Forest-Based Industrial Network

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Abstract

Following the literature on automation, we model the industrial network of the forest-based sector, with random demands, in presence of supply contracts. The economic network is composed of upstream, instream and downstream agents. Through the resolution of the variational inequality model, we investigate the network equilibrium flows and attempt to compute the prices at which the former could be attained. With respect to other results on optimal pricing of timber and wood products in France, the model outputs show that the forest resources may be overvalued by the market, while the manufactured products may be undervalued. The analysis in a dynamic setting corroborates these results. In case of network disequilibrium, the expected profits switch from increasing monotonic s-shaped functional forms to non-monotonic bell-shaped functional forms. Finally, we explicit the equilibrium conditions in case of vertical integration between the upstream and instream agents.

Keywords: Network Economics, Variational Inequality, Supply Chain, Forest-Based Sector
JEL Classification: C72, D85, D86, L11, L14, L73

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1 Introduction

The supply chain network is a complex object, and finding the equilibrium flows, along the chain value, is one of the fundamental problems in industrial organization. A decentralized supply chain network is a structure that involves several decision makers within the chain value. The steady behavior of each decision maker can be separately characterized by a series of equilibrium conditions (Yang and Zhao, 2011). The natural mathematical expression for doing so lies in variational inequality, with distinguishable states and controls (Friesz et al., 2006; Friez, 2010). Nagurney et al. (2002) first modeled the supply chain network equilibrium, and showed that it could be formulated as a variational inequality problem. Dong et al. (2004) and Yang and Zhao (2011) extended this approach to random demands with known probability distributions. Finally, Cojocaru et al. (2005), Yong-Hua et al. (2013) and Nagurney et al. (2014) provided a framework which allows to study the cases with time-dependent variational inequalities.

The forest-based sector can be thought of as an industrial network composed of: forest owners and managers, who not only manage forests, but also commercialize timber; lumber manufacturers, also known as the first processing subsector, who transform timber into lumber and commercialize it to lumber remanufacturers; the latter being part of what is called the second processing subsector. With respect to this configuration, we can reasonably speak in terms of upstream, instream and downstream agents, all being connected through a vertical supply chain and forming a network. Because of the strain that hangs over the forest resources, the equilibrium of wood flows in the French forest-based sector is one of the major challenges faced by the industry players (Sergent, 2010; Conseil National de l'Industrie, 2014), which makes the network framework even more suitable.

In France, the timber is sold through auction sales, over-the-counter sales and supply contracts, the latter being implemented by the French National Forestry Office (ONF),[1] but also, to a lesser extent, by the private forest cooperatives. In the past, the French Forest Code used to stipulate that the sales be on standing timber and by auctions exclusively. In order to structure the forest sector, the supply contracts have been

[1]The public agency is an industrial and commercial institution; it is also in charge of providing the services of general interest. As an important actor of the French forest-based sector, the manufacturing industry is relatively dependent upon the strategy pursued by the Office.

developed since a 2001 legislative reform, which removed the derogatory nature of the over-the-counter sales. The supply contracts provide for the provision of logs purchased in installments at several fixed dates. They can be annual or pluriannual, and cover the concepts of log volume, length, species, quality and price. The literature has shown that the contract is a common and effective supply chain coordination mechanism (Cachon and Lariviere, 2005; Yang and Zhao, 2011). Before the paper by Barkaoui and Dragicevic (2014), in which the authors proved that Nash bargaining with social preferences can lead to optimal contracting, Cachon and Lariviere (2005) have shown that profit-sharing contracts could theoretically coordinate the entire supply chain.

Unlike Nagurney (2009), who presented a framework that highlights the strategic advantages with horizontal mergers, we decide to analyze the impacts on equilibrium flows of a vertically integrated supply chain. Indeed, vertical integration occupies a central role in organizational economics. Williamson (2005) considers it to be the paradigm problem for explaining the distribution of firms and markets in modern economies. As such, the supply chain of a firm is owned by that firm, which reduces the number of intermediaries, and, in turn, cancels the transaction costs and solves the well-known double marginalization issue (Spengler, 1950). For example, Acemoglu et al. (2005) find that vertical integration is prevalent in countries with great contracting costs. With regard to the forest-based sector, Flückiger (2003) explains that, due to the lack of vertical integration, the forest-based sector suffers from the profitability shortfall. From the previous observation, Nordic countries have decided to move forward this industrial reconfiguration (Westholm et al., 2015).

Through the resolution of the variational inequality model, we investigate the network equilibrium flows and attempt to compute the prices at which the former could be reached. Compared with other results on optimal pricing of timber and wood products, the model outputs show that the forest resources may currently be overvalued by the market, while the manufactured products may currently be undervalued. The analysis of the forest-based sector in a dynamic setting corroborates these results. In case of network disequilibrium, we find that the expected profits switch from increasing monotonic s-shaped functional forms to non-monotonic bell-shaped functional forms. At last, we explicit the equilibrium outcomes in case of vertical integration between the upstream and instream agents.

The remainder of the paper is as follows. In Section 2, we outline, through a variational inequality model, a forest-based network with random demands, and analyze how supply contracts impact the equilibrium conditions. Sections 3 focuses on the network equilibrium in presence of vertical integration. Section 4 extends the equilibrium analysis in a time-varying setting. Section 5 is devoted to illustrating simulation examples. We briefly discuss the model outputs and conclude in Section 6.

2 Model

Following the modeling works on automation (Yang and Zhao, 2011; Setoodeh et al., 2012), consider a forest-based industrial network, such as the one depicted in Fig. 1, composed of U upstream agents or forest managers, each of which owns R_U resource units and provides them, in form of roundwood logs, to instream agents I, like the sawmills. The instream agents then sell their lumber production to downstream agents D or remanufacturers.

We denote a typical upstream agent by u, a typical instream agent by i and a typical downstrem agent by d.

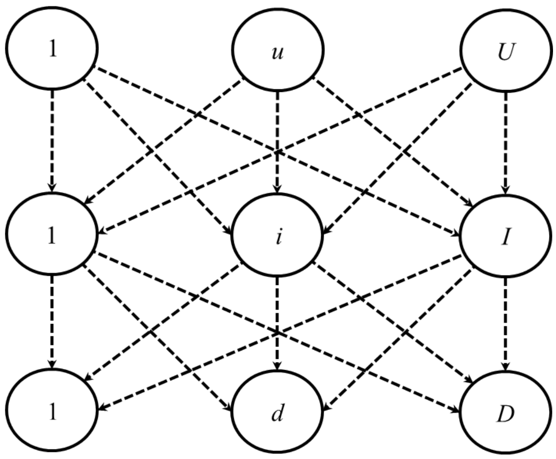


Figure 1: Forest-based industrial network

The volume owned by upstream agent u must satisfy the following conservation of flow

$$\sum_{r_u=1_u}^{R_u} q_u \geq \sum_{i=1}^{I} q_{ui} \tag{1}$$

The transaction volume between upstream agent u and instream agent i is denoted by q_{ui}. The inequality stipulates that the marketed volume cannot exceed the forest production.

