Renewable resources and inequality aversion: what consequences for the future?*

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Abstract

This paper addresses intragenerational and intergenerational issues about a renewable natural resource exploitation. In particular, we analyze how a change in the intragenerational inequality aversion influences the possible development paths for futures generations. We suppose an agent has access to a renewable resource and works to exploit it, while another agent does not have access to it. A social planner implements a transfer mechanism from the former to the latter. We show that if the worker is originally better off than the receiver, the inequality aversion has a negative effect on the resource stock with a lump-sum transfer, but potentially a positive effect with a proportional tax. Reciprocally, the higher the stock, the higher the possibilities for future consumptions. These links strongly suggest to deal jointly with the two equity dimensions in order to design consistent environmental policies.

Keywords: renewable resource, intragenerational equity, intergenerational equity.
JEL Classification: D63, Q20, Q56

1 Introduction

Emphasis has been put on futurity in the sustainability debate. Economists translated this question on how weighting the future compared to the present in the evaluation of different development paths. This way of evaluating generations that arise at different dates may be summarized under the vocable *intergenerational equity*. But if society cares about the "difference of well-being" between two generations, it cares also about that of two individuals from the same

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generation. This concern may be expressed as *intragenerational equity*. The economics literature has generally separated these two equity dimensions, but we can be interested in dealing with them together. First, it can be argued that an unjust society is likely to be unsustainable, either on the political side (revolution) or on the environmental side (degradation) (Haughton, 1999). Second, it seems curious to attach more importance to future generations, thus unborn, than to the current one (Solow, 1991; Anand and Sen, 2000). Finally, from a policy view, we may wonder if intragenerational and intergenerational concerns can be designed independently. On the contrary, we should be interested in how they interact to formulate consistent policies.

The interactions between these two dimensions are actually ever-present in economics. Think, for example, of a fiscal policy that aims to reduce the public debt. But alongside this debt there is the environmental debt, which can reduce the possibility of development for future generations. In this respect, three major dimensions can be taken into account: the climate change, with the question of burden-sharing between generations, but also into each one of them between expected losers and winers; the exhaustion of non-renewable resources, but this requires an understanding of the economic production processes. In particular, in what extend these resources can be substituted by manufactured capital; and, finally, the management of renewable resource stocks. We are interested in renewable resources in particular.

As these resources are not necessarily traded, the choice of the welfare criterion is of all importance. More precisely this may allow us to express the implicit values of stocks of natural resources. These *shadow prices* are essential to compute *genuine savings* (Hamilton, 1994; Pearce et al., 1996). The expenditures that enhance the environment are seen as savings and depletion of natural resources and the environmental degradations as dissavings. It generalizes the traditional concept of savings, and its positivity indicates that the welfare, however defined, is currently non-declining. The renewable resources have to be managed on the long run, but compared to non-renewable ones; they are generally directly consumed, they may have amenities and we can have "win-win" solutions. Further, some can be "essential" for life (less true in developed countries). For all of these reasons, the State intervention can be justified.

The purpose of this study is to analyze the intragenerational and the intergenerational equity trade-off. The question we are asking is what are the consequences of the intragenerational concern (broadly speaking, measured by the inequality aversion) on current and future welfares, when only some individuals have access to a renewable resource, and does the type of redistribution matters in such a context.

The linkages between the two equity dimensions have not been extensively studied in the economics literature, and in the environmental and resources economics literature in particular. Nonetheless, some authors have highlighted this question in the climate change debate for some time. For example, Schelling (1992) argues that the best way for developing countries to fight against the negative effects of climate change is to continue to develop. Heal (2009), on the opposite, explains that a preference for equality between generations and the preference for equality within each one of them may be opposed. If one expects consumption will grow, we can further discount the future, but this does not incite us to take preventive measures against the negative effects of climate change. Conversely, as developing countries are more vulnerable, more concerns about them would incite us to take actions more quickly. Kverndokk et al. (2014) proposed a model to analyze the burden-sharing between North and South in the reduction of greenhouse gas emissions through clean and dirty investments. These two dimensions are also present in the explanation of climate negotiations (e.g. Lecocq and Hourcade, 2012). Baumgärtner et al. (2012) proposed a framework to summarize the possible links between the two equity dimensions; independence, facilitation and/or rivalry. These features are detailed in the context of ecosystem services by Glotzbach and Baumgärtner (2012). We are not aware of any work that builds analytically the welfare possibility between individuals that depends on a renewable resource and has consequences on future generations.

In this paper, we assume the society has an inequality aversion. That is to say, it restrains the substitution of the "well-being" of one agent for the well-being of another. We have two limit cases. The first one, known as the "*utilitarism*" view[1], assumes the society has a nil inequality aversion; only the total of utility matters (Vickrey, 1945; Harsanyi, 1955, 1977). On the contrary, in the "*maximin*" view, inspired by Rawls (1971), the society is assumed to have an infinite inequality aversion; no substitution is possible between the individual utilities. Besides, the society is also averse to intertemporal inequalities. Several criteria summarized this concern. The most popular is the *discounted utilitarism*; every future generations are taken into account, but the further a generation arises, the less it counts. In this respect, the choice of the discount rate is of great importance. According to the criterion of the intragenerational equity, the criterion of the intergenerational equity may be constrained. Studying both together allows us to determine all possible choices of policy and to estimate opportunity costs. The renewable resource is a parable to links the two dimensions.

We use a social objective that allows us to deal with the two previous doctrines as well as all intermediary cases. Besides, we build a "set of possibilities" for the utilities. This tells us in what extend we can take from one agent to give to another agent. Then, optimal allocations will be represented by Pareto frontiers. From an allocation situated on that frontier, we cannot increase any more the utility of one agent without decreasing that of another. Afterward, we will be able to choose between these optimal allocations using the social criterion. To respond to our issue, we need, at least, two different agents. One agent has access to a resource and works to extract a part. The other agent does not have access to it, he entirely devotes his time to leisure. We introduce a redistribution mechanism that takes a part of the harvest from the worker to give it to the receiver. The utility of the worker depends on his leisure time and on his

[1] The term utilitarism may also simply refer to the utilization of the utility concept.

available consumption. The utility of the receiver depends only on the amount he receives. The redistribution mechanism is of two types. The first one is a lump-sum transfer; whatever the effort of the worker, the receiver will get the same amount of the resource caught. The second one is a tax. Whatever the effort of the first agent, the other will get the same *proportion* of the resource caught. We introduce a renewable resource which vary according to the catch. As the resource may affect the potential catch, it may impact the set of possibilities. First, we will be interested in how the instruments (the transfer and the tax) evolve according to a change in the inequality aversion, and their impact on stock paths. Second, we will be interested in the consequences of the evolution of the resources. In particular, will it benefit to every type of agents?

Our contribution is having stated clearly the assumptions underlying the construction of well-known utility possibility frontiers, in the context of two asymmetric agents. When the transfer is absolute (through a lump-sum), this does not pose any particular difficulties as long as leisure and consumption are normal goods. When the transfer is relative (proportional tax), we show that the disposition of the worker to substitute leisure for consumption matters *as well as* his labor supply in reaction to the tax. The more the first is high compared to the second, the more we are likely to get a "Laffer-like curve"; the amount received from a proportional tax is increasing, then decreasing with respect to its rate. Knowing that, we were able to analyze the impact of a change in the inequality aversion on the transfers. If the worker is better off than the receiver, we can expect the higher the inequality aversion, the higher the transfer (absolute or relative). If the lump-sum transfer rises, the worker harvests more (to compensate). But if the tax rate rises, under the restriction we made just before, the agent harvests less (discouragement). Thus, a higher inequality aversion affects negatively the resource stock in the case of lump-sum transfers, but *potentially* positively with proportional taxes. The consequences on the growth rate of the stock depend on the configuration of the economy.

The intragenerational dimension is built upon a framework proposed by Mas-Collel et al. (1995), borrowed itself from Atkinson (1973). We extend their example in two dimensions. First, we state clearly the conditions under their results. And second, we introduce a resource to take into account the intergenerational concern in a Gordon-Schaefer model (Clark, 1990). The welfare analysis is based upon social welfare functions (Bergson, 1938; Samuelson, 1966).

