What about polluting eco-industry?
Preliminary version

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Abstract

In this article we introduce a polluting eco-industry. Depending on the level of the damage we find two optimal equilibria. If the damage is low we generalize usual results in economics literature to polluting eco-industry: the dirty firm partially abates their emissions, only efficient eco-industry firms produce and the abatement level increases with the damage. However we obtain very specific results if the damage is high. In this case not all efficient eco-industry firms produce. The abatement level and the number of active eco-industry firms decrease with the damage. We finally show that a pigovian tax implements these equilibria in a competitive economy.

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1 Introduction

As a representative of the new energy, solar energy industry is presented as the leader in the market. But recent events make solar photovoltaic industry in the world of pollution exposure. On 2011, a river in Haining City - China - was completely polluted because of excessive levels of fluoride in the water. The polluting firm own to photovoltaic industry.

By and large this event raises the question of polluting eco-industry. Eco-industry is a new industry covering pollution and resource management activities, ranging from the development of clean technologies to the optimization of methods for monitoring and managing environmental impacts. This industry includes wastewater treatment, air treatment, waste treatment plants but as well photovoltaic industry.

Although the aim of these activities is to improve environmental quality, these production processes may also be polluting. According to Kyung and al. (2013), incineration facilities and wastewater treatment plants have been reported to emit significant amounts of GHGs. Recently, water treatments plants have also been categorized as one of the significant public facilities emitting important amounts of CO2 by consuming immense amounts of electricity and chemicals (Raucher and al. (2008), Rothausen and Conway (2011)). As far as energy conversion through photovoltaic system is concerned, it is one of the most important, more reliable and environment friendly renewable energy technology which has the potential to contribute significantly to a sustainable energy system. It also plays an important role to mitigate CO2 emissions. Once produced, they really enable to avoid GHGs, but a debate has emerged about pollution coming from the production process. This point raises the question of grey energy.

Grey energy is the hidden energy associated with a product, meaning the total energy consumed throughout the product life cycle from its production to its disposal. The question is to know wether the improvement in pollution reduction is more important than grey energy consumed. If it is the case we can infer that eco-industry is efficient.

This point is crucial for policy-makers because eco-industry firms always emerge following an environmental policy. Without environmental regulation, there is any incentives to reduce pollution. The economic literature is well documented about eco-industry. Most of these papers consider, as environmental tools, a Pigovian tax: see for instance David and Sinclair-Desgagné (2005, 2010), Nimubona and Sinclair-Desgagné (2005), Canton (2008), Canton *and al.* (2007) and David and *al.* (2011). Greaker (2006) and Greaker and Rosendahl (2008) introduce non tradeable quotas and Schwartz and Stahn (2014) retains a pollution permit market. As this industry is highly concentrated, the aim of all these articles is to analyze in which measure taking into account imperfect competition in the eco-industry challenge usual results about environmental policy.

But any studies investigate *polluting* eco-industry. However we can wonder if environmental regulation can lead to inefficient eco-industry. It is true that policy makers can base their decisions on scientific reports and forecasting. For example, Nawaza and Tiwarib (2006) find that the photovoltaic system is environment friendly in comparison to other source of energy used. That justifies public policies in favour of developing photovoltaic systems like subsidies. But in a real world information is obviously lacking about future technologies.

The aim of this paper is to investigate in which measure standard results in economic literature are challenged if eco-industry is polluting. Following almost all the papers quoted above we consider a vertical structure composed of a downstream polluting sector and of an upstream eco-industry. Contrary to the existing literature, in this article eco-industry firms are polluting and asymmetric - they are more or less polluting - and we assume that they cannot reduce their emissions. Under these new assumptions we first seek to define the centralized solution. Next, we wonder if this optimal policy can be decentralized *i.e.* without favoring inefficient eco-industry.

We find that two kinds of equilibria can emerge. The first equilibrium occurs if the marginal damage is not too high. In this case we extend usual results in economic literature to polluting eco-industry. We find that the optimal level of abatement is such that marginal social benefit and marginal social cost of abatement are equal to the marginal damage. The dirty firm partially abates its emissions and only efficient eco-industry firms produce. The more the marginal damage is the less the dirty firm produces and the more the abatement level is.

The second kind of equilibrium occurs if the marginal damage is very high: the dirty firm abates all its emissions and not all active firms in the first equilibrium produce. As pollution is very harmful for environment the only way to reduce even more damage is to reduce pollution coming from eco-industry. To do that in an efficient way, the regulator not only reduces the number of active firms but also modifies the distribution of abatement production among eco-industry. Finally we find the counter intuitive result that the number of active firms and the abatement production decrease with the marginal damage. The optimal abatement level is such that the marginal social benefit is equal to the marginal social cost but they are lower than the marginal damage. This second equilibrium is very specific to polluting eco-industry.

We finally show that a competitive economy reaches these optimal equilibria if the regulator implements a pigovian tax.

In Section 2, we present the model. Section 3 defines social benefits and social costs coming from pollution abatement. In Section 4 we determine the efficient outcome. Policy issues are presented in Section 5 and some concluding remarks are given in Section 6.

2 The basic assumptions

In order to keep the assumptions as simple as possible, we assume that the standard *polluting industry* is characterized by a representative firm which produces a quantity Q at a given cost $C(Q)$. This cost is increasing and convex (i.e. $C'(Q) > 0$ and $C"(Q) > 0$) and inaction is allowed (i.e. $C(0) = 0$). This activity is polluting. Emissions are given by $\varepsilon(Q)$, an increasing and convex function (i.e. $\varepsilon'(Q) > 0$ and $\varepsilon"(q) > 0$) with $\varepsilon(0) = 0$. This "end-of-pipe" pollution can be reduced by an abatement activity provided by specialized external firms which compose the eco-industry. So if we denote by A the total abatement realized by the polluting firm, the remaining pollution will be of $\max\{\varepsilon(Q) - A, 0\}$.