In parallel, let $S_i(q_i)$ be the expected sales of the instream agent i

$$S_i(q_i) = \mathbb{E}\left[\min(q_i, D_i)\right] = q_i - \int_0^{q_i} P_i(x)dx \tag{2}$$

where P_i is the demand distribution function of the instream agent i and p_i is its density function. The distribution function is considered to be differentble, strictly increasing and $P_i(0) = 0$. As well, its expectation equals $\mu_i = \mathbb{E}\left[D_i\right]$.

When supply exceeds demand, let $I_i(q_i)$ be the expected inventory of instream agent i

$$I_i(q_i) = \mathbb{E}\left[q_i - D_i\right] = q_i - S_i(q_i) \tag{3}$$

When demand exceeds supply, let $L_i(q_i)$ be the expected stockout of instream agent i

$$L_i(q_i) = \mathbb{E}\left[D_i - q_i\right] = \mu_i - S_i(q_i) \tag{4}$$

We introduce the supply contracts $\langle p_{ui}, \phi_{r_u i}\rangle$, defined over a price and a bargaining outcome, such as those presently in place in the French forest-based sector, into the network. Let $\phi \in [0,1]$ be the profit share that the instream agent keeps on the link (r_u, i), so $1 - \phi$ is the upstream agent share. The fractions are determined through Nash bargaining (Barkaoui and Dragicevic, 2014). The transfer adjustment between the upstream agent and the instream agent amounts to

$$T_{r_u i}(q_i) = (1 - \phi_{r_u i})\left(v_i - (v_i - p_{ui})\frac{q_{ui}}{q_i}S_i(q_i)\right) \tag{5}$$

where v_i represents the value per unit of resource, while p_{ui} stands for the price at which the resource is sold.

2.1 Upstream agents

Assume that upstream agent u charges instream agent i for resource unit r_u with unit price p_{ui}. Each upstream agent u is confronted with a production cost function f_u. His transaction cost with the instream agent is denoted by $c_u(q_u)$. For each gap between upstream offer and instream demand, there is a foregone benefit g_{ui}. This economic loss has to be accounted for as an impact on the supply chain equilibrium flows. The optimal prices are determined by finding the equilibrium point of the network. Upstream agents compete in a noncooperative way and each u solves the following optimization problem in order to maximize his profit Π_{ui}

$$\max \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} [p_{ui}q_{ui} - c_{r_u i}(q_{ui})] - \sum_{r_u=1_u}^{R_u} f_{r_u i}(q_{ui}) - \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} [g_{ui}L_i(q_i) - T_{r_u i}(q_i)] \tag{6}$$

subject to $q_{ui} \geq 0, \forall u = 1, ..., U, \forall r_u = 1, ..., R_u, \forall i = 1, ..., I.$

The Nash equilibrium of the noncooperative game between upstream agents coincides with the solution of the following variational inequality problem

$$\begin{aligned} &- \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[p^\star_{ui} - \frac{\partial c_{r_u i}(q^\star_{ui})}{\partial q_{ui}} \right] \times [q_{ui} - q^\star_{ui}] \\ &+ \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \left[\frac{\partial f_{r_u i}(q^\star_{ui})}{\partial q_{ui}} \right] \times [q_{ui} - q^\star_{ui}] \\ &+ \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[g_{ui} \left[1 - P_i(q^\star_i)\right] - (1 - \phi_{r_u i}) \left(v_i - (v_i - p_{ui}) \frac{\partial \frac{q^\star_{ui}}{q_i} S_i(q^\star_i)}{\partial q_{ui}} \right) \right] \times [q_{ui} - q^\star_{ui}] \geq 0 \end{aligned} \tag{7}$$

Lemma 1 *The Nash equilibrium is defined as the pair of flows* $\left(\boldsymbol{q}^{U\star}, \boldsymbol{q}^{I\star}\right) \in \Gamma^1$, *where* $\boldsymbol{q}^U$ *and* $\boldsymbol{q}^I$ *are respectively the* UR_U *and* UR_UI*-dimensional volume vectors, whose elements are* q_u *and* q_{ui}*, such that*

$$\Gamma^1 = \left\{ \left(\boldsymbol{q}^U, \boldsymbol{q}^I\right) \mid \left(\boldsymbol{q}^U, \boldsymbol{q}^I\right) \in \mathbb{R}_+^{UR_U + UR_U I} \right\} \tag{8}$$

is the upstream subspace of the network equilibrium, $\forall u = 1, ..., U, \forall r_u = 1, ..., R_u, \forall i = 1, ..., I$*, provided that the optimization constraints hold.*

2.2 Instream agents

Instream agents are involved in transactions with both upstream and downstream agents. Instream agent i faces a transaction cost $c_i(q_i)$. Each instream agent i is confronted with a production cost function f_i. Consider that instream agent i charges downstream agent d with unit price p_{id}. This price is obtained by finding the equilibrium point of the network. For each gap between instream offer and downstream demand, a shortfall penalty g_{id} is incurred. Again, the gap between supply and demand will create an imbalance in the network. Each i solves the following optimization problem in order to maximize his profit Π_{id}

$$\max \sum_{d=1}^{D} [p_{id} S_i(q_i) - c_{id}(q_{id})] - \sum_{i=1}^{I} f_{id}(q_{id}) - \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} [p_{ui} q_{ui} + c_{r_u i}(q_{ui})] - \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} [g_{id} L_i(q_i) - T_{r_u i}(q_i)] \quad (9)$$

subject to $q_{ui}, q_{id} \geq 0, \forall u = 1, ..., U, \forall r_u = 1_u, ..., R_u, \forall i = 1, ..., I, \forall d = 1, ..., D.$

