Optimal transition from coal to gas and renewable energy under capacity constraints and adjustment costs

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Given a cap on carbon emissions, what is the optimal transition from coal to gas and renewable energy? To answer this question, we model two critical features of the energy sector: (i) its dependence on polluting exhaustible resources and (ii) its dependence on long-lived capital, such as power plants. To minimize adjustment costs when building new capital, investment in and usage of renewable energy may start before investment in gas, and before phasing out fossil-fuel consumption. Gas-fired capital built in the short term can only be used temporarily. Simulations of the European Commission's Energy Roadmap illustrate these results.
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Many governments aim at stabilizing climate change to avoid important climate damages, which requires reaching near-zero greenhouse gas (GHG) emissions in the long term (Steinacher, Joos and Stocker, 2013; IPCC, 2014). Abating GHG emissions from power generation is key to reach this goal, as the power sector is currently responsible for nearly 40% of carbon emissions worldwide, and fuel switching to clean electricity is a major technical option to reduce emissions from other sectors (Williams et al., 2012; Audoly, Vogt-Schilb and Guivarch, 2014).

Two important features of the electricity sector are that (i) it depends on long-lived capital that in general is tied to a specific fuel, and (ii) today's power production greatly relies on polluting fossil fuels such as coal (IEA, 2014). Several alternatives are thus available to abate GHG emissions from electricity production. Emissions may be reduced by replacing coal plants with new gas power

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plants,[1] or with more-expensive but almost-carbon-free options such as renewable power. Also, decision makers can either wait for existing plants to reach their natural lifetime, or decide to decommission them earlier in order to switch faster to cleaner energy sources.

This paper analyzes the optimal transition from coal to gas and renewable energy under capacity constraints and a carbon budget. We model different types of nonrenewable resources, *à la* Chakravorty, Moreaux and Tidball (2008) (cheap coal and more expensive gas), and an expensive renewable source. All energy sources are subject to capacity constraints: consuming more energy first requires investment in coal, gas, or renewable power plants. Finally, investment is irreversible and bears *adjustment costs*. Adjustment costs are convex investment costs reflecting that there is an increasing opportunity cost to use scarce resources to build and deploy new capital faster (Lucas, 1967; Gould, 1968; Mussa, 1977).

We find that optimal investment in renewable can start early, that is before investment in new gas plants, and before coal and gas are phased out. The reason is that smoothing investment over time reduces adjustment costs. Loosely speaking, the availability of appropriate resources (skilled workers, production lines) sets an optimal speed at which to deploy renewable power. This speed, combined with the end goal of achieving carbon neutrality (to comply with the carbon budget), sets an optimal date to start investment in renewable power plants. Transient investment in new gas plants can be used to reduce (but not necessarily cancel) the need for investment in expensive renewable energy in the short term. But new gas-fired capital eventually needs to be decommissioned and give room to more carbon-free energy.

More generally, the transition from coal to gas and renewable energy can exhibit three different profiles. First, the social planner can sequentially switch from coal to gas, and only after from gas to renewable power. A sensitivity analysis reveals that this occurs for lax carbon budgets and low adjustment costs, giving time to switch entirely to gas before starting to invest in renewable power and reducing the need to smooth investment in renewable energy. Second, for intermediate adjustment costs and carbon budgets, the transition can start with investment in gas, and continue with investment in renewable before coal has been phased out. In the third type of transition profile, for stricter carbon budget and larger adjustment costs, investment in renewable power starts as soon as possible.

We illustrate and quantify these findings in a numerical simulation of the European Commission's Energy Roadmap (EU, 2011), whose aim is to fully decarbonize the European power sector by mid-century. According to our simulations, the ambitious goal of the European Commission implies that the optimal transition starts with investment in renewable power, except if adjustment costs are

[1]This paper assumes gas is a low-carbon substitute for coal. The relative carbon content of gas and coal may actually depend on the type of gas and coal and on the particular processes used for extracting and transporting the fuels (e.g., Alvarez et al., 2012). The relative merits of coal and gas also depend on factors disregarded here, such as impact on energy security or impact on local pollution (e.g., Shindell, 2015).

small and the cost of renewable energy is significantly higher than suggested by available data. With our reference calibration, it is optimal to start with investment in both renewable and gas. Optimal short-term investment in renewable capacity is comparable to actual figures. New gas power plants are built until 2040, to reduce the need for renewable power plants in the short-term, but electricity generation from gas is then phased out in about 15 years. These results shed light on technical choices (e.g. investors can consider gas plants with short scheduled lifetimes) as well as policy decisions (when setting milestones for carbon-free power generation capacity).

This paper relates to two different strands of the analytical literature. The first one studies the optimal usage of different fossil fuels under an environmental constraint, through the lens of nonrenewable resources theory (Hotelling, 1931; Herfindahl, 1967), with little attention to the dynamics of capital accumulation (e.g., Chakravorty, Moreaux and Tidball, 2008; van der Ploeg and Withagen, 2012). In this literature, renewable energy is modeled as a *clean backstop*, a technology that can immediately produce arbitrary quantities of energy at a fixed marginal cost. With these assumptions, renewable energy should never be used early, that is before fossil fuel consumption stops. We expand this literature by studying the effect of capacity constraints and adjustment costs limiting the extraction of all types of energies. We find that to reduce adjustment costs, it makes sense to start investing in both renewable and gas power plants early, and to use several energy types at the same time.

The other strand of the literature studies optimal investment in clean capital under adjustment costs, with little attention to fossil fuel resources, and neglecting intermediate alternatives to perfectly clean capital (such as switching from coal to gas). Fischer, Withagen and Toman (2004), Williams (2010) and Slechten (2013) all stress that to reduce adjustment costs, the optimal strategy is to smooth investment in clean capacity, anticipating future carbon prices. Vogt-Schilb, Meunier and Hallegatte (2014) compare abatement investment across sectors, and find that the abatement potential in each sector should also be anticipated: the same carbon price translates into more short-term investment in sectors with larger abatement potential. Here, we confirm that smoothing investment and anticipating future prices is critical. In particular we find that more investment should go to renewable power, which is built to last, than to gas power, which is only a bridge technology. Finally, Rozenberg, Vogt-Schilb and Hallegatte (2014) analyze the trade-off between early-scrapping existing dirty capital and investing in clean capital. They find that early-scraping part of the dirty capital, built before climate policies are announced, is optimal for achieving stringent climate targets. Here, we find that it also makes sense to build gas-fired capital after climate policies are imlemented, knowing that it will subsequently be underused, to move efforts from the short to the middle term.

Last but not least, other authors have previously investigated optimal capital accumulation and resource extraction under a single model. In a recent working paper, Amigues et al. (2013) study the extraction of a renewable and a non-renewable energy source, taking into account that the extraction of the renewable source requires to first invest in appropriate capital (renewable power plants) and pay for adjustment costs. They also find that optimal investment in renewable may start early, before fossil resources are exhausted.[2] But they model a single fossil resource, and they leave environmental constraints and capacity constraints limiting the extraction of fossil fuels for further research. This paper thus proposes the first analytical model able to assess the optimal transition from coal to gas and renewable power plants, taking into account capacity constraints on all energy sources and a carbon budget.[3]

The remainder of the paper is structured as follows. Section I details the model. Section II solves the model for the carbon and fossil energy costs (II.A), the optimal value of capacities and the cost of building new ones (II.B), the electricity price and capacity rents (II.C), and optimal investment trajectories (II.D). Section II.E discusses the optimal ordering of investment and possible transition profiles. Section III provides numerical simulations calibrated with data from the European electricity sector. Section IV discusses limitations of this paper and concludes.

I. Model

A social planner controls the supply of electricity, using and investing in three different technologies: an existing high-carbon technology (h, coal power), a fossil-fueled low-carbon technology (ℓ, gas), and an inexhaustible zero-carbon technology (z, renewable power).

At each time t, the social planner chooses a positive amount of physical investment $x_{i,t}$ in technology i. The investment adds to the installed capacity $k_{i,t}$, which otherwise depreciates at the constant rate δ (dotted variables denote tem-

[2] A whole literature started by Kemp and Long (1980) has established that if the extraction rate of an expensive energy source is constrained by an *exogenous* factor, it may be optimal to use that source simultaneously with cheaper alternatives (e.g., Amigues et al., 1998; Holland, 2003). Amigues et al. (2013) confirm this result for *endogenous* constraints.

Other reasons for not extracting resources according to a strict Herfindahl sequence include time to build (Winkler, 2007), imperfect substitution (Smulders and van der Werf, 2008), heterogeneity of producers, heterogeneity of consumers and transportation costs – Gaudet and Salant (2014) provide a review.