The *eco-industry* is composed of a continuum[1] $[0, 1]$ of firms indexed by i. Each of them supply $a(i)$ pollution reduction services produced at some cost $\kappa(a(i))$. They share the same increasing and convex cost function and inaction is allowed (i.e. $\kappa'(a) > 0$, $\kappa"(a) > 0$ and $\kappa(0) = 0$) We even assume that $\kappa'(0) = 0$ in order to ensure that there is, in a competitive setting, an offer for each positive price[2]. We however assume that this activity also pollutes and that these firms are heterogenous with respect to their emissions. This pollution is a proportion $\beta(i)$ of the production of firm i and is viewed as unavoidable (i.e. it cannot be abated) with $\beta(i) \in [\beta_{\min}, \beta_{\max}]$. Firms are ranked in $[0, 1]$ from the lowest to the highest $\beta(i)$ in a continuous and differentiable way. Moreover we assume $\beta_{\min} < 1$ that makes sure that at least some firms have a net contribution to global pollution reduction, while $\beta_{\max} > 1$ means that at least some of these firms contribute to pollution abatement in an inefficient way since their global contribution to the emissions per unit of output, $1 - \beta(i)$, is negative.

The *global emissions,* $E = \max\{\varepsilon(Q) - A, 0\} + \int_0^1 \beta(i)a(i)di$, are composed by the leftover pollution of the dirty industry and the emissions induced by the abatement activity. This means that we can have situations in which the dirty industry is clean and some pollution remains. So, contrary to most of the literature which does not introduce polluting eco-industry, it is now crucial to take into account that the abatement activity becomes inefficient when the pollution of the dirty industry is completely reduced. Moreover, it is rather obvious that the emergence of these various situations are linked to the size of marginal damage from pollution. This is why we assume that the social damage is measured by $D(E) = v.E$ with $v > 0$.

Finally, to close the model, we introduce an *inverse demand function* for the polluting goods

[1] This continuum assumption is essentially introduced in order to simplify the treatment of an industry composed by heterogeneous agents. The same arguments holds with a finite number of firms.

[2] A discussion about the emergence of an eco-industry related to the fact that $\kappa'(0) > 0$ can be found in Canton and al. (2007).

$P(Q)$ This function is decreasing (i.e. $P'(Q) < 0$) and verifies that $\lim_{Q \to 0} P(Q) = +\infty$ and $\lim_{Q \to +\infty} P(Q) = 0$.

3 Social benefits and costs from pollution abatement

This section is rather traditional. We fix a level A of production of abatement good and define, within our setting, the social benefits and costs of this abatement choice. The main difference with the usual approach is that the eco-industry is polluting. This has two upshots: (i) these goods only reduce the emissions of the polluting industry and (ii) this residual pollution must be integrated in the social cost of the abatement production.

The *social benefit* from a level A of pollution abatement is obtained by choosing the production of the dirty industry. This production level maximizes the welfare of consumers net of the production costs and of the pollution induced by this activity. This function is given by:

$$SB(A, v) = \max_{Q \geq 0} \left(\int_0^Q P(q)dq \right) - C(Q) - v \cdot \max \{\varepsilon(Q) - A, 0\} \tag{1}$$

Since abatement is inefficient when the pollution of the dirty industry is completely reduced, this optimization problem is typically non-smooth. We can however observe that:

- The production level should be set to $Q^* = Q_{\max}$ which stands for competitive production level without any pollution control. This however requires that $A \geq \varepsilon(Q_{\max})$.
- If there is only partial abatement, the optimal production level $Q^* = Q(v)$ only depends on the marginal damage v and solves

$$P(Q) - C'(Q) - v\varepsilon'(Q) = 0$$

  and this case occurs if $\varepsilon(Q(v)) > A$, a quantity which is by construction smaller the $\varepsilon(Q_{\max})$.
- Finally if $A \in [\varepsilon(Q(v)), \varepsilon(Q_{\max})]$, the pollution is fully reduced but the optimal production level without pollution cannot be reached. This is why the optimal production level will be in this case positively correlated with the abatement level i.e. $Q^* = \varepsilon^{-1}(A) \in [Q(v), Q_{\max}]$

These observations give us the social benefit from pollution abatement. However what really matter is the marginal social benefit, i.e. $\frac{\partial SB(A,v)}{\partial A}$. This one is clearly 0 for $A > \varepsilon(Q_{\max})$ because, in this case, additional abatement is fully inefficient. If there is only partial reduction, this marginal benefit will be as usually equal to the marginal damage. But for $A \in [\varepsilon(Q(v)), \varepsilon(Q_{\max}))$, it will be somewhere in between 0 and the marginal damage because within this range production and abatement are positively correlated.

More formally, we can state that:

Lemma 1 *If* $\xi(Q) = \frac{P(Q)-C'(Q)}{\varepsilon'(Q)}$, *an inspection of program 1 shows that:*

(i) The optimal production level is given by:

$$Q(v,A) = \begin{cases} \max\left\{\xi^{-1}(v), \varepsilon^{-1}(A)\right\} & \text{if } A < \varepsilon\left(Q_{\max}\right) \\ Q_{\max} & \text{if } A \geq \varepsilon\left(Q_{\max}\right) \end{cases} \tag{2}$$

(ii) The marginal social benefit is given by:

$$\frac{\partial SB(v,A)}{\partial A} = \max\left\{\min\left\{v, \xi(\varepsilon^{-1}(A))\right\}, 0\right\} \tag{3}$$

The *social cost* induced by the production of abatement goods in quantity A is obtained, as usually, by choosing an optimal sharing of the production between the different plants which compose the eco-industry. But in our case, this process not only involves the cost structure of these firms but also their pollution. This cost is defined by:

$$SC(A) = \min_{a(\cdot)\geq 0} \int_0^1 \kappa\left(a(i)\right) di + v \cdot \int_0^1 \beta(i)a(i)di \quad \text{s.t.} \quad \int_0^1 a(i)di = A \tag{4}$$

If we denote by λ the Lagrangian multiplier associated to the constraint, the first order conditions of this convex minimization problem are given by

$$\begin{cases} \forall i \in [0,1], \;\; \kappa'(a(i)) + v\beta(i) - \lambda \geq 0 \text{ (with equality if } a_i > 0) \\ \text{and } \int_0^1 a(i)di = A \end{cases} \tag{5}$$

From these FOC it is quite obvious that a given firm i is active if $\lambda - v\beta(i) > 0$ and in this case its production level is of $(\kappa')^{-1}(\lambda - v\beta(i))$. Since the emissions of these firms are increasing with their index i, this also means that there exists a pivotal firm[3] $i_0 = \beta^{-1}\left(\frac{\lambda}{v}\right)$ which is the first for which it is optimal to stop the production. If we now keep in mind that the total level A of production of abatement goods is given, the index of this firm can be obtained by making sure that the total level of production of firms $i \leq i_0$ is equal to A.