The Nash equilibrium of the noncooperative game between instream agents coincides with the solution of the following variational inequality problem

$$\begin{aligned}
- & \sum_{i=1}^{I} \sum_{d=1}^{D} \left[p_{id}^{\star} - \frac{\partial c_{id}(q_{id}^{\star})}{\partial q_{id}} \right] \times [q_{id} - q_{id}^{\star}] \\
+ & \sum_{i=1}^{I} \sum_{d=1}^{D} \left[\frac{\partial f_{id}(q_{id}^{\star})}{\partial q_{id}} \right] \times [q_{id} - q_{id}^{\star}] \\
+ & \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[p_{ui}^{\star} + \frac{\partial c_{r_u i}(q_{ui}^{\star})}{\partial q_{ui}} \right] \times [q_{ui} - q_{id}^{\star}] \\
+ & \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[g_{id} \left[1 - P_i(q_i^{\star})\right] - \phi_{r_u i} \left(v_i - (v_i - p_{ui}) \frac{\partial \frac{q_{ui}^{\star}}{q_i} S_i(q_i^{\star})}{\partial q_{ui}} \right) \right] \times [q_{ui} - q_{ui}^{\star}] \geq 0 \quad (10)
\end{aligned}$$

Lemma 2 *The Nash equilibrium is defined as the pair of flows* $\left(\boldsymbol{q}^{I\star}, \boldsymbol{q}^{D\star}\right) \in \Gamma^2$, *where* $\boldsymbol{q}^I$ *and* $\boldsymbol{q}^D$ *are respectively the* $UR_U I$ *and* ID*-dimensional volume vectors, whose elements are* q_{ui} *and* q_{id}, *such that*

$$\Gamma^2 = \left\{ \left(\boldsymbol{q}^I, \boldsymbol{q}^D\right) \mid \left(\boldsymbol{q}^I, \boldsymbol{q}^D\right) \in \mathbb{R}_+^{UR_U I + ID} \right\} \quad (11)$$

is the instream subspace of the network equilibrium, $\forall u = 1, ..., U, \forall r_u = 1_u, ..., R_u, \forall i =$

$1, ..., I, \forall d = 1, ..., D$, *provided that the optimization constraints hold.*

2.3 Downstream agents

Downstream agents compete with each other for resources in a noncooperative way. Each downstream agent d tries to maximize its share of resources q_d, subject to his budget constraint, by solving the following optimization problem.

$$\max\ q_d - \sum_{i=1}^{I} \left[p_{id} + c_{id}\right] S_i(q_i) \tag{12}$$

subject to $q_d \leq \sum_{i=1}^{I} S_i(q_i)$, $p^{\star}_{id} + c_{id} \leq \frac{S_i(q^{\star}_i)}{S_i(q^{\star}_i)-q_d}$, $0 \leq q_d \leq S_i(q^{\star}_i) - q_d$, $\forall i = 1, ..., I, \forall d = 1, ..., D$. $S_i(q^{\star}_i)$ is the downstream agent maximum budget.

The first inequality puts a limit on the price the downstream agent is willing to pay. The second inequality is the rationality constraint.

The Nash equilibrium of the noncooperative game between downstream agents is defined as the pair of levels $\left(\mathbf{q}^{D\star}, \mathbf{d}^{\star}\right) \in \Gamma^3$, coincides with the solution of the following variational inequality problem

$$\begin{aligned} + \quad & \sum_{i=1}^{I}\sum_{d=1}^{D} \left[p^{\star}_{id} + c_{id}\right] \times \left[q_{id} - q^{\star}_{id}\right] \\ - \quad & \sum_{d=1}^{D} \left[q_d - q^{\star}_d\right] \geq 0 \end{aligned} \tag{13}$$

Lemma 3 *The Nash equilibrium is defined as the pair of flows* $\left(\boldsymbol{q}^{D\star}, \boldsymbol{d}^{\star}\right) \in \Gamma^3$, *where* $\boldsymbol{q}^D$ *and* $\boldsymbol{d}$ *are respectively the* ID *and* D*-dimensional volume vectors, whose elements are* q_{id} *and* q_d, *such that*

$$\Gamma^3 = \left\{ \left(\boldsymbol{q}^D, \boldsymbol{d}\right) \mid \left(\boldsymbol{q}^D, \boldsymbol{d}\right) \in \mathbb{R}^{ID+D}_{+} \right\} \tag{14}$$

is the downstream subspace of the network equilibrium, $\forall i = 1, ..., I, \forall d = 1, ..., D$, *provided that the optimization constraints hold.*

2.4 Network equilibrium

The network reaches an equilibrium point if the optimality conditions of all agents are satisfied in a way that none of them has the incentive to change his strategy.

Proposition 1 *The equilibrium of the industrial network* $\left(\boldsymbol{q}^{U\star}, \boldsymbol{q}^{I\star}, \boldsymbol{q}^{D\star}, \boldsymbol{d}^{\star}\right) \in \Gamma$ *coincides with the solution of the variational inequality problem as follows*

$$
\begin{aligned}
&+ \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \left[\frac{\partial f_{r_u i}(q^{\star}_{ui})}{\partial q_{ui}}\right] \times [q_{ui} - q^{\star}_{ui}] \\
&+ \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[\frac{\partial c_{r_u i}(q^{\star}_{ui})}{\partial q_{ui}}\right] \times [q_{ui} - q^{\star}_{ui}] \\
&+ \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[(g_{ui} + g_{id})\left[1 - P_i(q^{\star}_i)\right] + \left(v_i - (v_i - p_{ui})\frac{\partial \frac{q^{\star}_{ui}}{q_i} S_i(q^{\star}_i)}{\partial q_{ui}}\right)\right] \times [q_{ui} - q^{\star}_{ui}] \\
&+ \sum_{i=1}^{I} \sum_{d=1}^{D} \left[\frac{\partial f_{id}(q^{\star}_{id})}{\partial q_{id}}\right] \times [q_{id} - q^{\star}_{id}] \\
&+ \sum_{i=1}^{I} \sum_{d=1}^{D} \left[\frac{\partial c_{id}(q^{\star}_{id})}{\partial q_{id}}\right] \times [q_{id} - q^{\star}_{id}] \\
&- \sum_{d=1}^{D} [q_d - q^{\star}_d] \geq 0
\end{aligned}
\tag{15}
$$

where

$$
\Gamma = \left\{ \left(\boldsymbol{q}^{U}, \boldsymbol{q}^{I}, \boldsymbol{q}^{D}, \boldsymbol{d}\right) \mid \left(\boldsymbol{q}^{U}, \boldsymbol{q}^{I}, \boldsymbol{q}^{D}, \boldsymbol{d}\right) \in \mathbb{R}_{+}^{(UR_U+D)(I+1)} \right\}
\tag{16}
$$

is the space of the network equilibrium, $\forall u = 1, ..., U, \forall r_u = 1_u, ..., R_u, \forall i = 1, ..., I, \forall d = 1, ..., D$*, provided that the optimization constraints hold.*

The variational inequality shows that the transactions shall be achieved when the prices paid by the upstream and instream agents are equal to the sums of the marginal production costs, the marginal transaction costs, and the foregone benefits when supplies and demands do not coincide. Otherwise, the deals shall not take place.

