Precautionary Motives Under Multiple Instruments

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Abstract

We study precautionary behavior when one or more instruments are available to the decision maker to optimize the expected utility of intertemporal consumption. Specifically, we analyze saving, self-protection, and self-insurance, as well as how the combinations of these instruments behave when employed simultaneously. We examine the reactions to a loss risk – which is prerequisite for self-protection and self-insurance – and a background income risk. When controlling for consumption smoothing, we find that each risk implies for each instrument a positive precautionary and a negative substitution effect. Consequently, the total precautionary motive, which is the net effect of all individual effects, can be decomposed into precautionary motives according to the different risks and instruments. We quantify the relative contributions to the total precautionary motive numerically. Instrument interaction has important implications for measuring the intensity of precautionary motives empirically and experimentally.

Keywords: prudence · saving · self-protection · self-insurance · precaution

JEL classification: D81, D90, D91

1 Introduction

Knowing the intensity of precautionary motives is crucial in many applications, including public policy. Investments to fight climate change, the design of social security systems, risk mitigation strategies by individuals and organizations, and recommendations for individual asset allocation decisions are but some contexts where spurious estimates of individual risk and time preferences may bias predictions. Various studies have sought to measure the precautionary saving motive from field observations, with diverse methodologies and datasets (*e.g.*, Kuehlwein 1991, Dynan 1993, Merrigan and Normandin 1996, Eisenhauer 2000, Ventura and Eisenhauer 2006, Lee and Sawada 2007). But the results have remained inconclusive regarding prevalence and strength.

Our analysis provides a potential explanation for the difficulty to quantify the intensity of precautionary motives using field data. Agents often dispose of a variety of channels to cope with risks. For example, they may reduce the probability or severity of potential damages, buffer risks using market

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instruments, or save or work more to prepare and forearm for future risks. These instruments will, in general, interact in their use and determine jointly an individual's effective risk exposure. Several instruments may pick up parts of the precautionary response. Consequently, if additional instruments are not (fully) identified, results will be biased against higher intensities of precautionary motives.

We study the intertemporal risk responses of a decision maker who disposes of one, two, or three instruments out of saving, self-protection, and self-insurance. Self-protection is an investment that allows the decision maker to decrease the probability of a potential loss, while self-insurance reduces the loss size keeping its probability (Ehrlich and Becker 1972). For observing the decision maker's precautionary motives in this context, two aspects are crucial at the outset. First, like all intertemporal decisions under risk, optimal behaviors involve a consumption-smoothing and a precautionary component. Second, the decision maker has two risks to cope with, the endogenous loss risk and an exogenous background risk. The two aspects are important because a change in the loss risk involves, by definition, a first-order effect and a change in pure riskiness. As a consequence, after controlling for consumption smoothing, already the loss risk activates an agent's precautionary preference. We model the background risk as an independent zero-mean risk on future income, and define an agent's precautionary motive as the extent to which the agent reacts in an instrument to a future risk. If there is only one risk-management instrument, the agent's total precautionary motive is thus made up of two components, the precautionary reactions to the loss risk and to the income risk. For the income risk, the agent's prudence is well known to be necessary and sufficient for a precautionary reaction. For the loss risk, the preference condition is the same for saving. Precautionary self-protection, however, requires a stronger condition involving prudence, whereas for precautionary self-insurance already risk aversion is necessary and sufficient.

Studies on risk management and underlying preferences should address the portfolio of instruments available to an agent and account for potential interactions between these. In a second step, we analyze, therefore, the interaction of precautionary choices, when more than one instrument is available. In the multi-instrument cases, the effects under one instrument persist in each instrument. However, in addition to the positive precautionary effects, now in each instrument also a negative substitution effect arises. Due to these substitution effects, the precautionary choices will be smaller at least in one instrument than in the corresponding single-instrument case. Indeed, the interaction between the instruments may be so strong that an instrument is used less in reaction to the future risk. We derive these results for the two-instrument cases as well as the case with all three instruments.

We finally quantify the precautionary motives in the different settings of our theoretical analysis numerically.[1] The simulations for some median agent yield several interesting results. In the single-instrument cases, saving shows by far the largest precautionary reaction. Self-protection reacts still more than self-insurance. Self-insurance is mostly used for consumption smoothing. In the two-instrument cases, the total precautionary motives are consistently lower than in each corresponding single-instrument case. Now, the precautionary motive in self-insurance dominates the ones in saving and self-protection, and the precautionary reaction in saving is still weaker than precautionary self-protection. When self-insurance is available, the total precautionary motives in saving or self-protection may even turn negative, if the loss size is large. In the three-instrument case, the effects of instrument interaction are still more pronounced. In particular, the total precautionary motive across all instruments is once more smaller than under two instruments. This suggests a negative link between the number of instruments available and the total precautionary motive.

Our paper contributes to several strands of literature. Precautionary choices in two variables have been studied in contexts complementary to ours. For example, Flodén (2006) shows in a two-period model with consumption and labor that, under reasonable conditions, a precautionary response in labor arises in reaction to a wage-rate risk and that saving is increased, if the labor choice is endogenous. Based on an analysis with recursive preferences, Nocetti and Smith (2011) add that labor endogenity tends to decrease saving, if a non-labor, instead of a labor, risk is considered. Low (2005) studies labor provision and saving in a calibrated lifecycle model and obtains results corresponding to Flodén's.[2] While the case of endogenous labor is complementary to our analysis, none of these papers considers self-protection or self-insurance technologies. Moreover, all focus on only one risk source at a time.

Aspects of the interaction of different risk-management instruments in intertemporal choice have been analyzed from different angles. For a two-period framework, Dionne and Eeckhoudt (1984) investigate the Hicksian demand for insurance and saving and show that under 'decreasing temporal risk aversion' the two instruments are pure substitutes.[3] Menegatti and Rebessi (2011) extend their result to self-protection and saving. For a setting with saving and demand for medical care, Nocetti and Smith (2010) show that, under recursive Selden (1978, 1979) preferences with decreasing absolute risk aversion, an increase in health risk will raise saving and second-period medical expenditures, while medical risk acts like interest-rate risk in pure precautionary-saving models. Moreover, they derive the determinants of optimal choices when admitting both risks and the additional possibility to invest in

[1] We are not aware of an empirical or numerical study on precautionary choices with respect to multiple risks or precautionary self-protection or self-insurance thus far.

[2] Marcet et al. (2007), Netzer and Scheuer (2007), and Heathcote et al. (2008) treat related aspects.

[3] This property is obtained by assuming that the curvature of utility over future consumption, as measured by the negative of its second over its first utility derivative, is non-increasing in future and non-decreasing in current consumption.

the first period in preventive care with risky effectiveness. Their first result on the positive effects of a health-risk increase on precautionary saving is in line with a number of empirical studies documenting that increased health insurance coverage reduces risk and implies less precautionary saving (*e.g.*, Levin 1995, Picone et al. 1998, Gruber and Yelowitz 1999, Chou et al. 2003, Hsu 2013, Kuan and Chen 2013). Another strand of literature uses continuous-time lifecycle frameworks. In this context, Briys (1988) finds a strong relation between consumption, portfolio, and insurance decisions. Gollier (1994, 2003) shows that precautionary capital accumulation will dominate insurance demand in the long run.[4] However, none of these papers, treats the interaction of precautionary motives between instruments.

While precautionary saving has long been analyzed (*e.g.*, Drèze and Modigliani 1966, 1972, Leland 1968, Sandmo 1970, Rothschild and Stiglitz 1971, Kimball 1990), only some recent contributions treat precautionary choices in self-protection (*e.g.*, Courbage and Rey 2012, Eeckhoudt et al. 2012) or self-insurance (*e.g.*, Hofmann and Peter 2012) in an intertemporal framework.[5] These latter papers, however, associate precautionary responses only with an exogenous future background risk and do not distinguish the consumption-smoothing and precautionary motivations for self-protection or self-insurance as arising from the loss risk. Independently, Menegatti (2009) investigates optimal saving in the presence two risks. Our case with saving only can be seen as an application of his analysis.

In Section 2, we introduce the two risks and treat the optimal intertemporal choices in the cases with one instrument. Section 3 studies instrument interaction when two instruments are available simultaneously. Section 4 treats the case with three instruments. We provide numerical comparative statics for the precautionary motives in the different theoretical settings in Section 5. Section 6 concludes. We gather the proofs, that are not in the text, in the appendix.

2 Intertemporal Decisions with One Instrument and Two Risks

Consider an agent who lives for two periods. The agent's intertemporal consumption stream $(c, \tilde{C})$ consists of certain consumption c in period one and potentially risky consumption $\tilde{C}$ in period two. We mark random variables with a tilde. The following intertemporal utility objective characterizes the agent's preferences:

$$U(c, \tilde{C}) = u(c) + \beta E v(\tilde{C}) \tag{1}$$

where u is the agent's first-period felicity, v the second-period felicity, and β the utility discount factor. We assume u and v are strictly increasing and concave.

[4] Futher papers in this vein are Somerville (2004), Moore and Young (2006), Perera (2010), and Lin and Lu (2012).

[5] For a monoperiodic setup, Eeckhoudt and Kimball (1992) establish that a zero-mean income risk creates a precautionary demand for insurance if the agent has a positive third utility derivative.

The agent receives the exogenous incomes y in period one and Y in period two. The riskiness of second-period consumption may arise from two sources. First, we consider the risk of a property loss of size L occurring with probability $p \in (0,1)$. The mitigation of this risk represents the focal aspect in the literature on self-protection or self-insurance. Second, there can be a background risk $\tilde{\varepsilon}$, $E\tilde{\varepsilon} = 0$, associated with the agent's second-period income Y.[6] A background risk of this kind is generally taken as the trigger for precautionary choices. Whereas the income risk, as a zero-mean risk, adds only riskiness to second-period consumption, the loss risk has also a first-order effect. We stick in this paper to the simple case where loss risk and income risk are uncorrelated.[7]

Given a loss risk, the agent can influence the intertemporal consumption stream in three ways, via saving, self-protection, or self-insurance. In contrast to saving, self-protection and self-insurance require the presence of the loss risk to become operative.

Saving. From the literature on intertemporal consumption, it is well known that agents commit to saving for two main reasons, to smooth consumption and for precautionary purposes. The origin of precautionary saving is risk on future resources for consumption. Given that the agent is prudent, that is, $v''' > 0$ (Kimball 1990), precautionary saving will in the present context already arise when only the loss risk is present. Let r be the interest rate in the market, which we take to be non-random. Then, the agent chooses saving s such as to maximize

$$U^l(s) \;=\; u(y-s) + \beta\left[pv(Y + s(1+r) - L) + (1-p)v(Y + s(1+r))\right]$$

where superscript l indicates that the utility objective includes the loss risk, but no other risk. We denote the optimal saving choice resulting from this problem by s^{l*}. As usual in treatments of consumption/saving problems, the consumption-smoothing component of optimal saving can be determined by maximizing

$$U^0(s) = u(y-s) + \beta v\left(Y + s(1+r) - pL\right)$$

Superscript 0 indicates that the riskiness of future consumption is set to zero by replacing the loss risk by its mean. We denote the optimal saving choice by s^{0*}. Precautionary saving is, in turn, the extra amount of saving that arises due to the riskiness of second-period consumption, $s^{l*} - s^{0*}$. As mentioned above it is positive under prudence.

[6] Gollier and Pratt (1996) study unfair background risks whose mean is allowed to be negative. To focus on the effects of riskiness we assume that expected consumption does not depend on the presence of the background risk, which is consistent with most of the literature.

[7] Several papers treat optimal choices in the face of correlated risks (*e.g.*, Tsetlin and Winkler 2005, Courbage and Rey 2007, Courbage et al. 2013).

The income risk constitutes either an alternative or an additional cause for precautionary saving, depending on whether it is considered as the only source of risk or the two risks are jointly admitted. We denote by superscript i the case where only the income risk is present (and the loss risk is replaced by its mean). Superscript li indicates the case with both loss risk and income risk. In the case with the income risk only, we denote the optimal saving choice by s^{i*}, so that precautionary saving due to the income risk arises as $s^{i*} - s^{0*}$. Here s^{i*} is obtained via maximization of

$$U^i(s) = u(y-s) + \beta E v\left(Y + \tilde{\varepsilon} + s(1+r) - pL\right)$$

In the case with both risks, optimal saving is s^{li*}, and precautionary saving due to the income risk is $s^{li*} - s^{l*}$. The latter case with loss risk and income risk is an example of optimal saving with two risks as in Menegatti (2009), and the optimal saving decision is given as the maximand of

$$U^{li}(s) = u(y-s) + \beta\left[pEv\left(Y + \tilde{\varepsilon} + s(1+r) - L\right) + (1-p)Ev\left(Y + \tilde{\varepsilon} + s(1+r)\right)\right]$$

In both cases precautionary saving will be positive under prudence.

Self-protection. The other two risk-management instruments endogenize the loss risk. Self-protection implies an investment to decrease the probability of loss. Formally, spending $x \geq 0$ dollars on self-protection reduces the loss probability to $p(x) < p$, with $p'(x) < 0$ and $p''(x) \geq 0$. The agent will choose self-protection expenditures x such as to maximize

$$U^l(x) \;=\; u(y-x) + \beta\left[p(x)v(Y-L) + (1-p(x))v(Y)\right]$$

yielding optimal self-protection expenditures of x^{l*}. Similar to the saving decision, also for self-protection consumption smoothing and precaution can be distinguished as basic motivations in the intertemporal case. The consumption-smoothing component of self-protection expenditures x^{0*} arises, in analogy to the saving case, by maximizing the utility objective with future utility depending on expected future consumption. The precautionary component is then $x^{l*} - x^{0*}$. Remark 1 gives the conditions for this component to be positive.

