On Abatement Services: Market Power and Efficient Environmental Regulation Preliminary version

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Abstract

In this paper, we study an eco-industry which provides an environmental service to a competitive polluting sector. We show that even if this eco-industry is highly concentrated, a standard environmental policy based on a Pigouvian tax or on a pollution permit market reaches the first-best, challenging the Tinbergen rule. To illustrate this point, we first consider an upstream monopoly which sells eco-services to a representative polluting firm. We progressively extend our result to heterogeneous downstream polluters and to upstream Cournot competitors. Finally, we underline some limits of this result. It does not hold assuming abatement goods or downstream market power. In this last case, we recover Barnett's result.

Key words: Environmental regulation, Eco-industry, Imperfect Competition, Abatement services

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1. Introduction

The so-called Environmental Goods and Services Sector (EGSS hereafter) and its link to environmental regulation is nowadays clearly recognized. This sector "*includes the provision of environmental technologies, goods and services for every kind of use, i.e. intermediate and final consumption as well as gross capital formation*" (Eurostats' handbook on EGSS [13]). Even if some methodological problems remain (see the UNEP report, [35]), the EGSS is largely documented by most of the statistical institutes.[4] They agree upon the fact that the size of the EGSS remains more or less moderated: the share of the EGSS gross value added in the GDP is around 2.0% in Europe as well in the US. They rather advocate the notable growth rate of this sector, its capacity to generate new job opportunities and its performance at the export. For instance, estimates for European Union show an increase of EGSS output per unit of GDP by 50 % between 2000 and 2011, while employment grew in the meantime of around 40%.

Two other observations concerning the EGSS seem to be important. First, it is widely acknowledged that this sector is controlled by worldwide firms like CH2M Hill, Veolia Environmental Services, Vivendi Environment's or Suez environment. The Ecorys report [11] on the European EGSS even mentions that 10% of the companies are responsible for almost 80% of the operating revenue. Secondly, a large part of the activity of the EGSS is dedicated to the furniture of environmental services. Environmental services represent more than 40% of the activity of the sector (see Sainclair Desgagné [33] table 2) and are largely involved in international trade.[5]

These two last observations motivate our paper. The question is quite simple: should we regulate a polluting industry in the same way when these firms access to an imperfect competitive eco-industry providing abatement goods or abatement services? This asks a second question: what is the difference between abatement goods and abatement services ? If we follow the Eurostats' handbook on EGSS [11] "*Services are outputs produced to order and which cannot be traded separately from their production. Services are not separate entities over which ownership rights can be established*". In the context of end-of-pipe abatement, this means that the polluter buys "pollution reduction" without taking care to the way in which this job will be done: he simply externalizes this activity. This is, for instance, the case for waste management, water sewerage and treatment, remediation and clean-up activities. In other words and since a service is *not a separate entity*, this facility can not be used as an input in the production process of the polluter contrary to environmental goods like filters, scrubbers or incinerators on which the polluter keeps

[4]Several empirical studies recently try to quantify the EGSS. For instance, the Canada statistical institute [34] has adopted a biennial survey of the EGGS (http://www.statcan.gc.ca/eng/survey/business/1209). In Europe, Eurostats initiates a study over 28 state members (see http://ec.europa.eu/eurostat/statistics-explained/index.php/Environmental_goods_and_services_sector) based on a methodology described in Eutostats [13]. The US department of commerce published in 2010 a survey called "Measuring the Green Economy" [36].

[5]See for instance the report of the US international trade commision [37].

some control. This simple observation fundamentally modifies his purchasing behavior. When he buys abatement services, *i.e.* a pollution reduction, he only makes an arbitrage between the price of this service and the compliance cost of an environmental regulation. While for environmental goods, he has also to care to the marginal rate of pollution abatement since these goods are viewed as inputs which reduce emissions.

If the eco-industry is imperfectly competitive, the difference between abatement goods and abatement services also affects the expected demand for abatement and therefore the strategic behaviors of the members of this industry. For instance, under a Pigouvian tax, the purchasing behavior of abatement services is only motivated by the difference between the environmental tax and the price of the abatement service. Thus, the regulator implicitly controls the market of abatement services by setting the level of the Pigouvian tax. By exploiting these particular features, we show that the regulator can restore the first best simply by setting a Pigovian tax equal to the marginal damage. In other words, the first best can be reach with only one economic policy tool although there are two market failures in this economy: market power on the market of abatement service and pollution. This result challenges the Tinbergen rule and suggests that the environmental agency has to make a distinction between environmental goods and services regulation.

In the best of our knowledge this distinction was not introduced in the eco-industry literature. If we invoke the previous Eurostats' definition, most of the contributions on these vertical structures are concerned with, on the one hand, environmental technologies and R&D or, on the another hand, with the provision of abatement goods.

The first branch considers the incentives provided by environmental policy instruments for both adoption and development of advanced abatement technology (see Requate [28] for an overview). Not all of these contributions explicitly introduce an EGSS since it requires that innovation is a private good. The studies which go in this direction often consider an innovative firm which invests in R&D to obtain a patent over a pollution-reducing new technology. In this context, the performance of taxes and tradeable permits are compared in various context. Denicolo [10] and Requate [29] make these comparisons under different timing and commitment regimes. A threat of imitation is introduced by Fisher *et al.* [14] while Perino [25] studies green horizontal innovation where new technologies reduce pollution of one type while causing a new type of damage. More recently, Perino [26] focuses on the second best policies for all combinations of emission intensity and marginal abatement costs.

The second branch which is closer to our contribution takes as given the existence of imperfect competition in the eco-industry which sells abatement goods to a polluting sector and explores the second best regulation policy under alternative instruments. Greaker [16] and Greaker and Rosendahl [17] introduce emission standards. David and Sinclair-Desgagné [7] and Canton *et al.* [6] explore the case of the Pigouvian tax with respectively imperfect competition upstream and both upstream and downstream whereas Schwartz and Stahn [32] introduces tradable pollution rights. Endres and Friehe [12] question the impact of environmental liability laws. Some other papers like David *et al.* [9] or Canton *et al.* [5] introduce entry or firm merging in the eco-industry.

Our contribution is also very close to Nimubona and Sinclair-Desgagné [24] which initiate a discussion about an internal abatement effort and an external procurement of abatement facilities. They however not leave the Katsoulacos and Xepapadeas [20] formulation of an end-of-pipe pollution which is common to almost all papers on the eco-industry and which states that emissions are a function depending on the levels of production and abatement. Most of these papers also assume that the abatement facilities have decreasing returns. In other words, the abatement facility works like an input and not as a service which should have constant returns.[6] This is why the first best can be restored with only one instrument contrary to David and Sinclair-Desgagné [8] who introduce Pigouvian taxation and a subsidy to the eco-industry.

The intuition beyond our main result is quite simple. If a Pigouvian tax is enforced, a polluter purchasing an *eco-service* either (i) prefers to pay the tax if the price of the abatement service is higher (ii) decides, in the opposite case, to fully abate the pollution issued from its equilibrium production level, or, (iii) is indifferent between both if the price and the tax are equal. This implies that the inverse demand for eco-services becomes inelastic over a range of quantities which depends from the tax level, so that any monopoly which sells these services looses - at least partially - its market power. Moreover, if the optimal provision of abatement services belongs to this range of quantities when the tax is equal to the marginal damage, the regulator will be able to implement the first best. This argument holds when we deal with homogeneous competitive polluters, an eco-service monopoly and an optimal abatement level which does not require full abatement. This is why we start with this benchmark case. We then show that our argument can be easily extended in order to (i) include the "boundary" solutions corresponding to efficient full pollution abatement[7] (ii) take into account regulation by pollution permit market (iii) allow heterogeneous polluters with regard to their production costs and emissions. The extension of our analysis to Cournot competition in the eco-industry is less obvious, this is why we have dedicated a whole section to this study. As both main assumptions nevertheless remain in this new case - upstream eco-services and downstream perfect competition - our result holds again. We finally exhibit some limits of our analysis, taking into account first abatement goods and then downstream imperfect competition. This last section enables us to underline on the one hand, that the existence of eco-services is very crucial and, on another hand, to extend Barnett's results *[2]*.

The structure of the article is as follows. Section 2 is dedicated to the presentation of the model. Section 3 presents the simplest case: downstream homogeneous polluters and an upstream monopoly. Section 4 introduces some straightforward extensions: downstream full abatement, pollution permit market and heterogeneous downstream firms. Section 5 is dedicated to Cournot competition in the eco-industry. Section 6 challenges

[6] Our end-of-pipe emission reduction technology can therefore be viewed as a particular case of the Katsoulacos and Xepapadeas [20] emission function in which the abatement good has constant return to scale. This case was, in the best of our knowledge, not explored for, we think, technical reasons: standard differential calculus does not really apply and corner solutions emerge.

[7] The case of efficient full downstream pollution abatement is particularly conceivable if the upstream eco-industry is also polluting like in Sans *et al.* [30].

both main assumptions: environmental services and downstream perfect competition. Finally, some concluding remarks are given in Section 7 and technical proofs are relegated to an appendix.

2. A basic model of environmental services

We first present the main assumptions of the model and then we characterize the first best allocation.

2.1. The main assumptions

We consider a standard *polluting industry* characterized by a representative firm which produces a quantity Q at a given cost $c(Q)$. This cost is increasing and convex (*i.e.* $c'(Q) > 0$ and $c''(Q) > 0$), inaction is allowed (*i.e.* $c(0) = 0$), $c'(0) = 0$ and $\lim_{q \to +\infty} c'(q) = +\infty$. This activity is polluting. Emissions are given by $\varepsilon(Q)$, an increasing and convex function (*i.e.* $\varepsilon'(Q) > 0$ and $\varepsilon''(Q) > 0$) with satisfies $\varepsilon(0) = 0$, $\varepsilon'(0) = 0$ and $\lim_{q \to +\infty} \varepsilon'(q) = +\infty$. This dirty firm has the ability to buy environmental services in order to reduce its "end-of-pipe" pollution. By doing so, a part A of *its* emissions is abated by a specialized external firm and the remaining pollution is $E = \max\{\varepsilon(Q) - A, 0\}$.

