Increases in reported oil reserves among Opec producers: a differential game approach

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Abstract

In the 1980's, the Opec's decision to set members' export quotas of oil in proportion of individual reserves volumes led to a general rise of the proved reserves: between 1982 and 1988, they jumped from 467,39 to 760,48 billions of barrels. This gap highlights the fact that nonrenewable resources reserves also depend on the market structure the producers choose. The purpose of this article is twofold: first, we analyze the consequences of a change of oligopoly on the level of reserves and second, we will explore what happen when countries can mis-report some of their resources for strategical reasons.

In a differential game framework, oil producing countries can produce and explore as in the seminal Pindyck model. But sharing the same global demand, they can choose between two different market structures. In the non cooperative oligopoly, they determine control variables - extraction and exploration - in order to maximize their individual profits. In the "coopetitive" oligopoly - the term 'coopetition', first coined by Barry Nalebuff and Adam Brandenburger, designates the phenomenon of firms that cooperate and compete at the same time -, they choose a global extraction that will maximize the global profit: individual extractions are decided through a 'rule of quotas', that is each country will extract in proportion of its individual reserves. Then, the choice of exploration is made in a competitive way.

Through this framework we will show that, with homogeneous producers and under some assumptions, a jump in the reserves still happen with a change of the oligopoly structure from non cooperation to coopetition. Moreover, with heterogeneous producers, this change is not exactly the same for every countries.

In a second framework, we will show that when producers are furthermore able to mis-report some resources, the amount of "cheated reserves" will highly depend on the shape of the marginal exploration cost. Then, by observing the new equilibrium situation we can determine which countries over or under report some of its reserves.

Keywords: differential game, non-renewable resource, oligopoly, Opec cartel, stock mis-reporting.
JEL classification: C70; C71; Q35

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1 Introduction

There is a consensus on the fact that reserves estimates published by reporting and information agencies are over-inflated. *Information agencies*, such as the International Energy Agency, the U.S Energy Information Agency or the BP statistical review often reproduce the data from *reporting agencies* such as Oil & Gas Journal, World Oil magazine and the Opec Secretariat. As the petroleum engineer Laherrère (2001) points out: "publishing is a political act and depends upon the image the author wants to give". Reserves reported in official publications often are politically or financially motivated so that there are too much optimistic and not reliable. Independent authors generally prefer using *technical data*: purchased from scouting companies, such as IHS energy or WoodMackenzie, it comes directly from individual fields and is considered as more objective.

Trying to explain the gap between public and technical data, Owen et al. (2010) proposed that it could arises from: *a lack of international standards* about oil (grade, volume...), *various definitions* of what is a 'proved reserve' (the point at which resources are commercially exploitable with a certain probability), *intentional mis-reporting* due to financial or political reasons and last *technical assessment uncertainty*. Often pointed as a perfect illustration of an *intentional mis-reporting*, the Opec 'fight for quotas' occurred in the 1980s after OPEC countries agreement to set their export quotas in proportion to reserve volumes. It provided strategical behavior inside the cartel: Opec members inflated reported reserves in order to gain market shares. As a result, between 1982 and 1988, the Opec's proven reserves jumped from 467,39 Gb [1] to 760,48 Gb (see figures 1 and 2). In 2008, the Agency (2008) acknowledged that: "proven reserves worldwide have almost doubled since 1980. Most of the changes result from increases in official figures from Opec countries [...]. They were driven by negotiations at that time over production quotas and have little to do with the discovery of new reserves".

Laherrère Laherrère and Campbell (1998), insisting on the fact that these countries can report the reserves which correspond to their wish since it is never audited, estimated that, between 1986 and 1990, more than 287 Gb were added without any significant discoveries. The oil economist Salameh (2004) even suggested that Opec's proven reserves could have been overstated by 300 Gb. The same result is obtained by Deffeyes (2001) by subtracting out any abrupt jump during 1982-1988 from the proven reserves. There is a wide literature on over-inflated reserve estimates: a lot of independent authors thus try to revise downward global oil reserves in order to give an approximation of the Ultimate Recoverable Reserves. But, most of the time, these authors study Opec like a "black box" and work directly on statistics by correcting every suspicious jumps in datas. They can not explain for instance why some countries did not increase their reserves at all during this period (see for instance group 2 in figure 2).

In this article, we wish to analyze the reserve gap, that occurred during the 'fight for quotas', as the result of inner non cooperative behaviors among members inside the cooperative structure of the Opec. We will thus consider oligopolistic producers who do not cheat, but have the possibility to explore to add some new reserves, choosing thus the level of reserves they want in order to increase their profit. We will study two types of competition between them and compare what would be the impact on the level of reserves. Our model is typically an "oil-igopoly" model. Many authors built this kind of model to study exhaustible resource market: Loury (1986), Polasky (1996), Hartwick and Sadorsky (1990), Salant (1976). Our purpose is to study the particular structure of Opec cooperation where countries choose cooperatively a global production but then decide their own individual explorations. This mix of cooperation

[1] Billion of barrels.

and competition can be called "coopetition"[2]: this particular structure will have all our interest.

We will show that the level of reserves highly depends on the competition structure that choose the producers. We will also determine how this level evolves with the different parameters of costs and the number of countries that compose the cartel.

Then in an other framework where countries can cheat on its reserves, we will show that we can determine, by observing the extractions of the different countries, which producers cheat on its reserves. We have also found that the real reserves of some countries might be even lower than before, which means that they cheated more than the jump of the reserves we can observe. This is an interesting result because it means that the studies we quoted before which tried to correct the level of reserves in order to know the real ones should be corrected by the fact that some of the countries may have not cheated and that some other may have cheated more than what we thought.

This paper is organized as follows: in the first section, we expose the general framework of the model. The second section presents results for the case of a linear market demand, hyperbolic extraction costs and quadratic exploration costs. In the final section, we expose a different framework in order to study a case of cheating producers.

[2]The term 'coopetition', coined by Nalebuff and Brandenburger (1996), designates the phenomenon of firms that cooperate and compete at the same time.

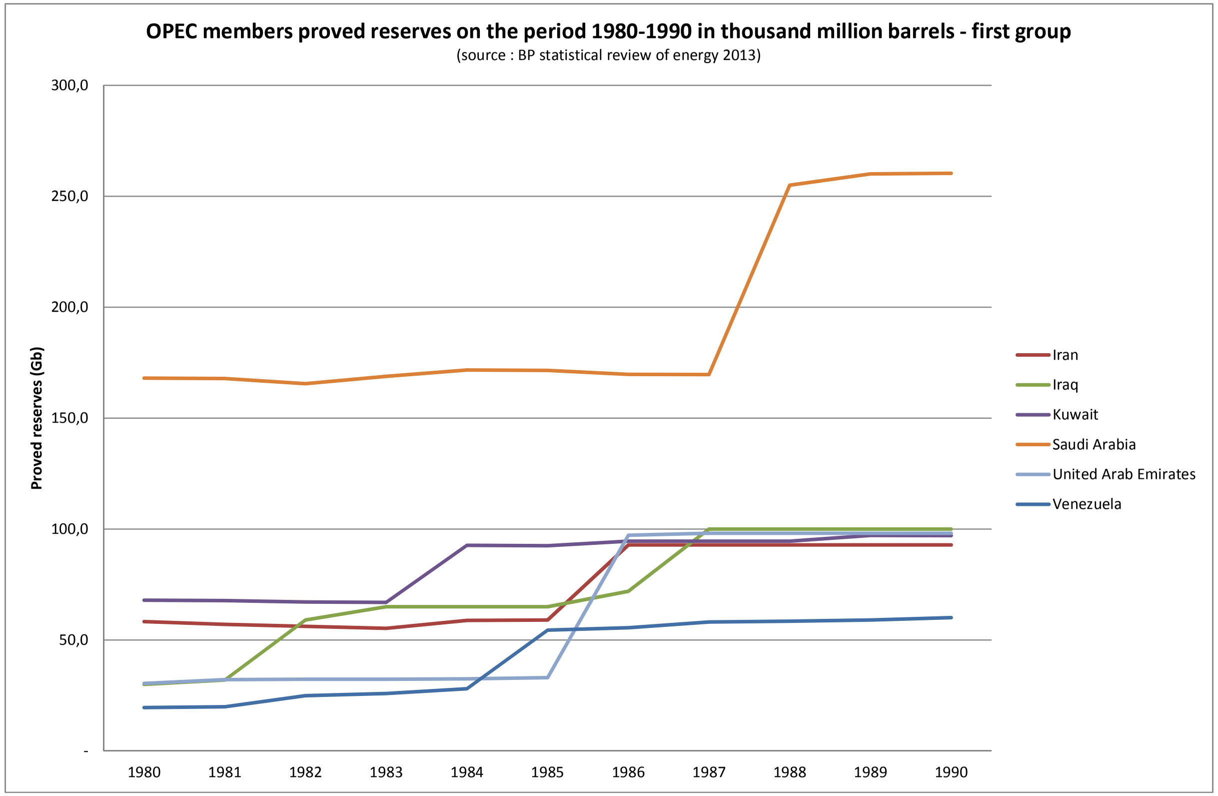

Figure 1: Opec members proven reserves on the period 1980-1990 (1/2).

