Climate Coalitions in a Mitigation-Adaptation Game
(First Unfinished Draft)

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June 4, 2014

Abstract

We study the strategic interaction between mitigation and adaptation strategies in the canonical model of international environmental agreements (IEAs). We show that these two strategies are strategic substitutes considering various definitions of substitutability, irrespective of the degree of cooperation. Moreover, different from a pure mitigation game, adaptation may cause mitigation levels between different countries to be strategic complements, generalizing a result by Ebert and Welsch (2011 and 2012) to more than two countries with the possibility that players form self-enforcing IEAs. We systematically analyze under which conditions this leads to more positive cooperative outcomes compared to the pure mitigation game. Particular emphasis is placed on sufficient conditions for the existence, and uniqueness

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of interior equilibrium strategies and how they link to the success of coalition formation.

Keywords: Climate Change, Mitigation-Adaptation Game, Agreements, Strategic Substitutes versus Complements

JEL-Classification: C71, D62, D74, H41, Q54

1 Introduction

Climate change is probably one of the most important challenges of human mankind. The Framework Convention on Climate Change (FCCC) in Rio de Janeiro in 1992 was the first conference at the global scale which addressed this issue. Despite the Kyoto Protocol being signed by 38 countries in 1997, it is clear that more mitigation measures must be implemented in order to avoid the most dramatic impacts of global warming. Global policy coordination is desperately needed in order to design an effective and comprehensive climate agreement succeeding the Kyoto Protocol, which ceased in 2012. To date, despite substantial international negotiations, a new treaty could not be concluded. Currently, the only hope is that, eventually, some climate agreement will be signed in Paris in 2015.

Clearly, mitigation to address the cause of global warming is costly, participation in a climate treaty is voluntary and compliance is difficult to enforce. Due to the lack of policy coordination and the first visible impacts of climate change, in particular in developing countries, the role of adaptation measures (like building dykes against flooding and installing air-conditioning devices against heat) receives increasingly attention in policy circles but also in the report of Working Group III by the Internal Panel on Climate Change released in 2014. In contrast to mitigation („reducing emissions“), which can be viewed as a non-excludable public good (everybody can benefit), adaptation („adjust or adapt to emissions“) is a private good; it only benefits the country in which adaptation measures are implemented.
The key research question is: how adaptation, as an additional strategy to mitigation, will affect the prospects of international policy coordination to tackle climate change?

The answer is not straightforward because the marginal benefits from mitigation measures are reduced by adaptation and vice versa, the marginal benefits from adaptation decrease with the level of mitigation. Thus, mitigation and adaptation interact strategically (Barrett 2008). On the one hand, intuition suggests that with an increasing focus on adaptation, the importance of mitigation diminishes, and hence less effort will be devoted to negotiations of an effective climate treaty in the future. On the other hand, one may suspect that having a larger portfolio of strategies available to address the impacts of climate change may reduce the costs of ameliorating the impacts of climate change, and hence a climate treaty is more likely to come about. Moreover, Ebert and Welsch (2011 and 2012) showed in a two-player model that reaction functions in mitigation space may be upward sloping in the presence of adaptation. They conjecture that in the context of coalition formation this may lead to larger stable coalitions than in a pure mitigation game. It is a central issue of this paper to test their conjecture. Similar attempts have been undertaken by Buob and Siegenthaler (2011), Marrouch and Chaudhuri (2011), Probst (2012) as far as we are aware. The general problem to obtain results is that coalition formation adds a lot of complexity. It is for this reason that we restrict ourself in this paper to the standard assumption of symmetric players. This also helps us to relate our results directly to the canonical model of international environmental agreements. Different from Ebert and Welsch, who employ a model in emission space, we specific our model in mitigation space. As providing a public good or ameliorating a public bad are dual problems, the two approaches do not differ. We are slightly more general as we do not assume a linear cost function of adaptation, but consider a general convex cost function.

Our paper is related to two strands in the literature. The first strand
on the game-theoretic analysis of IEAs can be traced back to Barrett (1994) and Carraro and Siniscalco (1993) which is summarized in Barrett (2003), Finus (2003) and Wagner (2001). Our model is particularly related to those recent papers, which analyze the impact of additional strategies to mitigation on the success of coalition formation, like breakthrough technologies (Barrett 2006 and Hoel and deZeeuw 2010), geo-engineering (Barrett 2007) or a modification of their impact, like Finus and Rubbelke (2012) considering ancillary benefits and Finus and Maus (2008) looking at modest emission reductions. Ancillary benefits also imply that strategies have not only a public but also a private benefit (i.e. impure public goods). However, ancillary benefits (co-benefits and secondary benefits are alternative terms used in the literature) imply one strategy (mitigation) having two effects (private and public), whereas a mitigation-adaptation game implies two strategies, one having a private and one having a public impact. Since we will show that mitigation and adaptation are substitutes, adaptation will reduce equilibrium mitigation and hence influences stability through the same channel as modest mitigation. Finally, as mentioned above, first attempts to consider a mitigation-adaptation game in the coalition context have been undertaken by Barrett (2008), Buob and Siegenthaler (2011), Marrouch and Chaudhuri (2011) and Probst (2012). Most of this work is very preliminary and does not cover the possibility of upward-sloping reaction functions, the most interesting case when it comes to coalition formation as we argue below.

The second strand of literature tries to understand the strategic interaction between adaptation and mitigation and to answer the question whether they are complements or substitutes, though abstracting from the issue of coalition formation, like in Ingham et al. (2005) and Tulkens and Steenberghe (2009). Buob and Stephan (2011) also look at the timing of mitigation and adaptation but in their model these strategies are always perfect substitutes. This is different in Zehaie (2009) who systematically analyzes the timing of adaptation and mitigation: a simultaneous choice and a sequential choice of
these two strategies, one version with adaptation first and mitigation second and a second sequential option where this is reversed. In a non-cooperative equilibrium, semi-cooperative equilibrium and in the social optimum he compares adaptation levels, though no mitigation levels. We consider two versions of the timing in Zehaie’s model, extend his model in a slightly different setting to more than two players, add coalition formation and capture as in Ebert and Welsch (2011, 2012) the possibility of strategic complements in mitigation space (which is absent in Zehaie’s model). Moreover, we are able to show that mitigation and adaptation are strategic substitutes, regardless whether countries behave non-cooperatively, partially cooperative or fully cooperative. We demonstrate this for different definitions of substitutability. We pay particular attention to the conditions that guarantee existence and uniqueness of interior equilibrium strategies and relate them to the success of coalition formation.

In what follows, we set out the model and its assumptions in Section 2. We present our general results in Section 3 and report on some simulations in Section 4. Section 5 summarizes our main results and draws some policy conclusions.

2 Model

We consider \( n \) players, which are countries in our context, \( i = 1, 2, \ldots, n \), with the payoff function of country \( i \) given by:

\[
\Pi_i(Q, q_i, x_i) = B_i(Q, x_i) - C_i(q_i) - D_i(x_i)
\]  

(1)

where we denote the set of players by \( N \). Country \( i \) can choose its individual mitigation and adaptation level \( q_i \) and \( x_i \), respectively, within the (compact and convex) strategy space of each country \( i \in N \), given by \( x_i \in [0, \bar{x}_i] \) and \( q_i \in [0, \bar{q}_i] \), with \( \bar{x}_i \) and \( \bar{q}_i \) sufficiently large. Country \( i \)'s payoff comprises benefits, \( B_i \), which depend on total mitigation, \( Q = \sum_{j=1}^{n} q_j \), and
its individual adaptation level, \( x_i \), the cost of mitigation, \( C_i \), and the cost of adaptation, \( D_i \). Apart from assuming that all functions are continuous in their respective variable(s), we make the following assumptions regarding the components of the payoff functions.

**General Assumptions**

For all players \( i \in N \):

\[ a) \ B_{iQ} = \frac{\partial B_i}{\partial Q} > 0, \ B_{iQQ} = \frac{\partial^2 B_i}{\partial Q^2} \leq 0, \ B_{ix_i} = \frac{\partial B_i}{\partial x_i} > 0, \ B_{ix_ix_i} = \frac{\partial^2 B_i}{\partial x_i^2} \leq 0. \]

\[ b) \ C_{qi} = \frac{\partial C_i}{\partial q_i} > 0, \ C_{qiq_i} = \frac{\partial^2 C_i}{\partial q_i^2} > 0, \ D_{ix_i} = \frac{\partial D_i}{\partial x_i} > 0, \ D_{ix_ix_i} = \frac{\partial^2 D_i}{\partial x_i^2} \geq 0. \]

\[ c) \text{If } \frac{\partial^2 B_i}{\partial x_i^2} = 0, \text{ then } \frac{\partial^2 D_i}{\partial x_i^2} > 0 \text{ and vice versa: if } \frac{\partial^2 D_i}{\partial x_i^2} = 0, \text{ then } \frac{\partial^2 B_i}{\partial x_i^2} < 0. \]

\[ d) \ B_{ix_iQ} = \frac{\partial^2 B_i}{\partial x_i \partial Q} = \frac{\partial^2 B_i}{\partial Q \partial x_i} = B_{iQx_i} < 0. \]

\[ e) \text{All countries are ex-ante symmetric, i.e. they have all the same benefit and cost functions.} \]

From a technical point of view, assumptions a) to d) reflect the standard assumptions of concave benefit and convex cost functions. Assumption a) allows for the possibility that benefit functions could be linear such that we can revisit some simple examples, which have been considered in the literature on IEAs in the context of a pure mitigation game. In assumption b) we assume the cost function of mitigation to be strictly convex in order to ensure unique equilibrium mitigation levels. (See more on uniqueness below.) For adaptation, it turns out that this is not necessary for our coalition formation game described below. However, in assumption c) we state that if benefit functions are linear in adaptation, then adaptation cost functions must be strictly convex and vice versa, if adaptation cost functions are linear in adaptation, then benefit functions need to be strictly concave. This is to
avoid corner solutions as for instance in Kolstad (2007) in a pure mitigation game and in Barrett (2008) in a mitigation-adaptation game. (See more on interior solutions below.)

