

The Orangutan Dilemma

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28th April 2015

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Abstract

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We study the optimal conservation of an endangered species threatened by the conversion of its natural territory to agriculture. The preservation of emblematic species like the orangutan in Borneo is a well-known example of this problem. Conversion of wild land to farmland incurs increasing conversion costs with the speed of the transformation process. The carriage capacity of the species population shrinks down with conversion and the population mortality rate increases with the speed of land conversion to farming. We first establish the optimal sharing rule of the territory between nature preservation and agriculture. We next show that without specific conversion costs, the society should choose to overshoot temporarily the sustainable land area allocated to the species conservation, converting back progressively farm land to wild land until the sustainable agricultural land share is attained. We show that this flexible land management policy is more protective to the species than a rigid rule conserving a fixed amount of land as a natural reserve. With positive conversion costs, we show that depending on the cost structure, either the land conversion process converges smoothly toward a sustainable state, either it overshoots in two different ways this state. The first way implies stopping conversion in finite time and let the population decline down to some long run stable state, the second one to stabilize the land allocation for some time and then revert some fraction of the agricultural land to nature until some another population steady state is reached.

Keywords: Conservation; land use; endangered species

JEL classifications: Q15, Q24, Q57

1 Introduction

The preservation of endangered species commonly implies not only the protection of the species itself against predatory human behavior but also the preservation of their natural resource base. This base is composed of natural resources *per se* : lands and forests, freshwater in wetlands, nutrients and biomass, together with various characteristics of the natural environment of the species: type of vegetation, climatic conditions, interactions with other species. Determining the ideal spatial characteristics of natural preservation sites is a key issue for endangered species management. Preservation areas must be able to provide an asylum to the species compatible with their sustained presence. However the determination of the spatial location of such preservation areas is not only a matter of ecological and biological ideal status but also of competition with other possible land uses, especially agriculture.

This is a well known problem in designing preservation plans of rainforests in tropical areas for biodiversity protection or preservation of their potential to supply various kinds of ecosystem services. The threats over the orangutans population living in the Borneo island is a famous example of this dilemma. It is currently estimated that the orangutans population could become extinct in the next 25 years. Over the last decades, the monkeys have lost 80% of their natural territory in Borneo and Sumatra, the only places they can be found on Earth. Conversion of wild land to farm land is considered as the main driver of this extinction process. In that respect, the responsibility of the fast development of palm trees cultivation is frequently advanced as a prominent factor of the Borneo deforestation process. The orangutans suffer from this situation in two ways: direct excess mortality due to fast deforestation, hunting, poaching and juvenile trafficking, and indirect impacts from the loss of their natural territory.

One main challenge of species preservation is to correctly incorporate not only the spatial aspects of the issue but also its intertemporal dimension. The quest for long run satisfactory protection of endangered species requires considering explicitly the dynamic interactions of the species with their environment over long time periods. Incorporating in a relevant way both the spatial and temporal dimensions into bio-economic models is a well identified loophole in the field of renewable resources economics (Brown,

2001). It is well known that spatial and dynamic aspects in meta-population models involve highly complex diffusion and growth processes, usually studied with the help of numerical simulation models. Here, we want to highlight the main economic aspects of the problem. To achieve this aim, we adopt the simplest structure able to catch its main ingredients. This allows providing analytical solutions which can then be discussed through a management perspective.

Concern for conservation has a long legacy in economics since the original contributions of Ciriacy-Wantrup (1953), who introduced intertemporal arbitrage reasoning in the design of optimal conservation policies; A. Scott (1955) who linked conservation to capital theory and capital markets, maybe imperfect, and Scott Gordon (1958) who derived the institutional and juridic implications of conservation. In their seminal paper, Fisher, Krutilla and Cicchetti (1972) (FKC thereafter) have described the optimal conversion of land from a wild state to a 'developed' state when taking into account the loss of environmental welfare resulting from this conversion. The contribution of FKC raises two difficulties. First they assume that land conversion (or land 'development') is an irreversible process. Secondly, they assume constant marginal 'development' costs, the two assumptions resulting into a sequence of alternating active development phases and non development phases, in accordance with the approach of irreversible investments policies first proposed by Arrow (1968).

Amigues, Ayong Le Kama, Moreaux (2013) study a similar conversion problem but with convex conversion costs. The FKC sequence disappears, conversion becoming a smooth process toward some long run stable share of land between agriculture (or a 'developed' status) and natural land. They show in addition that irreversibility of development has not to be introduced as an assumption inside the model but will result quite naturally from the conversion costs dynamics, land-owners deciding to convert a land plot at some point of time having no interest to convert back this plot to nature at a later date.

In this literature, environmental welfare is assumed to depend positively on the fraction of preserved land from economic development. Its main conclusion is that if nature has enough value, some part of the land should be kept in a wild state. Krautkramer (1985) considers to the contrary a case

where the environmental value comes from the flow of good and services extracted from the natural areas during the conversion process. Timber exploitation inside a deforestation process is a simple example of such situations. In this framework, natural land becomes some kind of an exhaustible resource, depleted in the course of conversion. This feature leads to the conclusion that without increasing conversion costs with cumulated conversion, wild land should be completely exhausted. However, Scott Barret (1992) making a synthesis of the FKC approach and the Krautkramer model shows that when natural land value derives both from the flow of goods it provides and the stock it represents, limited conversion can be optimal.

This conclusion is refined by Swallow (1990). In the Swallow model, a biological population is exploited in conjunction with some non renewable resource (may be land). The replenishment of the population is affected positively by the abundance of the non renewable resource. The consequence is a conflict between the depletion process of the exhaustible resource and the possibility to harvest the renewable resource. Swallow then describes different conditions under which the population could survive the optimal depletion process of its resource base or become extinct.

The objective of the paper is to stress the importance of a sound basis for the reasoning of land conservation to the benefit of endangered species. Assuming that the preservation value of a natural territory is some increasing function of its size can lead to misguiding conclusions for species protection¹. Natural reserves creation is generally justified by species conservation motives. But the real target is the sustained presence of a sufficient number of specimens of species of interest. Put differently, people care about African savannas because they are hosting elephants, and do not care of elephants because they live in savannas.

To deal with this issue, we assume that environmental welfare is an increasing function of the size of the population to be protected and not of the wild land area hosting the species itself. However this land area determines the carriage capacity of the species and conversion will reduce this capacity, inducing a decline of the population and thus environmental welfare losses.

¹The recent book of Maier (2013) offers a comprehensive critical account of the problems raised by the confusions appearing frequently in the biological and the economic literature between 'biodiversity' conservation motives, ecosystem services provision, protected land areas creation and endangered species conservation objectives.

These losses must be balanced with agricultural gains along the dynamic trajectory of land conversion and population evolution to define the optimal conversion and conservation policy.

Our model is close to the Swallow model in retaining the idea of a competition for land access between an economic activity (agriculture) and a species, although valued for itself alive in its natural environmental and as such not subjected to harvest. Another difference with the Swallow model is that we consider explicitly the conversion costs of land from wilderness to a 'developed' state. In addition we also model explicitly the impacts of the conversion speed itself over the mortality rate of the species, a feature we coined as 'biological adjustment cost'.

Our main findings are the following. Without specific conversion costs, the optimal policy consists in converting land in a bang-bang way at a level located above the long run sustainable sharing of land between agriculture and wildlife protection. Then land is converted back toward its natural state through a smooth process ending in the long run at the sustainable steady state. This is explained by the fact that the species population level does not react immediately to the shut down of its carriage capacity. The society should exploit this inertia of the malthusian trap to over-exploit temporarily the territory.

This solution is then contrasted with a 'once for all decision' problem where the society should initially decide over the fraction of land conceded to the species, a natural reserve creation problem. It is shown that the flexible conversion policy guarantees a better protection to the species in the long run than the rigid initial natural reserve creation policy. It is then shown that this qualitative feature of the conversion policies are conserved when marginal conversion costs are constant. When conversion costs are convex functions of the speed of land conversion, either land conversion increases up to the sustainable steady state either it temporarily overshoots this level, as in the no conversion cost case.

The paper is organized as follows. The next section 2 describes a model of competition for land between a species and agriculture. Section 3 describes the optimal reserve creation policy under a 'once for all' decision rule, the reserve size having to be fixed initially. Section 4 contrasts the land allo-

cation resulting from this rule with the land dynamics in a model of land conversion without conversion costs. The case of positive conversion costs is next examined in Section 5, first under a constant marginal conversion cost assumption and second for convex conversion costs. Section 6 concludes.

2 The model

Consider a territory of fixed size \bar{L} . At each time t , the territory is composed of a wild area, of size $W(t)$, and a cultivated area, of size $L(t)$, such that $\bar{L} = W(t) + L(t)$. The species to be conserved can only survive on the wild part of the territory. Let $N(t)$ denote the size of the species population. The growth rate of $N(t)$ over time is given by:

$$\frac{\dot{N}(t)}{N(t)} = \alpha - \frac{\delta(t)}{W(t)}N(t). \quad (2.1)$$

α measures the crude birth rate of the population while $\delta(t)N(t)/W(t)$ describes its recruitment rate, assumed proportional to its size to simplify.

