Semi-Endogenous Growth and Pollution:
No Double Dividend in the Long Term

Pascal da Costa*

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Abstract: Literature on endogenous growth shows that a polluting economy can grow sustainably and that a double-dividend (or win-win effect) boosting growth is possible. Even with a semi-endogenous growth approach - which occurs when the knowledge stock yield falls below the unit in the production of innovations - what happens to sustainability and the double dividend?
This paper presents the first semi-endogenous growth model with pollution which answers this very question. We first illustrate that the dynamics of this economy can be sustainable even if its long-term growth rate is exogenous. To ensure the latter, a knowledge stock yield that is greater than a certain strictly positive threshold is required. We then demonstrate that the double dividend and the Porter hypothesis are impossible. Indeed, the level of support for innovation has no positive impact on the long-term growth rate, and the environmental policy has a negative effect on growth.
JEL Codes: O41; Q20. **Key words:** Innovation; Pollution; Semi-Endogenous Growth; Double Dividend.

1 Introduction

Significant progress made in understanding growth mechanisms has been possible thanks to endogenous growth models (see the founding models by Romer 1990, Aghion and Howitt 1992, Grossman and Helpman 1991). The Jones growth model (1995 a, b) marks a turning point. In Jones' criticism, endogenous growth is based on the strong assumption that the yield of the knowledge stock in the production of innovations is unitary (i.e. unit yield on the accumulable factor; note that an equivalent criticism is maintained by Kortum 1997, Segerstrom 1998). Long-term growth becomes exogenous once the yield is considered below the unit. Intuitively, a yield below the unit indicates the level of difficulty in research: if the yield decreases, the research is more difficult.

To date no study has explicitly investigated the 'growth and environment' double-dividend or win-win effect of a polluting economy faced with the semi-endogenous growth approach. This double dividend appears when the economic policy makes it possible to both increase the growth rate of production and improve the quality of the environment in the long term. It is worth citing the existing semi-endogenous growth models by Eriksson and Zehaie (2005)[1] and Groth (2007)[2]. The economic policy issues related to semi-endogenous growth are worth studying in-depth, all the more so since numerous questions still remain unanswered: is the long-term efficiency of policies designed to reduce polluting emissions called into question by semi-endogeneity? Does the difficulty in research hinder the dissemination of technology and therefore represent an obstacle to sustainable growth? What are the new conditions (if they do indeed exist) for sustainable growth in the strongest sense of the term, i.e. leading to reduced emissions compatible with sustained growth? Is the 'growth and environment' double-dividend which is so dear

*Laboratoire Genie Industriel / Sustainable Economy team - Address: Ecole Centrale Paris - Grande Voie des Vignes - 92295 Chatenay-Malabry France Tel: +33 (0) 141 131 416 - Fax: (0) 141 131 272 - Email: pascal.da-costa@ecp.fr

[1]Eriksson and Zehaie (2005) take the specific stance that the damage resulting from pollution depends on the population density and the type of pollution.

[2]Groth (2007) studies the dynamics of economies exploiting non-renewable resources by disregarding the issues of pollution.

to endogenous growth economists (since it justifies pollution reduction efforts and current social costs, thanks to the future high growth rate of income) still possible?[3]

This paper develops and analyses the model of an economy experiencing both semi-endogenous growth and polluting emissions in an attempt to answer these questions. Our model is based on Stokey's polluting economy (1998) which is applied to Segerstrom's semi-endogenous growth (1998). Within this scope, the environment is degraded during the production process and the total factor productivity (TFP) increases thanks to R&D. The following new principles are enabled by our model.

It will be shown that with semi-endogenous growth, the new conditions to a path of sustainable growth in the strongest sense mainly depend on the difficulty in research (yield value on the accumulable factor in the innovation production function). A suitable substitution between the natural capital (source of polluting emissions) and intangible capital (TFP) can lead the economy onto a growth path that is advantageous to the environmental quality (reduced pollution stock) when the research difficulties are not excessive.

This appropriate substitution is made possible by the implementation of an incentive policy internalising the externalities: tax on polluting emissions (for negative externalities) and R&D subsidies or even taxation (for positive externalities). Like in other endogenous growth models with pollution (Aghion and Howitt 1998; Grimaud and Ricci 2004), the optimal taxation is increasing in our model. It has a negative effect on the production growth rate, while increasing the intensity of R&D. This paper shows that the last point nonetheless does not validate the Porter hypothesis (Porter 1991; Porter and van der Linde 1995) according to which environmental policy may improve the environmental quality while aiding the growth rate: the intensity of R&D under semi-endogenous growth has no effect on the long-term growth rate of innovation, nor does it affect that of production. R&D is no longer an endogenous drive for the growth of production in the long term. R&D subsidies only have an impact on the production level. The impact of economic policy is therefore significantly modified with semi-endogenous growth: the 'growth and environment' double dividend is impossible.