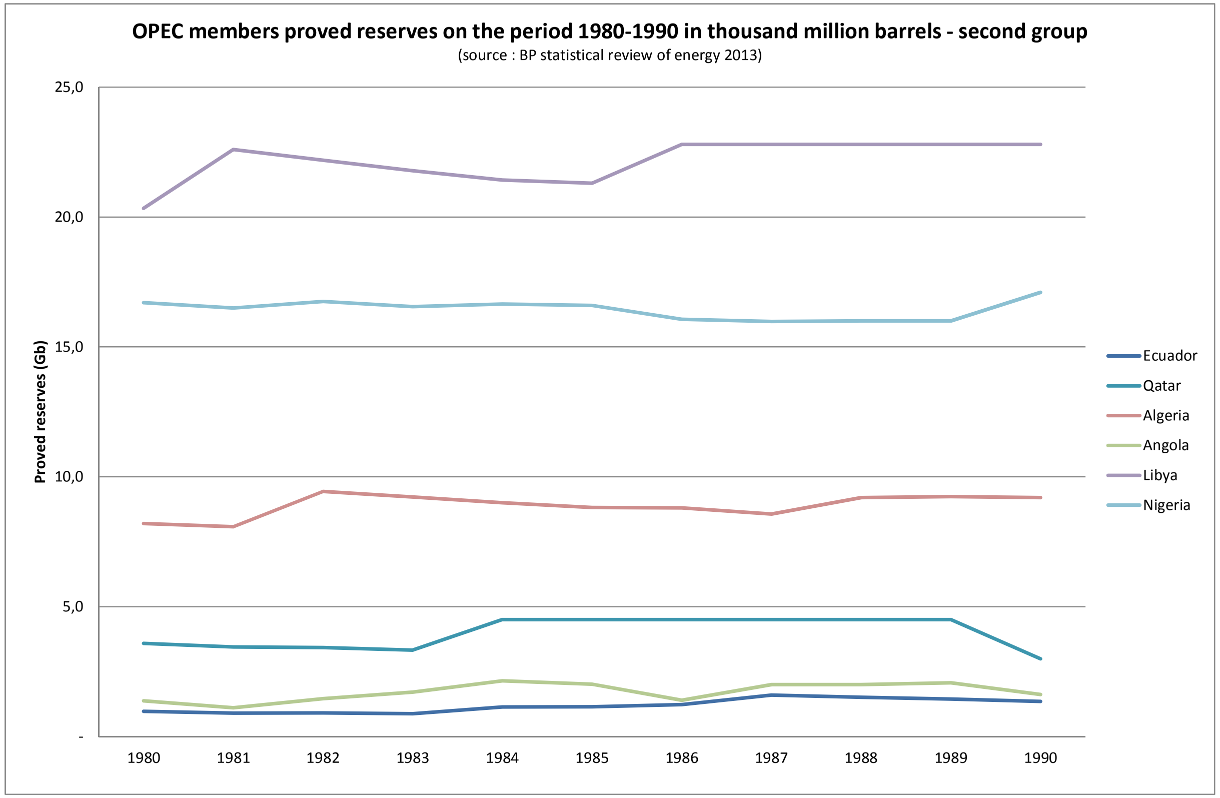

Figure 2: Opec members proven reserves on the period 1980-1990 (2/2).

2 General framework: a differential game

In our model, Opec members are in an oligopolistic competition. Each one is represented by an oil-extracting firm and begins with a known initial stock of oil. Knowing the market demand and facing average production costs increasing as the proved reserve is depleted, they make profit by extracting oil. As in the seminal paper of Pindyck Pindyck (1978), producers can also increase their proved reserves with exploration: the addition of new reserves x implies exploration costs $c_\beta(x)$. The exploration does not depend on cumulative discoveries - that is producers act as if the geological resources were infinite -: this leads to a long-run steady state level of reserve when extraction is exactly equal to exploration [3]. This model is then not a "depletion" model[4]. It is highly disputable to make such an assumption in the specific case of oil, but our concern is rather to compare equilibrium situations of different structures of oligopolies: these equilibria can be seen as "golden rules" that the producers would tend to reach in a short term period. The level of reserve becomes a variable that producers will indirectly control in order to maximize their profit. When the structure of the oligopoly changes or when the parameters are modified (with new technologies of extraction or new techniques of exploration for instance), the level of reserves will tend to an other equilibrium.

We will study two types of competitive structures[5]:

- *The non cooperative oligopoly* where each country chooses its extraction and exploratory effort to maximize its own profit function.
- *The coopetitive oligopoly* where extractions are chosen to maximize the global profit but, on the other hand, each country chooses its own exploration to maximize its individual profit. This is a partially cartelized organization.

This last structure, very particular, is similar to the current structure of the Opec. Members first decided production allocations at the 63rd meeting of the Opec Conference in 1982. Since then, they have regularly updated those allocations with the change in reserves of each countries. In this paper, we will consider that the gap occurred from a non cooperation situation to a coopetitive situation[6].

Producer i's profit can be written as:

$$\pi_i = \int_0^\infty e^{-\delta t}\left[p(q(t))q_i(t) - q_i(t)c_\alpha(R_i(t)) - c_\beta(x_i(t))\right]dt$$

(The "(t)" will be then omitted in the rest of the article). On the other hand, i's reserves will follow the following dynamic equation:

[3] This case is actually studied by Pindyck for nonrenewable such as Bauxite.

[4] Such issues as Peak Oil for instance cannot be analyzed with this approach.

[5] In a two-period, two-player exploration and extraction game, Hartwick (1991) made a similar comparison between three cases: price-taking producers, competition and the exact opposite of the case of 'coopetition' we discuss below.

[6] We consider that before the set of quotas, Opec was not really a cooperative oligopoly even it is true that between 1970 and 1980, it acted on price to influence the market.

$$\dot{R}_i = -q_i + x_i$$

Where q_i	:	country i's output.
x_i	:	country i's new additions to reserves.
R_i	:	country i's reserves.
$c_\alpha(R_i)$	:	country i's marginal extraction costs.
$c_\beta(x_i)$	:	country i's exploration costs.
$p(q)$	:	inverse demand function with q the total output.
δ	:	the discount rate.

The extraction cost is a decreasing and convex function of the level of reserves, that is $c'_\alpha < 0$ and $c''_\alpha \geq 0$: a producer extracts at first the cheapest oil thus the cost of extraction will increase as the proved reserve is depleted. When new wells are opened, it is equivalent to add new "cheap oil", then the extraction cost decreases. We also assume that $c_\alpha(0) = +\infty$: it means that a producer will never extract all the reserves. There is a choke price $p(0) \neq +\infty$ after which the producer will switch to a backstop technology. The price p is a decreasing and concave function of q, that is $p'(q) < 0$ and $p''(q) \leq 0$. The exploration cost is an increasing function of x, that is $c'_\beta > 0$.

In the next sections, we will present the two types of competition between producers and find the equilibrium situations.

2.1 Non cooperative oligopoly

In this competitive structure, the N producers do not cooperate at all: they choose their individual extractions and exploration in order to maximize their own individual profits:

$$\begin{cases} \max\limits_{q_i, x_i} \int_0^\infty e^{-\delta t} \left[p(q) q_i - q_i c_\alpha(R_i) - c_\beta(x_i) \right] dt \\ \text{s.t. } \dot{R}_i = -q_i + x_i \ \forall i \in [1, N] \end{cases}$$

As in the Hotelling model, we get this static efficiency condition:

$$p'(q)q_i + p(q) = c_\alpha(R_i) + \lambda_i$$

It means that the marginal revenue should be equal to the addition of the marginal cost of extraction and the shadow price of the resource extracted. We also get the following second static condition:

$$c'_\beta(x_i) = \lambda_i$$

It adds the fact that the marginal discovery cost c'_β should be also equal to the shadow price.

Then we get two dynamic efficiency conditions. The first one is:

$$\dot{\lambda}_i = \delta\lambda_i + q_i c'_\alpha(R_i)$$

It means that the shadow price evolves with the discount rate as in the Hotelling model. But now the term $q_i c'_\alpha(R_i)$ takes into account a stock effect on extraction costs: the fact that, because of today's extractions, the producer will face higher extraction costs in future time period. The second dynamic condition is:

$$\dot{R}_i = -q_i + x_i$$

It is simply the fact that when extraction is greater than addition to the reserves, the reserves decreases.

2.2 Coopetitive oligopoly

In this last oligopoly, the N producers only cooperate to decide the global production of the cartel. Then they chooses individually their own exploration decisions.

$$\begin{cases} \max\limits_{q} \int_0^\infty e^{-\delta t} \left[p(q)q - qC_\alpha((R_k)_{k\in[1,N]}) - \sum\limits_{k=1}^{N} c_\beta(x_k) \right] dt \\ \text{s.t. } \dot{R}_i = -q\frac{R_i}{R} + x_i \ \forall i \in [1,N] \end{cases}$$

Then, taking q as given from the Opec decision, each producer will chose his level of exploration in order to maximize his profit under his own resource constraint:

$$\begin{cases} \max\limits_{x_i} \int_0^\infty e^{-\delta t} \left[p(q)q\frac{R_i}{R} - q\frac{R_i}{R}c_\alpha(R_i) - c_\beta(x_i) \right] dt \\ \text{s.t. } \dot{R}_i = -q\frac{R_i}{R} + x_i \end{cases}$$

The main difference here with the non cooperation case is the existence of different shadow prices. It comes with the fact that producers defend at the same time the cartel's interest and their individual interests. Thus, the same stock of resource has two shadow price: from the viewpoint of the cartel and of the producer.

We get the following static efficiency conditions:

$$p'(q)q + p(q) - C_\alpha\left((R_k)_{k\in[1,N]}\right) = \sum_{k=1}^{N} \lambda_{opec,k}\frac{R_k}{R}$$

And:

$$c'_\beta(x_i) = \lambda_i$$

Then, we get three dynamic condition due to the double optimization the producers make (with the cartel and for themselves). The first one is the following:

$$\dot{\lambda}_{opec,i} = \delta\lambda_{opec,i} + q\frac{\partial C_\alpha\left((R_k)_{k\in[1,N]}\right)}{\partial R_i} + \frac{q}{R^2}\sum_{k\neq i} R_k\lambda_{opec,i} - \frac{q}{R^2}\sum_{k\neq i} R_k\lambda_{opec,k}$$

It describes the evolution of the shadow price from the cartel viewpoint:

- $q\frac{\partial C_\alpha\left((R_i)_{i\in[1,N]}\right)}{\partial R_i}$: the stock effect on extraction costs is now triple: because of today's extractions, the producer will face higher extraction costs but on the other hand with lower future reserves his quotas of extraction will become also lower and then it will tend to decrease his extraction costs. And third effect: the other producers' quotas of extraction will all increase and then it will tend to increase their extraction costs. Through the cartel viewpoint, each producer has to take into account what would be the consequences on other producers.
- $\frac{q}{R^2}\sum_{k\neq i}\lambda_{opec,i}R_k$: it takes into account that when the reserves of the country i decreases, this tend also - by the rule of quotas - to decrease its production $q_i = q\frac{R_i}{R}$. Then the physical scarcity of the resource will be lower in future periods.
- $-\frac{q}{R^2}\sum_{k\neq i}\lambda_{opec,k}R_k$: on the other hand, extractions of the other countries will tend - still by the rule of quotas - to increase. This increases the physical scarcity of their resources. Through the cartel viewpoint, it has to be taken into account by country i.