The introduction of a zero-mean income risk in the future raises the optimal investment in self-protection, if and only if the agent is prudent (Courbage and Rey 2012, Eeckhoudt et al. 2012). In analogy to before, we denote its level by x^{li*}. Because the presence of the loss risk is a prerequisite for self-protection, the precautionary demand for self-protection due to the income risk, $x^{li*} - x^{l*}$, is always additional to the precautionary demand already induced by the loss risk.

(Self-)insurance. Self-insurance involves an investment to mitigate the loss size. Formally, spending z dollars on self-insurance reduces the loss to $L(z)$, with $L'(z) < 0$ and $L''(z) \geq 0$.[8] The agent will choose self-insurance expenditures z such as to maximize

$$U^l(z) \;=\; u(y-z) + \beta\left[pv(Y - L(z)) + (1-p)v(Y)\right]$$

giving rise to the optimal level z^{l*}. The reasoning to obtain the consumption-smoothing component, z^{0*}, and precautionary component, $z^{l*} - z^{0*}$, of self-insurance, as well as the additional precautionary expenses due to the income risk, $z^{li*} - z^{l*}$, is analogous to the self-protection case. The aspect that adding a zero-mean income risk in the future raises the optimal self-insurance if the agent is prudent, is a special case of Hofmann and Peter (2012).

The following remark gives the conditions for positive precautionary choices of self-protection and self-insurance in response to the riskiness of the loss risk.

Remark 1 *Consider the effect of the riskiness of the loss risk in the absence of any other risk. There is a positive precautionary motive for self-protection, if the agent is prudent and* $p(x^{l*}) \leq 0.5$. *The precautionary motive for self-insurance is positive if and only if the agent is risk averse.*

Note that precautionary self-insurance in response to the loss risk occurs independent of whether or not the agent is prudent.

To clarify the different precautionary motives and their relations operative in the different cases, and to compare them later on across the different instruments, we introduce finally the following definition. This definition applies interchangeably to each instrument, because our modeling supposes that the agent's resources and choices are measured in the same dimension, namely money.

Definition 1 *Be instrument* a *either saving, self-protection, or self-insurance,* $a \in \{s, x, z\}$. *The*

- Loss-Risk Precautionary Motive in Instrument a *(LPM_a) is the additional use of this instrument beyond consumption smoothing due to the riskiness of the loss risk in the future,* $a^{l*} - a^{0*}$*;*

- Income-Risk Precautionary Motive in Instrument a *(IPM_a) is the additional use of this instrument due to the income risk in the future,* $a^{li*} - a^{l*}$*;*

- Total Precautionary Motive in Instrument a *(TPM_a) is the additional use of this instrument due to the riskiness of future consumption,* $TPM_a = LPM_a + IPM_a = a^{li*} - a^{0*}$*;*

[8] The coinsurance problem is a special case of optimal self-insurance. It is obtained by setting $z = \alpha(1+m)pL$ and $L(z) = (1-\alpha)L$ with $\alpha \in [0,1]$ being the coinsurance rate and $m \geq 0$ being a loading factor.

- shares *induced by loss risk, l, or income risk, i, respectively, in the total precautionary reaction of instrument a are, for $TPM_a > 0$,*

$$sh_a^l = \frac{LPM_a}{TPM_a} , \quad sh_a^i = \frac{IPM_a}{TPM_a}$$

Our definition of the IPM_a corresponds to the notion of precautionary effort found in the literature on self-protection and self-insurance. Alternatively, it could be defined as $a^{i*} - a^{0*}$, which would retain the first-order effect of the loss risk but not account for its riskiness. In the present paper we stick to the former definition. Our distinct definition of the LPM_a, however, points to the fact that in intertemporal decisions with risk, also in the cases with self-protection and self-insurance, a precautionary motive is operative already in response to the loss risk. Precautionary choices arise thus even when the second-period income risk is absent. As a consequence, the total precautionary response in instrument a to future risk is composed of the two components, LPM_a and IPM_a. Of course, the shares induced by loss and income risk in the total precautionary reaction of the instrument add up to one, $sh_a^l + sh_a^i = 1$. For the given risks, the TPM_a is positive as soon as the agent is prudent or the conditions of Remark 1 hold.

3 Precautionary Motives with Two Instruments

We turn to the settings with two instruments out of saving, self-protection, and self-insurance. We put a particular focus on the interaction of the choices within each instrument pair. In order to emphasize the structural similarity of the results, we start with an overarching analysis for two instruments a_1 and a_2, with $a_1, a_2 \in \{s, x, z\}$ and $a_1 \neq a_2$.

Starting from the intertemporal utility objective (1) for the case of certainty, we first consider the presence of the loss risk. As noted above, the loss risk will generate two sets of effects. On the one hand, the first-order effect in form of the lower expected future consumption will activate consumption smoothing. On the other hand, the associated riskiness may induce precautionary choices. In the two-instrument cases, the agent will, in general, use both instruments to deal with these two aspects, and the instruments may interact in a non-trivial way. Given our focus on precautionary motives, we compare the agent's choices in two situations: when facing the loss risk, and under its first-order effect only.

When the loss risk, but no further risk is present, the agent chooses the levels (a_1^{l*}, a_2^{l*}) of the two instruments maximizing the objective

$$U^l(a_1, a_2) \;=\; u(y - a_1 - a_2) + \beta V^l(a_1, a_2) \tag{2}$$

$V^l(.)$ represents second-period expected utility with the loss risk according to the instrument pair under consideration. If $\tilde{C}(.)$ denotes second-period consumption with the loss risk, we obtain that $V^l(a_1, a_2) = Ev\left(\tilde{C}^l(a_1, a_2)\right)$. V^l inherits its properties from its consituent functions. We show in Remark 2 that it has a negative cross-derivative for any choice of a_1 and a_2. The associated first-order conditions are, for $j = 1, 2$,[9]

$$-u'(y - a_1 - a_2) + \beta V^l_{a_j}(a_1, a_2) \;=\; 0 \tag{3}$$

We focus on interior solutions in this paper and assume throughout that the second-order conditions hold. Intuitively, conditions (3) claim that, at the optimum, the effect on expected utility of a marginal dollar is the same whichever instrument is used.

The consumption-smoothing and precautionary components of the choices of the two instruments can be distinguished, in analogy to above, by determining the optimal choices (a_1^{0*}, a_2^{0*}) maximizing utility under the first-order effect only:

$$U^0(a_1, a_2) \;=\; u(y - a_1 - a_2) + \beta V^0(a_1, a_2) \tag{4}$$

where $V^0(a_1, a_2) = v\left(E\tilde{C}^l(a_1, a_2)\right)$. This is the decision problem in which we "mute" the riskiness of the loss risk, and only its first-order effect, *i.e.*, the expected reduction of future income, is present. We denote the optimal solution (a_1^{0*}, a_2^{0*}).

Their potential interaction makes the optimal decisions non-separable. Indirect, or substitution, effects between the instruments are to be considered. We summarize the arising effects in

Proposition 1 *Assume prudence, a probability of loss not above* 0.5*, and* $V^l_{a_1 a_2} < 0$*. Then, the riskiness of the loss risk generates two effects. On each instrument, there is a positive precautionary effect and a negative substitution effect. At least one instrument will be used more compared to the situation where only the first-order effect is considered.*

The intuition is quite clear. As explained in Section 2, the riskiness of the loss risk induces a precautionary reaction in each instrument under the assumptions we make. According to this positive

[9] These conditions can also be stated in the Euler form, $\beta \frac{V^l_{a_j}(a_1, a_2)}{u'(y - a_1 - a_2)} = 1$.

direct effect, each instrument should be used more. However, the two instruments also compete for resources in the first period, and the use of one instrument diminishes the marginal benefit of using the other. Consequently, if one instrument is used more, this is bad news for the attractiveness of the other, which explains the negative indirect effects.

To derive formally the two kinds of effects, evaluate U^l in equation (2) at a_j^{0*} and solve for the other instrument, $\hat{a}_k = \arg\max_{a_k} U^l(a_j^{0*}, a_k)$, with $j, k = 1, 2,\ k \neq j$. Consider $j = 1$. To compare a_1^{0*} to a_1^{l*}, evaluate[10]

$$
\begin{aligned}
U^l_{a_1}(a_1^{0*}, \hat{a}_2) &= -u'(y - a_1^{0*} - \hat{a}_2) + \beta V^l_{a_1}(a_1^{0*}, \hat{a}_2) \\
&= \underbrace{\beta \left[V^l_{a_1}(a_1^{0*}, a_2^{0*}) - V^0_{a_1}(a_1^{0*}, a_2^{0*}) \right]}_{>0} \underbrace{-u'(\hat{c}) + u'(c^{0*})}_{<0} + \underbrace{\beta \left[V^l_{a_1}(a_1^{0*}, \hat{a}_2) - V^l_{a_1}(a_1^{0*}, a_2^{0*}) \right]}_{<0}
\end{aligned}
$$

The first summand is the positive *precautionary* effect. We can directly apply our observation from Section 2 that the riskiness of the loss risk induces an increase in each of the instruments under consideration when considered in isolation. Note that the second instrument is set to a_2^{0*} and that the marginal cost of using the first instrument is unaffected by the riskiness of the loss risk. Consequently, the marginal benefit of the first instrument increases when considering the risk associated with the loss risk. The second term refers to the marginal cost of using the first instrument. The pure change in the second instrument is positive ($\hat{a}_2 > a_2^{0*}$) so that $\hat{c} < c^{0*}$, because more money is spent on the second instrument. As a result, the marginal cost of using the first instrument is larger. Finally, the third term is negative due to the substitution between the two instruments. The negative cross-derivative ($V^0_{a_1 a_2} < 0$) indicates that the marginal benefit from using the first instrument is lower when the second instrument is used more. We see that the second together with the third term represent the negative *substitution* effect.

We can also analyze the components of $U^l_{a_1}(a_1^{0*}, \hat{a}_2)$ to determine the size of each effect and of the overall effect. The first bracket captures the effect of the riskiness of the loss risk on marginal utility of the first instrument. In case of saving and self-protection it is positively associated with the intensity of prudence as measured by the coefficient of absolute prudence (Kimball 1990), in case of self-insurance it is the curvature of v as measured by its second derivative that exerts a positive effect on it. The difference in marginal consumption utility today depends on the curvature of first-period utility and the pure change in the other instrument due to the riskiness of the loss risk. Put differently, the better the other instrument in isolation is suited to react to risk, the higher will be the marginal cost of using the instrument under consideration. The last bracket again depends on the effectiveness

[10] We obtain the second equation by adding and subtracting $V^l_1(a_1^{0*}, a_2^{0*})$ and by adding the first-order condition for the optimal use of the first instrument under the first-order effect of the loss risk.

of the other instrument when it comes to addressing riskiness of future consumption coupled with the degree of substitution between the two instruments.

The last statement in Proposition 1 follows by analyzing U^l in the (a_1, a_2)-plane. We take the directional derivative into the direction $\nu = (\nu_1, \nu_2) \in \mathbb{R}^2$ and evaluate it at (a_1^{0*}, a_2^{0*}):

$$\begin{aligned}\partial_\nu U^l(a_1^{0*}, a_2^{0*}) &= \nu_1 U_1^l(a_1^{0*}, a_2^{0*}) + \nu_2 U^l(a_1^{0*}, a_2^{0*}) \\ &= \nu_1 \beta \left[V_1^l(a_1^{0*}, a_2^{0*}) - V_1^0(a_1^{0*}, a_2^{0*})\right] + \nu_2 \beta \left[V_2^l(a_1^{0*}, a_2^{0*}) - V_2^0(a_1^{0*}, a_2^{0*})\right].\end{aligned}$$

The last equation follows by using the first-order conditions from the problem where the loss risk is replaced by its mean. Due to the assumption that the marginal expected utility benefit of each instrument is convex in wealth, both bracketed expressions are positive. As a result, not both ν_1 and ν_2 can be negative when moving towards the new equilibrium (a_1^{l*}, a_2^{l*}). Consequently, at most one of the two substitution effects can prevail at the new optimum. Figure 1 illustrates this.