We have to recognise that the complementarity of economic policy instruments in the double dividend is postulated in a great many models applied to long-run growth and the environment. Climate policy is the environmental policy of long-term excellence: the conclusions of numerical simulations by Acemoglu, Aghion, Bursztyn and Hemous (2012) can be cited to illustrate this statement.[4] They make it possible to gauge the effects economic instruments when several endogenous R&D sectors co-exist within the scope of a climate model (with two sectors aiming to improve the productivity of non-renewable and renewable resources respectively). These authors show that the R&D support policy alone, i.e. without the combined implementation of the carbon tax, does have a direct effect on the allocation of labour in favour of research sectors, but very little (or no) indirect effect on the polluting emissions. It should be pointed out that the polar case is also true. The combination of two tools appears to be the only efficient policy capable of producing a double-dividend effect in the long term according to Acemoglu, Aghion, Bursztyn and Hemous (2012). Though it is based on a single R&D sector, the theoretical approach applied in our article is sufficient to start a debate on such results.

This article is structured in the following manner: firstly, it determines the equilibrium of the optimal economy and the conditions required for an optimal sustainable growth path to exist (section 2), before defining the equilibrium of a decentralised economy (section 3) and then defining the optimal policy which matches with the two previous equilibriums (section 4). The conclusion is given in section 5.

[3]Ricci (2007) provides a literature review of the transmission channels of environmental policy with respect to economic growth.

[4]We could also cite the numerical simulations by Grimaud, Lafforgue and Magne (2011) whose conclusions are similar enough, or even those of Henriet, Maggiar and Schubert (2014).

2 Social Optimum

This section will first lay down the model's assumptions.

2.1 Final Good

The production function of the final good is given by the following relationship (Stokey 1998; Aghion and Howitt 1998):

$$Y_t = z_t \left[A_t \left(1 - n_t\right) L_t\right]^{1-\alpha} K_t^{\alpha} \tag{1}$$

with $\alpha \in [0,1]$, $z_t^{\gamma} \in [0,1]$ denoting the polluting intensity of the economy which is a controlled variable of the final goods sector, $(1 - n_t) \in [0,1]$ denoting the fraction of the labour input devoted to producing the final good, L_t denoting the size of the labour force, A_t denoting the average quality or TFP, and K_t denoting the capital. When $z_t = 1$, the polluting intensity is maximal and production of the final good reaches its potential level. On the contrary, however, when $z_t < 1$, the level of production is reduced.

The variation in the capital stock is given by: $\dot{K}_t = Y_t - L_t c_t$, with c_t representing the per capita consumption.

2.2 Households and Pollution

The pollution stock accumulation equation is written as follows: $\dot{S}_t = P_t - \Theta S_t$, with $\Theta \geq 0$ denoting the natural assimilation rate which is kept constant for the purpose of simplification, and P_t represents the pollution flow.

The representative consumer's utility function is given by: $U(c_t, S_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon} - \frac{S_t^{1+\varpi}}{1+\varpi}$. Its arguments are: per capita consumption c_t, the pollution stock level S_t, and parameters for which $\frac{1}{\varepsilon}$ is the intertemporal elasticity of substitution and $\varpi > 0$ so that: $\frac{\partial U(c_t,S_t)}{\partial c_t} > 0$, $\frac{\partial U(c_t,S_t)}{\partial S_t} < 0$, $\frac{\partial^2 U(c_t,S_t)}{\partial c_t \partial S_t} = 0$.

The pollution flow is expressed as follows (Stokey 1998; Aghion and Howitt 1998): $P_t = z_t^{\gamma} Y_t$, with $\gamma > 0$.

2.3 R&D

The R&D sector comprises a continuum of specialised laboratories. Each laboratory is looking to take into account the best available quality - i.e. the maximum productivity parameter or the frontier productivity - at date t, $\bar{A}_t = \max_j \{A_{jt}\}$, into its intermediate good j ($j \in [0,1]$ for the purpose of simplification). Under the semi-endogenous growth approach (Segerstrom 1998), the variation in the frontier quality $\bar{A}_t$ depends on both the labour flow allocated to the R&D sector and on the past frontier quality whose yield is below the unit:

$$\dot{\bar{A}}_t = \kappa \psi_t \equiv \delta \left(n_t L_t\right) \bar{A}_t^{\phi} \tag{2}$$

with $\kappa > 0$ and $\delta \equiv \kappa\beta > 0$ (β is relative to Poisson's process which we will discuss later on), ψ_t denotes the instantaneous flow of improvement in the frontier quality, while $n_t L_t$ denotes the R&D effort with $n_t \in [0,1]$ denoting the fraction of work allocated to R&D (intensity of R&D) and $\phi < 1$ the yield over the accumulable factor (unitary under endogenous growth hypothesis). This yield is the elasticity of the rate of knowledge accumulation to the level of existing knowledge. It can indicate the level of difficulty in R&D: the higher ϕ is, the easier research proves to be. Therefore, when $1 > \phi > 0$, the productivity of R&D increases with the innovations and an intertemporal spillover effect occurs which is called "standing on shoulders"; when $\phi = 0$, then the stock of innovations is no longer a factor of R&D; when $\phi < 0$, then any new innovation reduces the productivity of future research and the depletion of technology opportunities occurs which is known as a "fishing out" effect (Jones 1995 b).[5] The mean productivity index $A_t = \int_0^1 A_{jt} dj$, is then defined. It should be pointed out that the frontier quality and the mean quality increase at the same rate on the balanced-growth path because by nature $A_t = \frac{\bar{A}_t}{1+\kappa}$,