The *eco-services* are delivered on a non-competitive market at price p_A. We will frequently assume in this paper that these services are provided by a monopoly. This firm is characterized by an increasing and convex cost function and inaction is allowed (*i.e.* $\kappa'(a) > 0$, $\kappa''(a) > 0$ and $\kappa(0) = 0$). We also assume that $\kappa'(0) = 0$ in order to ensure that the eco-service market is activated when an environmental policy is implemented.[8]

The environmental damage induced by the remaining emissions E is measured by a standard damage function $D(E)$. As usually, this function is *i*ncreasing and convex (i.e. $D'(E) > 0$ and $D"(E) > 0$) and without emission there is no damage (*i.e.* $D(0) = 0$). We even set $D'(0) = 0$. This last assumption is essentially made for convenience: it ensures that full abatement never occurs at an efficient allocation.[9]

Finally, to close the model, we introduce an *inverse demand function* for the polluting goods $P(Q)$. This function is decreasing (*i.e.* $P'(Q) < 0$) and verifies that $\lim_{Q \to 0} P(Q) = +\infty$ and $\lim_{Q \to +\infty} P(Q) = 0$.

2.2. The first best allocation

Under these assumptions, a first best allocation is given by:

$$\left(Q^{opt}, A^{opt}\right) \in \arg\max_{Q,A \geq 0} \int_0^Q P\left(q\right) dq - c\left(Q\right) - D\left(\max\left\{\varepsilon\left(Q\right) - A, 0\right\}\right) - \kappa(A) \qquad (1)$$

[8] A discussion about the emergence of an eco-industry related to the fact that $\kappa'(0) > 0$ can be found in Canton *et al.* [6].

[9] If the marginal damage at zero is high enough and/or the marginal abatement cost is not to excessive, the "end-of-pipe" pollution assumption, *i.e.* $E = \max\{\varepsilon(Q) - A, 0\}$, can lead to an efficient allocation requiring full abatement (see Sans and al. [30] for a discussion). In Section 4.1, we extend our result to this case but that requires additional discussions which are not central to our main argument.

This is typically a non-smooth optimization problem, but remember that we have assumed that $D(0) = 0$ and $D'(0) = 0$. The first equality ensures that the optimal level of abatement cannot be larger than emissions since abatement is costly, hence $\varepsilon(Q^{opt}) - A^{opt} \geq 0$, while the second combined with the positivity of the marginal cost of abatement makes sure that this inequality holds strictly. Consequently, the first best allocation is characterized by the usual first order conditions:

$$P(Q^{opt}) - c'\left(Q^{opt}\right) - D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right).\varepsilon'\left(Q^{opt}\right) = 0 \tag{2}$$

$$D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) = 0 \tag{3}$$

Let us now introduce the function $\beta(Q) = \frac{P(Q)-c'(Q)}{\varepsilon'(Q)}$ defined on $[0, Q_{\max}]$ where $Q_{\max}$ stands for the optimal level of production without environmental damage (*i.e.* $P(Q_{\max}) = c'(Q_{\max})$). This function measures, for each $Q \leq Q_{\max}$, the marginal benefit from an additional unit of pollution. Therefore an optimal allocation has the property that the marginal benefit of pollution is equal (i) to the marginal damage and (ii) to the marginal cost of abating an additional unit of pollution:

$$\beta(Q^{opt}) = D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) = \kappa'(A^{opt}) \tag{4}$$

For latter use, let us also notice this marginal benefit is decreasing and $\beta(Q_{\max}) = 0$ so that $\beta^{-1} : [0, +\infty] \to [0, Q_{\max}]$ is defined.

3. Upstream monopoly power and first best regulation

Let us first show that a policy maker reaches the efficient allocation with a standard Pigouvian tax scheme even if the provider of environmental services has *a* monopoly power. To illustrate this point, we proceed in three steps. We first introduce a Pigouvian tax and compute the inverse demand for abatement services under a downstream market clearing assumption. This brings us, in a second step, to the characterization of the behavior of the upstream monopolist whatever the Pigouvian tax is. It remains, in a last step, to show that a Pigouvian tax equal to the marginal damage regulates both environmental and market power inefficiencies.

3.1. The (inverse) demand for abatement services

The competitive dirty firm chooses its production supply and its demand for abatement good by solving:

$$\max_{Q\geq 0}\left\{ p_Q \times Q - c(Q) - \underbrace{\min_{A\geq 0}\left\{p_A \times A + \tau \times \max\left\{\varepsilon(Q) - A, 0\right\}\right\}}_{=C_A(p_A,\tau,Q)} \right\} \tag{5}$$

An inspection of the cost minimization part of this program shows that the conditional demand for abatement services never exceeds $\varepsilon(Q)$ and that the objective function is linear in A on $[0, \varepsilon(Q)]$. Both properties imply that the conditional demand for abatement services is either 0 or $\varepsilon(Q)$ if respectively $p_A > \tau$ or $p_A < \tau$ and any quantity within $[0, \varepsilon(Q)]$ if $p_A = \tau$. Hence the abatement cost is given by $C_A(p_A, \tau, Q) = \min\{p_A, \tau\} \times \varepsilon(Q)$. The optimal product supply therefore solves the following FOC:

$$p_Q - c'(Q) - \min\{p_A, \tau\} \times \varepsilon'(Q) \leq 0 \text{ (with equality if } Q > 0) \tag{6}$$

If we now introduce the market clearing condition for the final good, we can replace p_Q by $P(Q)$, and, by the early definition of $\beta(Q)$, *i.e.* the marginal benefit of an additional unit of pollution, this quantity is given by:

$$\frac{P(Q) - c'(Q)}{\varepsilon'(Q)} = \min\{p_A, \tau\} \Rightarrow Q(p_A, \tau) = \beta^{-1}(\min\{p_A, \tau\}) \tag{7}$$

and the demand for abatement services becomes:

$$A^d(p_A, \tau) = \begin{cases} 0 & \text{if } p_A > \tau \\ [0, \varepsilon(\beta^{-1}(\tau))] & \text{if } p_A = \tau \\ \varepsilon(\beta^{-1}(p_A)) & \text{if } p_A < \tau \end{cases} \tag{8}$$

Both last equations (Eqs (7) and (8)) stress the consequence of the introduction of abatement services as opposed to abatement goods. In the former case, the dirty firm simply delegates its abatement activity to another firm, while, in the second one, the firm buy additional inputs which enter in the production process and which help to reduce pollution in a more or less efficient way. This means that the dirty firm considers that the marginal pollution abatement of an environmental service is constant (even equal to one). Its purchasing is therefore simply motivated by the difference between its price and the Pigouvian tax. If its price is higher than the Pigouvian fee, it is optimal to pay the tax and to adjust the output level due to this additional cost. In the opposite case, the firm totally abates its emissions but adjusts, as previously, the production level, since the abatement price now enters in the global marginal production cost.

By introducing abatement services instead of abatement goods, we therefore deal with a peculiar abatement demand curve (see Eq (8)) characterized by a flat part when the price is equal to the Pigouvian fee and a decreasing part for prices lower than this tax and which corresponds to full abatement. When this price goes to 0, we even notice, by our early definition of β, that the equilibrium production level will be equal to $Q_{\max}$, the production level without regulation. In other words, this demand has an upper bound given by $A^d(0, \tau) = \varepsilon(Q_{\max})$ which is independent of τ, so that the associated inverse demand function is:

$$P_A(A, \tau) = \max\left\{\min\left\{\tau, \beta\left(\varepsilon^{-1}(A)\right)\right\}, 0\right\} \tag{9}$$

3.2. The monopoly provision of environmental services

The question is now how behaves a monopoly facing such an inverse demand curve. Since the demand is bounded from above by $\varepsilon(Q_{\max})$ which is reached at a zero price, its production choice can be restricted to $A \in [0, \varepsilon(Q_{\max})]$ and its optimal decision solves:

$$\max_{A \in [0, \varepsilon(Q_{\max})]} \{\min\{\tau, p(A)\} \times A - \kappa(A)\} \tag{10}$$

where $p(A) = \beta(\varepsilon^{-1}(A))$ is the inverse demand curve corresponding to full pollution abatement occurring if the Pigouvian tax is larger than this price.

Since we maximize a continuous function on a compact set, existence is not a real issue. But the characterization of this solution requires some additional concavity properties. So let us assume, as usually for a monopoly, that $e_p(A) = \frac{p'A}{p}$, the elasticity of $p(A)$ is decreasing and $\lim_{A \to \varepsilon(Q_{\max})} e_p(A)$ is larger than -1.[10]

The whole inverse demand curve $\min\{\tau, p(A)\}$ nevertheless exhibits a flat part since the dirty firm only reacts to prices lower than the Pigouvian tax. We therefore deal with a non-smooth optimization problem which conducts to several regimes delineated by thresholds which are related to the levels of the Pigouvian fee.

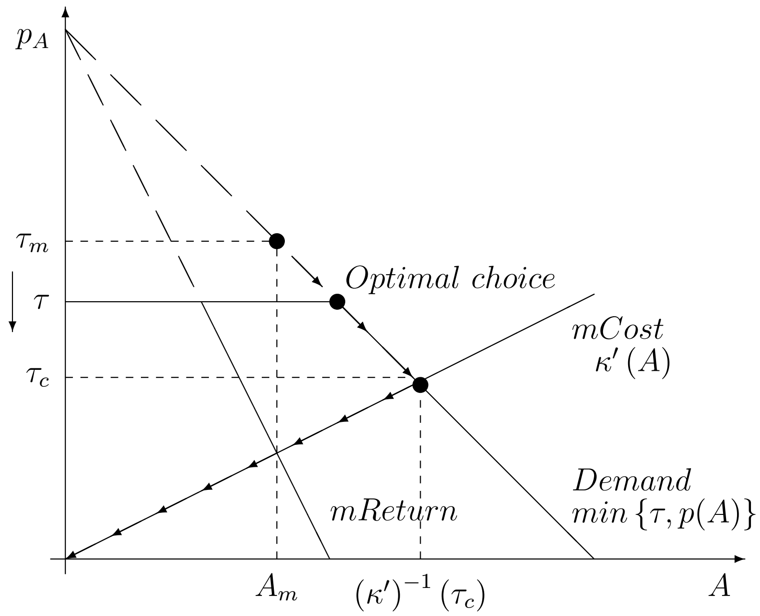

Figure 1: The monopoly solution as τ decreases

To get some intuitions (see Figure 1), let us first introduce the production level A_m. This one corresponds to the monopoly solution under a full pollution abatement behavior of the dirty firm, *i.e.*, the quantity which equates the marginal cost to the marginal revenue computed with $p(A)$. The price associated to this full abatement case is therefore given by $p(A_m) = \frac{1}{1+e_p(A_m)}\kappa'(A_m)$ where $\frac{1}{1+e_p(A_m)}$ stands for the standard margin taken by a monopolist. But this situation only occurs if the Pigouvian tax is larger than this

[10] Of course, the reader may object that these assumptions are not set on the primary data, especially if one bears in mind that $p(A) = \beta\left(\varepsilon^{-1}(A)\right)$. Other sufficient conditions can be introduced like for instance $2e_\varepsilon + e_{\beta'} - e_{\varepsilon'} > 0$ and $e_\varepsilon + e_\beta > 0$ where e denotes the elasticity.

price, *i.e.* $\tau \geq p(A_m)$, otherwise the dirty firm pays the tax instead of abating pollution. This means that there exists a tax rate t_m implicitly given by:

$$\tau_m = \frac{1}{1+e_p(p^{-1}(\tau_m))}\kappa'(p^{-1}(\tau_m)) \tag{11}$$

for which $p(A_m) = \tau_m$, and we can conclude that, for $\tau > \tau_m$, the monopoly always provides A_m units of abatement services.