The second dynamic efficiency we get is the following:

$$\dot{\lambda}_i = \delta\lambda_i + q\frac{R_i}{R}c'_\alpha(R_i) + \frac{q}{R^2}\sum_{k\neq i} R_k c_\alpha(R_i) - p(q)\frac{q}{R^2}\sum_{k\neq i} R_k + \frac{q}{R^2}\sum_{k\neq i} R_k\lambda_i$$

It describes the evolution of the shadow prices of the resources but view from each producer:

- $q\frac{R_i}{R}c'_\alpha(R_i)+\frac{q}{R^2}\sum_{k\neq i} R_k c_\alpha(R_i)$: the stock effect is different because, individually, the producer does not take into account the consequences on the other producers.
- $-p(q)\frac{q}{R^2}\sum_{k\neq i} R_k$: there is a new stock effect on the revenue because of the rule of quotas. Indeed, from the cartel viewpoint, this effect did not exist because a change of individual reserves had no impact on the global production but rather on the repartition through countries.
- $\frac{q}{R^2}\sum_{k\neq i} R_k\lambda_i$: as we mentioned previously, when the country i's reserves are depleted, this tend - by the rule of quotas - to decrease its extraction.

The last dynamic condition is always the same:

$$\dot{R}_i = -q\frac{R_i}{R} + x_i$$

2.3 Steady state

We can prove that (see A.1 and A.2) at the equilibrium situation:

$$\boxed{\begin{cases} \forall i \;\; p'(\bar{q})\bar{q}_i + p(\bar{q}) - c_\alpha(\bar{R}_i) = -\frac{\bar{q}_i}{\delta} c'_\alpha(\bar{R}_i) \\ \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta} c'_\alpha(\bar{R}_i) \\ \bar{x}_i = \bar{q}_i \end{cases}}$$

(In the non cooperative oligopoly).

$$\boxed{\begin{cases} \forall i \;\; p'(\bar{q})\bar{q} + p(\bar{q}) - C_\alpha((\bar{R}_i)_{i\in[1,N]}) = -\frac{\bar{q}}{\delta} \sum_k \frac{\bar{R}_k}{\bar{R}} \frac{\partial C_\alpha((\bar{R}_i)_{i\in[1,N]})}{\partial R_k} \\ \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} + \frac{\frac{\bar{q}}{\bar{R}}\left(1-\frac{\bar{R}_i}{\bar{R}}\right)}{\delta + \frac{\bar{q}}{\bar{R}}\left(1-\frac{\bar{R}_i}{\bar{R}}\right)} \left(p'(\bar{q})\frac{\bar{R}}{\bar{R}_i} + \phi_i((\bar{R}_i)_{i\in[1,N]})\right) = -\frac{1}{\delta} c'_\alpha(\bar{R}_i) \\ \bar{x}_i = \bar{q}_i \end{cases}}$$

With:

$$\phi_i((\bar{R}_i)_{i\in[1,N]}) = \frac{1}{\bar{q}_i}\left(-C_\alpha\left((\bar{R}_i)_{i\in[1,N]}\right) + c_\alpha(\bar{R}_i) - \bar{q}\frac{\bar{R}_i}{\bar{R}}\frac{1}{\delta}c'_\alpha(\bar{R}_i) + \frac{\bar{q}}{\delta}\sum_{k=1}^{N}\frac{\bar{R}_k}{\bar{R}}\frac{\partial C_\alpha\left((\bar{R}_i)_{i\in[1,N]}\right)}{\partial R_k}\right)$$

(In the coopetitive oligopoly).

3 Linear demand, hyperbolic extraction costs and quadratic exploration costs

In this section, we propose an analytical resolution of the basic model for N oil producing countries we exposed previously. We will study the case of a linear demand, hyperbolic extraction costs and quadratic exploration costs, that is :

$$\begin{cases} p(q) = A - Bq \\ c_\alpha(R) = \frac{\alpha}{R} \\ c_\beta(x) = \beta x^2 \end{cases}$$

3.1 Homogeneous producers

Assuming that producers are homogenous, that is they share exactly the same parameters, we can calculate equilibria in each cases we saw before in the general framework. In the non cooperative oligopoly:

$$\boxed{\begin{cases} \bar{q} = N\frac{A-\sqrt{2\alpha\delta\beta}}{2\beta+(N+1)B} \\ \bar{x} = N\frac{A-\sqrt{2\alpha\delta\beta}}{2\beta+(N+1)B} \\ \bar{R} = N\sqrt{\frac{\alpha}{2\delta\beta}} \end{cases}}$$

And, in the coopetitive oligopoly:

$$\boxed{\begin{cases} \bar{q} = \delta\bar{R}\frac{A\bar{R}-\alpha N}{\alpha N^2+2N\delta B\bar{R}^2} \\ \bar{x} = \delta\bar{R}\frac{A\bar{R}-\alpha N}{\alpha N^2+2N\delta B\bar{R}^2} \\ 2\beta - \frac{\bar{q}\frac{N-1}{N\bar{R}}}{\delta+\bar{q}\frac{N-1}{N\bar{R}}}NB = \frac{N^2\alpha}{\delta\bar{R}^2} \end{cases}}$$

The reserve gap In this specific case, we immediately verify that there is a gap in the reserves between the non cooperation situation to the coopetition situation. Indeed, as $\bar{q} \geq 0$, we get in the coopetition case that:

$$\frac{N^2\alpha}{\delta\bar{R}^2} \leq 2\beta$$

And then:

$$\bar{R} \geq N\sqrt{\frac{\alpha}{2\delta\beta}}$$

Which is the value of the reserves in the non cooperation case. Then we can explain the reserve gap with a change of the competition structure between producers.

We will see in the next section how each parameters influence this reserve gap.

Influence of the parameters on the reserves and the reserve gap The influence of the parameters on the reserves in the non cooperation case is obvious and can be directly seen through the formula we have exposed before.

Yet, for the coopetitive case, it is more difficult. We can calculate that the third equation in the coopetitive oligopoly with homogeneous producers can be written as the following polynomial:

$$\psi(\bar{R}) = \bar{R}^4 + \frac{\delta A(N-1)(2\beta - NB)}{4\beta\delta^2 BN}\bar{R}^3 + \frac{\alpha\delta N(2\beta - N(N+1)B)}{4\beta\delta^2 BN}\bar{R}^2 - \frac{A(N-1)\alpha N^2}{4\beta\delta^2 BN}\bar{R} - \frac{N^3\alpha^2}{4\beta\delta^2 BN}$$

The polynomial ψ has a unique real solution on $\mathbb{R}^+$. Moreover ψ is negative on $[0, \bar{R}]$ and positive on $[\bar{R}, +\infty[$. as we proved in B.1.

In the coopetitive case, we can show that:

Proposition 1. *Assuming a linear demand, hyperbolic extraction costs, linear exploratory effort costs and linear additions to reserves, the reserve in the coopetitive oligopoly:*

- *Increases with* α.
- *Decreases with* β.
- *Increases with* A.
- *Increases with* B.
- *Increases with* N.

Proof. See B.3. □

With the help of the following lemma:

Lemma 1. *When* $2\beta > NB$ *we get that :*

$$\bar{R}^2 < \frac{N^2\alpha}{\delta(2\beta - NB)}$$

Proof. See B.2. □

Now we want to study how the reserve gap between these oligopolies evolves with the parameters. We will then write the polynomial ψ making appear $R_0 = N\sqrt{\frac{\alpha}{2\delta\beta}}$ into it. We call this new polynomial χ whose the reserve gap is a solution:

$$\chi(\frac{R}{R_0}) = \left(\frac{R}{R_0}\right)^4 + \frac{A(N-1)(2\beta - NB)}{2BN^2\sqrt{2\alpha\delta\beta}}\left(\frac{R}{R_0}\right)^3 + \frac{2\beta - N(N+1)B}{2BN^2}\left(\frac{R}{R_0}\right)^2 - \frac{A(N-1)\beta}{BN^2\sqrt{2\alpha\delta\beta}}\left(\frac{R}{R_0}\right) - \frac{\beta}{BN^2}$$

As with the polynomial ψ, we get that the polynomial χ, whose the equilibrium reserve gap $\frac{\bar{R}}{R_0}$ is a root, has a unique real solution on $\mathbb{R}^+$. Moreover χ is negative on $[0, \frac{\bar{R}}{R_0}]$ and positive on $[\frac{\bar{R}}{R_0}, +\infty[$.

As for ψ, we can show that:

Proposition 2. *Assuming a linear demand, hyperbolic extraction costs, linear exploratory effort costs and linear additions to reserves, the reserve gap :*

- *Decreases with α.*
- *Decreases with β.*
- *Increases with A.*
- *Increases with B.*
- *Increases with N.*

Proof. See B.4. □

With the help of the following lemma:

Lemma 2. *When $2\beta > NB$ we get that :*

$$\frac{R}{R_0} < \sqrt{\frac{2\beta}{2\beta - NB}}$$

Proof. Directly from the lemma 1 □

The very last property of the influence of N is interesting. We saw that in the non cooperative case, the individual equilibrium reserves, $\frac{R_0}{N} = \sqrt{\frac{\alpha}{2\delta\beta}}$, do not depend on N thus, when a member leaves the cartel, there is no change for the other members. In the coopetitive oligopoly, things are different: as we have just seen that N influences positively the reserve gap, it means that individual equilibrium reserves are modified when the number of members is changed. The levels of individual reserves reflect then the strategical interactions that occur into the coopetitive oligopoly.

3.2 Heterogeneous case

Now, let's assume that each producer has his own parameters for every cost functions: extraction and exploration (coefficients α and β will be indexed by i). In the non cooperative oligopoly, we get that:

$$\boxed{\begin{cases} \bar{q}_i = \frac{p(\bar{q}) - \sqrt{2\alpha_i\beta_i\delta}}{2\beta_i + B} \\ \bar{x}_i = \frac{p(\bar{q}) - \sqrt{2\alpha_i\beta_i\delta}}{2\beta_i + B} \\ \bar{R}_i = \sqrt{\frac{\alpha_i}{2\delta\beta_i}} \\ \text{With } p(\bar{q}) = \frac{A + B\sum_{k=1}^{N}\frac{\sqrt{2\alpha_k\beta_k\delta}}{2\beta_k + B}}{1 + \sum_{k=1}^{N}\frac{B}{2\beta_k + B}} \end{cases}}$$

And in the coopetitive oligopoly:

$$\left\{\begin{array}{l} \bar{q} = \delta\bar{R}\frac{A\bar{R}-N\alpha}{N\alpha+2B\delta\bar{R}^2} \\ \bar{x} = \delta\bar{R}\frac{A\bar{R}-N\alpha}{N\alpha+2B\delta\bar{R}^2} \\ 2\beta_i + \frac{\frac{\bar{q}}{\bar{R}}\left(1-\frac{\bar{R}_i}{\bar{R}}\right)}{\delta+\frac{\bar{q}}{\bar{R}}\left(1-\frac{\bar{R}_i}{\bar{R}}\right)}\left[-B\frac{\bar{R}}{\bar{R}_i} + \phi((\bar{R}_i)_{i\in[1,N]})\right] = \frac{\alpha_i}{\delta\bar{R}_i^2} \end{array}\right.$$

Denoting $\alpha = \frac{1}{N}\sum_{k=1}^{N}\alpha_k$ and $1/\beta = \frac{1}{N}\sum_{k=1}^{N}1/\beta_k$.