From an economic point of view, assumption a) stresses that mitigation is a pure public good, i.e. the marginal benefit from mitigation only depends on the sum of total mitigation efforts. In contrast, adaptation is a pure private good, i.e. the marginal benefit from adaptation depends only on the individual adaptation level of a country. The interdependency between mitigation and adaptation is captured through assumption d). The marginal benefit from mitigation (adaptation) decreases with the level of adaptation (mitigation). For simplicity, such an interdependency is assumed away on the cost side.\footnote{This also means input markets are competitive without capacity constraints.} In order to stress this, we assume for clarity two separate cost functions, one for mitigation and one for adaptation (see assumption b)). The strategic interaction between countries is directly related to the (pure) public good nature of mitigation. Mitigation in country $i$ generates benefits in country $i$ but also in all other countries. Thus, mitigation levels generate positive externalities. Adaptation levels generate no direct externalities. However, they indirectly influence the strategic interaction among countries because, as will become apparent below: the higher the adaptation level in a country, the lower will be its mitigation level, irrespective whether country $i$ acts independently or joins a climate coalition. Note that conceptually, a pure mitigation game can be modelled by setting for all $i, j \in N, i \neq j, x_i = x_j = 0$ in the benefit function $B_i(Q, x_i)$ and cost function $D_i(x_i)$ in payoff function (1).

Finally, assumption e) reflects our focus on ex-ante symmetric players, i.e. all countries have the same benefit and cost functions, very much in the tradition of the literature of coalition formation in general (Bloch 2003 and Yi 1997 for overviews) and on IEAs in particular (Barrett 2003 and Finus 2003 for overviews). Note that this does not preclude that players
are ex-post asymmetric if they choose different mitigation and/or adaptation levels. For instance, in the context of coalition formation, and as will become apparent below, signatories and non-signatories will typically choose different mitigation levels and hence will receive different payoffs. As we have not yet established any form of ex-post symmetry, the subsequent definitions in this section cover the general case of asymmetric players; symmetry is established in the next section, in particular Subsection 3.1. We assume the General Assumptions to hold throughout the paper. If we make further assumptions, we will mention them explicitly.

Our two-stage coalition formation game unfolds as follows.

**Definition 1: Coalition Formation Game**

**Stage 1:** All countries simultaneously choose whether to join coalition $S \subseteq N$ or to remain a singleton player. Countries $i \in S$ are called signatories and countries $j \notin S$ are called non-signatories.

**Stage 2:** Version 1: Simultaneously, all non-signatories $j \notin S$ choose their mitigation and adaptation levels $q_j$ and $x_j$ in order to maximize their individual payoff $\Pi_j(Q, q_j, x_j)$ and all signatories $i \in S$ jointly choose their mitigation and adaptation levels $q_i$ and $x_i$ in order to maximize the aggregate payoff $\sum_{i \in S} \Pi_i(Q, q_i, x_i)$.

**Stage 2:** Version 2: a) First, simultaneously, all non-signatories $j \notin S$ choose their mitigation level $q_j$ in order to maximize their individual payoff $\Pi_j(Q, q_j, x_j)$ and all signatories $i \in S$ jointly choose their mitigation $q_i$ in order to maximize the aggregate payoff $\sum_{i \in S} \Pi_i(Q, q_i, x_i)$. b) Second, simultaneously, all non-signatories $j \notin S$ choose their adaptation level $x_j$ in order to maximize their individual payoff $\Pi_j(Q, q_j, x_j)$ and all signatories $i \in S$ jointly choose their adaptation level $q_i$ in order to maximize the aggregate payoff $\sum_{i \in S} \Pi_i(Q, q_i, x_i)$. 

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Stage 1 is the widely used cartel formation game, which originates from the literature in industrial organization (d’Aspremont et al., 1983) and which has been applied widely in this literature (e.g. Deneckere and Davidson 1985, Donsimoni et al. 1986 and Poyago-Theotoky 1995; see Bloch 2003 and Yi 1997 for surveys) but also in the literature on IEAs (e.g. Barrett 1994, Carraro and Siniscalco 1993 and Rubio and Ulph 2006; see Barrett 2003 and Finus 2003 for surveys). This game has also been called open membership single coalition game as membership in coalition $S$ is open to all players and players have only the choice between joining coalition $S$ or remaining a singleton.\footnote{See Finus and Rundshagen (2006) for a more formal exposition of the game. Surveys of coalition games with other membership rules, including exclusive membership and multiple coalitions, are provided in Bloch (2003) and Yi (2003) and a systematic comparison of equilibrium coalition structures under different membership rules is conducted in Finus and Rundshagen (2006 and 2009).} In the context of the provision of a public good, it appears that one is more concerned about players leaving a coalition than joining it. Moreover, most international environmental treaties are of the open membership type. The assumption of a single coalition simplifies the analysis but is also in line with the historical records of IEAs with a single treaty.

Stage 2 follows the standard assumption in the literature on coalition formation (see Bloch 2003 and Yi 2003 for surveys): the coalition acts as a kind of meta player (Haeringer 2004), internalizing the externality among its members, whereas non-signatories act selfishly, maximizing their own payoff. We always assume that signatories and non-signatories choose their economic strategies simultaneously.\footnote{This has been called Nash-Cournot assumption in the literature on IEAs and has been contrasted with the assumption of a sequential choice, called Stackelberg assumption, where signatories act as the Stackelberg leader. In the context of coalition formation, the Stackelberg assumption is not as innocuous as this may appear in a two player context. See Finus (2003) for details. The Stackelberg assumption has been made for instance in Barrett (1994) and Rubio and Ulph (2006) in a pure mitigation game.} Version 1 and 2 reflect different possible assumptions about the timing of mitigation and adaptation. As both versions lead
to the same second stage equilibrium economic strategies as we show below (see Subsection 3.1), our results are quite robust.\footnote{In principle, we could also consider a version 3 in which the timing is reversed compared to version 2. Version 3 is considered in Zehaie (2009), though it appears that this version is less plausible from a policy point of view than version 1 and 2. Note that version 3 leads generally to different results than version 1 and 2.}

The two-stage coalition formation game is solved by backwards induction. In the second stage, given that some coalition $S \subseteq N$ has formed in the first stage, version 1 determines simultaneously an equilibrium mitigation vector $q^*(S)$ and an equilibrium adaptation vector $x^*(S)$ as a Nash equilibrium between coalition $S$ and all remaining players not in $S$. Version 2 determines first the equilibrium adaptation vector, again, as a Nash equilibrium between coalition $S$ and the remaining singletons. Equilibrium adaptation levels in stage 2b will depend on the levels of mitigation chosen in stage 2a, which in turn depend on which coalition $S$ has formed in stage 1. Hence, in stage 2b, we can write $x^*(q(S))$. Substituting this into the payoff function (1), payoffs in stage 2a are only a function of mitigation levels. This allows us to solve stage 2a for equilibrium mitigation levels.

Regardless whether version 1 or 2 is played in stage 2, it is clear that for technical reasons we want that for each possible coalition $S$ a unique mitigation and adaptation vector exists.\footnote{It is interesting to note that almost all papers on coalition formation of which we are aware of either implicitly or explicitely assume existence and uniqueness of second stage equilibria without proof. Giving up uniqueness would complicate the analysis tremendously.} This allows us to write $\Pi^*_i(S)$ instead of $\Pi_i(q^*(S), x^*(S))$. Even though we provide sufficient conditions for existence and uniqueness only in the next section, we make already use of this assumption in order to save on notation and for clarity and define a stable coalition $S^*$ as follows:
internal stability: \[ \Pi^*_i(S^*) \geq \Pi^*_i(S^* \setminus \{i\}) \forall i \in S^* \text{ and} \]

external stability: \[ \Pi^*_j(S^*) \geq \Pi^*_j(S^* \cup \{j\}) \forall j \notin S \]

It is evident that the conditions of internal and external stability de facto define a Nash equilibrium in membership strategies in the first stage. Each player \(i\) who announced to join coalition \(S^*\) should not have an incentive to (unilaterally) change her strategy by leaving coalition \(S^*\) and each player \(j\) who announced not to join coalition \(S^*\) should not have an incentive to (unilaterally) change his strategy and join coalition \(S^*\), given the equilibrium announcements of all other players. Generally, there may be no, one or several stable coalitions. In case of several stable coalitions, we apply the selection criterion of Pareto-dominance, deleting all coalitions from the set of stable coalitions for which it is possible to move to another stable coalition where at least one player gains and no player is worse off. In Section 4, we discuss existence and uniqueness of stable coalitions.