[3] The interaction of investment and natural resources extraction is also the subject of the theory of the mine (e.g., Campbell, 1980; Gaudet, 1983; Lasserre, 1985), in which installed capital similarly limits the extraction rate of (a single type of) minerals. After reviewing this literature, Cairns (1998) notes that "*there can be three phases in the exploitation of the mine, namely (1) a period of positive investment after time t = 0, in which production is at full capacity, then (2) a period in which investment is zero and production is at full capacity, and finally (3) a period of declining production*". This paper is different as we model several resources and an environmental constraint; we find however a similar trajectory for exploitation of gas resources.

In addition, Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974) have started a literature that studies the impact of resource exhaustiblity on growth, in *green Ramsey models* that feature both capital accumulation and resource extraction — van der Ploeg and Withagen (2014) is a recent example. This literature also focuses on a single type of capital and a single fossil resource.

poral derivatives):[4]

$$\forall i, \quad \dot{k}_{i,t} = x_{i,t} - \delta k_{i,t} \tag{1}$$
$$x_{i,t} \geq 0 \tag{2}$$

Without loss of generality, we assume low-carbon and zero-carbon capacities to be nil at the beginning ($k_{\ell,t_0} = k_{z,t_0} = 0$).[5]

The constraint that investment is positive means that once a power plant has been built, it cannot be unbuilt to retrieve its cost. For instance, once workforce and cement have been used to build a gas power plant, the plant cannot be transformed back to manpower and raw cement to build something else. Other authors have referred to such constraints with the terms of *irreversible investment* (Arrow and Kurz, 1970), or *putty-clay capital* (Arltesou, 1999; Wei, 2003).[6]

Physical investment is made at a positive, increasing and convex cost c_i:

$$\forall x, \quad c_i(x) > 0,\ c_i'(x) > 0,\ c_i''(x) > 0 \tag{3}$$

This convexity captures the increasing opportunity cost to use scarce resources (skilled workers and appropriate capital) in order to build and deploy capacities faster. It has been labeled *adjustment costs* in the theory of the firm (Lucas, 1967; Gould, 1968). In general, the words *adjustment costs* cover many factors limiting the speed at which individual firms can adjust their stock of capital to new prices, such as the cost to purchase capital or to install it. In the literature on resource extraction (e.g., Gaudet, 1983; Amigues et al., 2013), including in this paper, adjustment costs are best understood as the production costs for the industries that manufacture each type of power plants — Mussa (1977) calls these *external* adjustment costs.[7]

A practical consequence of adjustment costs is that the faster the social planner builds new windmills or new gas plants, the higher the marginal cost of those power plants. In order to minimize the cost of the transition, the social planner will thus smooth investment over time.

[4] Through this paper, *capacity* is to be understood as equivalent capacity, e.g., in kWh/yr, unless otherwise specified. For instance if 2 kW of windmills are required to provide as much output per year as 1 kW of coal, then 2 kW of windmills are accounted as 1 kW_{eq}.

[5] The reason why this assumption does not result in a loss of generality is that our model studies the transition from the existing situation to cleaner and to clean power.

[6] As the irreversibility constraint is on the *building* process of power plants, it does not prevent private agents who own a power plant to sell it back to other private agents at any moment. During the analytical resolution of the problem, it will be useful to interpret some terms as the resale value of existing capacities.

[7] In this paper, the word *investment* refers to the building of new power plants. When private owners of power plants sell an existing plant to another electricity production firm, that latter firm is also performing investment. But at the social level, such investment is only a transfer, not a net accumulation of new capital.

We assume that, when both are built at the same pace, low-carbon capacity is cheaper than zero-carbon capacity:

$$\forall x \quad c'_\ell(x) < c'_z(x) \tag{4}$$

In general, we do not assume that $c'_i(0) = 0$; the strictly positive $c'_i(0)$ is the minimum cost at which power plants i may be built.[8]

The social planner also chooses how much output to produce from each technology. We assume production exhibits constant returns to scale: two gas plants can produce twice as much electricity as one gas plant.[9] The positive production $q_{i,t}$ with technology i cannot exceed the installed capacity $k_{i,t}$:

$$\forall i, \quad 0 \leq q_{i,t} \tag{5}$$

$$q_{i,t} \leq k_{i,t} \tag{6}$$

For simplicity, we assume the existing carbon-intensive capital is overabundant, such that (6) is not binding for coal; this assumption is relaxed and confirmed in the numerical application.

Let F_i be the carbon intensity (or emission factor) of technology i. The high-carbon technology is more carbon-intensive than the low-carbon technology:

$$F_h > F_\ell > F_z = 0 \tag{7}$$

The social planner is subject to an exogenous carbon budget (or emission ceiling), cumulative emissions cannot exceed a given ceiling $\bar{M}$:

$$\forall t, \quad m_t \leq \bar{M} \tag{8}$$

Where cumulative emissions m_t grow with emissions $F_i\, q_{j,t}$:

$$\dot{m}_t = \sum_i F_i\, q_{j,t} \tag{9}$$

[8]Some readers may prefer to think of our marginal investment cost $c'(x)$ decomposed as a "pure" investment cost $c'(0)$ and a "pure" adjustment cost $c'(x) - c'(0)$.

[9] We are neglecting any complementarity or imperfect substitution between gas and renewable plants that comes from their respective cost structures (in terms of fixed versus variable costs) and from the intermittency of renewable power availability. We are implicitly assuming that such considerations have more impact on the optimal electricity mix *at* the equilibrium than on the optimal transition from the existing state *to* that equilibrium, which is the focus of the paper.

Cumulative emissions have been found to be a good proxy for global warming (Allen et al., 2009; Matthews et al., 2009; IPCC, 2014). Some policy instruments, such as an emission trading scheme with unlimited banking and borrowing, set a similar constraint on firms (Slechten, 2013).

Using fossil fuel (gas or coal) requires to extract exhaustible resource from an initial stock, such that the current stock $S_{i,t}$ classically satisfies:

$$\begin{aligned} & S_{i,t_0} \text{ given} \\ & \dot{S}_{i,t} = -q_{i,t} \\ & S_{i,t} \geq 0 \end{aligned} \tag{10}$$

We assume that the zero carbon technology is renewable: $S_{z,t_0} = \infty$. While it is convenient to use the above general notations (indexed by i), parts of the analytical resolution will focus on the case where coal is overabundant ($S_{h,t_0} = \infty$), that is cases where the carbon budget is more stringent than the scarcity of coal resources (as in van der Ploeg and Withagen, 2012).

Consumers derive utility $u\left(\sum_i q_{i,t}\right)$ from electricity consumption, where u satisfies the Inada conditions and is smooth enough. The program of the social planner consists in determining the trajectories of investment $x_{i,t}$ and production $q_{i,t}$ that maximize discounted utility net from investment costs while complying with the carbon budget $\bar{M}$ and the various constraints:

$$\max_{x_{i,t},q_{i,t}} \int_0^\infty e^{-rt} \left[u\left(\sum_i q_{i,t}\right) - \sum_i c_i(x_{i,t}) \right] dt \tag{11}$$

$$\begin{aligned}
\text{s.t. } & \dot{k}_{i,t} = x_{i,t} - \delta k_{i,t} && (\nu_{i,t}) \\
& q_{j,t} \leq k_{j,t} && (\gamma_{i,t}) \\
& q_{j,t} \geq 0 && (\lambda_{i,t}) \\
& x_{i,t} \geq 0 && (\xi_{i,t}) \\
& \dot{m}_t = \sum_i F_i \, q_{j,t} && (\mu_t) \\
& m_t \leq \bar{M} && (\eta_t) \\
& \dot{S}_{i,t} = -q_{i,t} && (\alpha_{i,t}) \\
& S_{i,t} \geq 0 && (\beta_{i,t})
\end{aligned}$$

Where r is the constant discount rate and the Greek letters in parentheses are the costate variables and Lagrange multipliers (all chosen such that they are positive). A few of them play a key role in the analytical resolution: $\nu_{i,t}$, the shadow value of new power plants; $\gamma_{i,t}$, the social cost of the capacity constraint, which can also be interpreted as the shadow rental cost of power plants; μ_t, the shadow carbon price; and $\alpha_{i,t}$, the shadow price of resource i (all notations are gathered in Table A1).