Where the function ϕ here is equal to:

$$\phi_i((\bar{R}_i)_{i\in[1,N]}) = \frac{1}{\delta\bar{q}_i}\left(\delta + \frac{\bar{q}}{\bar{R}}\right)\left(\frac{\alpha_i}{\bar{R}_i} - \frac{N\alpha}{\bar{R}}\right)$$

In the coopetitive oligopoly, we can notice that:

$$\sum_{i=1}^{N}\left(\frac{\bar{R}_i}{\bar{R}}\right)^2\phi_i((\bar{R}_i)_{i\in[1,N]}) = 0$$

Thus this implies that ϕ_i will be positive for some producers and negative for others. The consequence is that when it is positive (it means when $\frac{\bar{R}_i}{\bar{R}} < \frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}$), the reserves can then be sometimes very close to the level of the non cooperative oligopoly as we can see on the figures 3 and 4. These figures represent the reserves of four countries with different value of α or β. We can see that some of them have almost the same level of reserves than before ($\beta = 0.3$ for instance). This could explain why some of the countries did not choose to increase their reserves during the fight for quotas period.

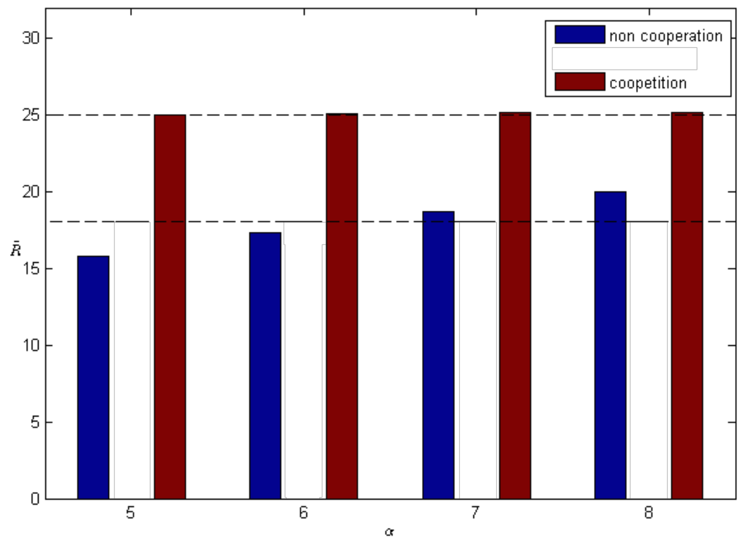

Figure 3: Influence of α on the reserve gap.

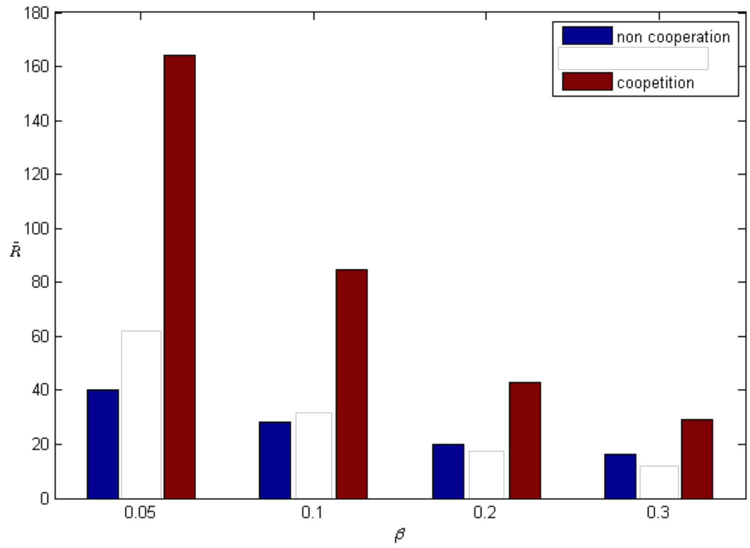

Figure 4: Influence of β on the reserve gap.

4 The case of cheating

We will now study a case where countries can mis-report their reserves. Indeed they will be able now to choose between two ways of changing their levels of reserves: exploration, as before, and an alternative way which is adding some new oil, but without any effect on the extraction cost. Then, there are "real reserves" which have an impact on extraction cost and "false reserves" which have not but are used only for strategical reasons.

4.1 About reserves declaration and misreporting

Having a closer look to data published about oil reserves, we can often notice great differences between sources. According to Owen et al. (2010), there is three types of sources. First, there are the *reporting agencies* (Oil & Gas Journal, World Oil magazine and Opec Secretariat): most of the available public data comes from the surveys they conduct. These sources are politically sensitive and often give optimistic estimates since, as Simmons (2007) pointed out, if a few countries have accepted a third party audit, none of the Opec producers did. The second type of sources are the *information agencies* such as International Energy Agency, the US Energy Information Agency and BP Statistical Review: they often take their data from *reporting agencies* with small corrections. The last source are technical data directly taken from individual fields by oil expert companies such as IHS or Wood Mackenzie.

At the end, most of the public information is coming from *reporting agencies* and there is a consensus that this kind of sources publish over-inflated reserve estimates. Owen et al. (2010) highlights four types of ambiguities that can explain differences in data through these sources:

1. A problem of definition of reserves as commercially extractable resource.
2. A lack of binding international standards to report oil reserve volume and grade.
3. Technical uncertainty.
4. Intentional mis-reporting for political reasons.

(1) The notion of reserves is indeed based on a probability system which splits resources into what is undiscovered, what is discovered but not economically extractable and reserves. Opec Secretariat does not include for instance unconventional oil such as tar sands in Canada.

(2) Reserves themselves are split into three categories (1P, 2P and 3P). Each of these categories correspond to a probability of successfully producing oil, 1P-reserves means for instance that you will be able to produce successfully 90 % of the amount considered. Laherrére (2009) points out that in Russia, companies declare 3P-reserves when US companies had to declare 1P-reserves until 2010. There is also a problem on measure unities: depending on the density we take, one barrel can represent many possible volumes.

(3) Technical problem can be a source of ambiguity in reserves estimates.

(4) This last point is the type of cheating on data we will focus on. Opec countries are indeed highly suspected to have increased their reserves during the period we focus on. As we say in introduction, a lot of studies estimate that 300 Gb could be withdrawn from the current reserves. Since there is no audit, countries can add the amount of reserves they want. Why won't they

add infinite reserves then ? Increasing global reserves has an impact on many variables as we saw in the case with no cheating: it implies a new distribution of quotas. A country has then not interest to increase indefinitely its reserves with cheated ones: there will be a level after which it will not be interesting anymore for its own profit.

4.2 Framework

The new profit function for a producer i will be:

$$\pi_i = \int_0^\infty e^{-\delta t} \left[p(q(t))q_i(t) - q_i(t)c_\alpha(R_i(t) - Z_i(t)) - c_\beta(x_i(t)) \right] dt$$

(The "(t)" will be then omitted in the rest of the article). The reserves will follow the following dynamic equation:

$$\begin{aligned} \dot{R}_i &= -q_i + x_i + z_i \\ \dot{Z}_i &= z_i \end{aligned}$$

$z_i(t)$ is the amount a producer chooses to add or remove from his reserves (this variable can be negative). The idea then is that through this variable, a producer can change his reserves uncostly but also with no impact on the extraction costs (because those reserves are not real ones). He will have thus to arbitrate between increasing his reserves by exploration - in order to decrease his extraction cost - or by cheating - in order to change his position in the cartel through the rule of quotas.

The incentive to cheat comes when reserves becomes a strategical variable. Then, we will only apply this framework to the coopetitive case. In the non cooperation case, producers have no incentive to cheat because their reserves are not linked two any control variables.

4.2.1 Dynamic equations

Now in the coopetitive oligopoly, we will see that producers have interest to cheat in order to gain market shares. The maximization program will be the following for the Opec:

$$\begin{cases} \max\limits_q \displaystyle\int_0^\infty e^{-\delta t} \left[p(q)q - qC_\alpha((R_k)_{k\in[1,N]}) - \sum_{k=1}^N c_\beta(x_k) \right] dt \\ \text{s.t. } \dot{R}_i = -q\frac{R_i}{R} + x_i + z_i \; \forall i \in [1, N] \end{cases}$$

In this case, the Opec takes the information which is given by producers. Then they calculate global production with the reserves the producers give, taking it as real reserves. The quotas of production are also calculated from the false reserves $q_i = q\frac{R_i}{R}$.

In the same time, taking q as given from the Opec decision, each producer will chose his exploratory effort and cheating in order to maximize his profit under his own resource constraint:

$$\begin{cases} \max\limits_{x_i, z_i} \int_0^{\infty} e^{-\delta t} \left[p(q) q \dfrac{R_i}{R} - q \dfrac{R_i}{R} c_\alpha (R_i - Z_i) - c_\beta (x_i) \right] dt \\ \text{s.t. } \dot{R}_i = -q \frac{R_i}{R} + x_i + z_i \\ \dot{Z}_i = z_i \end{cases}$$

The producer makes his calculations with the real reserves he has in the ground $R_i - Z_i$ when he declared R_i to the Opec.