Note that by the construction of the coalition game, the equilibrium economic strategy vectors in the second stage correspond to the Nash equilibrium known from games without coalition formation if coalition \(S\) is empty or contains only one player. We also call this the "all singleton coalition structure" or "no cooperation". By the same token, if coalition \(S\) comprises all players, i.e. the grand coalition forms, \(S = N\), this corresponds to the "social optimum". We also call this "full cooperation". Any non-trivial coalition which comprises more than one player but less than all players may be viewed as partial cooperation. The corresponding second stage equilibrium may be referred to as a coalitional Nash equilibrium.

In order to analyze the driving forces of coalition formation, we define some useful properties.
Definition 2: Superadditivity, Positive Externality and Cohesiveness

(i) A coalition game is (strictly) superadditive if for all $S \subseteq N$ and all $i \in S$:

$$\sum_{i \in S} \Pi_i^*(S) \geq (>) \sum_{i \in S \setminus \{i\}} \Pi_i^*(S \setminus \{i\}) + \Pi_i^*(S \setminus \{i\}).$$

(ii) A coalition game exhibits a (strict) positive externality if for all $S \subseteq N$ and for all $j \in N \setminus S$:

$$\Pi_j^*(S) \geq (> \Pi_j^*(S \setminus \{i\}).$$

(iii) A game is (strictly) cohesive if for all $S \subset N$:

$$\sum_{i \in N} \Pi_i^*\{N\} \geq (> \sum_{i \in S} \Pi_i^*(S) + \sum_{j \in N \setminus S} \Pi_j^*(S).$$

(iv) A game is (strictly) fully cohesive if for all $S \subseteq N$:

$$\sum_{i \in S} \Pi_i^*(S) + \sum_{j \in N \setminus S} \Pi_j^*(S) \geq (> \sum_{j \in S \setminus \{i\}} \Pi_i^*(S \setminus \{i\}) + \sum_{j \in N \setminus S \cup \{i\}} \Pi_j^*(S \setminus \{i\}).$$

All four properties are related to each other. For instance, a coalition game which is superadditive and exhibits positive externalities is fully cohesive and a game which is fully cohesive is cohesive. (See Cornet 1998 and Montero 2006 for applications.) Typically, a game with externalities is cohesive, with the understanding that in a game with externalities the strategy of at least one player has an impact on the payoff of at least one other player. The reason is that the grand coalition internalizes all externalities by assump-
Hence, cohesiveness motivates the choice of the social optimum as a normative benchmark, and it is the basic motivation to investigate stability and outcomes of cooperative arrangements. However, an even stronger motivation is related to full cohesiveness, as it provides a sound foundation for the search for large stable coalitions even if the grand coalition is not stable due to large free-rider incentives. The fact that large coalitions, including the grand coalition, may not be stable in coalition games with positive externalities is well-known in the literature (e.g. see the overviews by Bloch 2003 and Yi 1997). Examples of positive externality games include output and price cartels and the pure mitigation game. The positive externality can be viewed as a benefit generated by the coalition, which also accrues to outsiders as these benefits are non-excludable. This property makes it attractive to stay outside the coalition. This may be true despite superadditivity, a property which makes joining a coalition attractive. In the context of the pure mitigation game, stable coalitions are typically small because with increasing coalitions, the "push factor" positive externality dominates the "pull factor" superadditivity (e.g. see the overviews by Barrett 2003 and Finus 2003). Whether this is also the case if adaptation is added as a second strategy to mitigations is the key research question of this paper.

3 Results: Second Stage

3.1 Equivalence of Version 1 and 2 and Symmetry

In this subsection, we start to investigate the equivalence between version 1 and 2 in the second stage. Then, we have a look at the implication of our

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6Cohesiveness could fail if there are diseconomies of scale from cooperation, e.g. due transactions costs which increase in the number of cooperating players. Our model abstracts from such complications.

7This is quite different in negative externality games. In Eyckmans et al. (2012), it is shown that in a coalition game with negative externalities and superadditivity the grand coalition is the unique stable equilibrium.
assumption of ex-ante symmetric payoff functions as this will allow us to save on notation in the subsequent analysis. In this subsection, we assume existence of an interior second stage equilibrium for which we establish sufficient conditions in Subsection 3.2 and 3.3, respectively.

Consider the general situation that a coalition $S$ has formed and that the remaining countries $N \setminus S$ act as singletons. Consider version 1 in the second stage, assume an interior equilibrium. The first order conditions of signatories in terms of mitigation are given by

$$\sum_{i \in S} B_i Q_i(Q, x_{iS}) = C_{iq_i}(q_{iS}) \forall i \in S \tag{2}$$

and in terms of adaptation by

$$B_{ix_i} Q_i(Q, x_{iS}) = D_{ix_i}(x_{iS}) \forall i \in S \tag{3}$$

where the subscript $S$ stands for signatories. For non-signatories, using subscript $NS$, the first order conditions are given by

$$B_j Q_j(Q, x_{jNS}) = C_{jq_j}(q_{jNS}) \forall j \notin S \tag{4}$$

and

$$B_{jx_j} Q_j(Q, x_{jNS}) = D_{jx_j}(x_{jNS}) \forall j \notin S. \tag{5}$$

Now consider version 2 in the second stage, which allows us to state the following.

**Proposition 1: Equivalence of Version 1 and 2**

*In the coalition formation game described in Definition 1, version 1 and 2 are equivalent in terms of an interior second stage equilibrium.*

**Proof:** In the last stage, stage 2b, when signatories and non-signatories simultaneously choose their adaptation levels, the first order conditions of
signatories are given by (3) and those of non-signatories by (5), which implicitly determine adaptation $x_i$ as a function of total mitigation $Q$. Hence, using $x_i^*(Q)$, where the asterisk indicates the equilibrium value in stage 2b, the maximization problems, which signatories and non-signatories face at stage 2a, when choosing their mitigation levels, lead to the first order conditions for $i \in S$:

$$\sum_{i \in S} B_{ix_i}(Q, x_{iS}^*(Q)) + B_{ix_i}(Q, x_{iS}^*(Q)) \frac{\partial x_i^*}{\partial q_{iS}} - C_{iq_i}(q_{iS}) - D_{ix_i}(x_{iS}^*(Q)) \frac{\partial x_i^*}{\partial q_{iS}} - C_{ij}(q_{jNS}) - D_{jx_j}(x_{jNS}^*(Q)) \frac{\partial x_j^*}{\partial q_{jNS}} = 0 \quad \forall \ i \in S$$

Respectively, which, using the first order conditions (3) and (5), respectively, and rearranging terms, imply (2) and (4) from above. Q.E.D.

Turning now to the issue of symmetry, considering the set of first order conditions in (2) to (5), it is immediately clear that for ex-ante symmetric payoff functions within the group of signatories and within the group non-signatories players face exactly the same first order conditions. Moreover, and importantly, the assumption of a strictly convex cost function from mitigation (see General Assumptions, part b)) ensures that mitigation levels within the group of signatories and within the group of non-signatories are symmetric and unique, and that (3) and (4) can hold simultaneously (otherwise no interior equilibrium would exist). Finally, note that the first order conditions in terms of adaptation of signatories (3) and of non-signatories (5) are the same. Therefore, the set of first order conditions (2) to (5) simplify for ex-ante symmetric players as follows:

$$pB_Q(Q, x) = C_q(q_S)$$

(6)

$$B_Q(Q, x) = C_q(q_{NS})$$

(7)

$$B_x(Q, x) = D_x(x)$$

(8)

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where the all singleton coalition structure can be represented by either $p = 0$ or $p = 1$, but where we adopt the convention $p = 1$ henceforth and therefore we can state the following result.

**Proposition 2: Symmetry and Equilibrium Individual Mitigation and Adaptation Levels**

Consider an arbitrary non-trivial coalition of size $p$, $1 \leq p \leq n$ and an interior second stage equilibrium.

a) Nash equilibrium ($p = 1$): The mitigation and adaptation levels of all players are the same.

b) Coalitional Nash equilibrium ($1 < p < n$):

i) All signatories will choose the same individual mitigation level $q^*_S$ and all non-signatories will choose the same individual mitigation level $q^*_NS$ in equilibrium. The mitigation levels of signatories are strictly higher than those of non-signatories, i.e. $q^*_S > q^*_NS$.

ii) All signatories and all non-signatories will choose the same individual adaptation level $x^*$ in equilibrium, i.e. $x^*_S = x^*_NS = x^*$.

c) Social Optimum ($p = N$): The mitigation and adaptation levels of all players are the same.

**Proof:** Follows from the discussion above; $q^*_S > q^*_NS$ follows from comparing (6) and (7), noting that this implies $pC_q(qNS) = C_q(qS)$ in equilibrium and using the General Assumptions, in particular $Cqq > 0$. Q.E.D.