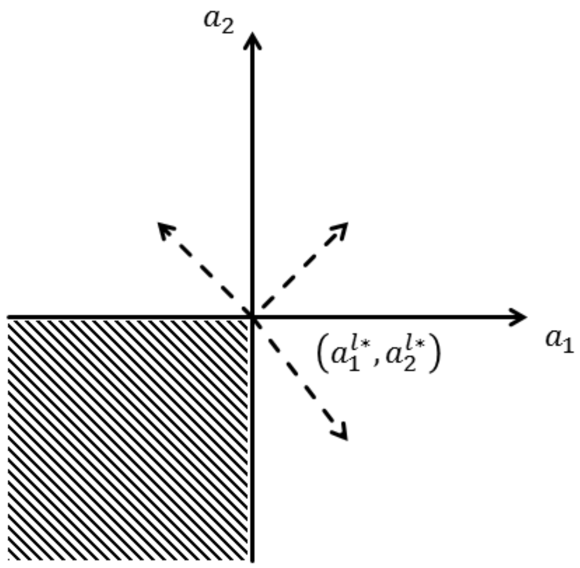

Figure 1: Illustration of possible reactions to the introduction of a future income risk

The next step is to introduce the income risk. The agent chooses the levels (a_1^{li*}, a_2^{li*}) maximizing

$$U^{li}(a_1, a_2) = u(y - a_1 - a_2) + \beta V^{li}(a_1, a_2)$$

where superscript li indicates the presence of both loss and income risk. V^{li} is expected second-period consumption utility where the expectation operator is taken with respect to the joint distribution of loss and income risk.[11] The first-order conditions arise, for $j = 1, 2$, as

$$-u'(y - a_1 - a_2) + \beta V^{li}_{a_j}(a_1, a_2) = 0. \tag{5}$$

The intuition is as before: at the optimum a marginal dollar has the same effect on intertemporal expected consumption utility whichever instrument is used. Also here instruments will, in general, interact in their use implying the presence of indirect effects. Due to the structural similarity we obtain the following corollary from Proposition 1.

Corollary 1 *Assume prudence and $V^{li}_{a_1 a_2} < 0$. Then, the income risk has on each instrument a positive* precautionary *effect and a negative* substitution *effect. At least one instrument will be used more.*

It is worthwhile to note that the negative cross-derivative of V^{li} as required for Corollary 1 is obtained from the negative cross-derivative of V^l in Proposition 1. The reason is that the income risk is assumed to be independent of the loss risk. Remark 2 states that this assumption holds in all three two-instrument cases.

Remark 2 *Proposition 1 and Corollary 1 apply in all three two-instrument cases.*

We renounce in this paper from stating a large set of theoretical comparative statics. While all single effects we obtain are intuitive, the overall effects are, in general, ambiguous. In Section 5, we study rather the risk reactions in the different settings numerically for a median agent. Preparing these quantitative comparisons, we extend now Definition 1 to the case with two instruments. The definitions of the loss-risk, the income-risk, and the total precautionary motive for one specific instrument naturally apply also in the two-instrument cases. We add the following concepts.

Definition 2 *For instruments a_j, $j = 1, 2$, with $a_j \in \{s, x, z\}$ and $a_1 \neq a_2$, the*

- Loss-Risk Precautionary Motive *(LPM) is the sum of the loss-risk precautionary motives in the two instruments, $LPM = LPM_{a_1} + LPM_{a_2}$;*
- Income-Risk Precautionary Motive *(IPM) is the sum of the income-risk precautionary motives in the two instruments, $IPM = IPM_{a_1} + IPM_{a_2}$;*
- Total Precautionary Motive *(TPM) arises, equivalently, as the sum of the loss-risk and income-risk precautionary motives across the two instruments or the sum of the total precautionary motives of the two instruments, $TPM = LPM + IPM = TPM_{a_1} + TPM_{a_2}$;*

[11] Due to the assumption of independence between the two risks, it is equal to the product of the two marginal distributions.

- share *that instrument a_j contributes to the TPM is, for $TPM > 0$, $sh_{a_j} = \frac{TPM_{a_j}}{TPM}$.*

We visualize the definitions for the precautionary motives in the two-instrument cases in Table 1. Obviously, across the instruments the shares add up to one, $\sum_{j=1}^{2} sh_{a_j}^{TPM} = 1$.

Table 1: Overview of Precautionary Motives

	Instrument 1	Instrument 2	Both Instruments
Loss Risk	LPM_{a_1}	LPM_{a_2}	LPM
Income Risk	IPM_{a_1}	IPM_{a_2}	IPM
Total Risk	TPM_{a_1}	TPM_{a_2}	TPM

Using these concepts, Proposition 1 and Corollary 1 have the following

Corollary 2 *Under prudence, the shares $sh_{a_j}^{TPM}$, $j = 1, 2$, in the total precautionary motive cannot be zero simultaneously for all instruments. Given a positive (negative) TPM, the share $sh_{a_j}^{TPM}$ is negative (positive) for instrument a_j if and only if the substitution effect prevails in that instrument.*

We finally turn to the question whether and to what extent the possibility of precautionary choices in a second instrument leads to systematic increases or decreases in precautionary choices in the first instrument. This has important implications for empirical identification strategies and the measurement of the strength of precautionary motives. If risks are endogenous, *e.g.*, via self-protection or self-insurance, but are treated as exogenous due to the lack of good proxies for individual measures of risk mitigation, this may lead to the systematic over- or underprediction of other precautionary choices. This in turn would imply that predictions about individual preference parameters deviate systematically from their true values. To achieve comparability, we compare the precautionary choice in one instrument in a setting with second instrument to a setting in which this second instrument is exogenous but the latter is fixed at the level it had in the endogenous case when the zero-mean income risk is present. This allows us to isolate the effect of neglecting the endogeneity of the second instrument on the choice of the first. The following proposition summarizes our results.

Proposition 2 *Treating the second instrument as exogenous leads to overestimating the first if and only if the expenditures on the second increase due to the introduction of a zero-mean income risk.*

The intuition is that when the second instrument is productive, in the sense that it is desirable for the decision maker to invest more in this instrument as a reaction to the zero-mean income risk, then the agent will depend less on precautionary choices in the first instrument and its level will be lower, compared to a situation in which the first instrument is indeed the only decision. Consequently, neglecting other endogenous determinants of risk exposure leads us to theoretically overestimate the precautionary response in the first instrument. For example, if in addition to saving there is self-protection or self-insurance as a second instrument, mere inference from consumption data can provide unrealistically low intensities of prudence.

4 Case with Three Instruments

The extension of the analysis for two instruments to three and more is formally straightforward. This section treats the case with all three instruments we consider in this paper. We discuss with respect to this case the arising particularities when there are more than two instruments.[12] If saving, self-protection, and self-insurance are available to optimize intertemporal consumption, the agent's problem reads

$$\max_{s,x,z} \left\{ u(y-s-x-z) + \beta \left[p(x)v(Y+s(1+r)-L(z)) + (1-p(x))v(Y+s(1+r)) \right] \right\},$$

with associated first-order conditions

$$\begin{aligned}
U^l_s &= -u'(c) + \beta(1+r)\left[p(x)v'(Y+s(1+r)-L(z)) + (1-p(x))v'(Y+s(1+r))\right] \;=\; 0,\\
U^l_x &= -u'(c) + \beta p'(x)\left[v(Y+s(1+r)-L(z)) - v(Y+s(1+r))\right] \;=\; 0,\\
U^l_z &= -u'(c) - \beta p(x)L'(z)v'(Y+s(1+r)-L(z)) \;=\; 0.
\end{aligned}$$

In terms of notation we use the same conventions as above. We state the analog of Proposition 1 about the precautionary motives associated with the riskiness of the loss risk in

Proposition 3 *Assume prudence and a probability of loss not above* 0.5. *Then, the riskiness of the loss risk generates three effects. On each instrument, there is a positive* precautionary *effect and two* substitution *effects, of which at least one is negative. At least one of the three instruments will be used more compared to the situation where only the first-order effect is considered.*

The analog of Corollary 1 is straightforward. To demonstrate Proposition 3, let us select saving. As before we evaluate U^l at s^{0*} and solve for the other two instruments via $\max_{x,z} U^l(s^{0*}, x, z)$. We denote the solutions $\hat{x}$ and $\hat{z}$. In order to compare s^{0*} to s^{l*}, evaluate

$$\begin{aligned}
U^l_s(s^{0*}, \hat{x}, \hat{z}) &= -u'(y - s^{0*} - \hat{x} - \hat{z}) + \beta V^l_s(s^{0*}, \hat{x}, \hat{z})\\
&= \beta \left[V^l_s(s^{0*}, x^{0*}, z^{0*}) - V^0_s(s^{0*}, x^{0*}, z^{0*}) \right] - u'(y - s^{0*} - \hat{x} - \hat{z}) + u'(y - s^{0*} - x^{0*} - z^{0*})
\end{aligned}$$

[12]Menegatti and Rebessi (2011) study the three instruments saving, self-protection, and (market) insurance, but do not consider future income risk or precautionary choices.

$$
\begin{aligned}
&+\beta\left[V_s^l(s^{0*},\hat{x},\hat{z})-V_s^l(s^{0*},x^{0*},z^{0*})\right]\\
=\;&\beta\left[V_s^l(s^{0*},x^{0*},z^{0*})-V_s^0(s^{0*},x^{0*},z^{0*})\right]\\
&-u'(y-s^{0*}-\hat{x}-\hat{z})+u'(y-s^{0*}-x^{0*}-\hat{z})+\beta\left[V_s^l(s^{0*},\hat{x},\hat{z})-V_s^l(s^{0*},x^{0*},\hat{z})\right]\\
&-u'(y-s^{0*}-x^{0*}-\hat{z})+u'(y-s^{0*}-x^{0*}-z^{0*})+\beta\left[V_s^l(s^{0*},x^{0*},\hat{z})-V_s^l(s^{0*},x^{0*},z^{0*})\right]
\end{aligned}
$$

The first equation is obtained by adding the first-order condition for optimal saving in the presence of the first-order effect only and by adding and subtracting $\beta V_s^l(s^{0*},x^{0*},z^{0*})$. The second equation follows by adding and subtracting $u'(y-s^{0*}-x^{0*}-\hat{z})$ and $V_s^l(s^{0*},x^{0*},\hat{z})$. The first term is the positive *precautionary* effect, the second and third term represent the first *substitution* effect, whereas the fourth and fifth term represent the second *substitution* effect. We know from Section 3 that, when looking at the pure change in self-protection and self-insurance, at least one instrument will be used more to react to the riskiness of the loss risk. Consequently, either $\hat{x}>x^{0*}$, or $\hat{z}>z^{0*}$, or both so that at least one substitution effect is negative. The fact that out of the three instruments at least one will be used more to address the riskiness of the loss risk can be obtained by inspecting the directional derivative of U^l in the direction $\nu=(\nu_1,\nu_2,\nu_3)\in\mathbb{R}^3$ in the (s,x,z)-plane.

When all three instruments can be used, also the effect of a precautionary response in one instrument to some additional risk to second-period consumption on the cross-effect between the two other instruments can be considered. The cross-effects are described by the cross-derivatives of the utility objective with respect to the two other variables as in system (8). Clearly, all instruments are mutual substitutes. In each case, the effect from a positive precautionary response in one variable on the cross-effect can be seen from the derivative of these cross derivatives with respect to the third decision variable:

$$
U_{sxz}^{li}=-u'''(c)-\beta(1+r)\,p'(x)\,L'(z)\,Ev''(\tilde{Y}+s(1+r)-L(z)) \tag{6}
$$

The effect from a positive precautionary response on the cross-effects on the marginal cost of the other instruments is negative, while its effect on the cross-effect between the marginal benefit of the other instruments is positive. The prevalence of one of these two effects determines the sign of derivative (6). Remark 3 states the arising endogenous condition.

Remark 3 *A positive precautionary response in the third instrument will strengthen (weaken) the cross-effect between the first and second variables if and only if Ross prudence of present consumption is sufficiently low (high) in the following sense:*

$$
-\frac{u'''(c)}{u'(c)}\;<(>)\;\beta(1+r)p'(x)L'(z)\frac{Ev''(\tilde{Y}+s(1+r)-L(z))}{Ev'(\tilde{Y}+s(1+r)-L(z))}\cdot\frac{Ev'(\tilde{Y}+s(1+r)-L(z))}{u'(c)}\;. \tag{7}
$$

The intuition for this finding is straightforward. The substitution effect between two instruments leads to lower precautionary responses than when each instrument is considered in isolation. If (Ross) prudence is very high,[13] the presence of the third instrument weakens the cross-effect between the two so that it is less likely that the substitution effect dominates. In this sense, high intensities of prudence give less room for weak or even negative precautionary responses in multiple-instrument situations.

Finally, we turn again to the question whether and to what extent the possibility to use other risk-management instruments leads to systematic increases or decreases in precautionary saving. The following proposition summarizes our results.

Proposition 4 *Treating the risk of loss as exogenous implies an overestimation of precautionary saving if and only if the sum of the expenditures on self-protection and self-insurance increases due to the introduction of a zero-mean income risk.*

Similar to the case with two decisions, it is apparent that the precautionary saving response will depend on the use of the other two instruments at hand. Consequently, these choices need to be fully identified to correctly quantify the overall precautionary motive.

5 Numerical Analysis

We study now numerically the precautionary motives and their interaction among the different risk-mitigation instruments from above. The goal is to quantify and compare the proportions the precautionary choices in the different instruments take in settings with one or more instruments. To analyze the determinants, we consider the comparative statics of the risk and technology parameters on the optimal choices of a median subject we define below.[14]

For the numerical analysis, we need to operationalize the decision maker's preferences, the technologies to influence the loss risk, and the income risk, and adopt appropriate parameter values. Regarding preferences, we consider Expected Utility in this paper. In that case, utility curvature represents mostly intertemporal, not risk, preferences, whence risk responses are somewhat stronger than under a utility specification that disentangles preferences in the risk and time domains (Bostian and Heinzel 2014). We use expo-power utility to specify preferences (Saha 1993, Holt and Laury 2002),

$$f(c) = \frac{1}{\alpha}\left[1 - \exp\left(-\alpha \cdot \frac{c^{1-\rho}}{1-\rho}\right)\right]$$

[13] $-\frac{u'''(.)}{u'(.)}$ as a coefficient of prudence has been treated by Modica and Scarsini (2005). The present terminology has been introduced by Denuit and Eeckhoudt (2010), who generalize coefficients of higher-order risk aversion that carry $u'(.)$ in the denominator to the nth order.

