Sustainable growth[*]

Geir B. Asheim[1]

January 20, 2016

Abstract

This paper explores the view that a criterion of intergenerational equity serves to make choices according to ethical intuitions on a domain of relevant technological environments. In line with this view I first calibrate different criteria of intergenerational equity in the AK model of economic growth, with a given productivity parameter A, and then evaluate their performance by mapping the consequences of the criteria in various technological environments. The evaluation is based on the extent to which they yield social choice mappings satisfying four desirable properties. The *Calvo criterion* as well as *sustainable discounted utilitarianism* and *rank-discounted utilitarianism* yield sustainable growth in the AK model, the Ramsey technology and the Dasgupta-Heal-Solow-Stiglitz technology for any specifications of these technological environments.

Keywords and Phrases: Intergenerational equity.
JEL Classification Numbers: D63, D71, O41, Q01.

[1]*Address*: Department of Economics, University of Oslo, P.O. Box 1095 Blindern, N-0317 Oslo, Norway. *Email*: g.b.asheim@econ.uio.no

1 Introduction

In a series of papers (Llavador, Roemer and Silvestre, 2010, 2011, 2013; Roemer, 2011, 2013) on the ethics of intertemporal distribution in a warming planet, John Roemer and his co-authors advocate a criterion of intergenerational distribution that allows for sustainable growth. Specifically, they suggest that current wellbeing[1] be maximized subject to assuring that wellbeing grows at rate g forever. Formally, in their *sustainable growth* criterion, a wellbeing stream $_1x = (x_1, x_2, \ldots, x_t, \ldots) \geq 0$ is selected to solve the following program (referred to as the g-SUS program):

$$\max \Lambda \quad \text{subject to} \quad _1x \text{ being feasible} \quad \text{and} \quad x_t \geq \Lambda(1+g)^{t-1} > 0 \text{ for all } t. \quad (1)$$

The growth rate g of wellbeing is assumed to be non-negative.

In general, John Roemer supports extreme egalitarianism in the sense of maximizing the wellbeing of the worst-off individual. However, like another proponent of the maximin criterion, John Rawls (1971), Roemer is willing to depart from the maximin criterion in the case of intergenerational distribution. He writes (Roemer, 2011, p. 378):

> A possible justification for choosing g greater than zero is that humans want their children to be better off than they are; indeed, they are willing to sacrifice their own [wellbeing] to make this possible—or, to state this less personally (so that childless adults are included) each generation wants 'human development' to take place, in the sense of increasing generational [wellbeing].

His position is clarified in his response (Roemer, 2013, p. 146) to Dasgupta (2011):

> No generation has an ethical license to violate the rights of future generations by choosing to discount their utility at a low rate, if doing so would render them worse off than their right grants them; but each generation does have the ethical permit not to enforce its own right to enjoy as much [wellbeing] as future generations. A person can voluntarily abstain for enforcing a right that applies to him, but he is not entitled to abrogate the rights of others.

This paper discusses how to capture Roemer's position through criteria of intergenerational equity.

---

[1]I use the term 'wellbeing' for what Roemer and his co-authors refer to as 'welfare'. It is meant to indicate the current living situation and thus includes more than material consumption. Sentiments like altruism is, however, assumed not to be included in this indicator.

My discussion is not based on an axiomatic approach. Rather, I explore the view that a criterion of intergenerational equity serves to make choices according to ethical intuitions on a domain of relevant technological environments. Each technological environment combined with an initial condition gives rise to a set of feasible streams of wellbeing. The role of the criterion is, according to this view, to select the first-ranked stream according to a ranking of wellbeing streams in each set of feasible streams, thus mapping an ethically commendable social choice as a function of sets of feasible streams in a domain of relevant technological environments.

In other words, in the setting of the present paper, criteria of intergenerational equity should be interpreted in an "as-if" sense: The ethically commendable social choice is made "as-if" the choice is selected by a criterion of intergenerational equity. My approach is based on *welfarism* (Sen, 1979) as the selection made by a criterion depends on a fixed ranking of wellbeing streams; hence, the ranking does not change with the sets of feasible streams as determined by the technological environments. Thus, widening the domain of technological environments increases the need for a criterion to be versatile. I understand that this represents a form of "intuitionism" (see Rawls, 1971, p. 34, and Dasgupta, 2011, p. 478) that John Roemer may not endorse.

When the sustainable growth criterion is used in Llavador, Roemer and Silvestre (2010, 2011), it is applied in a technological environment where the wellbeing function is linearly homogeneous, and where output is a constant-returns-to-scale function of inputs that grow with rate g along an efficient balanced growth path.[2] This means that the technological environments in which Llavador, Roemer and Silvestre apply the sustainable growth criterion essentially correspond to an AK model:

$$y_t = x_t + k_t = A \cdot k_{t-1} \quad \text{for all } t \geq 1, \text{ with } k_0 > 0 \text{ given,}$$

where A is an exogenous gross productivity parameter greater than 1, and where k_t and y_t denotes, respectively, capital and gross production as time t. In an efficient balanced growth path in the AK model, gross production, capital and wellbeing increase with a constant non-negative rate g smaller than $A - 1$.

Assume now that the gross productivity parameter takes on a particular value A^* and that in this technological environment it appeals to ethical intuitions that wellbeing

---

[2] The analysis of the sustainable growth criterion in Llavador, Roemer and Silvestre (2010) is based on a conjectured 'turnpike' result, entailing that such an efficient balanced growth path is approached when the inputs initially are not in the proportions needed for efficient balanced growth. In Llavador, Roemer and Silvestre (2011) there is in addition a stock of CO_2 in the atmosphere which is constant along the efficient balanced growth path.

grows by rate $g^* > 0$, where the condition $g^* < A^* - 1$ ensures that the resulting balanced growth path is feasible and efficient. In line with Roemer's position (see the quotes above) this may correspond to a belief that each generation t is willing to sacrifice some of their own wellbeing, compared to the maximum level $A^* \cdot k_{t-1} - k_{t-1}$ determined from their obligation not to violate the rights of future generations. It means that each generation chooses the same positive savings rate s_t (defined as the ratio of net capital accumulation and net production) at each time t:

$$s_t = \frac{k_t - k_{t-1}}{y_t - k_{t-1}} = \frac{(1 + g^*)k_{t-1} - k_{t-1}}{A^* \cdot k_{t-1} - k_{t-1}} = \frac{g^*}{A^* - 1}.$$

In the AK model where the gross productivity parameter equals A^*, it turns out that an efficient balanced growth path with constant growth rate g^* can be selected by a number of different criteria of intergenerational equity – in addition to the sustainable growth criterion with growth rate g^*, where a wellbeing stream is selected to solve the g^*-SUS program (1) – provided that their functional forms and the parameters are appropriately calibrated. Hence, the position that a given sustainable growth stream is a commendable ethical social choice in this particular technological environment cannot be used to evaluate the different criteria. Rather, such an evaluation can be done either by investigating the axiomatic basis—which is *not* the subject of the current paper—or by mapping the consequences of the different criteria in various technological environments. In the present paper I follow the latter strategy for evaluating criteria of intergenerational equity.

First, in Section 2 I calibrate undiscounted (or classical) utilitarianism, time-discounted utilitarianism, sustainable discounted utilitarianism (Asheim and Mitra, 2010), rank-discounted utilitarianism (Zuber and Asheim, 2012), the Calvo (1978) criterion and the Chichilnisky (1996) criterion so that they all lead to a constant growth rate g^* when the gross productivity parameter equals to A^*. Then, in accordance with welfarism, I investigate how these criteria – as calibrated to yield $g = g^*$ when $A = A^*$ – select optimal wellbeing streams in other technological environments: In Section 3 I consider the consequences of varying the gross productivity parameter A. In Section 4 I explore the consequences of these criteria in the Ramsey technology, while in Section 5 I do the same in the Dasgupta-Heal-Solow-Stiglitz technology of capital accumulation and resource depletion.

In the concluding Section 6 I discuss the result that three of the criteria – sustainable discounted utilitarianism, rank-discounted utilitarianism and the Calvo criterion – satisfy a number of desirable properties in all the technological environments that I

3

will consider: These three criteria are

- *effective* in the sense of making a unique selection of a time consistent stream,

- *non-wasteful* in the sense that the selected stream is efficient,

- *flexible* in the sense that the growth rate of the selected stream responds to changes in the technological environment,

- *sustainable* in the sense that the selected stream respect the rights of future generations by not rendering them worse off than the present generation.

In contrast, the other criteria that are treated in this paper seem not to be able to support ethical intuitions – as captured by the four properties above – over all the "relevant" technological environments. Finally, I argue that the SDU criterion or the Calvo criterion might be the ones that best correspond to Roemer's motivation for a positive growth rate g, namely that the generations are willing to sacrifice their own wellbeing for the benefit of future generations.

