \tag{A.20}$$

Using (A.18) and inserting into (A.7), we obtain

$$\frac{\partial S^*}{\partial t}(\theta, t) = -\frac{S^*(\theta, t) - S^*}{z^*} \tag{A.21}$$

which can be integrated to obtain (3.5). Inserting then (3.5) into (A.18) again, we find (3.6).

□

PROOF OF LEMMA 1. Absolute continuity follows from an argument in Milgrom and Segal (2003). The *Envelope Condition* for the maximization problem (4.4) then gives us (4.5).

Turning now to sufficiency. Absolute continuity allows us to use the integral representation of $U(\theta)$ as:

$$U(\theta) - U(\hat{\theta}) = -\int_{\theta}^{\hat{\theta}} \left(\int_0^{+\infty} C'(S(\tilde{\theta}, t)) q(\tilde{\theta}, t) exp(-rt) dt \right) d\tilde{\theta}. \tag{A.22}$$

Incentive compatibility in turn requires that for all pairs $(\theta, \hat{\theta})$, the following string of inequalities holds:

$$\begin{aligned} U(\theta) &= \int_0^{+\infty} (\omega(\theta, t) - C(S(\theta, t)) q(\theta, t)) exp(-rt) dt \\ &\geq \int_0^{+\infty} (\omega(\hat{\theta}, t) - C(\hat{S}(\theta, \hat{\theta}, t)) q(\hat{\theta}, t)) exp(-rt) dt \\ &= U(\hat{\theta}) + \int_0^{+\infty} C(\hat{\theta} - Q(\hat{\theta}, t)) q(\hat{\theta}, t) exp(-rt) dt - \int_0^{+\infty} C(\theta - Q(\hat{\theta}, t)) q(\hat{\theta}, t) exp(-rt) dt. \end{aligned}$$

Using (A.22), the later condition holds when:

$$\begin{aligned} \int_0^{+\infty} C(\theta - Q(\hat{\theta}, t)) q(\hat{\theta}, t) exp(-rt) dt - \int_0^{+\infty} C(\hat{\theta} - Q(\hat{\theta}, t)) q(\hat{\theta}, t) exp(-rt) dt \\ \geq -\int_{\theta}^{\hat{\theta}} \left(\int_0^{+\infty} C'(S(\tilde{\theta}, t)) q(\tilde{\theta}, t) exp(-rt) dt \right) d\tilde{\theta}. \end{aligned}$$

This last condition can be rewritten as:

$$(A.23)\quad -\int_{\theta}^{\hat{\theta}}\left(\int_0^{+\infty} C'(\tilde{\theta}-Q(\hat{\theta},t))q(\hat{\theta},t)exp(-rt)dt\right)d\tilde{\theta} \geq -\int_{\theta}^{\hat{\theta}}\left(\int_0^{+\infty} C'(S(\tilde{\theta},t))q(\tilde{\theta},t)exp(-rt)dt\right)d\tilde{\theta}$$

or

$$-\int_{\theta}^{\hat{\theta}}\left(\int_0^{+\infty}\left(C'(\tilde{\theta}-Q(\hat{\theta},t))q(\hat{\theta},t)-C'(\tilde{\theta}-Q(\tilde{\theta},t))q(\tilde{\theta},t)\right)exp(-rt)dt\right)d\tilde{\theta}\geq 0.$$

To prove this inequality, we integrate by parts the left-hand side to obtain:

$$r\int_{\theta}^{\hat{\theta}}\left(\int_0^{+\infty}(C(\tilde{\theta}-Q(\hat{\theta},t))-C(\tilde{\theta}-Q(\tilde{\theta},t)))exp(-rt)dt\right)d\tilde{\theta} = r\int_{\theta}^{\hat{\theta}}\left(\int_0^{+\infty}\left(\int_{Q(\hat{\theta},t)}^{Q(\tilde{\theta},t)} C'(\tilde{\theta}-\tilde{Q})\right)exp(-rt)dt\right)d\tilde{\theta}$$

Hence, we may rewrite (A.23) as:

$$\int_{\theta}^{\hat{\theta}}\left(\int_0^{+\infty}\left(\int_{Q(\hat{\theta},t)}^{Q(\tilde{\theta},t)} C'(\tilde{\theta}-\tilde{Q})d\tilde{Q}\right)exp(-rt)dt\right)d\tilde{\theta}\geq 0,$$

or using Fubini Theorem if (4.6) holds for all pairs $(\theta,\hat{\theta})$. Because C is convex, the latter condition holds provided that the monotonicity condition (4.7) is satisfied. □

Proof of Lemma 2. Differentiating (4.10) with respect to θ yields:

$$\frac{d\tilde{S}}{d\theta}(\theta)=\frac{\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)C'(S(\theta,t))}{C'(\tilde{S}(\theta))-\frac{1-F(\theta)}{f(\theta)}C''(S(\theta,t))}.$$

We conclude that $\frac{d\tilde{S}}{d\theta}(\theta)<0$ using the fact that $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\leq 0$, and $C''\geq 0>C'$. □

Proof of Proposition 2. This is the almost same proof as for Proposition 1 *mutatis mutandis* provided that $C(S)$ and S^* are respectively replaced by $\tilde{C}(\theta,S)$ and $\tilde{S}(\theta)$. Details are thus omitted. The only minor difference comes from Step 3 which has to be rewritten with some care. Indeed, for the optimal path of resource, we may again rewrite (2.2) as

$$(A.24)\quad \frac{\partial S^{sb}}{\partial t}(\theta,t)=-q^{sb}(\theta,t).$$

Replacing with $\tilde{C}(\theta,S)$, Equation (A.25) becomes:

$$(A.25)\quad \frac{\partial q^{sb}}{\partial t}(\theta,t)=\frac{r}{V''(q^{sb}(\theta,t))}(V'(q^{sb}(\theta,t))-\tilde{C}(\theta,S^{sb}(\theta,t))).$$