II. Analytical resolution

A. *Hotelling rents, carbon prices and energy costs*

The transition from coal to gas and renewable power is driven by the increase of three prices: the price of coal $\alpha_{h,t}$, the price of gas $\alpha_{\ell,t}$, and the price of carbon emissions μ_t. (In the following, we frequently omit the term "shadow" when referring to co-state variables and Lagrangian multipliers. We also freely switch between the social planner's perspective and the point of view of private agents facing actual energy and carbon prices.) All three prices classically correspond to scarcity rents, and follow the Hotelling rule (see the appendix for the full set of efficiency conditions). They grow over time at the interest rate:

$$\alpha_{h,t} = \alpha_h \, e^{rt}, \; \alpha_{\ell,t} = \alpha_\ell \, e^{rt}, \; \mu_t = \mu e^{rt} \tag{12}$$

In this model without extraction costs, $\alpha_{h,t}$, $\alpha_{\ell,t}$, and μ_t are non-zero only when the respective exhaustibility constraint for coal, gas, and the carbon budget is binding. Under the assumption that coal reserves are not binding, $\alpha_h = 0$; in addition, if there is more carbon contained in gas reserves underground that what the carbon budget allows to emit in the atmosphere, then $\alpha_{\ell,t} = 0$.

In the following, we continue the analytical resolution of the model using $\alpha_{h,t}$ and $\alpha_{\ell,t}$ — without substituting. With this more general notation, the resolution holds for cases with positive extraction, transformation and transportation costs that would be encompassed in the energy costs $\alpha_{h,t}$ and $\alpha_{\ell,t}$; or more generally for any exogenous energy price trajectory $\alpha_{h,t}$ and $\alpha_{\ell,t}$ — as will be used in Section III.

B. *Capacity rents, capacity value, and construction costs*

This subsection clarifies the link between capacity rents $\gamma_{i,t}$, capacity value $\nu_{i,t}$, and construction costs $c_i'(x_{i,t})$. Such clarification is essential to the rest of the resolution.

After deriving the first order conditions (see appendix), we find that along the optimal path, the cost of building new capacities $c_i'(x_{i,t})$ equals the shadow value of capacities $\nu_{i,t}$ plus the shadow cost of the irreversibility constraint $\xi_{i,t}$, that is $c_i'(x_{i,t}) = \nu_{i,t} + \xi_{i,t}$. This equation means that when the value of a power plant $\nu_{i,t}$ is higher than the minimal cost $c_i'(0)$ of constructing the same power plant from scratch, power plants are built up to the pace at which the building cost equals the value of power plants, and the irreversibility constraint is not binding

$(\xi_{i,t} = 0)$:

$$\nu_{i,t} > c_i'(0) \iff c_i'(x_{i,t}) = \nu_{i,t} \iff x_{i,t} > 0 \tag{13}$$

For values $\nu_{i,t}$ lower than $c_i'(0)$, it would be desirable to deconstruct the power plants back into raw resources, but the irreversibility constraint prevents that. First order conditions also show that the value of capacity $\nu_{i,t}$ is linked to the shadow cost of the capacity constraint $\gamma_{i,t}$ by the following relation:

$$\forall t, \quad \gamma_{i,t} = (\delta + r)\,\nu_{i,t} - \dot{\nu}_{i,t} \tag{14}$$

This is a classic equation in the theory of investment. It means that $\gamma_{i,t}$ is what Jorgenson (1967) calls the *implicit rental cost of capital*. It is the rental price that ensures agents are indifferent between buying capacity at $\nu_{i,t}$ and renting capacity at $\gamma_{i,t}$. As pointed by Jorgenson (1967), the relation translates the absence of profitable trade-off between the two following strategies: (i) buy a power plant at t at a cost $\nu_{i,t}$, rent it out during one period dt at the rental price $\gamma_{i,t}$, then sell the depreciated (δ) capacities at $t + dt$ at a price $\nu_{i,t} + \dot{\nu}_{i,t} dt$ or (ii) simply lend money at the interest rate r. In other words, the current price of capital, $\nu_{i,t}$, comes from the rent derived from the capital today, $\gamma_{i,t}$, and from future price changes $\dot{\nu}_{i,t}$.

Combining (13) and (14), we find that when power plants are being built, the building cost and the rental cost of a given type of power plant are linked in a similar way:

$$x_{i,t} > 0 \iff \gamma_{i,t} = (\delta + r)\, c_i'(x_{i,t}) - \frac{d}{dt} c_i'(x_{i,t}) \tag{15}$$

This relation means that on the optimal path, there are no profitable trade-offs between using a stock of money for building power plants in order to rent them, and simply lending money at rate r.

In Section II.D, we show how integrating (15) yields an equation analogous to $c_i'(x_{i,t}) = \int e^{-(r+\delta)t} \gamma_{i,t}\, dt$: power plants are built up to the pace for which the construction cost equals the discounted sum of future rental revenues from the depreciating marginal capacity. Before that, the next subsection studies the capacity rents $\gamma_{i,t}$.

C. *Electricity prices, Hotelling and capacity rents*

Let us continue the resolution with the equilibrium conditions between Hotelling rents, capacity rents, and electricity prices at each point in time. When power plants are being used, the first-order conditions simplify into (see appendix):

$$q_{i,t} > 0 \iff u_t' = \gamma_{i,t} + \alpha_{i,t} + \mu_t\, F_i \tag{16}$$

On the left hand side, u'_t is the competitive electricity price — u'_t stands for $u'\left(\sum_i q_{i,t}\right)$. On the right hand side, $\gamma_{i,t}$ is the capacity rent, $\alpha_{i,t}$ is the fuel cost, F_i the carbon intensity of technology i, and μ_t is the carbon price. Equation 16 means that, if a technology is used at time t, the full costs of producing electricity with that technology must be equal to the electricity price.

In addition, the capacity rent $\gamma_{i,t}$, as the Lagrange multiplier associated with the capacity constraint, is null when capacity i is underused (equation A11 in the appendix):

$$q_{i,t} < k_{i,t} \implies \gamma_{i,t} = 0 \tag{17}$$

Taken together, the last two equations imply that (in general) only one type of power plant may be used, but under full capacity, at the same moment t. Indeed, if two technologies i and j were both used under full capacity at on point of time, then the cost of fuel and carbon for these two technologies should be equal:

$$0 < q_{i,t} < k_{i,t} \;\&\; 0 < q_{j,t} < k_{j,t} \implies u'_t = \alpha_{i,t} + F_i\,\mu_t = \alpha_{j,t} + F_j\,\mu_t \tag{18}$$

As it would require a very specific set of parameters linking available coal resources, gas resources, and the carbon budget, we neglect this case in the rest of the paper.

The previous results lead to propose the following natural definitions, and an immediate proposition, both illustrated in Fig. 2:

DEFINITION 1: *1) We call* variable production costs *of technology i the sum of fuel prices and carbon prices: $\alpha_{i,t} + F_i\,\mu_t$*

2) We call marginal power plant at time t *the used capacity with the highest variable production costs at time t.*

Following these definitions, equation 16 allows to characterize the electricity price at each point in time t, depending on which one is the marginal power plant:

PROPOSITION 1: *At each point in time t, the electricity price u'_t is given by the variable costs of the marginal power plant, and a capacity rent which is strictly positive only when capacities are fully used. In particular:*

1) If coal capacities are used at less than full capacity, the electricity price equals variable costs from coal generation:

$$0 < q_{h,t} < k_{h,t} \implies u'_t = \alpha_{h,t} + F_h\,\mu_t \tag{19}$$

2) If gas capacities are used at less than full capacity, the electricity price equals variable costs from gas generation:

$$0 < q_{\ell,t} < k_{\ell,t} \implies u'_t = \alpha_{\ell,t} + F_\ell\,\mu_t \tag{20}$$

3) *If all capacities are either used at full or not used at all, rental costs adjusts such that the production cost, including the rental cost of capacities, is equal across technologies.*

$$q_{i,t} = k_{i,t} \implies \quad \gamma_{i,t} > 0 \quad \textit{and} \quad u'_t = \gamma_{i,t} + \alpha_{i,t} + \mu_t\, F_i \tag{21}$$