The next Section presents the model. We solve it and we present some welfare analyses. Section 3 exhibits the links between the two equity dimensions. Section 4 concludes.

2 The Model

2.1 The Framework

A social planner implements a mechanism of transfer at each period between the two agents. We consider two options: the first one is a lump-sum and the second one a tax. We note A the worker and B the receiver. We normalize their available time to unity. At a given date, the resource stock X is given. Its law of evolution is given by the gap between renewal and consumption; $\dot{X} = \phi(X) - c$. ϕ is represented by a "bell curve" ($\phi(0) = 0 = \phi(X_{sup})$) and reaches a maximum at $\bar{X}$ (the Golden Rule stock). The total consumption equals the sum of individuals ones; $c := c^A + c^B$. The utility of the worker depends on his leisure time and his effective consumption; $u_A\left(l^A, c^A\right)$; while the utility of the receiver depends only on his consumption; $u_B\left(c^B\right)$.[2]

The supply of this economy is given by the production (catch-effort) function of the worker; $f\left(l^A, X\right)$. We assume it depends linearly on labor (or leisure), and it is convex with respect to the resource; $f\left(l^A, X\right) = (1 - l^A)g(X)$. The production is bounded between zero (no work) and $g(X)$ (no leisure). We assume no loss, so that all what is produced is consumed; $c = f\left(l^A, X\right)$.

The agent A earns a virtual wage by hour worked; $y := w(1 - l^A)$; and can consume the value of his net production (the harvest minus the amount transfered); $e := pc^A$ (we normalize the price of the good to unity thereafter). As his utility is increasing with respect to consumption and leisure, and that he cannot save a part, he will spend all his income; $c^A = w\left(1 - l^A\right)$.

To represent the inequality aversion in a simple way, we use an ordinal social welfare function with a constant elasticity of substitution (CES) between individuals; $W(u_A, u_B) = \left(\frac{1}{2} \cdot u_A^\eta + \frac{1}{2} \cdot u_B^\eta\right)^{\frac{1}{\eta}}$. η (inferior to unity but non-zero) is a parameter of substitution from which we obtain the elasticity of substitution $\theta := \frac{1}{1-\eta}$. A decrease of the elasticity of substitution represents an increase of the inequality aversion.[3] Further, it is assumed to be nonpaternalist (only the utilities matter), paretian, symmetric and concave.[4]

The intergenerational dimension is dealt with through a value function of any maximal intertemporal welfare; $V = V(X, t)$. We do not solve such a problem, we simply assume its value function is non-declining with the stock.

2.2 Utility Possibility Frontier with Lump-Sum Transfer

The social planner can take an amount from what the agent A harvests, to give it to the agent B. For each amount he decides to transfer, we can determine the optimal utility of each

[2] The utility functions are assumed to be interpersonally comparable.

[3] It is well-known that a CES function may approach the symmetric minimum (respectively a weighted sum) when θ approaches zero (resp. the infinity).

[4] For more details, see for example Mas-Collel et al. (1995, pp. 825-826).

agent. Making the amount transfered vary, we can construct the "first-best Pareto frontier". This frontier bounds the utility possibility set. As leisure time of the worker is the only decision variable, we will see that we can express the problem and solve it in that variable. The frontier will then be parametrized by leisure time.

The transfer being absolute, the worker can consume what he harvests minus a constant transfer; $c^A = c - \bar{c}^B$. Substituting consumption of the agent A by his budget constraint and the total consumption by the market equilibrium condition, it is easy to find that the wage is equal to the marginal productivity minus the transfer per hour worked; $w = g(X) - \frac{\bar{c}^B}{1-l^A}$. The budget constraint can thus be rewritten as the maximal possible consumption minus the transfer: $c^A = \left(1 - l^A\right) g(X) - \bar{c}^B$.

2.2.1 Construction of the First-Best Frontier

For a given utility of the agent B (i.e. a given transfer), the agent A makes his trade-off labor/leisure so as to maximize his utility. We draw this problem in Figure 1.

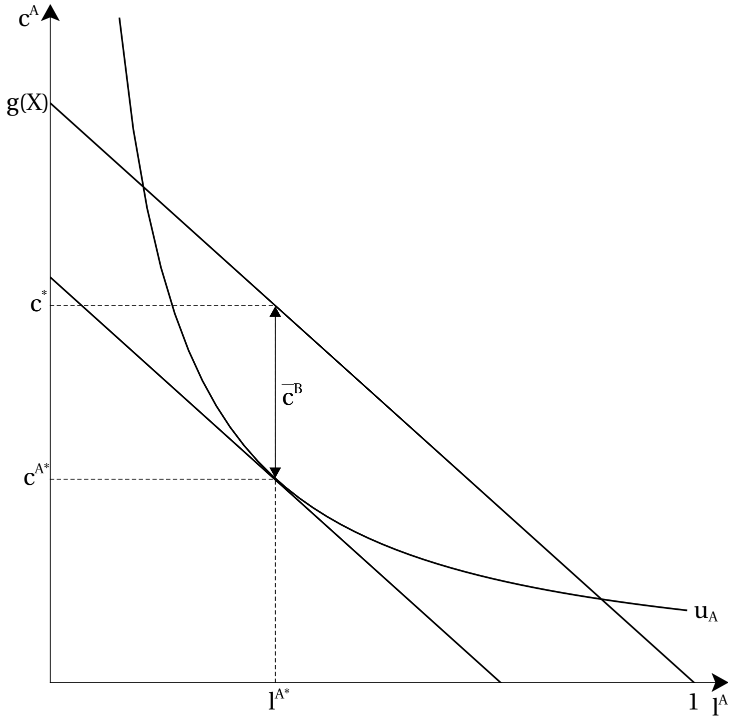

Figure 1: Construction of the utility possibility frontier with lump-sum transfers

We substitute the budget constraint in the utility, so as to maximize it in l^A.

$$\max_{l^A} \quad u_A\left(l^A, \left(1 - l^A\right) g(X) - \bar{c}^B\right). \tag{1}$$

A necessary condition to maximize the utility is that the marginal productivity of labor is equal to the marginal rate of substitution (MRS) of leisure for consumption; $g(X) = \frac{\partial u_A / \partial l^A}{\partial u_A / \partial c^A}$.

To get further in the treatment of this problem, we assume the utility function of the agent A to be homothetic. That is to say, for a given consumption per leisure time, the MRS is constant.[5] Therefore, we can express the optimal MRS as a function of this ratio. We note it Ω[6]. In this way, we can explicit the optimal consumption of the worker as a function of his leisure time.

$$\frac{c^A}{l^A} = \Omega\left(g(X)\right) \quad \Leftrightarrow \quad c^{A\star} = l^A \cdot \Omega\left(g(X)\right). \tag{2}$$

The optimal consumption of the agent A allows us to express his utility, as well as the one of the agent B, as only a function of leisure time (for a given resource stock); $u_A^\star\left(l^A, c^{A\star}\left(l^A, X\right)\right)$ and $u_B^\star\left(c^\star\left(l^A, X\right) - c^{A\star}\left(l^A, X\right)\right)$. We obtain two functions parametrized by leisure time. Then, making it vary, we can draw the first-best frontier of the utility possibility set in the (u_A, u_B) space.

2.2.2 Shape of the First-Best Frontier

Once we are able to construct the first-best frontier, a natural question that arises is about its shape. For that, we have to study the evolution of the optimal utility of each agent according to a virtual variation of leisure time.

It can be shown*[7] that the optimal utility of the agent A increases with his leisure, while the one of the agent B decreases.

For a constant MRS, the optimal consumption and leisure time of the agent A are positively linked. A higher utility is thus reached with both a higher consumption and a higher leisure time. Conversely, a higher leisure time, permitting less catch, corresponds to a lower utility for the agent B. Hence, the first-best possibility frontier is downward-sloping in (u_A, u_B).