In other words, even if these firms share the same cost function, the optimal sharing of the global production is not symmetric because they are heterogenous in their contribution to

[3] As long as i_0 is smaller then one.

pollution. We can therefore expect that not all firms will be selected at the efficient allocation. But to define this allocation, we also need some information about the social marginal cost of the production of an additional unit of abatement goods. This quantity which is given by $\frac{\partial SC(v,A)}{\partial A}$ is rather easy to construct. Since the constraint to the problem is given by $\int_0^1 a(i)di = A$, the envelop theorem tells us immediately that the social marginal cost is equal to the Lagrangian multiplier associated to this program. More precisely, we can say:

Lemma 2 *If $A = 0$ then $a(i) = 0$ for almost all i and $\lambda(A) \in (-\infty, v.\beta_{\min}]$ and for $A > 0$:*

(i) The productions of abatement goods are given by:

$$a^*(i, A, v) = \begin{cases} (\kappa')^{-1}(\lambda(A,v) - v\beta(i)) & \text{if } i \leq \beta^{-1}\left(\frac{\lambda}{v}\right) \\ 0 & \text{else} \end{cases} \tag{6}$$

(ii) $\lambda(A, v)$ is implicitly defined by:

$$\int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right),} (\kappa')^{-1}(\lambda(A,v) - v\beta(i))\, di = A \tag{7}$$

(iii) The marginal social cost is given by:

$$\frac{\partial SC(v,A)}{\partial A} = \lambda(A,v) \tag{8}$$

4 Efficient outcome and marginal damage

With regard to our previous results, let us first identify the optimal provision of abatement goods. This level results of a trade-off between the marginal social benefit and the marginal social cost. It is given by:

$$A^{opt}(v) = \arg\max_{A\geq 0} SB(A,v) - SC(A,v) \tag{9}$$

A quick inspection of this program shows that the optimal production of abatement goods is always smaller than the one which leads to the full abatement of the pollution induced by a pure competitive behavior of the agent i.e. $A^{opt}(v) < \varepsilon^{-1}(Q_{\max})$. In fact, in the opposite case, the marginal benefit of producing abatement good is typically zero (see (ii) of lemma 1) while the marginal cost of producing this good remain strictly positive.

In the same vein, we can also observe that the optimal provision of abatement goods is always positive, i.e. $A^{opt}(v) > 0$. If not, the marginal cost of abatement is, by lemma 2, smaller than $v\beta_{\min}$ and $\beta_{\min} < 1$ since we have assumed that at least some firms of the eco-industry reduce pollution in a larger extent than their own emission, while, the marginal benefit of an addition unit of abatement is of v unless there is no production (which is excluded due to the behavior of the demand function).

According to these observations, we can claim that the FOC associated to program 9 is given by:

$$\frac{\partial SB(A,v)}{\partial A} - \frac{\partial SC(A,v)}{\partial A} = \min\left\{v, \xi\left(\varepsilon^{-1}(A)\right)\right\} - \lambda(A,v) = 0 \tag{10}$$

This condition clearly suggests that two kinds of efficient outcome occur depending on the level of the marginal damage. The first equilibrium is a rather classical one: the dirty firm partially abates pollution and the marginal damage of pollution is equal both to the marginal benefit and the marginal cost of abating pollution. The second equilibrium occurs if the pollution of the dirty industry is totally reduced but some pollution persists. In this case the marginal benefit remains equal to the marginal cost, but their size is lower that the marginal damage induced by pollution.

The intuition suggests that there is a threshold of the marginal damage that makes us move from one case to the other. To get this intuition, let us start with a situation in which there is partial abatement in the dirty industry or, more formally, in which $\xi\left(\varepsilon^{-1}(A)\right) > v = \lambda(A,v)$. In this case we can use equation (7) to compute the optimal provision $A^{opt}(v)$ of abatement good by simply replacing $\lambda(A,v)$ by v. This quantity is given by:

$$A^{opt}(v) = \int_0^{\beta^{-1}(1)} \left(\kappa'\right)^{-1}\left(v \times (1-\beta(i))\right) di$$

and is obviously increasing with the marginal damage. But this case only hold seeing that $\xi\left(\varepsilon^{-1}(A^{opt}(v))\right) > v$, and since ξ is a decreasing function, the case of partial abatement cannot occur for sufficiently high marginal damage levels. This rather intuitive argument induces the following lemma:

Lemma 3 *There exists a unique threshold $\bar{v}$ given by:*

$$\varepsilon\left(\xi^{-1}(\bar{v})\right) - \int_0^{\beta^{-1}(1)} \left(\kappa'\right)^{-1}\left(\bar{v} \times (1-\beta(i))\right) di = 0 \tag{11}$$

with the property that there is only partial pollution reduction if $v < \bar{v}$ and full abatement of the pollution of the dirty industry else. The pollution of the eco-industry nevertheless remains in the last case.

If the marginal damage of pollution is lower than $\bar{v}$, we are in a rather standard case with respect to the literature. There is, at the optimal allocation, partial abatement of the pollution of the dirty industry and there is some additional pollution due to eco-industry. This additional pollution however contributes to a selection of firms among eco-industry which should produce. Only the firms which have a positive net contribution to pollution abatement (i.e. $(1-\beta(i))>0$) produces. Moreover, since one unit of abatement good removes on unit of pollution, we observe, that, at the efficient allocation, the marginal cost and benefit from the aggregate level of abatement are both equal to the marginal damage of pollution. Finally we have the usual arbitrage between the reduction of the final production and the increase in pollution abatement which depends on the level of the marginal damage since these quantities are negatively correlated when v changes. More precisely, we can say that:

Proposition 1 *If the marginal damage is not to high $v<\bar{v}$, i.e. only partial abatement of the pollution of the dirty firm occurs, the efficient allocation shares usual properties :*
(i) the marginal benefit as well as the marginal cost of pollution abatement are both equal to the marginal damage of pollution, i.e. $\frac{\partial SB(A^{opt},v)}{\partial A}=\frac{\partial SC(A^{opt},v)}{\partial A}=v$,
(ii) any firm of the eco-industry which efficiently reduces pollution, i.e. $\beta(i)\leq 1$, *contributes to the abatement but albeit in a different amount due to their own emission, i.e.* $\forall i\leq\beta^{-1}(1)$, $a^{opt}(i)=(\kappa')^{-1}(v\times(1-\beta(i)))$ *and this quantity is decreasing with* $\beta(i)$,
(iii) the optimal level of production $Q^{opt}=\xi^{-1}(v)$ *is decreasing with the marginal damage of pollution while the total production of abatement good*

$$A^{opt}(v)=\int_0^{\beta^{-1}(1)}\left(\kappa'\right)^{-1}(v\times(1-\beta(i)))\,di$$

as well as the individual one is increasing with the level of the marginal damage.