Proposition 2 implies that we can focus on symmetric second stage equilibria. That is, they are completely symmetric in the Nash equilibrium and the social optimum. In a coalitional Nash equilibrium, they are also completely symmetric with respect to adaptation and symmetric within the group of signatories and within the group of non-signatories with respect to mitigation. Note the result that signatories will abate more than non-signatories would also be true in a pure mitigation game. However, from the fact that
adaptation levels of signatories and non-signatories are the same, one should not mistakenly conclude that policy coordination is not required in terms of adaptation. We will show later that $x^*$ decreases in the size of the coalition and hence obtains its lowest level in the social optimum.

3.2 Existence of Second Stage Equilibria

In this subsection, we derive sufficient conditions for the existence of an equilibrium mitigation and adaptation vector in the second stage. Regarding existence, we may recall that a Nash equilibrium exists if a) the strategy space of each player is compact and convex, b) payoffs of all players are continuous and bounded for all strategies in the entire strategy space and c) payoffs of each player $i$ are concave with respect to own strategies. This standard theorem just needs to be adopted for our purposes in which the second stage can be viewed as a Nash equilibrium between coalition $S$, comprising $p$ players, though acting de facto as a single player, and the $n-p$ non-signatories, who play as singletons, which we called a coalitional Nash equilibrium. We notice that conditions a) and b) trivially hold, due to our definition of the payoff function in (1) and the associated definition of the strategy space. Hence, we can concentrate on condition c). For this, let us derive the second order conditions for a maximum by totally differentiating the first order conditions of signatories (6) and (8), and those of non-signatories (7) and (8), respectively, which delivers, by rearranging terms, and using short-hand notation, the Hessian matrix of signatories, $H^S$, and non-signatories, $H^{NS}$, respectively:

$$H^S = \begin{pmatrix} p^2 B_{QQ} - C_{qq} & p B_{Qx} \\ p B_{xQ} & B_{xx} - D_{xx} \end{pmatrix} \quad (9)$$

$$H^{NS} = \begin{pmatrix} B_{QQ} - C_{qq} & B_{Qx} \\ B_{xQ} & B_{xx} - D_{xx} \end{pmatrix} \quad (10)$$
where we drop the arguments in the derivatives of the benefit and cost functions and focus on strict concavity for convenience. The aggregate payoff function of signatories (the sum of the individual payoff functions of signatories) is strictly concave with respect to $x_i \in [0, \bar{x}_i]$ and $q_i \in [0, \bar{q}_i]$, $i \in S$, if $D_1^S = p^2 B_{QQ} - C_{qq} < 0$ and $D_2^S = [(p^2 B_{QQ} - C_{qq})(B_{xx} - D_{xx}) - (p B_{Qx})^2] > 0$. The individual payoff function of a non-signatory is strictly concave with respect to $x_j \in [0, \bar{x}_j]$ and $q_j \in [0, \bar{q}_j]$, $j \notin S$, if $D_1^{NS} = B_{QQ} - C_{qq} < 0$ and $D_2^{NS} = [(B_{QQ} - C_{qq})(B_{xx} - D_{xx}) - (B_{Qx})^2] > 0$. If $D_1^S < 0$ and $D_2^S > 0$ hold for all $i \in S$ and $D_1^{NS} < 0$ and $D_2^{NS} > 0$ for all $j \notin S$, then we know that an equilibrium in the second stage exist. Of course, as established above in Proposition 2, we know that within the group of signatories all players are ex-post symmetric and within the group of non-signatories the same is true. We further notice that the conditions for $D_1^S$ and $D_1^{NS}$ are satisfied due to the General Assumptions. In the pure mitigation game, these would be the only two conditions and hence strict concavity would follow immediately without any further assumptions. In the more complicated mitigation-adaptation game, there are two more conditions, $D_2^S > 0$ and $D_2^{NS} > 0$, for which the General Assumptions are not sufficient to establish that they are positive. If we define $A := B_{QQ} + \frac{(B_{Qx})^2}{D_{xx} - B_{xx}}$, then $D_2^S > 0$ implies $A < C_{qq}$ and $D_2^{NS} > 0$ implies $A < C_{qq}$. We note that if the term $A$ is negative this condition always holds. However, the term $A$ can also be positive in which case the condition becomes binding. A further analysis of this term and its implication will be deferred to Subsection 3.4. At this stage it is important to notice that the term $A$ is the same for signatories and non-signatories for any level of $Q$ and $x$. Hence, if we require $D_2^S$ and $D_2^{NS}$ to be positive over the entire strategy space $x_i \in [0, \bar{x}_i]$ and $q_i \in [0, \bar{q}_i]$ of player $i$, regardless whether he is a member of coalition $S$, then the condition for $D_2^S > 0$ is more restrictive than for $D_2^{NS} > 0$. Moreover, the most demanding condition for $D_2^S > 0$ is if $p = n$, i.e. the grand coalition forms, which corresponds to the social optimum, then $A < \frac{C_{qq}}{n^2}$ or $\frac{A}{C_{qq}} < 1$. We summarize our discussion in Assumptions 1 and
Lemma 1.

**Assumptions 1**

Let \( A := B_{QQ} + \frac{(B_{QQ})^2}{D_{xx} - B_{xx}} \). For all players \( i \in N \) and \( x_i \in [0, \bar{x}_i] \) and \( q_i \in [0, \bar{q}_i] \):

a) \( \frac{A}{C_{qq}} < 1 \);

b) \( \frac{Ap^2}{C_{qq}} < 1 \);

c) \( \frac{Ap^2}{C_{qq}} < 1 \).

**Lemma 1: Existence of an Equilibrium in the Second Stage.**

Consider an arbitrary coalition of size \( p, 1 \leq p \leq n \).

A sufficient condition for the existence of an equilibrium mitigation and adaptation vector in the second stage of the coalition formation game as defined in Definition 1 is:

a) Nash equilibrium \( (p = 1) \): Assumption 1 a);
b) Coalitional Nash equilibrium \( (1 < p < n) \): Assumption 1 b);
c) Social optimum \( (p = n) \): Assumption 1 c).

**Proof:** See the discussion above. Q.E.D.

Hence, if we assume Assumption 1, part c), then for every possible coalition a second stage equilibrium exist.

### 3.3 Uniqueness of Interior Second Stage Equilibria

In this subsection, we derive sufficient conditions for a unique interior second stage equilibrium for every possible coalition \( S \) of size \( p, 1 \leq p \leq n \).
Generally, the issue of an interior equilibrium and a unique equilibrium are separate issues. However, in the context of our mitigation-adaptation game with the possibility of upward-sloping reaction functions, they are closely linked. We will introduce the concept of reaction functions below, but only at this stage to stress their link to the concept of replacement functions as introduced by Cornes and Hartely (2007), as a convenient and elegant way to establish uniqueness of a Nash equilibrium. Again, we can extend their idea to any coalition, which is not necessarily the trivial coalition \((p = 1)\). We return to reaction functions in Subsection 3.4.

In order to fix ideas, consider the simplest case of the all singleton coalition structure. The first order conditions in terms of adaptation (8) implicitly define the equilibrium adaptation level as a function of total mitigation, \(x^*(Q)\). Consequently, the first order conditions in terms of mitigation are either (6) if we set \(p = 1\), or simply (7), which can be written now as \(B_Q(Q, x^*(Q)) = C_q(q)\). Noting that \(Q = q + Q_{-i}\), this first order condition implicitly defines \(q\) as a function of \(Q_{-i}\), which is the reaction function of an individual player \(i\) under Nash behavior, \(q_{NA} = r_{NA}^i(Q_{-i})\), where we use the subscript \(i\) to indicate an individual reaction function and use \(NA\) to indicate that this is the reaction function under Nash behavior. Note that \(q_{NA}\) only indicates the best response under Nash behavior as we reserve \(q_{NA}^*\) for the Nash equilibrium itself. Now if we use \(B_Q(Q, x^*(Q)) = C_q(q)\) directly, then we can derive \(q_{NA} = R_{NA}^i(Q)\) which is the individual replacement function of player \(i\). The aggregate replacement function is simply derived by summing over all individual replacement functions and hence \(Q_{NA} = R_{NA}(Q) = \sum_{i \in N} R_{NA}^i(Q)\). The idea is illustrated in Figure 1a for the assumption of downward sloping replacement function.\(^8\)

\(^8\)The graph assumes linear replacement functions but this does not necessarily has to be the case and is not crucial for the following arguments.
Figure 1a: Downward-sloping Replacement Functions in the Nash Equilibrium
The graphical determination of the Nash equilibrium works as follows. First, the aggregate replacement function is derived as the vertical summation of all individual replacement functions. Notice that due to symmetry all individual replacement functions are the same. Second, the intersection of the aggregate replacement function with the 45°-degree line, point $E$, determines the aggregate Nash equilibrium mitigation level because there $Q_{NA}^* = R_{NA}^*(Q_{NA}^*)$ by definition. Third, one draws a vertical line from point $E$ down to $Q_{NA}^*$ on the abscissa. Finally, from the intersection point with the individual replacement functions, point $e$ in the graph, one draws a horizontal line to the ordinate which gives the equilibrium individual mitigation level of player $i$, $q_{NA}^*$. We note that if all individual replacement functions are downward sloping...
over the entire strategy space, also the aggregate replacement function will have this property. If all replacement functions start at a positive value on the ordinate, all equilibrium mitigation levels will be strictly positive. Finally, the aggregate replacement function will only intersect once with the 45\degree-line if its slope is negative over the entire domain.