[14] Bostian and Heinzel (2014) study influences of preference and financial parameters on precautionary saving choices. The corresponding comparative-statics exercises would yield similar results here.

The expo-power form has the ability to capture increasing, decreasing, or constant shapes of the relative resistance to intertemporal substitution, and exhibits a decreasing absolute resistance to intertemporal substitution (except in the edge case of $\rho = 0$, where it is constant).[15]

For the self-protection and self-insurance technologies there is no standard modeling in the literature. In coherence with the assumptions above, we specify the self-protection technology as:

$$p(x) = p_0\, e^{-\mu x}$$

where $p_0 \in (0, 1)$ is the baseline probability of loss L and $\mu > 0$ is an efficiency parameter. For the self-insurance technology, we assume

$$L(z) = L_0\, e^{-\nu z}$$

where $L_0 < Y + s(1+r)$ is the baseline level of the potential loss and $\nu > 0$ is an efficiency parameter.

We implement the zero-mean risk on future income as mean-preserving spreads around the certain baseline level of Y using a factor $\delta \in (0, 1)$, which shrinks or increases Y by multiplication with $(1 - \delta)$ or $(1 + \delta)$, respectively, with probability $1/2$.

For ease of presentation, we consider a one-month time horizon between the decision and the realization of the risk. For longer time horizons, the effects will be qualitatively the same. We assume that the per-period felicity function does not change during this period, so that $u \equiv v$. Our baseline values for the utility and economic parameters are

$$\alpha = 3 \qquad \rho = 1.5 \qquad \beta = 0.99 \qquad y = Y = \$1,000 \qquad r = 0.01$$

The ρ parameter corresponds to typical empirical macroeconomic estimates, and the positive α adds a decreasing shape to the relative resistance to intertemporal substitution (increasing elasticity of intertemporal substitution) (*e.g.*, Meyer and Meyer 2005). We set the discount factor β to 0.99 (corresponding to a monthly discount rate of 1.01 percent and an annual rate of 12.82 percent) (*e.g.*, Andersen et al. 2008). The baseline income of $\$1,000$ is on the order of the monthly income for the first and second quintiles of the US population. The baseline return r to saving of 1 percent gives a fairly strong incentive to save.

[15] Its coefficients of absolute and relative resistance to intertemporal substitution are, respectively,

$$ARIS\,(c) = \frac{\alpha}{c^{\rho}} + \frac{\rho}{c}\,, \quad RRIS\,(c) = \alpha c^{1-\rho} + \rho$$

Excluding pathological parametrizations, the expo-power function has derivatives with alternating sign.

For the baseline risk and technology parameters, we assume

$$L_0 = \$250, \qquad p_0 = 0.5, \qquad \mu = \nu = 0.2, \qquad \delta = 0.3$$

The size of L_0 can be interpreted in relation to the assumed monthly income of \$1,000. Thus, the baseline loss risk we consider posits that the agent will suffer with a chance of one in two a loss on the order of a quarter of monthly income. For the efficiency parameters, we are not aware of studies that could give empirical support for one or another value. We choose the values of μ and ν so that coherent results arise across all scenarios we consider based on the modeling framework in this paper. We set them equal so as to allows to detect eventual differences in their comparative effectiveness. δ measures directly fractions of the monthly income. Income fluctuations on the order of 30 percent certainly correspond to the normal experience, for example, of self-employed persons, including small farmers or freelance workers in the service industry.

5.1 One Instrument and Two Risks

We study first the comparative statics of the precautionary motives in each of the three instruments separately. The theoretical literature has thus far focused on these single-instrument cases. To the best of our knowledge, past empirical or numerical studies have only concerned precautionary saving. Our saving case is still different from the typical approach in the literature, because we consider a setting where, with the potential loss, an additional risk is present. We start with the saving case as a benchmark and compare subsequently the cases of self-protection and self-insurance.

Saving

Figure 2 shows the comparative statics of the precautionary saving motives in reponse to the loss risk and the additional income risk as well as the total precautionary motive for $L \in (0, 800]$, $p \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $r \in (0, 1]$. The precautionary motives are represented as the precautionary fraction of the total choice in this instrument.[16] The two upper graphs show that, for a given income risk, increases in L or p_0 expand the share of the LPM_s in the TPM_s at the expense of the share of the IPM_s. In addition, due to the related first-order effect, the TPM_s tends to shrink in favor of consumption smoothing. For the considered parameter ranges, the first-order effect is stronger in the loss size than in its probability. Moreover, increases in loss size stimulate the LPM_s more strongly than increases in the loss probability. For the baseline parameter values, the IPM_s is clearly larger

[16]Note the slight abuse of notation. Whereas the precautionary motives are measured in Sections 2–4 in currency units, we give them here as percentages for ease of illustration and interpretation.

than the LPM_s. For example, if L or p_0 tend towards zero, the IPM_s is at about 0.9, whereas, if δ is zero, the LPM_s is at about 0.16. However, the LPM_s may exceed the IPM_s if the loss size is large enough. δ adds a pure risk to the decision maker's choice environment. Its expansion increases the IPM_s at the expense of both the LPM_s and consumption smoothing. The graph at the bottom-right shows the comparative statics is for a monthly interest rate of up to 100 percent. An expansion of r leaves the shares of LPM_s and IPM_s in the TPM_s unchanged, but the TPM_s decreases in favor of consumption smoothing due to the related first-order effect.

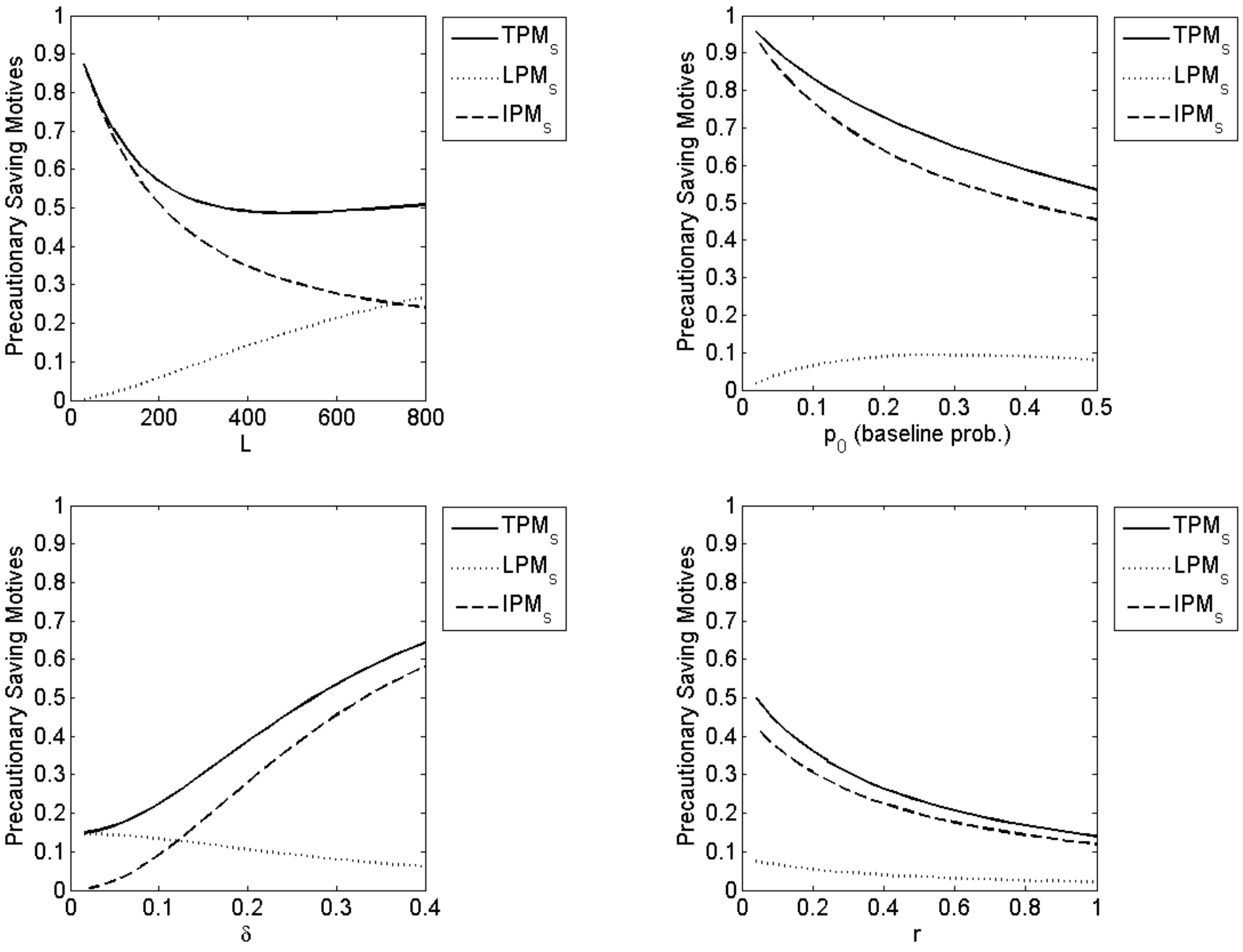

Figure 2: Comparative statics of precautionary saving motive for $L \in (0, 800]$, $p \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $r \in (0, 1]$.

Self-Protection

If self-protection is the agent's only instrument, the precautionary motives in response to the two risks are similarly operative (Figure 3). But the fractions of the loss-risk and income-risk as well as the total precautionary motives in the total choice are, in general, much lower than in the case with saving only. Thus, for L towards zero the IPM_x is at about 0.22, and for δ towards zero the LPM_x is at about 0.08. For the most part of the considered parameter ranges, the TPM_x stays below 0.25.

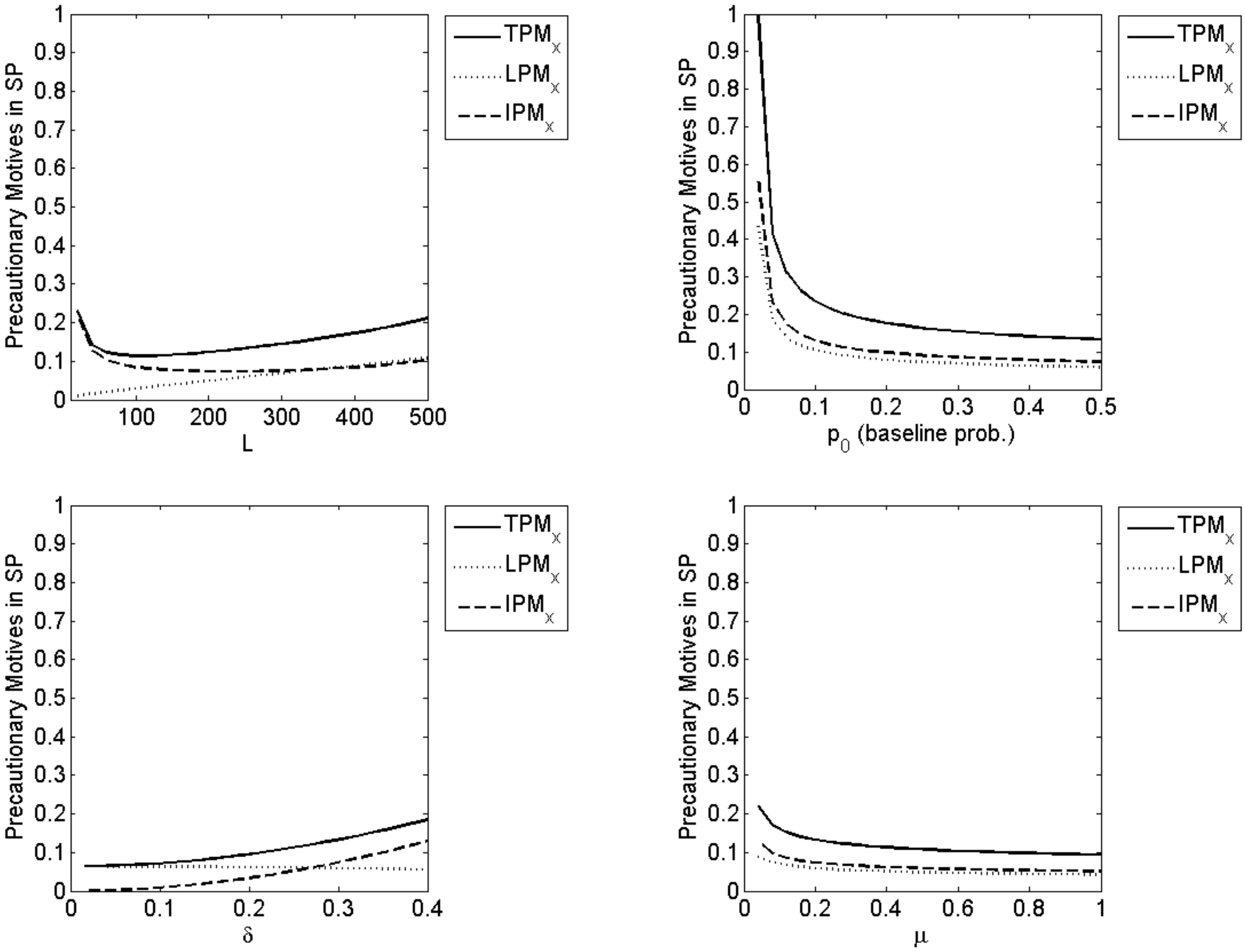

Figure 3: Comparative statics of precautionary motive in x for $L \in (0, 500]$, $p_0 \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $\mu \in (0, 1]$.