2.3 Utility Possibility Frontier with Tax

We study now the case where the social planner cannot transfer an absolute amount of the harvest from the agent A to the agent B. He decentralizes his choice through a tax, at the rate τ. This problem can be related to a second-best approach. This case is comparable to the previous one, but relatively different in its implications. Here we make the tax rate vary to construct the "second-best Pareto frontier". This frontier bounds the utility possibility set as before. But as leisure time depends on the tax rate, we will explicit every variable as a function of the tax to solve the problem in that variable. The frontier will now be parametrized by the tax rate.

The transfer being proportional, the worker can consume what he harvests minus the taxed

[5]By definition; $-\frac{\mathrm{d}c^A}{\mathrm{d}l^A}\Big|_{U\left(c^A, l^A\right)} = -\frac{\mathrm{d}c^A}{\mathrm{d}l^A}\Big|_{U\left(\alpha c^A, \alpha l^A\right)}$, $(\alpha > 0)$.

[6]The function is increasing due to the strict convexity of indifference curves.

[7]The asterisks indicate that the details on the calculations are exposed in the Appendix A.

part; $c^A = (1-\tau)c$. With the same substitution as before, we find that the wage is equal to the net marginal productivity; $w = (1-\tau)g(X)$. The budget constraint can thus be rewritten as the maximal possible consumption net of the tax; $c^A = (1-\tau)\left(1-l^A\right)g(X)$.

2.3.1 Construction of the Second-Best Frontier

For a given utility of the agent B (i.e. a given tax rate), the agent A makes his trade-off labor/leisure so as to maximize his utility. We draw this problem in Figure 2.

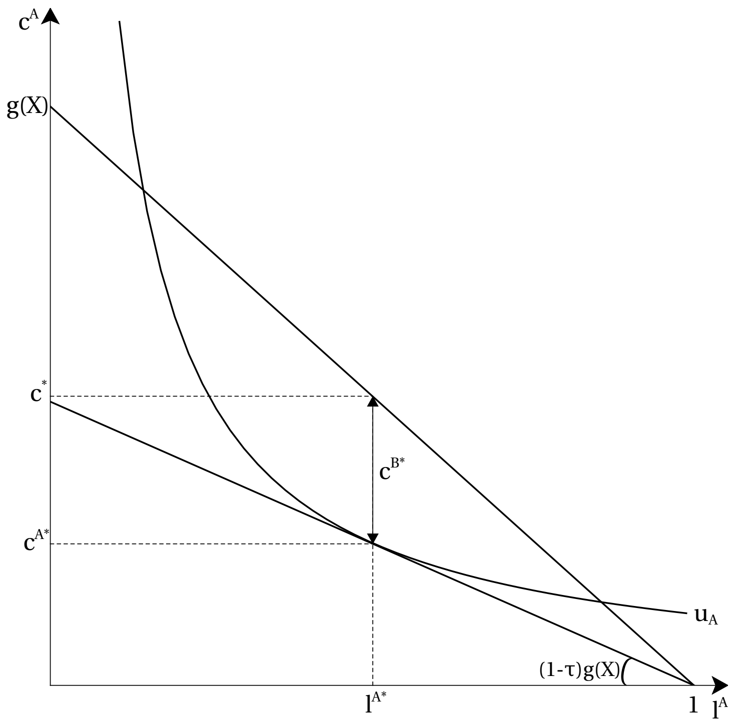

Figure 2: Construction of the utility possibility frontier with a proportional tax

We substitute, here also, the budget constraint in the utility, so as to maximize it with respect to leisure time.

$$\max_{l^A} \quad u_A\left(l^A, (1-\tau)\left(1-l^A\right)g(X)\right). \tag{3}$$

A necessary condition to maximize the utility is that the *net* marginal productivity of labor is equal to the MRS of leisure for consumption; $(1-\tau)g(X) = \frac{\partial u_A/\partial l^A}{\partial u_A/\partial c^A}$.

With the assumption on the utility made before (homothecy), we can express the optimal consumption as a function of leisure and of the tax rate.

$$\frac{c^A}{l^A} = \Omega\left((1-\tau)g(X)\right) \quad \Leftrightarrow \quad c^{A\star\star} = l^A \cdot \Omega\left((1-\tau)g(X)\right). \tag{4}$$

Here, and contrary to the first case, the optimal consumption depends on leisure time *and* on the tax rate. Nonetheless, we can take into account the optimal leisure time as a function of the tax rate in order to express utilities as a function of the tax rate only. This is done by equalizing consumption of the agent A (the budget constraint) with its optimal value (the equation (4)) and by solving it with respect to leisure time; $l^{A\star\star} = l^{A\star\star}(\tau)$.

We can now express, for a given stock, the optimal utility of the agents in terms of tax rate; $u_A^{\star\star}\left(l^{A\star\star}(\tau), c^{A\star\star}\left(l^{A\star\star}(\tau), X\right)\right)$ and $u_B^{\star\star}\left(\tau \cdot c^{\star\star}\left(l^{A\star\star}(\tau), X\right)\right)$. This time, the two utility functions are parametrized by the tax rate. Then, making it vary, we can draw the second-best frontier of the utility possibility set in the (u_A, u_B) space.

2.3.2 Shape of the Second-Best Frontier

Here, the evolution of the tax rate allows us to determine the shape of the frontier. It can be shown* that the tax is always negative for the worker. The tax reducing his budget constraint, it reduces the utility reached. But, for the other agent, it depends. If the worker works more when he is taxed more, the catch received by the second agent increases; we tax more a higher basis. If he works less (what should happen more generally), the effect is *a priori* ambiguous, because we tax more a lower basis. And if the worker is indifferent, the receiver is also better off, because we tax more a constant basis. Thus, we cannot directly conclude on the shape of this frontier. Nonetheless, we can explicit the condition under which the utility of the receiver evolves in a certain direction when the tax rate evolves.

It can be shown* that the amount received by the agent B increases if and only if the tax rate is not too high. The limit being given by the inverse of the difference between the elasticity of substitution between consumption and leisure; σ_{l^A,c^A}; and the cross-price elasticity of leisure (with respect to the virtual net price of consumption); $\varepsilon_{l^A,\frac{1}{1-\tau}}$; we note it $\bar{\tau}$.

$$\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad \tau < \bar{\tau} := \frac{1}{\sigma_{l^A,c^A} - \varepsilon_{l^A,\frac{1}{1-\tau}}}. \tag{5}$$

Note that this is close to the concept of the Laffer curve, the receiver playing the role of the State. If the threshold is strictly inferior to unity, the receiver takes advantage of a higher tax until a certain point ($\bar{\tau}$), from which on, a higher rate reduces the amount he perceives.

As the cross-elasticity depends on the substitution, we may expect the two parts of the denominator of the threshold be linked. Indeed, it can be shown* that if the elasticity of substitution is greater (or equal) to unity, the cross-price elasticity is greater (or equal) to zero.

$$\sigma_{l^A,c^A} \geq 1 \quad \Leftrightarrow \quad \varepsilon_{l^A,\frac{1}{1-\tau}} \geq 0. \tag{6}$$

Hence, if the worker has a high elasticity of substitution between leisure and consumption, he works less (takes more leisure) in reaction to a tax, and vice versa. We can then affirm that if the agent A considers his leisure and his consumption as quite complementary, he will always work more when he is taxed more, and then the agent B is getting better and better as the tax rate grows. Otherwise, it depends on the level of the threshold.

The table 1 summarizes the different situations.[8]

sign $\left(\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau}\right)$	$\sigma_{l^A,c^A} - \varepsilon_{l^A,\frac{1}{1-\tau}} < 1$	$\sigma_{l^A,c^A} - \varepsilon_{l^A,\frac{1}{1-\tau}} > 1$
$\sigma_{l^A,c^A} < 1$	+	Impossible[9]
$\sigma_{l^A,c^A} > 1$	+	+/-

Table 1: Evolution of the utility of the receiver when the tax rate rises

Thereafter, we restrict ourselves to the bottom right case, which seems to be the more plausible situation (the one described by a Laffer curve). To that, we assume the following. The elasticity of substitution of the worker between his consumption and his leisure is strictly greater than unity (necessary condition) and that one minus his elasticity of leisure with respect to the virtual price of consumption is strictly superior to unity (sufficient condition).[10]

To sum up, under our restrictions, the utility of the agent A is decreasing with respect to the tax rate while the utility of the agent B increases until a threshold and decreases afterward. Thus, we have a "bell curve" utility frontier. But we are considering only the decreasing part. The increasing one represents states where both agents can be better off (Pareto-dominated).