Let us now turn to the possibility of positively sloped replacement functions, which is illustrated in Figure 1b. The procedure of determining the equilibrium works exactly the same, as discussed above. However, different from above, the slope of the replacement functions matter. Figure 1b illustrates that the aggregate replacement function could have a slope larger than 1, in which case it will never intersect with the 45\degree-line, even if we assumed the upper bound of the strategy space to be infinite. We will show below that the sufficient conditions for an interior and unique equilibrium are closely related to the slope of the aggregate replacement function being smaller than 1. Moreover, as pointed out above, we need to establish sufficient conditions such that replacement functions start at a positive mitigation level on the ordinate. Finally, we have to make sure that equilibrium adaptation levels $x^*(Q^*)$ are unique and interior.

Before dealing with these issues, it is worthwhile to discuss briefly how the concept of replacement functions generalizes to any coalition of size $p$, $1 \leq p \leq n$. First notice that the first order conditions for adaptation are the same for any $p$, $1 \leq p \leq n$. Second, for the full cooperative or socially optimal behavior, we use (6), using $p = n$. Consequently, we can derive an individual replacement function $q_{SO} = R_{i}^{SO}(Q)$ and the aggregate replacement function $Q_{SO} = R^{SO}(Q) = \sum_{i \in N} R_{i}^{SO}(Q)$. Under partially cooperative behavior where a coalition forms where $1 < p < n$, we use (6) for signatories and we use (7) for non-signatories, leading to $q_{S} = R_{i}^{S}(Q)$ and $q_{NS} = R_{i}^{NS}(Q)$, respectively, with the aggregate replacement function of all signatories $Q_{S} = R^{S}(Q) = \sum_{i \in S} R_{i}^{S}(Q)$ and all non-signatories $Q_{NS} = R^{NS}(Q) = \sum_{j \notin S} R_{j}^{NS}(Q)$ and the overall aggregate replacement
function $Q_C = R^C(Q) = R^S(Q) + R^{NS}(Q)$ where the subscript/superscript $C$ stands for coalition and $Q_S = p \cdot q_S$ and $Q_{NS} = (n - p) \cdot q_{NS}$.

**Proposition 3: Slope of Replacement Functions in Mitigation Space**

Consider an arbitrary coalition of size $p$, $1 \leq p \leq n$, and let primes denote the slopes of replacement functions.

a) No cooperation ($p = 1$): The slope of the individual and the aggregate replacement function is given by $R_i^{NA'}(Q) = \frac{A}{c_{qq}(q_{NA})}$ and $R^{NA'}(Q) = \frac{nA}{c_{qq}(q_{NA})}$, respectively.

b) Partial Cooperation ($1 < p < n$): The slope of the individual and aggregate replacement function of signatories is given by $R_i^{SA'}(Q) = \frac{pA}{c_{qq}(q_{SA})}$ and $R^{SA'}(Q) = \frac{p^2A}{c_{qq}(q_{SA})}$, respectively. The slope of the aggregate replacement function is given by $R^{C'}(Q) = A \left[ \frac{p^2A}{c_{qq}(q_{SA})} + \frac{(n-p)pA}{c_{qq}(q_{NS})} \right]$.

c) Full Cooperation ($p = n$): The slope of the individual and the aggregate replacement function is given by $R_i^{SO'}(Q) = \frac{nA}{c_{qq}(q_{SO})}$ and $R^{SO'}(Q) = \frac{n^2A}{c_{qq}(q_{SO})}$, respectively.

**Proof:** We only sketch the proof for no cooperation as the derivation for partial and full cooperation proceeds exactly along the same lines and the relation between individual and aggregate replacement functions has been explained above. We rewrite $B_Q(Q, x^*(Q)) = C_q(q_{NA})$ to have $q_{NA} = R_i^{NA}(Q) = C_q^{-1}(B_Q(Q, x^*(Q)))$. From the theorem of inverse functions, we know that \( \frac{dC_q^{-1}(q)}{dq} = \frac{1}{c_{qq}(q)} \) and hence $R_i^{NA'}(Q) = \frac{dC_q^{-1}(B_Q(Q, x^*(Q)))}{dq} = \frac{1}{c_{qq}(q_{NA})} A$ because $\frac{dC_q(Q, x^*(Q))}{dq} = A$. In more detail, $\frac{dC_q(Q, x^*(Q))}{dq} = B_{QQ} + B_{Qx} \frac{dx}{dq}$. From the first-order condition (8), $x$ is an implicit function of $Q$. The total differential of this first-order condition is equal to: $B_{xx}(Q, x)dx + B_{xQ}(Q, x)dQ - D_{xx}(x)dx = 0$. Accord-
ingly to the implicit function theorem, we have: \[ \frac{dx}{dQ} = B_{xQ} D_{xx} - B_{xx}. \] Hence, \[ \frac{d(B_Q(Q, x^*(Q)))}{dQ} = B_{QQ} + \frac{B_{Qx^2}}{D_{xx} - B_{xx}} = A. \] Q.E.D.

We now turn to the conditions that individual replacement functions (and hence by symmetry and summation also aggregate replacement functions) start on the ordinate at positive mitigation levels. Considering the individual replacement functions under no cooperation, \( q_{NA} = R_{iNA}(Q) = C_q^{-1}(B_Q(Q, x^*(Q))) \), it is clear that \( B_Q(Q, x^*(Q)) \) needs to be positive as \( Q \) approaches 0, and \( C_q^{-1}(B_Q(0, x^*(0))) > 0 \), and \( \lim_{Q \to 0} B_Q(Q, x) > \lim_{q \to 0} C_q(q) > 0 \). Note that if this condition holds, then also all other individual replacement functions, under partial and full cooperation, start from positive mitigation levels. Finally, adaptation levels are unique for a given \( Q \). Recall that \( \frac{dx}{dQ} = \frac{B_{xQ}}{D_{xx} - B_{xx}} \). Because \( B_{xx} \leq 0 \) and \( D_{xx} \geq 0 \), and hence \( D_{xx} - B_{xx} \neq 0 \), then \( x(Q) \) is unique for a given \( Q \). Furthermore, adaptation levels are interior if we assume \( \lim_{x \to 0} B_x(Q, x) > \lim_{x \to 0} D_x(x) > 0 \). Our discussion is summarized in Assumptions 2 and Lemma 2.

**Assumptions 2**

For all players \( i \in N \) and \( x_i \in [0, \bar{x}_i] \) and \( q_i \in [0, \bar{q}_i] \):

a) \( \frac{A_{n}}{c_{qq}(QN)} < 1; \)

b) \( A \left[ \frac{\bar{p}^2}{c_{pq}(QS)} + \frac{(n-p)}{c_{pq}(QNS)} \right] < 1; \)

c) \( \frac{A_{n^2}}{c_{pq}(qSS)} < 1. \)

d) \( \lim_{Q \to 0} B_Q(Q, x) > \lim_{q \to 0} C_q(q) > 0 \) and \( \lim_{x \to 0} B_x(Q, x) > \lim_{x \to 0} D_x(x) > 0 \).
Lemma 2: Existence of a Unique Interior Equilibrium in the Second Stage

Consider an arbitrary coalition of size $p$, $1 \leq p \leq n$.

A sufficient condition for the existence of a unique interior equilibrium mitigation and adaptation vector in the second stage of the coalition formation game as defined in Definition 1, is Assumption 2d) and:

- a) Nash equilibrium ($p = 1$): Assumption 2a);
- b) Coalitional Nash equilibrium ($1 < p < n$): Assumption 2b);
- c) Social optimum ($p = n$): Assumption 2c).

Proof: See the discussion above. Q.E.D.

It is certainly not surprising that the conditions for a unique equilibrium in the second stage for a given coalition of size $p$ are more restrictive than those for existence, except in the social optimum when $p = n$, as it is evident by comparing Assumptions 1 and 2. In the social optimum, there is no strategic interaction by definition and hence the interest of an individual player is in line with the common interest of all players. Hence, strict concavity of the aggregate payoff function with respect to each individual player’s strategies is sufficient for a unique global optimum.

3.4 Reaction Functions

In this subsection, we briefly analyze the relation between replacement functions and reaction functions, where the latter are more common in the literature when it comes to the analysis of strategic interaction among players. Then, we provide an economic interpretation of upward sloping reaction (replacement) functions. In particular, we focus on the strategic aspect between mitigation and adaptation levels in different countries. Clearly, reaction functions under socially optimal behavior are rather uncommon, but we state
them here for completeness and to see the link between reaction and replacement functions. Clearly, this link must remain incomplete as aggregate reaction functions do not make much sense under Nash and socially optimal behavior. However, we can derive them for signatories, \( Q_S = r^S(Q^{NS}) \) and non-signatories, \( Q_{NS} = r^{NS}(Q^S) \) to study the strategic interaction between these two groups in a compact way.