The second case in which the marginal damage is higher then the threshold $\bar{v}$ is less usual. Its interpretation is related to the assumption that the activity of the eco-industry generates a pollution that cannot be reduced. In other words there exists an upper bound on the global pollution that can be cut down and that is given by the emissions of the dirty industry. So if the marginal damage is high enough, it may happen that the optimal abatement choice hits this upper bound and leads to a situation in which all the pollution of the dirty industry is removed. As pollution is very harmful this may perhaps not be enough to improve environment. As the remaining pollution coming from eco-industry is however irreducible, it becomes optimal to reallocate the production of abatement goods toward the less polluting firms of the eco-industry and to slow down the production of abatement goods. So it is not really surprising that (i) the number of active firms decreases with the marginal damage contrary to the previous case in which all efficient firms produce (ii) the level of final production is now positively correlated with the level of abatement simply because the maximal abatement level is reached and are both decreasing with the level of marginal damage

What is perhaps more surprising is that, at this efficient allocation, the marginal benefit and the marginal cost of pollution abatement must be lower than the marginal damage of pollution. In order to understand this property, let us fix a high marginal damage of pollution. This provides strong incentives to abate pollution and since the eco-industry also pollutes it could be efficient to reduce all the pollution of the dirty industry. So let us start with a level of production of the eco-industry corresponding to the total abatement and for which the marginal social benefit of pollution reduction is equal to the marginal damage. Since this one is large enough ($v > \bar{v}$), it may happen that the marginal cost associated to this level of abatement is lower than the marginal damage of pollution. This provides some incentives to produce additional abatement goods. But we are typically in a situation in which all the pollution of the dirty industry is exhausted. So any use of an additional unit of abatement good requires an increase of the production of the final good, hence of the emissions. This of course reduces the marginal benefit of this additional unit of abatement good, especially if full abatement of the pollution of the dirty industry should be achieved. Since on the same time the marginal cost of producing abatement goods increases, it is no more surprising that we end up in a situation in which the marginal benefit and cost of abatement are lower then the marginal damage. More precisely, we observe that:

Proposition 2 *If the marginal damage is high $v > \bar{v}$, all the pollution of the dirty firm is reduced. But the efficient allocation has less usual properties:*
(i) the marginal benefit remains equal to the marginal cost of abatement but this quantity is now smaller than the marginal damage, i.e.:

$$\frac{\partial SB(A^{opt}, v)}{\partial A} = \frac{\partial SC(A^{opt}, v)}{\partial A} = \lambda^{opt}(v) < v$$

(ii) not all efficiently firm to reduce pollution are active at the optimal allocation, i.e. i is active if $\beta(i) < \frac{\lambda^{opt\prime} v)}{v} < 1$. Their contribution to the production of abatement good, $a^{opt}(i) = (\kappa')^{-1}\left(\lambda^{opt}(v) - v\beta(i)\right)$, however remains decreasing with their emission rate. Moreover the number of active firm decreases with the marginal damage, i.e. $i^{opt}(v) = \beta^{-1}\left(\frac{\lambda^{opt\prime} v)}{v}\right)$ verifies $\frac{di^{opt}(v)}{dv} < 0$,
(iii) The optimal level of production $Q^{opt} = \varepsilon^{-1}(A^{opt})$ is become positively correlated with the optimal level of abatement and the total production of abatement goods is now decreasing with the marginal damage since the pollution of the eco-industry can only be reduced by a cut in the production of these goods, i.e. $\frac{dQ^{opt}(v)}{dv} \leq 0$ and $\frac{dA^{opt}(v)}{dv} \leq 0$.

5 The policy issues

At that point we have observed that there are two different kinds of efficient allocation depending on the level of the marginal damage. These allocations even have rather different properties. It therefore becomes important to verify that the usual instruments, like pigovian taxation or permit markets, are able to implement the efficient allocation in a competitive setting. This last point will be verified in two steps. We first assume that there is some price signal p_e which represents the price of the emission and compute the competitive allocation for each level p_e. This gives us the opportunity to show, in a second step, that these instruments implement the efficient allocation in both cases.

So let us begin to study the competitive behaviors and let us start with the dirty firm. If there is some price signal p_e associated to the emission, this one chooses its production supply and its demand for abatement good by solving:

$$\max_{Q\geq 0}\left\{pQ - C(Q) - \underbrace{\min_{A\geq 0}\left\{p_a A + p_e \max\left\{\varepsilon(Q) - A, 0\right\}\right\}}_{=C_A(p_a,p_e,Q)}\right\}$$

A quick inspection of this cost minimization part of this program tells us that our objective function is linear in A on $[0, \varepsilon(Q)]$. This typically implies that the optimal conditional demand for abatement goods will be either 0 or $\varepsilon(Q)$ if respectively $p_a > p_e$ or $p_a < p_e$ and any quantity within $[0, \varepsilon(Q)]$ if $p_a = p_e$. From that point of view the cost of abatement is given by $C_A(p_a, p_e, Q) = \min\{p_a, p_e\}\,\varepsilon(Q)$ and we can say that the FOC condition which characterizes the product supply is given by:

$$p - C'(Q) - \min\{p_a, p_e\}\,\varepsilon'(Q) \leq 0 \text{ (with equality if } Q > 0)$$