2 Calibration in the AK model

The purpose of this section is to calibrate the following criteria of intergenerational equity (they are presented in Appendix A),

(a) undiscounted utilitarianism (UU),

(b) time-discounted utilitarianism (TDU),

(c) sustainable discounted utilitarianism (SDU),

(d) rank-discounted utilitarianism (RDU),

(e) the Calvo criterion,

(f) the Chichilnisky criterion.

The criteria are calibrated so that they all lead to a constant growth rate g^* in the AK model when the gross productivity parameter equals A^*, where $0 < g^* < A^* - 1$. It is taken as a datum that $g = g^*$ when $A = A^*$ appeals to ethical intuitions.

Such a calibration is achieved by using a parameterized constant elasticity version of the function u that turns wellbeing x of each generation into transformed wellbeing (or generalized utility) $u(x)$:

$$u(x) = \frac{x^{1-\eta}}{1-\eta} \text{ , where } \eta > 0,$$

and where the case with $\eta = 1$ corresponds to $u(x) = \ln x$. Under criteria (a)–(d)+(f), the strict concavity of u (corresponding to $\eta > 0$) is an expression of inequality aversion between generations in social evaluation. In criterion (e) – the Calvo criterion – the strict concavity is an expression of inequality aversion between one generation and its successors in the "private" altruistic evaluation of the current generation. It is part of the calibration process to allow η to vary across the different criteria.

For the analysis of efficient balanced growth paths in the AK model, it is convenient to introduce the concept of a competitive stream. First, note that the supporting prices $_1p = (p_1, p_2, \dots)$ of wellbeing is exogenously determined due to the linearity of the AK technology:

$$p_t = \left(\tfrac{1}{A}\right)^t \text{ for all } t \geq 1.$$

A feasible wellbeing stream $_1x$ is *competitive with supporting prices* $_1\mu = (\mu_1, \mu_2, \dots)$ *of generalized utility* if,

$$x_t \text{ maximizes } \mu_t u(\tilde{x}) - p_t \tilde{x} \text{ over all wellbeing levels } \tilde{x}, \text{ for all } t \geq 1.$$

For any feasible stream $_1x$, we have that $k_0 = \sum_{t=1}^{\tau} p_t x_t + p_\tau k_\tau$ for all $\tau \geq 1$. Hence, if a wellbeing stream $_1x$ is competitive with supporting prices $_1\mu$, then

$$\sum_{t=1}^{\tau} \mu_t \big(u(x_t) - u(\tilde{x}_t)\big) \geq \sum_{t=1}^{\tau} p_t \big(x_t - \tilde{x}_t\big) \geq -p_\tau k_\tau$$

for all feasible streams $_1\tilde{x}$, by the definition of competitiveness since $k_0 = \tilde{k}_0$ and $p_\tau \tilde{k}_\tau \geq 0$. Therefore, if $p_\tau k_\tau \to 0$ as $\tau \to \infty$ holds as a transversality condition, then

$$\liminf_{\tau \to \infty} \sum_{t=1}^{\tau} \mu_t \big(u(x_t) - u(\tilde{x}_t)\big) \geq 0 \tag{2}$$

for all feasible streams $_1\tilde{x}$. Furthermore, by the iso-elastic form of the u-function with $\eta > 0$, it follows that $x_t > 0$ and $\mu_t u'(x_t) = p_t$ for all $t \geq 1$, since u is continuously differentiable and strictly concave with $\lim_{x \to 0} u'(x) = \infty$. Furthermore, $_1x$ is the unique stream having property (2) for all feasible streams $_1\tilde{x}$.

Note that if Λ^* is the maximum value of program (1) in the AK model, with $A = A^*$, $g = g^* \in (0, A^* - 1)$, and initial stock k_0 given, then

$$\Lambda^* = (A^* - 1 - g^*) k_0$$

and

$$p_\tau k_\tau = \left(\tfrac{1+g^*}{A^*}\right)^\tau \cdot k_0 \to 0 \text{ as } \tau \to \infty.$$

For the rest of this section, let $_1x$ denote the wellbeing stream defined by $x_t = \Lambda^*(1 + g^*)^{t-1}$ for all $t \geq 1$.

In the criterion of *undiscounted utilitarianism* (UU), wellbeing is transformed through a continuous, increasing and strictly concave u-function and summed *without* discounting. The strict concavity of the u-function leads to a decreasing marginal generalized utility along the geometrically growing wellbeing stream which counters the effect of having a gross rate of productivity A^* greater than one.

To determine how UU can select an efficient balanced growth path in the AK model, combine $x_t = \Lambda^*(1 + g^*)^{t-1}$ for all $t \geq 1$ with the requirement that $_1x$ be a competitive stream satisfying the condition that $\mu_t = \mu$ (constant) for all $t \geq 1$. Since u is continuously differentiable and strictly concave with $\lim_{x \to 0} u'(x) = \infty$ and $\lim_{x \to \infty} u'(x) = 0$, we must have that $\mu u'(x_t) = p_t$ for all $t \geq 1$, so that

$$(1 + g^*)^{-\eta} = \frac{x_{t+1}^{-\eta}}{x_t^{-\eta}} = \frac{u'(x_{t+1})}{u'(x_t)} = \frac{p_{t+1}}{p_t} = \frac{1}{A^*} \tag{3}$$

for all $t \geq 1$. This implies that the parameter of inequality aversion, η_U, in the case of UU is given by:

$$\eta_U = \frac{\ln A^*}{\ln(1 + g^*)}.$$

Since $0 < \ln(1 + g^*) < \ln A^*$, we have that $\eta_U > 1$. Furthermore, straight-forward calculations yield that

$$\mu = \frac{\left((A^* - 1 - g^*)k_0\right)^{\eta_U}}{A^*}.$$

The criterion of *time-discounted utilitarianism* (TDU) discounts future generalized utility by a discount factor $\rho \in (0, 1)$. Hence, in this criterion wellbeing is transformed through a continuous, increasing and strictly concave u-function and summed *with* discounting. The concavity of the u-function leads to a decreasing marginal generalized utility which together with a discount factor ρ smaller than one counters the effect of having a gross rate of productivity A^* greater than one. This means that the u-function will be less concave than in the case of UU.

To determine how TDU can select an efficient balanced growth path in the AK model, combine $x_t = \Lambda^*(1 + g^*)^{t-1}$ for all $t \geq 1$ with the requirement that $_1x$ be a competitive stream satisfying $\mu_t = \mu \rho^{t-1}$ (decreasing) for all $t \geq 1$. By the properties of the u-function, we must have that $\mu_t u'(x_t) = p_t$ for all $t \geq 1$, so that

$$\rho(1 + g^*)^{-\eta} = \frac{\rho x_{t+1}^{-\eta}}{x_t^{-\eta}} = \frac{\rho u'(x_{t+1})}{u'(x_t)} = \frac{p_{t+1}}{p_t} = \frac{1}{A^*} \tag{4}$$

for all t. This implies that the parameter of inequality aversion, η_T, in the case of TDU is given by:

$$\eta_T = \frac{\ln A^* + \ln \rho}{\ln(1 + g^*)}. \tag{5}$$

where ρ satisfies

$$\tfrac{1}{A^*} < \rho < 1 \,.$$

This condition implies that $\ln A^* + \ln \rho > 0$ to ensure that the u-function is strictly concave. Note that the u-function is less concave with discounting (i.e., $\eta_T < \eta_U$) as $\ln \rho < 0$. In particular, if $1 < A^* \rho \leq (1 + g^*)$, then $\eta_T \leq 1$. Hence, the requirement that η_U be greater than 1 does not hold for η_T. Furthermore,

$$\mu = \frac{\left((A^* - 1 - g^*)k_0\right)^{\eta_T}}{A^*} \,.$$

Appendix A presents the *sustainable generalized discounted utilitarian* (SDU) criterion (Asheim and Mitra, 2010) and the *rank-discounted generalized utilitarian* (RDU) criterion (Zuber and Asheim, 2012). Also in these criteria, the wellbeing x_t of each generation t is transformed into generalized utility $u(x_t)$ by an increasing, strictly concave and continuously differentiable function u. Both SDU and RDU discount future generalized utility by a discount factor $\rho \in (0, 1)$, as long as the future is better off than the present, thereby trading-off current sacrifice and future gain. Hence, if wellbeing is perfectly correlated with time, these criteria work as TDU. The important difference is that, in the criteria of SDU and RDU, the future is discounted because priority is given to the worse off earlier generations.

However, if the present is better off than the future, then priority shifts to the future. In this case, future generalized utility is not discounted, implying that zero relative weight is assigned to present wellbeing. The criteria of SDU and RDU can therefore capture the intuition that we should seek to assist future generations if they are worse off than us, while not having an unlimited obligation to save for their benefit if they turn out to be better off. Also, RDU is compatible with equal treatment of generations by being insensitive to all re-orderings of the wellbeing stream.