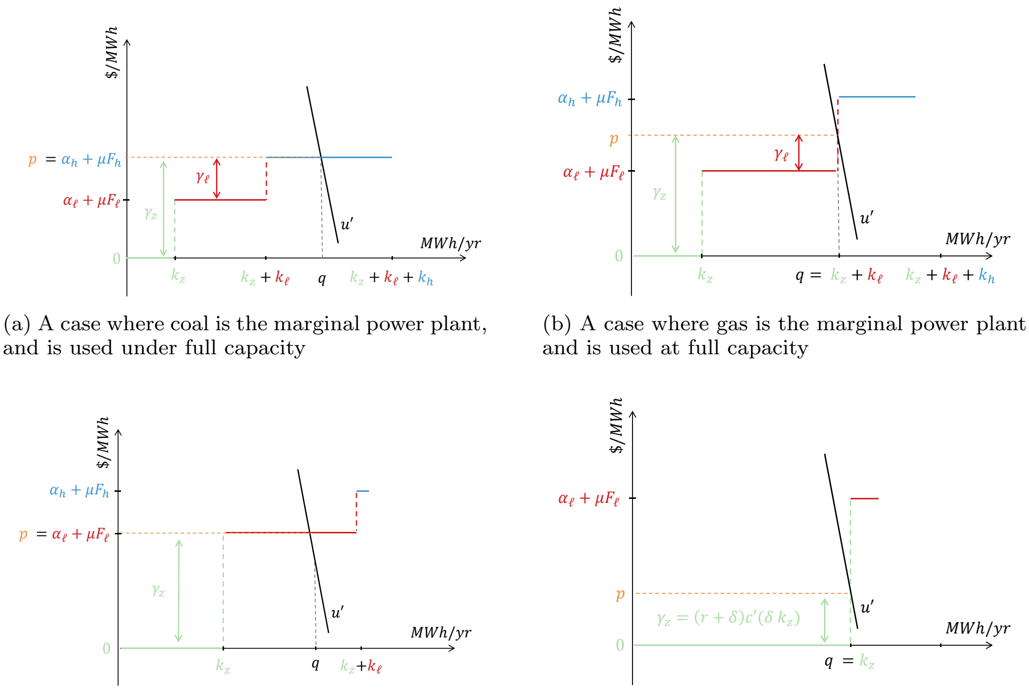
Fig. 1 and 2 illustrate this proposition. Fig. 1 shows the marginal costs of energy and the electricity price of electricity as a function of the installed capacity at one given point in time, as well as the downward sloping demand. Installed capacities are ranked in the *merit order*, that is according to variable production costs — again, excluding capacity rents. Price and total quantities are set where the demand function meets the merit order curve. The various panels represent different phases in the transition, and different cases of Proposition 1.

Fig. 1 also illustrates how fully-used capacities receive a capacity rent γ_i. Capacity rents provide incentive to invest and grow infra-marginal capacity. They also allow the market to clear when marginal capacities are fully used (Fig. 1b and Fig. 1d), and finance the maintenance of renewable capacities when all production comes from renewable power (Fig. 1d). Positive capacity rents are possible even in the absence of market imperfections (other than the unrelated GHG externality) because adjustment costs prevent total capacities to adjust immediately to a point where capacity rents would be null.[10]

As discussed in Section II.E, the ordering of investment in gas and renewable power may vary. Fig. 2 shows one particular situation where investment in both renewable and gas power start at the beginning of the period. Fig. 2a plots capacity and production against time; and Fig. 2b plots the electricity price. In a first phase, coal is progressively replaced by both renewable and gas, coal capacity is underused and the price of electricity is given by the variable cost of coal (as in Fig. 1a). During this phase, the electricity price increases as the cost of coal and the cost of carbon emissions increase.

When coal is entirely phased out follows a phase during which renewable power progressively replaces gas. During this phase, gas is used at full capacity, and the electricity price includes a rent for gas capacity (as in Fig. 1b). Investment in renewable and gas power first increases both renewable and gas capacity, reducing capacity rents, and decreasing the price of electricity. At one point, future rents for gas plants become too low to make new gas capacities profitable, and investment in gas stops. During the second part of this second phase, total capacity

[10] It is well-known that infra-marginal capacity rents allow power producers to finance investment (e.g., Houthakker, 1951; Boiteux, 1960; Biggar and Hesamzadeh, 2014). At our best knowledge, this paper is the first to propose that adjustment costs allow explicit (and parsimonious) modeling of the link between infra-marginal rents and investment decisions. Indeed, the most frequent practice to take into account the limited ability to switch quickly from high- to low-carbon capital is to use exogenous maximum investment speeds (see footnote 2 and Seebregts, Goldstein and Smekens, 2002; Loulou, 2008; Wilson et al., 2013; Iyer et al., 2014; Vogt-Schilb, Hallegatte and de Gouvello, 2014).



(a) A case where coal is the marginal power plant, and is used under full capacity

(b) A case where gas is the marginal power plant and is used at full capacity

(c) A case where gas is the marginal power plant, and is used under full capacity

(d) The steady state, when all production comes from renewable power

Figure 1. : Examples of the merit-order curve at different points of the transition from coal to gas and renewable power

Note: The horizontal axis shows the installed capacity for renewable power (z), gas (ℓ) and coal (h) at one given point in time. The vertical axis shows the marginal costs and the price of electricity. The production technologies are ranked according to their *variable production costs*, excluding capacity rents (in other words, accounting only for the resource costs α_i plus the cost of emissions μF_i). The price p and quantity q are set by the intersection of the demand curve u' and the merit-order curve. Such intersection can lead to only one type of power plants to be used, but under full capacity: the marginal capacity. All technologies used at full capacity receive a capacity rent $\gamma_i = p - \alpha_i - \mu F_i$.

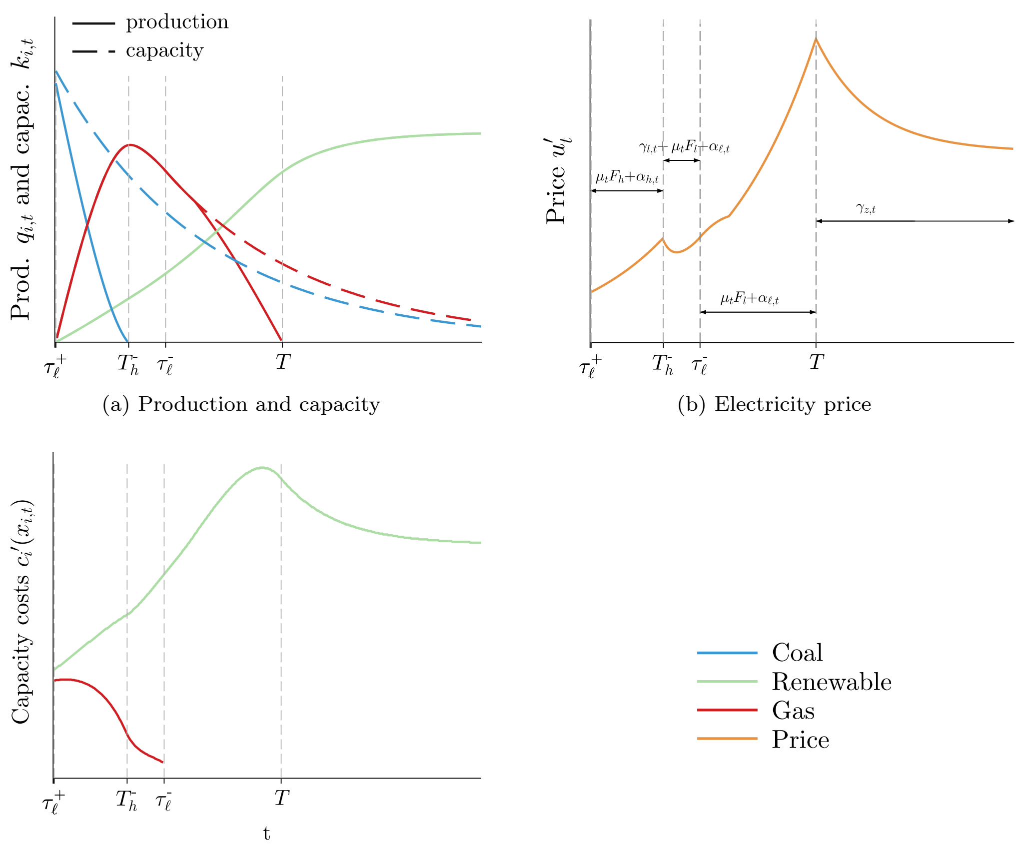


Figure 2. : Capacity, production, electricity price and investment in one possible transition from coal to gas and renewable power

decreases and electricity prices increase again.[11]

In a third phase, gas plants are used under full capacity, the price is set by variable costs from gas, and the price increases over time (as in Fig. 1c). During the final phase, gas plants are entirely phased out, carbon-free capacity receives a decreasing rent (and the electricity price decreases) until the point where capacity rents compensate exactly depreciation of carbon-free capacity, and the system has reached a steady state (as in Fig. 1d).

D. *Valuing investment in gas and renewable power plants*

In this section, we study optimal investment decisions. As clarified in Section II.B, the anticipation of all future prices and resulting capacity rents $\gamma_{i,t}$ translates into a value $\nu_{i,t}$ for current capacity. If that value is superior to the cost of building new capacities $\nu_{i,t} > c_i'(0)$, investment in type capacity i is desirable (and undertaken by well-functioning markets facing the right carbon price).