Since we know that the inverse demand is given by $P(Q)$, we can say that the quantity which cleans the commodity market is given by:

$$Q(p_a, p_e) = \xi^{-1}\left(\min\{p_a, p_e\}\right)$$

while the demand for abatement goods is:

$$A^d(p_a, p_e) = \begin{cases} 0 \text{ if } p_a > p_e \\ \left[0, \varepsilon\left(\xi^{-1}\left(\min\{p_a, p_e\}\right)\right)\right] \text{ if } p_a = p_e \\ \varepsilon\left(\xi^{-1}\left(\min\{p_a, p_e\}\right)\right) \text{ if } p_a < p_e \end{cases}$$

Let us now move to the study of the supply of abatement good, Each firm $i \in [0,1]$ of the eco-industry maximizes its profit:

$$\max_{a(i)\geq 0} \{p_a a(i) - \kappa(a(i)) - p_e \beta(i) a(i)\}$$

whose first order condition is given by:

$$p_a - \kappa'(a(i)) - p_e\beta(i) \leq 0 \text{ (with equality if } a(i) > 0)$$

We deduce that $a(i) = (\kappa')^{-1}(p_a - p_e\beta(i))$ for $i \leq \beta^{-1}\left(\frac{p_a}{p_e}\right)$ while for $i > \beta^{-1}\left(\frac{p_a}{p_e}\right)$, $a(i) = 0$ and that aggregated supply of abatement goods is:

$$A^s(p_a, p_e) = \int_0^{\beta^{-1}\left(\frac{p_a}{p_e}\right)} (\kappa')^{-1}(p_a - p_e\beta(i))\, di$$

It remains to study the equilibrium of the abatement good market. If we denote by $z(p_a, p_e) = A^d(p_a, p_e) - A^s(p_a, p_e)$ the excess demand correspondence, we note that:

- if $p_a > p_e$ there is no equilibrium since there is an excess supply (i.e. $z(p_a, p_e) < 0$)
- the price $p_a = p_e$ is an equilibrium price if and only if the maximal demand is larger than the supply i.e.
$$\varepsilon\left(\xi^{-1}(p_e)\right) \geq \int_0^{\beta^{-1}(1)} (\kappa')^{-1}(p_e(1 - \beta(i)))\, di \tag{12}$$
- If the previous condition is not met the equilibrium price is given by:
$$\varepsilon\left(\xi^{-1}(p_a)\right) = \int_0^{\beta^{-1}\left(\frac{p_a}{p_e}\right)} (\kappa')^{-1}(p_a - p_e\beta(i))\, di \tag{13}$$

We even observe that condition (12) is, at least formally, the same as the equation which define our threshold $\bar{v}$ (see equation (11)). We can therefore say that:

Lemma 4 *If pollution as an implicit price given by p_e, there are two kinds of competitive equilibria of the product and abatement good if p_e is either larger or lower than $\bar{p}_e = \bar{v}$. In fact:*
(i) If this implicit price $p_e < \bar{p}_e$, the aggregated market equilibrium production levels are given by $Q^c(p_e) = \xi^{-1}(p_e)$ and $A^c(p_e) = \int_0^{\beta^{-1}(1)} (\kappa')^{-1}(p_e(1 - \beta(i)))\, di$ and are traded, respectively, at prices $p^c = P\left(\xi^{-1}(p_e)\right)$ and $p_a^c(p_e) = p_e$. Moreover all efficient firms of the eco-industry are active (i.e. all $i \leq \beta^{-1}(i)$) and their individual productions are given by $a^c(i, p_e) = (\kappa')^{-1}(p_e(1 - \beta(i)))$.

(ii) If $p_e \geq \bar{p}_e$, the price of the abatement goods $p_a^c(p_e) \leq p_e$ is the unique solution to equation (13) and is lower than the the implicit price for emissions). The quantities which are traded on the commodity market are respectively given by $Q^c(p_e) = \xi^{-1}(p_a^c(p_e))$ and $A^c(p_e) = \int_0^{\beta^{-1}\left(\frac{p_a}{p_e}\right)} (\kappa')^{-1}(p_a(p_e) - p_e\beta(i))\,di$. The number of active firms in the eco-industry is given by $i^c(p_e) = \beta^{-1}\left(\frac{p_a^c(p_e)}{p_e}\right)$ each of them producing $a^c(i, p_e) = (\kappa')^{-1}(p_a^c(p_e) - p_e\beta(i))$.

Proposition 3 *From the previous Lemma and proposition 1, we can assert that a pigovian tax $t = v$, equal to the marginal damage decentralizes the efficient allocation in our competitive economy. Moreover if the marginal damage is high ($v > \bar{v}$) this pigovian tax is able to select firms of the eco-industry which should be active and the equilibrium price of these goods is lower than the pigovian tax.*

To be continued...

6 Concluding remarks

The aim of this article was to investigate in which measure polluting eco-industry could challenge usual results in economic literature. In order to do that we have considered a vertical structure composed of a polluting downstream firm and of an upstream eco-industry. We assumed that eco-industry firms are asymmetric because they are more or less polluting and that they cannot reduce their pollution level. Under these assumptions, we obtained two kinds of equilibria. The first equilibrium generalized well-known results in economic literature to polluting eco-industry. Only efficient eco-industry firms produce and the abatement level increases with the damage whereas the production of the final good decreases. The optimal level of abatement is such that the social benefit and the social cost are equal to the marginal damage. This equilibrium occurs if the damage is not to high. Our results are very different when the damage is high. In this case the dirty firm has to reduce all their emissions. As the remaining pollution coming from eco-industry is very harmful for environment the regulator seeks to reduce it. To do that in an efficient way not all efficient eco-industry firms produce and the level of production is modified among eco-industry firms compared to the first equilibrium. We found that the more the damage is the less the abatement level is and the number of producing eco-industry firms decreases. We finally show that a pigovian tax decentralizes both equilibria in a competitive economy.

Our results suggests that polluting eco-industry is not a problem for the regulator because the competitive equilibrium selects the good firms to produce provided that the regulator sets the good level of the pigovian tax. However this optimistic conclusion comes from the very crucial assumption of perfect information that we implicitly made in our model. In a real world the regulator is not able to well define this tax and our results may not hold. Moreover the main feature of the eco-industry is the fact that it is highly concentrated. In this latter case we can wonder if a pigovian tax would really decentralized the optimum. Further research is needed to investigate this question.