The static and dynamic efficiency conditions for the Opec will be the same than before:

$$\begin{cases} p'(q)q + p(q) - C_\alpha \left((R_k)_{k \in [1,N]} \right) = \sum\limits_{k=1}^{N} \lambda_{opec,k} \dfrac{R_k}{R} \\ \dot{\lambda}_{opec,i} = \delta \lambda_{opec,i} + q \frac{\partial C_\alpha \left((R_k)_{k \in [1,N]} \right)}{\partial R_i} + \frac{q}{R^2} \sum\limits_{k \neq i} R_k \lambda_{opec,i} - \frac{q}{R^2} \sum\limits_{k \neq i} R_k \lambda_{opec,k} \end{cases}$$

And almost as before, we get for an individual producer:

$$\begin{cases} \dot{\lambda}_i = \delta \lambda_i + q \frac{R_i}{R} c'_\alpha (R_i - Z_i) + \frac{q}{R} \left(1 - \frac{R_i}{R} \right) c_\alpha (R_i - Z_i) - p(q) \frac{q}{R} \left(1 - \frac{R_i}{R} \right) + \frac{q}{R} \left(1 - \frac{R_i}{R} \right) \lambda_i \\ c'_\beta (x_i) = \lambda_i \end{cases}$$

This is actually the same condition than in the no cheating model except that the extraction cost and marginal extraction cost take as argument the "real reserves" $(R_i - Z_i)$.

But now, there is also a new shadow price μ_i due to the new dynamic constraint $\dot{Z}_i = z_i$. We get then this new dynamic condition:

$$\dot{\mu}_i = \delta \mu_i - q \frac{R_i}{R} c'_\alpha (R_i - Z_i)$$

And also the following static condition:

$$0 = \mu_i + \lambda_i$$

4.2.2 Steady state

In the non cooperative case, the steady states will be exactly the same because countries will not cheat as we have seen before.

In coopetition, at the equilibrium, we can prove that (see C.1):

$$\begin{cases} \forall i \;\; p'(\bar{q})\bar{q} + p(\bar{q}) - C_\alpha((\bar{R}_i)_{i\in[1,N]}) = -\frac{\bar{q}}{\delta}\sum\limits_k \frac{\bar{R}_k}{\bar{R}}\frac{\partial C_\alpha((\bar{R}_i)_{i\in[1,N]})}{\partial R_k} \\ \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta}c'_\alpha(\bar{R}_i - \bar{Z}_i) \\ c'_\beta(\bar{x}_i) = -c_\alpha(\bar{R}_i - \bar{Z}_i) + p(\bar{q}) \\ \bar{x}_i = \bar{q}_i \end{cases}$$

4.3 Linear demand, hyperbolic extraction cost and linear marginal exploration cost

With the same specific functions we have taken in the analytical case as before, we can get that:

$$\begin{cases} \bar{q} = \delta\bar{R}\frac{A\bar{R}-N\alpha}{N\alpha+2B\delta\bar{R}^2} \\ \bar{x} = \delta\bar{R}\frac{A\bar{R}-N\alpha}{N\alpha+2B\delta\bar{R}^2} \\ \frac{\bar{R}_i}{\bar{R}} = \frac{1}{\bar{q}}\left[\frac{A-B\bar{q}}{2\beta_i} - \delta\sqrt{\frac{\alpha_i}{2\delta\beta_i}}\right] \\ \bar{R}^2\left[\delta A(2\beta - NB) + 2\delta^2 B\beta\sum\limits_{k=1}^{N}\sqrt{\frac{\alpha_i}{2\delta\beta_i}}\right] - \bar{R}\left[N\alpha(2\beta+NB)\right] - N\alpha\left[NA - 2\delta\beta\sum\limits_{k=1}^{N}\sqrt{\frac{\alpha_i}{2\delta\beta_i}}\right] = 0 \\ \bar{Z}_i = \bar{R}_i - \sqrt{\frac{\alpha_i}{2\delta\beta_i}} \end{cases}$$

With $\alpha = \frac{1}{N}\sum\limits_{k=1}^{N}\alpha_k$ and $1/\beta = \frac{1}{N}\sum\limits_{k=1}^{N}1/\beta_k$.

The last equation $\bar{Z}_i = \bar{R}_i - \sqrt{\frac{\alpha_i}{2\delta\beta_i}}$ is interesting because it means that, in this particular case of a linear marginal exploration cost, the increase of reserves from no cooperation to coopetition is only cheated. We can indeed recognize the equilibrium reserves of the non cooperation case $\sqrt{\frac{\alpha_i}{2\delta\beta_i}}$ then it means that the whole gap in reserves between non cooperation to coopetition is cheated. This would give support to Deffeyes (2001), Salameh (2004) and others authors who think that the so-called fight for quotas is a purely political period.

Then to correct the reserves and know the real ones, we would just have to remove every jumps that occurred during the fight for quotas. This would give us the real reserves of oil.

4.4 The case of non linear marginal exploration cost

In every analytical cases, we have assumed a linear marginal exploration cost ($c_\beta(x) = \beta x^2$). What happen if this cost is not linear, that is, if:

$$c'_\beta(x) = \beta x^\theta$$

We will assume that $\theta \geq 2$[7]. We can show that:

Proposition 3. *if and only if:*

- *If* $\bar{q}_i|_{coopetition} \leq \bar{q}_i|_{no\ cooperation}$*, the real reserves of a cheating producer will be greater in coopetition than in no cooperation.*
- *If* $\bar{q}_i|_{coopetition} \geq \bar{q}_i|_{no\ cooperation}$*, the real reserves of a cheating producer will be lower in coopetition than in no cooperation.*

Proof. (See C.2.) □

Thus, equilibrium productions together with the form of the marginal exploration cost reveal information about cheated reserves. A contraction of the production of a country together with an elasticity of marginal exploration cost greater than one reveals for instance that the increase of reserves is not totally cheated: a part of it really comes from exploration. At the opposite, if the production is, the real reserves of coopetition is lower than in non cooperation, it means that the increase of the reserves is totally cheated and, even worse, the producer has in fact decrease his real reserves. This can be seen on the figure 5 that summarizes this result. When the production of the coopetition case is greater than non cooperation production, the ratio of real reserves $\Delta\bar{R} = \bar{R}|_{\text{coopetition}} - \bar{R}|_{\text{non cooperation}}$ is negative.

If the assumption we made on $\theta \geq 2$ is verified, it means for instance that Saudi Arabia cheated on its reserves because its production increased (see figure 6) between 1988 and 1990. At the opposite, Kuwait did not cheat on its reserves because its production remained nearly the same.

[7]Without this assumption, the problem is much harder: we found multiple equilibria.

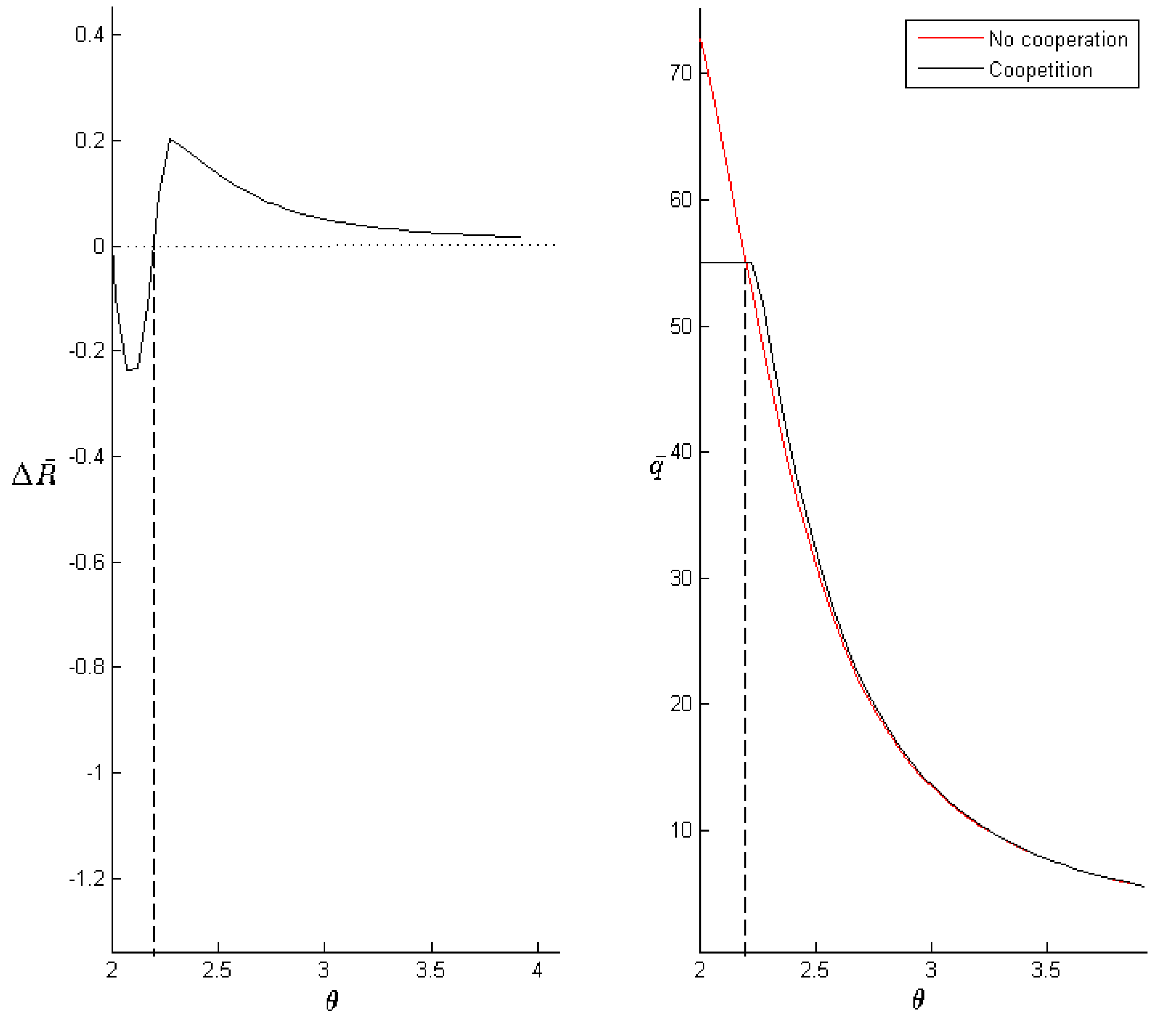

Figure 5: Variation of real reserves revealed by equilibrium production.