The system made of (A.24) and (A.25) admits a limit, which is necessarily the unique stationary point $(\tilde{S}(\theta),0)$. If convergence towards that point occurs in finite time $T^{sb}(\theta)<+\infty$, we demonstrate that output is necessarily null along the whole trajectory. Indeed, by Cauchy-Lipschitz

Theorem, there is a unique solution to the system now made of (A.24) and (A.25) with the initial condition $(S^{sb}(\theta, T^{sb}(\theta)), q^{sb}(\theta, T^{sb}(\theta))) = (\tilde{S}(\theta), 0)$ and this is $(S^{sb}(\theta, t), q^{sb}(\theta, t)) = (\tilde{S}(\theta), 0)$ itself. But this contradicts the initial condition $S^{sb}(\theta, 0) = \theta > \underline{\theta}$ if $\theta > \tilde{S}(\theta)$. Hence, the system cannot converge in finite time towards $(\tilde{S}(\theta), 0)$ and $q^{sb}(\theta, t)$ remains positive. Finally, gathering everything the system (A.24)-(A.25) converges towards $(\tilde{S}(\theta), 0)$ in infinite time when $\theta > \tilde{S}(\theta)$.

Suppose instead that $\theta \leq \tilde{S}(\theta)$, or equivalently (because $\tilde{C}(\theta, S)$ is decreasing in S)

$$\tilde{C}(\theta, \theta) \geq \tilde{C}(\theta, \tilde{S}(\theta)) = P(0).$$

Then extraction is not even valuable for the first unit and zero extraction takes place for those lower values of θ. □

PROOF OF PROPOSITION 3. We remind the following definition for $k^*(S)$ from (A.14) and $k^{sb}(\theta, S)$ (obtained *mutatis mudandis* after having noticed that the function φ is relevant in both cases):

$$\frac{dk^*}{dS}(S) = \frac{C(S)}{V'(\varphi(r(S - k^*(S))))} \text{ with } k^*(S^*) = S^* \tag{A.26}$$

and

$$\frac{\partial k^{sb}}{\partial S}(\theta, S) = \frac{\tilde{C}(\theta, S)}{V'(\varphi(r(S - k^{sb}(\theta, S))))} \text{ with } k^{sb}(\theta, \tilde{S}(\theta)) = \tilde{S}(\theta).\text{[40]} \tag{A.27}$$

From this, we construct $S^*(\theta, t)$ and $S^{sb}(\theta, t)$ as solutions to

$$\frac{\partial S^*}{\partial t}(\theta, t) = -\varphi(r(S^*(\theta, t) - k^*(S^*(\theta, t)))) \tag{A.28}$$

and

$$\frac{\partial S^{sb}}{\partial t}(\theta, t) = -\varphi(r(S^{sb}(\theta, t) - k^{sb}(\theta, S^{sb}(\theta, t)))) \tag{A.29}$$

with the same initial condition $S^*(\theta, 0) = S^{sb}(\theta, 0) = \theta$.

LEMMA A.1.

$$k^{sb}(\theta, S) > k^*(S) \quad \forall S \in \left[\tilde{S}(\theta)), \theta\right] \tag{A.30}$$

with equality only for $\theta = \overline{\theta}$ *in which case* $k^{sb}(\overline{\theta}, S) = k^*(S)$ *for all* $S \in \left[S^*, \overline{\theta}\right]$.

PROOF OF LEMMA A.1. Because $\tilde{S}(\theta) \geq S^*$ with equality only at $\theta = \overline{\theta}$, (A.30) certainly holds as an equality at $\theta = \overline{\theta}$ and for all S.

Consider now the case $\theta < \overline{\theta}$. Observe that $\tilde{S}(\theta) > S^*$ implies that $k^{sb}(\theta, \tilde{S}(\theta)) > k^*(\tilde{S}(\theta)) > k^*(S^*) = S^*$ since k^* is increasing. Moreover, by continuity, $k^{sb}(\theta, S) > k^*(S)$ in a right-neighborhood of $\tilde{S}(\theta)$. Suppose also that there exists $S_0 \in \left[\tilde{S}(\theta), \theta\right]$ such that $k^{sb}(\theta, S_0) = k^*(S_0)$, and if there are more than one such values, just denote by S_0 the lowest one. By construction, $S_0 > \tilde{S}(\theta) > S^*$. At S_0, we have:

$$\frac{\partial k^*}{\partial S}(S_0) = \frac{C(S_0)}{V'(\varphi(r(S_0 - k^*(S_0))))} < \frac{\tilde{C}(\theta, S_0)}{V'(\varphi(r(S_0 - k^*(S_0))))}.$$

[40]This boundary condition being now function of θ, we make the dependence of k^{sb} on θ explicit.

Therefore, $k^{sb}(\theta, S) < k^*(S)$ for S in a left-neighborhood of S_0. A contradiction with the definition of S_0. □

From Lemma A.1, (A.31) and (A.29), it follows that:

$$\frac{\partial S^*}{\partial t}(\theta, t) = -q^*(\theta, t) < \frac{\partial S^{sb}}{\partial t}(\theta, t) = -q^{sb}(\theta, t) < 0. \tag{A.31}$$

Hence, (4.22). Taking into account that $S^*(\theta, 0) = S^{sb}(\theta, 0) = \theta$, yields (4.21). □

PROOF OF PROPOSITION 4. As a preliminary, we prove the following Lemma.

LEMMA A.2. *k^{sb} satisfies the following properties:*

1.
$$k^{sb}(\theta, S) \leq S \quad \forall S \in \left[\tilde{S}(\theta), \theta\right].$$

2.
$$\frac{\partial k^{sb}}{\partial S}(\theta, S) \leq 1 \quad \forall S \in \left[\tilde{S}(\theta), \theta\right].$$
with equality at $\tilde{S}(\theta)$ only.