In the present case of the wellbeing stream $_1x$ defined by $x_t = \Lambda^*(1 + g^*)^{t-1}$ for all $t \geq 0$, wellbeing is perfectly correlated with time. Hence, SDU and RDU yields the same result as TDU: The stream $_1x$ is the unique SDU optimum and the unique RDU optimum if the u-function is parameterized by $\eta_S = \eta_R = \eta_T$ as given by eq. (5).

To formally demonstrate that $_1x$ is a SDU when the u function is parameterized in this way, note first that it follows from Asheim and Mitra (2010, Proposition 2(i)) that TDU welfare $w_\rho^T(_1\tilde{x})$ is always as great as SDU welfare $w_\rho^S(_1\tilde{x})$ for any wellbeing stream $_1\tilde{x}$, where the welfare functions w_ρ^T and w_ρ^S are defined in Appendix A. However, as stated in Asheim and Mitra (2010, Proposition 2(ii)) $w_\rho^S(_1x^+) = w_\rho^T(_1x^+)$ for a non-decreasing stream $_1x^+$ like $_1x$. Finally, it follows from the fact that $_1x$ is the unique

TDU optimum that $w_\rho^T(_1x) > w_\rho^T(_1\tilde{x})$ for any feasible stream $_1\tilde{x}$ not identical to $_1x$. Combining these results yields:

$$w_\rho^S(_1x) = w_\rho^T(_1x) > w_\rho^T(_1\tilde{x}) \geq w_\rho^S(_1\tilde{x})$$

for any feasible stream $_1\tilde{x}$ not identical to $_1x$, there establishing that $_1x$ is the unique SDU optimum if the u-function is parameterized by $\eta_S = \eta_T$ as given by eq. (5).

The corresponding result for RDU optimality can be established by adapting Zuber and Asheim (2012, Proposition 10) to the AK model.

Turn next to the *Calvo criterion* (Calvo, 1978) which maximizes the infimum of TDU welfare of all generations. It applies the maximin criterion, not on the wellbeing stream $_1x$, but on the stream of TDU welfare $_1w = (w_1, w_2, \dots)$. The interpretation is that each generation exhibits a simple recursive form of *nonpaternalistic* altruism (Ray, 1987), where the welfare of each generation t is an additively separable function of its own wellbeing and the welfare of the next generation $t + 1$:

$$w_t = (1 - \rho)u(x_t) + \rho w_{t+1}\,.$$

Thus, the welfare of each generation t, w_t, equals $w_\rho^T(_tx)$.

Along the increasing stream $_1x$, infimum of TDU welfare over all generations equals $w_\rho^T(_1x)$, the welfare of generation 1. However, $_1x$ uniquely maximizes $w_\rho^T(_1x)$ if the u-function is parameterized by η_T. It is therefore a trivial observation that $_1x$ is the unique Calvo optimum with ρ satisfying $1/A^* < \rho < 1$ and η_T being given by (5).

The *Chichilnisky criterion* (Chichilnisky, 1996) evaluates streams according to a convex combination of TDU welfare and the limit of transformed wellbeing as time goes to infinity. It might be interpreted as a convex combination of TDU and the long-term average criterion, thereby avoiding dictatorship of both the present and the future. It follows that $_1x$ is the unique Chichilnisky optimum with ρ satisfying $1/A^* < \rho < 1$ and η_T being given by (5), since then $_1x$ is a TDU optimum and $\lim_{t\to\infty} x_t = \infty$.

The results of the present section can be summarized as follow:

Proposition 1 *Let $_1x$ denote the wellbeing stream selected by the sustainable growth criterion in the AK model, when the gross productivity parameter equals A^*, and the growth rate in program (1) equals g^*, where $0 < g^* < A^* - 1$. It is taken as a datum that $g = g^*$ when $A = A^*$ appeals to ethical intuitions. The UU, TDU, SDU, RDU, Calvo and Chichilnisky criteria can be calibrated so that they all uniquely select $_1x$ as the optimal wellbeing stream.*

3 Varying the rate of productivity

In the present section I investigate how the different criteria behave when the gross productivity parameter A of the AK is varied and takes on values that differ from A^*, but where I stick to the calibrations obtained in Section 2. These criteria are, in addition to the sustainable growth criterion with growth rate g^*: (a) UU, (b) TDU, (c) SDU, (d) RDU, (e) the Calvo criterion, and (f) the Chichilnisky criterion

In the sustainable growth criterion where a wellbeing stream is selected to solve the g^*-SUS program (1), a balanced growth path with $\Lambda > 0$ exists if and only if $A > 1 + g^*$. Furthermore, this solution is time consistent. Thus, sustainable growth criterion is *effective* and *non-wasteful* under this condition, but not otherwise. There is a balanced growth path also if $A = 1 + g^*$, but then the savings rate

$$s_t = \frac{g^*}{A - 1} = 1 \quad \text{for all } t,$$

meaning that wellbeing equals zero at any point in time. Clearly, this is wasteful. The sustainable growth criterion is not *flexible* since the growth rate does not respond to different technological environments. Although the sustainable growth criterion is *sustainable* by respecting the rights of future generations whenever the rate of gross productivity is sufficiently high for the criterion to be *effective* and *non-wasteful*, it does not solve the intergenerational conflict for low rates of gross productivity.

By substituting A for A^* and η_U for η, the condition for competitiveness under UU is changed from (3) to:

$$(1 + g)^{-\eta_U} = \frac{x_{t+1}^{-\eta_U}}{x_t^{-\eta_U}} = \frac{u'(x_{t+1})}{u'(x_t)} = \frac{p_{t+1}}{p_t} = \frac{1}{A}.$$

Solving for g yields:

$$g_U = A^{\frac{1}{\eta_U}} - 1 = A^{\frac{\ln(1+g^*)}{\ln A^*}} - 1.$$

This uniquely determines the efficient balanced growth path, $_1 x^U$, defined by $x_t^U = \Lambda_U (1 + g_U)^{t-1}$ for all $t \geq 1$, where $\Lambda_U = (A - 1 - g_U) k_0$. Since $A > 1$ and $0 < \ln(1 + g^*) < \ln A^*$, it follows that $0 < g_U < A - 1$. Hence, UU is *effective* and *non-wasteful* for all $A > 1$. Furthermore, it is *flexible* since the growth rate g_U responds to changes in A, with a higher A leading to a higher g_U. Finally, the UU criterion is *sustainable* for all $A > 1$ in the sense of respecting the rights of future generations.

By substituting A for A^* and η_T for η, the condition for competitiveness under TDU

is changed from (4) to:

$$\rho(1+g)^{-\eta_T} = \frac{\rho x_{t+1}^{-\eta_T}}{x_t^{-\eta_T}} = \frac{\rho u'(x_{t+1})}{u'(x_t)} = \frac{p_{t+1}}{p_t} = \frac{1}{A}.$$

Solving for g yields:

$$g_T = (A\rho)^{\frac{1}{\eta_T}} - 1 = (A\rho)^{\frac{\ln(1+g^*)}{\ln A^*}} - 1.$$

This uniquely determines the efficient balanced growth path, $_1x^T$, defined by $x_t^T = \Lambda_T(1+g_T)^{t-1}$ for all $t \geq 1$, where $\Lambda_T = (A - 1 - g_T)k_0$. Since $0 < \ln(1+g^*) < \ln A^*$, it follows that $0 < g_T < A\rho - 1 < A - 1$ if $A > 1/\rho$, $g_T = 0$ if $A = 1/\rho$ and $g_T < 0$ if $A < 1/\rho$. Hence, TDU is *effective* and *non-wasteful* for all $A > 1$. Furthermore, it is *flexible* since the growth rate g_T responds to changes in A, with a higher A leading to a higher g_T. However, the TDU criterion is *sustainable* in the sense of respecting the rights of future generations only if $A \geq 1/\rho$, but not if $1 < A < 1/\rho$.

It now follows from the argument for SDU optimality in Section 2 that $_1x^T$ is SDU optimal whenever the efficient balanced growth path is increasing (i.e., in the case where $A > 1/\rho$), because then the inequalities $w_\rho^S(_1x^T) = w_\rho^T(_1x^T) > w_\rho^T(_1\tilde{x}) \geq w_\rho^S(_1\tilde{x})$ hold for any feasible stream $_1\tilde{x}$ not identical to $_1x^T$. However, for $A \leq 1/\rho$, then it follows from Asheim and Mitra (2010, Proposition 3) that SDU optimality corresponds to the equalitarian stream $_1x^e$, defined by $x_t^e = (A - 1)k_0$ for all $t \geq 1$, ensuring that capital is maintained at its original level ($k_t = k_0$ for all $t \geq 1$) and corresponding to a growth rate $g_S = 0$.[3] Hence, SDU is *effective* and *non-wasteful* for all $A > 1$. Furthermore, it is *flexible* since the growth rate g_S responds to changes in A for $A > 1/\rho$, with a higher A leading to a higher $g_S = g_T$, even though g_S is constant and equal to 0 for all $A \leq 1/\rho$. Finally, the SDU criterion is *sustainable* in the sense of respecting the rights of future generations for all $A > 1$.