[5] It should be pointed out that $\phi > 1$ is disregarded otherwise growth becomes explosive.

i.e. $\left.\frac{\dot{A}}{A}\right|_t = \left.\frac{\dot{\bar{A}}}{\bar{A}}\right|_t$. The appearance of innovations in the laboratories follows Poisson's stochastic approach but it can be considered as smoothed out for the R&D sector (Aghion and Howitt 1992).

Lastly, in the model's steady state, $\left.\frac{\dot{\bar{A}}}{\bar{A}}\right|_t$ (Equation (2)) is constant if and only if:

$$\frac{\dot{\bar{A}}}{\bar{A}} = \frac{l}{1-\phi} \tag{3}$$

with $l > 0$ denoting the constant growth rate of the labour force such that $L_t = L_0 e^{lt}$. Equation (3) shows that the long-term growth rate under the semi-endogenous growth approach is exogenous (and strictly positive). The population growth rate l and the yield of research ϕ are the determinants of this growth rate. Conversely, in an endogenous growth model with $\phi = 1$, Equation (2) becomes $\frac{\dot{\bar{A}}}{\bar{A}} = \delta n L$ in steady-state conditions with n and L remaining constant. This growth rate is directly proportional to the resources allocated to R&D. Therefore, it is possible to evidence the existence of two long-term effects: that of the economic policy through the intensity of R&D (n) and a scale effect with the size of the labour force (L). Yet since Jones' review, it is believed that this scale effect must be disregarded because it contradicts the facts (an economy with twice the population of another does not necessarily growth twice as fast).[6]

This paper now sets out to answer the following questions. How will the factors of production be distributed in an optimal version of this economy? What is the objective of production? Under what conditions can the optimal path to semi-endogenous growth become sustainable in the strong sense of the word?

2.4 Maximising Well-Being and the Optimal Path to Sustainable Growth

The social planner maximises the well-being of the representative agent with the aim of determining the optimal values of per capital consumption (c_t), R&D intensity (n_t) and the polluting intensity of the economy (z_t). The state variables are the physical capital stock (K_t), the qualiy (A_t) and the pollution stock (S_t). ρ denotes the time preference rate. The planner's programme is written as follows:

$$\left\{\begin{array}{l|l} \max\limits_{\{(c_t,n_t,z_t)\}_{t=0}^{+\infty}} & \int\limits_0^\infty e^{-\rho t} U(c_t, S_t) dt \\ & \dot{K}_t = Y_t - L_t c_t \\ & \dot{A}_t = \delta n_t L_t A_t^\phi \\ s.t./ & \dot{S}_t = P_t - \Theta S_t \\ & K_t \geq 0, A_t \geq 0, S_t \geq 0 ... \\ & K_0, A_0, S_0 \text{ known} \end{array}\right. \tag{4}$$

Lemma 1 *The long-term optimal growth rates for the model's variables (written g^o) after having resolved the planner's programme (4) are as follows:*

$$\left\{\begin{array}{l} g_A = \frac{l}{1-\phi} > 0 \\ g_c^o = \frac{\gamma(1-\alpha)g_A - l}{\gamma(1-\alpha) + \frac{\varepsilon+\varpi}{1+\varpi}} \\ g_z^o = -\frac{1}{\gamma}\left(\frac{\varepsilon+\varpi}{1+\varpi} g_c^o + l\right) \\ g_S^o = \frac{1-\varepsilon}{1+\varpi} g_c^o \\ n^o = \frac{1}{1+Z^o} \ \textit{with}\ Z^o = \frac{(\varepsilon-1)g_c^o + \rho + l}{g_A} \end{array}\right. \tag{5}$$

[6] Some papers endorsing endogenous growth without scale effects nonetheless recommend models that combine horizontal and vertical differentiations so economic policy has an impact on the long-term growth without this producing, however, any scale effect. It is worth referring to the canonical models by Aghion and Howitt (1998), Peretto (1998) and the review by Jones (1999). Lastly, it is worth citing the econometric estimations by Sedgley and Elmslie (2010) who set out to measure the parameter ϕ.

Proof. See Appendix A for resolution of the social optimum. ■

Next, we recall the fact that sustainable growth in the strong sense is defined by a strictly positive production growth rate ($g_c^o > 0$) and a strictly negative pollution stock growth rate ($g_S^o < 0$), with respect to sign and limit constraints.