3.
$$\frac{\partial k^{sb}}{\partial \theta}(\theta, S) \leq 0 \quad \forall S \in \left[\tilde{S}(\theta), \theta\right].$$
with equality at $\tilde{S}(\theta)$ only.

PROOF OF LEMMA A.2.

1. Remember that k^{sb} which is defined over the domain $[\tilde{S}(\theta), \theta]$ satisfies (A.27). It follows that $\frac{\partial k^{sb}}{\partial S}(\theta, \tilde{S}(\theta)) = 1$. Differentiating (A.27) with respect to S once yields:

$$\frac{\partial^2 k^{sb}}{\partial S^2}(\theta, S) = \frac{\frac{\partial \tilde{C}}{\partial S}(\theta, S)}{V'(\varphi(r(S - k^{sb}(\theta, S))))}$$
$$-\frac{r\varphi'(r(S - k^{sb}(\theta, S)))V''(\varphi(r(S - k^{sb}(\theta, S))))C(\theta, S)}{(V'(\varphi(r(S - k^{sb}(\theta, S)))))^2}\left(1 - \frac{\partial k^{sb}}{\partial S}(\theta, S)\right)$$

where $\frac{\partial \tilde{C}}{\partial S}(\theta, S) = C'(S) - \frac{1-F(\theta)}{f(\theta)}C''(S) < 0$. From this, we deduce that $\frac{\partial^2 k^{sb}}{\partial S^2}(\theta, \tilde{S}(\theta)) < 1$. So that, in a right neighborhood of $\tilde{S}(\theta)$, $k^{sb}(\theta, S) \leq S$ with equality only at $\tilde{S}(\theta)$.

Suppose now that there exists $S_0 > \tilde{S}(\theta)$ such that $k^{sb}(\theta, S_0) = S_0$ and, if there are many such values, relabel those such as S_0 is the lowest one which implies that $k^{sb}(\theta, S) < S$ for $S < S_0$ but close enough. Then, (A.27) implies that:

$$\frac{\partial k^{sb}}{\partial S}(\theta, S_0) = \frac{C(\theta, S_0)}{P(0)} < \frac{C(\theta, \tilde{S}(\theta))}{P(0)} = 1$$

where the last inequality follows from the fact that $C(\theta, S)$ is decreasing in S over the relevant domain. Hence, $k^{sb}(\theta, S) > S$ for $S < S_0$ but close enough. A contradiction. Item 1. follows from there.

2. We denote $y^{sb}(\theta, S) = 1 - \frac{\partial k^{sb}}{\partial S}(\theta, S)$. Differentiating (A.27) with respect to S gives us:

$$\frac{\partial y^{sb}}{\partial S}(\theta, S) = -\frac{\frac{\partial \tilde{C}}{\partial S}(\theta, S)}{V'(\varphi(r(S - k^{sb}(\theta, S))))} - \frac{r\tilde{C}(\theta, S)}{V'(\varphi(r(S - k^{sb}(\theta, S))))} y^{sb}(\theta, S). \tag{A.32}$$

Observe also that, from a step in the proof of Item 1., $y^{sb}(\theta, \tilde{S}(\theta)) = 0$. Moreover, (A.32) implies that $\frac{\partial y^{sb}}{\partial S}(\theta, \tilde{S}(\theta)) > 0$ and thus $y^{sb}(\theta, S) > 0$ for $S > \tilde{S}(\theta)$ but close enough. Now, (A.32) also implies that $\frac{\partial y^{sb}}{\partial S}(\theta, S_0) > 0$ at any S_0 such that $y^{sb}(\theta, S_0) = 0$. Hence, $y^{sb}(\theta, S) < 0$ for $S < S_0$ but close enough. Putting together these two findings, there cannot exist such $S_0 > \tilde{S}(\theta)$. Hence, $y^{sb}(\theta, S) > 0$ for all $S > \tilde{S}(\theta)$ which proves Item 2..

3. We denote $z^{sb}(\theta, S) = \frac{\partial k^{sb}}{\partial \theta}(\theta, S)$. Differentiating (A.27) with respect to θ gives us:

$$\frac{\partial z^{sb}}{\partial \theta}(\theta, S) = -\frac{\frac{\partial \tilde{C}}{\partial \theta}(\theta, S)}{V'(\varphi(r(S - k^{sb}(\theta, S))))} + \frac{r\tilde{C}(\theta, S)}{V'(\varphi(r(S - k^{sb}(\theta, S))))} z^{sb}(\theta, S). \tag{A.33}$$

By definition,

$$\tilde{S}(\theta) = k^{sb}(\theta, \tilde{S}(\theta)).$$

Differentiating with respect to θ yields

$$\dot{\tilde{S}}(\theta) y^{sb}(\theta, \tilde{S}(\theta)) = z^{sb}(\theta, \tilde{S}(\theta))$$

and thus, using some finding from the proof of Item 2., $z^{sb}(\theta, \tilde{S}(\theta)) = 0$. Moreover, (A.33) implies that $\frac{\partial z^{sb}}{\partial S}(\theta, \tilde{S}(\theta)) < 0$ since $\frac{\partial \tilde{C}}{\partial \theta}(\theta, S) < 0$ from the Monotone Hazard Rate Property. Hence, $z^{sb}(\theta, S) < 0$ for $S > \tilde{S}(\theta)$ but close enough. Now, (A.33) also implies that $\frac{\partial z^{sb}}{\partial S}(\theta, S_0) < 0$ at any S_0 such that $z^{sb}(\theta, S_0) = 0$. Hence, $z^{sb}(\theta, S) > 0$ for $S < S_0$ but close enough. Putting together these two findings, there cannot exist such $S_0 > \tilde{S}(\theta)$. Hence, $z^{sb}(\theta, S) < 0$ for all $S > \tilde{S}(\theta)$ which proves Item 3..