The same conclusions hold for the RDU and Calvo criteria. Again, the result for RDU optimality can be established by adapting Zuber and Asheim (2012, Proposition 10) to the AK model.[4] The result for Calvo optimality can be established by adapting the proof of part (2) of Calvo (1978, Proposition 2) to the AK model. Hence, both the TDU and Calvo criteria are *effective* and *non-wasteful* for all $A > 1$, *flexible* in that the

---

[3]Conditions (2) and (3) of Asheim and Mitra (2010, Proposition 3) are satisfied since $p_t/p_{t-1} = 1/A \geq \rho$ for all $t \geq 1$, writing $p_0 = 1$, and $\sum_{t=1}^{\infty} p_t x_t^e = k_0 = \sum_{t=1}^{\tau} p_t \tilde{x}_t + p_\tau \tilde{k}_\tau \geq \sum_{t=1}^{\infty} p_t \tilde{x}_t$ for any feasible stream $_1\tilde{x}$ and for all $\tau \geq 1$.

[4]Zuber and Asheim (2012, Proposition 10) is based on Asheim (1991, Proposition 6), where the proof of Case 2 must be adapted to the AK model to show that $_1x^e$ maximizes $w_\rho^T(_1x^+)$ over all non-decreasing streams $_1x^+$ when $A \leq 1/\rho$.

sense the growth rate responds to changes in A if and only if $A > 1/\rho$, and *sustainable* in the sense of respecting the rights of future generations for all $A > 1$.

The TDU optimal stream $_1x^T$ is also Chichilnisky optimal if $A > 1/\rho$. However, if $A \leq 1/\rho$, there does not exist a Chichilnisky optimal stream. The problem is that the TDU part of the Chichilnisky criterion is increased by following $_1x^T$ for a longer period of time, before ensuring that $\lim_{t\to\infty} x_t = \infty$ to "maximize" the second limiting part of the Chichilnisky criterion. However, if $_1x^T$ is followed forever, then $\lim_{t\to\infty} x_t = (A-1)k_0$ for $A = 1/\rho$ and $\lim_{t\to\infty} x_t = 0$ for $A < 1/\rho$. Hence, the Chichilnisky criterion behaves like TDU for $A > 1/\rho$, while not being *effective* otherwise.

The results of the present section can be summarized as follow:

Proposition 2 *Consider the sustainable growth criterion as well as the UU, TDU, SDU, RDU, Calvo and Chichilnisky criteria with the calibrations from Section 2. Allow the gross productivity parameter A of the AK to be varied, taking on values that differ from A^*. Only the UU, SDU, RDU and Calvo criteria satisfy all four desirable properties in the AK model when the gross productivity parameter A is varied, by being effective, non-wasteful, flexible and sustainable.*

4 The Ramsey technology

Assume that the technology is given by an increasing, strictly concave, and twice continuously differentiable production function $f : \mathbb{R}_+ \to \mathbb{R}_+$, satisfying $f(0) = 0$ and $\lim_{k\to\infty} f'(k) = 0$. A wellbeing stream $_1x = (x_1, x_2, \dots) \geq 0$ is feasible given an initial capital stock $k_0 > 0$ if there exists a capital stream $_1k = (k_1, k_2 \dots) \geq 0$ such that

$$x_t + k_t = k_{t-1} + f(k_{t-1})$$

for all $t \geq 1$. This is the *Ramsey technology*, which is the standard technological environment for the analysis of economic growth.

With the assumption that the net marginal productivity of capital, $f'(k)$, approaches 0 as $k \to \infty$, it follows that the ratio of net product to capital, $f(k)/k$, approaches 0 as $k \to \infty$. The implication is that no stream with positive wellbeing growing at a constant positive rate g is feasible.[5] This leads to the conclusion that

---

[5]Asheim and Mitra (2010, Lemma 1) is a formal demonstration of this result, as $\sum_{t=1}^{\tau} \rho^{t-1} \Lambda (1+g)^{t-1}$ would diverge for any ρ satisfying $1/(1+g) \leq \rho < 1$ if a wellbeing stream defined by $x_t = \Lambda (1+g)^{t-1}$ for all t were feasible with $\Lambda > 0$ and $g > 0$.

the sustainable growth criterion of selecting a wellbeing stream that solves the g^*-SUS program (1) selects no stream in the Ramsey technology with this assumption on the net marginal productivity of capital. Hence, the sustainable growth criterion is not *effective* in the Ramsey technology.

Also the UU criterion need not be *effective* in the Ramsey technology, even with the requirement that η_U be greater than 1. A closed-form demonstration of this fact is most easily done in the continuous time version of the Ramsey technology: A non-negative wellbeing stream $(x(t))_{t=0}^{\infty}$ is feasible given an initial capital stock $k(0) > 0$ if there exists a non-negative capital stream $(k(t))_{t=0}^{\infty}$ such that

$$x(t) + \dot{k}(t) = f(k(t)) \tag{6}$$

for all $t \geq 0$. By giving the production function a iso-elastic form: $f(k) = k^{\alpha}$, where $0 < \alpha < 1$, the assumptions on f are fulfilled and a closed-form solution can be given. Note that the net marginal productivity of capital $f'(k)$ then equals $\alpha k^{\alpha-1}$, so that the Keynes-Ramsey rule becomes:

$$\alpha k(t)^{\alpha-1} = \eta_U \frac{\dot{x}(t)}{x(t)} \tag{7}$$

Combining the Keynes-Ramsey rule (7) with

$$\lim_{\tau \to \infty} e^{-\int_0^{\tau} \alpha k(t)^{\alpha-1}dt} k(\tau) = 0 \tag{8}$$

as a transversality condition is sufficient and necessary for UU optimality. The technological relation (6), where $f(k) = k^{\alpha}$, together with the Keynes-Ramsey rule (7) imply that the savings rate $s(t)$ defined by $\dot{k}(t) = s(t)f(k(t))$ is constant and equal to $1/\eta_U$.[6] Substituting $1/\eta_U$ for $s(t)$ and $k(t)^{\alpha}$ for $f(k(t))$ in $\dot{k}(t) = s(t)f(k(t))$ and integrating yields:

$$k(\tau) = e^{\int_0^{\tau} \frac{1}{\eta_U} k(t)^{\alpha-1}dt} k(0) \,.$$

Hence, for the transversality condition (8) to hold, it must be that $\alpha > 1/\eta_U$.

Hence, the UU criterion is *effective* if and only $\eta_U > 1/\alpha > 1$, implying that $\eta_U > 1$ is not sufficient for an UU optimum to exist. In this case, with $\eta_U > 1/\alpha$, the UU criterion is *non-wasteful* for all $k(0) > 0$, *flexible* in the sense that the growth rate responds to changes in $k(0)$, with a higher $k(0)$ leading to a lower growth rate due to lower net marginal productivity of capital. Finally, the UU criterion is *sustainable* for all $k(0) > 0$ in the sense of respecting the rights of future generations.

---

[6]Use (6) combined with $f(k) = k^{\alpha}$ and $\dot{k} = sf(k)$ to obtain $\dot{x}/x = s\alpha k^{\alpha-1}$ and insert in (7).

However, if $\eta_U \leq 1/\alpha$, then there exists no UU optimum. The problem is that short-run optimality, as dictated by the Keynes-Ramsey rule, leads to over-accumulation of capital. Hence, a given elasticity of marginal generalized utility $\eta_U > 1$ – as an expression of inequality aversion in social evaluation – fails to deliver a UU optimal stream in all the technological environments that satisfy the assumptions of the Ramsey technology. If $f(k)$ is interpreted as a reduced form constant-returns-to-scale production function of capital and labor, with the labor force constant and normalized to 1, then a sufficiently high functional share of capital α is needed for the UU criterion to be *effective* in the sense of being able to select an optimal stream.

Return now to the Ramsey technology in discrete time. For analyzing the properties of the remaining criteria in this technological environment, we need to define the *modified golden rule*. Define $k_\infty : (0, 1) \to \mathbb{R}_+$ by, for all $\rho \in (0, 1)$,

$$k_\infty(\rho) = \min\{k \geq 0 : \rho\left(1 + f'(k)\right) \leq 1\}.$$

It follows from the properties of f that k_∞ is well-defined and continuous, and it is strictly increasing for all ρ for which there exists $k \geq 0$ such that $\rho(1 + f'(k)) = 1$. For given $\rho \in (0, 1)$, $k_\infty(\rho)$ is the capital stock corresponding to the modified golden rule.