**Proposition 4: Slopes of Reaction Functions in Mitigation and Adaptation Space**

Consider an arbitrary coalition of size \( p \), \( 1 \leq p \leq n \), and let primes denote the slopes of reaction functions. The slopes of the reactions functions in mitigation and adaptation space are respectively:

a) No cooperation \((p = 1)\): The slope of an individual reaction function is given by
\[
r^{NA^i}(Q_{-i}) = \frac{A}{c_{qS(qNA)} - A}.
\]

b) Partial Cooperation \((1 < p < n)\): The slopes of the individual and aggregate reaction functions of signatories are given by
\[
r^{S^i}(Q_{-i}) = \frac{pa}{c_{qS(qNS)} - pA}
\]
and
\[
r^{S^S}(Q^{NS}) = \frac{p^2A}{c_{qS(qNS)} - p^2A},
\]
respectively, and the slopes of non-signatories’ reaction functions are given by
\[
r^{NS^i}(Q_{-i}) = \frac{A}{c_{qS(qNS)} - A}
\]
and
\[
r^{NS^S}(Q^S) = \frac{(n-p)A}{c_{qS(qNS)} - (n-p)A}.
\]

c) Full Cooperation \((p = n)\): The slope of an individual reaction functions is given by
\[
r^{SO^i}(Q_{-i}) = \frac{nA}{c_{qS(qSO)} - nA}.
\]
d) For each possible coalition, the slope of the reaction function \( x = f_i(Q) \) is given by
\[
\frac{Bq_i}{Bxx - B_{xx}} < 0.
\]

**Proof:** The derivation follows the same lines as described for replacement functions in Subsection 3.3, in particular Proposition 3 and is therefore omitted. **Q.E.D.**

The economic interpretation of Proposition 4 are summarized in the following Corollary.
Corollary 1: Strategic Substitutes and Complements

1) Adaptation and (total) mitigation are strategic substitutes for every possible coalition \( p, 1 \leq p \leq n \).

2) Under Assumptions 1 for the existence of a second stage equilibrium:
   i) the signs of the slopes of reaction functions and corresponding replacement functions in mitigation space are the same;
   ii) the signs of the slopes of the reaction functions in mitigation space are the same for all players;
   iii) the signs of the slopes of the reaction and replacement functions in mitigation space depend on the sign of the term \( A \); functions are downward sloping if \( A < 0 \) in which case mitigation levels are strategic substitutes, and they are upward sloping if \( A > 0 \) in which case mitigation levels are strategic complements;
   iv) downward sloping reaction functions in mitigation space have a slope strictly larger than -1;
   v) upward sloping reactions functions in mitigation space may have a slope larger than 1.

3) Under Assumptions 2 for the existence of a unique interior second stage equilibrium:
   upward sloping individual reaction functions in mitigation space have a slope strictly smaller than 1. For the slope of the aggregate reaction functions of signatories and non-signatories this does not have to be the case, though the sum of the slopes of the two aggregate reaction functions is strictly less than 2.

Proof: Follows directly from Proposition 4 using some basic algebra, noting that the denominator of all slopes of the reaction functions in mitigation space are positive due to Assumptions 1 and 2. Q.E.D.

Part 1 of Corollary 1 gives a clear answer to the question whether adaptation and total mitigation are substitutes or complements. They are al-
ways substitutes, irrespective whether considered under non-, partial- or full-
cooperative behavior.

Part 2 stresses the direct link between reaction and replacement func-
tions. More important from an economic point of view, mitigation levels can be strategic substitutes or complements. This is related to the sign of the term $A := B_{QQ} + \frac{(B_{xQ})^2}{D_{xx}-B_{xx}}$. The first part of $A$, $B_{QQ} < 0$, is the part which would only appear in a pure mitigation game. Therefore, in a pure mitigation $A < 0$ always holds and hence all slopes of the reaction functions listed in Proposition 4 are always negative. That is, mitigation levels are always strategic substitutes. The second part of $A$, $\frac{(B_{xQ})^2}{D_{xx}-B_{xx}} > 0$, comes from the interaction between mitigation and adaptation. If this term is large enough, then $A > 0$ is possible, without violating the even most restrictive Assumptions 2. Simply speaking, the stronger the strategic interaction between mitigation and adaptation, the larger this term.

This is also evident, when considering the first order conditions in terms of mitigation for instance under Nash behavior $B_Q(q_{NA}+Q_{-i}, x^*(q_{NA}+Q_{-i})) = C_q(q_{NA})$, using the information that optimal adaptation is a function of total mitigation. Increasing $Q_{-i}$ has a direct negative effect on $B_Q$, namely $B_{QQ} < 0$, which, anything else being equal, would call to lower $q_{NA}$ in order for the equality to be satisfied. However, there is also the indirect effect because increasing $Q_{-i}$ calls for a lower $x^*$, which in turn has an indirect positive effect on $B_Q$ because $B_{Qx} < 0$. This second indirect effect is exactly $\frac{(B_{xQ})^2}{D_{xx}-B_{xx}}$.

It is worthwhile pointing out that despite the possibility of $A > 0$, in which case the indirect effect is stronger than the direct effect, neither Assumptions 1, the sufficient conditions for strict concavity, nor Assumptions 2, the sufficient conditions for the existence of a unique interior equilibrium need to be violated. Thus, we generalize the result of Ebert and Welsch (2011, 2012) to more than two countries, considering not only non-cooperative behavior of players but also any degree of partial cooperation, including full
Ebert and Welsch conjecture (without proof) that upward sloping reaction functions could lead to more optimistic outcomes in a coalition formation game (i.e. larger coalitions). The intuition (which we analyze in more detail in Subsection 3.5 and Section 4) is that downward sloping reaction functions imply that any additional increase of signatories’ mitigation efforts is countered by a decrease of non-signatories’ mitigation efforts. In the context of climate change, this has been called (carbon) leakage. Thus, upward sloping reaction functions may be viewed as a form of anti-leakage or matching.

Ebert and Welsch additionally conjecture that in terms of large stable coalitions, signatories’ reaction functions should be downward sloping and non-signatories should be upward sloping such that signatories have a relative strategic advantage over non-signatories and additionally benefit from the matching behavior of non-signatories, thereby reducing the free-rider incentive. Unfortunately, as Part 2 ii) in Corollary 1 makes clear, we cannot validate this conjecture because the signs of the slopes of the reaction functions of all players are the same, even under the assumption of partial cooperation, due to our assumption of ex-ante symmetric players. However, given the widespread assumption of ex-ante symmetric players in the literature of coalition formation and the complications involved in any analysis departing from this assumption, it seems sensible to focus first on an exhaustive analysis under this standard assumption.

Finally, note that Part 3 relates to the common notion that an interior unique equilibrium requires reaction functions with slope less than 1 in absolute terms. However, as the second sentence related to the aggregate reaction functions of signatories and non-signatories highlights, things are less straightforward as commonly believed. The reason is that this situation can be viewed as a game between two possibly asymmetric players (all signatories versus all non-signatories) for which it suffices for uniqueness if the sum of the slopes of the reaction functions is smaller than 2 over the entire strategy.
3.5 Coalition Formation and Second Stage Equilibria

In this subsection, we have a closer look at some important properties related to the second stage, which help us to clarify the incentives to form coalitions in the first stage. In a first step, we analyze how mitigation and adaptation levels change with the degree of cooperation, related to the size of coalition $S$, $p$. In a second step, we consider how this relates to the properties which we defined in Section 2. In a third step, we consider how these properties relate to the stability of coalitions. Note that any discrete change of $p$ (because the number of signatories must be an integer value) can be approximated by a continuous change and hence we can use the differential with respect to $p$.

**Proposition 5: Equilibrium Mitigation and Adaptation Levels and the Degree of Cooperation**

Consider an arbitrary coalition of size $p$, $1 \leq p \leq n$. Further assume Assumptions 1 and 2 and let an asterisk denote equilibrium values for a given $p$.

a) Non-signatories
i) $\frac{dq^*_S}{dp} > 0$ if $A > 0$ and $\frac{dq^*_S}{dp} < 0$ if $A < 0$.
ii) $\frac{dQ^*_S}{dp} > 0$ if $A > 0$ and $\frac{dQ^*_S}{dp} < 0$ if $A < 0$.

b) Signatories
i) $\frac{dq^*_S}{dp} > 0$ if $A > 0$ and $\frac{dq^*_S}{dp} > 0$ if $A < 0$.
ii) $\frac{dQ^*_S}{dp} > 0$.

c) Signatories and Non-signatories
$\frac{dx^*_i}{dp} < 0$.

d) Signatories and Non-signatories at the aggregate
i) $\frac{dQ^*_c}{dp} > 0$
Moreover, that tion cost functions because are the same for signatories and non-signatories, but keep them for mitiga-
differentiation gives the pure mitigation game.