Self-protection seems thus, in general, much more effective to cope with the loss and income risks than saving. An exception is the case of small baseline loss probabilities ($p_0 < 0.05$), where with decreasing p_0 also the LPM_x rises and the TPM_x can reach 100 percent. In general, the LPM_x is smaller than the IPM_x, but may exceed the IPM_x for L large enough or δ small enough. The graphs show only very few interaction between the risk-wise precautionary motives. Notably, δ has not a large influence on the LPM_x. An increase in the efficiency of self-protection μ reduces the TPM_x, decreasing similarly LPM_x and IPM_x.

Self-Insurance

Also if the agent disposes only of self-insurance, both the loss- and the income-risk precautionary motives are active (Figure 4). However, the total precautionary motive is still smaller than in the case with self-protection only. While the IPM_x is at about 0.22 when the baseline loss tends towards zero, the LPM_z remains tiny unless for loss probabilities $p < 0.2$. Self-insurance occurs thus as a most effective instrument to cope with the loss risk and, to a lesser degree, also with the income risk. By implication, self-insurance is mostly used to smooth consumption. Similar to μ in the self-protection case, ν has a monotonically decreasing effect on the precautionary motives. The share of the LPM_z in the TPM_z tends towards zero fairly quickly in favor of the IPM_z.

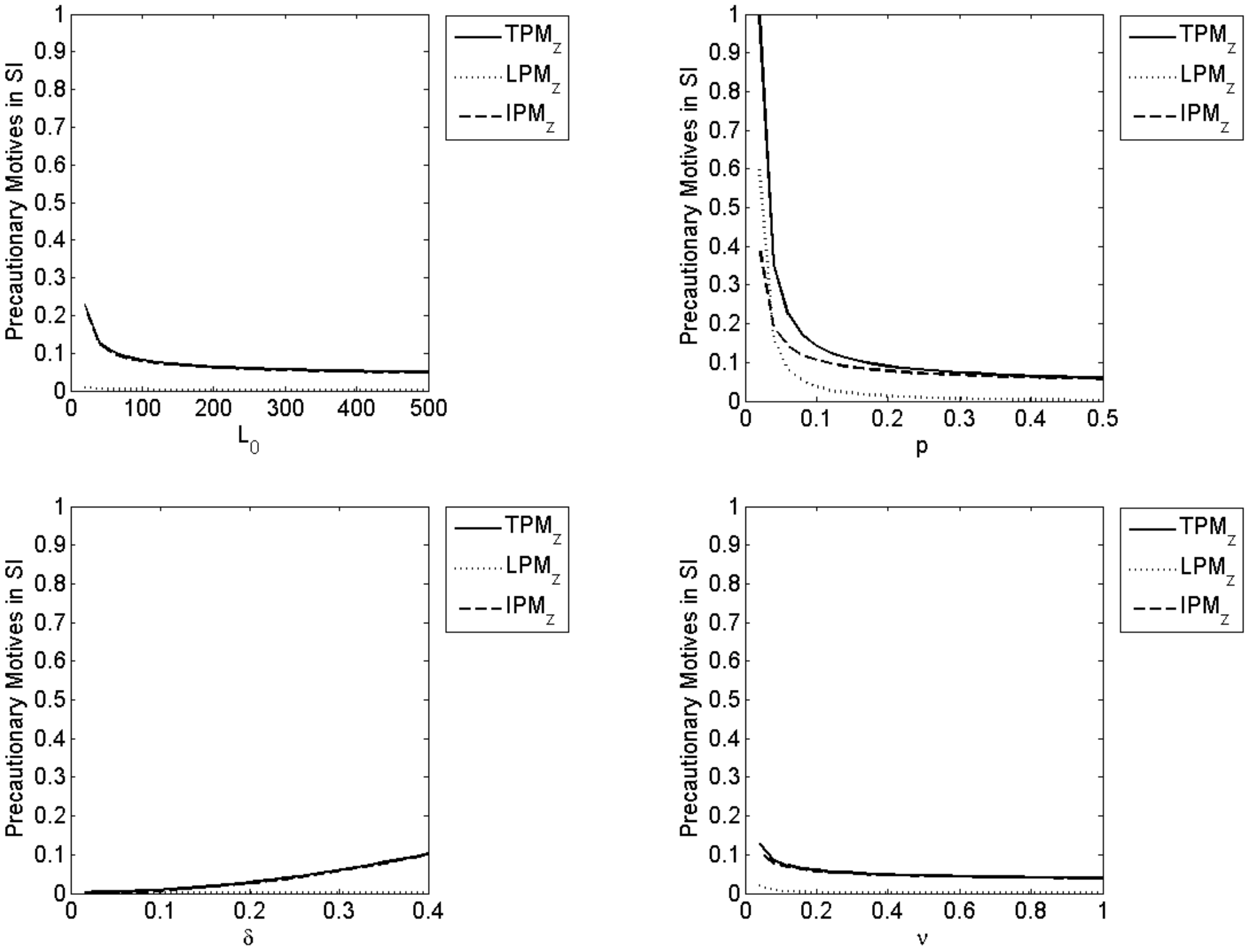

Figure 4: Comparative statics of precautionary motive in z for $L_0 \in (0, 500]$, $p \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $\mu \in (0, 1]$.

5.2 Instrument Interaction

From a real-world perspective, assuming that an agent disposes only of one instrument to cope with future risks occurs as stylized. The graphs in this section present the TPM_a for each instrument $a \in \{s, x, z\}$ and the TPM across all instruments. We also comment on the risk-wise precautionary motives and show the LPM_a, if of particular interest. (The IPM_a is then implicit as $TPM_a - LPM_a$.)

Two Instruments

Because the precautionary motives are generally lower than in the single-instrument cases, we reduce the scale of the precautionary motives in the graphs for two instruments to the range of $[-0.05, 0.2]$.

Saving and Self-Protection. Figure 5 shows the comparative statics of the precautionary motives for $L \in (0, 600]$, $p_0 \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $\mu \in (0, 1]$. Generally, the precautionary motives are much smaller than in the cases with saving or self-protection only. Consistent with the single-instrument cases, also here the total as well as the risk- and instrument-wise precautionary motives decrease with the baseline loss probability and the efficiency parameter μ and increase with δ. However, in contrast to before, the IPM_s and IPM_x as well as, for L above about \$100, the TPM, TPM_s, and TPM_x rise with the loss size. At the same time, for a high enough loss size the LPM_x crowds out the LPM_s and, thus, precautionary saving. A similar instrument interaction can be observed in the comparative statics for δ and μ. An increase in δ makes the IPM_s and IPM_x and the TPM_s and TPM_x consistently rise, slightly more for self-protection than for saving, whereas the LPM_s and LPM_x shrink slightly. Increases in μ consistently decrease the precautionary motive across all instruments and risk sources. More efficient self-protection reduces precautionary saving more strongly than precautionary self-protection investments.

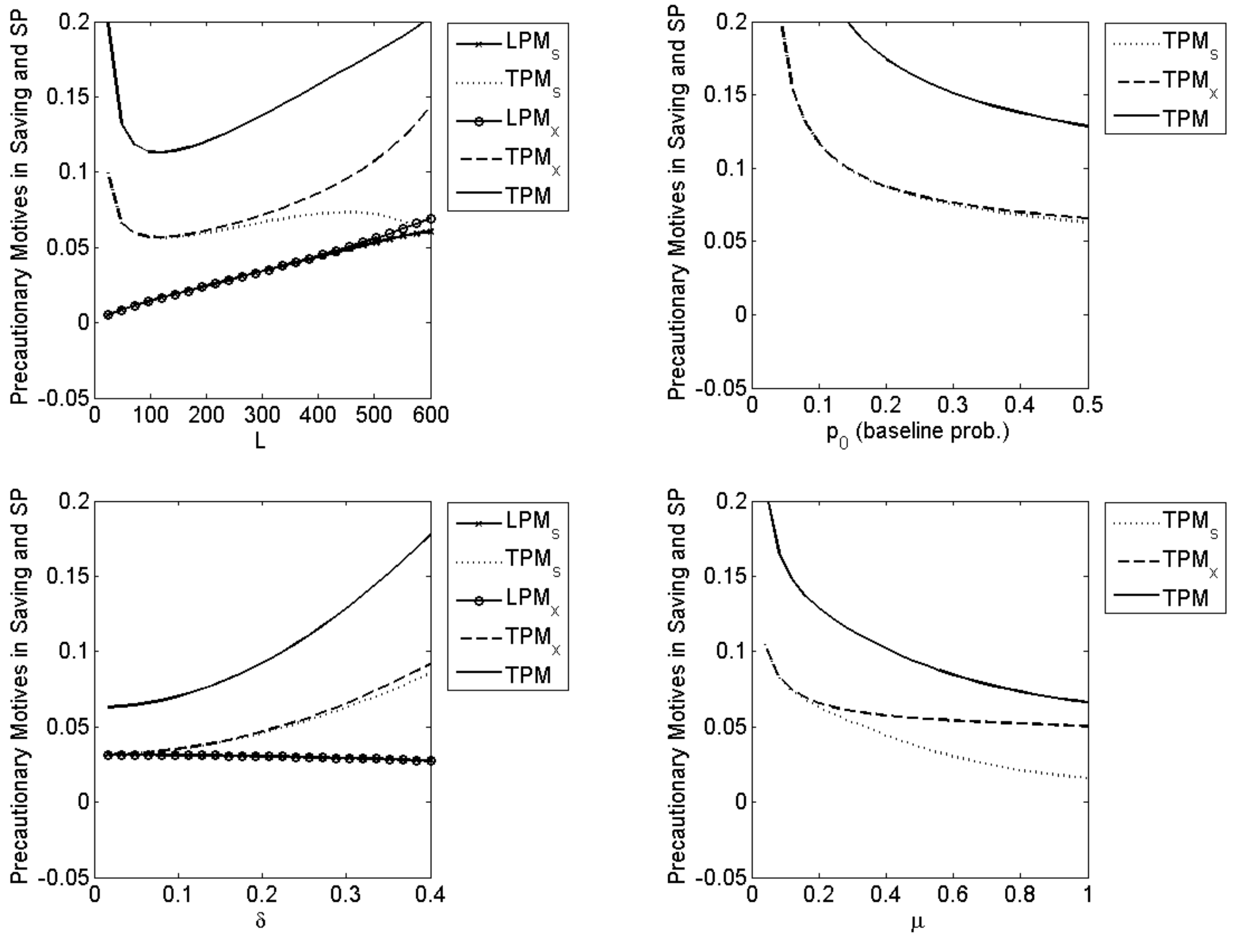

Figure 5: Comparative statics of precautionary motives for $L \in (0, 600]$, $p_0 \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $\mu \in (0, 1]$.

Saving and Self-Insurance. Under saving and self-insurance (Figure 6), most effects resemble the case with saving and self-protection, but they are typically more pronounced and involve, in general, lower precautionary choices. Contrary to the previous case, with an increase in loss size now all precautionary motives consistently decrease. For L high enough, precautionary investments in self-insurance crowd out precautionary saving. At a loss size of around \$340, the TPM_s becomes even negative. Similarly, if the efficiency parameter ν is above 0.4, the TPM_s turns negative. In the δ comparative statics, the TPM response is for the most part carried by the IPM_z and only to some extent by IPM_s. The LPM_z is consistently close to zero, and the LPM_s is also slightly negative.