According to the foregoing, we can also see that the network equilibrium is independent of $\phi_{r_u i}$. The following corollary ensues.

Corollary 1 *Within the industrial network, the equilibrium flows do not depend on the bargaining outcome between the upstream and instream agents.*

From the equilibrium conditions, the upstream prices $p_{ui}^{\star}$ amount to

$$p_{ui}^{\star} = \frac{\partial c_{r_u i}(q_{ui}^{\star})}{\partial q_{ui}} + \frac{\partial f_{r_u i}(q_{ui}^{\star})}{\partial q_{ui}} + g_{ui}\left[1 - P_i(q_i^{\star})\right] - (1 - \phi_{r_u i})\left(v_i - (v_i - p_{ui})\frac{\partial \frac{q_{ui}^{\star}}{q_i} S_i(q_i^{\star})}{\partial q_{ui}}\right) \tag{17}$$

such that $q_{ui} > 0$. With the sharing revenue contract, the agents decide on a profit allocation so as to achieve some equilibrium state. The equilibrium prices are endogenous to the optimal transaction quantities.

Likewise, the optimal conditions of downstream prices $p_{id}^{\star}$ lead to

$$p_{id}^{\star} = \frac{\partial c_{id}(q_{id}^{\star})}{\partial q_{id}} + \frac{\partial f_{id}(q_{id}^{\star})}{\partial q_{id}} + g_{id}\left[1 - P_i(q_i^{\star})\right] \tag{18}$$

such that $q_{id} > 0$.

The state vector of the network is defined as

$$\mathbf{z} = \left(\mathbf{q}^{U\star}, \mathbf{q}^{I\star}, \mathbf{q}^{D\star}, \mathbf{d}^{\star}\right)^T \tag{19}$$

By concatenating the terms of the network equilibrium, we fall on

$$\mathbf{F}(\mathbf{z}) = \begin{bmatrix} \left[\left[\left[\frac{\partial f_{r_u i}(q_{ui}^{\star})}{\partial q_{ui}}\right]_{i=1}^{I}\right]_{r_u=1}^{R_u}\right]_{u=1}^{U} \\ \left[\left[\left[\frac{\partial c_{r_u i}(q_{ui}^{\star})}{\partial q_{ui}}\right]_{i=1}^{I}\right]_{r_u=1}^{R_u}\right]_{u=1}^{U} \\ \left[\left[\left[(g_{ui} + g_{id})\left[P_i(q_i^{\star}) - 1\right] - \left(v_i - (v_i - p_{ui})\frac{\partial \frac{q_{ui}^{\star}}{q_i} S_i(q_i^{\star})}{\partial q_{ui}}\right)\right]_{i=1}^{I}\right]_{r_u=1}^{R_u}\right]_{u=1}^{U} \\ \left[\left[\frac{\partial f_{id}(q_{id}^{\star})}{\partial q_{id}}\right]_{d=1}^{D}\right]_{i=1}^{I} \\ \left[\left[\frac{\partial c_{id}(q_{id}^{\star})}{\partial q_{id}}\right]_{d=1}^{D}\right]_{i=1}^{I} \\ -\mathbf{1}_D \end{bmatrix} \tag{20}$$

where $\mathbf{1}_D$ is an D-dimensional vector, whose elements are all 1. Rewritten in the compact form, the network variational inequality yields a Nash equilibrium if and only if $\mathbf{z}^{\star} \in \Gamma$ and, $\forall \mathbf{z} \in \Gamma$, we have

$$(\mathbf{z} - \mathbf{z}^\star)^T \, \mathbf{F}(\mathbf{z}^\star) \geq 0 \tag{21}$$

3 Vertical integration

Consider now that the industrial network is subjected to vertical integration, in which case the upstream agents and the instream agents merge (Fig. 2).

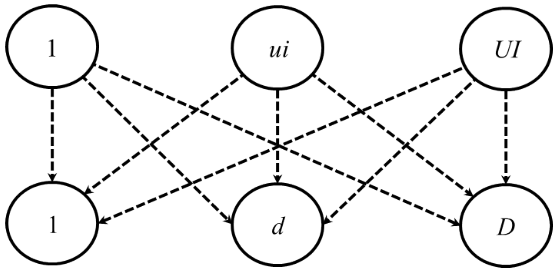


Figure 2: Forest-based integrated industrial network

In this case, the supply contract $\langle p_{ui}, \phi_{r_u i} \rangle$ disappears. The second proposition follows.

Proposition 2 *The equilibrium of the integrated industrial network* $\left(\boldsymbol{q}^{U\star}, \boldsymbol{q}^{I\star}, \boldsymbol{q}^{D\star}, \boldsymbol{d}^\star\right) \in \Gamma$ *coincides with the solution of the variational inequality problem as follows*

$$\begin{aligned}
- & \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \left[\frac{\partial c_{r_u i}(q_{ui}^\star)}{\partial q_{ui}}\right] \times [q_{ui} - q_{ui}^\star] \\
+ & \sum_{i=1}^{I} \sum_{d=1}^{D} \left[\frac{\partial f_{id}(q_{id}^\star)}{\partial q_{id}}\right] \times [q_{id} - q_{id}^\star] \\
+ & \sum_{i=1}^{I} \sum_{d=1}^{D} \left[\frac{\partial c_{id}(q_{id}^\star)}{\partial q_{id}}\right] \times [q_{id} - q_{id}^\star] \\
- & \sum_{d=1}^{D} [q_d - q_d^\star] \geq 0
\end{aligned} \tag{22}$$

where

$$\Gamma = \left\{ \left(\boldsymbol{q}^U, \boldsymbol{q}^I, \boldsymbol{q}^D, \boldsymbol{d}\right) \mid \left(\boldsymbol{q}^U, \boldsymbol{q}^I, \boldsymbol{q}^D, \boldsymbol{d}\right) \in \mathbb{R}_+^{(UR_U+D)(I+1)} \right\} \tag{23}$$

$\forall u = 1, ..., U, \forall r_u = 1_u, ..., R_u, \forall i = 1, ..., I, \forall d = 1, ..., D$*, provided that the optimization constraints hold.*

The state vector of the network becomes

$$\mathbf{w} = \left(\mathbf{q}^{U\star}, \mathbf{q}^{I\star}, \mathbf{q}^{D\star}, \mathbf{d}^{\star}\right)^T \tag{24}$$