The case of renewable power plants is the simplest one. Because they bare no variable costs, existing renewable power plants are always used and always receive a rent equal to the electricity price : $\gamma_{z,t} = u_t'$. The value of renewable power plants is the solution of the differential equation (14), and simply reads:

$$\forall t, \quad \nu_{z,t} = \int_t^{\infty} e^{-(r+\delta)(\theta-t)} u_\theta' \, d\theta \tag{22}$$

It is equal to the present value of all future revenues from selling the electricity (u_θ') produced by the depreciated marginal unit of capacity ($e^{-(r+\delta)(t-\theta)}$). We denote τ_z^+ the date when investment in renewable power starts; following (13):

$$\forall t \geq \tau_z^+, \quad c_z'(x_{z,t}) = \int_t^{\infty} e^{-(r+\delta)(\theta-t)} u_\theta' \, d\theta \tag{23}$$

Equation 23 simply means that when windmills are built, they are built up to the pace where the marginal cost of producing windmills equals discounted revenues.

The case of gas capacity is more complex. As the previous subsection has shown, gas capacity may be underused during the optimal transition from coal to gas and renewable power. We denote T_γ the date when gas is underused. Gas capacities receive a rent only when they are fully used ($t > T_\gamma \implies \gamma_{\ell,t} = 0$), leading to the following value for gas capacities:

[11] Fig. 1b shows that when gas is used at full capacity, the increasing cost of carbon and gas fuel (which translates as an increasing height for the red rectangle) cannot be passed to the consumer through the electricity price; while the changes of wind and gas capacity (modifying the width of the green and red rectangles) do change capacity rents and resulting electricity prices.

$$\nu_{\ell,t} = \int_t^{T_\gamma} e^{-(r+\delta)(\theta-t)}(u'_\theta - \mu_\theta\, F_\ell - \alpha_{\ell,\theta})d\theta \tag{24}$$

The value of gas power plants is the discounted sum of the electricity price net of production costs over the period when gas power plants are used at full capacity. Investment in gas power plants optimally happens when the value of gas plants exceed the minimal value of producing a gas plant $\nu_{\ell,t} > c'_\ell(0)$.[12]

Comparing equations 23 and 24 shows that renewable power plants are more valuable than gas power plants:

$$\nu_{z,t} - \nu_{\ell,t} = \underbrace{\int_t^{T_\gamma} e^{-(r+\delta)(\theta-t)}\,(\mu_\theta\, F_\ell + \alpha_{\ell,\theta})\,d\theta}_{\Delta\gamma} + \underbrace{c'_z(x_{z,T_\gamma})e^{(r+\delta)(t-T_\gamma)}}_{\Delta\nu} \geq 0 \tag{26}$$

The difference in the values of renewable and gas power plants breaks down into:

- $\Delta\gamma$, the discounted value of emissions and fossil fuels that the marginal renewable capacity built at time t saves before τ^-_ℓ, when compared to a marginal gas capacity built at the same time; and
- $\Delta\nu$, the difference between the discounted values of the capacities at T_γ. Indeed, after T_γ, gas plants are underused and receive no rents, such that their value is null; while renewable power plants have a positive value $\nu_{z,t} = c'_z(x_{z,T_\gamma}) > c'_z(0)$.

While the value of renewable plant is higher than the value of gas power plants, gas plants are cheaper to build than windmills. As a result, the optimal ordering of investment in both types of power plant is not trivial, as discussed in the next subsection.

E. *Ordering investment in low- and zero-carbon capacity*

Three dates are of importance to classify transition profiles from coal to gas and renewable power: the date τ^+_z when investment in renewable starts, the date τ^+_ℓ when investment in gas power plants starts and the date T^-_h when coal production is phased out.

[12]Note that investment in gas stops at τ^-_ℓ when future revenues are lower than the minimal building cost, such that:

$$\int_{\tau^-_\ell}^{T_\gamma} e^{-(r+\delta)(\theta-\tau^-_\ell)}(u'_\theta - \mu_\theta\, F_i - \alpha_{i,\theta})d\theta = c'_\ell(0) \tag{25}$$

In particular, if $c'_\ell(0) = 0$ then $\tau^-_\ell = T_\gamma$ (this is the case in Fig. 2).

In general, three given dates can be ordered in six different ways. Two cases ($T^{-}_{h} < \tau^{+}_{z} \leq \tau^{+}_{\ell}$ and $T^{-}_{h} < \tau^{+}_{\ell} \leq \tau^{+}_{z}$) are never optimal with standard utility functions, because they imply that coal is phased out before investment in renewable or gas have started, so that energy consumption tends to 0 when t tends to T^{-}_{h}. This leaves us with four cases, as stated in the following proposition:

PROPOSITION 2: *Depending on the parameters, investment phases may be ordered in three ways:*

1) *Two successive transitions ($\tau^{+}_{\ell} \leq T^{-}_{h} < \tau^{+}_{z}$). Gas first completely replaces coal, then renewable power replaces gas. In this case, investment in renewable power starts after coal is phased out, but before gas is phased out (Fig. 3a).*
2) *Gas and wind simultaneously replace coal, starting with investment in gas ($\tau^{+}_{\ell} \leq \tau^{+}_{z} \leq T^{-}_{h}$). In this case, investment in renewable starts before production from any fossil resources is phased-out (Fig. 3b).*
3) *Starting with investment in renewable power, either using some gas ($\tau^{+}_{z} \leq \tau^{+}_{\ell} \leq T^{-}_{h}$, as in Fig. 2) or without using any gas ($\tau^{+}_{z} \leq T^{-}_{h} \leq \tau^{+}_{\ell}$,Fig. 3c).*

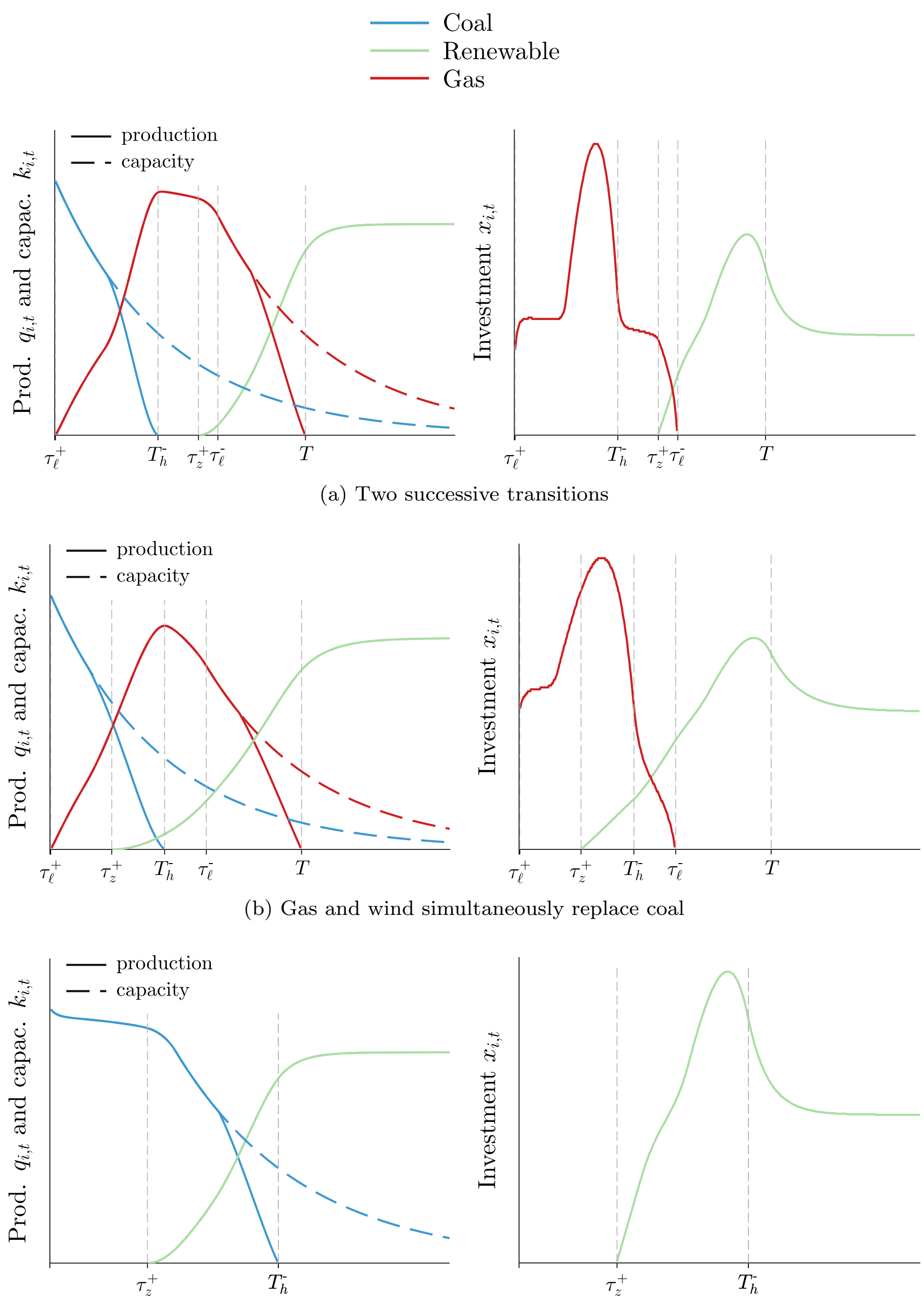
We perform a sensitivity analysis to investigate how the stringency of the carbon budget and the adjustment costs lead to different transitions profiles. To quantify the stringency of adjustment costs, we write the cost functions as:

$$c_i(x_{i,t}) = C_i \times \left((1-A)\, x_{i,t} + \frac{1}{2} A x_{i,t}^2 \right) \tag{27}$$

$$\implies c_i'(x_{i,t}) = C_i \times ((1-A) + A x_{i,t}) \tag{28}$$

where $A \in (0,1)$ is a measure of adjustment costs, and is equal across technologies for simplicity, and C_i is a scaling parameter ($C_z > C_\ell$ as renewable capacity is more expensive than gas capacity). When $A = 0$, marginal investment costs are constant, there is no adjustment cost, and optimal investment pathways can exhibit jumps. When $A = 1$, the full cost is purely quadratic, marginal costs are linear, capacity accumulated at very low speed is almost free ($\lim_{x_{i,t} \to 0; A=0} \; c_i'(x_{i,t}) = 0$), and the cost of new capacity doubles when the investment pace doubles. For intermediate value $A \in (0,1)$, new capacity is always costly, and the marginal cost of new capacity increases with the investment pace.

Results are displayed in Fig. 4. The first case discussed in Prop. 2 reminds an Herfindahl sequence, where energy sources are used one after the other (Fig. 3a). The sensitivity analysis reveals that this case requires a large carbon budget—which "gives time" to switch entirely to gas before starting to invest in renewable power, and low adjustment costs—which make it cheap to invest in each of the transitions (red triangles in Fig. 4). In the limit case without adjustment costs, the optimal strategy is to replace all coal with gas overnight at a date $T_{h\ell} = T^{-}_{h} = \tau^{+}_{\ell}$,



(a) Two successive transitions

(b) Gas and wind simultaneously replace coal

(c) Wind alone replaces coal

Figure 3. : Numerical simulations of three possible transition profiles.

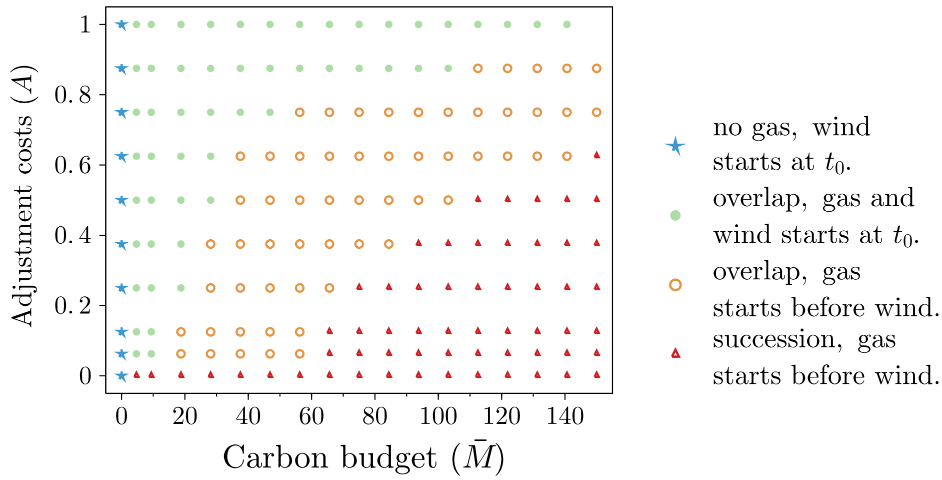


Figure 4. : How transition profiles depend on the stringency of the carbon budget and the adjustment costs.

and then replace all gas with renewable at a latter date $T_{\ell z} = T_\gamma = \tau_z^+$.

The second case happens with any non-zero carbon budget, provided adjustment costs are large enough, while not being too large (orange circles in Fig. 4). In this case, renewable power enters early to smooth investment and reduce adjustment costs. In addition, transient investment in gas allows for reducing investment in expensive renewable power plants in the short term, moving some efforts to the medium term, when gas power is under-used and replaced by renewable power (Fig. 3b).

With even higher adjustment costs, it becomes even more profitable to smooth investment in renewable power, and investment in renewable power starts as soon as possible. In this case, investment in gas and wind start simultaneously at the beginning of the transition ($\tau_z^+ = \tau_\ell^+ = t_0 \leq T_h^-$, green dots in Fig. 4). This case is listed as the third one in Prop. 2, as the transition begins with investment in renewable power. Finally, for very low carbon budgets, investment still starts with renewable power, but gas becomes unhelpful (Fig. 3c and blue stars in Fig. 4).[13]

[13]In the limit case with no adjustment costs, and very low carbon budgets, switching overnight from coal to gas at $T_{h\ell}$ and then overnight to renewable at $T_{\ell z}$ would not be optimal, because the time span during which gas power plants would operate $T_{\ell z} - T_{h\ell}$ would be too short for the initial investment to be profitable. This case, mentioned for the sake of completeness, is not distinguishable from the case of a 0 carbon budget in Fig. 4.

III. Numerical application to the European electricity sector

A. Functional forms, data and calibration

We calibrate the model with raw data from the European power sector as described in the European 2050 Energy Roadmap (EU, 2011). In this numerical application, efficient gas power plants (the low-carbon technology) and onshore wind (the zero-carbon technology) are used to phase out the existing polluting capacities represented as the average legacy thermal production mix (composed of coal, oil and gas Table 1).

We modify the model from Section I to take into account that Europe is price-taker for fossil resources. We assume exogenous fossil resources costs $\alpha_{i,t}$, calibrated from the European 2050 Roadmap. $\alpha_{h,t}$ is computed as the weighted average of the price of coal, gas and oil (Table 2).

For the utility function, we start from a demand function calibrated against D_t, the projected demand in the Roadmap. We assume this demand corresponds to a reference price P, taken as the average electricity price to households in all European countries $P = 90\$/\text{MWh}$ (Eurostat, 2014). We then assume a short-term linear elasticity $e = .1$, meaning that increases of 10% of the electricity price above this reference level would lead demand to be reduced by 1% compared to the reference demand D_t. The demand function reads:

$$q(p) = D_t - e \cdot (p - P) \tag{29}$$

Substituting the price p for $u_t'(q)$ yields:

$$u(q) = \left(P + \frac{D_t}{e}\right) q - \frac{q^2}{2e} \tag{30}$$

The reference electricity consumption point D_0 is calibrated as the reference fossil energy production (from coal, oil and gas) in 2008, that is $D_0 = 1\,940$ TWh/yr (ENERDATA, 2012). The central scenario of the Roadmap envisages that electricity demand will increase by 700 TWh/yr between 2008 and 2050. We model that as a linear growth of $G = 16.5$ TWh/yr^2:

$$D_t = D_0 + t \cdot G \tag{31}$$

To better fit the data, we express installed capacity $k_{i,t}$ in peak capacity (GW), and production $q_{i,t}$ in GWh/yr. Production is constrained by a maximum number of operating hours per year H_i (Table 3). For instance, a given windmill will produce electricity only at the moments where it is windy, which expectedly happens a given number of hours per year. We thus ignore the effect of *unpredicted* intermittency issues, which add a cost to renewable sources, and are a rationale to use back-up generation along with renewable power (Ambec and Crampes,

Table 1—: Technology sets considered in the numerical model

Set	Abbreviation	Composition
High carbon technology	Legacy	Average thermal production mix in 2008: 40% gas, 50% coal, 10% oil
Low carbon technology	Gas	Efficient gas
Zero carbon technology	Wind	Onshore wind

Source: (ENERDATA, 2012)

Table 2—: Fuel price trajectories of the fossil technology sets in \$/MWh

	2008	2025	2035	2050
Legacy	40	43	45	37
Gas	58	68	67	54

Source: (EU, 2011)