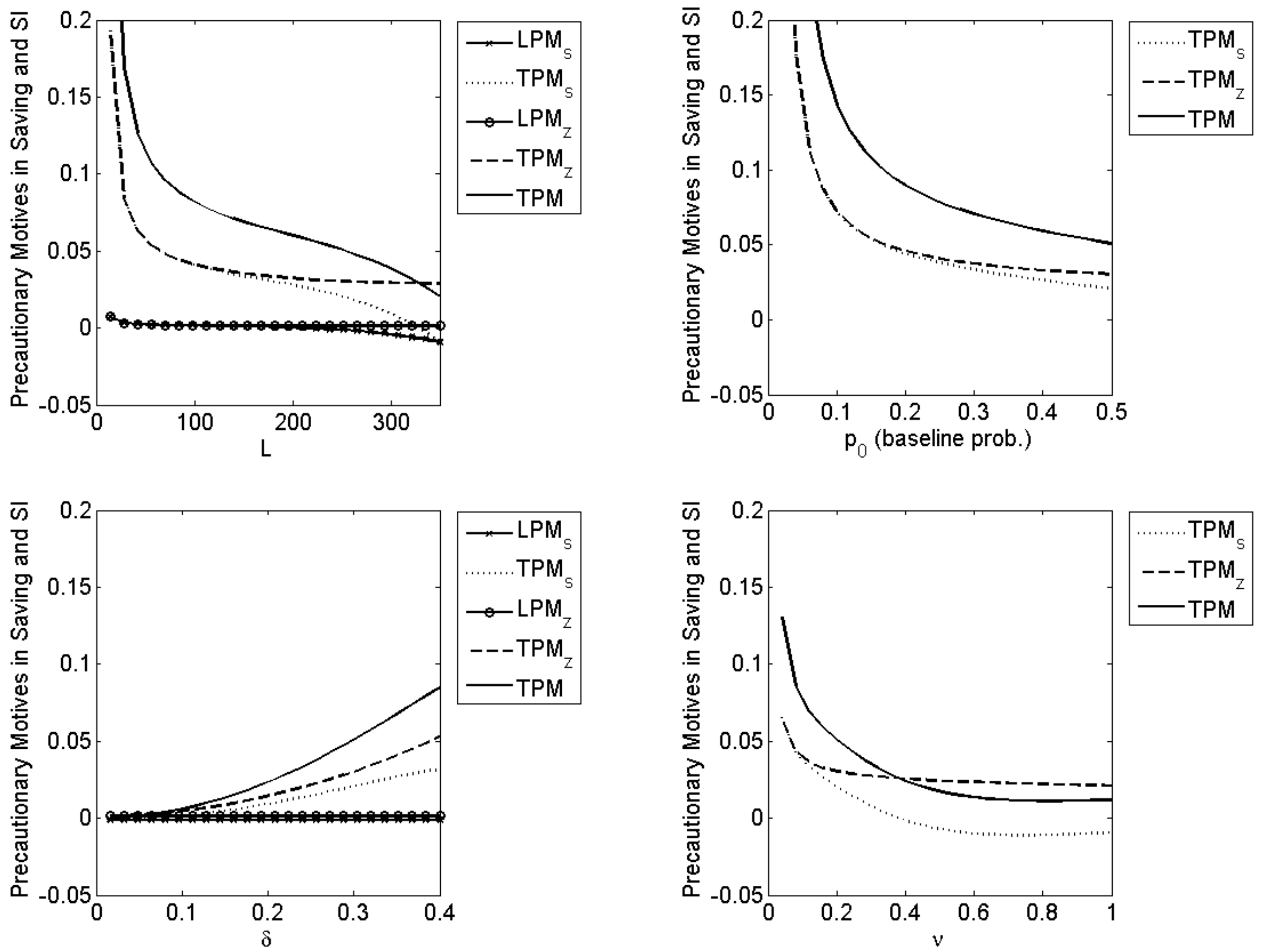

Figure 6: Comparative statics of precautionary motives for $L_0 \in (0, 350]$, $p \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $\nu \in (0, 1]$.

Self-Protection and Self-Insurance. The graphs for the case with self-protection and self-insurance (Figure 7) resemble the case with saving and self-insurance, with some exceptions. The comparative statics in the loss size exhibits a strong crowding out of the precautionary investments in self-protection by those in self-insurance. For L_0 above \$200, the IPM_x and the LPM_x start to shrink, whereas the IPM_z and the LPM_z rise. In both instruments, the former effect is stronger than the latter. At a loss size of about \$480, the TPM_x turns negative. Self-insurance seems, thus, clearly more effective than self-protection for precautionary purposes. Increases in the baseline loss probability make all precautionary motives monotonically decrease. When δ increases, the rise of the TPM is again mostly carried by the IPM_x and the IPM_z. The LPM_x and the LPM_z are very small but stay positive. The efficiency parameter of self-protection μ has only a very mild influence on the precautionary motives. For about $\mu > 1.2$, the LPM_z decreases and with it the TPM. All precautionary motives stay positive. An increase of ν decreases clearly the TPM by reducing the LPM_x from about 0.04 to almost zero, the IPM_x is slightly negative from ν of about 0.4 on.

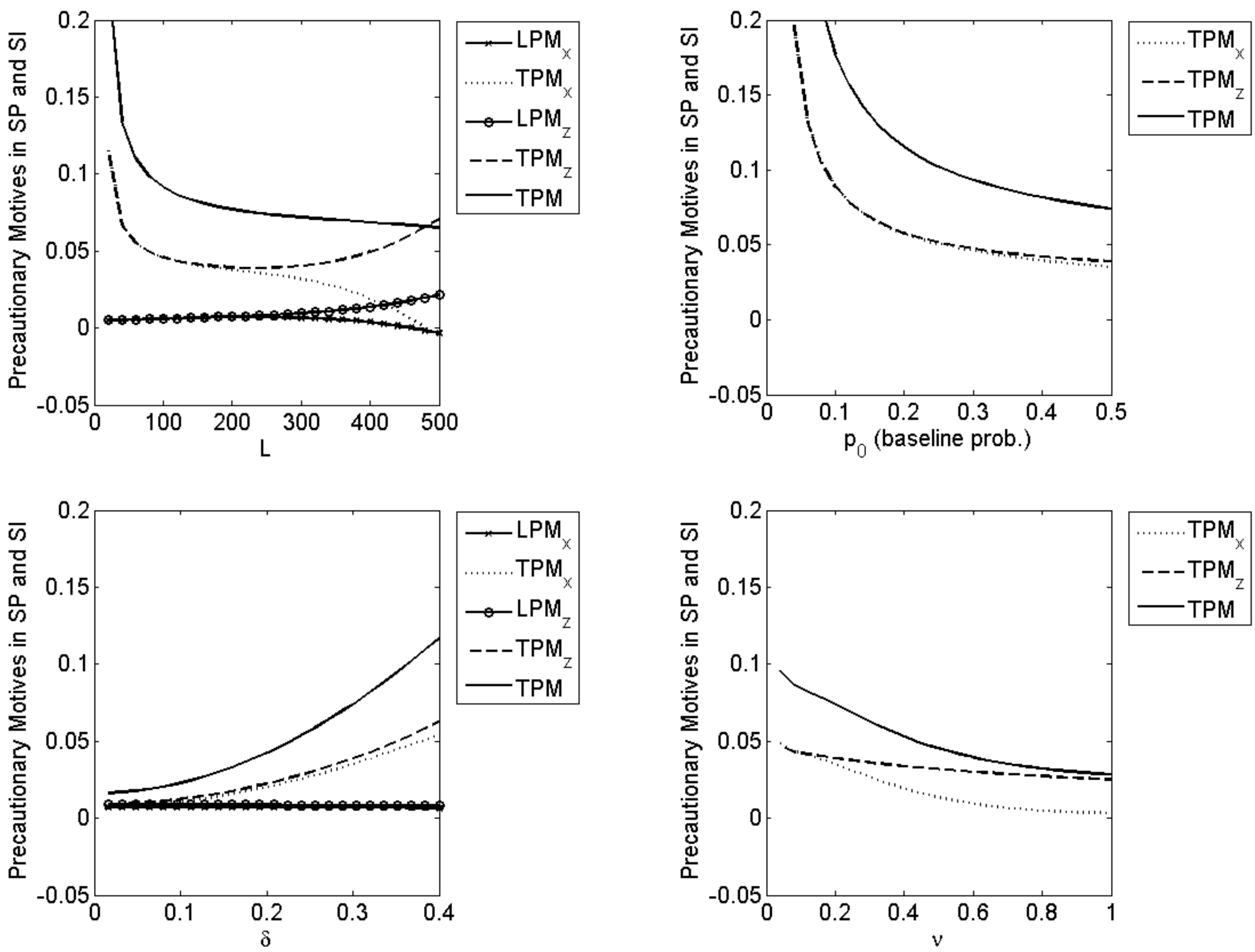

Figure 7: Comparative statics of precautionary motives for $L_0 \in (0, 500]$, $p_0 \in (0, 0.5]$, $\delta \in (0, 0.4]$, and $\nu \in (0, 1]$.

Saving, Self-Protection, and Self-Insurance

Figures 8-10 show the comparative statics of the precautionary motives in the three instruments for $L_0 \in (0, 500]$, $p_0 \in (0, 0.5]$, $\mu \in (0, 1]$, and $\nu \in (0, 1]$. For a better summary impression, we add graphs with the shares that the instruments contribute to the TPM. Because the precautionary motives are generally lower than in the two-instrument cases, we reduce the scale of the precautionary motives in the graphs, in general, to the range of $[-0.05, 0.15]$. Similar to the cases with self-insurance before, the comparative statics in the loss size exhibit a monotonically decreasing shape of the TPM. Now, investments in self-insurance crowd out both saving and self-protection, the former more strongly than the latter. At a loss size of \$500 even both the TPM_s and the TPM_x are negative, with both of their components. The shares in the TPM show how strongly self-insurance appropriates the precautionary reaction at the expense first of saving, but also of self-insurance. The comparative statics in the loss probability show the same consistently declining shape of all precautionary motives as in all cases with endogenous risk before. The crowding among the three instruments has the same tendency as under loss-size increases but is much less pronounced.

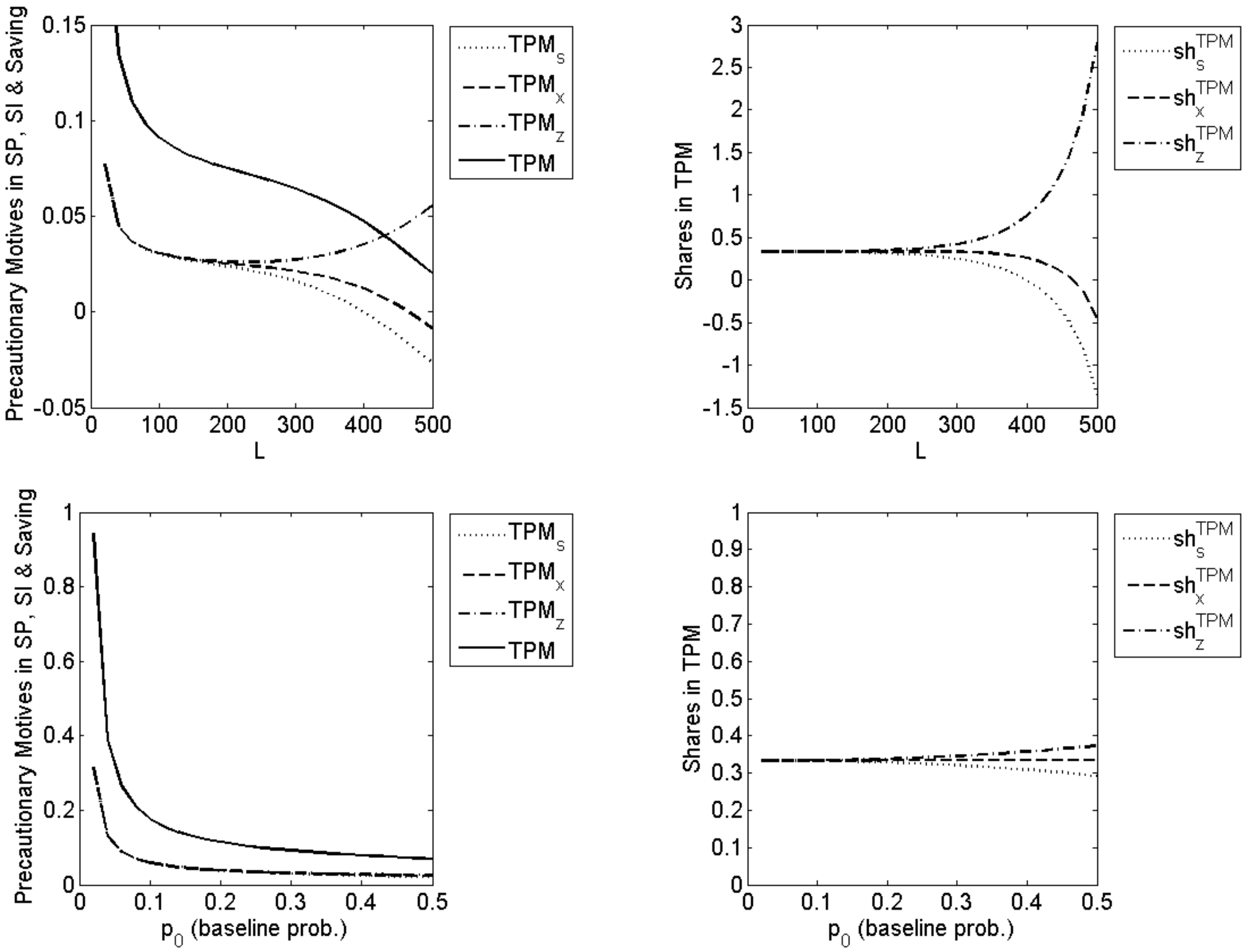

Figure 8: Comparative statics of precautionary motives and shares in TPM for $L_0 \in (0, 500]$ and $p_0 \in (0, 0.5]$.

The comparative statics in δ is very similar to the case with self-protection and self-insurance. However, the TPM are slightly, and the TPM_x and the TPM_z are clearly lower. Again, most precautionary reaction is taken up by the IPM_a and the LPM_a for $a \in \{s, x, z\}$ stay virtually unchanged in the considered range of δ. At the same time, the shares in the TPM stay mostly the same for all three instruments.