□

Observe now that:

$$Q^{sb}(\theta, 0) = 0 \tag{A.34}$$

and

$$\frac{\partial Q^{sb}}{\partial t}(\theta, t) = -\frac{\partial S^{sb}}{\partial t}(\theta, t) = \varphi(r(\theta - Q^{sb}(\theta, t) - k^{sb}(\theta, \theta - Q^{sb}(\theta, t)))). \tag{A.35}$$

Hence, we get:

$$\frac{\partial Q^{sb}}{\partial t}(\theta, 0) = \varphi(r(\theta - k^{sb}(\theta, \theta))). \tag{A.36}$$

Differentiating, we obtain:

$$\frac{d}{d\theta}(\theta - k^{sb}(\theta, \theta)) = y^{sb}(\theta, \theta) - z^{sb}(\theta, \theta) > 0 \tag{A.37}$$

from Items 2. and 3. in Lemma A.2.

Inserting into (A.36), we deduce that:

$$\frac{\partial^2 Q^{sb}}{\partial \theta \partial t}(\theta, 0) > 0. \tag{A.38}$$

From (A.34), we deduce that locally, around $t = 0$,

$$\frac{\partial Q^{sb}}{\partial \theta}(\theta, t) > 0. \tag{A.39}$$

Take now two values $\theta_1 < \theta_2$. From (A.34) and (A.39), locally around $t = 0$, we have $Q^{sb}(\theta_1, t) < Q^{sb}(\theta_2, t)$ with equality only at $t = 0$. Suppose that there exists $t_0 > 0$ such that $Q^{sb}(\theta_1, t_0) = Q^{sb}(\theta_2, t_0)$ and, again take the lowest such value if there are several. Using (A.35), we have:

$$\frac{\partial Q^{sb}}{\partial t}(\theta_1, t_0) = \varphi(r(\theta_1 - Q^{sb}(\theta_1, t_0) - k^{sb}(\theta_1, \theta_1 - Q^{sb}(\theta_1, t_0)))). \tag{A.40}$$

By a proof similar to that used to prove (A.41), we can prove that:

$$\frac{d}{d\theta}(\theta - Q - k^{sb}(\theta, \theta - Q)) > 0 \tag{A.41}$$

for any Q. Inserting into (A.40) for $Q = Q^{sb}(\theta_1, t_0) = Q^{sb}(\theta_2, t_0)$ yields:

$$\frac{\partial Q^{sb}}{\partial t}(\theta_1, t_0) < \frac{\partial Q^{sb}}{\partial t}(\theta_2, t_0).$$

Thus, for $t < t_0$ but close enough, we would have $Q^{sb}(\theta_1, t) > Q^{sb}(\theta_2, t)$. A contradiction which proves that (4.23) holds. □

Proof of Proposition 5. The nonlinear schedule $T(q_0)$ implements the optimal choice $q_0^{sb}(\theta)$ when Problems (4.24) and (4.25) have the same optimality condition. The first-order condition for (4.24) writes as

$$T'(q_0^{sb}(\theta)) = \frac{\partial \mathcal{C}}{\partial q_0}(\theta, q_0^{sb}(\theta)),$$

which may be rewritten using the linear functional forms of our Running Example as

$$T'(q_0^{sb}(\theta)) = \frac{C(S^*) + C'(S^*)(\theta - S^*)}{r + \frac{1}{z^*}} - \frac{2C'(S^*)q_0^{sb}(\theta)}{\left(r + \frac{1}{z^*}\right)\left(r + \frac{2}{z^*}\right)}.$$

Observe that $T'(q_0^{sb}(\theta)) > 0$ since the two terms above are positive.

The first-order condition for (4.25) is

$$\mathcal{V}'(q_0^{sb}(\theta)) = \frac{\partial \mathcal{C}}{\partial q}(\theta, q_0^{sb}(\theta)) + \frac{1 - F(\theta)}{f(\theta)} \frac{\partial^2 \mathcal{C}}{\partial \theta \partial q}(\theta, q_0^{sb}(\theta))$$

or again, using the linearity of the marginal cost function,

$$\mathcal{V}'(q_0^{sb}(\theta)) = \frac{C(S^*) + C'(S^*)(\theta - S^*)}{r + \frac{1}{z^*}} - \frac{2C'(S^*)q_0^{sb}(\theta)}{\left(r + \frac{1}{z^*}\right)\left(r + \frac{2}{z^*}\right)} - \frac{1 - F(\theta)}{f(\theta)} \frac{C'(S^*)}{r + \frac{1}{z^*}}.$$

Identifying these first-order conditions above requires that, for all $\theta \in \Theta$, the following identity holds:

$$T'(q_0^{sb}(\theta)) = \mathcal{V}'(q_0^{sb}(\theta)) + \frac{1 - F(\theta)}{f(\theta)} \frac{C'(S^*)}{r + \frac{1}{z^*}} = \frac{\partial \mathcal{C}}{\partial q}(\theta, q_0^{sb}(\theta)).$$

In particular that, with our functional forms, we have:

$$\mathcal{V}'(q_0) = \int_0^{+\infty} P\left(q_0 exp\left(-\frac{t}{z^*}\right)\right) exp\left(-\frac{t}{z^*}\right) exp(-rt)dt = \frac{P(0)}{r+\frac{1}{z^*}} + \frac{P'(0)q_0}{r+\frac{2}{z^*}}.$$

Eliminating θ from the above expression immediately gives us the following expression of the marginal reward

$$T'(q_0) = \mathcal{V}'(q_0) + \frac{1-F(\vartheta_0(q_0))}{f(\vartheta_0(q_0))}\frac{C'(S^*)}{r+\frac{1}{z^*}}. \tag{A.42}$$

Finally, the value of $T(q_0)$ is anchored by the fact that the firm with initial stock $\underline{\theta}$ makes no profit, i.e.,

$$T(q_0^{sb}(\underline{\theta})) = \mathcal{C}(\underline{\theta}, q_0^{sb}(\underline{\theta})). \tag{A.43}$$