If the tax rate becomes smaller than τ_m, the monopoly is unable to reach this optimal outcome simply because the monopoly price associated to full abatement is not reachable. In this case, this firm has an incentive to choose the solution which leads to the highest price $p = \tau$ at the production level $A = p^{-1}(\tau)$ *i.e.* to stick at the kink of the demand function. But this behavior is only optimal for prices $p = \tau$ which are larger than the marginal production cost, *i.e.* $\kappa'(p^{-1}(\tau))$. If it is not the case, the firm adjusts its behavior in a way that equates the tax rate to the marginal cost. This means that there exists another threshold $\tau_c < \tau_m$ which the property that $\forall \tau < \tau_c$, the monopoly adopts, in some sense, a competitive behavior. This new threshold is given by:

$$\tau_c = \kappa'(p^{-1}(\tau_c)) \tag{12}$$

From this discussion, we conclude that:

Lemma 1. *Under our assumptions, (i) the monopoly problem (Eq 10) has a unique solution for each tax rate, (ii) there exists two unique thresholds τ_c and τ_m which respectively solve Eq (12) and Eq (11), (iii) the monopoly provision of abatement services is, for any tax τ, a continuous function given by:*

$$A^m(\tau) = \begin{cases} (\kappa')^{-1}(\tau) & \text{if } \tau < \tau_c \\ p^{-1}(\tau) = \varepsilon(\beta^{-1}(\tau)) & \text{if } \tau \in [\tau_c, \tau_m] \\ A_m = p^{-1}(\tau_m) = \varepsilon(\beta^{-1}(\tau_m)) & \text{if } \tau > \tau_m \end{cases} \tag{13}$$

(iv) the price of these services is $P_A^m(\tau) = \min\{\tau, \tau_m\}$ and *(v) from Eq (7), the production of the dirty good is of:*

$$Q^m(\tau) = \beta^{-1}(\min\{\tau, \tau_m\}) \tag{14}$$

3.3. The efficient regulation of the emissions

The previous lemma has an interesting consequence: for any tax rate lower than τ_c, the monopoly behaves like a competitive firm. This firm equates its marginal cost to the tax rate which is nothing else than the price of the abatement services. Since the polluting firm also behaves competitively, the regulator should be able to implement the first best allocation, by selecting, as in a competitive case, a Pigouvian tax equal to the marginal damage of pollution, *i.e.* by setting $\tau^{opt} = D'(\varepsilon(Q^{opt}) - A^{opt})$.

This point is obvious as long as $\tau^{opt} < \tau_c$. In this case we know, from Eq (13), that the monopoly provision of environmental services verifies $\kappa'(A^m(\tau^{opt})) = \tau^{opt}$ while Eq (14) says that $\beta(Q^m(\tau^{opt})) = \tau^{opt}$, and since β and κ' are both monotonic, we conclude, by identification to Eq (4), that $Q^m(\tau^{opt}) = Q^{opt}$ and $A^m(\tau^{opt}) = A^{opt}$, i.e. that the first best allocation is reached.

It therefore remains to make sure that $\tau^{opt} < \tau_c$. If this is not the case, we know from the definition of the threshold τ_c (see Eq (12)) that

$$\tau^{opt} \geq \kappa'(p^{-1}(\tau^{opt})) \Leftrightarrow (\kappa')^{-1}(\tau^{opt}) \geq \varepsilon\left(\beta^{-1}(\tau^{opt})\right) \text{ since } p(A) = \beta\left(\varepsilon^{-1}(A)\right)$$

Moreover, by Eq (4) which characterizes the first best, $(\kappa')^{-1}(\tau^{opt}) = A^{opt}$ and $\beta^{-1}(\tau^{opt}) = Q^{opt}$, so that $A^{opt} \geq \varepsilon(Q^{opt})$. But, from our discussion about the efficient allocation, we know that the assumptions $D(0) = D'(0) = 0$ make sure that there is always a residual pollution at the optimum, *i.e.* that $\varepsilon(Q^{opt}) - A^{opt} > 0$. We can therefore say:

Proposition 1. *Even if an upstream monopoly controls the price of the environmental services while the downstream commodity market remains competitive, the regulator reaches the first-best by setting the Pigouvian tax at the marginal damage of the emissions (evaluated at the first-best), i.e. by setting* $\tau^{opt} = D'(\varepsilon(Q^{opt}) - A^{opt})$.

4. Some straightforward extensions

The previous result suggests that the existence of a residual pollution at the optimal outcome is a crucial assumption. But, as we will see, this is only a simplifying assumption: an identical result can be obtained for $D'(0) \neq 0$. We can also ask if the result is maintained if the regulator uses another incentive based mechanism like tradable pollution permits. The answer is again yes since *under perfect competition both instruments work in the same way.* Finally we relax the representative polluting firm assumption and investigate heterogeneous polluters.

4.1. Efficient regulation and full abatement

To illustrate this point let us come back to the construction of the efficient outcome by relaxing $D'(0) \neq 0$. This outcome solves the optimization program (Eq (1)) introduced in section 2.2. But if we only assume that $D(0) = 0$, we can only argue that $\varepsilon(Q) - A \geq 0$, (*i.e.* without a strict inequality). The interior first order optimality conditions given by Eqs (2) and (3) must therefore be amended. If λ denotes the associated Lagrangian multiplier, the new FOCs become:

$$\begin{cases} P(Q^{opt}) - c'(Q^{opt}) - (D'(\varepsilon(Q^{opt}) - A^{opt}) - \lambda)\,\varepsilon'(Q^{opt}) = 0 \\ D'(\varepsilon(Q^{opt}) - A^{opt}) - \kappa'(A^{opt}) - \lambda = 0 \\ \lambda(\varepsilon(Q^{opt}) - A^{opt}) = 0 \text{ and } \lambda \geq 0 \end{cases} \quad (15)$$

If the constraint is not binding, we are, of course, back to partial abatement, a case previously analyzed. So let us concentrate on the case in which $\lambda > 0$. In this situation, the first and the second condition of system (15) suggest that an efficient allocation has the property that the marginal benefit $\beta(Q^{opt})$ of an additional unit of pollution must be equal to the marginal abatement cost. But to achieve full abatement, this marginal benefit needs only to be smaller than the marginal damage of the first unit of pollution. This situation essentially occurs if $D'(0)$ is high enough. In this case, the efficient allocation verifies:

$$\begin{cases} E^{opt} = \varepsilon\left(Q^{opt}\right) - A^{opt} = 0 \\ \beta(Q^{opt}) = \kappa'(A^{opt}) < D'(0) \end{cases} \tag{16}$$

instead of the interior condition introduced in Eq. (4)

Let us now come back to the monopoly case. Since the marginal damage never enters in the definition of the different behaviors, the monopoly outcome depicted in Lemma 1 remains unchanged. This means that we simply have to make sure that the regulator is able to implement the first best when it is optimal to abate all the pollution (*i.e.* for $\lambda > 0$).

So let us assume that he sets the Pigouvian tax at $\tau^{opt} = \tau^c$ given by Eq (12). From Lemma 1, the equilibrium abatement and production levels are $A^m(\tau^c) = \varepsilon\left(\beta^{-1}\left(\tau^c\right)\right)$ and $Q^m(\tau^c) = \beta^{-1}\left(\tau^c\right)$ so that the first optimality condition of Eq (16) is satisfied. It remains to use the definition of τ^c in order to verify the second condition. This one is $\tau_c = \kappa'(p^{-1}\left(\tau_c\right))$ so that $\beta^{-1}\left(Q^m(\tau^c)\right) = \kappa'\left(A^m\left(\tau_c\right)\right)$. We can therefore note:

Proposition 2. *Assume that the marginal damage of the first unit of pollution is sufficiently large so that full abatement becomes the efficient outcome. If the regulator sets the Pigouvian tax at $\tau^{opt} = \tau^c$ given by Eq (12), he again implements the first best outcome.*

4.2. Pollution permit market

Let us now verify that our result is also maintained if the regulator implements a pollution permit market instead of a Pigouvian tax. To illustrate this point let us come back to the monopoly case depicted in Section 3 and let us introduce a competitive market of pollution rights. The regulator sets the pollution cap E. Without loss of generality, we assume that pollution permits are sold by means of an auction.[11] One right corresponds to one unit of emission and the competitive price of these rights is denoted by p_E.

The competitive price works, at the agent level, like a Pigouvian tax. Under our assumptions, the results obtained in Section 3 concerning the inverse demand and the provision of abatement services by the monopoly extend to this case: it simply remains to replace the Pigouvian tax τ by the price p_E of the emission rights. Thus, our result is maintained if there exists a pollution cap E^{opt} with the property that the equilibrium price of the pollution rights is equal to the optimal level of the Pigouvian tax introduced in Proposition 1. This nevertheless leaves two questions open: (i) what is this pollution cap? and (ii), more crucially, is $p_E = \tau^{opt}$ the unique equilibrium of the pollution permit market when this cap is set? Otherwise there may be several equilibria, some of them being inefficient.

[11] For simplicity, we do not introduce explicitly initial distribution of pollution permits. Following Montgomery [22], the competitive equilibrium of a pollution permit market is irrespective of the mechanism beyond the initial distribution of permits.