At each time, there is land conversion from wild land to farm land or the reverse. Let $\dot{L}(t) = l(t)$ be the land conversion rate to farming at time t . $l(t)$ may be either positive or negative. Then $\delta(t)$ is defined as:

$$\delta(t) = \begin{cases} \delta(l(t)) & \text{if } l(t) > 0 \\ \delta & \text{if } l(t) \leq 0 \end{cases}.$$

In case of a strictly positive conversion of wild land to farm land, the $\delta(l(t))$ function satisfies, $\delta(0^+) = \delta > 0$, $\delta'(l) > 0$, $\delta'(0^+) = \underline{\delta}' > 0$, $\delta''(l) > 0$. When submitted to the expansion of the farm land area, the species suffers from an excess mortality at a rate increasing with the speed of land conversion itself, $l(t)$. Without land conversion, δ measures the natural decay rate of the population. To simplify, we assume that a reverse conversion from farm land to wild land has no effect over this decay rate. However, mortality is of course reduced by a larger wild territory, $W(t)$, which here stands as the carriage capacity of the population. We call *biological adjustment cost* the specific mortality cost paid by the species when facing fast land conversion to farming. Thus, the population is threatened in two ways by farm land expansion: first through the loss of its natural habitat and second

through the perturbations induced by the conversion process itself. Other specifications of such perturbations are of course possible. For example, if the reproductive success of the species is highly sensitive to human pressure over its habitat, it makes sense to assume that the crude birth rate is depending upon l , the speed of the conversion process. A form $\alpha(l)$ with $\alpha'(l) < 0$ and $\alpha''(l) < 0$ would fit into this rationale. For the sake of analytical tractability, we dispense from considering explicitly this kind of problem.

Initially, the territory is not cultivated (e.g. a virgin island) so that $W(0) = \bar{L}$ and the population has historically stabilized to the long run level \bar{N} compatible with the carriage capacity \bar{L} : $\bar{N} = \alpha\bar{L}/\delta$, hence $N(0) = \bar{N}$.

The society gives a value to the protection of the species. Let $h(N)$ be the environmental welfare assumed to depend from the size of the population. More complex formulations of the environmental welfare, for example h functions depending not only on N but also of W could be envisioned. However, as emphasized in the introduction, by giving an intrinsic value to the wild territory, the logic of land competition between agriculture and protection areas to the benefit of the species would be blurred. The same applies to environmental value functions depending of the richness index of the wild territory, $N(t)/W(t)$. Assuming that the society takes care only of the number of preserved individuals, social concern for natural environments then affects only indirectly land valuation. The environmental value function $h(N)$ is twice continuously differentiable, strictly increasing, $h'(N) > 0$, strictly concave, $h''(N) < 0$, and $\lim_{N \downarrow 0} h'(N) = +\infty$. Social concern for the population decreases at the margin when the population becomes very large. However, the society is willing to give an infinite amount of wealth to preserve the last specimens of the population, a way to introduce an existence value component into the specification of the environmental welfare.

Denote by $u(L)$ the net surplus from farm land exploitation. Inside the relevant range $[0, \bar{L}]$, the surplus function satisfies: $u'(L) > 0$, $u''(L) < 0$ while $\lim_{L \downarrow 0} u'(L) > 0$. This last assumption is made in order to drive an interesting discussion. Whatever be the value given by the society to the protection of the species, there will be some land conversion. Since $L(0) = 0$ by assumption, this means that the conversion rate will be adjusted to a strictly positive level immediately after the initial time: $l(0^+) > 0$. On the other hand, the assumption $u'(\bar{L}) > 0$ implies that in a 'laissez faire'

context, where farmers convert the land to agriculture without taking care of the environmental impacts of their decisions, the whole territory could be devoted to farming in the long run and the species would become extinct.

The strict concavity of the net surplus function may result from increasing marginal farming exploitation costs or from demand effects. For example, 85 % of the world palm trees cultivation area is concentrated in Indonesia, Malaysia and New Guinea. It is thus possible that strong preservation actions against land conversion result into price hikes on the palm oil market.

The land conversion process to farming incurs a conversion cost $C(l)$. If $l > 0$, it is assumed that $C(l)$ is a twice continuously differentiable function such that $C(0^+) = 0$, $dC(l)/dl \equiv c(l) > 0$, and $c'(l) > 0$ while $c(0^+) = \underline{c} > 0$. If $l \leq 0$, $C(l) = 0$. Hence it appears that the land conversion process is submitted to two types of adjustment costs. The first one is the biological adjustment cost, the excess mortality of the population resulting directly from land conversion. The second one is the agricultural conversion cost, covering all the expenses needed to put the land into productive status: transportation infrastructures, buildings and facilities, timber cutting. These expenses rise with the scale of the conversion process measured by $l(t)$, the number of hectares of land converted to agriculture at each time.

Let $\rho > 0$ be the assumed positive constant level of the social discount rate. The objective of the society is to maximize the cumulated sum of the discounted instantaneous net welfare, itself the sum of the agricultural net welfare and the environmental welfare, that is solve the following (*S.P.*) problem:

$$\begin{aligned} \max_l \quad & \int_0^\infty [u(L(t)) - C(l(t)) + h(N(t))] e^{-\rho t} dt \\ \text{s.t.} \quad & \frac{\dot{N}(t)}{N(t)} = \alpha - \frac{\delta(t)}{W(t)} N(t) \quad N(0) = \bar{N} \\ & \dot{L}(t) = l(t) \quad L(0) = 0 \\ & \bar{L} - W(t) - L(t) \geq 0 \end{aligned}$$

Before turning to the study of the solutions of the (*S.P.*) problem, it is worth discussing some interesting benchmarks. The first one is to consider what we call a 'once for all' decision problem of land allocation, neglecting temporarily the conversion costs. The second one is to allow the land allocation to vary freely at each time without taking into account adjustment costs, either

economical or biological. With the help of these benchmarks, the interpretation of the results of the general problem will prove much easier. Another interesting aspect of this progressive approach is to allow interpreting its results in a land management perspective, by comparing the resulting long run allocations of the territory to agriculture and wildlife protection under different decision settings.

3 'Once for all' land allocation decisions

Neglect the specific conversion costs and assume that the society has to take a decision at time 0 about the sharing of the territory between agriculture and environmental preservation. Let $L^0 = L(0^+)$ be the outcome of this decision so that $W^0 = \bar{L} - L^0$ is the size of the wild territory conceded to the protected species. Such a decision rule looks superficially static but it is not the case. By setting abruptly the carriage capacity of the population to W^0 , the land allocation decision initiates a convergence process of the population from its initial level \bar{N} toward the new long run level $\hat{N}(W^0) = \alpha W^0 / \delta < \bar{N}$. This means a permanent environmental welfare loss all along this transition process that the society should take into account when choosing the amount of land converted to farming.

A pure static decision rule could be to maximize the welfare level in the long run situation emerging at the end of the transition in capitalized terms. This would be equivalent to solve the static problem:

$$\begin{aligned} \max_L \quad & u(L) + h\left(\frac{\alpha}{\delta}W\right) \\ \text{s.t.} \quad & \bar{L} - L - W \geq 0 \end{aligned}$$

Let λ be the Lagrange multiplier associated to the land constraint. Then the optimal agricultural land area resulting from the static decision rule satisfies:

$$u'(L) = \lambda = \frac{\alpha}{\delta} h'\left(\frac{\alpha(\bar{L} - L)}{\delta}\right).$$

As usual, λ may be interpreted as the farm land rent, hence the first equality relating the land rent to the marginal surplus from farming. The farm land rent must also be equalized to the rental opportunity cost of land conversion, that is the environmental marginal cost $\alpha h' / \delta$. Since $u(\cdot)$ and $h(\cdot)$ are strictly

concave, the above equalities define a unique level of L , we denote by L^S , and λ , we denote by λ^S . The main difference between the allocation resulting from this purely static decision rule and the 'once for all' land allocation problem is that the static rule ignores the fact that the population only progressively declines toward its long run stationary level. This means a higher environmental welfare level at each time than the one entering the pure static decision problem, and hence a higher conversion rate of land to agriculture, $L^S < L^0$.

To be more specific, let $V_N(L^0)$ denote the environmental cumulated welfare resulting from the initial land decision allocation.

$$V_N(L^0) = \int_0^\infty h(N(t))e^{-\rho t} dt .$$

The optimal sharing of land between agriculture and the environment satisfies:

$$\frac{u'(L^0)}{\rho} = -\frac{\partial}{\partial L^0} V_N(L^0) . \quad (3.1)$$

(3.1) states that the society has to be indifferent between converting more land to agriculture, increasing the capitalized welfare by $u'(L)dL/\rho$ for an extra conversion dL , to the loss of environmental value resulting from the reduction of the species territory, $-(\partial V_N(L)/\partial L)dL$. Applying the dynamic envelope theorem:

$$\frac{\partial}{\partial L^0} V_N(L^0) = \int_0^\infty \frac{\partial \mathcal{H}_N(t)}{\partial L^0} e^{-\rho t} dt ,$$

where $\mathcal{H}_N(t)$ denotes the following Hamiltonian function in current value:

$$\mathcal{H}_N(t) = h(N(t)) + \lambda_N(t)N(t) \left[\alpha - \frac{\delta}{\bar{L} - L^0} N(t) \right] . \quad (3.2)$$

$\lambda_N(t)$ measures the current shadow marginal value of the species population. The motion of $\lambda_N(t)$ satisfies the following adjoint equation:

$$\begin{aligned} \dot{\lambda}_N(t) &= \rho \lambda_N(t) - \frac{\partial \mathcal{H}_N(t)}{\partial N(t)} \\ &= (\rho - \alpha) \lambda_N(t) + \frac{2\delta}{\bar{L} - L^0} \lambda_N(t) N(t) - h'(N(t)) . \end{aligned} \quad (3.3)$$

Computing $\partial \mathcal{H}_N / \partial L^0$ from (3.2), (3.1) is equivalent to:

$$\frac{u'(L^0)}{\rho} = \frac{\delta}{(\bar{L} - L^0)^2} \int_0^\infty \lambda_N(t) N^2(t) e^{-\rho t} dt . \quad (3.4)$$

Let $\Phi(t) \equiv \lambda_N(t)N^2(t)$. It results from (2.1) and (3.3) that:

$$\begin{aligned}\dot{\Phi}(t) &= \dot{\lambda}_N(t)N^2(t) + 2\lambda_N(t)N(t)\dot{N}(t) \\ &= (\rho + \alpha)\Phi(t) - h'(N(t))N^2(t) .\end{aligned}\quad (3.5)$$

The dynamics of $N(t)$ and $\Phi(t)$ are solution of the following autonomous differential system:

$$\begin{aligned}\frac{\dot{N}(t)}{N(t)} &= \alpha - \frac{\delta}{W^0}N(t) \\ \dot{\Phi}(t) &= (\rho + \alpha)\Phi(t) - h'(N(t))N^2(t) .\end{aligned}\quad (3.6)$$

In the phase plane (N, Φ) , the locus $\dot{N} = 0$ is the vertical line $N = \hat{N}(L^0)$ where $\hat{N}(L^0) \equiv \alpha(\bar{L} - L^0)/\delta$. $N(t)$ decreases to the right of the vertical and increases to the left of it. The locus $\dot{\Phi} = 0$ defines a curve of equation $\Phi = h'(N)N^2/(\rho + \alpha)$. Assume that $h(N)$ is sufficiently concave for the function $h'(N)N^2$ to be decreasing with N .² Other assumptions may lead to the non monotonicity of $h'(N)N^2$, complicating the discussion without adding much to its qualitative conclusions. Fixing N and increasing slightly Φ above the $\dot{\Phi} = 0$ locus, $\dot{\Phi} > 0$ above the locus and $\dot{\Phi} < 0$ below it.