The network equilibrium concatenation yields

$$\mathbf{F}(\mathbf{w}) = \begin{bmatrix} -\left[\left[\left[\frac{\partial c_{r_u i}(q^{\star}_{ui})}{\partial q_{ui}}\right]_{i=1}^{I}\right]_{r_u=1}^{R_u}\right]_{u=1}^{U} \\ \left[\left[\frac{\partial f_{id}(q^{\star}_{id})}{\partial q_{id}}\right]_{d=1}^{D}\right]_{i=1}^{I} \\ \left[\left[\frac{\partial c_{id}(q^{\star}_{id})}{\partial q_{id}}\right]_{d=1}^{D}\right]_{i=1}^{I} \\ -\mathbf{1}_D \end{bmatrix} \tag{25}$$

Rewritten in the compact form, the network variational inequality yields a Nash equilibrium if and only if $\mathbf{w}^{\star} \in \Gamma$ and, $\forall \mathbf{w} \in \Gamma$, we have

$$(\mathbf{w} - \mathbf{w}^{\star})^T \, \mathbf{F}(\mathbf{w}^{\star}) \geq 0 \tag{26}$$

4 Dynamic network equilibrium

By using the time-dependent variational inequality, let us now reframe the equilibrium flows and place them in a dynamical system (see Fig. 3).

The dynamic model describes the evolution of the equilibrium flows. Consider a network in which the equilibrium prices evolve according to the variations in flows. The flows dynamics are issued from the formalization of the variational inequalities previously used in the static framework. By making the flows time-dependent, and after summing and simplifying, the following proposition ensues.

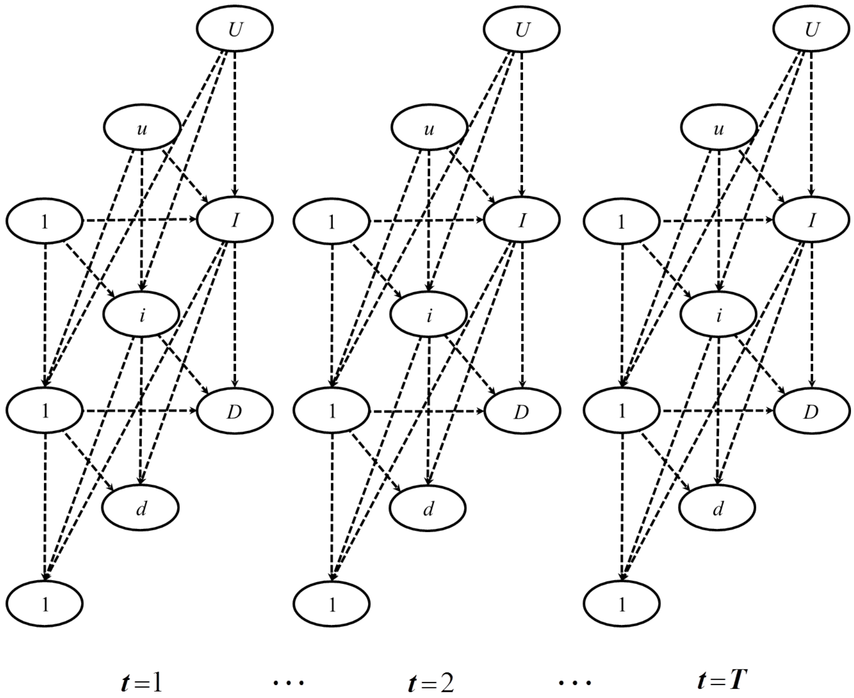


Figure 3: Dynamic forest-based industrial network

Proposition 3 *For the time interval $t \in [1, T]$, the equilibrium of the industrial network $\left(\boldsymbol{q}^{U\star}(t), \boldsymbol{q}^{I\star}(t), \boldsymbol{q}^{D\star}(t), \boldsymbol{d}^{\star}(t)\right) \in \Gamma_t$ coincides with the solution of the dynamic variational inequality problem as follows*

$$
\begin{aligned}
+ & \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{t=1}^{T} \left[\frac{\partial f_{r_u i}(q_{ui}^{\star}(t))}{\partial q_{ui}(t)} \right] \times [q_{ui}(t) - q_{ui}^{\star}(t)] \\
+ & \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \sum_{t=1}^{T} \left[\frac{\partial c_{r_u i}(q_{ui}^{\star}(t))}{\partial q_{ui}(t)} \right] \times [q_{ui}(t) - q_{ui}^{\star}(t)] \\
+ & \sum_{u=1}^{U} \sum_{r_u=1_u}^{R_u} \sum_{i=1}^{I} \sum_{t=1}^{T} \left[(g_{ui} + g_{id}) \left[1 - P_i(q_i^{\star}(t))\right] + \left(v_i - (v_i - p_{ui}) \frac{\partial \frac{q_{ui}^{\star}(t)}{q_i(t)} S_i(q_i^{\star}(t))}{\partial q_{ui}(t)} \right) \right] \times [q_{ui}(t) - q_{ui}^{\star}(t)] \\
+ & \sum_{i=1}^{I} \sum_{d=1}^{D} \sum_{t=1}^{T} \left[\frac{\partial f_{id}(q_{id}^{\star}(t))}{\partial q_{id}(t)} \right] \times [q_{id}(t) - q_{id}^{\star}(t)] \\
+ & \sum_{i=1}^{I} \sum_{d=1}^{D} \sum_{t=1}^{T} \left[\frac{\partial c_{id}(q_{id}^{\star}(t))}{\partial q_{id}(t)} \right] \times [q_{id}(t) - q_{id}^{\star}(t)] \\
- & \sum_{d=1}^{D} \sum_{t=1}^{T} [q_d(t) - q_d^{\star}(t)] \geq 0
\end{aligned}
\tag{27}
$$

where

$$\Gamma_t = \bigcup_{t \in [1,T]} \left\{ \left(\boldsymbol{q}^U, \boldsymbol{q}^I, \boldsymbol{q}^D, \boldsymbol{d} \right) \mid 0 \leq \left(\boldsymbol{q}^{U\star}(t), \boldsymbol{q}^{I\star}(t), \boldsymbol{q}^{D\star}(t), \boldsymbol{d}^\star(t) \right) \leq \mu(t) \right\} \tag{28}$$

is the extended space of the network equilibrium and $\mu(t) \in \Gamma_t$ is the boundary in the constraint set, $\forall u = 1, ..., U, \forall r_u = 1_u, ..., R_u, \forall i = 1, ..., I, \forall d = 1, ..., D$, provided that the optimization constraints hold.

Again, the dynamic variational inequality shows that, in any period, the transactions shall be executed when the prices paid by the upstream and instream agents amount to the sums of the marginal production costs, the marginal transaction costs, as well as the penalties related to the revenue shortfalls.