To answer both questions, let us observe that the quantities introduce in Eqs (13) and (14) of Lemma 1 describe in our new setting the equilibrium allocation conditionally to each pollution permit price p_E. So if we want to look at the global equilibrium of this vertical structure with tradable rights, it remains to clear the permit market. The demand for pollution rights is given by:

$$E^D(p_E) = \varepsilon\left(Q^m\left(p_E\right)\right) - A^m\left(p_E\right) \tag{17}$$

and we observe that for any price $p_E \geq \tau_c$ there is full abatement, hence

$$E^D(p_E) = \begin{cases} \varepsilon\left(\beta^{-1}\left(p_E\right)\right) - \left(\kappa'\right)^{-1}\left(p_E\right) \text{ if } p_E < \tau_c \\ 0 \text{ if } p_E \geq \tau_c \end{cases}$$

Now let us remember from our early discussion, that $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) < \tau_c$. Hence the optimal pollution cap must be:

$$E^{opt} = \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right) - \left(\kappa'\right)^{-1}\left(\tau^{opt}\right)$$

Moreover, in order to make sure that $p_E = \tau^{opt}$ is the unique equilibrium, let us observe that the demand for tradable rights is decreasing for all $p_E < \tau_c$, *i.e.*

$$\frac{dE^D(p_E)}{dp_E} = \frac{\varepsilon'\left(\beta^{-1}(p_E)\right)}{\beta'(\beta^{-1}(p_E))} - \frac{1}{\kappa"\left((\kappa')^{-1}(p_E)\right)} < 0 \tag{18}$$

since under our assumptions $\varepsilon' > 0$, $\beta' < 0$. We can therefore say:

Proposition 3. *If pollution is regulated by a pollution permit market, the regulator is also able to implement the efficient allocation by choosing the optimal pollution cap* E^{opt}.

4.3. Heterogeneous polluters

Finally, it can also be interesting to verify if this result extends to heterogeneous polluters. So let us introduce m polluting firms, indexed by j, with different cost and emission functions, $c_j(q)$ and $\varepsilon_j(q)$, each of them satisfying the assumptions introduced in Section 2. All the other assumptions are maintained, especially those concerning the marginal damage at 0, so that an efficient allocation is now given by:

$$\forall j \quad P(\textstyle\sum_{j=1}^m q_j^{opt}) - c_j'\left(q_j^{opt}\right) - D'\left(\sum_{j=1}^m \varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right).\varepsilon_j'\left(q_j^{opt}\right) = 0 \tag{19a}$$

$$D'\left(\textstyle\sum_{j=1}^m \varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) = 0 \tag{19b}$$

The intuition beyond this extension is quite simple. Even if the polluting firms are heterogeneous in costs and/or in emissions, they invariably choose their level of abatement by comparing the price p_A to the Pigouvian tax τ. One therefore expects that the *aggregated* demand for abatement goods behaves in the same way: no abatement if $p_A > \tau$, full pollution abatement noted $A_f(p_A)$ if $p_A < \tau$ and any situation between both if $p_A = \tau$. Moreover if the part of the demand corresponding to full abatement is again decreasing and bounded from above, the inverse demand has the same structure as the one obtained in Section 3.1. So, with similar assumption on its elasticity, the properties of the monopoly outcome provided in Lemma 1 should extend to the case of heterogeneous polluters.

The main weakness of this argument is that the computation of the aggregated level of abatement corresponding to full pollution reduction ($A_f(p_A)$) and, more generally, the construction of the market clearing production levels for all τ and p_A, is now more tricky. In fact - as in Section 3.1 - it is easy to compute the individual conditional demand for abatement services and the cost function related to this activity. But when we seek for the market clearing production level, we face now a system of m equations since the price of the polluting good equates, for each firm, the full marginal cost including abatement cost. In other words, these individual production levels solve:

$$\forall j = 1, \ldots, m \quad P\left(\sum_{j=1}^{m} q_j\right) = c_j'(q_j) + \min\{p_A, \tau\} \times \varepsilon_j'(q_j) \tag{20}$$

instead of the single equation given by Eq. (7). It can however be shown:

Lemma 2. *Under our assumptions on the demand, the costs and the emissions, the system of Eqs. (20) admits a unique solution $(q_j(k))_{j=1}^{m}$ for each constant $k = \min\{p_A, \tau\} \geq 0$. Moreover $A_f(p_A) = \sum_{j=1}^{m} \varepsilon_j(q_j(p_A))$ - the total quantity of abatement good which induces full pollution reduction - is decreasing (for all $p_A \leq \tau$) and bounded from above by $A_{\max} = \sum_{j=1}^{m} \varepsilon_j(q_j(0))$.*

It finally remains to verify that the Pigouvian fee $\tau^{opt} = D'\left(\sum_{j=1}^{m} \varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right)$ (i) is lower than the highest tax τ_c^h which induces a competitive behavior of the abatement producer and (ii) is able to implement the first best.[12] The implementation part is obvious. It $\tau^{opt} < \tau_c^h$, the eco-service firm equates its marginal cost to the Pigouvian tax, *i.e.*, $\tau^{opt} = k'(A)$ so that that the second efficiency condition (Eq. (19b)) is satisfied. Since, in this case, the price of the abatement good is τ^{opt}, the set of Eqs 20 which describes the equilibrium production levels corresponds exactly to the first efficiency condition (Eqs. (19a)).

If $\tau^{opt} \geq \tau_c^h$, this implies, by the definition of τ_c^h, that the tax τ^{opt} is larger than the marginal abatement cost which induces full abatement at price $p_A = \tau^{opt}$, *i.e.* $\tau^{opt} > k'(A_f(\tau^{opt}))$. This again implies that one reduces, at the optimum, more pollution than the existing one which is impossible. We can therefore claim that:

Proposition 4. *Even if the polluting sector is composed of heterogeneous firms, especially concerning their emissions, the regulator is able to neutralize the monopoly power on the abatement service market and to implement the first best by setting the tax rate at the marginal damage.*

[12] This threshold τ_c^h is now defined $\tau_c^h = \kappa'\left(A_f(\tau_c^h)\right)$ and it can be shown with a similar argument as in the proof of Lemma 1 that it exists and is unique.

5. Cournot competition in the eco-industry

Let us now restore the representative polluting firm assumption, but instead let us introduce Cournot competition in the eco-service industry. They are now n firms indexed by i in this industry, each of them being characterized by a cost function $\kappa_i(a)$. Seeing that all the other assumptions are maintained, an efficient allocation now verifies:

$$\forall i = 1\ldots, n \quad \beta(Q^{opt}) = D'\left(\varepsilon\left(Q^{opt}\right) - \textstyle\sum_{i=1}^{n} a_i^{opt}\right) = \kappa_i\left(a_i^{opt}\right) \tag{21}$$

Could we again implement the first best allocation by setting the Pigouvian tax at $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - \sum_{i=1}^{n} a^{opt}\right)$? To answer this question, we first move to the study of the best reply of these Cournot players, the behavior of the polluting firm being by construction unchanged.

5.1. The best reply of a firm

If we denote by $A_{-i} = \sum_{j=1, j\neq i}^{n} a_j^{opt}$ the aggregated abatement supply of the opponents, this best response is given by:

$$BR_i(A_{-i}, \tau) \in \arg\max_{a_i} \left\{\min\left\{\tau, p(a_i + A_{-i})\right\} \times a_i - \kappa_i\left(a_i\right)\right\} \tag{22}$$

where $p(A) = \beta\left(\varepsilon^{-1}\left(A\right)\right)$ stands for the inverse demand part corresponding to full abatement behavior of the polluting industry. To gain some intuition about $BR_i(A_{-i}, \tau)$, we introduce $br(A_{-i})$, the best response obtained when the demand always corresponds to a full abatement behavior, *i.e.* with $p(a)$. This one is defined by:

$$br(A_{-i}) = \max_{a_i \in [0, \varepsilon(Q_{\max})]} p(a_i + A_{-i}) \times a_i - \kappa_i\left(a_i\right) \tag{23}$$

and solves:

$$p'(a_i + A_{-i}) \times a_i + p(a_i + A_{-i}) - \kappa'\left(a_i\right) = 0 \tag{24}$$

We even know that (i) $br_i(\varepsilon(Q_{\max})) = 0$ since $p\left(\varepsilon(Q_{\max})\right) = 0$, (ii) $br_i(0) = A_m < \varepsilon(Q_{\max})$ the monopoly solution and (iii) br_i is decreasing as long as $e(p') > -1$, the elasticity of p' is larger than -1.

We now try to understand, at least graphically - see Figure 2 - what happens when the constraint on the price (*i.e.* $p(A) \leq \tau$) begins to matter.

We consider in Figure 2 the unconstrained best response $br_i(A_{-i})$ and the $-45°$ line given by $a_i + A_{-i} = p^{-1}(\tau)$. Since $p(a)$ is decreasing, this linear constraint provides, for each A_{-i}, the minimum production level of firm i which ensures that the price is lower than τ, or in other words, the minimal production level which preserves market power. So, as long as the best reply $br_i(A_{-i})$ lies above this line, the firm is able to exert market powers, *i.e.* his best reply is $BR_i(A_{-i}, \tau) = br_i(A_{-i})$. This remains true until $br_i(A_{-i})$ cuts this line. This intersection occurs when the A_{-i} is equal to $M_i\left(\tau\right)$ which is defined by:

$$p'(p^{-1}(\tau)) \times \left(p^{-1}(\tau) - M_i\left(\tau\right)\right) + \tau - \kappa_i'\left(p^{-1}(\tau) - M_i\left(\tau\right)\right) = 0 \tag{25}$$

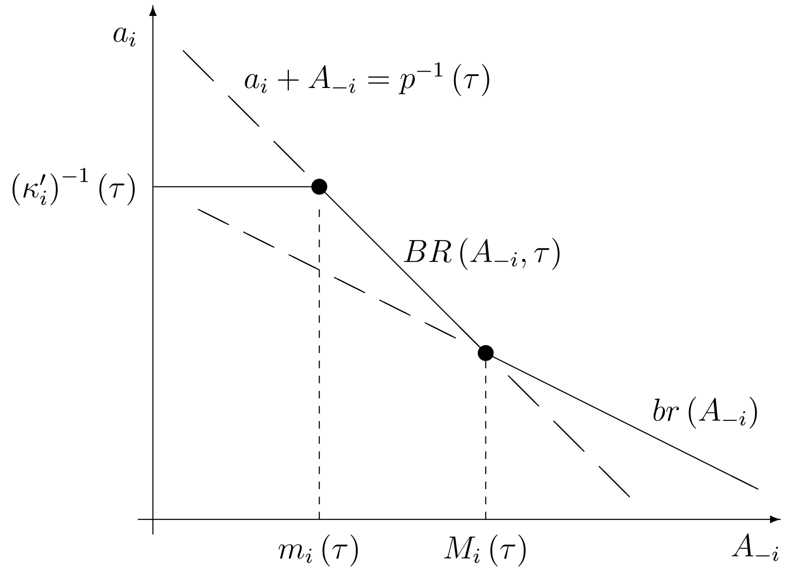

Figure 2: The best response of Firm i

If $A_{-i} \leq M_i(\tau)$, Firm i is, as in the monopoly case, unable to manipulate the price. This is why it becomes optimal to produce the quantity which keeps the price at τ. In other words, the best reply is $BR_i(A_{-i}, t) = p^{-1}(\tau) - A_{-i}$. But, as A_{-i} decreases, this firm increases its market share while the price remains constant at τ. It may therefore happen that the marginal production cost becomes larger than the price. This situation occurs for all $A_{-i} < m_i(\tau)$ with $m_i(\tau)$ solution to:

$$\kappa_i'\left(p^{-1}(\tau) - m_i(\tau)\right) = \tau \Leftrightarrow m_i(\tau) = p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) \tag{26}$$

In this last case, the best reply is simply $BR_i(A_{-i}, \tau) = (\kappa_i')^{-1}(\tau)$. Moreover, by construction, it is immediate that $m_i(\tau) \leq M_i(\tau)$.