Beals and Koopmans (1969) show that there is a unique TDU optimal capital stream $_1k^T$, with associated TDU optimal wellbeing stream $_1x^T$, for given initial capital stock k_0. Furthermore, $_1k^T$ and $_1x^T$ are monotone, converging to $k_\infty(\rho)$ and $f(k_\infty(\rho))$, respectively. In particular, wellbeing is increasing if $k_0 < k_\infty(\rho)$, constant (and equal to $f(k_\infty(\rho))$) if $k_0 = k_\infty(\rho)$ and decreasing if $k_0 > k_\infty(\rho)$. Hence, TDU is *effective* and *non-wasteful* for all $k_0 > 0$. Furthermore, it is *flexible* in the sense that the growth rate responds to changes in k_0; in particular, increasing k_0 from below to above $k_\infty(\rho)$ changes positive growth in wellbeing to negative growth in wellbeing due to lower net marginal productivity of capital. However, the TDU criterion is *sustainable* in the sense of respecting the rights of future generations only if $k_0 \leq k_\infty(\rho)$, but not if $k_0 > k_\infty(\rho)$.

As in the case of the AK model, the TDU optimal stream is SDU optimal whenever the TDU optimal stream is increasing (i.e., the case where $k_0 < k_\infty(\rho)$). However, for $k_0 \geq k_\infty(\rho)$, then it follows directly from Asheim and Mitra (2010, Theorem 2) that SDU optimality corresponds to the equalitarian stream $_1x^e$, defined by $x_t^e = f(k_0)$ for all $t \geq 1$, ensuring that capital is maintained at its original level ($k_t = k_0$ for all $t \geq 1$). Hence, SDU is *effective* and *non-wasteful* for all $k_0 > 0$. Furthermore, it is *flexible* in the sense that the growth rate responds to changes in k_0; in particular, increasing k_0 from below to above $k_\infty(\rho)$ changes a stream with increasing wellbeing to a stream of

constant wellbeing due to lower net marginal productivity of capital. Finally, the SDU criterion is *sustainable* in the sense of respecting the rights of future generations for all $k_0 > 0$.

The same conclusions hold for the RDU and Calvo criteria. The result for RDU optimality follows directly from Zuber and Asheim (2012, Proposition 10), while the result for Calvo optimality follows direcly from Calvo (1978, Proposition 2). Hence, both RDU and the Calvo criterion are also *effective* for all $k_0 > 0$. Furthermore, they are both *non-wasteful*, *flexible* in the sense that the growth rate responds to changes in k_0, and *sustainable* in the sense of respecting the rights of future generations for all $k_0 > 0$.

There is no Chichilnisky optimum in the Ramsey technology (see e.g. Asheim and Ekeland, 2015, Proposition 3). As for the AK model, the existence problem arises since, by the Chichilnisky criterion, it is socially valuable to extend the period of time for which the TDU optimal stream is followed, while it is not optimal to follow the TDU optimal stream forever. However, in addition to not yielding an optimum, the Chichilnisky criterion is also time inconsistent. Therefore, in Asheim and Ekeland (2015), we investigate stationary Markov equilibria in the game that generations with Chichilnisky preferences play in the continuous time version of the Ramsey technology. We argue for plausibility of a particular equilibrium where behavior

- corresponds to the TDU optimum if the TDU optimum is increasing; in continuous time version this corresponds to small k_0 satisfying $f'(k_0) > \delta$ where δ is the continuous time discount rate;

- corresponds to the SDU/TDU/Calvo optimum, yielding a constant stream, for intermediate values of k_0 satisfying $\delta \geq f'(k_0) \geq (1 - \gamma)\delta$, where $(1 - \gamma)$ is the weight on the TDU part of the Chichilnisky criterion;

- is unsustainable, leading to a decreasing stream, for large values of k_0 satisfying $f'(k_0) < (1 - \gamma)\delta$.

Hence, in this equilibrium, the Chichilnisky criterion is *effective* and *non-wasteful* for all $k_0 > 0$. Furthermore, it is *flexible* in the sense that the growth rate responds to changes in k_0, so that the selected wellbeing stream is increasing for small values of k_0, constant for intermediate values of k_0, and decreasing for large values of k_0. However, in this equilibrium, the Chichilnisky criterion is *sustainable* in the sense of respecting the rights of future generations only for small and intermediate values of k_0, but not for large values of k_0.

The results of the present section can be summarized as follow:

Proposition 3 *Consider the sustainable growth criterion as well as the UU, TDU, SDU, RDU, Calvo and Chichilnisky criteria with the calibrations from Section 2. Only the SDU, RDU and Calvo criteria satisfy all four desirable properties in the Ramsey technology when the initial capital stock $k_0 > 0$ is varied, by being effective, non-wasteful, flexible and sustainable.*

Note in particular that the UU criterion, which passed this test in the AK model, need not be effective in the Ramsey technology if the functional share of capital is too small.

5 The Dasgupta-Heal-Solow-Stiglitz technology

Dasgupta and Heal (1974, 1979), Solow (1974) and Stiglitz (1974) developed the standard technological environment for the analysis of economic growth when natural resources are important. In their model, production depends on man-made physical capital k_t^m, the extraction d_t of a natural exhaustible resource k_t^n, and the labor supply ℓ_t. The natural resource is depleted by the resource use, so that $k_{t+1}^n = k_t^n - d_t$. The production function $\hat{f}(k_t^m, d_t, \ell_t)$ is concave, non-decreasing, homogeneous of degree one, and twice continuously differentiable. It satisfies $(\hat{f}_{k^m}, \hat{f}_d, \hat{f}_\ell) \gg 0$ for all $(k^m, d, \ell) \gg 0$ and $\hat{f}(k^m, 0, \ell) = \hat{f}(0, d, \ell) = 0$ (both the physical capital and the natural resource are essential in the production). Moreover, given $(\tilde{k}^m, \tilde{d}) \gg 0$, there exists a scalar $\tilde{\chi}$ such that $\left(d\hat{f}_d(k^m, d, 1)\right)/\left(\hat{f}_\ell(k^m, d, 1)\right) \geq \tilde{\chi}$ for (k^m, d) satisfying $k^m \geq \tilde{k}^m$ and $0 \leq d \leq \tilde{d}$ (the ratio of the share of the resource to the share of labor is bounded away from zero when labor is fixed at unit level).

Assume, as before, that the labor force is constant and normalized to 1. Write $f(k^m, d) := \hat{f}(k^m, d, 1)$. Also assume that f is strictly concave and that the cross partial derivative satisfies $f_{k^m, d}(k^m, d) \geq 0$ for all $(k^m, d) \gg 0$. A wellbeing stream $_1x = (x_1, x_2, \dots) \geq 0$ is feasible given initial stocks $(k_0^m, k_0^n) \gg 0$ if there exist streams of capital, $_1k^m = (k_1^m, k_2^m, \dots) \geq 0$, and resource, $_1k^n = (k_1^n, k_2^n, \dots) \geq 0$, such that

$$x_t + k_t^m \leq f(k_{t-1}^m, k_{t-1}^n - k_t^n) + k_{t-1}^m$$

for all $t \geq 1$. Hence, net production $f(k_{t-1}^m, k_{t-1}^n - k_t^n)$ is split between wellbeing x_t and capital accumulation $k_t^m - k_{t-1}^m$ at each time t.

The assumptions made so far do not ensure that it is feasible to maintain a constant and positive wellbeing level forever. Therefore, assume in addition that there

exists from any $(k_0^m, k_0^n) \gg 0$ a constant stream with positive wellbeing. Cass and Mitra (1991) give a necessary and sufficient condition on f for this assumption to hold.[7] Under this additional assumption there exists an *efficient* constant wellbeing stream from any $(k_1^m, k_1^n) \gg 0$ (see Dasgupta and Mitra, 1983, Proposition 5). A technology satisfying the above assumptions is referred to as a *Dasgupta-Heal-Solow-Stiglitz (DHSS) technology*.

Even though a stream with constant and positive wellbeing is feasible, no stream with positive wellbeing growing at a constant positive rate g is feasible.[8] This leads to the conclusion that the sustainable growth criterion of selecting a wellbeing stream that solves the g^*-SUS program (1) selects no stream in the DHSS technology with the assumptions above. Hence, as for the Ramsey technology, the sustainable growth criterion is not *effective* in the DHSS technology.