The first order conditions (6) and (7) imply the mitigation-adaptation game and the subscript M

Proof: We will omit the arguments in the benefit functions because they are the same for signatories and non-signatories, but keep them for mitigation cost functions because \( q_s > q_{NS} \) for every \( p \neq 1 \) from Proposition 2. Moreover, in order to save on notation, we drop the asterisk in the following derivations. We use four pieces of information in order to obtain the following results. 1) Total differentiating the first order conditions (6), using \( x(Q_C) \), and recalling that \( \frac{dB}{dQ} x(Q_C) = A \), i.e. \( pBQ(Q_C, x(Q_C)) = C_q(q_s) \), gives \( dp \cdot BQ + p \cdot A \cdot dQ = C_q(q_s) \cdot dq_s \). 2) Total differentiating of the first order conditions (7) gives \( A \cdot dQ = C_q(q_{NS}) \cdot dq_{NS} \). 3) Noting that the first order conditions (6) and (7) imply \( p \cdot C_q(q_{NS}) = C_q(q_s) \), total differentiation gives \( dp \cdot C_q(q_{NS}) + p \cdot C_q(q_{NS}) \cdot dq_{NS} = C_q(q_s) \cdot dq_s \). 4) Moreover, \( \frac{dQ_s}{dp} = q_s + p \cdot \frac{dq_s}{dp}, \frac{dQ_{NS}}{dp} = -q_N + (n - p) \cdot \frac{dq_{NS}}{dp} \) and \( \frac{dC}{dp} = \frac{dC_s}{dp} + \frac{dC_{NS}}{dp} \). Now, some basic manipulations lead to the following results. a) \( \frac{dq_{NS}}{dp} = \frac{C_{(q_s)}}{C_{(q_{NS})}} \frac{dQ}{dp} \), implying because \( C_{qq}(q_{NS}) > 0 \) by the General Assumptions and \( \frac{dC}{dp} > 0 \) as we show below that the sign depends on the sign of \( A \). aii) \( \frac{dQ_{NS}}{dp} = -q_{NS} \left( \frac{1 - A \frac{q_s}{C_{qq}(q_s)}}{1 - A \left( \frac{q_s}{C_{qq}(q_s)} + \frac{n - p}{C_{qq}(q_{NS})} \right)} \right) \) where we note that \( 1 - A \frac{q_s}{C_{qq}(q_s)} > 0 \) by Assumptions 1, \( 1 - A \left( \frac{q_s}{C_{qq}(q_{NS})} + \frac{n - p}{C_{qq}(q_{NS})} \right) > 0 \) by Assumptions 2 and hence the denominator is always positive. Consequently, if \( A < 0 \), the nominator is negative and hence \( \frac{dQ_{NS}}{dp} < 0 \) follows. If \( A > 0 \), the expression cannot be signed because an increase in \( p \) by one unit implies one non-signatory less but \( q_{NS} \) increase in \( p \) as shown above. b) \( \frac{dq_s}{dp} = \frac{B \frac{q_s}{C_{(q_s)}} + A \frac{q_s}{C_{(q_{NS})}}}{C_{(q_s)}} \) where we note that the first term on the R.H.S. is positive by the General Assumptions and the sign of the second term depends on \( A \). Hence if \( A > 0 \), then \( \frac{dq_s}{dp} > 0 \), otherwise if \( A < 0 \) this expression cannot be signed. bii) \( \frac{dq_s}{dp} = \frac{(qs-q_{NS}) + p \frac{C_{qs}(q_{NS})}{C_{qs}(q_s)} + q_s \left( 1 - \frac{Aq_s^2}{C_{qs}(q_s)} \right) - \frac{(n-p)A}{C_{qs}(q_{NS})} \left( q_{NS} + p \frac{C_{qs}(q_{NS})}{C_{qs}(q_s)} \right)}{1 - A \left( \frac{q_s}{C_{qs}(q_s)} + \frac{n - p}{C_{qs}(q_{NS})} \right)} \)
where the denominator is positive by Assumptions 2, $q_S - q_{NS} > 0$ from Proposition 2, $1 - \frac{Ap^2}{C_{qq}(q_S)} > 0$ by Assumption 1, and $q_S + p \frac{C_{qq}(q_S)}{C_{qq}(q_S)} > 0$ by the General Assumptions. Consequently, if $A < 0$, the nominator is positive and $\frac{dQ_S}{dp} > 0$ is evident. In case $A > 0$, we use simply $\frac{dQ_S}{dp} = q_S + p\left[\frac{B_Q}{C_{qq}(q_S)} + \frac{Ap}{C_{qq}(q_S)} \frac{dQ_C}{dp}\right] > 0$. e) $\frac{dx}{dp} = \frac{B x Q}{D_{xx} - B_{xx}} \frac{dQ_C}{dp}$ where the first term on the R.H.S is negative by the General Assumptions and $\frac{dQ}{dp} > 0$ as shown below. di) $\frac{dQ}{dp} = \frac{(q_S - q_{NS}) + p \frac{C_{qq}(q_S)}{C_{qq}(q_S)}]}{1 - \frac{Ap^2}{C_{qq}(q_S)} [p^2 \frac{C_{qq}(q_S)}{C_{qq}(q_S)} + (n-p)]}$ where we notice that the nominator is obviously positive and the denominator is positive by Assumptions 2. dii) Because $x^*_{C,M+A}(p) > x^*_{C,M}(p) = 0$ trivially by definition and a unique interior equilibrium by Assumptions 2, $Q^*_{C,M+A}(p) < Q^*_{C,M}(p)$ follows trivially from the first order conditions (6), and the General Assumptions. Q.E.D.

Part ai) confirms that the reaction of non-signatories when the degree of cooperation increases depends on the sign of the slope of their reaction functions. If a non-signatory joins the coalition such that $p + 1$, the total mitigation level of signatories, $Q_S$, increases (Part bi)), and the remaining individual non-signatories match this behavior if mitigation levels are strategic complements and countervail this behavior if they are substitutes.

Clearly, moving from $p$ to $p + 1$, means one non-signatory less and hence if individual non-signatories equilibrium provision level $q_{NS}$ drops as $p$ increases ($A < 0$), the total provision level of non-signatories, $Q_{NS}$, will have dropped. In case of strategically complementary mitigation levels, we have two opposing effects and hence overall predictions are difficult (Part aii)).

Interestingly, despite signatories’ total mitigation level always increases when a non-signatory joins their coalition (Part bi)), individual mitigation levels do not necessarily have to increase (Part bi)). On the one hand, one more member calls for higher individual provision levels because more players internalize the externality among them. On the other hand, before the expansion of the coalition, the new joining member had lower marginal mitigation
costs than the old members; now when joining the coalition, the equalization of marginal mitigation costs (as a result of cost-effectiveness within the coalition) may call for lower mitigation levels of old signatories compared to the initial situation. However, at the aggregate things are clear-cut: total mitigation levels increase with the size of the coalition (Part di)). As total mitigation and adaptation are substitutes, it is not surprising that the opposite holds for adaptation levels (Part c)).

Viewing Part di) and Part c) together, suggests that not only in a pure mitigation game (henceforth abbreviated M-game), total mitigation increases with the degree of cooperation and is highest in the social optimum, but also in the mitigation-adaptation game (henceforth abbreviated M+A-game). Because of the substitutional relation between adaptation and mitigation, for any degree of cooperation, total mitigation will be lower in the M+A-game than in the M-game (Part dii)). Since the equivalent term for $A$ in the M-game is always negative, i.e. $B_{QQ} < 0$, $\frac{dq_{NS}}{dp} < 0$ and $\frac{dQ_{NS}}{dp} < 0$ would always hold.

We now have a look at the some of the properties we defined in Section 2 and how they relate to the incentives to form coalitions. In a first instance, one may clarify the normatively motivated question why we should care about the degree of cooperation. At the most basic level, total payoffs under full cooperation (social optimum) should be larger than under any other degree of cooperation (partial and no cooperation), which is called cohesiveness in the literature on coalition formation. As pointed out already in Section 2, this condition trivially holds in the M+A-game, but also holds in the M-game as in any externality game without further complication. The more interesting question relates to whether total payoffs continuously increase in the degree of cooperation for all $1 \leq p \leq n$, called full cohesiveness such that it is worthwhile to analyze conditions that favour large stable coalitions.

Unfortunately, it is very difficult to establish full cohesiveness at a general level, using some or all assumptions mentioned before. The reason is that
if mitigation levels are strategic substitutes, an expansion of the coalition means on the one hand higher total mitigation levels but on the other hand drives an increasing wedge between the mitigation levels of signatories and non-signatories, causing cost-ineffectiveness from a global point of view (as cost functions are symmetric by assumption and marginal cost equalization requires the same mitigation levels across all players). However, if we can derive conditions when the positive externality and superadditivity conditions hold, then we have sufficient for full cohesiveness.

In the M-game, the positive externality property holds generally, and as will become evident from our proof of Proposition 6 below, this is also true in the M+A-game. In order to establish full cohesiveness, this is certainly good news (but bad news in terms of large stable coalitions as argued below). Thus, the focus must be on establishing superadditivity.