2.4 Comparison of the Frontiers

The two instruments have different implications. In particular, it can be shown that the second-best utility possibility set is included in the first-best one* (see Figure 3). That is to say, for a given utility of the worker, the tax is always less favorable for the receiver than the lump-sum transfer. Indeed, the tax on the production discourage the worker and can even lead to worsen their both situations. From a social planner point of view, the lump-sum is always better than the tax since it permits more choice.

Since the impact of the tax depends on the reaction of the worker, we may expect the difference between the two frontiers, i.e. the inefficiency of the tax (represented by the gray area in Figure 3), being dependent on the worker preferences. Indeed, if they are characterized by a constant elasticity of substitution, it can be shown that that the lower it is, the lower is the gap between the two instruments[11]*. At the limit, if leisure and consumption are perfectly complementary, the instrument used doest no longer matter, the two frontier merge together.[12] The

[8]We assume the denominator of the ratio in the inequality (17) is strictly positive.

[9]When the elasticity of substitution is strictly inferior to unity, we know that the sign is always positive. Then, the threshold is necessarily greater than one, what implies its denominator is inferior to unity.

[10]We implicitly assume here that the worker works less when he is taxed more.

[11]Rigorously speaking, the more the worker is efficient (high $g(X)$) and weights consumption compared to leisure, the more we are likely to be in a situation where a lower elasticity of substitution implies *always* a lower inefficiency of the tax. Otherwise, this is true only if the elasticity of substitution is under a threshold ($\bar{\sigma}$).

[12]We exclude the perfect substitutes case to avoid unrealistic corner solutions.

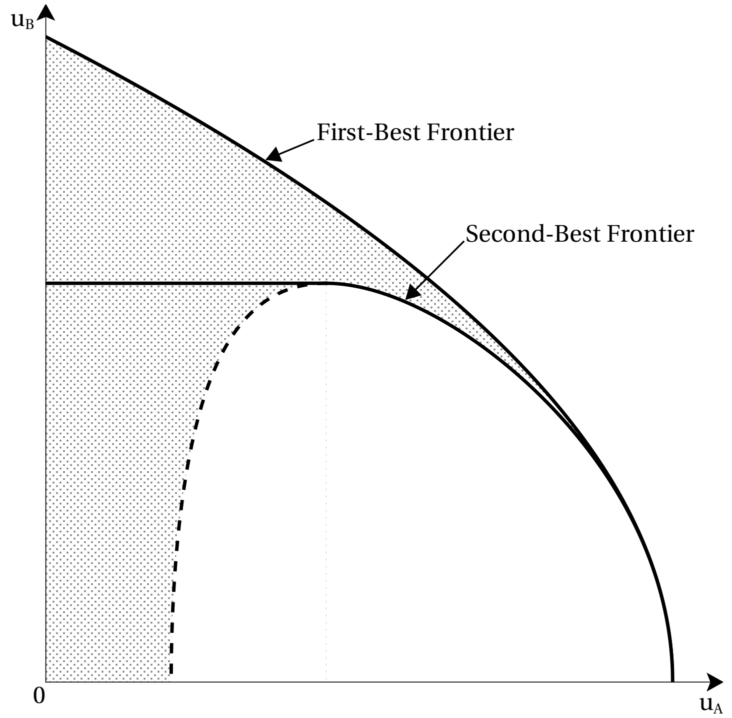

Figure 3: First and Second-Best Frontiers

latter result is not surprising since in this case the (implicit) relative prices does not matter for the worker choice.

2.5 Welfare Analysis

We built our utility possibility sets (bounded by the frontier). We are now interested in finding the optimal allocations of utility, which indicate the optimal transfer (absolute or proportional). We will also see the consequence of a change in the inequality aversion on such an allocation. Let us recall that a higher inequality aversion corresponds to a lower elasticity of substitution. We begin the welfare analysis with the lump-sum transfer. Then we will do it with the tax.

In general terms, we have to seek the maximal welfare subject to the fact that utilities are elements of the utility possibility set. Here, as we solved already a maximization problem, we can maximize directly the welfare through leisure time. We will always be situated on the frontier.

$$\max_{l^A} \quad W^\star(l^A) = \left(\frac{1}{2} \cdot u_A^\star(l^A)^\eta + \frac{1}{2} \cdot u_B^\star(l^A)^\eta\right)^{\frac{1}{\eta}}. \tag{7}$$

Let us define the marginal social rate of substitution (MSRS) as the willingness of the society to increase marginally the utility of the agent B taking from the utility of the agent A, keeping his global satisfaction equals. A necessary condition for the welfare to be maximal implies that the utility ratio (the one of *B* over the one of *A*) is a function of the MSRS and of the inequality aversion*.

At the optimum, the higher the slope of the frontier in absolute value, the higher the utility ratio. The easier is to get a high increasing of the utility of the agent *B* taking from the utility of the agent *A*, the more we will move toward a situation where the agent *B* has a high utility compared to that of the agent *A*.

The impact of the inequality aversion depends on the initial situation*. Two types of frontier illustrate it in Figure 4. If the agent *B* is relatively better off in the original situation, the higher

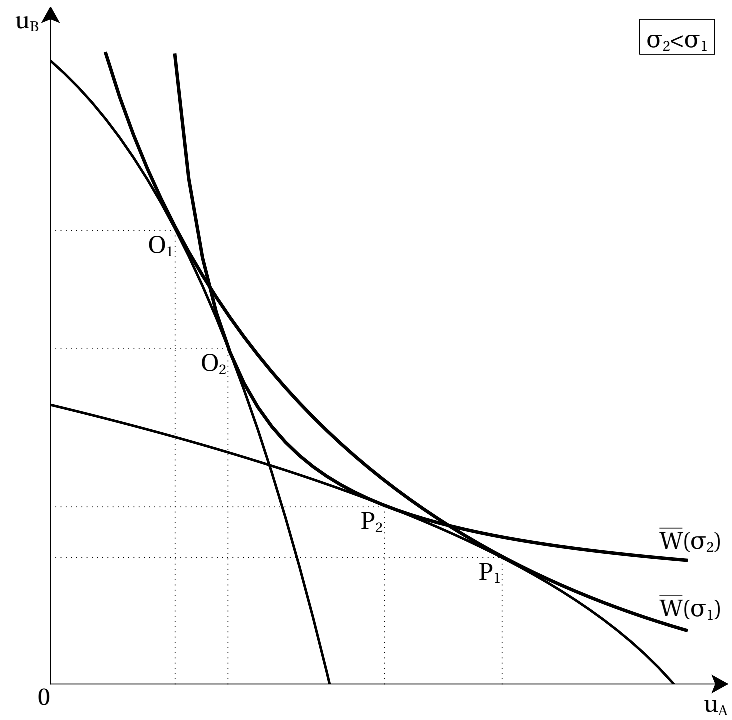

Figure 4: Variation of the inequality aversion with two types of frontier

the inequality aversion, the lower the utility ratio (from O_1 to O_2). This implies to reduce the optimal transfer from *A* to *B*. On the contrary, if the agent *A* is relatively better off at the beginning, the higher the inequality aversion the higher the utility ratio (from P_1 to P_2). This implies, then, to rise the optimal transfer. And finally, if we have a perfect equality between individuals (ratio equals to one), the inequality aversion has no influence. Anyways, a higher inequality aversion implies always a more egalitarian situation.[13]

We have the same analysis with the second mechanism except that we maximize with respect to the tax rate.

[13]Note that according to the shape of the frontier, a perfect equality may never be obtained.

3 Intragenerational and Intergenerational Considerations

From a social planner point of view, we can express the intergenerational consideration through a value function (V), non-declining with respect to the resource stock. This minimalist view allows us to simply answer the question: does the *possibilities* of development have increased after the intragenerational redistribution?