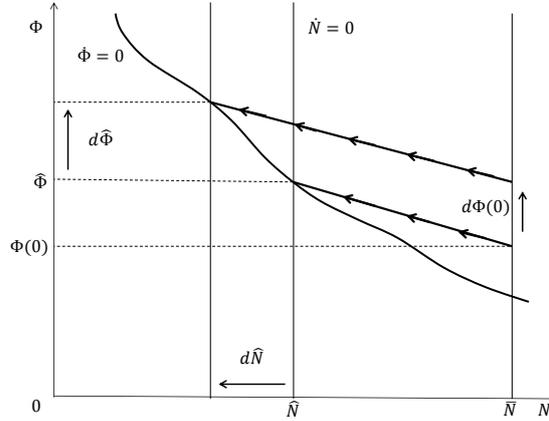


Figure 1: **Phase Diagram in the (N, Φ) plane.**

The phase diagram is drawn in Figure 1. Starting from $N(0) = \bar{N}$, the optimal trajectory follows the saddle branch converging toward $(\hat{N}, \hat{\Phi})$ such

²A sufficient condition for this to be the case is $-h''(N)N/h'(N) > 2$.

that $\hat{\Phi} = h'(\hat{N})\hat{N}^2/(\rho + \alpha)$. $N(t)$ decreases over time while $\Phi(t)$ increases. This shows that $\lambda_N(t) = \Phi(t)/N^2(t)$ increases along the optimal trajectory, reflecting the increased environmental value of an ever decreasing species population.

Since $\hat{N} = \alpha W^0/\delta$ is an increasing function of W^0 , the long run level of the population is a strictly decreasing function of L^0 , a function we denote by $\hat{N}(L^0)$. Now consider a slight increase of L^0 , $\hat{N}(L^0)$ being decreased, the locus $\dot{N} = 0$ is thus shifted to the left. By a standard argument (see Caputo, Chap 18, p 481), $\hat{\Phi}$ shifting upward, $\Phi(0)$ is increased together with $\Phi(t)$. Let $\Phi_A(L^0) = \Phi(0)$ denote the relationship between the initial level of Φ , $\Phi(0)$, and the initial land conversion decision, L^0 . Since the Φ trajectory is shifted upward in the phase plane, $\Phi_A(L^0)$ is an increasing function of L^0 .

Furthermore, the whole $\Phi(t)$ trajectory being shifted upward, $\int_0^\infty \Phi(t)e^{-\rho t} dt$ is an increasing function of L^0 varying between $\underline{\Phi} \equiv h'(\bar{N})\bar{N}^2/\rho$ when $L^0 = 0$ and infinity when $L^0 \rightarrow \bar{L}$. This implies that the r.h.s. of (3.4) is an increasing function of L^0 varying between $\delta\underline{\Phi}/\bar{L}^2$ when $L^0 = 0$ and infinity when $L^0 \rightarrow \bar{L}$. Since $u'(L)$ decreases from infinity when $L^0 = 0$ down to $u'(\bar{L})$ when $L^0 \rightarrow \bar{L}$, (3.4) defines a unique value of L^0 .

To check that $L^S < L^0$ remark that since the ratio $\hat{N}/\hat{W} = \alpha/\delta$ is independent from L and $\Phi(t)$ is a time increasing function:

$$\begin{aligned} u'(L^0) &= \frac{\rho\delta}{(\hat{W}^0)^2} \int_0^\infty \Phi(t)e^{-\rho t} dt < \frac{\rho\delta}{(\hat{W}^0)^2} \frac{h'(\hat{N})\hat{N}^2}{\rho(\rho + \alpha)} \\ &= \frac{\alpha}{\delta} h'(\hat{N}) \frac{\alpha}{\rho + \alpha} < \frac{\alpha}{\delta} h'(\hat{N}) = u'(L^S). \end{aligned}$$

The agricultural net surplus function, $u(L)$, being a strictly concave function of L , $u'(L^0) < u'(L^S)$ implies that $L^S < L^0$ as was intuitively expected.

Shifting abruptly the farm land area from nothing up to some $L^0 > 0$ has no immediate effect over the species population level. The environmental welfare loss induced by the initial conversion decision is only progressive until the new steady state is attained. This implies to convert more land that would be prescribed by a decision rule based on the rental value of the environmental loss evaluated at the new steady state. Note also that the land rent is lower under the 'once for all' decision rule with respect to the long run static one. The next section will exhibit similar conclusions in

a more spectacular way. It will appear that the optimal policy consists in over-exploiting initially the territory before reverting progressively the land to wildlife.

4 Land conversion without adjustment costs

As before, assume the absence of specific conversion costs. $L(t)$ becomes the control variable of the economy. The corresponding current value Hamiltonian reads:

$$\mathcal{H}(t) = u(L(t)) + h(N(t)) + \lambda_N(t)N(t) \left[\alpha - \frac{\delta}{\bar{L} - L(t)}N(t) \right].$$

The optimal choice of $L(t)$ at any time t verifies:

$$u'(L(t)) = \frac{\delta}{(\bar{L} - L(t))^2} \lambda_N(t)N^2(t) = \frac{\delta}{(\bar{L} - L(t))^2} \Phi(t). \quad (4.1)$$

This defines implicitly a relationship between Φ and L , denoted by $\varphi(L)$:

$$\varphi(L) \equiv (\bar{L} - L)^2 \frac{u'(L)}{\delta} = \Phi.$$

The function $\varphi(L)$ varies between infinity when $L = 0$ and 0 when $L \rightarrow \bar{L}$ and $d\varphi(L)/dL < 0$ because of the strict concavity of $u(L)$, the agricultural net surplus function.

Time differentiating yields $\dot{\Phi} = \varphi'(L)\dot{L}$, thus the optimal trajectory $\{N(t), L(t), t \geq 0\}$ is the solution of the following autonomous differential system:

$$\begin{aligned} \dot{L}(t) &= \frac{1}{\varphi'(L(t))} \{(\rho + \alpha)\Phi(L(t)) - h'(N(t))N^2(t)\} \\ \frac{\dot{N}(t)}{N(t)} &= \alpha - \frac{\delta}{\bar{L} - L(t)}N(t). \end{aligned} \quad (4.2)$$

The $\dot{N} = 0$ locus is a line of equation:

$$N^N(L) = \frac{\alpha}{\delta} (\bar{L} - L). \quad (4.3)$$

$N^N(L)$ is a decreasing function of L varying between \bar{N} when $L = 0$ and 0 when $L = \bar{L}$. Fixing L and increasing slightly N above $N^N(L)$, $\dot{N} < 0$ above the $\dot{N} = 0$ locus and $\dot{N} > 0$ below it.

The locus $\dot{L} = 0$ defines implicitly $N^L(L)$. Remembering that optimality requires that $\Phi = \varphi(L)$, the locus $\dot{L} = 0$ defines implicitly a relation between N and L through (4.2). Let $\Theta(L, N) \equiv (\rho + \alpha)\varphi(L) - h'(N)N^2 = 0$ denote this relationship. Then, assuming that $h'(N)N^2$ is a decreasing function of N :

$$\frac{\partial \Theta(N, L)}{\partial N} = -\frac{d}{dN}h'(N)N^2 > 0 \quad \text{and} \quad \frac{\partial \Theta(N, L)}{\partial L} = (\rho + \alpha)\varphi'(L) < 0 .$$

Thus:

$$\frac{dN^L(L)}{dL} = -\frac{\partial \Theta / \partial L}{\partial \Theta / \partial N} > 0 .$$

$N^L(L)$ is an increasing function of L varying between 0 when $L = 0$ and infinity when $L \rightarrow \bar{L}$. Fixing L and increasing slightly N above $N^L(L)$ it appears that, because $\Phi'(L) < 0$, $\dot{L} < 0$ above the $\dot{L} = 0$ locus and $\dot{L} > 0$ below it. The corresponding phase diagram is drawn in Figure 2.

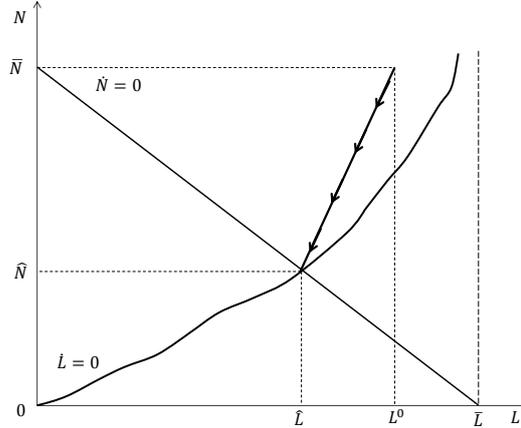


Figure 2: **Land and Species Dynamics without Conversion Costs.**

The graph shows that the dynamical system is saddle path stable, the optimal path converging toward the steady state (\hat{N}, \hat{L}) . Let $N^*(L)$ be the implicit equation defining the stable arm converging from above toward (\hat{N}, \hat{L}) . Starting from $L(0) = 0$ and $N(0) = \bar{N}$, the optimal policy is to set immediately L to the level L^0 defined by $N^*(L) = \bar{N}$, $L(0^+) = L^0$. Next the optimal trajectory follows forever the stable arm converging from above toward (\hat{N}, \hat{L}) . This means a relaxation of the control after the initial bang-bang impulse of the farm land area from nothing to L^0 .