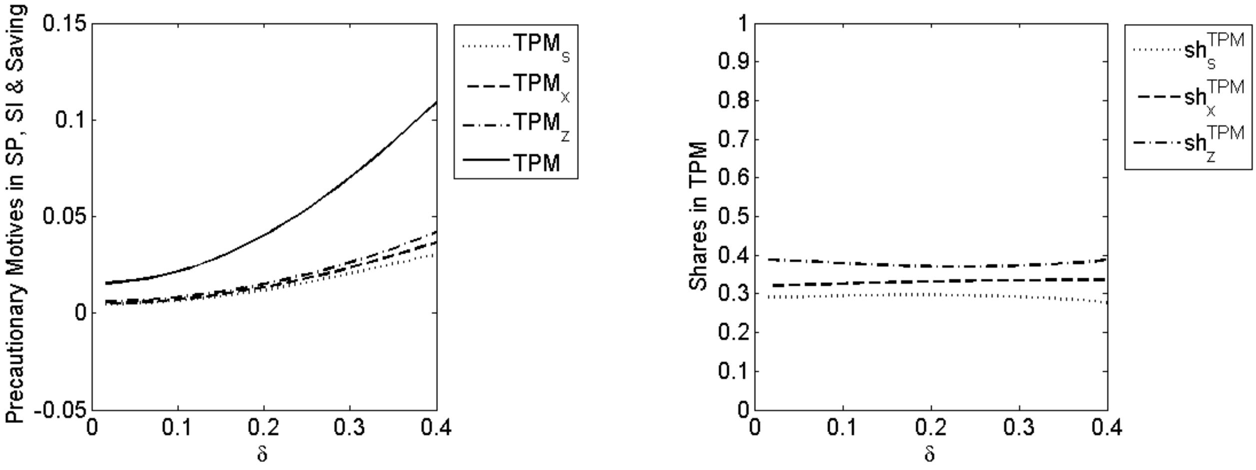

Figure 9: Comparative statics of precautionary motives and shares in TPM for $\delta \in (0, 0.4]$.

Also the comparative statics in the efficiency parameters μ and ν resemble previous cases. Like under self-protection and self-insurance, the precautionary motives first increase up to a level of μ of about 0.1 and then consistently decrease. The initial increase contrasts with the case with saving and self-protection. Generally, the levels of the precautionary motives are lower. Self-protection crowds out saving and self-insurance, the former more than the latter. For μ high enough, self-protection will be used the most for precautionary purposes. The IPM stay tiny for all instruments. With ν all precautionary motives consistently decrease. The higher ν the more the TPM is mostly determined by the TPM_z. For ν high enough, the IPM_s and the IPM_x as well as the TPM_s and the TPM_x go negative, but less strongly than, where this occurs, in the two-instrument cases. The comparison of the comparative statics of the shares in the TPM in the two cases shows how much more strongly self-insurance may appropriate the precautionary reaction than self-protection for high levels of the efficiency parameters. In the presence of self-protection and self-insurance, saving has the least important role to play in the precautionary reaction in both cases, which still decreases with the efficiency levels.

5.3 Summary

We summarize our above simulation results and discuss some implications. In the cases with one instrument, each instrument reacts to each of the two risks but to different degrees. The largest precautionary-saving motives by far are in saving. Self-protection and self-insurance react much less, self-protection still more than self-insurance. The IPM_s clearly dominates the LPM_s in the most part of the considered parameter ranges. While the IPM_x and the LPM_x are roughly equal, the LPM_z is only tiny. In all cases, the first-order effect associated with the loss risk is clearly visible.

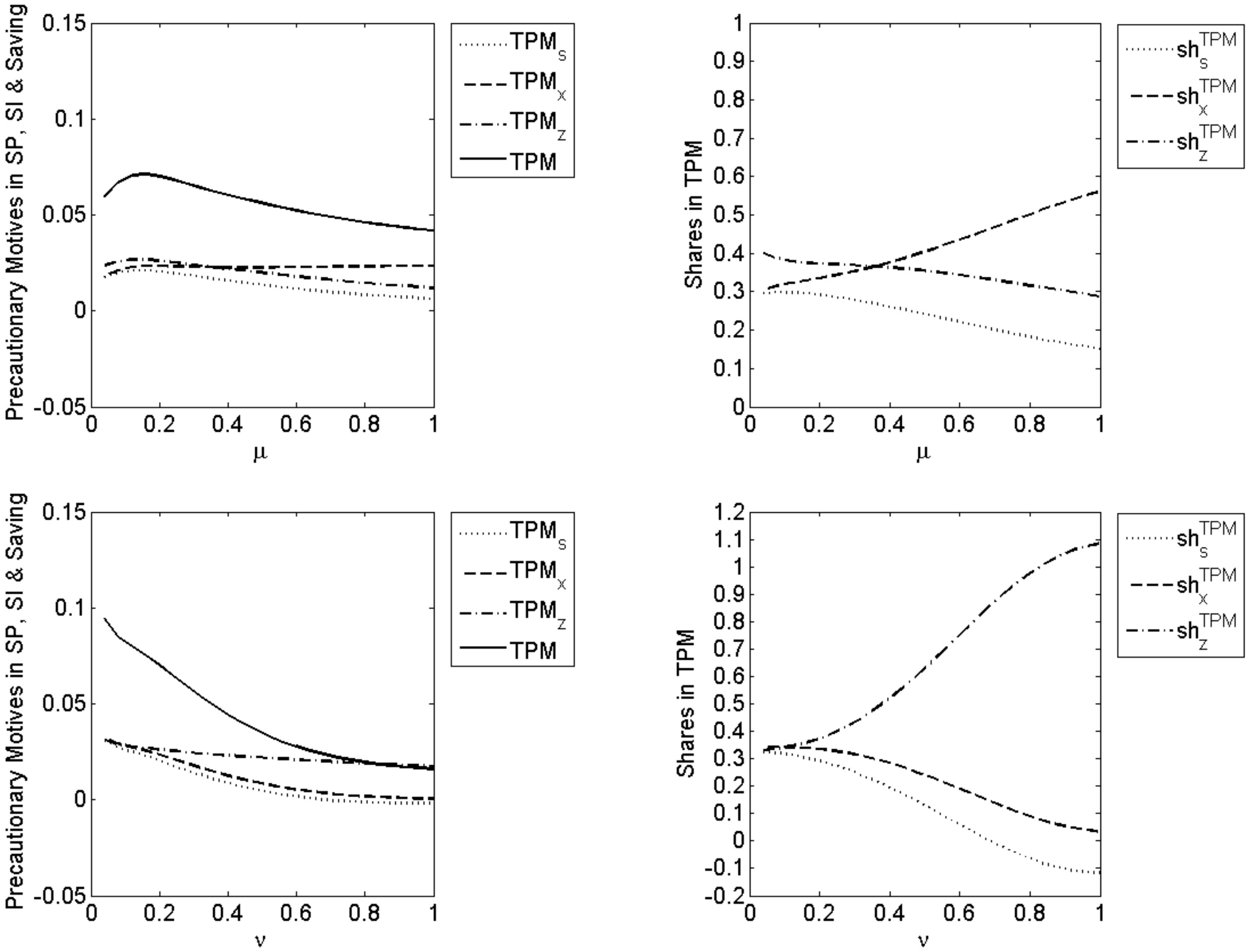

Figure 10: Comparative statics of precautionary motives and shares in TPM for $\mu \in (0, 1]$, and $\nu \in (0, 1]$.

With rising loss size or loss probability, consumption smoothing prevails at the expense of precaution. Self-insurance is mostly used for consumption smoothing.

When a second risk-management instrument is admitted, the total precautionary motive is, in each case, lower than in the corresponding cases with only one instrument. The precautionary reactions in self-protection and in self-insurance dominate precautionary saving. Similarly, precautionary self-insurance dominates precaution in self-protection. If in the scenarios with self-insurance the loss size is large enough, the total precautionary motive in saving or self-protection, respectively, may even be negative. Thanks to instrument interaction, the LPM_z may be very small and the LPM_s even negative for rising income risk δ in the setting with saving and self-insurance.

If all three instruments are available, all effects tend to be still more pronounced. The total precautionary motive across all instruments is once more smaller than in the corresponding two-instrument cases. Self-insurance choices clearly dominate the precautionary reactions in saving and self-protection, unless self-protection is particularly productive. While, in general, much smaller than self-insurance, self-protection is still more used than saving.

These results hinge, obviously, on the chosen parameter values. Especially, the implementation and parameterization of the self-protection and self-insurance technologies are purely illustrative in this paper. At the same time, the strong predicted interactions between the instruments in precautionary choices suggest the importance to control for a maximum of instruments if precautionary behaviors are studied from field data.

6 Conclusion

We study situations where a decision maker has one, two, or three instruments available to maximize the expected intertemporal consumption utility in the face of a future loss risk and a future income risk. As is well known, saving, self-protection, and self-insurance each exhibit a precautionary increase after the introduction of a zero-mean income risk when the agent is prudent. We argue that the loss risk already stimulates consumption smoothing (due to its first-order effect) and precaution (due to its riskiness). Controlling for consumption smoothing, we distinguish thus precautionary motives in response to the loss risk and to the income risk, which both add up to the total precautionary motive in an instrument. In our numerical implementation for some median agent, all instruments react in the single-instrument cases generally more to the income risk than to the loss risk. Total precautionary choices are by far the largest in saving before self-protection. Self-insurance is mostly used for consumption smoothing.

When the agent uses more than one instrument, after controlling for consumption smoothing, each risk implies in each instrument a positive precautionary effect and a negative substitution effect. Consequently, in the multi-instrument cases the total precautionary motive, which is the net of all individual effects, will typically differ from a mere summation of the effects in the cases with one instrument. In our simulations for the multi-instrument cases, the total precautionary motives are consistently lower than in each single-instrument case. Saving is now used the least among the three instruments, and self-insurance tends to be more used than self-protection. In the three-instrument case, the effects are more pronounced than if two instruments compete for resources. In particular, the total precautionary motive is still lower.

While our numerical results are bound to a particular parameterization, they still illustrate that instrument interaction may have important repercussions for the strength of the total precautionary motive and the expression of individual risk and time preferences. When other instruments are not well identified, the precautionary response of the observed variables will systematically deviate from its true value. Hence, when self-protection or self-insurance are productive in the sense that they react positively to the introduction of a risk, neglecting this endogeneity will systematically overpredict the precautionary saving response. This is a potential explanation for the difficulty that arises in attempts to measure the intensity of prudence from field data.

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A Appendix

A.1 Proof of Remark 1

Optimal *self-protection* expenditures x^{l*} are implicitly determined by the first-order condition:

$$U_x^l = -u'(y-x) + \beta p'(x)\left[v(Y-L) - v(Y)\right] = 0$$

The consumption-smoothing component x^{0*} is obtained from

$$U_x^0 = -u'(y-x) - \beta p'(x) L v'(Y - p(x)L) = 0$$

The riskiness of the loss risk implies a positive precautionary motive, $x^{l*} > x^{0*}$, if and only if $U_x^0\left(x^{l*}\right) < 0$. By using the first-order condition for x^{l*}, we obtain that

$$U_x^0\left(x^{l*}\right) = -\beta p'(x^{l*})L\left[v'\left(Y - p(x^{l*})L\right) - \frac{v(Y) - v(Y-L)}{L}\right]$$

Drawing on prudence and $p(x^{l*}) \leq 0.5$, we can determine the sign of the bracketed expression:

$$\begin{aligned} v'\left(Y - p(x^{l*})L\right) - \frac{v(Y) - v(Y-L)}{L} &= v'\left(Y - p(x^{l*})L\right) - \frac{1}{L}\int_{Y-L}^{Y} v'(C)dC \\ < v'\left(Y - p(x^{l*})L\right) - v'\left(\frac{1}{L}\int_{Y-L}^{Y} CdC\right) &= v'\left(Y - p(x^{l*})L\right) - v'\left(Y - \frac{1}{2}L\right) \leq 0 \end{aligned}$$

Together with $-\beta p'(x^{l*})L > 0$, this implies $U_x^0\left(x^{l*}\right) < 0$ as required.

For *self-insurance*, we obtain z^{l*} according to the first-order condition:

$$U_z^l = -u'(y-z) - \beta p L'(z) v'(Y - L(z)) = 0$$

The consumption-smoothing component z^{0*} is given by

$$U_z^0 = -u'(y-z) - \beta p L'(z) v'(Y - pL(z)) = 0$$

The riskiness of the loss risk implies a positive precautionary motive, $z^{l*} > z^{0*}$, if and only if $U_z^0\left(z^{l*}\right) < 0$. By using the first-order condition for z^{l*}, we obtain that

$$U_z^0\left(z^{l*}\right) = -\beta p L'(z^{l*})\left[v'\left(Y - pL(z^{l*})\right) - v'\left(Y - L(z^{l*})\right)\right] < 0$$

for, $-\beta p L'$ is positive and the bracketed expression is negative due to risk aversion. Consequently, in the case of self-insurance risk aversion over future consumption alone implies a precautionary reaction with respect to the riskiness associated with the loss risk.