This explanation nevertheless requires that (i) the line $a_i + A_{-i} = p^{-1}(\tau)$ crosses $br_i(A_{-i})$ at a unique positive point and (ii) $p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) \geq 0$ otherwise some of these cases are vacuous. A formal construction of the best response is provided in Appendix. We even show that the case depicted in Figure 1 only occurs when the rate τ is lower than τ_c^i given by $\tau_c^i = \kappa_i'(p^{-1}(\tau_c^i))$, *i.e.* the upper bound of the tax rates at which Firm i adopts a competitive behavior. But these are basically the taxes in which we are interested in. This is why we only spell out the characterization of the best reply for $\tau \leq \tau_c^i$.

Lemma 3. *For any tax rate $\tau < \tau_c^i$ and any $A_{-i} \in [0, \varepsilon^{-1}(Q_{\max})]$ the best response of an eco-service firm is given by:*

$$BR(A_{-i}, \tau) = \begin{cases} (\kappa_i')^{-1}(\tau) \ \text{if } A_{-i} < m_i(\tau) \\ p^{-1}(\tau) - A_{-i} \ \text{if } m_i(\tau) \leq A_{-i} \leq M_i(\tau) \\ br_i(A_{-i}) \ \text{if } A_{-i} \geq M_i(\tau) \end{cases} \tag{27}$$

Moreover this best response is continuous and, since $e(p') > -1$, it is also non increasing with A_{-i}.

5.2. Cournot equilibrium and efficient taxation

From the previous Lemma, we essentially learn that the competitive behavior is a part of the best response of Firm i when $\tau \leq \tau_c^i$. So we will concentrate now on taxes smaller than $\tau_c^{\min} = \min_{i=1,\ldots,n} \{\tau_c^i\}$. In this case, it can be shown that:

Lemma 4. *For any tax $\tau < \tau_c^{\min}$, the unique equilibrium provision of the eco-services is, for each firm, $a_i^C(\tau) = (\kappa_i')^{-1}(\tau)$ while the price of these services is $P_A^C(\tau) = \tau$. From Eq (7), we observe that the production of the dirty good is $Q^C(\tau) = \beta^{-1}(\tau)$.*

The existence part of this result is obvious. Since $\tau < \tau_c^{\min}$, then $\forall i$, $\tau < \kappa_i'(p^{-1}(\tau))$ by the definition of τ_c^i or equivalently $\forall i$, $(\kappa_i')^{-1}(\tau) < p^{-1}(\tau)$. We can therefore say that:

$$\forall i \quad A_{-i}^c = \sum_{j=1, j\neq i}^{n} \left(\kappa_j'\right)^{-1}(\tau) < p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) = m_i(\tau) \tag{28}$$

This means from Eq. (27) that playing $a_i^C = (\kappa_i')^{-1}(\tau)$ is for each firm a best response. Concerning uniqueness, let us first observe that the best response is bounded from above by $(\kappa_i')^{-1}(\tau)$. So if there exists another equilibrium, say b^C, there must be at least one firm i_0 such that $b_{i_0}^C < \left(\kappa_{i_0}'\right)^{-1}(\tau)$ and, due to the upper bound, $B_{-i_0}^c \leq \sum_{j=1,j\neq i_0}^{n} \left(\kappa_j'\right)^{-1}(\tau)$. But this leads to a contradiction since for $\tau < \tau_c^{\min}$, we have as before that $B_{-i_0}^c < m_{i_0}(\tau)$ so that $b_{i_0}^C = \left(\kappa_{i_0}'\right)^{-1}(\tau)$ should be the best response.

Let us not assume that the regulator sets $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - \sum_{i=1}^{n} a_i^{opt}\right)$. If $\tau^{opt} \leq \tau_c^{\min}$, the Cournot equilibrium meets the first best conditions given by Eq. (21) since, by Lemma 4, we have*:*

$$\forall i = 1\ldots,n \quad \beta\left(Q^C(\tau^{opt})\right) = \tau^{opt} = \kappa_i'\left(a_i^C(\tau^{opt})\right) \tag{29}$$

It remains to verify that $\tau^{opt} \leq \tau_c^{\min}$. If it is not the case, there exists at least one agent, say i_0, for which $\tau^{opt} > \kappa_{i_0}'(p^{-1}(\tau^{opt}))$. But this implies for our characterization of an optimal allocation (Eq. (21)) that

$$a_{i_0}^{opt} = \left(\kappa_{i_0}'\right)^{-1}(\tau^{opt}) > p^{-1}\left(\tau^{opt}\right) = \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right) = \varepsilon(Q^{opt}) \tag{30}$$

so that $\sum_{i=1}^{n} a_{i_0}^{opt} > \varepsilon(Q^{opt})$ since all the $a_i^{opt} \geq 0$. In other words there is, at the optimum, more abatement than emissions, which is a contradiction. We can therefore say:

Proposition 5. *If there is Cournot competition in an eco-service industry and pure competition in the polluting sector, the first best allocation can be reached by setting as usually the tax rate to the marginal damage.*

6. Two main limits of the result

Up to now we have extended our leading case of Section 3 to various settings. Both main assumptions nevertheless remained: an upstream sector of eco-services and downstream perfect competition. In this section, we show that both assumptions are very crucial. We first introduce a counter example which shows that our result cannot be extended to abatement goods. In a second step we introduce downstream monopoly power. Due to this new market imperfection the first best cannot be implemented. In this case, we show that the optimal Pigouvian tax must be lower than the marginal damage, recovering Barnett result [2].

6.1. Abatement goods versus abatement services

Our results only apply for environmental services whereas most of the literature considers environmental goods. In this latter case and under an "end-of-pipe" pollution assumption (see Katsoulacos and Xepapadeas [20]), the emissions $\varepsilon(Q, A)$ are for the dirty firm negatively correlated to the use of abatement goods. The marginal productivity of abatement goods $(-\partial_A \varepsilon(Q, A))$ therefore matters in the abatement choice contrary to abatement services for which the purchase decision in only based on the difference between the environmental tax and the price. As we will show, this strongly reduces the control that the regulator has over the equilibrium and limits the implementation of the first best.

As the case of abatement goods is largely documented in the literature under alternative sets of assumptions, we simply illustrate our purpose by a (counter) example to highlight what changes compared to eco-services.

Example 1. *We consider (i) quadratic cost functions, i.e.* $c(Q) = \frac{1}{2}Q^2$ *and* $\kappa(A) = \frac{1}{2}A^2$*, (ii) a linear demand* $P = 1 - Q$*, (iii) a linear damage function* $D(E) = 0.2E$ *and (iv) an emission function* $\varepsilon(Q, A) = \max\left\{Q - \sqrt{A}, 0\right\}$ *which is now "non-linear" in abatement.*

If we construct the inverse demand for abatement goods as in Section 3.1, we observe, after some computations, that the conditional demand for abatement goods and the cost associated to this activity are given by:

$$A(p_A, \tau, Q) = \min\left\{Q^2, \left(\frac{\tau}{2p_A}\right)^2\right\} \qquad C_A(p_A, \tau, Q) = \begin{cases} p_A Q^2 \text{ if } Q \leq \frac{\tau}{2p_A} \\ \tau Q - \frac{\tau^2}{2p_A} \text{ if } Q > \frac{\tau}{2p_A} \end{cases} \tag{31}$$

As expected, the conditional demand does not move from full abatement (here Q^2) to no abatement since $\left(\frac{\tau}{2p_A}\right)^2$ is a demand for partial pollution reduction. Moreover, the marginal cost $\partial_Q C_A$ associated to this activity is no more linear in quantities since $\partial_Q C_A = \min\{2p_A Q, \tau\}$. This drastically modifies the computation of the dirty good market clearing condition, which is given by:

$$P(Q) = \frac{dc}{dQ} + \frac{\partial C_A}{\partial Q} \Leftrightarrow 1 - Q = Q + \min\{2p_A Q, \tau\}$$

$$\Leftrightarrow Q(p_A, \tau) = \begin{cases} \frac{1}{2(1+p_A)} \text{ if } p_A \leq \tau(1+p_A) \\ \frac{1-\tau}{2} \text{ else} \end{cases} \tag{32}$$

It follows that the demand for abatement goods consistent with market clearing and the inverse demand curve are:

$$A(p_a, \tau) = \min\left\{\left(\tfrac{1}{2(1+p_A)}\right)^2, \left(\tfrac{\tau}{2p_A}\right)^2\right\} \qquad P_A(A, \tau) = \begin{cases} \frac{\tau}{2\sqrt{A}} \text{ if } A < \left(\frac{1-\tau}{2}\right)^2 \\ \frac{1}{2\sqrt{A}} - 1 \text{ if } A \geq \left(\frac{1-\tau}{2}\right)^2 \end{cases} \tag{33}$$

Since $\frac{1}{2\sqrt{A}} - 1$ stands for $p(A)$, the inverse demand under full abatement, this inverse demand can be written as $P_A(A, \tau) = \min\left\{p(A), \frac{\tau}{2\sqrt{A}}\right\}$. Clearly this expression differs from $\min\{p(A), \tau\}$. The monopoly is now able to exert market power on the whole range of its inverse demand. The flat part vanishes but a kink remains. This is why the monopoly solution leads nevertheless to three different outcomes (i) the monopoly solution under partial abatement for small taxes (ii) a solution which sticks in the kink of the inverse demand curve and (iii) the monopoly strategy under full abatement for high taxes. It can be show, in this example, that:

$$A^m(\tau) \approx \begin{cases} \left(\frac{\tau}{4}\right)^{2/3} \text{ for } \tau < 0.2291 \\ \left(\frac{1-\tau}{2}\right)^2 \text{ for } \tau \in [0.2291, 0.5265] \\ 0.0561 \text{ for } \tau > 0.5265 \end{cases} \quad \text{and } Q^m(\tau) \approx \begin{cases} \left(\frac{1-\tau}{2}\right) \text{ for } \tau \leq 0.5265 \\ 0.2368 \text{ for } \tau > 0.5265 \end{cases} \tag{34}$$

Moreover a simple exercise of computation shows that $A^{opt} = 0.2$ and $Q^{opt} = 0.4$. As $A^m(\tau)$ is bounded from above by 0.1485, the first best is unreachable simply because the monopoly can now take a margin in the case of full and partial pollution abatement.