Farm land is converted back toward wilderness, $L(t)$ being permanently decreasing from L^0 toward \hat{L} . As in the preceding section, there is no initial reaction of the species population. Facing an abrupt shortage of its carriage capacity, the population starts a decreasing move sustained until \hat{N} is attained. With respect to a scenario where the converted land would remain fixed at the L^0 level, the permanent increase of $W(t)$, the wild land area, after the initial impulse $W(0^+) < \bar{L}$, allows for a higher long run size of the species population. Put differently, the society should initially over-exploit the land with respect to a long run sustainable situation. This over-exploitation decreases over time with land set aside for biological conservation motive.

The idea behind this result is quite simple. Cutting the carriage capacity of the species has only a delayed effect on the population and hence on environmental welfare. In other words, the Malthusian trap closes only slowly on the species. The society should take advantage of this inertia to push the land conversion to agriculture above its optimal sustainable long run level. Note however that this sustainable level itself takes into account the inertia of the Malthusian trap. To reduce the negative impact over environmental welfare of the initial land conversion decision, the society should decrease progressively its pressure on the territory by letting expand the wild areas at the benefit of the species.

A last feature worth pointing out is the sensitivity of the long run sustainable state with respect to the demographic parameters of the population and the discount rate. The formal details are presented in the Appendix A.3. It appears that both \hat{N} and \hat{L} are increased by a higher crude birth rate α and decreased by a higher mortality parameter δ . These results should probably be expected. A more expansive population resists better to land conversion, allowing to devote a larger share of the territory to agriculture. The reverse happens for a more vulnerable population. A higher level of the rate of impatience of the society makes decrease the optimal sustainable population level and increase the size of the agricultural area. Thus, despite the fact that the society takes care of the population through the environmental welfare it generates, more impatient societies should convert more land to farming. Once again this behavior is explained by the inertia of the Malthusian trap, the welfare loss induced by the population decline being delayed with respect to the immediate welfare gain that can be obtained through more land conversion to farming activities.

4.1 Comparing the 'once for all' decision rule with the continuous conversion policy

Let h_A and h_B be respectively the levels of any variable h under the 'once for all' land allocation decision scheme and under the continuous land allocation scheme without adjustment costs. In the two models, the long run relations:

$$\Phi = \frac{h'(N)N^2}{\rho + \alpha} \quad \text{and} \quad N = \frac{\alpha}{\delta} (\bar{L} - L)$$

define implicitly a function $\hat{\Phi}(L)$ such that $\hat{\Phi}(0) = \underline{\Phi} > 0$, $\hat{\Phi}(\bar{L}) = +\infty$ and $d\hat{\Phi}(L)/dL > 0$. Denote $\theta(L) \equiv \hat{\Phi}(L) - \varphi(L)$. Since $\hat{\Phi}(L)$ is an increasing function of L and $\varphi(L)$ is a decreasing function of L , $\theta(L)$ is itself an increasing function of L .

The long run level of L in the continuous conversion model, \hat{L}_B , is the unique solution of $\varphi(L) = \hat{\Phi}(L)$, that is of $\theta(L) = 0$. On the other hand, L_A^0 , the optimal land allocation in the 'once for all' decision model satisfies:

$$\frac{u'(L)}{\rho} = \frac{\delta}{(\bar{L} - L)^2} \int_0^\infty \Phi(t)e^{-\rho t} dt \iff \varphi(L) = \rho \int_0^\infty \Phi(t)e^{-\rho t} dt .$$

Since the optimal $\Phi(t)$ function is a strictly increasing time function in the 'once for all' decision model:

$$\Phi(0) = \Phi_A(L_A^0) < \varphi(L_A^0) < \hat{\Phi}(L_A^0) .$$

Comparing the long run situations in the two models, $\theta(\hat{L}_B) = 0$ and $\theta(L_A^0) = \hat{\Phi}(L_A^0) - \varphi(L_A^0) > 0$ imply that $\hat{L}_B < L_A^0$ and thus that $\hat{N}_A < \hat{N}_B$. On the other hand, the initial level of agricultural land in the continuous conversion model, L_B^0 , verifies $\Phi_A(L_B^0) = \varphi(L_B^0)$. The function $\Phi_A(L)$ being increasing in L , the difference $\theta_A(L) \equiv \Phi_A(L) - \varphi(L)$ is also an increasing function of L . Since $\theta_A(L_A^0) = \Phi_A(L_A^0) - \varphi(L_A^0) < 0$, it may thus be concluded that $L_A^0 < L_B^0$. The optimal land conversion to agriculture under the 'once for all' decision rule stands in between the initial farm land allocation and its long run level under the continuous conversion decision rule, $\hat{L}_B < L_A^0 < L_B^0$.

The continuous decision rule conserves a higher population in the long run than the 'once for all' decision rule. In terms of agricultural surplus, by converting less land to agriculture initially, the 'once for all' decision rule

induces first a lower farming surplus. In the long run the result is inverted, since the reversal of land toward wilderness in the continuous decision rule model ends with a smaller agricultural territory. Hence, the 'once for all' decision rule has a pivotal effect on the agricultural exploitation plan, lowering initially the farm surplus before increasing it with respect to the continuous decision rule.

The comparison illustrates interesting features of species preservation policies. It is widely acknowledged that endangered species preservation implies specific protection measures with respect to their spatial distribution. Preservation policies thus become an issue of appropriate definition of preserved areas to the benefit of target species. The 'once for all' decision scheme previously studied fits inside this rationale.

However by constraining the decision problem to the definition of a fixed preserved area, such decision rules lose the flexibility allowed by policies targeting first the preservation of the species itself rather than its environment. This is illustrated by the fact that continuous decision schemes are more protective to the species than fixed spatial preservation rules. When the species dynamics are explicitly taken into account, flexible rules appear superior both to protect endangered species and exploiting the agricultural potential of the land than fixed allocation of land to agriculture and protected areas. Of course, flexible schemes are also generally easier to enforce since they do not require the acceptance by the landowners of a permanent set-aside of their properties.

5 General case

For the sake of clarity, we split the discussion of the general case in two parts. We first examine in the next subsection a model with constant marginal conversion costs under a land conversion constraint. This simpler benchmark allows identifying the main conversion scenarios. With these results in hand, we next turn to the study of the solutions of the (*S.P.*) problem.

5.1 Constant marginal conversion costs

Assume that when $l > 0$, $C(l) = cl$. To avoid infinite conversion rates that should result from such an assumption, assume in addition an upper technically possible conversion rate \bar{l} . As before $C(l) = 0$ for $l \leq 0$. Neglect the biological conversion cost. It is immediate that if the society decides to convert land to agriculture at some point of time, it should do so at the maximum possible rate \bar{l} . Since agricultural production has been assumed essential and $L(0) = 0$, we can thus conclude that $l(0^+) = \bar{l}$. From the preceding discussion, it may be also concluded that conversion should occur initially at the rate \bar{l} over some finite time interval $[0, T]$. Let $L_T \equiv L(T)$ denote the maximum extension of the agricultural domain. Since $L(0) = 0$, $L_T = \bar{l}T$.

With the initial condition $N(0) = \bar{N}$ and $L(t) = \bar{l}t$, (2.1) defines the population trajectory $\{N(t), 0 \leq t \leq T\}$, $t \in [0, T]$. The integration of the population dynamics (2.1) over the time interval $[0, T]$ yields:

$$N(t) = \frac{\bar{N}e^{\alpha t}}{1 + \delta\bar{N} \int_0^t \frac{e^{\alpha\tau}}{\bar{L} - \bar{l}\tau} d\tau} \quad t \in [0, T]. \quad (5.1)$$

The current value Lagrangian of the corresponding optimization problem reads:

$$\mathcal{L} = u(L) + h(N) - cl + \lambda_L l + \lambda_N N \left[\alpha - \frac{\delta}{\bar{L} - L} N \right] + \bar{\gamma}_l (\bar{l} - l). \quad (5.2)$$

$\lambda_L(t)$ is the costate variable associated to the land conversion dynamics, the marginal land value, while $\bar{\gamma}_l(t)$ is the Lagrange multiplier associated to the land conversion constraint $\bar{l} - l(t) \geq 0$. Optimality requires that:

$$\lambda_L(t) = c + \bar{\gamma}_l(t) \quad (5.2)$$

$$\bar{\gamma}_l(t) \geq 0 \quad ; \quad \bar{\gamma}_l(t)(\bar{l} - l(t)) = 0 \quad (5.3)$$

$$\dot{\lambda}_L(t) = \rho\lambda_L(t) - \left[u'(L(t)) - \frac{\delta\Phi(t)}{(\bar{L} - L)^2} \right]. \quad (5.4)$$

(5.2) and (5.3) imply that $\lambda_L(t) \geq c$, $t \in [0, T]$. On the other hand $l(t) = \bar{l}$ as stated before while the dynamics of $\Phi(t)$ still obeys (3.5).

The optimal policy can be identified in two steps. First, it can be shown that the population cannot converge toward a steady state before the territory is entirely converted to agriculture. But since a complete conversion of the territory would drive the species to extinction and thus induce an infinite environmental marginal welfare loss, such an outcome cannot be optimal. Hence the conversion process must be closed down in finite time for a finally converted agricultural land area strictly lower than \bar{L} . This implies that the optimally converted land has to be identified with respect to some final phase where the population continues to decline and the cultivated area either stays constant or either decreases. The Appendix A.1 checks this point.