Proposition 2 *It is necessary and sufficient to combine all three of the following conditions to achieve optimal growth in the long term that is sustainable in the strong sense:*

i. *The parameter ϕ must be greater than a given strictly positive threshold:*

$$g_c^o > 0 \textbf{\textit{ 0 if and only if }} \phi > 1 - \gamma(1-\alpha) > 0 \tag{6}$$

ii. *The intertemporal elasticity of substitution must be less than the unit:*

$$g_S^o < 0 \textbf{\textit{ if and only if }} \varepsilon > 1 \tag{7}$$

iii. *The natural assimilation rate must not be too low:*

$$\tfrac{P}{S} > 0 \textbf{\textit{ if and only if }} \Theta > \frac{\varepsilon - 1}{\varpi + 1} \frac{\gamma(1-\alpha) g_A - l}{\gamma(1-\alpha) + \frac{\varepsilon + \varpi}{1+\varpi}} \tag{8}$$

Proof. See Appendix B on the existence of the optimal path. ■

The first Condition (6) is specific to sustainable semi-endogenous growth: the parameter ϕ must be strictly positive and not too low, failing which the economy's growth which reduces its polluting intensity can be negative in the long term (see impact of z on Y in (1)). It can also be seen that the exhaustion of technology opportunities (fishing-out effect: $\phi < 0$) is incompatible with the criterion of strong sustainability. The second Condition (7) is, however, similar to that of sustainable endogenous growth (Aghion and Howitt 1998; Grimaud and Ricci 2004): since households have a very strong preference for consumption smoothing, if consumption is declining then they let emissions increase on the balanced-growth path in order to favour future consumption (recall that pollution affects only utility, not consumption), whereas if consumption is increasing then they sacrifice more and more consumption over time, leading to decreasing emissions and hence a decreasing stock. The third Condition (8) can be summarised as follows: a higher long-term production growth rate can be accompanied by a reduced pollution stock if and only if the natural regeneration rate increases.[7]

Under these three conditions, the level variables show coherent values, particularly the level of production which remains strictly positive in the long term despite a polluting intensity that tends asymptotically towards zero: its reduction is therefore offset by the continuous increase - at an acceptable rate - in the quality of intermediate goods.

3 Decentralised Economy

In a second-best economy such as the one we are investigating here, three instruments are needed to get as close as possible to the optimal target (like Grimaud and Ricci 2004): subsidising the purchase of intermediate goods (written τ), taxing polluting emissions (h), and subsidising or taxing the cost of R&D (σ). The economy experiences market distortion and two external effects: the potential monopoly of producers over intermediate goods, polluting emissions from the final goods sector, and R&D activities. It should be pointed out that a fourth instrument is required to balance the Government's budget: a non-distortive lump sum (T) for households to avoid resorting to debt.

[7]Condition (8) can be written $\Theta > -g_S^o$ and knowing that $g_S^o = \frac{1-\varepsilon}{1+\varpi} g_c^o$.

3.1 R&D Sector

To integrate the best available quality $\bar{A}_t$ into the intermediate good j, each laboratory uses the quantity of labour consumed within the labour input to directly determine its probability for innovation ψ_j. If successful, the R&D project is rewarded by a local monopoly rent resulting from its protection by a patent. The programme for maximising a laboratory j is written: $\begin{cases} \max\limits_{(n_{jt})} V_{jt}\psi_{jt} - (1-\sigma_t)\, w_t n_{jt} L_t \\ s.t./\ \psi_{jt} = \widetilde{\beta}\,(n_{jt}L_t) \end{cases}$,

with V_{jt} denoting the value of innovation, w_t denotes the nominal wage, and $\widetilde{\beta}= \frac{\beta}{\bar{A}_t^{1-\phi}}$ denotes exogenous data for the innovator. This parameter takes into account all the knowledge spillover in R&D. From an atomistic viewpoint, the R&D yields are constant because the laboratories see their marginal product as being equal to the mean product (Segerstrom 1998). The programme leads to the following free-entry condition: $V_{jt} = \left(\frac{1-\sigma_t}{\widetilde{\beta}}\right) w_t$. $\widetilde{\beta}$ is replaced to obtain the following:

$$V_{jt} = \left(\frac{1-\sigma_t}{\beta}\right)(n_t L_t)\,\bar{A}_t^{1-\phi} w_t = V_t \tag{9}$$

Equation (9) shows that the value of innovation is the same for all innovators since they achieve an identical frontier quality in the case their project is successful.