6.2. Downstream market power and the Barnett's result

As we will see our result is also limited by the number of market failures and/or imperfections that the regulator control. We showed that one instrument, the Pigouvian tax, was able to regulate the environmental externality in the dirty sector and the imperfect competition problem in the eco-service industry. But, this result will fail if a new market imperfection is introduced. In this case the regulator will be able to reach only a second best policy. In order to illustrate this problem, let us move back to the basic case of Section 3, and let us introduce monopoly power in the dirty sector instead of pure competitive[13].

[13]This is clearly a simplifing assumption. Similar results can be obtained by introducing Cournot competition on the downstream poluting good market. We neverthless only introduce monopoly power to stay as closed as possible to Barnett's paper [2].

Our result will recover Barnett's results [2] under this new framework since we will show that the second best Pigouvian tax must be lower than the marginal damage. This result therefore contrasts with Canton et al. ([6]). To fit with both papers, we assume a linear damage function $D(E) = vE$.

Let us first quickly revisit Section 3 in order to see what changes. Since the dirty firm remains competitive on the abatement market, its conditional demand as well as the abatement cost $C_A(p_A, \tau, Q) = \min\{p_A, \tau\} \cdot \varepsilon(Q)$ remain unchanged. As this firm now exerts a monopoly power on the output market, its output choice therefore satisfies:

$$Q^m \in \arg\max_Q P(Q) \times Q - c(Q) - \min\{p_A, \tau\} \times \varepsilon(Q) \tag{35}$$

If we assume, as usually for a monopoly, that the elasticity $e_P = \frac{QdP}{PdQ}$ of the inverse demand curve belongs to $[-1, 0)$ and is decreasing, the solution of this program can be summarized by the following FOC:

$$P'(Q) \times Q + P(Q) - c'(Q) - \min\{p_A, \tau\} \times \varepsilon'(Q) \leq 0 \text{ (with equality if } Q > 0) \tag{36}$$

This equation is relatively similar to the competitive FOC (see Eq. 6). So, let us now introduce

$$\beta_m(Q) = \frac{P'(Q) \times Q + P(Q) - c'(Q)}{\varepsilon'(Q)}$$

instead of the marginal benefit for pollution $\beta(Q)$. This function shares similar properties with $\beta(Q)$: (i) it is positive up to $\bar{Q}_m$ which is now the monopoly production level without environmental regulation and (ii) it is decreasing on $\left[0, \bar{Q}_m\right]$ under our restriction on the elasticity e_P.[14] It follows that $Q = \beta_m^{-1}(\min\{p_A, \tau\})$ and the rest of the argument of Section 3.1 and 3.2 extends as long as we replace the function β by β_m and $p(A)$ by $p_m(A) = \beta_m(\varepsilon^{-1}(A))$. Hence:

Lemma 5. *If the elasticity of $p_m(A)$ is decreasing and belong to $(-1, 0)$, the equilibrium quantities with upstream and downstream monopoly power are given by:*

$$A^*(\tau) = \begin{cases} (\kappa')^{-1}(\tau) \text{ for } \tau < \tau_c \\ \varepsilon(Q^*(\tau)) \text{ for } \tau \geq \tau_c \end{cases} \text{ and } Q^*(\tau) = \beta_m^{-1}(\min\{\tau, \tau'_m\}) \tag{37}$$

with τ_c defined as in Eq. (12) and τ'_m given by $\tau'_m = \frac{1}{1+e_{p_m}(p_m^{-1}(\tau'_m))}\kappa'(p^{-1}(\tau'_m))$. Moreover the price of the abatement services is $P_A^* = \min\{\tau, \tau'_m\}$.

If we now seek for the second best Pigouvian tax, we have to solve:

$$\max_\tau \underbrace{\int_0^{Q^*(\tau)} P(q)dq - c(Q^*(\tau)) - \kappa(A^*(\tau)) - D(\max\{\varepsilon(Q^*(\tau)) - A^*(\tau), 0\})}_{=SB(\tau)} \tag{38}$$

[14] This follows from computation and the fact that (i) $P"(Q)Q + 2P'(Q) = P'(Q)(1 + e_P) + P\frac{de_P}{dQ}$ and (ii) $\xi_m(Q) \geq 0$ on $\left[0, \bar{Q}_m\right]$.

As the equilibrium quantities $A^*(\tau)$ and $Q^*(\tau)$ are constant $\forall\tau \geq \tau_m'$ (see Eq. 37), $\forall\tau \geq \tau_m'$, $SB(\tau)$ is also constant. We can therefore restrict our attention to tax rates $\tau \in [0, \tau_m']$. So consider now a tax $\tau \in (\tau_c, \tau_m']$. In this case, the monopoly provision of abatement services totally reduces pollution. It follows that $\forall\tau \in (\tau^c, \tau^m)$:

$$\frac{dSB(\tau)}{d\tau} = \left(P\left(Q^*(\tau)\right) - c'\left(Q^*(\tau)\right) - \kappa'\left(\varepsilon\left(Q^*(\tau)\right)\right)\varepsilon'\left(Q^*(\tau)\right)\right)\frac{dQ^*}{d\tau} \tag{39}$$

Since the FOC of the polluting firm (see Eq. (36)) is satisfied at equilibrium and $P_A^* = \min\{\tau, \tau_m'\}$, Eq. (39) becomes:

$$\frac{dSB(\tau)}{d\tau} = \left(-P'\left(Q^*(\tau)\right)Q^*(\tau) + \left(\tau - \kappa'\left(\varepsilon\left(Q^*(\tau)\right)\right)\right)\varepsilon'\left(Q^*(\tau)\right)\right)\frac{dQ^*}{d\tau} \tag{40}$$

Moreover $\frac{dQ^*(\tau)}{d\tau} = 1/\left(\beta_m'\left(\beta_m^{-1}(\tau)\right)\right) < 0$ since β_m is decreasing and by the definition of τ_c (see Eq. (12)) and we know that $\forall\tau \in (\tau^c, \tau^m)$, $\tau > k'\left(A^*(\tau)\right)$. We can therefore assert that $\forall\tau \in (\tau^c, \tau^m)$, $\frac{dSB(\tau)}{d\tau} < 0$.

Following these developments a second best solution necessarily belongs to $[0, \tau_c]$. If this solution is an interior one, we can write:

$$\frac{dSB(\tau)}{d\tau} = \left(P(Q^*(\tau)) - c'\left(Q^*(\tau)\right) - v\varepsilon'\left(Q^*(\tau)\right)\right)\frac{dQ^*}{d\tau} - \left(\kappa'\left(A^*(\tau)\right) - v\right)\frac{dA^*}{d\tau} = 0 \tag{41}$$

By using again the FOC of the polluting firm (see Eq. (36)), we have:

$$\left(-P'\left(Q^*(\tau)\right)Q^*(\tau) + (\tau - v)\,\varepsilon'\left(Q^*(\tau)\right)\right)\frac{dQ^*}{d\tau} - (\tau - v)\frac{dA^*}{d\tau} = 0 \tag{42}$$

which implies that:

$$\tau^{sb} - v = \frac{P'\left(Q^*(\tau)\right)Q^*(\tau)\frac{dQ^*}{d\tau}}{\varepsilon'\left(Q^*(\tau)\right)\frac{dQ^*}{d\tau} - \frac{dA^*}{d\tau}} < 0 \tag{43}$$

since $\forall\tau \in [0, \tau_c]$, $\frac{dA^*}{d\tau} = 1/k''\left((k')^{-1}(\tau)\right) > 0$ and $\frac{dQ^*(\tau)}{d\tau} = 1/\left(\beta_m'\left(\beta_m^{-1}(\tau)\right)\right) < 0$. We can therefore claim:

Proposition 6. *If there is a monopoly power on the final good and on the abatement service market, the second best taxation rule neutralizes the market power on the abatement service market (since $\tau^{sb} \leq \tau_c$) but remains lower than the marginal damage in order to limit the reduction of the production of the final good induced by the monopoly power.*

7. Concluding remarks

The EGSS is highly concentrated. The economic literature has mainly analyzed the design of the environmental regulation taking into account this feature. However, no study tried to analyze in which measure distinguishing between abatement goods and abatement services matters for environmental regulation. It was the topic of this paper.

We found a very interesting result for policy makers. Whereas there are two market failures in our economy - market power on the abatement service market and pollution generated by downstream firms - the regulator can reach the first best with only one tool, the environmental regulation. This result challenges the Tinbergen rule.

The abatement services introduce a flat part in the inverse demand curve. As the polluting firm makes only an arbitrage between the price of the abatement services and the Pigovian tax to be in conformity with the environmental regulation, it is indifferent between both choices if the price of the abatement service equals the Pigovian tax. An accurate choice of the Pigovian tax can lead the monopoly to choice the first best level of production. We showed that if this tax is set such that it is equal to the marginal damage, the economy reaches the first best.

We then extended our model taking into account different assumptions in order to verify the stability of our result. We first set assumptions such as total abatement is allowed, second we considered a pollution permit market instead the Pigovian tax and, third, we investigated heterogenous polluters. We finally assumed that instead of a monopoly, a Cournot competition characterizes the eco-industry. We finished by underlining some limits of our result. It does not hold anymore if we consider abatement goods instead of abatement services and if we add another market failure in the output market.