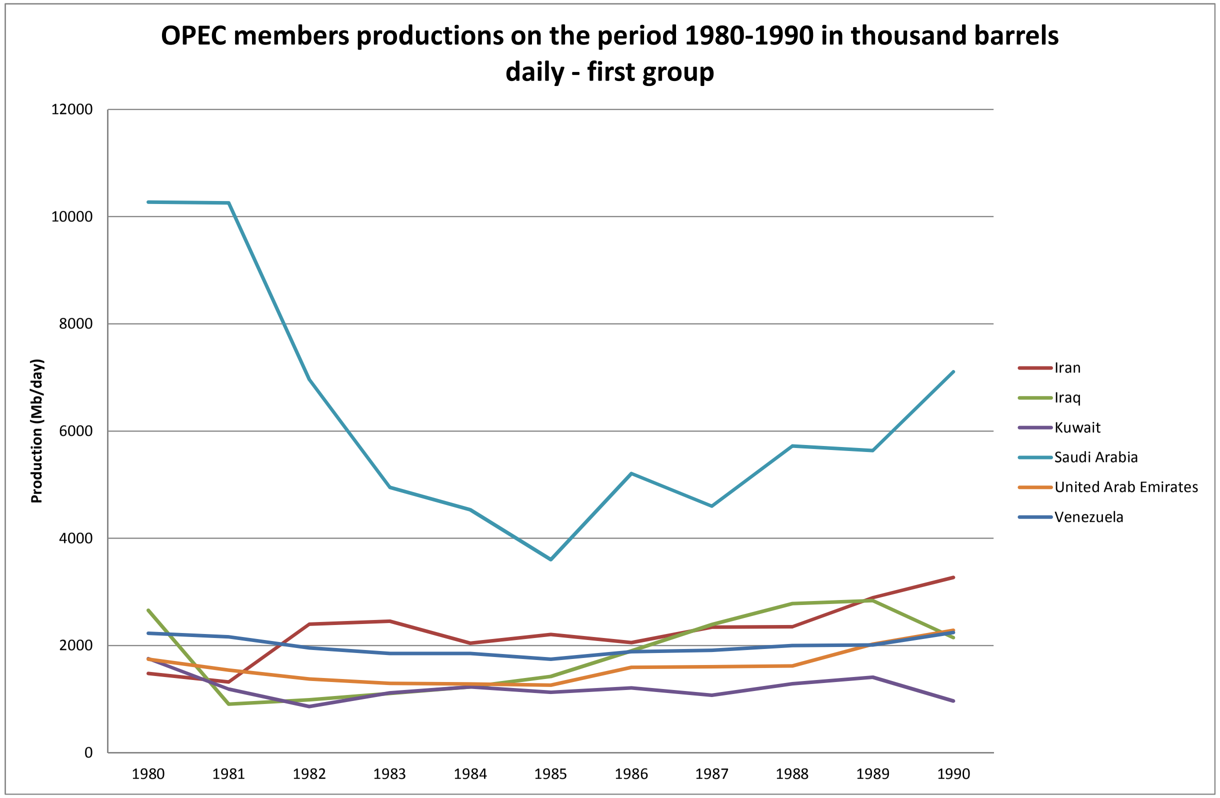

Figure 6: Opec members production on the period 1980-1990 (1/2).

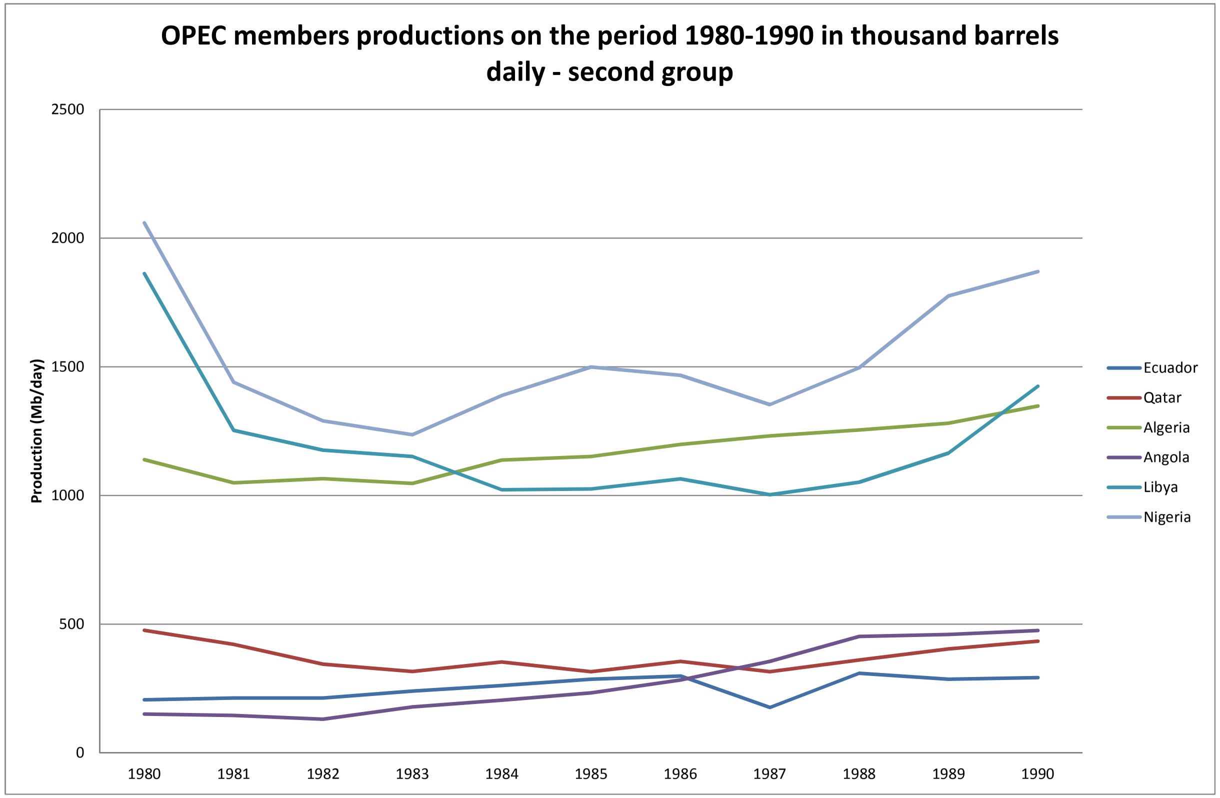

Figure 7: Opec members production on the period 1980-1990 (2/2).

5 Conclusion

In this article, we showed first that the level of reserves of a non renewable resource highly depends on the structure of the competition between producers. We found that in a coopetitive structure, like the one that runs currently in the Opec cartel, the level of reserves will be higher than in a non cooperative oligopoly for strategical reasons. We detailed how each parameters influence the level of reserves that will be optimum for producers and we saw that if some of them have interest to highly change their reserves with a switch of competition structure, other will keep the same level.

Then, in a second part of the paper, we found that the incentive to add some cheated reserves to the total amount depends on the shape of the marginal discovery cost function. Then, knowing this shape for a country and observing his choice of production, we can know what will be the amount of cheated reserves. Then it means that in order to correct the reserves to know the real ones, we should take into account that some of countries did not cheat and that at the opposite some of them even decrease their real reserves.

A General framework: a differential game

A.1 Non cooperative oligopoly equilibrium situation

We saw that the differential game is described in this case by the following system:

$$\begin{cases} p'(q)q_i + p(q) - c_\alpha(R_i) = \lambda_i & (1) \\ \lambda_i = c'_\beta(x_i) & (2) \\ \dot{\lambda}_i = \delta\lambda_i + q_i c'_\alpha(R_i) & (3) \\ \dot{R}_i = -q_i + x_i & (4) \\ \forall i \in [1, N] \end{cases}$$

At the equilibrium, we get that:

$$\begin{cases} \dot{\lambda}_i = 0 \Leftrightarrow \bar{\lambda}_i = -\frac{\bar{q}_i}{\delta} c'_\alpha(\bar{R}_i) \\ \dot{R}_i = 0 \Leftrightarrow \bar{q}_i = \bar{x}_i \\ \forall i \in [1, N] \end{cases}$$

And then, directly, we finally get that:

$$\begin{cases} \forall i \;\; p'(\bar{q})\bar{q}_i + p(\bar{q}) - c_\alpha(\bar{R}_i) = -\frac{\bar{q}_i}{\delta} c'_\alpha(\bar{R}_i) \\ \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta} c'_\alpha(\bar{R}_i) \\ \bar{x}_i = \bar{q}_i \end{cases}$$

A.2 Coopetitive oligopoly equilibrium situation

We saw that the differential game is described in this case by the following system:

$$\begin{cases} p'(q)q + p(q) - C_\alpha\left((R_k)_{k\in[1,N]}\right) = \sum_{k=1}^{N} \lambda_{opec,k} \frac{R_k}{R} & (1) \\ \lambda_i = c'_\beta(x_i) & (2) \\ \dot{\lambda}_{opec,i} = \delta\lambda_{opec,i} + q\frac{\partial C_\alpha\left((R_k)_{k\in[1,N]}\right)}{\partial R_i} + \frac{q}{R^2}\sum_{k\neq i} R_k \lambda_{opec,i} - \frac{q}{R^2}\sum_{k\neq i} R_k\lambda_{opec,k} & (3) \\ \dot{\lambda}_i = \delta\lambda_i + q\frac{R_i}{R}c'_\alpha(R_i) + \frac{q}{R}\left(1 - \frac{R_i}{R}\right)c_\alpha(R_i) - p(q)\frac{q}{R}\left(1 - \frac{R_i}{R}\right) + \frac{q}{R}\left(1 - \frac{R_i}{R}\right)\lambda_i & (4) \\ \dot{R}_i = -q\frac{R_i}{R} + x_i & (5) \\ \forall i \in [1, N] \end{cases}$$

At the equilibrium, we get that:

$$\begin{cases} \dot{\lambda}_{opec,i} = 0 \Leftrightarrow \bar{\lambda}_{opec,i}(\delta + \frac{\bar{q}}{R}) = -\bar{q}\frac{\partial C_\alpha\left((\bar{R}_i)_{i\in[1,N]}\right)}{\partial R_i} + \frac{\bar{q}}{R}\sum_{k=1}^{N} \bar{\lambda}_{opec,k}\frac{\bar{R}_k}{R} \\ \dot{\lambda}_i = 0 \Leftrightarrow \bar{\lambda}_i\left[\delta + \frac{\bar{q}}{R}\left(1 - \frac{\bar{R}_i}{R}\right)\right] = -\bar{q}\frac{\bar{R}_i}{R}c'_\alpha(\bar{R}_i) - \frac{\bar{q}}{R}\left(1 - \frac{\bar{R}_i}{R}\right)c_\alpha(\bar{R}_i) + p(\bar{q})\frac{\bar{q}}{R}\left(1 - \frac{\bar{R}_i}{R}\right) \\ \dot{R}_i = 0 \Leftrightarrow \bar{q}\frac{\bar{R}_i}{R} = \bar{q}_i = \bar{x}_i \\ \forall i \in [1, N] \end{cases}$$