Integrating (A.42) and taking into account (A.43) yields (4.26). □

PROOF OF PROPOSITION 6. The regulator's problem can now be written as:

$$(\mathcal{P}^*): \quad \max_{T,q\geq 0,S,Q} \int_0^{+\infty} \left(V(Q(t)) - \int_{\underline{\theta}}^{\overline{\theta}} C(S(\theta,t))q(\theta,t)f(\theta)d\theta\right) exp(-rt)dt$$

subject to (2.1), (2.2) and

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta,t))f(\theta)d\theta = Q(t). \tag{A.44}$$

Let denote by $p^*(t)$ the multiplier of the feasibility condition (A.44). Optimality with respect to $Q(t)$ immediately gives us (5.1). Denoting now by $\lambda(\theta,t)$ the costate variable for (2.1), the necessary conditions for optimality (which again turn out to be also sufficient) can be written as:

- *Costate variables:*

$$\frac{\partial \lambda}{\partial t}(\theta,t) = exp(-rt)q^*(\theta,t)C'(S^*(\theta,t)) \tag{A.45}$$

- *Control variables:*

$$exp(-rt)(p^*(t) - C(S^*(\theta,t))) - \lambda(\theta,t) \leq 0 \text{ with } = 0 \text{ if } q^*(\theta,t) > 0. \tag{A.46}$$

- *Free-end point and transversality condition:* Let us now denote by $\overline{T}^*(\theta) = \sup\{t \geq 0 \text{ such that } q^*(\theta,t) > 0\}$ and $\underline{T}^*(\theta) = \inf\{t \geq 0 \text{ such that } q^*(\theta,t) > 0\}$.
  1. If $\overline{T}^*(\theta) < +\infty$, we should have:

$$\lambda(\theta, \overline{T}^*(\theta)) = 0 \text{ and } p^*(\overline{T}^*(\theta)) - C(S^*(\theta, \overline{T}^*(\theta)))q^*(\theta, \overline{T}^*(\theta)) = 0. \tag{A.47}$$

  2. If $\overline{T}^*(\theta) = +\infty$, we should have:

$$\lim_{t\to+\infty} \lambda(\theta,t) = 0 \text{ and } \lim_{t\to+\infty} exp(-rt)(p^*(t) - C(S^*(\theta,t)))q^*(\theta,t)) = 0. \tag{A.48}$$

3. If $\underline{T}^*(\theta) > 0$, we should have:

$$exp(-r\underline{T}^*(\theta))(p^*(\underline{T}^*(\theta)) - C(S^*(\theta, \underline{T}^*(\theta))) = \lambda(\theta, \underline{T}^*(\theta)). \tag{A.49}$$

Several facts follow from those optimality conditions.

1. First, observe that differentiating (A.46) with respect to t when $q^*(\theta, t) > 0$ yields:

$$\dot{\lambda}(t) + r\lambda(t) = exp(-rt)\left(\dot{p}^*(t) + C'(S^*(\theta, t))q^*(\theta, t)\right).$$

   Using (A.45) and (A.46) and simplifying yields at any point where $q^*(\theta, t) > 0$:

$$\dot{p}^*(t) = r(p^*(t) - C(S^*(\theta, t))). \tag{A.50}$$

2. From (A.50), it follows that all firms which are active at date t and extract a positive amount ($q^*(\theta, t) > 0$) have the same remaining stock at such t:

$$S^*(\theta, t) = \sigma^*(t) \tag{A.51}$$

   where $\sigma^*(t)$ is defined as the lowest level of resource currently extracted date t. Formally, $\sigma^*(t) = \min\{\theta \in \Theta \text{ such that } S^*(\theta, t) = \sigma^*(t)\}$ when $\sigma^*(t) \geq \underline{\theta}$ and $\sigma^*(t) = S^*(\underline{\theta}, t)$ otherwise. From (A.50), we obtain that $p^*(t)$ satisfies:

$$\dot{p}^*(t) = r(p^*(t) - C(\sigma^*(t))). \tag{A.52}$$

3. Differentiating (A.51) with respect to time, it follows from (2.2) that

$$q^*(\theta, t) = -\dot{\sigma}^*(t) \quad \forall \theta \in \left[\sigma^*(t), \overline{\theta}\right]. \tag{A.53}$$

   Together with (A.44), we deduce (5.5) or

$$\dot{\sigma}^*(t)(1 - F(\sigma^*(t)) = -D(p^*(t)) \tag{A.54}$$

   where $D = P^{-1}$.

4. Consider the system made of (A.52) and (A.54) together with the initial condition $\sigma^*(0) = \overline{\theta}$. By an argument that replicates that in the Proof of Proposition 1, this system $(\sigma^*(t), p^*(t))$ converges in infinite time towards $(S^*, p^* = P(0))$. In particular, $\dot{\sigma}^*(t) < 0$ for all t and thus, from (A.53), $q^*(\theta, t) > 0$ for t large enough (indeed, $t \geq \underline{T}^*(\theta)$ and for all θ.

5. $\underline{T}^*(\theta)$ is determined by the simple condition:

$$\sigma^*(\underline{T}^*(\theta)) = \theta. \tag{A.55}$$

□

PROOF OF PROPOSITION 7. The proof is to a large extent similar to the proof of Proposition 6 *modulo* the fact that the virtual cost of extraction must now be taken into account. The regulator's problem can thus be written as:

$$(\mathcal{P}^{sb}): \quad \max_{T, q \geq 0, S, Q} \int_0^{+\infty} \left(V(Q(t)) - \int_{\underline{\theta}}^{\overline{\theta}} \tilde{C}(\theta, S(\theta, t))q(\theta, t)f(\theta)d\theta\right) exp(-rt)dt$$

subject to (2.1), (2.2) and

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta,t))f(\theta)d\theta = Q(t). \tag{A.56}$$

Let us now denote by $p^{sb}(t)$ the multiplier of the feasibility condition (A.44).