If we essentially explore the case of upstream market power in Section 6.2, other market imperfections would also be considered. If a pollution permit market is organized, a polluting firm can exert a dominant position on this market i.e. a simple manipulation (see Hahn ([19]) and Westskog *[39]),* or try to manipulate the cost of its opponents on the output market what we call an exclusionary manipulation (see Misiolek and Elder [21], Sartzetakis [31] or Von der Fehr [38]). Some other externalities may also modify our result, like a polluting eco-industry (see Sans et al. [30]). In this case, the Pigouvian taxation not only modifies the demand for the abatement firms but also the production cost of the abater.

In this article, we have implicitly considered a benevolent regulator which controls a closed economy. However, it is well-known that lobbies matter in the definition of an environmental policy (Aidt [1]) and abatement services are often exchanged on an international market. Canton ([4]) studies the role of lobbies in the case of eco-industry providing environmental goods. In an open economy, each firm has to be submitted at national environmental regulations. In this case, environmental policies can be used in a strategic way (see for instance Barrett [3] or Hamilton and Requate [18]). Nimubona [23] studies effects of reductions in trade barriers on the eco-industry sector decided in the Doha Round.

Some further research is needed to investigate whether our result would be challenged or not taking into account these new features or how it should be amended.

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A. Proof of Lemma 1

We need to solve:

$$\max_{A\in[0,\varepsilon(Q_{\max})]} \underbrace{\left[\min\left\{\tau, p\left(A\right)\right\} A - \kappa\left(A\right)\right]}_{\pi(A,\tau)} \text{ where } p\left(A\right) = \beta\left(\varepsilon^{-1}\left(A\right)\right)$$

<u>*Step 1*</u>*: Existence of a unique solution*

Since we maximize $\pi(A,\tau)$ over a compact set, it remains to verify that $\pi(A,\tau)$ is strictly concave in A. Moreover, $\kappa\left(A\right)$ being strictly convex, we only need to check that $\min\left\{\tau.A, p\left(A\right).A\right\}$ is concave.

But let us first observe that, under the assumption that $e_p(\varepsilon(Q_{\max})) > -1$ and $\frac{de_p(A)}{dA} \leq 0$, $p(A).A$ is concave since

$$\frac{d^2}{(dA)^2}(p(A).A) = p'(A)(e_p(A)+1) + p(A)\frac{de_p(A)}{da} \leq 0$$

It follows that $\forall\lambda \in [0,1]$ and $\forall A_1, A_2 \in [0, \varepsilon(Q_{\max})]$

$$\begin{aligned}
&\min\{\tau(\lambda A_1 + (1-\lambda)A_2), p((\lambda A_1 + (1-\lambda)A_2))(\lambda A_1 + (1-\lambda)A_2)\}\\
&\geq \min\{\lambda\tau A_1 + (1-\lambda)\tau A_2, \lambda p(A_1)A_1 + (1-\lambda)p(A_2)A_2\} \text{ (concavity of } p(A)A)\\
&\geq \lambda\min\{\tau A_1, p(A_1)A_1\} + (1-\lambda)\min\{\tau A_2, p(A_2)A_2\} \text{ (concavity of the } \min\{x,y\}
\end{aligned}$$

Step 2: *Construction of the thresholds*

This program is not smooth but nevertheless concave. This means (see Rockafellar [27]) that an optimum is reached iff $0 \in \partial_A\pi$ where $\partial_A\pi$ denotes the sub-derivative of $\pi(A,\tau)$ with respect to A. By computation, we get:

$$\partial_A\pi = \begin{cases} \tau - \kappa'(A) & \text{if } A < p^{-1}(\tau)\\ \left[\underbrace{\tau + p'(p^{-1}(\tau))p^{-1}(\tau) - \kappa'(p^{-1}(\tau))}_{:=\phi_m(\tau)}, \underbrace{\tau - \kappa'(p^{-1}(\tau))}_{:=\phi_c(\tau)}\right] & \text{if } A = p^{-1}(\tau)\\ p(A) + p'(A)A - \kappa'(A) & \text{if } A > p^{-1}(\tau)\end{cases} \tag{44}$$

Now observe that $\phi_m(\tau) = 0$ and $\phi_c(\tau) = 0$ implicitly defines the two thresholds τ_c and τ_m introduced in Eqs (11) and (12). It remains to verify that these thresholds exist, are unique and that $\tau_c < \tau_m$. These results directly follow from the next observations:

(i) ϕ_c and ϕ_m are both increasing. More precisely, $\phi_c'(\tau) = -\frac{\kappa"(p^{-1}(\tau))}{p'(p^{-1}(\tau))} > 0$, and

$$\begin{aligned}
\phi_m'(\tau) &= \tfrac{d}{d\tau}\left((p(A) + p'(A).A) - \kappa(A)|_{A=p^{-1}(\tau)}\right)\\
&= \left.\underbrace{\left(\tfrac{d^2}{(dA)^2}(p(A).A) - \kappa"(A)\right)}_{<0 \text{ (second order condition)}}\right|_{A=p^{-1}(\tau)} \times \tfrac{1}{p'(p^{-1}(\tau))} > 0
\end{aligned}$$

(ii) $\lim_{\tau\to0}\phi_c(\tau) < 0$ and $\lim_{\tau\to0}\phi_m(\tau) < 0$. Let us remember that $p(\varepsilon(Q_{\max})) = 0$, it follows that $\lim_{\tau\to0}\phi_c(\tau) = -\kappa'(\varepsilon(Q_{\max})) < 0$ and $\lim_{\tau\to0}\phi_m(\tau) = p'(\varepsilon(Q_{\max}))\varepsilon(Q_{\max}) - \kappa'(\varepsilon(Q_{\max})) < 0$

(iii) $\lim_{\tau\to\tau_m}\phi_c(\tau) > 0$ and $\lim_{\tau\to+\infty}\phi_m(\tau) > 0$. From the implicit definition of τ_m, we observe that $\lim_{\tau\to\tau_m}\phi_c(\tau) = -p'(p^{-1}(\tau_m))p^{-1}(\tau_m) > 0$. Concerning the second limit, we note:

$$\lim_{\tau\to+\infty}\phi_m(\tau) = \lim_{\tau\to+\infty}\tau\left(1 + e_p(A)|_{A=p^{-1}(\tau)}\right) - \lim_{\tau\to+\infty}\kappa'(p^{-1}(\tau))$$

The second term of the r.h.s. is clearly bounded since $p^{-1}(\tau) \in [0, \varepsilon(Q_{\max})]$. If we now remember that $e_p(A)$ is decreasing and $e_p(\varepsilon(Q_{\max})) > -1$, we have $\lim_{\tau\to+\infty}\phi_m(\tau) = +\infty$.

Step 3: *The optimal provision of abatement services*

Let us come back to the subdifferential given by Eq (44). With similar arguments as in (i) of Step 2, it can now be argued that the first and the last equation of Eqs (44) are both decreasing function. Let us also note that (i) $\lim_{A\to0}(\tau - \kappa'(A)) = \tau \geq 0$ and (ii)

$$lim_{A\to\varepsilon(Q_{\max})}(p(A) + p'(A)A - \kappa'(A)) = lim_{A\to\varepsilon(Q_{\max})}(p(A)(1+e_p(A)) - \kappa'(A)) = -\kappa'(\varepsilon(Q_{\max}))$$

since by construction $p(\varepsilon(Q_{\max})) = 0$ and $e_p(A)$ bounded. From these observations and the fact that at a maximum $0 \in \partial_A\pi$, we can immediately say that:

(i) if $\phi_c(\tau) < 0$ or, equivalently, $\tau < \tau_c$, the zero of $\partial_A \pi$ is given by $\tau - \kappa'(A) = 0$, so that $A = (\kappa')^{-1}(\tau)$

(ii) if $\phi_m(\tau) \leq 0$ and $\phi_c(\tau) \geq 0$, or $\tau \in [\tau_c, \tau_m]$, the zero is obtained for $A = p^{-1}(\tau)$

(iii) if $\phi_m(\tau) > 0$, *i.e.* $\tau > \tau_m$, the optimal provision solves the last equation and this is nothing else than the standard monopoly solution associated to $p(A)$ (*i.e.* without the kink introduced by the min function).

B. Proof of Lemma 2

Step 1: *Existence of a solution*

Let us denote by $Q = \sum_{j=1}^m q_j$ and let take $\min\{\tau, p_A\}$ as given. We observe that (i) the r.h.s. of each equation of Eqs (20) is increasing in q_j since $c_j^{"}, \varepsilon_j^{"} > 0$ (ii) the range of these functions is $[0, +\infty)$ since $c_j'(0) = \varepsilon_j'(0) = 0$ and both functions goes to $+\infty$ as $q_j \to +\infty$. We can therefore reverse the function given by the r.h.s. and say that $q_j = \phi_j(Q)$. Moreover, we also observe that (i) $\lim_{Q\to 0} \phi_j(Q) = +\infty$ since $\lim_{Q\to 0} P(Q) = +\infty$ so that the equality (20) requires that $q_j \to +\infty$, and (ii) $\lim_{Q\to+\infty} \phi_j(Q) = 0$ since $\lim_{Q\to+\infty} P(Q) = 0$ and therefore $q_j \to 0$ to maintain the equality.

Let us now aggregate over the q_j. We obtain $Q = \sum_{j=1}^m \phi_j(Q)$. So if there exists a solution in Q to this equation our existence problem is solved. It remains to observe that (i) $\Phi(Q) = Q - \sum_{j=1}^m \phi_j(Q)$ is continuous (ii) $\lim_{Q\to 0} \Phi(Q) = -\infty$ and (iii) $\lim_{Q\to+\infty} \Phi_j(Q) = +\infty$.

Step 2: *Uniqueness of the solution*

Let us set $K = \min\{\tau, p_A\}$ and *write the system* (20) as:

$$\Psi\left((q_j)_{j=1}^m, K\right) = \left(c_j'(q_j) + K.\varepsilon_j'(q_j)\right)_{j=1}^m - P\left(\sum_{j=1}^m q_j\right) e \text{ with } e' = (1, \ldots, 1)$$

By computation, we observe that $\partial_{(q_j)_{j=1}^m} \Psi = D - P'\left(\sum_{j=1}^m q_j\right) e \cdot e'$ where D is a diagonal matrix whose generic term is $c_j^{"}(q_j) + K.\varepsilon_j^{"}(q_j)$. This symmetric matrix is clearly positive definite since $c_j^{"}$, $\varepsilon_j^{"} > 0$ and $P' < 0$. It follows from Gale-Nikaido (1965 Theorem 6) that the solution $(q_j(K))_{j=1}^m$ of $\Psi\left((q_j)_{j=1}^m, K\right) = 0$ is unique for every K.