The state vector of the network is defined as

$$\mathbf{y}(t) = \left(\mathbf{q}^{U\star}(t), \mathbf{q}^{I\star}(t), \mathbf{q}^{D\star}(t), \mathbf{d}^\star(t) \right)^T \tag{29}$$

The concatenation of the terms of the network equilibrium gives

$$\mathbf{F}(\mathbf{y}(t)) = \begin{bmatrix} \left[\left[\left[\left[\frac{\partial f_{r_u i}(q^\star_{ui}(t))}{\partial q_{ui}(t)} \right]_{t=1}^{T} \right]_{i=1}^{I} \right]_{r_u=1}^{R_u} \right]_{u=1}^{U} \\ \left[\left[\left[\left[\frac{\partial c_{r_u i}(q^\star_{ui}(t))}{\partial q_{ui}(t)} \right]_{t=1}^{T} \right]_{i=1}^{I} \right]_{r_u=1}^{R_u} \right]_{u=1}^{U} \\ \left[\left[\left[\left[(g_{ui} + g_{id}) \left[P_i(q^\star_i(t)) - 1 \right] - \left(v_i - (v_i - p_{ui}) \frac{\partial \frac{q^\star_{ui}(t)}{q_i(t)} S_i(q^\star_i(t))}{\partial q_{ui}(t)} \right) \right]_{t=1}^{T} \right]_{i=1}^{I} \right]_{r_u=1}^{R_u} \right]_{u=1}^{U} \\ \left[\left[\left[\frac{\partial f_{id}(q^\star_{id}(t))}{\partial q_{id}(t)} \right]_{t=1}^{T} \right]_{d=1}^{D} \right]_{i=1}^{I} \\ \left[\left[\left[\frac{\partial c_{id}(q^\star_{id}(t))}{\partial q_{id}(t)} \right]_{t=1}^{T} \right]_{d=1}^{D} \right]_{i=1}^{I} \\ -\mathbf{1}_D \end{bmatrix} \tag{30}$$

The compact form of the dynamic variational inequality represents a Nash equilibrium if and only if $\mathbf{y}^\star(t) \in \Gamma_t$ and, $\forall \mathbf{y}(t) \in \Gamma_t$, we have

$$(\mathbf{y}(t) - \mathbf{y}^\star(t))^T \mathbf{F}(\mathbf{y}^\star(t)) \geq 0 \tag{31}$$

5 Simulations

With respect to the industrial network equilibrium properties, this section covers, by means of a series of quadratic functions, the analysis of the market values.

5.1 Assumptions

Assume a density function of instream agent i demand given by

$$p_i(x) = \begin{cases} \frac{1}{a_i} & \text{if } x \in [0, a_i] \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

Hence, the distribution function is expressed as

$$P_i(x) = \begin{cases} \frac{x}{a_i} & \text{if } x \in [0, a_i] \\ 1 & \text{otherwise} \end{cases} \tag{33}$$

The expected sales of instream agent i are equal to

$$S_i(q_i) = q_i - \int_0^{q_i} P_i(x)dx = q_i - \frac{q_i^2}{2a_i} \tag{34}$$

Let f_u be the cost function of upstream agent u

$$f_u = q_i^2 + 10q_i \tag{35}$$

Let f_i be the cost function of instream agent i

$$f_i = 20q_i^2 + 300q_i \tag{36}$$

Finally, assume equivalent upstream and downstream transaction costs in form of

$$c_{ui} = c_{id} = q_i^2 + q_i \tag{37}$$

5.2 Equilibrium values

With $a_i = 35$,[2] $g_{ui} = 10$ and $g_{id} = 100$, the outputs of the explicit functional forms are shown in Fig. 4.

Subfig. 4.a. illustrates, on the scale of the industrial network, the equilibrium prices per cubic meters of resource (ordinates) as functions of production units in cubic meters (abscissa). We can observe an increasing upstream supply curve, in conformity with the microeconomic market theory. Thereby, the higher the price, the higher the profitability of the silvicultural production, and hence the higher the volume intended to be marketed. In detail, the mean volume amounts to 17.50 cubic meters ($\sigma_{q_{ui}} = 10.54$) for a mean price of 86.00 Eur per cubic meter ($\sigma_{p_{ui}} = 39.13$).[3]

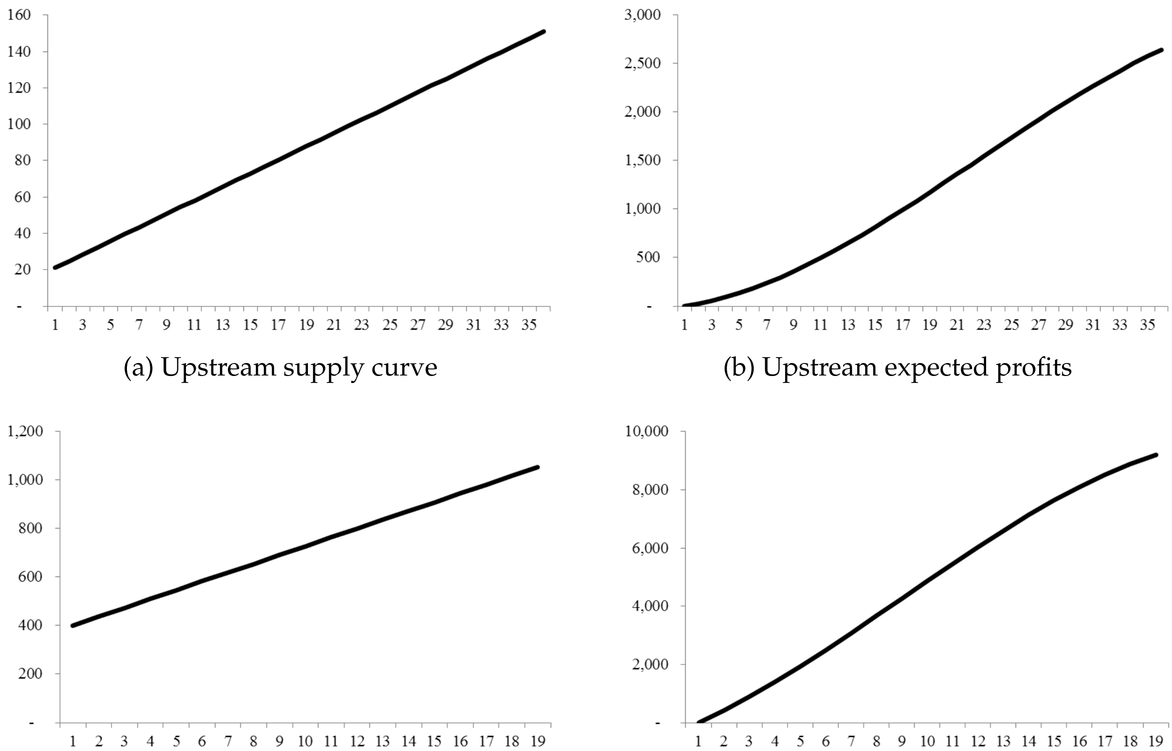
Within the industrial network, Subfig. 4.c. depicts the equilibrium prices per cubic meters of manufactured wood product (ordinates), as functions of production units in cubic meters (abscissa). Again, the instream supply curve is increasing with price. This time, the mean volume of 8.75 cubic meters ($\sigma_{q_{id}} = 5.63$) ends up with a mean price of 727.57 Eur per cubic meter ($\sigma_{p_{id}} = 204.19$).