Table 3—: Technology-specific data used in the numerical application

	Description	Unit	Legacy	Gas	Wind	Source
C_i^m	Nominal investment costs	\$/kW	2 100	1 400	2 500	EU (2011); IEA (2010)
X_i	Average annual new capacity in Europe	GW/yr	4.2	11	10	ENERDATA (2012)
H_i	Average annual operating hours	h/yr	7 500	7 500	2 000	EU (2011); IEA (2010)
F_i	Carbon intensity	gCO_2/kWh	530	330	0	ENERDATA (2012); Trotignon and Delbosc (2008)

Table 4—: General parameter values used in the numerical application

	Description	Unit	Value	Source
r	Discount rate	%/yr	5	
$\bar{M}$	Carbon budget (central value)	$GtCO_2$	22	UE (2011)
D_0	Electricity demand in 2008	TWh/y	1 940	ENERDATA (2012); EU (2011)
G	Annual growth of demand	TWh/y	16.5	EU (2011)
P	Average electricity price in 2008	\$/MWh	90	Eurostat (2014)
e	Short term linear price elasticity of electricity demand	(TWh/y)/(\$/MWh)	0.1	Eurostat (2014)
δ	Depreciation rate	%/yr	3.33	IEA (2010)
A	Convexity parameter (central value)	·	1	

2012).[14]

The initial capacity of wind and gas are by assumption zero, and we calibrate the initial capacity of the legacy technology as the level of capacity needed to produce the initial reference electricity consumption point D_0 at the given capacity factor:

$$k_{z,0} = k_{l,0} = 0; \quad k_{h,0} = \frac{D_0}{H_h}$$

To calibrate the cost functions, we assume that when investment equals the actual average annual investment flow in Europe between 2009 and 2011 (X_i), the marginal investment cost C_i^m is equal to the OECD average value for 2010 (Table 3). We thus re-write the cost function used in the previous section as:

$$c_i(x_{i,t}) = C_i^m \cdot X_i \cdot \left((1 - A) \frac{x_{i,t}}{X_i} + \frac{A}{2} \left(\frac{x_{i,t}}{X_i} \right)^2 \right) \tag{32}$$

$$\implies c_i'(X_i) = C_i^m \tag{33}$$

According to Trotignon and Delbosc (2008), emission allowances allocated to the power sector in 2008 amounted to $E_{\text{ref}} = 1.03$ GtCO$_2$/yr , leading to a reference carbon intensity of $F_h = 530\,\text{tCO}_2/\text{GWh}$. A linear decrease of these emissions until 2050, as planned in the Roadmap, yields a carbon budget of $\bar{M} = 22$ GtCO$_2$. (A sensitivity analysis on $\bar{M}$ is performed later.) The carbon intensity of gas is taken equal to $F_l = 330\,\text{tCO}_2/\text{GWh}$.

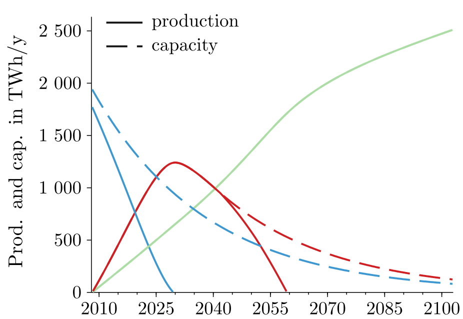
We use $r = 5$ %/yr for the social discount rate. We assume for simplicity that all technologies have the same depreciation rate δ, calibrated as $\delta = 1/\text{lifetime}$ assuming a lifetime of 30 years (IEA, 2010).

B. Results

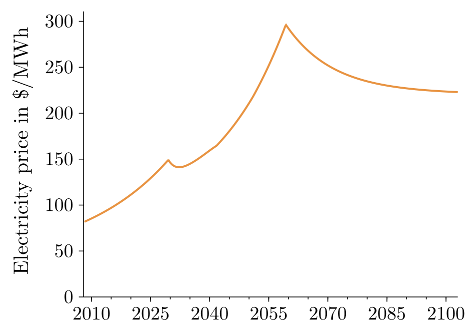
Fig. 5 shows production, investment and the electricity price obtained in the numerical application. For the carbon budget consistent with the roadmap, the social planner does not invest in the legacy capacity. Moreover, existing legacy plants start being decommissioned as soon as the climate policy is implemented (Fig. 5a). In this simulation, legacy fossil-fueled plants are entirely phased out in 2028. With our technology assumptions, the carbon budget is slightly too loose to justify a complete decarbonization by 2050, as it is the European Commission's objective; production from fossil fuel is phased out by 2059 instead.[15] The carbon

[14] Alternatively, our renewable technology may be interpreted as a mix of intermittent renewable and carbon-free back-up, as provided for instance, by large hydro, nuclear and biomass.

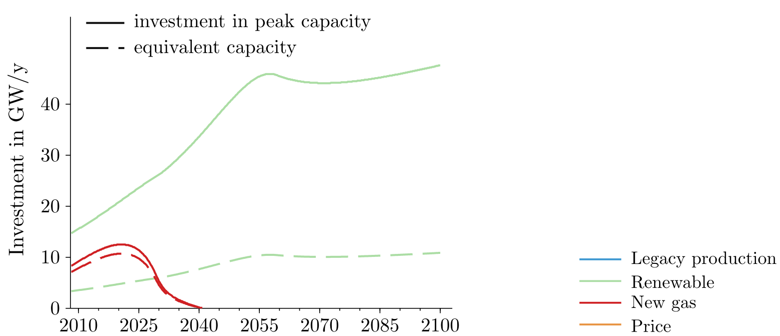
[15] Note that we ignored technical progress reducing the cost of renewable energy in the future, which could be a rationale to accelerate decarbonization.



(a) Production and capacity



(b) Electricity price



(c) Investment

Figure 5. : Numerical application to the European electricity sector

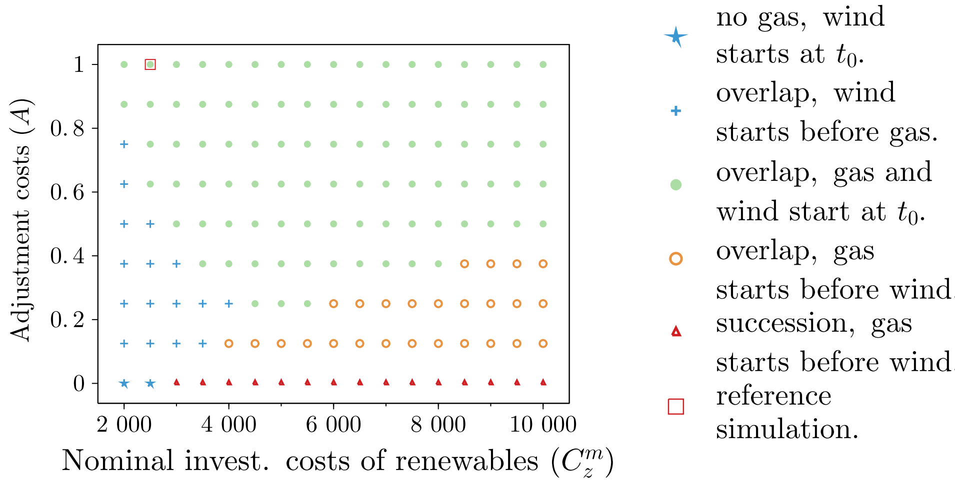


Figure 6. : Transition types for different values of the cost of renewables and the adjustment costs.

price consistent with the carbon budget is 108 \$/tCO_2.[16] Electricity prices start at 90\$/MWh (close to the actual average EU level), increase with the carbon price to 300 \$/MWh by the end of the transition in 2059, and decrease again to reach a plateau around 220 \$/MWh by the end of the century.

Investment in renewable starts as soon as possible (Fig. 5c), and grows over time, until all production comes from renewable power plants (Fig. 5a). Optimal investment in renewable grows from 15 to 18 GW/yr between 2008 and 2013. These figures are close to actual numbers, as investment in renewable capacity in Europe fluctuated between 15 to 34 GW/yr during this same period (EWEA, 2014). (All these figures are expressed in peak capacity; Fig. 5c also reports investment in equivalent capacity, taking into account the lower numbers of hours that renewable power can be used per year.)

Investment in new gas plants also start as soon as possible, smoothing the need for more expensive investment in renewable in the short term (Fig. 5c). But new gas plants play a transient role. In 2040, investment in gas stops and gas capacities start being underused, allowing gas to be replaced faster by renewable power. In this simulation, up to 80 GW of gas power plants (or the capacity to produce 600 TWh/y) are underused during the simulation.

We performed various sensitivity analysis to investigate under which conditions it would be optimal to wait before building renewable power in Europe. A first one varying adjustment costs A and the carbon budget $\bar{M}$ revealed that for any

[16]This is much higher than the current carbon price in the EU-ETS. Note that the European governments currently rely on other policies — feed-in-tariffs, mandates and auctioned pilot projects — to enforce investment in renewable power.

carbon budget lower than 45 $GtCO_2$ (twice the budget implied in the Roadmap), and for any adjustment cost, it is optimal to start building renewable power as soon as possible.