A.2 Proof of Remark 2

Consider expected utility of future consumption in the presence of the loss risk, V^l. Due to the functional properties defined in Section 2, it holds for saving and self-protection that

$$V_{sx}^l(s,x) = (1+r)p'(x)\left[v'(Y + s(1+r) - L) - v'(Y + s(1+r))\right] < 0$$

for saving and self-insurance that

$$V^l_{sz}(s,z) = -(1+r)L'(z)v''(Y + s(1+r) - L(z)) \;<\; 0$$

and for self-protection and self-insurance that

$$V^l_{xz}(x,z) = -p'(x)L'(z)v'(Y - L(z)) \;<\; 0.$$

As a result, for each selection of two of the three instruments the cross-partial derivative of expected future consumption utility with respect to the two instruments is negative.

In the presence of an income risk on top of the loss risk, we find for saving and self-protection that

$$V^{li}_{sx}(s,x) = (1+r)p'(x)\left[Ev'(Y + \tilde{\varepsilon} + s(1+r) - L) - v'(Y + \tilde{\varepsilon} + s(1+r))\right] \;<\; 0$$

for saving and self-insurance that

$$V^{li}_{sz}(s,z) = -(1+r)L'(z)Ev''(Y + \tilde{\varepsilon} + s(1+r) - L(z)) \;<\; 0$$

and for self-protection and self-insurance that

$$V^{li}_{xz}(x,z) = -p'(x)L'(z)Ev'(Y + \tilde{\varepsilon} - L(z)) \;<\; 0$$

Again, in all three two-instrument cases the cross-partial derivative of expected future consumption utility with respect to the two instruments is negative. We note that the fact that the indirect utility function exhibits the same signs of successive derivatives as the original utility function implies that the negative cross-partials without income risk induce negative cross-partials in its presence.

A.3 Proof of Proposition 2

Because we fix the second instrument in the case of its exogeneity at its optimal level in the case of endogeneity, we obtain that the levels of the first instrument in the presence of the zero-mean income risk are identical in both cases. Consistent with the notation above they are denoted by a_1^{r*}. Consequently, we need to compare choices of a_1 in the absence of the zero-mean income risk with and without the opportunity to vary a_2. Consider the problem

$$\max_{a_1,a_2} U(a_1,a_2) \qquad \text{s.t.} \qquad a_2 = a_2^{r*}.$$

The Lagrangian is given by

$$\mathcal{L}(a_1,a_2;\lambda) = U(a_1,a_2) + \lambda(a_2^{r*} - a_2)$$

with associated first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial a_1} &= U_1 = 0,\\
\frac{\partial \mathcal{L}}{\partial a_2} &= U_2 - \lambda = 0,\\
\frac{\partial \mathcal{L}}{\partial \lambda} &= a_2^{r*} - a_2 = 0.
\end{aligned}$$

We denote the associated Lagrange multiplier $\hat{\lambda}$. Consequently, we obtain that $a_1^*(\hat{\lambda})$ is the level of a_1 when a_2 is fixed, whereas $a_1^*(0)$ is the level of the first instrument when the second is allowed to vary. Table 2 summarizes this approach.

Table 2: Approach in Proof of Proposition 2

Instrument(s)	a_1	a_1 and a_2
No income risk	$a_1^*(\hat{\lambda})$, $\overline{a}_2 = a_2^{r*}$	(a_1^*, a_2^*)
Income risk	a_1^{*r}, $\overline{a}_2 = a_2^{r*}$	(a_1^{r*}, a_2^{r*})

To compare the two cases, we treat λ as exogenous and use the Implicit Function Theorem:

$$\begin{pmatrix} \frac{da_1}{d\lambda} \\ \frac{da_2}{d\lambda} \end{pmatrix} = -\frac{1}{U_{11}U_{22} - U_{12}^2} \begin{pmatrix} U_{22} & -U_{12} \\ -U_{12} & U_{11} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial \mathcal{L}}{\partial a_1 \partial \lambda} \\ \frac{\partial \mathcal{L}}{\partial a_2 \partial \lambda} \end{pmatrix} = \begin{pmatrix} -\frac{U_{12}}{U_{11}U_{22}-U_{12}^2} \\ \frac{U_{11}}{U_{11}U_{22}-U_{12}^2} \end{pmatrix}.$$

Consequently, we obtain that

$$a_1(0) - a_1(\hat{\lambda}) = \int_{\hat{\lambda}}^{0} \frac{\partial a_1}{\partial \lambda} d\lambda.$$

Due to the substitution between saving and self-protection, it is

$$U_{12} = u''(w) + \beta E_L V_{12}(a_1, a_2) < 0 \ ,$$

so that the integrand is unambiguously positive. Hence, the sign entirely depends on $\hat{\lambda}$. Now $a_1^*(0) > a_1^*(\hat{\lambda})$ which is equivalent to the fact that the precautionary choice of a_1 is larger if a_2 is exogenous, if and only if $\hat{\lambda}$ is negative, which is the case if and only if $a_2^* < a_2^{r*}$.

A.4 Proof of Proposition 3

The proof for the effects on the single decision variables is analogous to the ones for Proposition 1 and Corollary 1. In particular, the substitutive relationship between each two instruments can be seen from the respective cross-derivatives of the utility objective:

$$U_{xz}^{li} = u''(c) - \beta p'(x) L'(z) E v'(\tilde{Y} + s(1+r) - L(z)) \ < \ 0 \tag{8a}$$

$$U_{xs}^{li} = u''(c) + \beta(1+r)p'(x)E\left(\tilde{Y} + s(1+r) - L(z)) - v'(\tilde{Y} + s(1+r))\right) \ < \ 0 \tag{8b}$$

$$U_{zs}^{li} = u''(c) - \beta(1+r)p(x)L'(z)Ev''(\tilde{Y} + s(1+r) - L(z)) \ < \ 0 \tag{8c}$$

For the statement on the prevalence of positive precautionary responses in equilibrium, consider U^{li} in the (x, z, s)-plane and evaluate the directional derivative into the direction $\nu = (\nu_1, \nu_2, \nu_3) \in \mathbb{R}^3$ at (x^*, z^*, s^*):

$$\partial_\nu U^{li}(x^*, z^*, s^*) = \nu_1 \frac{\partial U^{li}}{\partial x}(x^*, z^*, s^*) + \nu_2 \frac{\partial U^{li}}{\partial s}(x^*, z^*, s^*) + \nu_3 \frac{\partial U^{li}}{\partial s}(x^*, z^*, s^*)$$

Under prudence, $\frac{\partial U^{li}}{\partial x}(x^*, z^*, s^*)$, $\frac{\partial U^{li}}{\partial z}(x^*, z^*, s^*)$, and $\frac{\partial U^{li}}{\partial s}(x^*, z^*, s^*)$ are each positive so that at least one of ν_1, ν_2, or ν_3 must be positive. Consequently, at most two of the three variables can be lower in the new optimum.

A.5 Proof of Proposition 4

Assuming that the probability and size of the loss are identical in the cases with and without self-protection and self-insurance, savings in the presence of the zero-mean income risk, denoted s^{li*}, are identical in both cases. Compare the saving choices in the absence of the zero-mean income risk with and without the opportunity to self-protect and self-insure. Consider the problem

$$\max_{x,z,s} U^{li}(x,z,s) \qquad \text{s.t.} \quad x = x^{li*}\;, \quad z = z^{li*}$$

The Lagrangian is given by

$$\mathcal{L}(x,z,s;\lambda_1,\lambda_2) = U^{li}(x,z,s) + \lambda_1\left(x^{r*} - x\right) + \lambda_2\left(z^{r*} - z\right) \tag{9}$$

and the first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = U_x^{li} - \lambda_1 = 0 \tag{10a}$$
$$\frac{\partial \mathcal{L}}{\partial z} = U_x^{li} - \lambda_2 = 0 \tag{10b}$$
$$\frac{\partial \mathcal{L}}{\partial s} = U_s^{li} = 0 \tag{10c}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = x^{li*} - x = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = z^{li*} - z = 0$$

The constrained optimum is given by $(x^{li*}, z^{li*}, \hat{s})$ where $\hat{s}$ maximizes $U^{li}(x^{li*}, z^{li*}, s)$. The associated Lagrange multipliers $\hat{\lambda}_i$, $i = 1,2$, equal $U_x^{li}(x^{li*}, z^{li*}, \hat{s})$ and $U_z^{li}(x^{li*}, z^{li*}, \hat{s})$, respectively.

Now assume λ_1 and λ_2 are exogenous to (9) and define the optimal x and z as functions of (λ_1, λ_2). Then, obviously, $\left(x(\hat{\lambda}_1, \hat{\lambda}_2),\, z(\hat{\lambda}_1, \hat{\lambda}_2)\right) = \left(x^{li*}, z^{li*}\right)$ and $(x(0,0),\, z(0,0)) = \left(x^{li*}, z^{li*}\right)$. To see how variations in (λ_1, λ_2) affect the optimal values of x, z, and s, consider the following transformation of system (10):

$$\begin{pmatrix} dx \\ dz \\ ds \end{pmatrix} = -H^{-1}\left(U^{li}(x,z,s)\right) \cdot \begin{pmatrix} \frac{\partial \mathcal{L}}{\partial x \partial \lambda_1} & \frac{\partial \mathcal{L}}{\partial x \partial \lambda_2} \\ \frac{\partial \mathcal{L}}{\partial z \partial \lambda_1} & \frac{\partial \mathcal{L}}{\partial z \partial \lambda_2} \\ \frac{\partial \mathcal{L}}{\partial s \partial \lambda_1} & \frac{\partial \mathcal{L}}{\partial s \partial \lambda_2} \end{pmatrix} \begin{pmatrix} d\lambda_1 \\ d\lambda_2 \end{pmatrix} = \frac{adj\,(H)}{|H|} \cdot \begin{pmatrix} d\lambda_1 \\ d\lambda_2 \\ 0 \end{pmatrix}$$

$$= \frac{\begin{pmatrix} U_{zz}U_{ss} - U_{zs}^2 & U_{xs}U_{zs} - U_{xz}U_{ss} & U_{xz}U_{zs} - U_{xs}U_{zz} \\ U_{xs}U_{zs} - U_{xz}U_{ss} & U_{xx}U_{ss} - U_{xs}^2 & U_{xz}U_{xs} - U_{xx}U_{zs} \\ U_{xz}U_{zs} - U_{xs}U_{zz} & U_{xz}U_{xs} - U_{xx}U_{zs} & U_{xx}U_{zz} - U_{xz}^2 \end{pmatrix}}{U_{xx}U_{zz}\left(U_{zs} - U_{ss}\right) + U_{xz}^2\left(U_{xs} - U_{zs}\right) + U_{xs}\left(U_{xz}U_{ss} - U_{xs}U_{zs}\right)} \cdot \begin{pmatrix} d\lambda_1 \\ d\lambda_2 \\ 0 \end{pmatrix}$$

$$= \frac{\begin{pmatrix} \left(U_{zz}U_{ss} - U_{zs}^2\right) d\lambda_1 + \left(U_{xs}U_{zs} - U_{xz}U_{ss}\right) d\lambda_2 \\ \left(U_{xs}U_{zs} - U_{xz}U_{ss}\right) d\lambda_1 + \left(U_{xx}U_{ss} - U_{xs}^2\right) d\lambda_2 \\ \left(U_{xz}U_{zs} - U_{xs}U_{zz}\right) d\lambda_1 + \left(U_{xz}U_{xs} - U_{xx}U_{zs}\right) d\lambda_2 \end{pmatrix}}{U_{xx}U_{zz}\left(U_{zs} - U_{ss}\right) + U_{xz}^2\left(U_{xs} - U_{zs}\right) + U_{xs}\left(U_{xz}U_{ss} - U_{xs}U_{zs}\right)}$$

where $H(.)$ is the Hessian matrix of the objective $U^{li}(x,z,s)$ and $adj\,(H)$ its adjugate.

Consequently,

$$ds = \frac{\left(U_{xz}U_{zs} - U_{xs}U_{zz}\right) d\lambda_1 + \left(U_{xz}U_{xs} - U_{xx}U_{zs}\right) d\lambda_2}{U_{xx}U_{zz}\left(U_{zs} - U_{ss}\right) + U_{xz}^2\left(U_{xs} - U_{zs}\right) + U_{xs}\left(U_{xz}U_{ss} - U_{xs}U_{zs}\right)} \ , \tag{11}$$

so that the sign of ds is equal to the sign of the numerator in equation (11),

$$sgn\left[ds\right] = sgn\left[\left(U_{xz}U_{zs} - U_{xs}U_{zz}\right) d\lambda_1 + \left(U_{xz}U_{xs} - U_{xx}U_{zs}\right) d\lambda_2\right]. \tag{12}$$

Given that $\left(U_{xz}U_{zs} - U_{xs}U_{zz}\right)$ and $\left(U_{xz}U_{xs} - U_{xx}U_{zs}\right)$ are positive, $ds > 0$, i.e., precautionary saving is larger in the absence of self-protection or self-insurance, if and only if the sum of $d\lambda_1$ and $d\lambda_2$ weighted, respectively, by $\left(U_{xz}U_{zs} - U_{xs}U_{zz}\right)$ and $\left(U_{xz}U_{xs} - U_{xx}U_{zs}\right)$ is positive. More generally, it holds thus that

$$ds \gtreqless 0 \quad \Leftrightarrow \quad d\lambda_1 \gtreqless -\frac{U_{xz}U_{xs} - U_{xx}U_{zs}}{U_{xz}U_{zs} - U_{xs}U_{zz}}\ d\lambda_2\ .$$