Also the UU criterion need not be *effective* in the DHSS technology. A closed-form demonstration of this fact is most easily done in the continuous time version of the DHSS technology: A non-negative wellbeing stream $(x(t))_{t=0}^{\infty}$ is feasible given an initial stock $(k^m(0), k^n(0)) \gg 0$ if there exists non-negative streams of capital $(k^m(t))_{t=0}^{\infty}$ and resource $(k^n(t))_{t=0}^{\infty}$ such that

$$x(t) + \dot{k}^m(t) = f(k^m(t), -\dot{k}^n(t))$$

for all $t \geq 0$. By giving the production function a Cobb-Douglas form: $f(k^m, -\dot{k}^n) = (k^m)^{\alpha}(-\dot{k}^n)^{\beta}$, where $0 < \beta < \alpha < 1$ and $\alpha + \beta < 1$, the assumptions on f are fulfilled and a closed-form solution can be given, as demonstrated by Asheim et al. (2007). Furthermore, it follows from Asheim et al. (2007, Theorem 10 and 12) (see also Dasgupta and Heal, 1979, pp. 303–308) that a UU optimum exists if and only if

$$\alpha - \beta > \frac{1 - \beta}{\eta_U} \, .$$

Hence, the UU criterion is *effective* if and only $\eta_U > (1-\beta)/(\alpha-\beta) > 1$, implying that $\eta_U > 1$ is not sufficient for an UU optimum to exist in the DHSS technology, echoing the similar result in the Ramsey technology. A sufficiently large functional share of capital α and a sufficiently small functional share of resource input β is needed for an UU optimum to exist. With $\eta_U > (1-\beta)/(\alpha-\beta)$, the UU criterion is *non-wasteful* and *sustainable* in the sense of respecting the rights of future generations for all $(k^m(0), k^n(0)) \gg 0$,

---

[7]Mitra et al. (2013) do likewise in the continuous time version of the model.

[8]Asheim and Mitra (2010, Lemma 2) is a formal demonstration of this result, as $\sum_{t=1}^{\tau} \rho^{t-1} \Lambda (1+g)^{t-1}$ would diverge for any ρ satisfying $1/(1+g) \leq \rho < 1$ if a wellbeing stream defined by $x_t = \Lambda(1+g)^{t-1}$ for all t were feasible with $\Lambda > 0$ and $g > 0$.

and *flexible* in the sense that the growth rate responds to changes in $(k^m(0), k^n(0))$. However, no UU optimum exists with $\eta_U \leq (1 - \beta)/(\alpha - \beta)$.

Already Dasgupta and Heal (1974) demonstrated in the continuous time version of the DHSS model that wellbeing is forced towards 0 as time goes to infinity under the TDU criterion for any positive discount rate δ and specification of the u-function that turns wellbeing into generalized utility. The same conclusion holds in the discrete time version for any discount factor ρ between 0 and 1. The reason is that the net productivity of capital is forced towards zero in any stream where wellbeing is bounded away from zero. So, even though the TDU criterion is *effective* and *non-wasteful* for all initial stocks $(k_0^m, k_0^n) \gg 0$, it is never *sustainable* in the sense of respecting the rights of future generations.

In contrast, the SDU, RDU and Calvo criteria give always rise to a sustainable wellbeing stream, for any $(k_0^m, k_0^n) \gg 0$. This wellbeing stream has the property of maximizing the TDU criteria over all non-decreasing wellbeing streams. Since the TDU optimum is not non-decreasing, the stream selected by the SDU, RDU, and Calvo criteria differs from the stream selected under TDU. To describe this wellbeing stream, denote by $x^e(k_0^m, k_0^n)$ the positive and constant level of wellbeing that can be sustained forever along an efficient constant consumption stream from $(k_0^m, k_0^n) \gg 0$. It is possible to attach a sequence of shadow prices

$$(p_1(k_0^m, k_0^n), p_2(k_0^m, k_0^n), \ldots, p_t(k_0^m, k_0^n), \ldots)$$

to the corresponding stationary consumption stream (for a characterization of the prices, see Asheim and Mitra, 2010, Lemma 3). Write

$$\rho_\infty(k_0^m, k_0^n) = \frac{\sum_{t=1}^{\infty} p_t(k_0^m, k_0^n)}{\sum_{t=0}^{\infty} p_t(k_0^m, k_0^n)},$$

where $p_0(k_0^m, k_0^n) = 1$, for the long-run discount factor at time 0 supporting this stationary stream.

The unique SDU optimal wellbeing stream $_1x^S$ is characterized as follows:

(a) If $\rho_\infty(k_0^m, k_0^n) < \rho$, then $_1x^S$ is the non-decreasing stream maximizing TDU welfare over all feasible and non-decreasing streams. This stream has the property that there exists $\tau = \min\{t \geq 0 : \rho_\infty((k_t^m)^S, (k_t^n)^S) \geq \rho\}$ such that

$$x_t^S < x_{t+1}^S \text{ for } t \leq \tau,$$
$$x_t^S = x^e((k_\tau^m)^S, (k_\tau^n)^S) \text{ for all } t > \tau.$$

17

(b) If $\rho_\infty(k_0^m, k_0^n) \geq \rho$, then $_1x^S$ is the egalitarian stream with $x_t^S = x^e(k_0^m, k_0^n)$ for all $t \geq 1$.

The result that $_1x^S$ is the unique SDU optimal wellbeing stream follows directly from Asheim and Mitra (2010, Theorem 3). The result that $_1x^S$ is also the unique RDU optimal wellbeing stream follows directly from Zuber and Asheim (2012, Proposition 11). The analysis with the Calvo criterion is more complicated, as an optimal time consistent stream does not exist in case (a) (see Asheim, 1988, Theorem 1). However, $_1x^S$ is the unique equilibrium of the intergenerational game, under the assumption that the optimal stream is followed as soon as one exists (see Asheim, 1988, Theorem 2). To see that stream $_1x^S$ is not optimal in case (a), but an equilibrium, observe that generation $\tau + 1$ would want the generation $\tau + 2$ to enjoy wellbeing on its behalf as ρ times the gross marginal instantaneous productivity of capital exceeds 1. However, if generation $\tau + 1$ makes such a sacrifice, generation $\tau + 2$ would want to share this wellbeing with its descendants. However, if generation $\tau + 2$ does so, then it is better for generation $\tau + 1$ to stick to $_1x^S$ since the gross marginal productivity of capital is decreasing along $_1x^S$.

Therefore, the SDU, RDU and Calvo criteria are *effective* in the DHSS technology – in the sense of selecting a time consistent stream – for all $(k_0^m, k_0^n) \gg 0$. They are also *non-wasteful*, *flexible* in the sense that the growth rate responds to changes in (k_0^m, k_0^n), and *sustainable* in the sense of respecting the rights of future generations for all $(k_0^m, k_0^n) \gg 0$.

As for the Ramsey technology, there is no Chichilnisky optimum in the DHSS technology. There is yet no game theoretic analysis of the Chichilnisky criterion in the DHSS technology, not even in the continuous time version of the model, so it is premature to speculate on the properties of such a Chichilnisky equilibrium in this technological environment.

The results of the present section can be summarized as follow:

Proposition 4 *Consider the sustainable growth criterion as well as the UU, TDU, SDU, RDU, Calvo and Chichilnisky criteria with the calibrations from Section 2. Only the SDU, RDU and Calvo criteria satisfy all four desirable properties in the DHSS technology when the initial stocks $(k_0^m, k_0^n) \gg 0$ are varied, by being effective, non-wasteful, flexible and sustainable.*

6 Discussion

In this paper I have evaluated different criteria of intergenerational equity by mapping the consequences of the criteria in various technological environments, and observed to which extent they yield social choice mappings that are *effective* (in the sense of selecting a time consistent stream), *non-wasteful* (in the sense that the selected stream is efficient), *flexible* (in the sense that the growth rate of the selected stream responds to changes in the technological environment), and *sustainable* (in the sense that the selected stream respect the rights of future generations by not rendering them worse off than the present generation). The different criteria was calibrated so that they give rise to the same efficient balanced growth path in the AK model for a given value A^* of the gross productivity parameter A, before being subjected to the technological environments that arise when varying A in the AK model, as well as to the Ramsey and DHSS technologies. The results are summarized in Table 1.

[Table 1 about here.]

Being *effective* and *non-wasteful* are basic and undisputable properties. Flexibility seems necessary for ensuring that a criterion is *effective*, as is illustrated in Table 1 by the results on the sustainable growth criterion of selecting a wellbeing stream that solves the g^*-SUS program (1). Lastly, sustainability is the position that Roemer (2011, 2013) argues for in the quotes of the introduction; this is a position that I support.

The table shows that the UU criterion is not a viable alternative, as it need not be *effective* in the Ramsey and DHSS technologies. The criterion that economists usually endorse, TDU, does fine in the AK model and in the Ramsey technology as long the economy is sufficiently productive. Hence, the endorsement of TDU might from a pragmatic perspective be based on a view that these are the "relevant" technological environments and on the fact that it is not to the disadvantage of future generations in these settings that they are discriminated against by TDU. However, if productivity is low, or if natural resources are important (as they are in the DHSS technology), then the TDU criterion does not yield sustainable outcomes.

The Chichilnisky criterion is not *effective*, at least not unless a game theoretic analysis is invoked in response to the time inconsistency of this criterion. And even in the game that generations with Chichilnisky preferences play in the continuous time version of the Ramsey technology, the plausible equilibrium does not lead to a sustainable outcome if productivity is low.

Table 1 shows that the remaining criteria, SDU, RDU, and the Calvo criterion, do well with respect to all the four desirable properties in all the considered technological environments. In these criteria, discounting of generalized utility is not allowed to be to the disadvantage of future generations, at least not in technological environments (like the ones I consider) with positive net marginal productivity of capital.