Such as pointed out in Subfigs. 3.b. and 3.d., the market equilibrium values are confirmed by the observation of the expected profits (ordinates), for different production levels (abscissa), of both upstream and instream agents. Indeed, we observe flat sigmoid curves and inflection points, with respective x-axes coordinates of 17.50 cubic meters and 8.75 cubic meters, from which the expected profits grow less rapidly, given that they turn into concave functions of volumes.

[2] For the instream agents, we apply a transformation coefficient of 0.5, such as in Peyron et al. (2005), meaning that a cubic meter of log yields half a cubic meter of sawnwood.

[3] In absence of data on the demand side, the mean values are used as substitutes for the market equilibrium values.

At the equilibrium price of 86.00 Eur per cubic meter, the upstream agent expects a profit of $1,196.58$ Eur ($\sigma_{\Pi_{ui}} = 861.66$); at the equilibrium price of 727.57 Eur per cubic meter, the instream agent expects a profit of $4,769.44$ Eur ($\sigma_{\Pi_{id}} = 3,044.19$).



(a) Upstream supply curve

(b) Upstream expected profits

(c) Instream supply curve

(d) Instream expected profits

Figure 4: Network equilibrium outputs

In a previous work conducted on the roundwood log supply contracts, Barkaoui and Dragicevic (2014) have shown, through a Nash bargaining game, that the equilibrium exchange between the upstream and instream forest-sector firms, in a scenario with respective unit prices of standing timber and roundwood logs of 100.00 Eur and 130.00 Eur, should stand at a contract price of 126.93 Eur per cubic meter ($\sigma_{p_{ui}} = 7.66$), for an invariable contract volume of 28.92 cubic meters. As for the downstream exchange, they assumed, from the observation of the publicly available market data, a mean lumber volume of 5.00 cubic meters ($\sigma_{q_{id}} = 1.00$) for a mean wood product sale of 600.00 Eur per cubic meter ($\sigma_{p_{id}} = 50.00$).

If we were to draw a parallel between the bargaining game and the variational inequality model, we could observe that their results almost fall within the confidence intervals of the prices obtained in the present study. Nevertheless, the average values in expectation do not match. We could therefore conclude that the bargaining upstream price is at the upper endpoint of the industrial network equilibrium price, while the bargaining instream price is at its lower endpoint equilibrium price. This leads us to the following result.

Result 1 *The distance comparison to the forest-based network equilibrium reveals that (1) the forest resources may be overvalued; (2) the manufactured wood products may be undervalued.*

Put differently, to ensure that the network equilibrium arises, the model outputs show that the prices of forest resources should, on average, decrease by 47.59%;[4] the prices of manufactured wood products should, on average, increase by 21.26%.

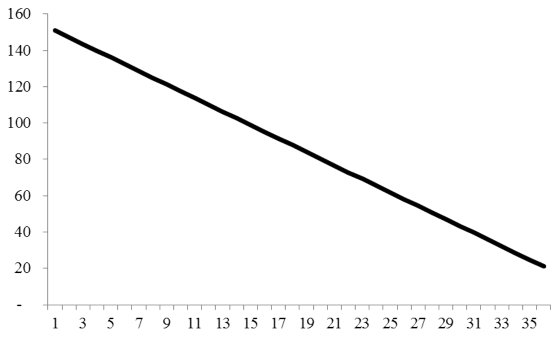
5.3 Disequilibrium values

Knowing that the network equilibrium coordinates have been identified, let us now analyze the inventory and stockout scenarios, namely: all values above or below the equilibria.

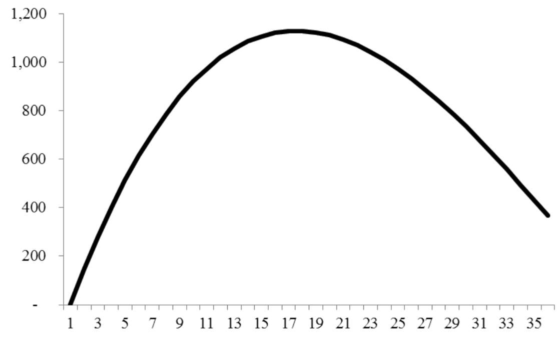
Fig. 5 illustrates the prices and expected profits (ordinates) in disequilibra. The volumes (abscissa) less than 17.50 cubic meters or 8.75 cubic meters correspond to stockouts; otherwise, we are in presence of inventories.

In detail, Subfigs. 5.a. and 5.c. show that prices increase with scarcity and decrease with overflow. When $q_{ui}, q_{id} \rightarrow 0.00$ cubic meters, the prices respectively increase by 41.82% and 30.98%. As the volumes go to their maxima, that is $q_{ui} \rightarrow 35.00$ cubic meters and $q_{id} \rightarrow 17.50$ cubic meters, the prices respectively fall by 76.10% and 44.89%. With regard to the expected profits portrayed in Subfigs. 5.b. and 5.d., we observe bell-shaped curves, because the rises in commercialization volumes less than compensate the falls in prices. Indeed, when $\lim(q_{ui}) = 35.00$ cubic meters and $\lim(q_{id}) = 17.50$ cubic meters, the expected profits respectively decrease by 67.50% and 27.93%. It follows that the maximum values are to be found at the network equilibrium, as noted on the subfigures.

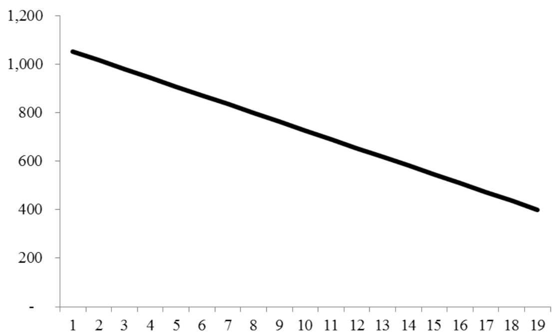
[4] This result is consistent with the study performed by Peyron (2002), who shows that, since 1920, timber prices undergo asynchronous bubble cycles, with an average increase of 1% or quasi-stability.