Formally, the evolution of the value function over time is given by the direct influence of the time and by its indirect influence through the stock.

$$\dot{V}(X,t) = \frac{\partial V(X,t)}{\partial t} + \underbrace{\frac{\partial V(X,t)}{\partial X}}_{\nu} \dot{X}. \tag{8}$$

For an autonomous problem, the first term is nil. The value function is increasing over time if the genuine savings is positive (shadow price (ν) times the evolution of the stock).

We will see first how the inequality aversion impacts both the current stock (through the current consumption) and its evolution. Second, we will see how the resource transforms the welfare possibilities.

3.1 Inequality Aversion, the Current Consumption and the Evolution of the Stock

3.1.1 Inequality Aversion and the Current Consumption

To understand the consequences of a variation of the inequality aversion, say an increase, on the current global consumption, we need to analyze several stages. Let us see it for our two redistribution mechanisms.

Lump-sum Transfer The inequality aversion has ambiguous effects on the repartition. From the previous section, we know that we will converge toward a more egalitarian situation. That is, the evolution of the utility ratio depends on the initial situation. If the utility of the agent A is high compared to the utility of the agent B in the initial situation, the utility ratio grows with a higher inequality aversion. In this case, the agent A works more and gets a lower utility, while the agent B gets a higher utility.

As the global catch increases with the work, global consumption rises (in the case where A is originally better off).

Tax This case is much more harder since the reaction to the tax may be ambiguous (contrary to the transfer). Be that as it may, we still restrict ourselves to the case of a decreasing supply of labor with respect to the tax.

The inequality aversion has still ambiguous effect here. According to the initial situation, a higher inequality aversion may advantage one or the other agent. Let us analyze, as before, the case that benefits to the receiver. In this case, the tax rate rises. With our restriction, the agent A works less and gets a lower utility, while the agent B gets a higher utility.

As the global catch increases with the work, global consumption decreases here (still in the same case).

Whatever the mechanism, if the utilities are equal, as the inequality aversion has no influence on the redistribution, it has no influence on the global consumption too.

The table 2 summarizes the different situations.[14]

$\text{sign}\left(\frac{\mathrm{d}c^{\text{opt}}}{-\mathrm{d}\theta}\right)$	$\frac{u_B}{u_A} < 1$	$\frac{u_B}{u_A} = 1$	$\frac{u_B}{u_A} > 1$
Lump-sum	+	0	-
Tax	-	0	+

Table 2: Evolution of the global consumption when the inequality aversion rises

We know that a higher catch will affect negatively the stock. It will also converges to a lower steady state.

3.1.2 Inequality Aversion and the Evolution of the Resource Stock

We analyze now the impact of a variation of the inequality aversion on the growth rate of the resource. Let us recall that this one is given by the gap between its natural growth and consumption; $\dot{X} = \phi(X) - c$, and let us differentiate it with respect to $-\theta$;

$$-\frac{\mathrm{d}\dot{X}}{\mathrm{d}\theta} = -\left(\phi'(X)\frac{\mathrm{d}X}{\mathrm{d}\theta} - \frac{\mathrm{d}c}{\mathrm{d}\theta}\right) \quad \Leftrightarrow \quad -\frac{\mathrm{d}\dot{X}}{\mathrm{d}\theta} = -\frac{\mathrm{d}c}{\mathrm{d}\theta}\left(\frac{\phi'(X)}{\frac{\mathrm{d}c}{\mathrm{d}X}} - 1\right). \tag{9}$$

As long as the marginal renewal of the resource is higher than the marginal contribution of the stock to the optimal consumption (which might be expected if the stock is low), the term inside the parentheses is positive. Thus, the evolution of the growth rate has the same sign as the evolution of consumption due to a change of the inequality aversion.

3.2 Evolution of the Resource and the Possibilities for Futures Welfares

The current consumption has consequences on the resource stock. And reciprocally, the resource stock has consequences on the possibilities of consumption.

[14] $-\mathrm{d}\theta > 0$ describes a higher inequality aversion.

If the resource stock varies, we have to determine beforehand to whom will go that supplement. This should be done according to the social welfare criterion chosen. But, to begin, we are interested in the "deformation" of the frontier due to the variation of the resource stock. Let us see it for our two mechanisms.

3.2.1 Lump-sum Transfer

To study the evolution of the frontier due to the evolution of the resource stock, we consider that an agent keeps constant his utility, while we maximize the one of the other. For example, we may consider the worker is indifferent whatever the stock. In this case we have to make him work more*. Indeed, the desutility of the work compensates the higher available utility due to a higher stock. For the receiver the result is *a priori* ambiguous. On the one hand, more resource are present and then he can receive a greater part of them. On the other hand, as the worker works more, he will receive less than he could expect. It can be shown that the first effect outweighs always the second one*. That is to say, the agent *B can* take advantage of a higher resource for every level of the utility of the agent *A*. We cannot conclude that everybody will be better off with more resource, but that they have the possibility to, since the first-best frontier is expansing.[15]

3.2.2 Tax

We have the same reasoning as before with the tax. But to make the worker indifferent when the stock evolves, we have to modify the tax rate. It is not surprising that the tax rate has to move in the same direction as the stock*. Indeed, if the resource grows, we have to take him that supplement. The frontier is therefore always expansing in the second-best approach too.

3.2.3 Welfare

As the frontiers are shifting outwards with an increase of the resource stock, the welfare is unambiguously increasing too.[16]

Hence, a variation of the inequality aversion has a direct effect on the individuals through the transfer and an indirect one through the evolution of the stock.

[15] Strictly speaking, as individuals are supposed to live "one instant", more resources allows more possibilities for their descendants.

[16] At this stage, we were not able to prove that the optimal utilities are effectively increasing. We suppose they are.

4 Conclusion

If one type of agent has access to a resource and another does not have access to it, the society may want to implement a transfer of a part of the harvest from the first one to the second one. We studied the effects of an absolute transfer and of a relative one based on the resource caught. We constructed utility possibility frontiers in each case. They represent the necessarily trade-off the society has to make when it wants to enhance one agent at the expense of another. Not surprisingly, the set of possibilities with a tax is lower than with a lump-sum transfer. The inefficiency of the tax depends on the preferences of the worker. Besides, the social criterion allowed us to find the social optimum, and we were particularly interested in its variation due to a change in the inequality aversion. This depends on the initial situation of the economy and on the mechanism of transfers used. In particular, we found that the society may afford a higher inequality aversion without worsening the future in two situations. First, using an absolute transfer to reduce the inequality if the worker is originally worse off than the receiver. And second, using a tax to reduce the inequality if the worker is originally better off than the receiver. Finally, if they are as well off as each other, the inequality aversion has no effect on the global consumption. If consumption rises, we might expect the resource grows faster if the stock is low.

Our asymmetry was intentionally radical; one agent has access to a resource and another does not. Nonetheless it could be interesting to analyze the effects of a transfer between two agents having a different access. The redistribution could have influences on both of them.

A Details on the calculations

A.1 Shape of the First-Best Frontier

$$\frac{\mathrm{d}u_A^\star}{\mathrm{d}l^A} = \frac{\partial u_A^\star}{\partial l^A} + \frac{\partial u_A^\star}{\partial c^A}\frac{\mathrm{d}c^{A\star}}{\mathrm{d}l^A} = \underbrace{\frac{\partial u_A^\star}{\partial l^A}}_{>0} + \underbrace{\frac{\partial u_A^\star}{\partial c^A}}_{>0}\left(\underbrace{\Omega(\cdot)}_{>0} + l^A \cdot \underbrace{\Omega^{-1\prime}(\cdot)}_{>0} \cdot \underbrace{\left(-\frac{\partial^2 f\left(l^A, X\right)}{\left(\partial l^A\right)^2}\right)}_{=0}\right). \tag{10}$$

$$\frac{\mathrm{d}u_B^\star}{\mathrm{d}l^A} = \frac{\partial u_B^\star}{\partial c^B}\frac{\mathrm{d}c^B}{\mathrm{d}l^A} = \frac{\partial u_B^\star}{\partial c^B}\left(\frac{\mathrm{d}c^\star}{\mathrm{d}l^A} - \frac{\mathrm{d}c^{A\star}}{\mathrm{d}l^A}\right) = \underbrace{\frac{\partial u_B^\star}{\partial c^B}}_{>0}\left(\underbrace{\frac{\partial f(\cdot)}{\partial l^A}}_{<0} - \underbrace{\frac{\mathrm{d}c^{A\star}}{\mathrm{d}l^A}}_{>0}\right). \tag{11}$$