The Calvo criterion is of particular interest, as it combines the extreme egalitarianism of the maximin principle – which is endorsed by John Roemer – with altruism for future generations. In fact, Calvo's (1978) contribution arose as a response to Rawls's (1971) discussion of intergenerational equity, as interpreted by Arrow (1974) and Dasgupta (1974). It shows how sustainable growth can be combined with the maximin principle, because generations with altruism choose to let their descendants enjoy wellbeing on their behalf.

Also the SDU criterion has a motivation that is close to that conveyed by Roemer (2011, 2013) in the quotes of the introduction. It captures the intuition that we have an obligation to assist future generations if they are worse off than us, but we may choose to give them the means for achieving a higher wellbeing than the one enjoyed by ourselves.

The RDU has a different motivation, as generations are treated completely symmetric, with rank-discounting being an expression of social aversion to inequality. In particular, with negative net marginal productivity of capital between two periods, RDU would allow a generation a wellbeing level that exceeds the wellbeing that some future generation will enjoy. However in the technological environments that I have considered in this paper, RDU yields the same outcome as the SDU and Calvo criteria.

From a technical point of view, the discount factor ρ between 0 and 1 that enters into the SDU, RDU and Calvo criteria has property to ensure that the criteria are *effective*. When ρ approaches 1, the outcome approaches the UU optimum when this exists. For ρ sufficiently small, the outcome is an egalitarian wellbeing stream, hence, the outcome that would have arisen if the maximin criterion had been applied.

As for the choice of the growth rate g^* in the g^*-SUS problem when applied to the AK model, the parameters of the SDU, RDU and Calvo criteria, the discount factor ρ and the parameter of inequality aversion η, must be chosen according to ethical intuitions—or perhaps, in line with the quotes of the introduction, how we believe that generations will choose to make sacrifices for their descendants. Even for a given choice of the growth rate g^* in the AK model with with the gross productivity parameter A equal to a particular value A^*, there is trade-off between ρ and η as shown by eq. (5): More discounting (in the form of a lower discount factor ρ) corresponds to less

inequality aversion (in the form a lower parameter of inequality aversion η) and less responsiveness to changes in productivity.[9]

Throughout this paper I have abstracted from exogenous or endogenous variation of population size. In particular, population is been assumed to be constant and normalized to 1, implying that the analysis has abstracted from *intra*generational inequality. Moreover, I have assumed zero probability of extinction, and that generations succeed each other without overlapping in a deterministic setting. This setting has allowed a focus on the ethics of intergenerational distribution.

As Stéphane Zuber and I show in Asheim and Zuber (2014, 2015), the RDU can be generalized to settings with variable population and intergenerational risk. I do the same for SDU in Dietz and Asheim (2011). Practical use of the Calvo criterion requires that such issues be resolved also for this criterion of intergenerational equity. The properties of these criteria may well differ when expanding the domain of technological environments by considering also population issues and intergenerational risk.

---

[9]The two parameters can be calibrated independently if there are two different combinations of gross productivity and growth rate, (A^*, g^*) and (A^{**}, g^{**}), that appeal to ethical intuitions in the AK model.

Appendix A

Definitions of UU optimality, TDU optimality, SDU optimality, RDU optimality, Calvo optimality, and Chichilnisky optimality

Let X be a set of feasible wellbeing streams for given initial condition. That is, the wellbeing stream $_1x = (x_1, x_2, \dots)$ is feasible if and only if $_1x \in X$. Let $\mathcal{X}$ be the union of X when varying the initial conditions in both models. For all elements X in $\mathcal{X}$, X contains only streams where wellbeing x_t at each time t is non-negative. Let $u : \mathbb{R}_{++} \to \mathbb{R}$ be a increasing, strictly concave and continuously differentiable function with $\lim_{x \to 0} u'(0) = \infty$ that maps positive wellbeing into *transformed wellbeing* (which can also be referred to as *generalized utility*). In the parameterized versions of u in the main text, u need not be defined for zero wellbeing.

Undiscounted generalized utilitarianism (UU). Say that $_1x \in X$ is an *undiscounted generalized utilitarian* (TDU) *optimum* given the set of feasible utility streams X if

$$\liminf_{\tau \to \infty} \sum_{t=1}^{\tau} \big(u(x_t) - u(\tilde{x}_t) \big) \geq 0 \quad \text{for all } _1\tilde{x} \in X \,.$$

Time-discounted generalized utilitarianism (TDU). Define the TDU welfare function $w_\rho^T : \mathcal{X} \to \mathbb{R}$ for the discount factor $\rho \in (0,1)$ as follows:

$$w_\rho^T(_1x) = (1 - \rho) \sum_{t=1}^{\infty} \rho^{t-1} u(x_t) \,.$$

Say that $_1x \in X$ is a *time-discounted generalized utilitarian* (TDU) *optimum* given the set of feasible utility streams X if

$$w_\rho^T(_1x) \geq w_\rho^T(_1\tilde{x}) \quad \text{for all } _1\tilde{x} \in X \,.$$

Sustainable discounted generalized utilitarianism (SDU) (see Asheim and Mitra, 2010, for a presentation and analysis of this criterion, including an axiomatization). Under SDU, the future is discounted if and only if the future is better off than the present. Define the SDU welfare function $w_\rho^S : \mathcal{X} \to \mathbb{R}$ for $\rho \in (0,1)$ as follows: $w_\rho^S(_1x) = \lim_{\tau \to \infty} z(1, \tau)$, where $z(1, \tau)$ is constructed as follows:

$$z(\tau, \tau) = w_\rho^T(_\tau x)$$
$$z(\tau - 1, \tau) = \min\{(1 - \rho)u(x_{\tau-1}) + \rho z(\tau, \tau), \, z(\tau, \tau)\}$$
$$\dots$$
$$z(1, \tau) = \min\{(1 - \rho)u(x_1) + \rho z(2, \tau), \, z(2, \tau)\} \,.$$

Say that $_1x \in X$ is a *sustainable generalized discounted utilitarian* (SDU) *optimum* given the set of feasible utility streams X if

$$w_\rho^S(_1x) \geq w_\rho^S(_1\tilde{x}) \quad \text{for all } _1\tilde{x} \in U.^{[10]}$$

Rank-discounted generalized utilitarianism (RDU) (see Zuber and Asheim, 2012, for a presentation and analysis of this criterion, including an axiomatization). Under RDU, streams are first reordered into a non-decreasing stream, so that discounting becomes according to rank, not according to time. The definition takes into account that streams like $(1, 0, 0, 0, \dots)$, with elements of infinite rank, cannot be reordered into a non-decreasing stream. Therefore, let $\ell(_1x)$ denote $\liminf$ of $_1x$ if it exists (set $\ell(_1x) = \infty$ otherwise), and let $L(_1x) := \{t \in \mathbb{N} \mid x_t < \ell(_1x)\}$. If $|L(_1x)| = \infty$, then let $_{[1]}u = (x_{[1]}, x_{[2]}, \dots)$ be a non-decreasing reordering of all elements x_t with $t \in L(_1x)$ (so that $x_{[r]} \leq x_{[r+1]}$ for all ranks $r \in \mathbb{N}$). If $|L(_1x)| < \infty$, then let $(x_{[1]}, x_{[2]}, \dots, x_{[|L(_1x)|]})$ be a non-decreasing reordering of all elements x_t with $t \in L(_1x)$ (so that $x_{[r]} \leq x_{[r+1]}$ for all ranks $r \in \{1, \dots, |L(_1x)|\}$), and set $x_{[r]} = \ell(_1x)$ for all $r > |L(_1x)|$.

Define the RDU welfare function $w_\rho^R : \mathcal{X} \to \mathbb{R}$ for $\rho \in (0, 1)$ as follows:

$$w_\rho^R(_1x) = w_\rho^T(_{[1]}x).$$

Say that $_1x \in X$ is a *rank-discounted generalized utilitarian* (RDU) *optimum* given the set of feasible utility streams X if

$$w_\rho^R(_1x) \geq w_\rho^R(_1\tilde{x}) \quad \text{for all } _1\tilde{x} \in X.$$

It follows from the assumptions made in Sections 2, 3, 4 and 5 that the generalized utilitarian welfare functions w_ρ^T, w_ρ^S and w_ρ^R are well-defined on $\mathcal{X}$.

The *Calvo criterion* (see Calvo, 1978, for a presentation and analysis of this criterion) evaluates streams according to the infimum of the non-paternalistic altruistic welfare. Hence, the Calvo welfare function is defined on $\mathcal{X}$ as follows:

$$\inf_{t \geq 1} w_\rho^T(_tu).$$

Say that $_1x \in X$ is a *Calvo optimum* if

$$\inf_{t \geq 1} w_\rho^T(_tx) \geq \inf_{t \geq 1} w_\rho^T(_t\tilde{x}) \quad \text{for all } _1\tilde{x} \in X.$$

---

[10] Asheim and Mitra (2010, Section 2) use the construction presented here to establish the existence of a SDU welfare function, while using their requirements (W.1)–(W.4) as the primitive definition.