3.2 Final Goods Sector

The producer's final goods programme involves maximising his instant profit[8] by determining the polluting intensity of his production, his labour demand and the desired quantity of intermediate goods, i.e. the following first-order conditions:

$$\begin{aligned} z_t &= \begin{cases} [(\gamma+1)\,h_t]^{-\frac{1}{\gamma}} \text{ if } h_t > \frac{1}{\gamma+1} \\ 1 \text{ if } h_t \leq \frac{1}{\gamma+1} \end{cases} \\ w_t &= (1-\alpha)\left(\frac{\gamma}{\gamma+1}\right)\frac{Y_t}{(1-n_t)\,L_t} \\ p_{jt} &= \left(\frac{\alpha}{1-\tau_t}\right)\left(\frac{\gamma}{\gamma+1}\right) z_t\,[(1-n_t)\,L_t]^{1-\alpha} A_{jt} x_{jt}^{\alpha-1} \end{aligned} \tag{10}$$

with x_{jt} denoting the volume of the intermediate good j.

3.3 Intermediate Goods Sector

Resolving the programme of a monopolistic company j in the intermediate goods sector[9] leads to $\begin{cases} p_{jt} = \frac{r_t A_{jt}}{\alpha} \\ x_{jt} = (1-n_t)\,L_t\left[\alpha^2\left(\frac{\gamma}{\gamma+1}\right)\frac{z_t}{(1-\tau_t)r_t}\right]^{\frac{1}{1-\alpha}} = x_t, \forall j \end{cases}$. It can be seen that $x_{jt} = x_t$, $\forall j$: this result makes sector aggregation possible. The following interest rate is obtained: $r_t = \frac{\alpha^2}{1-\tau}\frac{\gamma}{\gamma+1}\frac{Y_t}{K_t}$. The company's profit is therefore written:

$$\Pi_{jt} = (1-\alpha)\,p_{jt} x_{jt} \tag{11}$$

[8]i.e. $Y_t - w_t(1-n_t)\,L_t - \int_0^1 p_{jt}(1-\tau_t)\,x_{jt}dj - h_t\int_0^1 P_{jt}dj$, with p_{jt} denoting the price of the intermediate good j, x_{jt} denoting the volume of the intermediate good j such that $x_{jt} = \frac{K_{jt}}{A_{jt}}$ and the aggregate capital: $K_t = \int_0^1 A_{jt}x_{jt}dj$.

[9]i.e. $\max \Pi_t = p_{jt}x_{jt} - r_t A_{jt} x_{jt}$ under (10) with r_t denoting the cost of capital.

With Equation (10), we can rewrite Equation (11) as a function of the frontier quality:

$$\Pi_t\left(\overline{A}_t\right) = (1+\kappa)\,\frac{(1-\alpha)\,\alpha}{1-\tau_t}\left(\frac{\gamma}{\gamma+1}\right)Y_t\left(\overline{A}_t\right) \tag{12}$$

3.4 Households and the Governement

The representative consumer maximises his intertemporal utility function under a dynamic budget constraint without being able to influence the amount of emissions to which he is subjected. The consumer's programme is given by the following dynamic problem:

$$\begin{cases} \max(c_t)_{t=0}^{+\infty} \int\limits_0^{\infty} e^{-\rho t}\left(\frac{c_t^{1-\varepsilon}}{1-\varepsilon}-\frac{S_t^{1+\varpi}}{1+\varpi}\right)dt \\ s.t./\left| \begin{array}{l} \dot{W}_t = w_t L_t + r_t W_t - L_t c_t + T_t \\ \text{and } \lim_{t\to+\infty} e^{-\int\limits_0^{\infty} r_s ds} W_t \geq 0 \end{array}\right. \end{cases} \tag{13}$$

with $\dot{W}_t$ denoting the dynamic budget constraint, where W_t represents the household wealth and T_t, the tax or subsidy making it possible to balance the government budget which is written: $T_t + \sigma w_t n_t L_t + \tau r_t K_t = h_t P_t$. Resolving Problem (13) implies using the Hamiltonian: $H = \mathrm{e}^{-\rho t}U\left(c,S\right) - \chi\dot{W} = e^{-\rho t}\left(\frac{c^{1-\varepsilon}}{1-\varepsilon}-\frac{S^{1+\varpi}}{1+\varpi}\right) - \chi\left(wL+rW-C+T\right)$.

The first-order conditions are: $\begin{cases} \frac{\partial H}{\partial c} = 0 \\ \dot{\chi} = -\frac{\partial H}{\partial W} \end{cases} \Leftrightarrow \begin{cases} e^{-\rho t}c^{-\varepsilon} = \chi \\ \frac{\dot{\chi}}{\chi} = -r \end{cases}$. The previous system leads to the Keynes-Ramsey rule: $g_{c,t} = \frac{r_t-\rho-l}{\varepsilon}$. Calculation of the single interest rate for the market (r_t) follows on from this rule.[10]

3.5 Determining the R&D Intensity and Growth Rates of the main Aggregates in a Steady State

The free-entry condition for invention based on which we will determine the R&D intensity and the interest rate in a steady state can be written:

$$V\left(\bar{A}_t\right) = \frac{\Pi\left(\bar{A}_t\right)}{r_t + p\left(\bar{A}_t\right) - \left.\frac{\dot{V}}{V}\right|_t} \tag{14}$$

with $\Pi\left(\bar{A}_t\right)/V_t = \delta\frac{\alpha\frac{1+\kappa}{\kappa}}{(1-\sigma)(1-\tau)}\,\frac{1-n_t}{n_t}\,g_{A_t}$ and $V_t = (1-\alpha)\,\frac{\gamma}{\gamma+1}\,\frac{1-\sigma_t}{\beta}\,\frac{n_t}{1-n_t}\,\frac{Y_t}{g_{A_t}}$ (Equations (12) and (9)). Equation (14) is common to all growth models with vertical differentiation (Aghion and Howitt 1992).