By multiplying by $\frac{R_i}{R}$ and then summing the expression of $\bar{\lambda}_{opec,i}$, we get that:

$$\sum_{i=1}^{N} \frac{R_i}{R} \bar{\lambda}_{opec,i}(\delta + \frac{\bar{q}}{\bar{R}}) = -\bar{q} \sum_{i=1}^{N} \frac{R_i}{R} \frac{\partial C_\alpha \left((\bar{R}_i)_{i\in[1,N]}\right)}{\partial R_i} + \frac{\bar{q}}{\bar{R}} \sum_{i=1}^{N} \frac{R_i}{R} \sum_{k=1}^{N} \bar{\lambda}_{opec,k} \frac{\bar{R}_k}{\bar{R}}$$

And then:

$$\sum_{k=1}^{N} \bar{\lambda}_{opec,k} \frac{\bar{R}_k}{\bar{R}} = -\frac{\bar{q}}{\delta} \sum_{k=1}^{N} \frac{\bar{R}_k}{\bar{R}} \frac{\partial C_\alpha \left((\bar{R}_i)_{i\in[1,N]}\right)}{\partial R_k}$$

We can the write that:

$$p'(\bar{q})\bar{q} + p(\bar{q}) - C_\alpha((\bar{R}_i)_{i\in[1,N]}) = -\frac{\bar{q}}{\delta} \sum_{k} \frac{\bar{R}_k}{\bar{R}} \frac{\partial C_\alpha((\bar{R}_i)_{i\in[1,N]})}{\partial R_k}$$

At the equilibrium, we also get that:

$$\bar{\lambda}_i \left[\delta + \frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)\right] = -\bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i) \left[\delta + \frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)\right] - \frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right) \left[c_\alpha(\bar{R}_i) - p(\bar{q}) - \bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i)\right]$$

Thus:

$$\bar{\lambda}_i = -\bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i) - \frac{\frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)}{\delta + \frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)} \left[c_\alpha(\bar{R}_i) - p(\bar{q}) - \bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i)\right]$$

By substituting the expression of $p(\bar{q})$ in this last expression, we can write that:

$$\bar{\lambda}_i = -\bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i) - \frac{\frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)}{\delta + \frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)} \left[p'(\bar{q})\bar{q} + c_\alpha(\bar{R}_i) - C_\alpha((\bar{R}_i)_{i\in[1,N]}) + \frac{\bar{q}}{\delta} \sum_{k} \frac{\bar{R}_k}{\bar{R}} \frac{\partial C_\alpha((\bar{R}_i)_{i\in[1,N]})}{\partial R_k} - \bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i)\right]$$

And then, remembering that $\bar{\lambda}_i = c'_\beta(\bar{x}_i)$:

$$\frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta} c'_\alpha(\bar{R}_i) - \frac{\frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)}{\delta + \frac{\bar{q}}{\bar{R}} \left(1 - \frac{\bar{R}_i}{\bar{R}}\right)} \left[p'(\bar{q}) \frac{\bar{R}}{\bar{R}_i} + \frac{1}{\bar{q}_i} \left(c_\alpha(\bar{R}_i) - C_\alpha((\bar{R}_i)_{i\in[1,N]}) + \frac{\bar{q}}{\delta} \sum_{k} \frac{\bar{R}_k}{\bar{R}} \frac{\partial C_\alpha((\bar{R}_i)_{i\in[1,N]})}{\partial R_k} - \bar{q} \frac{\bar{R}_i}{\bar{R}} \frac{1}{\delta} c'_\alpha(\bar{R}_i)\right)\right]$$

B Analytical case: linear demand, hyperbolic extraction cost and linear marginal exploration cost

B.1 ψ has a unique positive real solution

The polynomial that verifies $\bar{R}$ can be written as:

$$\bar{R}^4+\frac{\delta A(N-1)(2\beta-NB)}{4\beta\delta^2BN}\bar{R}^3+\frac{\alpha\delta N(2\beta-N(N+1)B)}{4\beta\delta^2BN}\bar{R}^2-\frac{A(N-1)\alpha N^2}{4\beta\delta^2BN}\bar{R}-\frac{N^3\alpha^2}{4\beta\delta^2BN}$$

Thus the signs of the coefficients give us informations on solutions. There are three types of solutions for this kind of polynomial:

- Four complex solutions. Here it is impossible because of the negative constant which means that the product of the solutions is negative.
- Two complex and two real solutions. This case is possible and therefore the negative product of solutions implies that the real solutions have opposite signs (then there is a unique positive solution).
- Four real solutions. There are two possibilities: a unique positive solution and three negative or three positive solutions and a unique negative. We will now show that this last case is actually impossible.

We have to study the derivative of the polynomial:

$$4\bar{R}^3+\frac{3\delta A(N-1)(2\beta-NB)}{4\beta\delta^2BN}\bar{R}^2+\frac{2\alpha\delta N(2\beta-N(N+1)B)}{4\beta\delta^2BN}\bar{R}-\frac{A(N-1)\alpha N^2}{4\beta\delta^2BN}$$

The negative constant allows us to say that the product of solutions would be this time positive. In the case of 'four real solutions with a unique negative' we focus on, there should be then three positive real solutions to this derivative polynomial.

But if the coefficient in $\bar{R}^2$ is positive (that is $2\beta - NB > 0$), then the sum of the solutions is negative and there is a contradiction with the fact that there all are positive.

If the coefficient in $\bar{R}^2$ is negative (that is $2\beta - NB < 0$), then the coefficient in $\bar{R}$ is also negative (because $2\beta - N(N+1)B < 2\beta - NB < 0$) and the 2 by 2 solutions products should be negative which is also in contradiction with the fact that there all are positive.

In conclusion, this case is impossible. The two cases that remain are:

- One positive real solution, one negative real solution and two complex solutions.
- One positive real solution and three negative real solutions.

In these two cases, there is a unique positive real solution.

B.2 Proof of lemma 1

We can calculate that :

$$\psi\left(N\sqrt{\frac{N^2\alpha}{\delta(2\beta-NB)}}\right)=N^5\alpha^2B\frac{2\beta+NB}{(2\beta-NB)^2}>0$$

Then with the property of ψ, we can conclude that $\bar{R}^2<\frac{N^2\alpha}{\delta(2\beta-NB)}$.

B.3 Proof of proposition 1

Influence of α We can use the implicit functions theorem to know how the extraction cost influences the equilibrium level of reserves. We know that :

$$\frac{d\bar{R}}{d\alpha}=-\frac{\frac{\partial\psi}{\partial\alpha}}{\frac{\partial\psi}{\partial\bar{R}}}$$

We already know that $\frac{\partial\psi}{\partial\bar{R}}>0$ because we previously showed that ψ is increasing in the neighborhood of its unique positive real root. Thus the sign of $\frac{d\bar{R}}{d\alpha}$ is the opposite sign of :

$$\frac{\partial\psi}{\partial\alpha}=\left[\delta N(2\beta-NB)\bar{R}^2-N^3\alpha\right]-\delta N^3B\bar{R}^2-A(N-1)N^2\bar{R}-N^3\alpha$$

- When $2\beta<NB$, we directly get that $\frac{\partial\psi}{\partial\alpha}<0$ and thus $\frac{\bar{R}}{d\alpha}>0$.
- When $2\beta>NB$, we know from lemma 1 that $\bar{R}^2<\frac{N^2\alpha}{\delta(2\beta-NB)}$ then $\frac{d\bar{R}}{d\alpha}>0$.

Finally the parameter α influences positively the equilibrium reserves in the coopetitive case.

$$\boxed{\frac{d\bar{R}}{d\alpha}>0}$$

Influence of β As previously we know that the sign of $\frac{d\bar{R}}{d\beta}$ is the opposite sign of :

$$\frac{\partial \psi}{\partial \beta} = 4\delta^2 BN\bar{R}^4 + 2\delta A(N-1)\bar{R}^3 + 2\alpha\delta N\bar{R}^2$$

This last expression is always positive. Then the coefficient β influences negatively the equilibrium reserves in the coopetitive case.

$$\boxed{\frac{d\bar{R}}{d\beta} < 0}$$

Influence of A As previously we know that the sign of $\frac{d\bar{R}}{dA}$ is the opposite sign of :

$$\frac{\partial \psi}{\partial A} = \bar{R}\left[\delta(N-1)(2\beta - NB)\bar{R}^2 - (N-1)\alpha N^2\right]$$

This is the same reasoning than for the influence of α, we can use the lemma 1 to show finally that A influences positively the reserve gap.

$$\boxed{\frac{d\bar{R}}{dA} > 0}$$

Influence of B As previously we know that the sign of $\frac{d\bar{R}}{dB}$ is the opposite sign of :

$$\frac{\partial \psi}{\partial B} = 4\beta\delta^2 N\bar{R}^4 - \delta AN\bar{R}^3 - \alpha`\delta N^2(N+1)\bar{R}^2$$

Knowing that $\psi(\bar{R}) = 0$, we can write that this is equal to :

$$\begin{aligned} \frac{\partial \psi}{\partial B} &= \frac{1}{B}\left[-\delta A(N-1)2\beta\bar{R}^3 - \alpha\delta N2\beta\bar{R}^2 + A(N-1)\alpha N^2\bar{R} + N^3\alpha^2\right] \\ &= \frac{\bar{R}A(N-1)+N\alpha}{B}(\alpha N^2 - 2\delta\beta\bar{R}^2) \end{aligned}$$

We know that $\bar{R} > \bar{R}_0$, thus we have $\alpha N^2 - 2\delta\beta\bar{R}^2 < 0$. This last expression is then always negative. The coefficient B influences positively the equilibrium reserves in the coopetitive case.

$$\boxed{\frac{d\bar{R}}{dB} > 0}$$

Influence of N As previously we know that the sign of $\frac{d\bar{R}}{dN}$ is the opposite sign of :

$$\frac{\partial\psi}{\partial N} =$$

The condition of existence of a steady state is :

$$A > \sqrt{2\alpha\delta\beta}$$

Moreover, we know that $\frac{R}{R_0} > 1$, then we get that $2\beta(N-2) - (2\beta(N-2) + NB)\left(\frac{R}{R_0}\right)^2 < 0$ because $\frac{2\beta(N-2)}{2\beta(N-2)+NB} < 1$. Finally we can write that :

$$\frac{\partial\psi}{\partial N} < \frac{1}{2BN^3}\left[2\beta(N-2) - (2\beta(N-2) + NB)\left(\frac{R}{R_0}\right)^2\right] + \frac{1}{2BN^3}\left[4\beta - (4\beta - NB)\left(\frac{R}{R_0}\right)^2\right]$$