Step 3: $A_f(K) = \sum_{j=1}^m \varepsilon_j(q_j(K))$ *is decreasing*

Let us first observe from the implicit theorem applied to $\Psi\left((q_j)_{j=1}^m, K\right) = 0$ that $\frac{\partial (q_j)_{j=1}^m}{\partial K} = -\left(\partial_{(q_j)_{j=1}^m} \Psi\right)^{-1} \cdot \left(\varepsilon_j'(q_j)\right)_{j=1}^m$. It follows that:

$$\frac{dA_f}{dK} = \left(\left(\varepsilon_j'(q_j)\right)_{j=1}^m\right)' \cdot \frac{\partial (q_j)_{j=1}^m}{\partial K} = -\underbrace{\left(\left(\varepsilon_j'(q_j)\right)_{j=1}^m\right)' \cdot \left(\partial_{(q_j)_{j=1}^m} \Psi\right)^{-1} \cdot \left(\varepsilon_j'(q_j)\right)_{j=1}^m}_{>0} < 0$$

since the inverse of a positive definite matrix remains positive definite.

C. Proof of Lemma 3

We need to solve for all $A_{-i} \in [0, \varepsilon(Q_{\max})]$

$$\max_{a_i \in [0, \varepsilon(Q_{\max}) - A_{-i}]} \underbrace{\left[\min\{\tau, p(a_i + A_{-i})\}(a_i + A_{-i}) - \kappa_i(a_i)\right]}_{\pi_i(a_i, A_{-i}, \tau)} \text{ where } p(A) = \beta\left(\varepsilon^{-1}(A)\right)$$

Step 1: $\pi_i(a_i, A_{-i}, \tau)$ *is strictly concave in* a_{-i}

Under the assumption that $e_p(\varepsilon(Q_{\max})) > -1$ and $\frac{de_p(A)}{dA} \leq 0$, we have for $a_i > 0$:

$$\begin{aligned} 0 &> \tfrac{a_i}{A}\left(p'(A)\left(e_p(A)+1\right)+p(A)\frac{de_p(A)}{da}\right) = P"(A)a_i + \tfrac{2a_i}{A}P'(A) \\ &> P"(A)a_i + 2P'(A) = \tfrac{\partial^2}{(\partial a_i)^2}\left[p\left(a_i + A_{-i}\right)a_i\right] \end{aligned}$$

We can therefore use the similar argument as in Step 1 of Lemma 1 in order to show that $\pi_i(a_i, A_{-i}, \tau)$ is strictly concave in a_i. We simply need to decompose A in $(a_i + A_{-i})$ and take a convex combination of two a_i.

Step 2: *the subdifferential and the thresholds*

Let us now compute the sub-derivate of $\pi_i(a_i, A_{-i}, \tau)$ with respect to a_i. For $A_{-i} < p^{-1}(\tau)$, we obtain:

$$\partial_{a_i}\pi = \begin{cases} \tau - \kappa_i'(a_i) & \text{if } a_i < p^{-1}(\tau) - A_{-i} \\ \left[\underbrace{\tau + p'(p^{-1}(\tau))\left(p^{-1}(\tau) - A_{-i}\right) - \kappa_i'\left(p^{-1}(\tau) - A_{-i}\right)}_{:=\varphi_m^i(\tau, A_{-i})}, \underbrace{\tau - \kappa_i'\left(p^{-1}(\tau) - A_{-i}\right)}_{:=\varphi_c^i(\tau, A_{-i})}\right] & \text{if } a_i = p^{-1}(\tau) - A_{-i} \\ \underbrace{p\left(a_i + A_{-i}\right) + p'\left(a_i + A_{-i}\right)a_i - \kappa_i'\left(a_i\right)}_{:=\psi_i(a_i, A_{-i})} & \text{if } a_i > p^{-1}(\tau) - A_{-i} \end{cases} \tag{45}$$

If $A_{-i} \geq p^{-1}(\tau)$, the first and even the second line (if $A_{-i} > p^{-1}(\tau)$) are simply vacuous.

So let us for the moment assume that $A_{-i} < p^{-1}(\tau)$ and let us introduce the thresholds $m_i(\tau)$ and $M_i(\tau)$ given by $\varphi_c^i(\tau, m_i(\tau)) = 0$ and $\varphi_m^i(\tau, M_i(t)) = 0$. Concerning $m_i(\tau)$, we observe that (i) $\partial_{A_{-i}}\varphi_c^i(\tau, A_{-i}) = \kappa_i"\left(p^{-1}(\tau) - A_{-i}\right) > 0$, (ii) $\lim_{A_{-i}\to p^{-1}(\tau)}\varphi_c(\tau, A_{-i}) = \tau > 0$ and (iii) $\lim_{A_{-i}\to 0}\varphi_c^i(\tau, A_{-i}) = \tau - \kappa_i'\left(p^{-1}(\tau)\right) = \phi_c^i(\tau)$ this last function being the same as in Eq. (44)) but now indexed by agent i. So by using the Step 2 of the proof of Lemma 1, we know that $\left(\phi_c^i\right)' > 0$ and that there exists a unique τ_c^i such that $\phi_c^i(\tau_c^i) = 0$. We can therefore say that:

$$\begin{cases} \forall\tau \leq \tau_c^i, \ \exists m_i(\tau) \in \left[0, p^{-1}(\tau)\right], \ \ \varphi_c\left(\tau, m_i(\tau)\right) = 0 \\ \forall\tau > \tau_c^i, \ \ \varphi_c\left(\tau, A_{-i}\right) > 0 \text{ for all } A_{-i} \in \left[0, p^{-1}(\tau)\right) \end{cases}$$

Concerning $M_i(\tau)$, we now observe that (i) $\partial_{A_{-i}}\varphi_m\left(\tau, A_{-i}\right) = \kappa"\left(p^{-1}(\tau) - A_{-i}\right) > 0$, (ii) $\lim_{A_{-i}\to p^{-1}(\tau)}\varphi_m(\tau, A_{-i}) = \tau > 0$ and (iii) $\lim_{A_{-i}\to 0}\varphi_m(\tau, A_{-i}) = \phi_m^i(\tau)$ this last function being again the same as in Eq (44)). Using again Step 2 of the proof of Lemma 1, we have:

$$\begin{cases} \forall\tau \leq \tau_m^i, \ \exists M_i(\tau) \in \left[0, p^{-1}(\tau)\right], \varphi_m\left(\tau, M_i(\tau)\right) = 0 \\ \forall\tau > \tau_m^i, \varphi_m^i\left(\tau, A_{-i}\right) > 0 \text{ for all } A_i \in \left[0, p^{-1}(\tau)\right) \end{cases}$$

Finally since $\varphi_m^i\left(\tau, A_{-i}\right) < \varphi_c^i\left(\tau, A_{-i}\right)$ and both are increasing we can say that for $\tau \leq \tau_c^i$, $m_i(\tau) < M_i(\tau)$.

Step 3: *The unconstraint best response*

Let us concentrate on the last equation of Eq. (45). If we compute the associated best response (without taking care to $a_i > p^{-1}(\tau) - A_{-i}$) we obtain a standard best response $br_i(A_{-i})$ which corresponds to a Cournot game in which the inverse demand is $p(A)$. This function exists for all $A_{-i} \in \left[0, \varepsilon^{-1}(Q_{\max})\right]$, since (i) $\psi_i\left(a_i, A_{-i}\right)$ is decreasing in a_i (see Step 1 and remember that $\kappa" > 0$), (ii) $\lim_{a_i\to 0}\psi_i\left(a_i, A_{-i}\right) = p(A_{-i})\left(1 + e_p(A_{-i})\right) > 0$ since $e_p > -1$ by assumption and (iii) $\lim_{a_i\to(\varepsilon^{-1}(Q_{\max}) - A_{-i})}\psi_i\left(a_i, A_{-i}\right) = -\kappa_i'\left(\varepsilon^{-1}(Q_{\max}) - A_{-i}\right) < 0$ for $A_{-i} < \varepsilon^{-1}(Q_{\max})$ while for $A_{-i} = \varepsilon^{-1}(Q_{\max})$ the best response is $a_i = 0$.

Step 4: *The best response*

Three cases must be distinguished.

Case 1: $\tau > \tau_m^i$

In this case we have $\varphi_c^i(\tau, A_{-i}) > \varphi_m^i(\tau, A_{-i}) > 0$. If we now keep in mind that $(\tau - \kappa'(a_i))$ is decreasing and converges to $\varphi_c^i(\tau, A_{-i})$ as $a_i \to \left(p^{-1}(\tau) - A_{-i}\right)$, $\partial_{a_i}\pi$ only admits a zero in third case of Eq (45). In other words the best response is $BR_i(A_{-i}, \tau) = br_i(A_{-i})$ defined in Step 3.

Case 2: $\tau \in \left(\tau_c^i \tau_m^i\right]$

Here we know that $\varphi_c^i(\tau, A_{-i}) > 0$ and therefore $\tau - \kappa_i'(a_i) > 0$ for all $a_i < p^{-1}(\tau) - A_{-i}$, but $\exists M_i(\tau) \in \left[0, p^{-1}(\tau)\right]$, $\varphi_m^i(\tau, M_i(\tau)) = 0$. This means that the best response is given by:

$$BR_i(A_{-i}, \tau) = \begin{cases} p^{-1}(\tau) - A_{-i} \text{ for all } A_{-i} \leq M_i(\tau) \\ br_i(A_{-i}) \text{ else} \end{cases}$$

Case 3: $\tau \leq \tau_c^i$

In this case both thresholds matter so that the best response is given by:

$$BR_i(A_{-i}, \tau) = \begin{cases} (\kappa_i')^{-1}(\tau) \text{ for all } A_{-i} < m_i(\tau) \\ p^{-1}(\tau) - A_{-i} \text{ for all } A_{-i} \in [m_i(\tau), M_i(\tau)] \\ br_i(A_i) \text{ else} \end{cases}$$