We have

$$\frac{\mathrm{d}u_A^\star}{\mathrm{d}l^A} > 0 \quad \text{and} \quad \frac{\mathrm{d}u_B^\star}{\mathrm{d}l^A} < 0.$$

A.2 Shape of the Second-Best Frontier

A.2.1 Evolution of the Utilities According to the Tax Rate

$$\frac{\mathrm{d}u_A^{\star\star}}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_A^{\star\star}}{\partial l^A}}_{>0}\underbrace{\frac{\mathrm{d}l^{A\star\star}}{\mathrm{d}\tau}}_{?} + \underbrace{\frac{\partial u_A^{\star\star}}{\partial c^A}}_{>0}\underbrace{\frac{\mathrm{d}c^{A\star\star}}{\mathrm{d}\tau}}_{<0^{17}}. \tag{12}$$

Actually, since the constraint set is diminishing with the tax, his utility will reduce.

$$-\left.\frac{\mathrm{d}c^A}{\mathrm{d}l^A}\right|_{\tau_1} > -\left.\frac{\mathrm{d}c^A}{\mathrm{d}l^A}\right|_{\tau_2>\tau_1} \quad \Rightarrow \quad -\frac{\mathrm{d}c^{A\star\star}}{\mathrm{d}l^A} > \frac{\frac{\partial u_A^{\star\star}}{\partial l^A}}{\frac{\partial u_A^{\star\star}}{\partial c^A}} \quad \Leftrightarrow \quad \frac{\mathrm{d}u_A^{\star\star}}{\mathrm{d}\tau} < 0. \tag{13}$$

$$\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} = \frac{\partial u_B^{\star\star}}{\partial c^B}\frac{\mathrm{d}c^{B\star\star}}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_B^{\star\star}}{\partial c^B}}_{>0}\left(\underbrace{f(\cdot)}_{>0} + \tau\underbrace{\frac{\partial f(\cdot)}{\partial l^A}}_{<0}\underbrace{\frac{\mathrm{d}l^{A\star\star}}{\mathrm{d}\tau}}_{?}\right). \tag{14}$$

It is straightforward to see that if $\frac{\mathrm{d}l^A}{\mathrm{d}\tau} \leq 0$ then $\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0$.

[17] From the equation (4).

A.2.2 How Evolves the Utility of the Receiver When the Tax Rate Varies

To that purpose, let us recall that $c^B = \tau c$ and $c^A = (1-\tau)c$, hence $c^B = \frac{\tau}{1-\tau}c^A$. Then,

$$
\begin{aligned}
&\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \Rightarrow \quad \frac{\mathrm{d}c^{B\star\star}}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad \frac{1-\tau+\tau}{(1-\tau)^2}c^{A\star\star} + \frac{\tau}{1-\tau}\frac{\mathrm{d}c^{A\star\star}}{\mathrm{d}\tau} > 0;\\
&\Leftrightarrow \quad -\frac{\mathrm{d}c^{A\star\star}}{\mathrm{d}\tau}\frac{1-\tau}{c^{A\star\star}} < \frac{1}{\tau};\\
&\Leftrightarrow \quad -\left(\left(\Omega(\cdot) + l^A\cdot\Omega^{-1\prime}(\cdot)\cdot(1-\tau)\left(-\frac{\partial^2 f(\cdot)}{(\partial l^A)^2}\right)\right)\frac{\mathrm{d}l^A}{\mathrm{d}\tau} + l^A\cdot\Omega^{-1\prime}(\cdot)\cdot\frac{\partial f(\cdot)}{\partial l^A}\right)\\
&\times\frac{1-\tau}{l^A\cdot\Omega(\cdot)} < \frac{1}{\tau};\\
&\Leftrightarrow \quad -\left(\left(\frac{1}{l^A} + \frac{\Omega^{-1\prime}(\cdot)}{\Omega(\cdot)}\cdot(1-\tau)\left(-\frac{\partial^2 f(\cdot)}{(\partial l^A)^2}\right)\right)\frac{\mathrm{d}l^A}{\mathrm{d}\tau} + \frac{\Omega^{-1\prime}(\cdot)}{\Omega(\cdot)}\cdot\frac{\partial f(\cdot)}{\partial l^A}\right)(1-\tau) < \frac{1}{\tau}. \quad (15)
\end{aligned}
$$

As we assumed the catch to be linear with respect to the labor;

$$
\Leftrightarrow \quad \frac{1-\tau}{l^A}\frac{\mathrm{d}l^A}{\mathrm{d}(1-\tau)} + \frac{\Omega^{-1\prime}(\cdot)}{\Omega(\cdot)}(1-\tau)\left(-\frac{\partial f(\cdot)}{\partial l^A}\right) < \frac{1}{\tau}. \tag{16}
$$

The first term of the left-hand side is the elasticity of leisure with respect to $(1-\tau)$. We note it $\varepsilon_{l^A,(1-\tau)}$. It can be shown[18] that $\varepsilon_{l^A,(1-\tau)} = -\varepsilon_{l^A,\frac{1}{1-\tau}}$. So, the first term is the opposite of the cross-price elasticity of leisure.[19] The second term of the left-hand side is equal to the elasticity of substitution between consumption and leisure, we note it $\sigma_{l^A,c^A} > 0$.[20]

Thus, we have

$$
\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad \tau < \frac{1}{\sigma_{l^A,c^A} - \varepsilon_{l^A,\frac{1}{1-\tau}}}. \tag{17}
$$

A.2.3 Links of the Two Parts of the Threshold

Let us explore the sign of $\frac{\mathrm{d}l^A}{\mathrm{d}\tau}$. Let us equalize the budget constraint and the equation (4);

$$
\begin{aligned}
(1-\tau)f\left(l^A,X\right) &= l^A\cdot\Omega\left((1-\tau)g(X)\right) \qquad (18)\\
\Leftrightarrow \qquad \frac{f\left(l^A,X\right)}{l^A} &= \frac{\Omega(\cdot)}{1-\tau};
\end{aligned}
$$

18 $\frac{\mathrm{d}(1-\tau)}{1-\tau} = \frac{1}{1-\tau}\frac{1}{\frac{\mathrm{d}\left(\frac{1}{1-\tau}\right)}{\mathrm{d}(1-\tau)}}\mathrm{d}\left(\frac{1}{1-\tau}\right) = -\frac{\mathrm{d}\left(\frac{1}{1-\tau}\right)}{\frac{1}{1-\tau}}$.

19 Note that $c^A = (1-\tau)\left(1-l^A\right)g(X) \Leftrightarrow \frac{1}{1-\tau}c^A = \left(1-l^A\right)g(X)$, so $\frac{1}{1-\tau}$ is the virtual net price of consumption.

20 $\sigma_{l^A,c^A} = \frac{\mathrm{d}\Omega(\cdot)}{\mathrm{dMRS}}\frac{\mathrm{MRS}}{\Omega(\cdot)}$.

differentiate both sides;

$$\begin{aligned}
\frac{\frac{\partial f(\cdot)}{\partial l^A} l^A - f(\cdot)}{(l^A)^2} \mathrm{d}l^A &= \frac{\Omega^{-1\prime}(\cdot)\left(-\frac{\partial f(\cdot)}{\partial l^A}\right)(1-\tau) - \Omega(\cdot)}{(1-\tau)^2} \mathrm{d}(1-\tau);\\
\Leftrightarrow \qquad \frac{\mathrm{d}l^A}{\mathrm{d}(1-\tau)} \frac{1-\tau}{l^A} &= \frac{\Omega^{-1\prime}(\cdot)\left(-\frac{\partial f(\cdot)}{\partial l^A}\right)(1-\tau) - \Omega(\cdot)}{\frac{\partial f(\cdot)}{\partial l^A} l^A - f(\cdot)} \cdot \frac{l^A}{1-\tau};\\
\Leftrightarrow \qquad -\varepsilon_{l^A, \frac{1}{1-\tau}} &= \frac{\sigma_{l^A, c^A} - 1}{\frac{\partial f(\cdot)}{\partial l^A} l^A - f(\cdot)} \cdot \frac{l^A}{\Omega(\cdot)(1-\tau)}. \qquad (19)
\end{aligned}$$

The denominator of the first term of the right-hand side being negative;

$$\sigma_{l^A, c^A} \geq 1 \quad \Leftrightarrow \quad \varepsilon_{l^A, \frac{1}{1-\tau}} \geq 0. \qquad (20)$$

Let us recall that $\frac{\mathrm{d}l^A}{\mathrm{d}\tau} \leq 0 \Rightarrow \frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0$. Besides, note that $\varepsilon_{l^A,\tau}$ has the same sign as $\varepsilon_{l^A, \frac{1}{1-\tau}}$.[21] We have then

$$\sigma_{l^A, c^A} \leq 1 \quad \Rightarrow \quad \frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} > 0. \qquad (21)$$

A.2.4 Comparison of Utility Possibility Sets

General We want to compare the two utility possibility sets. Let us do it through their frontier.