Turn to the second step. Assume that conversion stops at some farm land area L_0 and is maintained at this level forever. To this L_0 is associated some $N_0 = N(T_0)$ where $T_0 = L_0/\bar{l}$. After T_0 , the economy faces a 'once for all' like decision problem. To the arbitrary level L_0 is thus also associated a long run level of the population $\hat{N}(L_0) = \alpha(\bar{L} - L_0)/\delta$ and a long run level of Φ , $\hat{\Phi}$, itself given by $\hat{\Phi}(L_0) = \hat{\Phi}$. On the other hand, the level of Φ at time T_0 is also an increasing function of L_0 , a function we denoted previously by $\Phi_A(L_0)$, so that $\Phi(T_0) = \Phi_A(L_0)$.

Let $\underline{\Phi} = h'(\bar{N})\bar{N}^2/(\rho + \alpha)$, then $\hat{\Phi}(0) = \Phi_A(0) = \underline{\Phi}$. On the other hand $\hat{\Phi}$ and Φ_A both converge toward infinity when $L_0 \rightarrow \bar{L}$ and $\hat{\Phi}(L_0) \geq \Phi_A(L_0)$ by construction. The optimal level of L_0 is then determined by the condition $\lambda_L = c$ and thus $\dot{\lambda}_L = 0$, which implies that:

$$\Phi_A(L_0) = \frac{1}{\delta} (\bar{L} - L_0)^2 (u'(L_0) - \rho c) \equiv \varphi^c(L_0) .$$

It is immediate that $\varphi^c(L_0)$ is a decreasing function of L_0 varying between infinity, when $L_0 \rightarrow 0$, and 0, when \bar{L} increases up to \tilde{L} , the unique solution of $c = u'(L)/\rho$. Thus, it can be concluded that the equation $\Phi_A(L) = \varphi^c(L)$ admits a unique solution, we denote by \hat{L}^c . This solution corresponds to the maximal extension of the cultivated domain.

However, there exists another possible long run optimal solution. Consider the outcome of the continuous conversion policy without conversion costs. As before denote by \hat{L}_B the sustainable converted land area. This land area is the unique solution of $\hat{\Phi}(L) = \varphi(L)$. By construction, the conversion cost c being positive, $\varphi^c(L) < \varphi(L)$. Then, depending on the level of c , two situations can arise.

If c is sufficiently low, $\hat{L}_B < \hat{L}^c$, while the contrary happens for higher levels of c . In the high conversion cost case, the optimal policy requires to stop the conversion process whence the level \hat{L}^c has been attained, that is at time $\hat{T}^c = \hat{L}^c/\bar{l}$. Then the population continues to decrease from the level $N(\hat{L}^c)$ and converges in infinite time toward the long run level $\hat{N}^c = \hat{N}(\hat{L}^c)$. We call Scenario 1 the optimal policy in this high conversion cost situation.

In the low conversion cost case, the optimal policy is slightly more complex. As in the preceding case, conversion stops whence the level \hat{L}^c has been reached and the population has declined to the level $N(\hat{T}^c) = N(\hat{L}^c)$ where $\hat{T}^c \equiv \hat{L}^c/\bar{l}$. After \hat{T}^c , the species population continues to decline. But since now $\hat{L}_B < \hat{L}^c$, there exists a critical level of the population $N^*(\hat{L}^c)$ located along the stable arm converging toward \hat{L}_B in a no conversion cost model. Whence this critical level has been attained, the economy should start to reduce the size of the cultivated domain below \hat{L}^c , follow the stable arm, $N^*(L)$, and make converge in infinite time the farm land area toward \hat{L}_B . The population will finally stabilize in the long run at the level $\hat{N}_B = \alpha(\bar{L} - \hat{L}_B)/\delta$. We call Scenario 2 the optimal policy in this low conversion cost case. The below Figure 4 illustrates the construction of the two optimal scenarios. The top panel shows the optimal dynamics in the (L, Φ) plane and the lower panel its counterpart in the (L, N) space.

In the Scenario 1, the conversion process is monotonous, land being converted to agriculture until the limit \hat{L}^c . The scenario 2 is by contrast a three phases scenario. During a first time phase, land is converted to agriculture up to the same limit \hat{L}^c . During a second time phase, the farm land area is maintained constant at this level. During a third time phase, farm land is returned to wilderness while the population continues to decline. The process ends when the sustainable farm land area of the no conversion costs model, \hat{L}_B , has been attained. The cut-off conversion cost level is that level for which $\hat{L}_B = \hat{L}^c$. Let \tilde{c} denote this cost threshold. If $c < \tilde{c}$, the optimal policy is to follow the Scenario 2 while it is to follow the Scenario 1 if $c > \tilde{c}$.

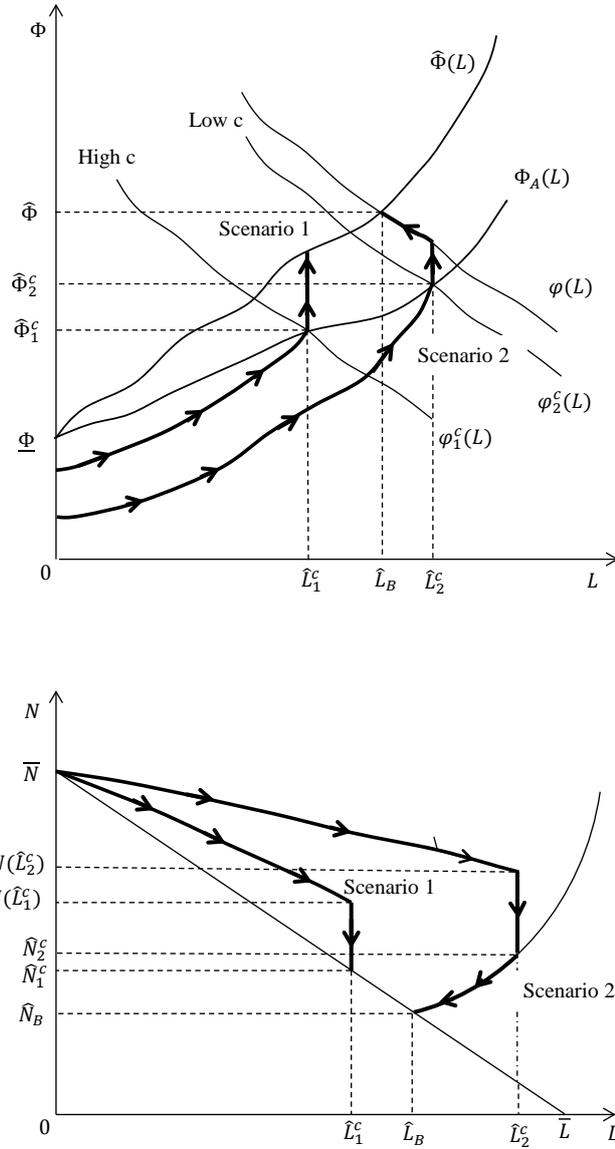


Figure 3: Dynamics in the Constant Unitary Conversion Cost Case.

5.2 Convex adjustment costs

The current value hamiltonian of the (*S.P.*) problem reads (dropping time dependency):

$$\mathcal{H} = u(L) - C(l) + h(N) + \lambda_N N \left(\alpha - \frac{\delta(l)}{L - L} N \right) + \lambda_L l .$$

Within any time interval \mathcal{T}_l when $l(t) > 0$:

$$\lambda_L(t) = c(l(t)) + \frac{\delta'(l(t))\Phi(t)}{\bar{L} - L(t)} \quad t \in \mathcal{T}_l. \quad (5.5)$$

On the other hand:

$$\frac{\dot{N}(t)}{N(t)} = \alpha - \frac{\delta(l(t))N(t)}{\bar{L} - L(t)} \quad \text{and} \quad \dot{\Phi}(t) = (\rho + \alpha)\Phi(t) - h'(N(t))N^2(t).$$

The motion of $\lambda_L(t)$ satisfies when $\lambda_L(t)$ is time differentiable:

$$\dot{\lambda}_L(t) = \rho\lambda_L(t) - u'(L(t)) + \frac{\delta(l(t))\Phi(t)}{(\bar{L} - L(t))^2}. \quad (5.6)$$

Under our assumption over the net agricultural surplus, it can be guessed that the society will start converting the land at some strictly positive level initially. The species population will thus experience a progressive decline. Intuition suggests that with high adjustment costs, the conversion of land to agriculture should be rather slow, giving time to the population to adjust in size in such a way that the two processes of land conversion and population decline converge in the very long run toward a sustainable steady state, an outcome we refer as the Scenario 1.

With almost flat marginal adjustment costs, the situation should resemble the constant conversion cost model. The conversion speed should be too high to allow for a population stabilization before the gains from land conversion to agriculture have been reaped. In this case, we refer as the Scenario 2, conversion will happen during a first time phase of finite duration. After this first time phase, the economy will maintain a constant sharing of the land between farming and wilderness, the species population continuing to decline toward the steady state compatible with this land sharing, as in the 'once for all' conversion model.

However, it may be also the case that with sufficiently low conversion costs, the farm land area should be maintained constant only during a finite time period. At the end of this period, the economy will start to reduce the farm land area and make converge the land sharing toward the sustainable state of the no conversion costs model. We call Scenario 3 this possible outcome. The analysis is complicated by the fact that the population can converge in the long run toward three different sustainable steady states, a

consequence of the assumed positivity of the minimal marginal adjustment costs in the biological and agricultural dimensions and of the convexity of the adjustment costs structure.