Lemma 3 *The R&D intensity and the interest rate of a steady growth path in a decentralised economy are equivalent to (15) and (16) respectively:*

$$n_{ss}^d = \frac{1}{Z_{ss}^d+1} \tag{15}$$

with $Z_{ss}^d = \frac{(1-\sigma)(1-\tau)}{\alpha\delta\frac{1+\kappa}{\kappa}}\left[(\varepsilon-1)+\frac{\delta}{\kappa}-\frac{\frac{\varepsilon-1}{\gamma(1-\alpha)}g_h-\rho}{\frac{l}{1-\phi}}\right]$, *and*

$$r_{ss}^d = \varepsilon\frac{l}{1-\phi} - \frac{\varepsilon}{\gamma\left(1-\alpha\right)}g_h + \rho + l \tag{16}$$

[10] The optimal programme for the representative consumer also leads to the following condition: $\lim_{t\to+\infty} e^{-\int\limits_0^{\infty} r_s ds} W_t = 0$.

Equation (16) shows that the higher growth rate of the environmental tax, the lower the interest rate. Conversely, the higher the exogenous growth rate of innovation, the higher the interest rate. The cost of capital r^d_{ss} increases when the TFP rises at a faster rate but it drops when the pollution generated by production and thus by capital is taxed even more.

Equation (15) shows that the R&D intensity increases with the three economic instruments. Internalising externalities of an environmental and technological nature while resolving distortion therefore boosts R&D investments, n^d_{ss}.

Lemma 4 *The long-term growth rates in a decentralised economy (written g^d) are as follows:*

$$\begin{cases} g_A = \frac{l}{1-\phi} \\ g_c^d = g_A - \frac{1}{\gamma(1-\alpha)} g_h \\ g_S^d = g_A - \frac{\gamma(1-\alpha)+1}{\gamma(1-\alpha)} g_h + l \\ g_z^d = -\frac{g_h}{\gamma} \ \textit{if } g_h > 0 \end{cases} \tag{17}$$

The second and fourth lines of System (17) show that the pollution emitted by the final goods producer is taxed such that it is in his every interest to reduce the polluting intensity of his production in a continuous manner. Reducing z mechanically reduces the producer's level of actual output. The final goods sector therefore requires less labour and thus fewer intermediate goods. Concerning R&D, neither the reduced wage level (due to the reduced labour demand from the final goods sector), nor the reduced profit of intermediate goods producers (related to the reduced demand for intermediate goods by the final goods sector) can affect the growth rate of innovations g_A , which is independent of all endogenous parameters (first line of System (17)).

The 'growth and environment' double-dividend or win-win effect can therefore be defined as follows: the double dividend exists in the regulated economy when there is both an improvement in the environmental quality (i.e. $g_S^d < 0$) and a production growth rate greater than that of the laissez-faire economy (no instruments).

Proposition 5 *In the steady state of the semi-endogenous growth model with pollution, the 'growth and environment' double-dividend does not operate.*

Proof. $g_c^d(g_h), g_c^{d\prime} < 0$: see second line of System (17). Thus the growth rate of a laissez-faire economy is always greater than the regulated economy: $g_c^d\big|_{(g_h=0)} > g_c^d\big|_{(g_h>0)}$. ■

Contrary to endogenous growth, the fact that an environmental tax increases the intensity of R&D (Equation (15): Lemma (3)) does not validate the Porter hypothesis in our model. The R&D intensity has no impact on the growth rate of innovations in the long term or on that of production. The long-term economic growth rate decreases with the increasing taxing of polluting emissions (second line of System (17)), regardless of the level of support for R&D (constant g_A, first line of System (17)). In the long term, R&D subsidies thus only have an impact on the economy's level of production but not on its growth rate.

4 Optimal Economic Policy

This section determines the levels of three instruments which can be used to match the long-term growth rates of optimal economies and decentralised economies. Whenever possible, we will only refer to the impact of the parameter characteristic of semi-endogenous growth, ϕ, on the instruments; other parameters of the model and their impact have already been discussed in the literature.