For a given utility of the agent A, let us seek the maximum of amounts we can transfer, whatever the mechanism used. If we note $\tilde{c}^A(l^A)$ the image of an indifference curve of the agent A, we have to maximize the gap between the production and that curve.

$$\max_{l^A} \quad c^B = (1-l^A)g(X) - \tilde{c}^A(l^A). \qquad (22)$$

It is not hard to show that a necessary condition of this problem is the equalization of the MRS with the marginal productivity; $g(X) = \mathrm{MRS}$. As this is done with the lump-sum transfer, this mechanism is then the most favorable for the agent B. For a given utility of the agent A, the first-best frontier corresponds to the highest utility for the agent B. Thus, the utility possibility set with the lump-sum transfer contains the set with the tax. The two frontiers coincide well when no transfer occurs, since net and gross productivity are equivalent.

CES For a strictly convex indifference curve, the difference between the budget constraint (a straight line) and the indifference curve is strictly concave in (l^A, c^B). We know that the optimum is reached at the first-best optimal consumption-leisure ratio; $\Omega(g(X))$. So, for a

[21] $\mathrm{sign}\left(\varepsilon_{l^A,\tau}\right) = -\mathrm{sign}\left(\varepsilon_{l^A,(1-\tau)}\right) = -\mathrm{sign}\left(-\varepsilon_{l^A,\frac{1}{1-\tau}}\right) = \mathrm{sign}\left(\varepsilon_{l^A,\frac{1}{1-\tau}}\right)$.

given utility of the worker, the higher is the difference between the first-best and the second-best optimal ratios, the higher is the distance between the two frontiers.

Let us assume here the worker has a constant elasticity of substitution of leisure for consumption[22]; $u_A(l^A, c^A) = \left(\alpha \cdot l^{A\,\rho} + (1-\alpha) \cdot c^{A\,\rho}\right)^{\frac{1}{\rho}}$. The function that links the MRS and the optimal consumption-leisure ratio is then of the form; $\Omega(z) = \left(\frac{1-\alpha}{\alpha} z\right)^{\sigma}$.

When the elasticity of substitution evolves, we want to determine the evolution of the difference between the two optimal ratios; $D(\sigma) := \Omega(g(X)) - \Omega((1-\tau)g(X))$. Let us note the first-best one $r_1 := \frac{1-\alpha}{\alpha} g(X)$ and the second-best one $r_2 := \frac{1-\alpha}{\alpha}(1-\tau)g(X)$. We have:

$$\frac{\mathrm{d}D(\sigma)}{\mathrm{d}\sigma} \geq 0 \quad \Leftrightarrow \quad \ln(r_1) r_1^{\sigma} - \ln(r_2) r_2^{\sigma} \geq 0. \tag{23}$$

- $r_1, r_2 \geq 1$. $\frac{\mathrm{d}D}{\mathrm{d}\sigma} \geq 0 \Leftrightarrow \frac{r_1^{\sigma}}{r_2^{\sigma}} \geq \frac{\ln(r_2)}{\ln(r_1)}$. The inequality always holds.
- $r_1 \geq 1, r_2 < 1$. The inequality always holds.
- $r_1, r_2 < 1$. $\frac{\mathrm{d}D}{\mathrm{d}\sigma} \geq 0 \Leftrightarrow \frac{r_1^{\sigma}}{r_2^{\sigma}} \leq \frac{\ln(r_2)}{\ln(r_1)} \Rightarrow \sigma \leq \bar{\sigma} := \frac{\ln\left(\frac{\ln(r_2)}{\ln(r_1)}\right)}{\ln\left(\frac{r_1}{r_2}\right)}$. The inequality holds as long as the elasticity of substitution is not too high.

Besides, $\lim_{\sigma \to 0} D(\sigma) = 0$.

A.3 Welfare Analysis

A.3.1 Lump-Sum Transfer

Welfare Optimization

$$\max_{l^A} \quad W^{\star}(l^A) = \left(\frac{1}{2} \cdot u_A^{\star}(l^A)^{\eta} + \frac{1}{2} \cdot u_B^{\star}(l^A)^{\eta}\right)^{\frac{1}{\eta}}. \tag{24}$$

First-order condition:

$$\begin{aligned}
& & \frac{\partial W^{\star}(l^A)}{\partial l^A} &= 0; \\
\Leftrightarrow & \quad \frac{1}{\eta}\left(\frac{1}{2} \cdot u_A^{\star\eta} + \frac{1}{2} \cdot u_B^{\star\eta}\right)^{\frac{1-\eta}{\eta}} & \left(\frac{\eta}{2} u_A^{\star\eta-1} u_A^{\star\prime} + \frac{\eta}{2} u_B^{\star\eta-1} u_B^{\star\prime}\right) &= 0; \\
\Leftrightarrow & & u_A^{\star\eta-1} u_A^{\star\prime} + u_B^{\star\eta-1} u_B^{\star\prime} &= 0; \\
\Leftrightarrow & & \left(\frac{u_B^{\star}}{u_A^{\star}}\right)^{1-\eta} &= -\frac{u_B^{\star\prime}}{u_A^{\star\prime}}; \\
\Leftrightarrow & & \frac{u_B^{\star}}{u_A^{\star}} &= \left(-\frac{u_B^{\star\prime}}{u_A^{\star\prime}}\right)^{\theta}.
\end{aligned} \tag{25}$$

[22] Given by $\sigma := \frac{1}{1-\rho}$.

The MSRS is by definition the marginal increase of the utility of the agent B when we decrease marginally the utility of the agent A, along a social indifference curve; $\text{MSRS}\rho \left| -\frac{\mathrm{d}u^B}{\mathrm{d}u^A} \right|$. It makes no difficulties to show that it equals the ratio of social marginal welfare with respect to A above the one with respect to B; $\text{MSRS} = \frac{\partial W/\partial u_A}{\partial W/\partial u_B}$.

Let us differentiate the optimal welfare;

$$\mathrm{d}W^\star(l^A) = \frac{\partial W(\cdot)}{\partial u_A}\frac{\partial u_A}{\partial l^A}\mathrm{d}l^A + \frac{\partial W(\cdot)}{\partial u_B}\frac{\partial u_B}{\partial l^A}\mathrm{d}l^A. \tag{26}$$

Along a social indifference curve, the welfare is constant, then:

$$\begin{aligned} \frac{\partial W(\cdot)}{\partial u_A}\frac{\partial u_A}{\partial l^A}\mathrm{d}l^A + \frac{\partial W(\cdot)}{\partial u_B}\frac{\partial u_B}{\partial l^A}\mathrm{d}l^A &= 0; \\ \Leftrightarrow \qquad \frac{\partial W/\partial u_A}{\partial W/\partial u_B} &= -\frac{u_B^{\star\prime}}{u_A^{\star\prime}}. \end{aligned} \tag{27}$$

Let λ be the utility ratio; $\lambda := \frac{u_B}{u_A}$. λ is a function of the optimal MSRS; $\lambda = \text{MSRS}^{\star\theta}$.