The Scenario 1 may be identified along the same lines as for the constant conversion cost case. In this scenario, all the model variables converge asymptotically toward a steady state. Let $(\hat{N}_1, \hat{L}_1, \hat{\Phi}_1)$ be the steady state values of respectively N , L and Φ in the Scenario 1. It has been shown in the preceding subsection that $\hat{\Phi}_1 = \hat{\Phi}(\hat{L}_1)$ and $\hat{N}_1 = \alpha(\bar{L} - \hat{L}_1)/\delta$. Hence, once the optimal land conversion level, \hat{L}_1 , has been determined, the long run characteristics of the species population, that is its sustainable level \hat{N}_1 and its long run marginal value $\hat{\Phi}_1/\hat{N}_1^2$, are identified.

On the other hand, $\dot{L} = l = 0$ and $\dot{\lambda}_L = 0$ at the steady state. Thus (5.5) and (5.6) take the following expressions:

$$\lambda_L = \underline{c} + \frac{\delta' \Phi}{\bar{L} - L} \quad (5.7)$$

$$0 = \rho \lambda_L - u'(L) + \frac{\delta \Phi}{(\bar{L} - L)^2} . \quad (5.8)$$

Inserting the expression of λ_L resulting from (5.7) into (5.8) yields the following relationship between L and Φ having to be satisfied at the end of the land conversion process:

$$\Phi = (\bar{L} - L)^2 \frac{u'(L) - \rho \underline{c}}{\rho \delta' (\bar{L} - L) + \delta} \equiv \varphi^c(L) \quad (5.9)$$

It is immediately checked that $\varphi^c(L)$ is a strictly decreasing function of L with limits: $\lim_{L \downarrow 0} \varphi^c(L) = +\infty$, $\lim_{L \uparrow \bar{L}} \varphi^c(L) = 0$.

In the (Φ, L) plane, the steady state locus in a Scenario 1 corresponds to the intersection between the $\hat{\Phi}(L)$ curve and the $\varphi^c(L)$ curve, equivalently \hat{L}_1 is the unique solution of $\hat{\phi}(L) = \varphi^c(L)$. Once \hat{L}_1 is determined, $\hat{\Phi}_1$ and \hat{N}_1 are also determined while $\lim_{t \uparrow \infty} \lambda_L(t) \equiv \hat{\lambda}_{L1}$ is given by (5.7) evaluated at $(\hat{\Phi}_1, \hat{L}_1)$. Taking the steady state vector $(\hat{L}_1, \hat{\lambda}_{L1}, \hat{\Phi}_1, \hat{N}_1)$ as a particular solution of the differential system, the whole optimal trajectory in a Scenario 1 is determined.

However, this kind of long run outcome is not the only one possible. To show this, consider the following alternative policy. During a first finite time

period, $[0, t_0)$, the economy converts the wild territory to agriculture until some farm land area L_0 has been established. This land area is next maintained constant after t_0 while the species population continues to decline. Note that this scenario is only possible in a situation where at time t_0 , the level of the population $N(t_0)$ is higher than $\hat{N}(L_0)$, the long run population level corresponding to the land area L_0 . Denote by $N_0 \equiv N(t_0)$ the population level inherited from the past history of land conversion from wilderness to farming before t_0 . Let $V(N_0, L_0)$ be the continuation value function after t_0 in current terms:

$$\begin{aligned} V(L_0, N_0) &\equiv \int_{t_0}^{\infty} [u(L_0) + h(N(t))] e^{-\rho(t-t_0)} dt \\ &= \frac{u(L_0)}{\rho} + \int_{t_0}^{\infty} h(N(t)) e^{-\rho(t-t_0)} dt . \end{aligned}$$

Assume that t_0 is finite, then $t_0 \leq T < \infty$. Then the optimal conversion stopping time, t_0 , satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\mathcal{H}(t_0) \geq \rho V(L_0, N_0) \quad \text{and} \quad = \text{ if } t_0 < T .$$

Since $l(t_0) = 0$, we get $C(l(t_0)) = 0$ and $\delta(l(t_0)) = \delta$. Thus the HJB equation takes the following expression:

$$u(L_0) + h(N_0) + \Phi(t_0) \left[\frac{\alpha}{N_0} - \frac{\delta}{\bar{L} - L_0} \right] \geq \rho V(L_0, N_0) .$$

Denote $\Phi_0 \equiv \Phi(t_0)$. Since conversion is closed down at time t_0 , its results from $l(t_0) = 0$ and $\dot{\lambda}_L(t_0) = 0$ that $\Phi_0 = \varphi^c(L_0)$. Thus, it may be concluded that, if $t_0 < T$, the HJB equation defines an implicit relationship between N_0 and L_0 , a relationship we denote by $\Omega(N_0, L_0) = 0$. It is easily checked that:

$$\begin{aligned} \frac{\partial \Omega(L_0, N_0)}{\partial L_0} &= \frac{\delta}{(\bar{L} - L_0)^2} \left\{ \rho \int_{t_0}^{\infty} \Phi(t) e^{-\rho(t-t_0)} dt - \Phi_0 \right\} \\ &\quad + \frac{d\varphi^c(L_0)}{dL_0} \left[\frac{\alpha}{N_0} - \frac{\delta}{\bar{L} - L_0} \right] > 0 \\ \frac{\partial \Omega(L_0, N_0)}{\partial N_0} &= -\frac{1}{N_0^2} [(\rho + \alpha)\Phi_0 - h'(N_0)N_0^2] = -\frac{\dot{\Phi}(t_0)}{N_0^2} < 0 . \end{aligned}$$

Let $\tilde{N}(L_0) = N_0$ be the implicit function so defined. It is immediate that $\tilde{N}(L_0)$ is a strictly increasing function of L_0 . Furthermore, λ_L takes the

following value at t_0 :

$$\begin{aligned}\lambda_L(t_0) &= \underline{c} + \frac{\delta' \varphi^c(L_0)}{\bar{L} - L_0} \\ &= \underline{c} + (u'(L) - \rho \underline{c}) \frac{\delta'(\bar{L} - L_0)}{\rho \delta'(\bar{L} - L_0) + \delta} \equiv \lambda_{L_0}(L^0).\end{aligned}$$

It is easily checked that $\lambda_{L_0}(L_0)$ is a decreasing function of L_0 . Thus to any L_0 is associated a unique vector $(L_0, N_0(L_0), \Phi_0(L_0), \lambda_{L_0}(L_0))$. To this particular solution of the differential system is associated a unique trajectory $\{(L(t), N(t), \Phi(t), \lambda_L(t)), 0 \leq t \leq t_0\}$. Then t_0 is defined as the solution of $L(t_0) = L_0$. It results that $N(t_0) = N_0(L_0)$ defines a unique L_0 . Since $N(t_0)$ is itself inherited from the land conversion history described by the trajectory $\{(L(t), N(t), \Phi(t), \lambda_L(t))\}$ depending on L^0 , either the solution exists from a finite t_0 , either its exists only asymptotically. In the first case, denote by \hat{L}_2 the corresponding value of L^0 .

Furthermore, it may be the case that \hat{L}_2 is higher or lower than \hat{L}_B , the long run land area in the no conversion cost model. If $\hat{L}_2 < \hat{L}_B$, then the optimal policy is a Scenario 2 where the farm land conversion process stops in finite time \hat{t}_2 and next the agricultural land area is maintained constant forever at the level \hat{L}_2 while the species population continues to decline toward the steady state level $\hat{N}_2 = \tilde{N}(\hat{L}_2)$. If $\hat{L}_2 > \hat{L}_B$, then the optimal policy is the Scenario 3. In this scenario, conversion stops when the land area \hat{L}_2 has been allocated to farming. Then the agricultural land area is maintained constant until the population has declined toward the level $N^*(\hat{L})$. The economy should then start a process of land set aside in favor of the species conservation, reducing the farm land area asymptotically down toward the area \hat{L}_B .

Turn now to the features of the transition toward the maximal extension of the agricultural domain in any possible scenario. The formal details are presented in Appendix A.2. It may be shown that in all scenarios, the marginal value of agricultural land, $\lambda_L(t)$, should permanently decrease until the conversion attains a steady state. Moreover, the speed of the conversion process of wild land to farm land, $l(t)$, should also steadily decline down to zero in the course of the conversion.

This behavior is easily explained. On the one hand, $L(t)$ being increasing through time, the marginal net surplus $u'(L(t))$ decreases, thus lowering the

profitability prospect of more land conversion to agriculture. On the other hand, the environmental loss progressively increases with the decline of the population, even if the extinction process is somewhat delayed by the demographic inertia of the population with respect to the progressive destruction of its natural habitat. Last, slowing down conversion allows reducing the conversion costs while reducing the impact of the perturbation generated by the conversion itself upon the species. All these drivers combine themselves to reduce the speed of land conversion.

6 Conclusion

The economics of endangered species conservation is commonly studied in two ways. Most studies focus on the impact of human predation or perturbing activities over the fate of the species. The conservation problem is here seen as one of internalizing the various externalities created by human presence over the abundance of the species population or its survival probability. The issue is usually treated in static models or less commonly inside a dynamic population framework in the line of bioeconomic models of renewable resources management. The main trade-off is thus between the harvesting value of the population and its *in situ* value inside the environment.

Other authors have stressed the importance of the relationship between the species and its environment for its sustained presence. This 'environment' is commonly assimilated to the characteristics of the species habitat, especially its size. Such analysis incorporate explicitly spatial considerations into the design of endangered species conservations policies. Some rare studies have tried to incorporate both the dynamic and spatial aspects of the conservation issue. In most case, the 'value' of the species is attributed to its habitat, usually by postulating an environmental value function depending positively on the size on this natural habitat.