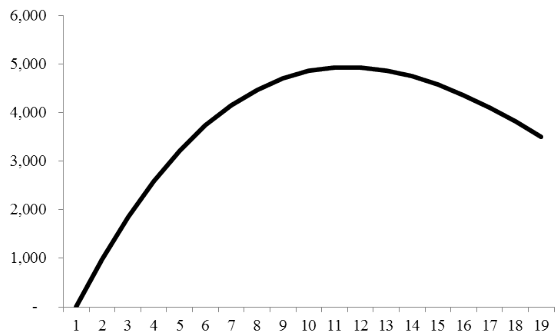
(a) Upstream prices



(b) Upstream expected profits



(c) Instream prices



(d) Instream expected profits

Figure 5: Network disequilibrium outputs

Result 2 *When the network is in disequilibrium, the expected profits switch from increasing monotonic s-shaped functions to non-monotonic bell-shaped functions.*

5.4 Vertical integration

In Fig. 6, the full curve represents the network supply of the integrated firm. For comparison purposes, the dashed curve corresponds to the unintegrated instream supply.

The market price decreases to 708.57 Eur per cubic meter ($\sigma_{p_{id}} = 192.94$), which amounts to a price reduction of 2.61%.[5] In detail, it can be emphasized that the greater the market volume, the larger the price gap between the separated and integrated firms. The third result can be stated.

Result 3 *The vertical integration of the forest-based sector appears relevant for a large market size. Otherwise, the benefits do not seem significant.*

[5]The price reduction serves as a strategic advantage of the integrated agent over the competitors.

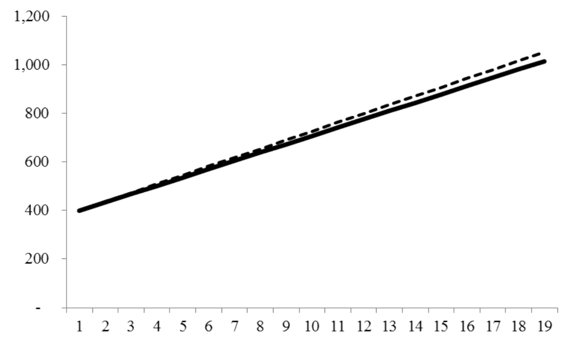
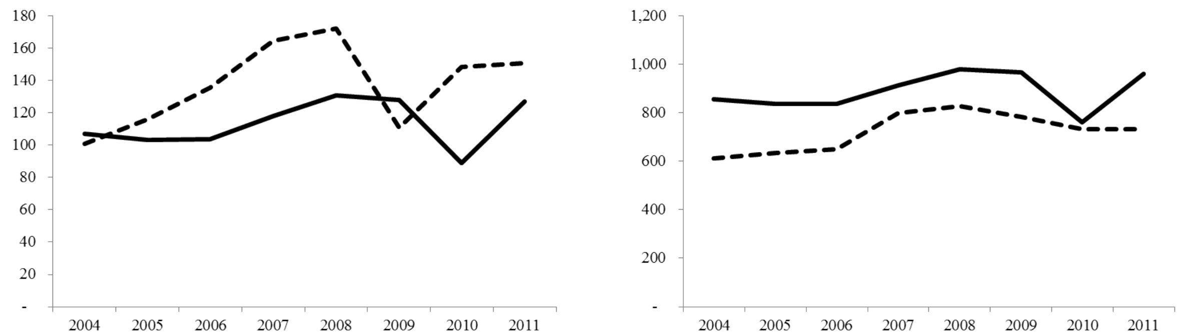


Figure 6: Integrated firm supply curve

5.5 Dynamic analysis

Such as illustrated in Fig. 7, we now make use of the time-varying variational inequality for the sake of measuring, over a period ranging from 2004 to 2011 (abscissa), the gaps between the network equilibrium prices and the observed French market prices[6] (ordinates). The theoretical equilibrium values are represented by full curves; the observed values are represented by dashed curves.



(a) Upstream price dynamics

(b) Instream price dynamics

Figure 7: Network price dynamics

Along the lines of what has been previously revealed, the upstream observed prices (Subfig. 7.a.) are higher than what would have been obtained in a state of equilibrium at the scale of the industrial network. Likewise, the instream observed prices (Subfig. 7.b.) are below those computed at the network theoretical equilibrium.

Although we can notice that, during the study period, the upstream theoretical equilibrium prices tie in with the observed prices on three occasions, the latter exceed

[6] Given the time-scale, current and constant prices substantially give the same results.

the former by 21.31% on average. In parallel, when compared to the theoretical equilibrium prices, the instream observed prices are, on average, inferior by 18.83%, even though they are comparable in 2010.

Result 4 *The temporal distance comparison to the forest-based network equilibrium reveals that (1) the forest resources are regularly overvalued; (2) the manufactured wood products are invariably undervalued.*

6 Conclusive remarks

In this paper, we first show that the equilibrium flows of the forest-based network do not depend on the bargaining in the roundwood log supply contracts, which are being promoted by the French authorities (Office National des Forêts, 2013). Counter to some stakeholders who condemn the development of supply contracts, for they consider that their characteristics impede competition, the model outputs imply that their presence should not unbalance the equilibrium flows along the chain value.

Second, by comparing, through a numerical example, our results with those obtained in a Nash bargaining game, we unfold that the industrial network equilibrium could be attained in theory, would the prices of forest resources and wood manufactured products respectively decrease and increase. The dynamic analysis corroborates these findings. *De facto*, what could be economically sound at a local scale, across two firms, could be unsound at the scale of the whole industrial sector.

Third, we find that the network disequilibrium causes a switch in the expected profits, which pass from an increasing monotonic s-shaped functional form to a non-monotonic bell-shaped functional form. This result is in line with other works (Rosser, 2012).

Fourth, with reference to the vertical integration, we find that the strategic advantages apply to sole large market volumes. This could mean that the vertical integration should be preceded by a horizontal one, in order to reach a critical size in the production capacities, which would ultimately permit to obtain the well-documented benefits of the upstream integration.

From the discussion, an overall conclusion could be drawn. If the equilibrium of the industrial network were to be attained in favor of the instream agents and at the expense of the upstream agents, the latter would all the more have an incentive to vertically integrate with the former.

Acknowledgment

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