Lemma 6 *The optimal subsidy for purchasing intermediate goods, the subsidy (or tax) enabling R&D to use the optimal number of researchers ($n^o = n^d$), and the growth of tax on emissions equating the* *pollution stock growth rates of optimal and decentralised economies ($g_S^o = g_S^d$) can be written respectively:*

$$\begin{cases} \tau^o = 1-\alpha \\ 1-\sigma^o = \delta \frac{\alpha}{1-\tau^o} \frac{1+\kappa}{\kappa} \frac{(\varepsilon-1) g_A \left(\frac{\gamma(1-\alpha) - \frac{l}{g_A}}{\gamma(1-\alpha) + \frac{\varepsilon+\varpi}{1+\varpi}} \right) + \rho + l}{g_A \left[(\varepsilon-1) + \frac{\delta}{\kappa} - \frac{\frac{\varepsilon-1}{\gamma(1-\alpha)} g_h^o - \rho}{g_A} \right]} \\ g_h^o = \frac{\gamma(1-\alpha)\frac{\varepsilon+\varpi}{1+\varpi} g_A + \gamma(1-\alpha) l}{\gamma(1-\alpha) + \frac{\varepsilon+\varpi}{1+\varpi}} \\ g_A = \frac{l}{1-\phi} \end{cases} \tag{18}$$

As expected, the optimal subsidy for purchasing intermediate goods (first line of System (18)) is equal to $1-\alpha$ (Barro and Sala-i-Martin 1995). With this subsidy, the final goods producer pays $p_j \alpha$ for the intermediate good j which is equal to the marginal cost. α denotes the mark-up rate applied by the intermediate goods company which is equal to the elasticity of substitution between intermediate goods.[11]

The second line of System (18) shows that the optimal instrument for R&D has an undefined sign: it is possible that the instrument be a tax (negative σ^o). This case arises when decentralised research efforts are excessive with respect to what is required by the Pareto optimal. Intuitively, an economy can over-invest in R&D due to the creative destruction inherent to the vertical differentiation model. It enables the innovator to capture the entire rent flow. Yet this rent can be over-estimated. In this case, the creative destruction increases the private rate of return of R&D and positions it above the social rate of return.

Proposition 7 *In the long term, the optimal environmental policy is stricter in the semi-endogenous case, in the sense that the tax on polluting emissions increases when R&D is easier.*

Proof. $g_h^o(\phi), g_h^{o\prime} > 0$: see third line of System (18). ■

The easier research ($\phi \to 1$) proves to be, the more innovations will be produced, which increases the growth rate and the pollution (System (17)). The growth rate of the incentive tax must increase to limit the latter. Furthermore, we obtained the following environmental policy rule (like Grimaud and Ricci 2004): $h_t^o P_t^o / Y_t^o = 1/(1+\gamma)$, which can be re-written as $g_{h_t}^o = g_{Y_t}^o - g_{P_t}^o$. This rule is in line with Proposition (7).

5 Conclusion

In a growth model with pollution and a function of the production of innovations with a non-unitary yield on the accumulable factor, we have demonstrated that the growth path can become sustainable in the strong sense of the word under certain conditions. To be positive, the long-term exogenous growth rate therefore requires a knowledge stock yield above a given, strictly positive threshold (our first proposition: Proposition (2)). Intuitivelly, this threshold corresponds to a low level of difficulty in research. We have also demonstrated that a double dividend is impossible since the level of support for innovation no longer has a positive impact on the long-term growth rate (our second proposition: Proposition (5)). The environmental policy has a negative impact on the long-term growth and becomes more severe when R&D is easier (our third and last proposition: Proposition (7)). The long-term efficiency of policies designed to reduce polluting emissions is therefore seriously called into question by the semi-endogeneity of growth.

The ‘growth and environment’ double-dividend for endogenous growth configurations is no longer possible on a steady semi-endogenous growth path, but must we nevertheless consider that economic policy is futile if it does not increase growth in the long term? Most certainly not since the environmental policy will have achieved its objective, i.e. increasing social well-being. Sustainable semi-endogenous growth therefore provides us with the perfect opportunity to stress the importance of well-being as an ultimate economic objective.

[11] In this case, there is perfect substitutability between intermediate goods (Cobb-Douglas production function: Equation (1)), which reduces the mark-up rate and the market power of intermediate goods companies.

Finally, we would like to mention a few avenues of research which are worth exploring in the future. Firstly, it could be worth presenting the transition simulations calculated by our model, particularly when studying the transitory effect of the double dividend. Even if there is no double dividend in the long term, nothing proves that it will not be operational beyond ten or so years for example. But this criticism is probably not decisive in the case of climate change: very long-run pollutions require policies whose effects are persistent! Furthermore, empirical studies should be used this time to check that the semi-endogenous approach to growth is more plausible than the fully-endogenous approach when describing polluting economies.

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Appendices

Appendix A: Optimal Equilibrium

The common Hamiltonian resulting from programme (4) is equivalent to: $H = U(c,S) + \vartheta \dot{K} + \mu \dot{A} - \zeta \dot{S} = \frac{c_t^{1-\varepsilon}}{1-\varepsilon} - \frac{S_t^{1+\varpi}}{1+\varpi} + \vartheta \left[Y - Lc\right] + \mu\left[\delta n_t L_t A^{\phi}\right] - \zeta\left[P - \Theta S\right]$. ϑ_t represents the implicit cost of capital (or shadow price), while μ_t denotes the implicit price of innovations, and ζ_t the implicit price of the pollution stock.