1. We proceed as in the proof of Proposition Proposition 6 and obtain that, for $q^{sb}(\theta,t) > 0$:

$$\dot{p}^{sb}(t) = r(p^{sb}(t) - \tilde{C}(\theta, S^{sb}(\theta,t))). \tag{A.57}$$

Define now $\sigma^{sb}(t)$ is as the lowest level of resource at any date t. Formally, $\sigma^{sb}(t) = \min\left\{\theta \in \Theta \text{ such that } S^{sb}(\theta,t) = \sigma^{sb}(t)\right\}$ when $\sigma^{sb}(t) \geq \underline{\theta}$ and $\sigma^{sb}(t) = S^{sb}(\underline{\theta},t)$ otherwise. From (A.57), we obtain that $p^{sb}(t)$ satisfies:

$$\dot{p}^{sb}(t) = r(p^{sb}(t) - \tilde{C}(\sigma^{sb}(t), \sigma^{sb}(t))). \tag{A.58}$$

From (A.57), it also follows that all firms which are active at date t and extract a positive amount ($q^{sb}(\theta,t) > 0$) are such that:

$$S^{sb}(\theta,t) = \Phi(\theta, \sigma^{sb}(t)) \tag{A.59}$$

where Φ is implicitly defined as the solution to

$$\tilde{C}(\theta, \Phi(\theta,\sigma)) = \tilde{C}(\sigma,\sigma).$$

Note that the function $\Phi(\theta,\cdot)$ is defined over a range of possible values of σ since $\tilde{C}(\sigma,\sigma)$ is strictly decreasing in σ (since $\frac{d}{d\sigma}\left(\tilde{C}(\sigma,\sigma)\right) = C'(\sigma) - \frac{1-F(\sigma)}{f(\sigma)}C''(\sigma) - \frac{d}{d\sigma}\left(\frac{1-F(\sigma)}{f(\sigma)}\right)C'(\sigma) < 0$ when $C' < 0$, $C'' \geq 0$ and the *Monotone Hazard Rate Property* holds) and $\tilde{C}(\theta, S)$ is strictly decreasing in S. It follows that $\frac{\partial \Phi}{\partial \sigma}(\theta,\sigma) > 0$.

2. Differentiating (A.59) with respect to time, it follows from (2.2) that

$$q^{sb}(\theta,t) = -\dot{\sigma}^{sb}(t)\frac{\partial \Phi}{\partial \sigma}(\theta, \sigma^{sb}(t)). \tag{A.60}$$

Together with (A.44), we deduce:

$$\dot{\sigma}^*(t)\int_{\sigma^{sb}(t)}^{\overline{\theta}} \frac{\partial \Phi}{\partial \sigma}(\theta, \sigma^{sb}(t))f(\theta)d\theta = -D(p^*(t)). \tag{A.61}$$

3. Consider the system made of (A.58) and (A.61) together with the initial condition $\sigma^{sb}(0) = \overline{\theta}$. By an argument that replicates that in the Proof of Proposition 1, this system $(\sigma^{sb}(t), p^{sb}(t))$ converges in infinite time towards $(\tilde{S}^*, P(0))$ where $\tilde{S}^* \geq S^*$ is defined as:

$$\tilde{C}(\tilde{S}^*, \tilde{S}^*) = P(0). \tag{A.62}$$

In particular, $\dot{\sigma}^{sb}(t) < 0$ for all t and thus, from (A.60), $q^*(\theta,t) > 0$ for $t \geq \underline{T}^*(\theta)$ and for all θ.

From this, it follows that $S^{sb}(\theta,t)$ defined in (A.59) converges in infinite time also but now towards $\tilde{S}(\theta)$. Thus $\overline{T}^{sb}(\theta) = +\infty$.

4. $\underline{T}^{sb}(\theta)$ is determined by the simple condition:

$$\tilde{C}(\sigma^{sb}(\underline{T}^{sb}(\theta)), \sigma^{sb}(\underline{T}^{sb}(\theta))) = \tilde{C}(\theta, \theta).$$

Taking into account the fact that $\tilde{C}(\sigma, \sigma)$ is strictly decreasing in σ (since $\frac{d}{d\sigma}\left(\tilde{C}(\sigma, \sigma)\right) = C'(\sigma) - \frac{1-F(\sigma)}{f(\sigma)}C''(\sigma) - \frac{d}{d\sigma}\left(\frac{1-F(\sigma)}{f(\sigma)}\right)C'(\sigma) < 0$ when $C' < 0$, $C'' \geq 0$ and the *Monotone Hazard Rate Property* holds), we simplify this condition as:

$$\sigma^{sb}(\underline{T}^{sb}(\theta)) = \theta. \tag{A.63}$$

Because $\sigma^{sb}(t)$ is decreasing, this condition implies that the subset of active types at any date t is $\left\{\sigma^{sb}(t), \overline{\theta}\right\}$.

□

PROOF OF PROPOSITION 8. The system (5.4)-(5.5) can be rewritten with the new variables $\tilde{p}^*(t) = \frac{p^*(t)-P(0)}{P'(0)}$ and $\tilde{\sigma}^*(t) = \sigma^*(t) - S^*$ as:

$$\dot{\tilde{p}}^*(t) = r(\tilde{p}^*(t) - \kappa^*\tilde{\sigma}^*(t)) \tag{A.64}$$

and

$$\dot{\tilde{\sigma}}^*(t) = -\frac{\tilde{p}^*(t)}{1 - \tilde{\sigma}^*(t)} \text{ with } \tilde{\sigma}^*(0) = 1$$

Define

$$x^*(t) = \tilde{\sigma}^*(t) - \frac{\tilde{\sigma}^{*2}(t)}{2} \Leftrightarrow \tilde{\sigma}^*(t) = 1 - \sqrt{1 - 2x^*(t)} \text{ with } x^*(0) = \tfrac{1}{2}.$$