Note that though superadditivity cannot be violated over the entire range of \( p \) (otherwise cohesiveness could not hold), it may nevertheless be violated for some \( p \). For instance, in the M-game, it is easy to construct examples where for instance starting from the singleton coalition structure, and then forming a two-player coalition both players are worse off than initially. The reason is that if the total number of players is large enough, there are too many free-riders who countervail the efforts of the small group of signatories; in other words, the leakage effect is too strong to make it worthwhile for signatories to set a good example. In the M-game, this leakage is zero if the benefit function is linear in mitigation, i.e. \( B_{QQ} = 0 \), as then reaction functions are orthogonal (have a slope of zero). In the M+A-game, \( A > 0 \) causes anti-leakage, which is true for \( B_{QQ} = 0 \), but much more generally, as discussed above, namely as long as the indirect effect from mitigation via adaptation is stronger than the direct effect. It is exactly these conditions, which allow us to establish superadditivity in the M- and M+A-game, respectively.
Proposition 6: Cohesiveness, Positive Externality and Superadditivity

Let Assumptions 1 and 2 hold and let an asterisk denote equilibrium values for a given $p$.

a) The M-game and the M+A-game, are cohesive.

b) In the M-game and the M+A-game, the positive externality property as defined in Definition 1, holds. That is, \( \frac{\partial \Pi^*_S(p)}{\partial p} > 0 \) for all $p$, $1 \leq p < n$.

c) A sufficient condition for superadditivity in the M-game is $B_{QQ} = 0$ and in the M+A-game it is $A > 0$ which are also sufficient for full cohesiveness.

Proof: a) is obvious. b) Derivations similar to those described in the proof of Proposition 5 deliver \( \frac{\partial \Pi^*_S(p)}{\partial p} = B_Q \left[ \frac{\partial Q^*_t}{\partial p} \left( 1 - \frac{A}{C_{qq}(q^*_NS)} \right) \right] \) which is positive because \( \frac{\partial Q^*_t}{\partial p} > 0 \) from Proposition 5 and \( 1 - \frac{A}{C_{qq}(q^*_NS)} > 0 \) by Assumptions 1. For the M-game, replace $A$ by $B_{QQ}$ in the differential. c) Consider the M-game for which \( \frac{\partial q^*_S}{\partial p} = 0 \) if $B_{QQ} = 0$ is true. That is, all non-signatories not involved in the move from $S$ to $S \cup \{i\}$ will not change their mitigation levels. Hence, \( \max_{q^S \cup \{i\}} \sum_{i \in S \cup \{i\}} \Pi_S(q^S(S \cup \{i\}), q^{-s}) > \max_{q^S \cup \{i\}} \sum_{i \in S} \Pi_S(q^S(S), q_i, q^{-s}) + \max_{q^S \cup \{i\}} \Pi_S(q^S(S), q_i, q^{-s}) \) must be true where superscripts indicate vectors. The proof in the M+A game is slightly more involved but proceeds along the same lines. Q.E.D.

The importance of superadditivity for internal stability has two dimensions. The first dimension is at the intuitive level: only if superadditivity holds, players have an incentive to join a coalition. The second dimension is at the more specific level by considering that the internal stability condition can be simplified in the context of ex-ante symmetric players:

\[
\Pi_S(p) \geq \Pi_{NS}(p - 1)
\] (11)

and, similarly, superadditivity:
\[ p \Pi_S(p) \geq (p - 1) \Pi_S(p - 1) + \Pi_{NS}(p - 1) . \] (12)

First note that the focus on internal stability can be justified on two grounds. First, in the context of a public good it seems "natural" to be more concerned about a player leaving the coalition than joining it. Second, note that internal and external stability are directly linked for symmetric players (Carraro and Siniscalco 1993). If coalition \( p \) is not externally stable, then coalition \( p + 1 \) is internally stable and under full cohesiveness, the aggregate payoff over all players is higher under \( p + 1 \) than \( p \).

Now consider that we rearrange the superadditivity condition (12) such that we have:
\[ p \cdot \Pi_S(p) - (p - 1) \cdot \Pi_S(p - 1) \geq \Pi_{NS}(p - 1) \text{ or } \Pi_S(p) + (p - 1) \cdot (\Pi_S(p) - \Pi_S(p - 1)) \geq \Pi_{NS}(p - 1). \]
We know that \( \Pi_{NS}(p - 1) > \Pi_S(p - 1) \) because all players have the same benefits from mitigation and adaptation and the same costs from adaptation but signatories have higher mitigation levels and hence higher mitigation costs than non-signatories. So we have: \( \Pi_S(p) + (p - 1) \cdot (\Pi_S(p) - \Pi_S(p - 1)) \geq \Pi_{NS}(p - 1) > \Pi_S(p - 1). \)

If we compare this condition of superadditivity with the internal stability condition (11) from above, then we see that the superadditivity condition has an additional term on the left hand side, namely \((p - 1) \cdot (\Pi_S(p) - \Pi_S(p - 1))\). This term needs to be positive and therefore superadditivity is a necessary condition for internal stability.

Why does this term need to be positive? If \( \Pi_S(p) - \Pi_S(p - 1) > 0 \) does not hold, then \( \Pi_S(p) \geq \Pi_{NS}(p - 1) \geq \Pi_S(p - 1) \) is not possible and hence superadditivity cannot hold. Thus, superadditivity is a necessary (though not sufficient) condition for internal stability. From this argument we can also immediately conclude that \( \Pi_S(p) - \Pi_S(p - 1) > 0 \) is a necessary condition for superadditivity to hold and this condition is nothing else, expressed as a continuous instead of a discrete change of \( p \) as \( \frac{\partial \Pi_S(p)}{\partial p} > 0 \). We summarize our discussion in Proposition 7.
Proposition 7: Signatories’ Payoff, Superadditivity and Internal Stability

a) A sufficient condition for \( \frac{\partial \Pi^*_S(p)}{\partial p} > 0 \) in the M-game is \( B_{QQ} = 0 \) and in the M+A-game it is \( A > 0 \).

b) \( \frac{\partial \Pi^*_S(p)}{\partial p} > 0 \) is a necessary condition for superadditivity.

c) Superadditivity is a necessary condition for internal stability.

d) Superadditivity is a sufficient condition for the existence of a non-trivial stable coalition.

Proof:

a) Some basic algebra delivers:
\[
\frac{\partial \Pi^*_S(p)}{\partial p} = \frac{B_Q(q_S - q_{NS}) \left(1 - \frac{2A}{e_{qq}(q_{NS})^2}\right) + p(n-p) \frac{C_q(q_{NS})}{e_{qq}(q_S)} \frac{A}{1 - \frac{A}{e_{qq}(q_{NS})^2}(p^2 - e_{qq}(q_{NS})^2 + (n-p))}}{1 - \frac{A}{e_{qq}(q_{NS})^2}(p^2 - e_{qq}(q_{NS})^2 + (n-p))}.
\]
which can be easily signed if \( A \geq 0 \), noting that in the M-game we can substitute \( B_{QQ} = 0 \) for \( A \) in this expression. b) and c) have been proved in the text above. d) Is proved in Eyckmans et al. (2012).

Though part d) of Proposition is interesting, it is still not clear how large stable coalitions will be and whether they are larger in the M+A-game than in the M-game and if so on what this depends. The reason is that superadditivity is only a necessary condition but not a sufficient condition for internal stability. If we reconsider the internal stability condition (11), recalling from Proposition 6 that \( \Pi^*_{NS} \) increases in \( p \) due to the positive externality property, starting from \( p = 1 \) in which case \( \Pi^*_S(1) = \Pi^*_{NS}(1) \), gradually increasing \( p \), in order to have large stable coalitions, we need that \( \Pi^*_S(p) \) increases faster than \( \Pi^*_{NS}(p - 1) \). The question is what "faster" means. Moreover, we cannot rule out the possibility that \( \Pi^*_S(p) \) decreases first and then, only at some level \( p \), may increase. We also have to be aware that \( \Pi^*_S(p) < \Pi^*_{NS}(p) \) for any \( p \) and hence the "fast" increase of \( \Pi^*_S(p) \) to have internal stability at \( p + 1 \), i.e. \( \Pi^*_S(p + 1) \geq \Pi^*_{NS}(p) \), must happen within a very short interval.
In order to shed light on this question, we will have to consider some examples in Section 4.

4 Conclusion

In this paper, we have analyzed how adaptation, as an additional strategy to mitigation, affects the prospects of international policy coordination to tackle climate change. More specifically, we have studied the strategic interaction between mitigation and adaptation strategies in the canonical model of international environmental agreements (IEAs). We have shown that these two strategies are strategic substitutes considering various definitions of substitutability, regardless whether countries behave non-cooperatively, partially cooperative or fully cooperative. Moreover, different from a pure mitigation game, adaptation may cause mitigation levels between different countries to be strategic complements, generalizing a result by Ebert and Welsch (2011 and 2012) to more than two countries and the possibilities of players to form self-enforcing IEAs. Under which conditions this leads to more positive cooperative outcomes compared to the pure mitigation game has been analysed. Particular emphasis has been placed on the sufficient conditions for the existence and uniqueness of interior equilibrium strategies, how they link to the slopes of the reactions and eventually to the success of coalition formation.

In order to have more insights on the conditions which ensure the stability of coalitions, especially internal stability, we will have to consider some examples and carry out simulations runs.

References