Thus :

$$\frac{\partial\chi}{\partial N} < 0$$

Then the number of members influences positively the reserve gap.

$$\boxed{\frac{d\bar{R}}{dN} > 0}$$

B.4 Proof of proposition 2

Influence of α We can use the implicit functions theorem to know how the extraction cost influences the reserve gap. We know that :

$$\frac{d\frac{R}{R_0}}{d\alpha} = -\frac{\frac{\partial\chi}{\partial\alpha}}{\frac{\partial\chi}{\partial\frac{R}{R_0}}}$$

We already know that $\frac{\partial\chi}{\partial\frac{R}{R_0}} > 0$ because we previously showed that χ is increasing in the neighborhood of its unique positive real root. Thus the sign of $\frac{\partial\frac{R}{R_0}}{\partial\alpha}$ is the opposite sign of :

$$\frac{\partial\chi}{\partial\alpha} = \frac{R}{R_0}\frac{A(N-1)}{4BN^2\sqrt{2\alpha^3\beta\delta}}\left[2\beta - (2\beta - NB)\left(\frac{R}{R_0}\right)^2\right]$$

- When $2\beta < NB$, we directly get that $\frac{\partial \chi}{\partial \alpha} > 0$ and thus $\frac{d\frac{R}{R_0}}{d\alpha} < 0$.
- When $2\beta > NB$, we know from lemma 2 that $\left(\frac{R}{R_0}\right)^2 < \frac{2\beta}{2\beta - NB}$ then $\frac{d\frac{R}{R_0}}{d\alpha} < 0$.

Finally the parameter α influences negatively the reserve gap.

$$\boxed{\frac{d\frac{R}{R_0}}{d\alpha} < 0}$$

Influence of β As previously we know that the sign of $\frac{d\frac{R}{R_0}}{d\beta}$ is the opposite sign of :

$$\frac{\partial \chi}{\partial \beta} = \frac{A(N-1)}{2BN^2\sqrt{2\alpha\beta\delta}} \frac{R}{R_0} \left[\frac{2\beta + NB}{2\beta} \left(\frac{R}{R_0}\right)^2 - 1 \right] + \frac{1}{BN^2} \left[\left(\frac{R}{R_0}\right)^2 - 1 \right]$$

As we have shown that $\frac{R}{R_0} > 1$, this last expression is then always positive. Then the coefficient β influences negatively the reserve gap.

$$\boxed{\frac{d\frac{R}{R_0}}{d\beta} < 0}$$

Influence of A and B We know that $\bar{R}_0$ does not depend on A and B. Then the variation of the reserve gap with respect to these variables will be the same as $\bar{R}$:

$$\boxed{\frac{d\frac{R}{R_0}}{dA} > 0}$$

$$\boxed{\frac{d\frac{R}{R_0}}{dB} > 0}$$

Influence of N As previously we know that the sign of $\frac{d\frac{R}{R_0}}{dN}$ is the opposite sign of :

$$\frac{\partial\chi}{\partial N} = \frac{A}{2BN^3\sqrt{2\alpha\beta\delta}}\frac{R}{R_0}\left[2\beta(N-2) - (2\beta(N-2) + NB)\left(\frac{R}{R_0}\right)^2\right] + \frac{1}{2BN^3}\left[4\beta - (4\beta - NB)\left(\frac{R}{R_0}\right)^2\right]$$

The condition of existence of a steady state is :

$$A > \sqrt{2\alpha\delta\beta}$$

Moreover, we know that $\frac{R}{R_0} > 1$, then we get that $2\beta(N-2) - (2\beta(N-2) + NB)\left(\frac{R}{R_0}\right)^2 < 0$ because $\frac{2\beta(N-2)}{2\beta(N-2)+NB} < 1$. Finally we can write that :

$$\frac{\partial\chi}{\partial N} < \frac{1}{2BN^3}\left[2\beta(N-2) - (2\beta(N-2) + NB)\left(\frac{R}{R_0}\right)^2\right] + \frac{1}{2BN^3}\left[4\beta - (4\beta - NB)\left(\frac{R}{R_0}\right)^2\right]$$

Thus :

$$\frac{\partial\chi}{\partial N} < 0$$

Then the number of members influences positively the reserve gap.

$$\boxed{\frac{d\frac{R}{R_0}}{dN} > 0}$$

C About cheating

C.1 Coopetitive oligopoly with cheating

We saw that the differential game is described in this case by the following system:

$$\begin{cases} p'(q)q + p(q) - C_\alpha\left((R_k)_{k\in[1,N]}\right) = \sum_{k=1}^{N} \lambda_{opec,k}\frac{R_k}{R} \\ 0 = \mu_i + \lambda_i \\ c'_\beta(x_i) = \lambda_i \\ \dot{\lambda}_{opec,i} = \delta\lambda_{opec,i} + q\frac{\partial C_\alpha\left((R_k)_{k\in[1,N]}\right)}{\partial R_i} + \frac{q}{R^2}\sum_{k\neq i} R_k\lambda_{opec,i} - \frac{q}{R^2}\sum_{k\neq i} R_k\lambda_{opec,k} \\ \dot{\lambda}_i = \delta\lambda_i + q\frac{R_i}{R}c'_\alpha(R_i - Z_i) + \frac{q}{R}\left(1 - \frac{R_i}{R}\right)c_\alpha(R_i - Z_i) - p(q)\frac{q}{R}\left(1 - \frac{R_i}{R}\right) + \frac{q}{R}\left(1 - \frac{R_i}{R}\right)\lambda_i \\ \dot{\mu}_i = \delta\mu_i - q\frac{R_i}{R}c'_\alpha(R_i - Z_i) \\ \dot{R}_i = -q\frac{R_i}{R} + x_i + z_i \\ \dot{Z}_i = z_i \end{cases}$$

As in ?? and A.2, we get directly:

$$p'(\bar{q})\bar{q} + p(\bar{q}) - C_\alpha((\bar{R}_i)_{i\in[1,N]}) = -\frac{\bar{q}}{\delta}\sum_k \frac{\bar{R}_k}{\bar{R}}\frac{\partial C_\alpha((\bar{R}_i)_{i\in[1,N]})}{\partial R_k}$$

And at the equilibrium:

$$\begin{cases} \dot{\lambda}_i = 0 \Leftrightarrow \bar{\lambda}_i\left[\delta + \frac{\bar{q}}{\bar{R}}\left(1 - \frac{\bar{R}_i}{\bar{R}}\right)\right] = -\bar{q}\frac{\bar{R}_i}{\bar{R}}c'_\alpha(\bar{R}_i - \bar{Z}_i) - \frac{\bar{q}}{\bar{R}}\left(1 - \frac{\bar{R}_i}{\bar{R}}\right)c_\alpha(\bar{R}_i - \bar{Z}_i) + p(\bar{q})\frac{\bar{q}}{\bar{R}}\left(1 - \frac{\bar{R}_i}{\bar{R}}\right) \\ \dot{\mu}_i = 0 \Leftrightarrow \delta\bar{\mu}_i = q\frac{\bar{R}_i}{\bar{R}}c'_\alpha(\bar{R}_i - \bar{Z}_i) \\ \dot{R}_i = 0 \Leftrightarrow \bar{q}\frac{\bar{R}_i}{\bar{R}} = \bar{q}_i = \bar{x}_i + \bar{z}_i \\ \dot{Z}_i = 0 \Leftrightarrow \bar{z}_i = 0 \\ \forall i \in [1, N] \end{cases}$$

Then:

$$\bar{\lambda}_i = -c_\alpha(\bar{R}_i - \bar{Z}_i) + p(\bar{q})$$

Finally, we get that:

$$\begin{cases} \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta}c'_\alpha(\bar{R}_i - \bar{Z}_i) \\ c'_\beta(\bar{x}_i) = -c_\alpha(\bar{R}_i - \bar{Z}_i) + p(\bar{q}) \end{cases}$$

C.2 Real reserves gap

Remembering that:

$$\begin{cases} \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta}c'_\alpha(\bar{R}_i - \bar{Z}_i)|_{\text{coopetition}} \\ \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i} = -\frac{1}{\delta}c'_\alpha(\bar{R}_i)|_{\text{no cooperation}} \end{cases}$$

Then, knowing that the marginal extraction cost is an increasing function, we get that:

$$\left(\bar{R}_i - \bar{Z}_i\right)|_{\text{coopetition}} \geq \bar{R}_i|_{\text{no cooperation}} \Leftrightarrow c'_\alpha(\bar{R}_i - \bar{Z}_i)|_{\text{coopetition}} \geq c'_\alpha(\bar{R}_i)|_{\text{no cooperation}}$$

It means that "the real reserves" - reserves that really come from exploration - in the coopetitive oligopoly are greater that in the no cooperative oligopoly if and only if marginal extraction costs are ordered in the same way at the equilibrium, and then:

$$\left(\bar{R}_i - \bar{Z}_i\right)|_{\text{coopetition}} \geq \bar{R}_i|_{\text{no cooperation}} \Leftrightarrow \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i}|_{\text{coopetition}} \leq \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i}|_{\text{no cooperation}}$$

Then if for instance the function $\frac{c'_\beta(x)}{x}$ is an increasing function, we get that:

$$\frac{c'_\beta(\bar{x}_i)}{\bar{x}_i}|_{\text{coopetition}} \leq \frac{c'_\beta(\bar{x}_i)}{\bar{x}_i}|_{\text{no cooperation}} \Leftrightarrow \bar{q}_i|_{\text{coopetition}} \leq \bar{q}_i|_{\text{no cooperation}}$$

Last, we can notice that:

$$\frac{d}{dx}\left(\frac{c'_\beta(x)}{x}\right) \geq 0 \Leftrightarrow \frac{c''_\beta(x)x - c'_\beta(x)}{x^2} \geq 0 \Leftrightarrow \frac{c''_\beta(x)x}{c'_\beta(x)} \geq 1$$

Then $\frac{c'_\beta(x)}{x}$ is increasing if and only if its elasticity is greater than one.

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