We have:

$$\dot{x}^*(t) = -\tilde{p}^*(t). \tag{A.65}$$

Taking the ratio between (A.64) and (A.65), we may express $\tilde{p}^*$ in terms of x^* and observe that the corresponding function $P^*(x)$ solves the following ordinary differential equation:

$$\frac{dP^*}{dx}(x) = -r\left(1 - \frac{\kappa^*}{P^*(x)}(1 - \sqrt{1-2x})\right) \text{ with } P^*(0) = 0. \tag{A.66}$$

Importing this finding above, $x^*(t)$ now solves:

$$\dot{x}^*(t) = -P^*(x^*(t)). \tag{A.67}$$

It is immediate from this equation that $x^*(t)$ is decreasing and, since it remains non-negative, converges as t goes to $+\infty$ and its limit is $\lim_{t\to+\infty} x^*(t) = 0$.

Using L'Hospital Rule in (A.66) to determine the derivative $\dot{P}^*(0)$, we find $\dot{P}^*(0) = \frac{1}{z^*}$ Thus, when t goes to $+\infty$, $P^*(x) \sim_{x\to 0} \frac{x}{z^*}$. Thus, fix an arbitrary $\epsilon > 0$, because $x^*(t)$ converges towards zero, there exists t_0 such that for $t \geq t_0$,

$$\frac{1-\epsilon}{z^*}x^*(t) \leq P^*(x^*(t)) = -\dot{x}^*(t) \leq \frac{1+\epsilon}{z^*}x^*(t).$$

By Gronwall Lemma, we deduce that, for $t \geq t_0$, the following inequalities hold:

$$Bexp\left(-\frac{1-\epsilon}{z^*}t\right) \leq x^*(t) \leq Aexp\left(-\frac{1-\epsilon}{z^*}t\right).$$

for some constants A and B. From this, it follows that for t large enough, we can write:

$$-\frac{2\epsilon}{z^*} \leq \frac{\log(x^*(t))}{t} + \frac{1}{z^*} \leq \frac{2\epsilon}{z^*}$$

for any $\epsilon > 0$; or

$$\lim_{t\to+\infty} \frac{\log(x^*(t))}{t} + \frac{1}{z^*} = 0.$$

Thus, there exists a function $\eta^*(t)$ such that $\lim_{t\to+\infty} \eta^*(t) = 0$ and such that one can write:

$$x^*(t) = exp\left(-\frac{t}{z^*} + t\eta^*(t)\right).$$

Therefore,

$$\tilde{p}^*(t) = P^*(x^*(t)) \sim_{t\to+\infty} \frac{1}{z^*} exp\left(-\frac{t}{z^*} + t\eta^*(t)\right)$$

as claimed.

Consider now the asymmetric information scenario. We now define $\tilde{p}^{sb} = \frac{p^{sb}(t)-P(0)}{P'(0)}$ and $\tilde{\sigma}^{sb}(t) = \sigma^{sb}(t) - \tilde{S}^*$. Notice that $\Phi(\theta, \sigma) = 2\sigma - \theta$ and that $\tilde{S}^* = S^* + \frac{1}{2}$. These new variables solve the system:

$$\dot{\tilde{p}}^{sb}(t) = r(\tilde{p}^{sb}(t) - 2\kappa^* \tilde{\sigma}^{sb}(t)) \quad \text{(A.68)}$$

and

$$\dot{\tilde{\sigma}}^{sb}(t) = -\frac{\tilde{p}^{sb}(t)}{1 - 2\tilde{\sigma}^{sb}(t)} \text{ with } \tilde{\sigma}^{sb}(0) = 1.$$

Let us now define

$$x^{sb}(t) = \tilde{\sigma}^{sb}(t) - \tilde{\sigma}^{sb2}(t) \Leftrightarrow \tilde{\sigma}^{sb}(t) = 1 - \sqrt{1 - 4x^{sb}(t)} \text{ with } x^{sb}(0) = \tfrac{1}{4}.$$

We now have:

$$\dot{x}^{sb}(t) = -P^{sb}(x^{sb}(t)). \quad \text{(A.69)}$$

Taking now the ratio between (A.68) and (A.69), we may express $\tilde{p}^{sb}$ in terms of x^{sb} namely $P^{sb}(x)$, and observe that this function solves the following ordinary differential equation:

$$\frac{dP^{sb}}{dx} = -r\left(1 - \frac{\kappa^*}{P^{sb}(x)}(1 - \sqrt{1-4x})\right) \text{ with } P^{sb}(0) = 0. \quad \text{(A.70)}$$

Importing this finding above, $x^{sb}(t)$ now solves:

$$\dot{x}^{sb}(t) = -P^{sb}(x^{sb}(t)).$$

It is immediate from this equation that $x^{sb}(t)$ is decreasing and, since it remains non-negative, converges as t goes to $+\infty$ and its limit is $\lim_{t\to+\infty} x^{sb}(t) = 0$.

Using L'Hospital Rule in (A.70) to determine the derivative $\dot{P}^{sb}(0)$, we find $\dot{P}^{sb}(0) = \frac{1}{z^{sb}}$ with z^{sb} being defined in the text. Thus, locally for x small enough, we have: $P^{sb}(x) \sim_{x\to 0} \frac{x}{z^{sb}}$. Proceeding as in the complete information scenario, we demonstrate that

$$\tilde{p}^{sb}(t) = P^{sb}(x^{sb}(t)) \sim_{t\to+\infty} \frac{1}{z^{sb}} exp\left(-\frac{t}{z^{sb}} + t\eta^{sb}(t)\right)$$

as claimed. □

APPENDIX B: COMPARISON WITH THE CASE OF AN UNREGULATED MONOPOLY

As a benchmark to compare with our analysis of the case of asymmetric information, we now provide the analysis of the optimal path of resource extraction under monopoly. There also there are distortions away from the standard *Hotelling Rule* although, as we argued in the text, those distortions are of a different nature, more related to demand specification than to costs as under asymmetric information. This path solves the following problem:[41]

$$(\mathcal{P}^m(\theta)): \quad \max_{q,S} \int_0^{+\infty} (R(q(\theta,t)) - C(S(\theta,t))q(\theta,t))exp(-rt)dt$$

subject to (2.1) and (2.2)

where $R(q) = qP(q)$ stems now for the monopolist's revenue.