We thus perform another sensitivity analysis, holding the carbon budget constant and varying adjustment costs and nominal investment costs for renewable power (Fig. 6). For high adjustment costs ($A > 0.5$), it is optimal to start with renewable power even if investment costs where as high as 10\$/W (four times our estimation of actual costs). For low adjustment costs ($A = .125$), it is optimal to delay investment in clean electricity if costs are higher than 4\$/W, that is 60% more expensive than our reference calibration. Finally, if adjustment costs are entirely absent, the optimal strategy is to first replace existing capacity with gas, and only later replace all gas power with renewable power overnight.

IV. Conclusion

Our results should be interpreted cautiously, as our analysis makes several simplifications. Among them, we did not account for knowledge accumulation and directed technical change, which have been found to play a key role in the optimal transition from fossil to renewable energy (Tahvonen and Salo, 2001; Acemoglu et al., 2012; André and Smulders, 2014; Gerlagh, Kverndokk and Rosendahl, 2014). Knowledge spillovers would tend to increase the short-term gap between optimal investment in renewable and gas energy (Rosendahl, 2004; Bramoullé and Olson, 2005; del Rio Gonzalez, 2008), adding to the effect of adjustment costs studied here.

We also disregarded uncertainty, known to play a key role in optimal accumulation of capital (Pindyck, 1991), optimal emission reduction pathways (Ha Duong, Grubb and Hourcade, 1997), and optimal extraction of several energy sources (Gaudet and Lasserre, 2011). While it is well known that uncertainty provides strong rationale for avoiding irreversible activities, the net effect of uncertainty is not trivial in our setting. Indeed, extracting exhaustible resources, releasing long-lived GHG in the atmosphere, investing in carbon-intensive power plants, and investing in expensive renewable capacities are all irreversible activities. Moreover, in the presence of adjustment costs, waiting makes subsequent investment more expensive. We let the question of uncertainty for further research.

Another limitation is that, we did not explicitly model the intermittency of renewable sources, which may require to keep flexible gas turbines (or other flexible but carbon-free options such as large hydro or biomass-fueled plants) in operation. Finally, we disregarded the possibility to retrofit existing plants (notably with carbon capture and storage), and the possibility to use cleaner fuels (in particular derived from biomass) in existing power plants.

Notwithstanding these limitations, our parsimonious model suggests that capacity constraints and adjustment costs play an essential role in the transition from coal to gas and renewable energy. Our results contrast with those derived from "pure" Hotelling models, where capital accumulation and adjustment costs are neglected (or that focus on generic capital that can be fueled with any type of energy). In particular, we find that it makes sense to start investment in, and usage of, renewable energy before coal and gas resources are phased out, even if renewable energy first appears to be more expensive.

We also find that the transition to carbon-free energy may benefit from temporary investment in intermediate technologies such as gas, to decrease (but not necessarily cancel) the need for costlier renewable in the short term. The resulting plants would however need to be subsequently decommissioned, to give room to more carbon-free energy in the medium term.

Our results shed light on technical choices (e.g. investors can consider gas plants with shorter scheduled lifetimes or an option to retrofit) as well as policy decisions (when setting milestones for carbon-free power). Finally, while we used power generation as an obvious illustration, our paper more broadly informs on the ordering and assessment of investment in polluting fossil-fueled and clean capital. Its results could be extended to other capitalistic and energy-intensive sectors, such as buildings and transportation.

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Efficiency conditions

Table A1—: Variables and parameters notations used in the model.

	Description	Dimension
i	technology index	
h	coal (high-carbon technology,h)	
l	gas (low-carbon technology, ℓ)	
z	renewables (zero-carbon technology, z)	
$k_{i,t}$	capacity of technology i at time t	GW
$q_{i,t}$	production with technology i at time t	GW
$x_{i,t}$	physical investment in technology i at time t	GW/yr
$c_i(x_{i,t})$	cost of investment in technology i at time t	\$/yr
$\nu_{i,t}$	shadow price of capacity $k_{i,t}$	\$/GW
μ_t	shadow cost of emissions (carbon price)	\$/tCO_2
$\alpha_{i,t}$	shadow cost of resource used by technology i (fuel price)	\$/GWh
$\gamma_{i,t}$	shadow rental cost of existing capacity i	\$/(GW· yr)
u'_t	shadow electricity price (or willingness to pay)	\$/GWh
m_t	stock of atmospheric carbon	tCO_2
δ	depreciation rate	yr^{-1}
r	discount rate	yr^{-1}
F_i	emission rate of technology i	tCO_2/GWh
D_t	demand level at time t	GW
$\bar{M}$	carbon budget	tCO_2
$u\left(\sum_i q_{i,t}\right)$	consumer utility	\$/yr

The Hamiltonian associated with Problem 11 reads:

$$\begin{aligned}\mathcal{H} = e^{-rt} \Bigg[& u\left(\sum_i q_{i,t}\right) - \sum_i c_i(x_{i,t}) - \sum_i \nu_{i,t}\left(\delta\, k_{i,t} - x_{i,t}\right) \\ & - \mu_t \sum_i F_i\, q_{j,t} - \eta_t \left(m_t - \bar{M}\right) - \sum_i \left(\alpha_{i,t}\, q_{i,t} - \beta_{i,t}\, S_{i,t}\right) \\ & - \sum_i \gamma_{i,t}\left(q_{j,t} - k_{j,t}\right) + \sum_i \lambda_{i,t}\, q_{j,t} + \sum_i \xi_{i,t}\, x_{i,t} \Bigg]\end{aligned} \tag{A1}$$

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial x_i} = 0 \iff c_i'(x_{i,t}) = \nu_{i,t} + \xi_{i,t} \tag{A2}$$

$$\frac{\partial \mathcal{H}}{\partial q_i} = 0 \iff \lambda_{i,t} - \mu_t R_i - \alpha_{i,t} + u_t' = \gamma_{i,t} \tag{A3}$$

$$\frac{\partial \mathcal{H}}{\partial k_i} = \frac{-d\left(e^{-rt}\nu_{i,t}\right)}{dt} \iff (\delta + r)\, \nu_{i,t} - \dot{\nu}_{i,t} = \gamma_{i,t} \tag{A4}$$

$$\frac{\partial \mathcal{H}}{\partial m_t} = \frac{-d\left(e^{-rt}\mu_t\right)}{dt} \iff \dot{\mu}_t - r\, \mu_t = -\eta_t \tag{A5}$$

$$\frac{\partial \mathcal{H}}{\partial S_i} = \frac{-d\left(e^{-rt}\alpha_{i,t}\right)}{dt} \iff \dot{\alpha}_i - r\, \alpha_{i,t} = -\beta_{i,t} \tag{A6}$$

The complementary slackness conditions are:

$$\forall i,t, \quad \xi_{i,t} \geq 0, \quad x_{i,t} \geq 0 \quad \text{and} \quad \xi_{i,t}\, x_{i,t} = 0 \tag{A7}$$

$$\forall i,t, \quad \lambda_{i,t} \geq 0, \quad q_{i,t} \geq 0 \quad \text{and} \quad \lambda_{i,t}\, q_{i,t} = 0 \tag{A8}$$

$$\forall i,t, \quad \eta_t \geq 0, \quad \bar{M} - m_t \geq 0 \quad \text{and} \quad \eta_t \left(\bar{M} - m_t\right) = 0 \tag{A9}$$

$$\forall i,t, \quad \beta_{i,t} \geq 0, \quad S_{i,t} \geq 0 \quad \text{and} \quad \beta_{i,t}\, S_{i,t} = 0 \tag{A10}$$

$$\forall i,t, \quad \gamma_{i,t} \geq 0, \quad k_{i,t} - q_{i,t} \geq 0 \quad \text{and} \quad \gamma_{i,t}\, \left(k_{i,t} - q_{i,t}\right) = 0 \tag{A11}$$

The transversality condition is replaced by the terminal condition that at some point the atmospheric carbon reaches its ceiling (8).

A1. Scarcity rents of the exhaustible resources and the carbon budget

From (A9) and (A10), we see that while the carbon budget and the resource stocks are not exhausted, their non-negativity dual variable η_t and $\beta_{i,t}$ are zero. From (A5) (A6), we then get the classic Hotelling rule:

$$\alpha_{h,t} = \alpha_h\, e^{rt}$$

$$\begin{aligned}\alpha_{\ell,t} &= \alpha_{\ell}\, e^{rt}\\ \mu_t &= \mu e^{rt}\end{aligned}$$