We have borrowed different elements of this literature while departing from it in significant ways. We assume that the society values for itself the species in its natural environment and not in the form of an harvested resource. Thus 'value' applies here only to the 'stock' of specimens (its *in*

situ value) and not to a flow of wealth that can be obtained from harvesting them. The species population has its own dynamics, only indirectly dependent from the dynamics of human settlement on its natural habitat. Thus the environmental value dynamics is here partially disconnected from the dynamics of land occupation. We kept the idea of human perturbations, or 'anthropic pressure', by making the species mortality rate dependent from the speed of wild land conversion to human activities. We modeled the land artificialisation process in a more convincing way than the earlier literature by introducing convex conversion costs of land development. Our framework thus incorporates explicitly both the spatial and temporal dimension of the endangered species conservation problem.

Despite long lasting efforts since forty years, both spatial and dynamic modeling of natural resources management remains largely underdeveloped. In particular, the literature lacks from basic analytical foundations for conservation economics in such settings. This may be explained by the high degree of complexity of the models and their mathematical intractability even in highly stylized frameworks. Because of these computational intricacies, most of the literature has focused on the description of steady states, or long run balanced outcomes (the so-called 'sustainable' situations). We have departed from this tradition by focusing instead on the transitory dynamics of a both spatial and dynamic endangered species management model. This comes at a cost since to achieve this aim we had to build a highly simplified model of the conservation problem, much simpler than the ones that can be found in the biodiversity management literature.

This research strategy allowed us to unveil a much richer set of potential optimal management policies than the earlier literature focusing only on steady states. Without adjustments costs, we have shown that optimal land use requires to temporally overshoot what will be the sustainable land sharing between nature and human activities in the very long run (the 'steady state' outcome). We have also shown that such policies dominate the traditional approach of the conservation problem as one of an optimal design of the size of conserved areas, a reserve creation problem. This domination appears not only with respect to the human valuation of land for its own sake, as should be intuitively expected, but also with respect to the long run prospects of the population.

With positive conversion costs of wild land to a 'developed' state, we have shown that several kinds of long run steady state can arise. The most commonly expected policy is an asymptotic convergence of both the species population and the size of its natural habitat toward some sustainable outcome, conversion costs reducing progressively the economic prospects of further land conversion to zero in the very long run. But such an outcome is not the only possible one. With relatively low conversion costs, the optimal land conversion process may be so fast that the induced decline of the population will be relatively slow transitorily. In this case, the optimal conservation rule will dictate to stop in finite time the land conversion process. After this time, the land sharing may be maintained forever. But it may be also the case that the optimal conservation policy implies to reduce after some time the developed area at the benefit of the species. The population will then converge toward an higher long run size thanks to this ecological land set aside.

The present work may be extended in different directions. The issue of the implementation of the optimal conservation and land allocation policy remains to be settled down carefully. Several policy tools may be considered: land use fees, exploitation licenses, either permanent or temporary, subsidies to environmental preservation. Ahead of the implementation problem, it is well known that conservation of emblematic endangered species faces strong acceptability issues by local settlers communities. In particular, the imposition of fixed land sharing or land sparing rules have proven typically difficult to enforce. Our work has shown that these rigid sharing rules may be dominated, even from an environmental point of view, by more flexible rules, as such more acceptable for land settlers.

For tractability motives, we have modeled the competition for land between human activity and natural life through the impact on the species population dynamics of land conversion over their carriage capacity. The strength of this approach is to emphasize the importance of the time lags between the population dynamics and the land conversion dynamics. Its main weakness is to overlook the real complexity of the competition for natural resources access between ecological communities and human activities. To encompass this limitation, the analysis should be extended to several species and several natural resources, at the cost of a significant increase of the computational burden of the resulting model.

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Appendix

A.1 Constant marginal conversion costs

Denote $n(t) = \alpha(\bar{L} - \bar{l}t)/\delta$. $\dot{N}(t) = 0$ if $N(t) = n(t)$. Initially, the population is at a rest point and $n(0) = \bar{N}$. Denote by $h(t) = N(t) - n(t)$ the lag between the state of the population at time t and its steady state level evaluated at $L(t)$. Since the population is initially stationary, $h(0) = 0$. Time differentiating:

$$\begin{aligned} \dot{h} &= \dot{N} - \dot{n} = N \left(\alpha - \frac{\delta N}{\bar{L} - L} \right) - \left(-\frac{\alpha \bar{l}}{\delta} \right) \\ &= N \left(\alpha - \alpha \frac{N}{\frac{\alpha}{\delta}(\bar{L} - \bar{l}t)} \right) + \frac{\alpha \bar{l}}{\delta} \\ &= \alpha N \left(1 - \frac{N}{n} \right) + \frac{\alpha \bar{l}}{\delta} \\ &= \alpha \left[\frac{N}{n} (n - N) + \frac{\bar{l}}{\delta} \right]. \end{aligned}$$

Since $h = N - n$ and $N/n = 1 + h/n$, the above expression is equivalent to:

$$\begin{aligned} \dot{h} &= \alpha \left[-h \left(1 + \frac{h}{n} \right) + \frac{\bar{l}}{\delta} \right] \\ &= \alpha \left[-\frac{h^2}{n} - h + \frac{\bar{l}}{\delta} \right]. \end{aligned}$$

Let $P(h; n)$ be the second order polynomial into brackets in the previous expression of \dot{h} . The dynamics of $h = N - n$ can thus be studied by describing the behavior of $P(h; n)$ over time.

First note that at time 0, $h(0) = 0$ implies that $\dot{h}(0) = \alpha \bar{l}/\delta > 0$. Thus the difference initially increases. The polynomial $P(h; n)$ admits a unique positive root $h_0(n)$ defined as:

$$h_0(n) = \frac{n}{2} \left[\left(1 + \frac{4\bar{l}}{\delta n} \right)^{\frac{1}{2}} - 1 \right].$$

It can thus be concluded that $\dot{h}(t) = \alpha P(h(t); n(t)) > 0$ if $h(t) < h_0(n(t))$ and $\dot{h}(t) < 0$ if $h(t) > h_0(n(t))$. Next it is easily checked that h_0 decreases

through time. An equivalent expression of h_0 is:

$$\begin{aligned} h_0(n) &= \left[\left(1 + \frac{4\bar{l}}{\delta n} \right) \frac{n^2}{4} \right]^{\frac{1}{2}} - \frac{n}{2} \\ &= \left[\frac{n^2}{4} + n \frac{\bar{l}}{\delta} \right]^{\frac{1}{2}} - \frac{n}{2}. \end{aligned}$$

Differentiating with respect to n this expression gets:

$$\frac{dh_0(n)}{dn} = \frac{1}{2} \left[\left(\frac{n}{2} + \frac{\bar{l}}{\delta} \right) \left(\frac{n^2}{4} + \frac{n\bar{l}}{\delta} \right)^{-\frac{1}{2}} - 1 \right].$$

It is immediate that $dh_0(n)/dn > 0$. Assume to the contrary that $dh_0(n)/dn < 0$ then this would require that:

$$\begin{aligned} \frac{n}{2} + \frac{\bar{l}}{\delta} &< \left(\frac{n^2}{4} + \frac{n\bar{l}}{\delta} \right)^{\frac{1}{2}} \\ \iff \frac{n^2}{4} + \frac{n\bar{l}}{\delta} + \left(\frac{\bar{l}}{\delta} \right)^2 &< \frac{n^2}{4} + \frac{n\bar{l}}{\delta}, \end{aligned}$$

a contradiction. Since $\dot{n} < 0$, it results that $dh_0(t)/dt < 0$. Since $n \rightarrow 0$ when $L(t) \rightarrow \bar{L}$ on the one hand and $h_0(0) = 0$ on the other hand, $h_0(t)$ is a time decreasing function converging toward zero when $t \rightarrow \bar{T}$, $\bar{T} \equiv \bar{L}/\bar{l}$ being the time needed to convert the whole territory to farmland.

Hence it may be concluded that $h(t)$ is a time function first increasing up to some time t_0 when $h(t_0) = h_0(t_0)$ and next decreasing but remaining strictly positive at least until \bar{T} . Hence, $h(\bar{T}) = N(\bar{T}) - n(\bar{T}) > 0$ implies that even if the whole territory is converted to agriculture, the population could not reach in the limited time \bar{T} a steady state situation. Since a complete conversion should involve the extinction of the population, which cannot be optimal, the conversion process should stop at some converted area $\hat{L}^c < \bar{L}$. To this converted land corresponds some $\hat{T}^c = \hat{L}^c/\bar{l}$, the time needed to convert the area \hat{L}^c to agriculture and thus a size of the population $\hat{N}^c = N(\hat{T}^c)$ at the end of the conversion process. At this level $\hat{N}^c = N(\hat{T}^c)$, the species population is still in transition toward a steady state. This ends the first step of the identification of the optimal scenario.

A.2 Identification of the optimal policy in the general case

(5.5) defines an implicit function $\dot{L} = l(\lambda_L, L, \Phi)$. Let $D \equiv c'(l)(\bar{L} - L) + \delta''(l)\Phi > 0$, then differentiating (5.5) yields:

$$\begin{aligned}\frac{\partial l(\lambda_L, L, \Phi)}{\partial \lambda_L} &= \frac{\bar{L} - L}{D} > 0 \\ \frac{\partial l(\lambda_L, L, \Phi)}{\partial L} &= -\frac{\delta'(l)\Phi}{(\bar{L} - L)D} < 0 \\ \frac{\partial l(\lambda_L, L, \Phi)}{\partial \Phi} &= -\frac{\delta'(l)}{D} < 0\end{aligned}$$

On the other hand, when $l > 0$, the dynamics of $\lambda_L(t)$ is defined as a function of (λ_L, L, Φ) as:

$$\begin{aligned}\dot{\lambda}_L(t) &= \rho\lambda_L(t) - u'(L(t)) + \frac{\delta(l(\lambda_L(t), L(t), \Phi(t)))}{(\bar{L} - L(t))^2}\Phi(t) \\ &\equiv \Lambda(\lambda_L(t), L(t), \Phi(t))\end{aligned}\tag{A.2.1}$$

Differentiating the expression (A.2.1) of $\Lambda(\lambda_L, L, \Phi)$ yields:

$$\begin{aligned}\frac{\partial \Lambda(\lambda_L, L; \Phi)}{\partial \lambda_L} &= \rho + \frac{\delta'(l)\Phi}{(\bar{L} - L)D} > 0 \\ \frac{\partial \Lambda(\lambda_L, L; \Phi)}{\partial L} &= -u''(L) + \frac{2\delta(l)\Phi}{(\bar{L} - L)^3} - \frac{(\delta'(l)\Phi)^2}{(\bar{L} - L)^3 D} (?) \\ \frac{\partial \Lambda(\lambda_L, L; \Phi)}{\partial \Phi} &= \frac{\delta(l)}{(\bar{L} - L)^2} - \frac{(\delta'(l))^2\Phi}{(\bar{L} - L)^2 D} (?)\end{aligned}$$

Assume that the function $\delta(l)$ is sufficiently convex for $\delta''(l)/\delta'(l) > \delta'(l)/\delta(l)$, then it is easily checked that $\partial \Lambda/\partial L > 0$ and $\partial \Lambda/\partial \Phi > 0$.