APPENDIX

A Proof of Lemma 1

Step 0: Some notations

(i) $Q_{\max} = \arg\max_{Q\geq 0}\left(\left(\int_0^Q P(q)dq\right) - C(Q)\right)$ is given by $P(Q_{\max}) - C'(Q_{\max}) = 0$. This quantity exists and is unique since $(P(Q) - C'(Q))$ is, under our assumptions, a continuous and decreasing function with the property that $\lim_{Q\to 0}(P(Q) - C'(Q)) = +\infty$ and $\lim_{Q\to+\infty}(P(Q) - C'(Q)) = \lim_{Q\to+\infty}(-C'(Q)) < 0$

(ii) $\forall Q \in [0, Q_{\max}]$, $\xi(Q) := \frac{P(Q)-C'(Q)}{\varepsilon'(Q)}$ is invertible and $\xi^{-1} : \mathbb{R}_+ \to [0, Q_{\max}]$. Under our assumptions $\forall Q \in [0, Q_{\max}]$,

$$\xi'(Q) = \frac{(P'(Q) - C\text{"}(Q))\,\varepsilon'(Q) - \varepsilon\text{"}(Q)\,(P(Q) - C'(Q))}{(\varepsilon'(Q))^2} < 0$$

and $\lim_{Q\to 0}\xi(Q) = +\infty$ and $\xi(Q_{\max}) = 0$.

Step 1: The existence of a unique solution $Q(v, A)$ to program (1)

This result follows from the fact that we maximize (i) a strictly concave function on (ii) a domain which can be reduced to the compact convex set $[0, Q_{\max}]$. Let us check these two points:

(i) Let us first observe that $\left(\left(\int_0^Q P(q)dq\right) - C(Q)\right)$ is a strictly concave function since its second derivative is given by $(P'(Q) - C\text{"}(Q)) < 0$. Now, remark that $(\varepsilon(Q) - A)$ is convex in Q while $v\max\{x, 0\}$ is convex and increasing (for $v > 0$), hence their combination $v \times \max\{(\varepsilon(Q) - A), 0\}$ is convex. We therefore conclude that:

$$\phi_1(Q; A, v) = \left(\left(\int_0^Q P(q)dq\right) - C(Q)\right) - v\max\{(\varepsilon(Q) - A), 0\} \tag{14}$$

is strictly concave.

(ii) By (i) of step 0, and since $v\max\{(\varepsilon(Q) - A), 0\}$ is non decreasing, $\phi_1(Q; A, v)$ decreases after $Q_{\max}$. We can therefore reduce the maximization domain to $[0, Q_{\max}]$.

Step 2: The characterization of the solution $Q(v, A)$

Even if this problem is non-smooth, we can always define the subdifferential (see Rockafellar XXX part V) of $\phi_1(Q; A, v)$ (see (14)) with respect to Q. This quantity is given by:

$$\partial_Q \phi_1 = \begin{cases} P(Q) - C'(Q) \text{ if } Q < \varepsilon^{-1}(A) \\ \left[P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v\varepsilon'\left(\varepsilon^{-1}(A)\right), P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A))\right] \text{ if } Q = \varepsilon^{-1}(A) \\ P(Q) - C'(Q) - v\varepsilon'(Q) \text{ if } Q \geq \varepsilon^{-1}(A) \end{cases}$$

Since a maximum is reached if and only if $0 \in \partial_Q \phi_1$, this one is given by:

$$Q(v, A) = \begin{cases} Q_{\max} \text{ if } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) < 0 \\ \varepsilon^{-1}(A) \text{ if } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) \geq 0 \text{ and } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v\varepsilon'\left(\varepsilon^{-1}(A)\right) \leq 0 \\ \xi^{-1}(v) \text{ if } P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v\varepsilon'\left(\varepsilon^{-1}(A)\right) > 0 \end{cases}$$

Now let us notice that:

- $\left[P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) - v\varepsilon'\left(\varepsilon^{-1}(A)\right) \lesseqgtr 0\right] \Leftrightarrow \left[\varepsilon^{-1}(A) \gtreqless \xi^{-1}(v)\right]$ (by (i) of step 0)
- $\left[P(\varepsilon^{-1}(A)) - C'(\varepsilon^{-1}(A)) \lesseqgtr 0\right] \Leftrightarrow \left[\varepsilon^{-1}(A) \gtreqless Q_{\max}\right]$ (by (ii) of step 0)

Thus, we can say that:

$$Q(v, A) = \begin{cases} \max\left\{\varepsilon^{-1}(A), \xi^{-1}(v)\right\} \text{ if } A \leq \varepsilon(Q_{\max}) \\ Q_{\max} \text{ if } A > \varepsilon(Q_{\max}) \end{cases}$$

Step 3: The computation of $\frac{\partial SB(A,v)}{\partial A}$

If we replace Q by $Q(v, A)$ in $\phi_1(Q; A, v)$ (see (14)) and remember, by step 0, that $\forall v > 0$, $\xi^{-1}(v) < Q_{\max}$, we obtain:

$$SB(A, v) = \begin{cases} \left(\int_0^{\xi^{-1}(v)} P(q)dq\right) - C(\xi^{-1}(v)) - v\left(\varepsilon\left(\xi^{-1}(v)\right) - A\right) \text{ if } A < \varepsilon\left(\xi^{-1}(v)\right) \\ \left(\int_0^{\varepsilon^{-1}(A)} P(q)dq\right) - C(\varepsilon^{-1}(A)) \text{ if } A \in \left[\varepsilon\left(\xi^{-1}(v)\right), \varepsilon(Q_{\max})\right] \\ \left(\int_0^{Q_{\max}} P(q)dq\right) - C(Q_{\max}) \text{ if } A > \varepsilon(Q_{\max}) \end{cases}$$

Moreover if we differentiate this function piecewise with respect to A, we observe that:

$$\frac{\partial SB(A, v)}{\partial A} = \begin{cases} v \text{ if } A < \varepsilon\left(\xi^{-1}(v)\right) \\ \frac{P\left(\varepsilon^{-1}(A)\right) - C(\varepsilon^{-1}(A))}{\varepsilon'\left(\varepsilon^{-1}(A)\right)} \text{ if } A \in \left(\varepsilon\left(\xi^{-1}(v)\right), \varepsilon(Q_{\max})\right) \\ 0 \text{ if } A > \varepsilon(Q_{\max}) \end{cases}$$

is a continuous function (remember step 0) which can be summarized by $\frac{\partial SB(A,v)}{\partial A} = \max\left\{\min\left\{v, \xi(\varepsilon^{-1}(A))\right\}, 0\right\}$.