23

The Chichilnisky criterion (see Chichilnisky, 1996, for a presentation and analysis of this criterion, including an axiomatization) evaluates streams according to a convex combination of TDU welfare and the limit of transformed wellbeing:

$$(1 - \gamma)w_\rho^T({}_1x) + \gamma \lim_{t\to\infty} u(x_t)\,,$$

where $\gamma \in (0,1)$.

Say that ${}_1x \in X$ is a *Chichilnisky optimum* if

$$(1 - \gamma)w_\rho^T({}_1x) + \gamma \lim_{t\to\infty} u(x_1) \geq (1 - \gamma)w_\rho^T({}_1\tilde{x}) + \gamma \lim_{t\to\infty} u(\tilde{x}_t) \quad \text{for all } {}_1\tilde{x} \in X\,,$$

or if ${}_1x$ is a TDU optimum and $\lim_{t\to\infty} x_t = \infty$.

Remark on the axiomatic foundation. The UU, TDU and Chichilnisky criteria satisfy the strong Paretian principle, while the SDU and RDU criteria do not. Pareto efficiency is not a straight-forward concept in the case of the Calvo criterion, since efficiency in wellbeing need not imply efficiency in altruistic utility. Moreover, the UU and RDU criteria treat generations equally in the sense of satisfying Anonymity (RDU in the strong sense of being invariant even to infinite permutations), while the remaining criteria do not treat generations equally. Finally, the UU, TDU and SDU induce time consistent preferences, while the RDU, Calvo and Chichilnisky criteria do not.

References

Arrow, K.J. (1974), Rawls' principle of just saving. *Swedish Journal of Economics* **75**, 323–335.

Asheim, G.B. (1988), Rawlsian intergenerational justice as a Markov-perfect equilibrium in a resource technology. *Review of Economic Studies* **55**, 469–483.

Asheim, G.B. (1991), Unjust intergenerational allocations. *Journal of Economic Theory* **54**, 350–371.

Asheim, G.B., Buchholz, W., Hartwick, J., Mitra, T., and Withagen, C. (2007), Constant savings rates and quasi-arithmetic population growth under exhaustible resource constraints. *Journal of Environmental Economics and Management* **53**, 213229.

Asheim, G.B., and Ekeland, I. (2015), Resource conservation across generations in a Ramsey-Chichilnisky model. Memorandum 17/2015, Department of Economics, University of Oslo.

Asheim, G.B., and Nesje, F. (2015), Destructive intergenerational altruism. Memorandum 22/2015, Department of Economics, University of Oslo.

Asheim, G.B., and Mitra, T. (2010), Sustainability and discounted utilitarianism in models of economic growth. *Mathematical Social Sciences* **59**, 148–169.

Asheim, G.B., and Zuber, S. (2014), Escaping the repugnant conclusion: rank-discounted utilitarianism with variable population. *Theoretical Economics* **9**, 629–650.

Asheim, G.B., and Zuber, S. (2015), Evaluating intergenerational risks. CESifo Working Paper No. 4728. Revised December 2015.

Beals, R., and Koopmans, T.C. (1969), Maximizing stationary utility in a constant technology. *SIAM Journal of Applied Mathematics* **17**, 1001–1015.

Calvo, G. (1978), Some notes on time inconsistency and Rawls' maximin criterion. *Review of Economic Studies* **45**, 97–102.

Cass, D., and Mitra, T. (1991), Indefinitely sustained consumption despite exhaustible natural resources. *Economic Theory* **1**, 119–146.

Chichilnisky, G. (1996), An axiomatic approach to sustainable development. *Social Choice and Welfare* **13**, 231–57.

Dasgupta, P.S. (1974), On some alternative criteria for justice between generations. *Journal of Public Economics* **3**, 405–423.

Dasgupta, P.S. (2011), The Ethics of Intergenerational Distribution: Reply and Response to John E. Roemer. *Environmental and Resource Economics* **50**, 475–493.

Dasgupta, P.S., and Heal, G.M. (1974), The optimal depletion of exhaustible resources. *Review of Economic Studies* (Symposium), 3–28.

Dasgupta, P.S., and Heal, G.M. (1979), *Economic Theory and Exhaustible Resources.* Cambridge University Press, Cambridge, UK.

Dasgupta, S., and Mitra, T. (1983), Intergenerational equity and efficient allocation of exhaustible resources. *International Economic Review* **24**, 133–153.

Dietz, S., and Asheim, G.B. (2012), Climate policy under sustainable discounted utilitarianism. *Journal of Environmental Economics and Management* **63**, 321–335.

Llavador, H., Roemer, J.E., and Silvestre, J. (2010), Intergenerational Justice when Future Worlds Are Uncertain. *Journal of Mathematical Economics* **46**, 728–761.

Llavador, H., Roemer, J.E., and Silvestre, J. (2011), A dynamic analysis of human welfare in a warming planet. *Journal of Public Economics* **95**, 1607–1620.

Llavador, H., Roemer, J.E., and Silvestre, J. (2013), Should We Sustain? And if so, Sustain What? Consumption or the Quality of Life? In Fouquet, R. (ed.), *Handbook on Energy and Climate Change*, Cheltenham, UK: Edward Elgar.

Mitra, T., Asheim, G.B., Buchholz, W., and Withagen, C. (2013), Characterizing the sustainability problem in an exhaustible resource model. *Journal of Economic Theory* **148**, 2164–2182.

Ray, D. (1987), Nonpaternalistic intergenerational altruism. *Journal of Economic Theory* **41**, 112–132.

Rawls, J. (1971). *A Theory of Justice*, 1999 revised edition, Cambridge (Mass.): The Belknap Press of the Harvard University Press.

Roemer, J.E. (2011), The Ethics of Intertemporal Distribution in a Warming Planet. *Environmental and Resource Economics* **48**, 363–390.

Roemer, J.E. (2013), Once More on Intergenerational Discounting in Climate-Change Analysis: Reply to Partha Dasgupta. *Environmental and Resource Economics* **56**, 141–148.

Sen, A. (1979), Utilitarianism and Welfarism. *The Journal of Philosophy* **76**, 463–489.

Solow, R.M. (1974), Intergenerational equity and exhaustible resources. *Review of Economic Studies* (Symposium), 29–45.

Stiglitz, J. (1974), Growth with exhaustible natural resources: Efficient and optimal growth paths. *Review of Economic Studies* (Symposium), 123–137.

Zuber, S., and Asheim, G.B. (2012), Justifying social discounting: the rank-discounted utilitarian approach. *Journal of Economic Theory* **147**, 1572–1601.

Table 1: The implications of criteria in various technological environments

	AK A large	AK A small	$Ramsey$ k_0 small	$Ramsey$ k_0 large	$DHSS$
g^*-SUS					
Effective	$+$	$-$	$-$	$-$	$-$
Non-wasteful	$+$	n.a.	n.a.	n.a.	n.a.
Flexible	$-$	n.a.	n.a.	n.a.	n.a.
Sustainable	$+$	n.a.	n.a.	n.a.	n.a.
$Undisc.\ util.\ (UU)$					
Effective	$+$	$+$	$(+)^{a}$	$(+)^{a}$	$(+)^{b}$
Non-wasteful	$+$	$+$	$(+)^{a}$	$(+)^{a}$	$(+)^{b}$
Flexible	$+$	$+$	$(+)^{a}$	$(+)^{a}$	$(+)^{b}$
Sustainable	$+$	$+$	$(+)^{a}$	$(+)^{a}$	$(+)^{b}$
$Time\text{-}disc.\ util.\ (TDU)$					
Effective	$+$	$+$	$+$	$+$	$+$
Non-wasteful	$+$	$+$	$+$	$+$	$+$
Flexible	$+$	$+$	$+$	$+$	$+$
Sustainable	$+$	$-$	$+$	$-$	$-$
$Sust.\ disc.\ util.\ (SDU)$					
$Rank\text{-}disc.\ util.\ (RDU)$					
$Calvo\ criterion$					
Effective	$+$	$+$	$+$	$+$	$+$
Non-wasteful	$+$	$+$	$+$	$+$	$+$
Flexible	$+$	$+$	$+$	$+$	$+$
Sustainable	$+$	$+$	$+$	$+$	$+$
$Chichilnisky\ criterion$					
Effective	$+$	$-$	$(+)^{c}$	$(+)^{c}$	$-$
Non-wasteful	$+$	n.a.	$(+)^{c}$	$(+)^{c}$	n.a.
Flexible	$+$	n.a.	$(+)^{c}$	$(+)^{c}$	n.a.
Sustainable	$+$	n.a.	$(+)^{c}$	$(-)^{c}$	n.a.

(a) For high functional share of capital only. (b) For high high functional share of capital and low functional share of resource only. (c) In the equilibrium of an intergenerational game.

27