The existence and uniqueness of the optimal scenario results from standard properties of the differential system:

$$\begin{aligned}\dot{L}(t) &= l(L(t), \lambda_L(t), \Phi(t)) \\ \dot{\lambda}_L(t) &= \Lambda(L(t), l(t), \Phi(t)) \\ \dot{\Phi}(t) &= (\rho + \alpha)\Phi(t) - h'(N(t))N^2(t) \equiv \psi(\Phi(t), N(t)) \\ \frac{\dot{N}(t)}{N(t)} &= \alpha - \frac{\delta(l(L(t), \lambda_L(t), \Phi(t)))N(t)}{\bar{L} - L(t)} \equiv n(L(t), \lambda_L(t), \Phi(t), N(t)),\end{aligned}$$

with the particular solution $(0, \lambda_{L0}, \Phi_0, \bar{N})$ where $\lambda_{L0} = \lambda_L(0)$ and $\Phi_0 = \Phi(0)$. Existence results from the continuity of the time differentials of the

variables $(L(t), \lambda_L(t), \Phi(t), N(t))$, while the differentiability of the functions $l(L, \lambda_L, \Phi)$, $\Lambda(L, \lambda_L, \Phi)$, $\psi(\Phi, N)$ and $n(L, \lambda_L, \Phi, N)$ with respect to their arguments implies that the Lipschitz condition for uniqueness is satisfied.

Turn to the dynamics of the rest points in the phase plane (L, λ_L) . Let $(\hat{L}^0, \hat{\lambda}_L^0)$ be the solution of the system $\dot{L} = 0$ and $\dot{\lambda}_L = 0$. Differentiating the system $[l = 0, \Lambda_L = 0]$ yields in matrix form:

$$\begin{bmatrix} \frac{\partial l}{\partial \lambda_L} & \frac{\partial l}{\partial L} \\ \frac{\partial \Lambda}{\partial \lambda_L} & \frac{\partial \Lambda}{\partial L} \end{bmatrix} \begin{bmatrix} d\lambda_L \\ dL \end{bmatrix} = \begin{bmatrix} -\frac{\partial l}{\partial \Phi} \\ -\frac{\partial \Lambda}{\partial \Phi} \end{bmatrix} d\Phi$$

Let Δ be the determinant of this system.

$$\Delta = \rho \frac{\delta' \Phi}{(\bar{L} - L)D} + \frac{2\delta\Phi}{(\bar{L} - L)^2 D} - \frac{u''(L)(\bar{L} - L)}{D} > 0$$

Applying the Cramer rule:

$$\begin{aligned} \frac{d\hat{\lambda}_L^0}{d\Phi} &= \frac{1}{\Delta} \begin{vmatrix} -\partial l/\partial \Phi & \partial l/\partial L \\ -\partial \Lambda/\partial \Phi & \partial \Lambda/\partial L \end{vmatrix} \\ &= \frac{\delta'}{\Delta D} \left\{ \frac{\delta\Phi}{(\bar{L} - L)^3} - u''(L) \right\} > 0 \\ \frac{d\hat{L}^0}{d\Phi} &= \frac{1}{\Delta} \begin{vmatrix} \partial l/\partial \lambda_L & -\partial l/\partial \Phi \\ \partial \Lambda/\partial \lambda_L & -\partial \Lambda/\partial \Phi \end{vmatrix} \\ &= -\frac{1}{\Delta D} [\delta + \rho\delta'] < 0 \end{aligned}$$

As illustrated in Figure A. 1, the rest points $(\hat{L}^0, \hat{\lambda}_L^0)$ are shifted in the north west direction as $\Phi(t)$ increases over time. When $t \rightarrow \infty$, $\Phi(t) \rightarrow \hat{\Phi}_1$ implies that $\hat{L}^0 \rightarrow \hat{L}_1$ while $\hat{\lambda}_L^0 \rightarrow \hat{\lambda}_{L1}$, $(\hat{L}_1, \hat{\lambda}_{L1})$ being the steady state values in the Scenario 1. There exists a unique trajectory $\{L(t), \lambda_L(t)\}$ moving in the south east direction and pointing toward the steady state locus $(\hat{L}_1, \hat{\lambda}_{L1})$. Other trajectories cannot converge toward the steady state locus and thus cannot be optimal. In the scenario 1 the conversion process induces a permanent decrease of the implicit price of agricultural land.

Since $\lambda_L(t)$ decreases while $L(t)$ increases together with $\Phi(t)$, it may be concluded from the signs of the partial derivatives of l with respect to

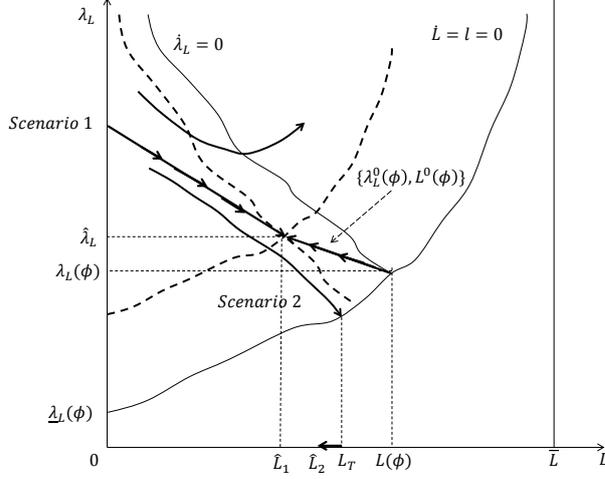


Figure A.1: **Conversion and Land Price Dynamics.**

(λ_L, L, Φ) that $l(t)$ should permanently decrease along the transition path. Hence the optimal land conversion path slows down progressively.

A.3 Comparative statics of the steady state

In the no conversion cost case, the steady state (\hat{N}_B, \hat{L}_B) solves the system of conditions:

$$\begin{aligned} N &= \frac{\alpha}{\delta}(\bar{L} - L) \\ \varphi(L) &= \frac{h'(N)N^2}{(\rho + \alpha)}. \end{aligned}$$

Denote $H(N) \equiv h'(N)N^2$. Remember that it has been assumed that the function $h'(N)$ is sufficiently concave for $H'(N) < 0$. Then differentiating leads to the following system in matrix form:

$$\begin{bmatrix} 1 & \frac{\alpha}{\delta} \\ -\frac{H'(N)}{(\rho + \alpha)} & \varphi'(L) \end{bmatrix} \begin{bmatrix} dN \\ dL \end{bmatrix} = \begin{bmatrix} \frac{N}{\alpha} \\ -\frac{\varphi(L)}{(\rho + \alpha)} \end{bmatrix} d\alpha + \begin{bmatrix} -\frac{N}{\delta} \\ \frac{\varphi(L)}{\delta} \end{bmatrix} d\delta + \begin{bmatrix} 0 \\ -\frac{\varphi(L)}{(\rho + \alpha)} \end{bmatrix} d\rho$$

The determinant of this system is given by:

$$\Delta_S = \varphi'(L) + \frac{\alpha H'(N)}{(\rho + \alpha)} < 0 .$$

Applying the Cramer rule:

$$\frac{d\hat{N}_B}{d\alpha} = \frac{1}{\delta\Delta_S} \left[\left(\frac{\alpha}{\rho + \alpha} - 2 \right) (\bar{L} - \hat{L}_B)^2 \frac{u'(\hat{L}_B)}{\delta} + (\bar{L} - \hat{L}_B)^3 \frac{u''(\hat{L}_B)}{\delta} \right] > 0$$

$$\frac{d\hat{N}_B}{d\delta} = -\frac{\alpha}{\delta^2\Delta_S} \left[(\bar{L} - \hat{L}_B)^3 \frac{u''(\hat{L}_B)}{\delta} - (\bar{L} - \hat{L}_B)^2 \frac{u'(\hat{L}_B)}{\delta} \right] < 0$$

$$\frac{d\hat{N}_B}{d\rho} = \frac{\alpha}{\delta\Delta_S} \frac{\varphi(\hat{L}_B)}{(\rho + \alpha)} < 0 .$$

On the other hand:

$$\frac{d\hat{L}_B}{d\alpha} = \frac{1}{(\rho + \alpha)\Delta_S} \left[-\varphi(\hat{L}_B) + H'(\hat{N}_B)\hat{N}_B \right] > 0$$

$$\frac{d\hat{L}_B}{d\delta} = -\frac{1}{\delta\Delta_S} \left[\varphi(\hat{L}_B) - \frac{H'(\hat{N}_B)\hat{N}_B}{(\rho + \alpha)} \right] < 0$$

$$\frac{d\hat{L}_B}{d\rho} = \frac{1}{(\rho + \alpha)\Delta_S} \varphi(\hat{L}_B) > 0 .$$