Variation of the inequality aversion Let us differentiate the optimal utility ratio with respect to the elasticity of substitution;

$$\frac{\mathrm{d}\lambda^\star}{\mathrm{d}\theta} = \text{MSRS}^{\star\theta}\ln\left(\text{MSRS}^{\star\theta}\right). \tag{28}$$

Note that a MSRS higher than one corresponds to a λ higher than one too. Then, the optimal utility ratio depends positively on the elasticity of substitution (negatively on the inequality aversion) if it is originally higher than one, and vice versa.

Note that if the optimal MSRS is equal to one, the utility ratio is equal to one too. The optimal ratio is then indifferent to the inequality aversion in such a case.

A.4 Evolution of the Resource

A.4.1 Lump-Sum Transfer

We are considering that the agent A keeps a fixed utility, and we study how evolves the optimal utility of the agent B.[23]

[23] Note that doing the converse is equivalent.

The worker keeps constant his utility

$$
\begin{aligned}
& \frac{\mathrm{d}u_A^\star}{\mathrm{d}X} = 0;\\
\Leftrightarrow\quad & \frac{\partial u_A^\star}{\partial l^A}\frac{\mathrm{d}l^A}{\mathrm{d}X} + \frac{\partial u_A^\star}{\partial c^A}\frac{\mathrm{d}c^{A\star}}{\mathrm{d}X} = 0;\\
\Leftrightarrow\quad & \underbrace{\frac{\partial u_A^\star}{\partial l^A}}_{>0}\frac{\mathrm{d}l^A}{\mathrm{d}X} + \underbrace{\frac{\partial u_A^\star}{\partial c^A}}_{>0}\\
& \times\left(\left(\underbrace{\Omega(\cdot)}_{>0} + l^A\cdot\underbrace{\Omega^{-1\prime}(\cdot)}_{>0}\cdot\underbrace{\left(-\frac{\partial^2 f\left(l^A,X\right)}{\left(\partial l^A\right)^2}\right)}_{=0}\right)\frac{\mathrm{d}l^A}{\mathrm{d}X} + l^A\cdot\underbrace{\Omega^{-1\prime}(\cdot)}_{>0}\cdot\underbrace{\left(-\frac{\partial^2 f\left(l^A,X\right)}{\partial X\partial l^A}\right)}_{>0}\right) = 0;
\end{aligned}
\tag{29}
$$

$$\Rightarrow \frac{\mathrm{d}l^{A\star}}{\mathrm{d}X} < 0.$$

Evolution of the utility of the receiver The evolution of the utility of the agent B is given by;

$$\frac{\mathrm{d}u_B^\star}{\mathrm{d}X} = \frac{\partial u_B^\star}{\partial c^B}\frac{\mathrm{d}c^B}{\mathrm{d}X} = \underbrace{\frac{\partial u_B^\star}{\partial c^B}}_{>0}\left(\underbrace{\frac{\mathrm{d}c^\star}{\mathrm{d}X}}_{>0} - \underbrace{\frac{\mathrm{d}c^{A\star}}{\mathrm{d}X}}_{>0}\right). \tag{30}$$

We have to know when the inequality $\frac{\mathrm{d}c^\star}{\mathrm{d}X} > \frac{\mathrm{d}c^{A\star}}{\mathrm{d}X}$ holds. Note that $f\left(l^A,X\right) = \left(1-l^A\right)g(X)$. The previous inequality becomes

$$
\begin{aligned}
& -g(X)\frac{\mathrm{d}l^A}{\mathrm{d}X} + \left(1-l^A\right)g'(X) > \Omega(\cdot)\frac{\mathrm{d}l^A}{\mathrm{d}X} + l^A\cdot\Omega^{-1\prime}(\cdot)g'(X);\\
\Leftrightarrow\quad & \left(1-l^A\right)g'(X) > \left(\Omega(\cdot)+g(X)\right)\frac{\mathrm{d}l^A}{\mathrm{d}X} + l^A\cdot\Omega^{-1\prime}(\cdot)g'(X).
\end{aligned}
\tag{31}
$$

Besides, the equation (29) can be rewritten as

$$\frac{\mathrm{d}l^A}{\mathrm{d}X} = -\frac{\frac{\partial u_A^\star}{\partial c^A}\cdot l^A\cdot\Omega^{-1\prime}(\cdot)g'(X)}{\frac{\partial u_A^\star}{\partial l^A} + \frac{\partial u_A^\star}{\partial c^A}\cdot\Omega(\cdot)}. \tag{32}$$

Substitute it into the inequality (31), we have

$$
\begin{aligned}
& \left(1-l^A\right) g'(X) > (\Omega(\cdot)+g(X)) \left(-\frac{\frac{\partial u_A^\star}{\partial c^A} \cdot l^A \cdot \Omega^{-1\prime}(\cdot) g'(X)}{\frac{\partial u_A^\star}{\partial l^A} + \frac{\partial u_A^\star}{\partial c^A} \cdot \Omega(\cdot)}\right) + l^A \cdot \Omega^{-1\prime}(\cdot) g'(X); \\
\Leftrightarrow \quad & \frac{1-l^A}{l^A} > -(\Omega(\cdot)+g(X)) \frac{\frac{\partial u_A^\star}{\partial c^A} \cdot \Omega^{-1\prime}(\cdot)}{\frac{\partial u_A^\star}{\partial l^A} + \frac{\partial u_A^\star}{\partial c^A} \Omega(\cdot)} + \Omega^{-1\prime}(\cdot); \\
\Leftrightarrow \quad & l^A < \frac{1}{1+\Omega^{-1\prime}(\cdot) - \frac{\frac{\partial u_A^\star}{\partial c^A} \cdot \Omega^{-1\prime}(\cdot)(\Omega(\cdot)+g(X))}{\frac{\partial u_A^\star}{\partial l^A} + \frac{\partial u_A^\star}{\partial c^A} \cdot \Omega(\cdot)}}; \\
\Leftrightarrow \quad & l^A < \frac{\frac{\partial u_A^\star}{\partial l^A} + \frac{\partial u_A^\star}{\partial c^A} \cdot \Omega(\cdot)}{\frac{\partial u_A^\star}{\partial l^A}\left(1+\Omega^{-1\prime}(\cdot)\right) + \frac{\partial u_A^\star}{\partial c^A}\left(\Omega(\cdot) - \Omega^{-1\prime}(\cdot) g(X)\right)}. \qquad (33)
\end{aligned}
$$

Note that, under our assumptions; $\frac{\frac{\partial u_A}{\partial l^A}}{\frac{\partial u_A}{\partial c^A}} = g(X)$. Thus,

$$
\frac{\mathrm{d}u_B^\star}{\mathrm{d}X} > 0 \quad \Leftrightarrow \quad l^A < 1. \qquad (34)
$$

A.4.2 Tax

We seek the tax rate that makes the agent A indifferent to a variation of the resource stock. Doing so, the agent has no reason to change his offer of labor.[24]

$$
\frac{\mathrm{d}u_A^{\star\star}}{\mathrm{d}X} = \underbrace{\frac{\partial u_A^{\star\star}}{\partial l^A}}_{>0} \underbrace{\frac{\mathrm{d}l^A}{\mathrm{d}X}}_{=0} + \underbrace{\frac{\partial u_A^{\star\star}}{\partial c^A}}_{>0} \underbrace{\frac{\mathrm{d}c^{A\star\star}}{\mathrm{d}X}}_{>0} + \underbrace{\frac{\partial u_A^{\star\star}}{\partial \tau}}_{<0} \frac{\mathrm{d}\tau}{\mathrm{d}X} = 0. \qquad (35)
$$

$$
\Rightarrow \quad \frac{\mathrm{d}\tau}{\mathrm{d}X} > 0. \qquad (36)
$$

$$
\frac{\mathrm{d}u_B^{\star\star}}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_B^{\star\star}}{\partial c^B}}_{>0} \cdot \underbrace{\frac{\mathrm{d}c^{B\star\star}}{\mathrm{d}X}}_{>0} \cdot \underbrace{\frac{\mathrm{d}X}{\mathrm{d}\tau}}_{>0} > 0. \qquad (37)
$$

[24] Contrary to the case of absolute transfer, here the shape of the budget constraint remains constant.

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