B Proof of Lemma 2

Step 1: The solution to program (4)

Let us remember that the FOCs of program (4) are given by:

$$\forall i \in [0, 1], \ \kappa'(a(i)) + v\beta(i) - \lambda \geq 0 \ (= \text{ if } a_i > 0) \tag{15a}$$

$$\int_0^1 a(i)di = A \tag{15b}$$

It is a matter of fact to observe that if $A = 0$, almost all $a(i) = 0$, and equation (15a) requires that $\lambda \leq v\beta_{\min}$ since $\kappa'(0) = 0$ and $\beta(i)$ increasing. So let us us concentrate on the situations in which $A > 0$ and $\lambda > v\beta_{\min}$.

From equation (15a), we observe (i) that for $\lambda < v\beta_{\max}$, only the firms $i \in \left[0, \beta^{-1}\left(\frac{\lambda}{v}\right)\right)$ produces while, for $\lambda \geq v\beta_{\max}$, all firms are active and (ii) that their individual production is given by $(\kappa')^{-1}(\lambda - v\beta(i))$. It remains to use equation (15b) in order to get λ. This one is implicitly defined by:

$$\phi_2(\lambda, A, v) = A - \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} (\kappa')^{-1}(\lambda - v\beta(i))\, di = 0 \tag{16}$$

Let us now observe that $\forall (A, v) \gg 0$, (i) $\lim_{\lambda \to v\beta_{\min}} \phi_2 = A > 0$, (ii) $\lim_{\lambda \to +\infty} \phi_2 = -\infty$ and (iii) $\partial_\lambda \phi_2 = -\int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} \left(\kappa"\left((\kappa')^{-1}(\lambda - v\beta(i))\right)\right)^{-1} di < 0$. It therefore exists a unique $\lambda(A, v)$ which solves equation (16) for each (A, v) and the optimal solution to program (4) is given by:

$$\forall i \in [0,1], \quad a^*(i, A, v) = \begin{cases} (\kappa')^{-1}(\lambda(A, v) - v\beta(i)) & \text{if } i \leq \beta^{-1}\left(\frac{\lambda}{v}\right) \\ 0 \text{ else} \end{cases} \tag{17}$$

<u>*Step 2:*</u> The computation of $\frac{\partial SC(A,v)}{\partial A}$

$$\begin{aligned} \frac{\partial SC(A, v)}{\partial A} &= \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} \left(\kappa'\left(a^*(i, A, v)\right) + v\beta(i)\right) \frac{\partial a^*(i, A, v)}{\partial A} di \\ &= \lambda(v, A) \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} \frac{\partial a^*(i, A, v)}{\partial A} di \qquad \text{by equation (15a)} \\ &= \lambda(v, A) \qquad \text{since } \int_0^1 a(i)di = A \end{aligned}$$

<u>*Step 3:*</u> Additional results for latter use.

Let us observe that:

- $\frac{\partial \phi_2(\lambda, A, v)}{\partial \lambda} = -\int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} \left(\kappa"\left((\kappa')^{-1}(\lambda - v\beta(i))\right)\right)^{-1} di < 0$ (remember that $\kappa'(0) = 0$).
- $\frac{\partial \phi_2(\lambda, A, v)}{\partial A} = 1 > 0$
- $\frac{\partial \phi_2(\lambda, A, v)}{\partial v} = \int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)} \beta(i) \left(\kappa"\left((\kappa')^{-1}(\lambda - v\beta(i))\right)\right)^{-1} di > 0$ (remember that $\kappa'(0) = 0$).

It follows that :

$$\frac{\partial \lambda(v, A)}{\partial v} = -\frac{\partial_v \phi_2(\lambda, A, v)}{\partial_\lambda \phi_2(\lambda, A, v)} > 0 \text{ and } \frac{\partial \lambda(v, A)}{\partial A} = -\frac{1}{\partial_\lambda \phi_2(\lambda, A, v)} > 0 \tag{18}$$

C Proof of Lemma 3

<u>*Step 1:*</u> There exists a unique threshold $\bar{v}$

Let us verify that:

$$\phi_3(v) = \varepsilon\left(\xi^{-1}(v)\right) - \int_0^{\beta^{-1}(1)} (\kappa')^{-1}(v \times (1 - \beta(i)))\, di = 0 \tag{19}$$

admits a unique solution $\bar{v}$. This result as rather immediate since:

- $\phi_3(v)$ is a continuous and decreasing. Indeed :

$$\phi_3'(v) = \frac{\varepsilon'\left(\xi^{-1}(v)\right)}{\xi'\left(\xi^{-1}(v)\right)} - \int_0^{\beta^{-1}(1)} \left(\frac{(1 - \beta(i))}{\kappa"\left((\kappa')^{-1}(v \times (1 - \beta(i)))\right)}\right) di$$

  Moreover we know that (i), by assumption, $\varepsilon'(Q) > 0$, (ii), by step 0 of the proof of lemma 1), $\xi'(Q) < 0$, and (iii), by the range of the integral, $1 - \beta(i) \geq 0$. Hence $\phi_5'(v) < 0$.