Assuming that R is increasing and concave (which requires the familiar condition $qP''(q) + 2P'(q) < 0$, the dynamics under monopoly is very similar to that described in the competitive scenario. Observe in particular that

$$R'(0) = P(0) = C(S^*).$$

Hence, the marginal revenue at zero is again equal to the choke price and it is thus the same for the monopolist and for the competitive firm. As a result, there is in the long run as much resource extraction under monopoly provision than with a competitive market. *Mutatis mutandis*, we may state the main feature of the dynamics under monopoly as follows.

PROPOSITION B.1. *Under complete information, the path of resource extraction under monopoly provision converges towards no extraction and a finite stock of resource in infinite time:*

$$\lim_{t\to+\infty} q^m(\theta,t) = 0 \ \textit{and} \ \lim_{t\to+\infty} S^m(\theta,t) = S^*. \tag{B.1}$$

The marginal revenue of the monopolist $\gamma(\theta,t) = R'(q^m(\theta,t)) = P(q^m(\theta,t)) + q^m(\theta,t)P'(q^m(\theta,t))$[42] *evolves according to the modified Hotelling Rule:*

$$\frac{\partial \gamma}{\partial t}(\theta,t) = r(\gamma(\theta,t) - C(S^*(\theta,t))). \tag{B.2}$$

From the earlier work of Stiglitz (1976) and Salant, Eswaran and Lewis (1983), the comparison of the trajectories under monopoly and perfect competition is well known to be, in general, complex and linked to intricate details of demand elasticity and cost function. To illustrate, let denote the elasticity of the inverse demand function by $\varepsilon(q) = -\frac{qP'(q)}{P(q)} = -\frac{qV''(q)}{V'(q)}$ and let φ be the inverse function for $\int_0^q \varepsilon(\tilde{q})d\tilde{q}$. We could proceed *mutatis mutandis* in the case of a monopoly and define accordingly $\varepsilon^m(q) = -\frac{qR''(q)}{R'(q)}$ and by φ^m the inverse function for $\int_0^q \varepsilon^m(\tilde{q})d\tilde{q}$. Equipped with these notations, we now compare extraction under monopoly with the optimal one.

[41] Monopoly extraction of resources has been analyzed by Stiglitz (1976), Salant, Eswaran and Lewis (1983) and Pindyck (1987) under various assumptions on uncertainty, demand and technologies of extraction.

[42] Of course, this marginal revenue is less than the market price $\gamma(\theta,t) < P(q^m(\theta,t))$ because demand is downward sloping.

PROPOSITION B.2. *Suppose that:*

$$V'(\varphi(y)) \geq R'(\varphi^m(y)) \text{ and } \varphi^m(y) \leq \varphi(y) \text{ (with equality only at } y = 0). \tag{B.3}$$

Then, the stock of resources remains greater under monopoly than at the optimum.

$$S^*(\theta, t) \leq S^m(\theta, t) \text{ (with equality at } t = 0 \text{ and } +\infty). \tag{B.4}$$

This proposition shows that the unregulated monopolist slows down resource extraction under reasonable assumptions on preferences. Our running example which is developed in the main text nicely illustrates this feature.

PROOF OF PROPOSITION B.1. The proof is similar to that of Proposition 1 and is thus omitted. □

PROOF OF PROPOSITION B.2. We proceed *mutatis mutandis* as in the last item of the proof of Proposition B.2 to get the following differential equation

$$q^m(\theta, S) = \varphi^m(r(S - k^m(S))) \tag{B.5}$$

where $k^m(\theta, S)$ solves

$$\frac{dk^m}{dS}(S) = \frac{C(S)}{R'(\varphi^m(r(S - k^m(S))))} \tag{B.6}$$

with the initial condition $k^m(S^*) = S^*$. (Observe again that $\frac{dk^m}{dS}(S^*) = 1$ so that the Cauchy-Lipschitz Theorem applies and such solution is unique.)

Finally, inserting the solution of (B.6) into (A.12) (now indexed with the superscript m) gives that $S^m(\theta, t)$ solves

$$\frac{\partial S^m}{\partial t}(\theta, t) = -\varphi^m(r(S^m(\theta, t) - k^m(S^m(\theta, t)))) \tag{B.7}$$

with the initial condition $S^m(\theta, 0) = \theta$.

The first observation is that $k^*(S^*) = k^m(S^*)$ and $\frac{dk^*}{dS}(S^*) = \frac{dk^m}{dS}(S^*) = 1$. By the first item in (B.3), we have

$$\frac{dk^m}{dS}(S) = \frac{C(S)}{R'(\varphi^m(r(S - k^m(S))))} \geq \frac{C(S)}{V'(\varphi(r(S - k^m(S))))}.$$

From which it follows that $k^m(S) \geq k^*(S)$ for all S and thus $\varphi^m(r(S^m(\theta, t) - k^m(S^m(\theta, t))) \leq \varphi^m(r(S^m(\theta, t) - k^*(S^m(\theta, t)))$. Inserting this finding into (B.7), we obtain:

$$\frac{\partial S^m}{\partial t}(\theta, t) \geq -\varphi^m(r(S^m(\theta, t) - k^*(S^m(\theta, t)))) \geq -\varphi(r(S^m(\theta, t) - k^*(S^m(\theta, t))))$$

where the second inequality follows from the second item in (B.3). Since $S^m(\theta, 0) = S^*(\theta, 0) = \theta$, we deduce from that (B.4). □