The first-order conditions are as follows:
$$\begin{cases} c_t^{-\varepsilon} = \vartheta_t L_t \\ \vartheta_t = (\gamma+1)\,\zeta_t z_t^{\gamma} \\ \mu_t \delta L_t A_t^{\phi} = \vartheta_t\left(\frac{\gamma}{\gamma+1}\right)(1-\alpha)\,\frac{Y_t}{1-n_t} \\ \frac{\dot{\vartheta}_t}{\vartheta_t} = \rho - \left(\frac{\gamma}{\gamma+1}\right)\alpha \frac{Y_t}{K_t} \\ \frac{\dot{\mu}_t}{\mu_t} = \rho - g_A\left[\left(\frac{1-n_t}{n_t}\right) + \phi\right] \\ \frac{\dot{\zeta}_t}{\zeta_t} = \rho + \Theta - \frac{S_t^{\varpi}}{\zeta_t} \\ z_t = \begin{cases} 1 \text{ if } \vartheta_t \geq (\gamma+1)\,\zeta_t \\ \left[\frac{\vartheta_t}{(\gamma+1)\zeta_t}\right]^{\frac{1}{\gamma}} \text{ if } \vartheta_t < (\gamma+1)\,\zeta_t \end{cases} \\ \vartheta_t - (\gamma+1)\,\zeta z_t^{\gamma} \geq 0 \text{ if } z_t < 1 \end{cases}$$

Resolving this equation makes it possible to determine the optimal growth rates (5), knowing that in the long term $g_y = g_c = g_k$ (long-term condition associated with the capital variation equation) and $g_P = g_S$ (long-term condition associated with the pollution stock variation equation).

Appendix B: The Existence of the Optimal Growth Path

The conditions - three in total - are as follows:

- Growth rates and all sign or limit constraints on variables must be met.

Proof. Conditions (6), (7) and (8) concern the validity of these constraints.
With the first two Conditions (6) and (7), the following is obtained: $g_c^o > 0$, $g_S^o < 0$, $g_A^o > 0$ and $n^o \in [0,1]$.
For g_z^o : $g_z^o \leq 0$ if and only if $-\frac{\gamma(1-\alpha)l}{\frac{\varepsilon+\varpi}{1+\varpi}} \leq g_A$; this inequality is always verified if $g_A > 0$ (Equation (3)).
The remaining constraint $\frac{P}{S} > 0$ results in Condition (8). Under conditions (6) and (7): Condition (8) - corresponding to the minimum natural regeneration rate - is positive. ■

- The intertemporal utility of consumers must be limited: $\lim_{t\to+\infty} e^{-\rho t} U(c_t, S_t) = 0$.

Proof. $\lim_{t\to+\infty} e^{-\rho t} U(c_t, S_t) = 0 \iff \lim_{t\to+\infty} e^{-\rho t}\left[\frac{c_t^{1-\varepsilon}}{1-\varepsilon} - \frac{S_t^{1+\varpi}}{1+\varpi}\right] = 0 \iff \lim_{t\to+\infty} e^{-\rho t}\frac{\left(e^{g_c^o \cdot t} c_0\right)^{1-\varepsilon}}{1-\varepsilon} - \lim_{t\to+\infty} e^{-\rho t}\frac{\left(e^{g_S^o \cdot t} S_0\right)^{1+\varpi}}{1+\varpi} = 0$. A sufficient condition to meet the above-mentioned constraint consists in having: $g_c^o(1-\varepsilon) < 0$ and $g_S^o(1+\varpi) < 0$, which is always true with Conditions (6) and (7). ■

- The conditions of transversality must be verified: $\begin{cases} \lim_{t\to+\infty} e^{-\rho t}\vartheta_t K_t = 0 \ \textit{(i)} \\ \lim_{t\to+\infty} e^{-\rho t}\mu_t A_t = 0 \ \textit{(ii)} \\ \lim_{t\to+\infty} e^{-\rho t}\zeta_t S_t = 0 \ \textit{(iii)} \end{cases}$

Proof. For *(i)* : $\vartheta_t K_t$ increases at the long-term rate $(1-\varepsilon)\,g_c^o - l < 0$ therefore Condition *(i)* is verified. For *(ii)* : $\mu_t A_t$ increases at the long-term rate $(1-\varepsilon)\,g_c^o < 0$ therefore Condition *(ii)* is verified. For *(iii)* : $-\rho + g_\zeta + g_S < 0$ is necessary, which is equivalent to $\rho > \left(\frac{1-\varepsilon}{1+\varpi}\right) g_c$: *(iii)* is verified for $\varepsilon > 1$ (Condition (7)). ■

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