- $\lim_{v\to 0}\phi_3(v) = \varepsilon\left(Q_{\max}\right) > 0$. More precisely $\lim_{v\to 0}\phi_3(v) = \lim_{v\to 0}\varepsilon\left(\xi^{-1}(v)\right)$ since $\kappa'(0) = 0$. Moreover, using again step 0, we know that $\xi^{-1}(0) = Q_{\max} > 0$. The result follows.
- $\lim_{v\to +\infty}\phi_3(v) < 0$. By step 0 of the proof of lemma 1, we know that $\lim_{Q\to +\infty}\xi^{-1}(Q) = 0$. It remains to remember that $\varepsilon(0) = 0$ in order to conclude that :

$$\lim_{v\to +\infty}\phi_3(v) = -\lim_{v\to +\infty}\int_0^{\beta^{-1}(1)}\left(\kappa'\right)^{-1}\left(v\times(1-\beta(i))\right)di < 0$$

<u>*Step 2:*</u> If $v < \bar{v}$ then the optimal abatement provision only partially reduces the emissions of the dirty firm

By contraposition, assume that the efficient solution requires full pollution abatement of the dirty industry. At this optimal allocation λ^{opt} and A^{opt} verifies:

$$\lambda^{opt} = \xi\left(\varepsilon^{-1}(A^{opt}\right) \leq v \text{ and } \underbrace{A^{opt} - \int_0^{\beta^{-1}\left(\frac{\lambda^{opt}}{v}\right)}\left(\kappa'\right)^{-1}\left(\lambda^{opt} - v\beta(i)\right)di}_{=\phi_2(\lambda^{opt},A^{opt},v)\text{ (see equation(16))}} = 0$$

From the first equation we get that $\lambda^{opt} \leq v$ and $A^{opt} \geq \varepsilon\left(\xi^{-1}(v)\right)$. It follows, from step 3 of the proof of lemma 2, that:

$$0 = \phi_2(\lambda^{opt}, A^{opt}, v) \geq \phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v) = \phi_3(v)$$

Since $\phi_3'(v) < 0$, this implies that $v \geq \bar{v}$.

<u>*Step 3:*</u> If $v \geq \bar{v}$ then the optimal abatement provision requires full pollution abatement of the dirty industry

By contraposition, assume now that the efficient solution requires partial abatement. At this optimal allocation λ^{opt} and A^{opt} verifies:

$$\lambda^{opt} = v < \xi\left(\varepsilon^{-1}(A^{opt}\right) \text{ and } A^{opt} = \int_0^{\beta^{-1}(1)}\left(\kappa'\right)^{-1}\left((1-\beta(i))\,v\right)di$$

Since now $A^{opt} < \varepsilon\left(\xi^{-1}(v)\right)$, we can say by using the second condition and step 3 of the proof of lemma 2 that:

$$0 = \phi_2(v, A^{opt}, v) < \phi_2(v, \varepsilon\left(\xi^{-1}(v)\right), v) = \phi_3(v)$$

which implies that $v < \bar{v}$.

D Proof of proposition 1

<u>*Point (i):*</u> This result follows from the definition of the case

<u>*Point (ii):*</u> This follows from the proof of step 1 of lemma 2 for $\lambda = v$

<u>*Point (iii):*</u> By step 0 of the proof of lemma 1, we can say that $\frac{dQ^{opt}}{dv} = \left(\xi'\left(\xi^{-1}(v)\right)\right)^{-1} < 0$ and by computation

$$\frac{dQ^{opt}}{dv} = \int_0^{\beta^{-1}(1)}(1-\beta(i))\,\kappa\text{"}\left(\left(\kappa'\right)^{-1}\left(v\times(1-\beta(i))\right)\right)^{-1}di > 0$$

since $\forall i \in \left[0, \beta^{-1}(1)\right)$, $(1-\beta(i)) > 0$.

E Proof of proposition 2

<u>*Point (i):*</u> This result follows from the definition of the case

Point (ii): Since $\lambda^{opt}(v) < v$, the proof of the first part of the result directly follows from equation (17). It remains to verify that $\frac{di^{opt}(v)}{dv} = \frac{d\beta^{-1}\left(\frac{\lambda^{opt\prime}(v)}{v}\right)}{dv} < 0$. Since $\beta(i)$ is increasing let us compute:

$$\frac{d\left(\lambda\left(v, A^{opt}(v)\right)/v\right)}{dv} = \frac{1}{v}\left(\underbrace{\left(\frac{\partial\lambda\left(v,A\right)}{\partial A}\Big|_{A=A^{opt}(v)} \cdot \frac{dA^{opt}(v)}{dv}\right)}_{<0} + \underbrace{\left(\frac{\partial\lambda\left(v,A\right)}{\partial v}\Big|_{A=A^{opt}(v)} - \frac{\lambda\left(v, A^{opt}(v)\right)}{v}\right)}_{=A}\right)$$

By point (iii) of this proof and equation (18), we know that the first term of the previous equation is negative. Now let us observe, by equation (18), that the second term can be written as:

$$A = \left(-\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}\right)^{-1}\left(\frac{\partial\phi_2(\lambda, A, v)}{\partial v} + \frac{\lambda}{v}\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}di\right)$$

If we now replace the derivatives of ϕ_2 by their value (see step 3 of the proof of Lemma 2), we obtain:

$$A = \underbrace{\left(-\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}\right)^{-1}}_{>0}\left(\int_0^{\beta^{-1}\left(\frac{\lambda}{v}\right)}\underbrace{\left(\beta(i) - \frac{\lambda}{v}\right)}_{<0}\underbrace{\left(\kappa"\left(\left(\kappa'\right)^{-1}\left(\lambda - v\beta(i)\right)\right)\right)^{-1}}_{>0}di\right) < 0$$

Point (iii): Since $Q^{opt} = \varepsilon^{-1}(A^{opt})$ it is obvious that if $\frac{dA^{opt}(v)}{dv} \leq 0$ then $\frac{dQ^{opt}(v)}{dv} \leq 0$. So let us check that $\frac{dA^{opt}(v)}{dv} \leq 0$. To verify this point, let us remember that an optimal allocation is in this case defined by

$$\phi_2(\lambda^{opt}, A^{opt}, v) = 0 \text{ and } \lambda^{opt} = \xi\left(\varepsilon^{-1}(A^{opt})\right)$$

By differentiation, this implies that:

$$\left(\underbrace{\frac{\partial\phi_2(\lambda, A, v)}{\partial\lambda}}_{<0} \times \underbrace{\frac{\xi'\left(\varepsilon^{-1}(A)\right)}{\varepsilon'\left(\varepsilon^{-1}(A)\right)}}_{<0} + \underbrace{\frac{\partial\phi_2(\lambda, A, v)}{\partial A}}_{>0}\right)dA + \underbrace{\frac{\partial\phi_2(\lambda, A, v)}{\partial v}}_{>0}dv = 0$$

and the result follows from the proof of Lemma 1 (step 0) and Lemma 2 (step 3).

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