Spatial concessions with limited tenure[☆]

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Abstract

We examine theoretically a system of spatial property rights over a mobile renewable resource. The resource grows and is harvested in each area, but harvest in one patch imposes an externality on other patches through resource movement. This externality gives rise to over-extraction by non-cooperative patch owners. We propose a new institutional arrangement to internalize this externality. The instrument involves granting limited-duration tenure, with the possibility of renewal, to each of the N interconnected concessionaries. Renewal of concession i is predicated on concessionaire i having never driven the stock below a pre-defined target $\mathcal{S}_i$. We show that not only can this instrument improve upon the decentralized property right solution, but under general conditions it can replicate the socially optimal extraction in every patch and every time period, in perpetuity. The duration of tenure and the dispersal of the resource play pivotal roles in whether this instrument achieves socially optimal resource use over time.

1. Introduction

There is mounting scientific evidence that many natural resources previously thought of as *aspatial* are in fact mobile. Fish swim, water flows, animals migrate, and pollutants are carried through the air. This paper addressed the assignment of property rights for spatially-connected renewable resources. This issue has great contemporary policy appeal. We are amidst a privatization revolution for natural resources in which spatial property rights feature prominently. Forests, game, waterfowl, and water are prominent terrestrial examples of resources governed and extracted by private owners with some degree of spatial ownership over the resource. Even in the ocean, which has traditionally been viewed as a non-excludable, spatial property rights are emerging. For example, the Chilean coast contains over

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700 territorial user right fisheries (TURFs) where firms manage and extract fishery resources; more broadly the world's oceans consist of about 200 spatial exclusive property right assignments (the exclusive economic zones) which are traversed by migratory species such as tuna, sharks, and whales. These examples contain three common features: (1) A renewable resource (2) is spatially mobile and (3) spatial property rights holders extract the resource on their property thus imposing a spatial externality on one another. Improved scientific understanding of the interconnectedness of these resources combined with increasing policy interest in spatial property rights for managing them, has given rise to a large and quickly growing literature on the economic consequences of spatially-connected natural resource use.

The purpose of this paper is to propose and analyze a limited-tenure concession instrument that can be overlayed on a spatial property rights system to internalize spatial externalities. Concessions have been applied and analyzed extensively for forest management. Smith et al. (2003) discuss the long history of forest concessions in Indonesia, where illegal logging and corruption are common practice. Karsenty (2007) review concessions used in Central and West Africa (mainly related to rubber and ivory, rather than timber). The literature focuses on the form of concession design that will best allow government to earn rent from the private use of the public forest, and how concessionaires will respond to those constraints. A notable part of the related studies has generally focused on the choice of royalties (or fees) for harvesting rights (Gray, 2000; Hardner and Rice, 2000; Vincent, 1990; Giudice et al., 2012). Among others, Vincent (1990) highlights the failure of royalty systems because of undervaluation and analyzes how the inefficiency of this system affects forest management (with an application to Malaysia). Giudice et al. (2012) develop a timber rent model that estimates optimal fees and apply it to the forestry sector in Madre de Dios, Peru. They find that current fees could be increased to a maximum annual average of US$ 23,4 million over a 20-year period. While royalty structure and efficient rent capture are important dimensnions of concession design, our focus is instead on analyzing whether (and under what conditions) limited-duration tenure can be designed to correct for market failures.

A related literature focuses on illegal exploitation (Amacher et al., 2001; Barbier and Burgess, 2001; Boscolo and Vincent, 2000; Clarke et al., 1993). For instance, Barbier and Burgess (2001) analyze the effects of tenure insecurity on the migrants' decision to convert tropical frontier forestland to unsustainable agriculture, while, recently, Amacher et al. (2012) analyze how corruption impacts on the decision of a central planner to implement various types of concessions (royalties rates, land size, logging).

The fishery is perhaps the best example of a resource for which property rights will lead to spatial externalities. While this problem has been identified , the literature on fisheries concessions is scarce. An exception is Costello and Kaffine (2008) who explore the effects of limited-tenure property rights for a single aspatial resource. They find that limiting tenure weakens the incentive to steward one's own resource, but that fully efficient extraction may still be possible under this weakened right. But because they did not consider the possibility of resource mobility, and multiple spatially connected property rights, spatial externalities were absent, so they could not examine the effects of limited tenure on efficiency for the class of resources considered here.

In practice, concessions are increasingly used in spatially-connected resources, but are implemented with little or no guidance from economic theory. In 2000, a 10th TURF was granted to several cooperatives and each of them has a 20-year species concession in Mexico. The EU Commission has suggesting introducing Transferable Fishing Concessions (TFCs) under which the resources remain public goods (no property rights over marine resources), but user rights were granted to exploit the resources. Concessions can only be sold to fishermen who are registered. These TFCs apply to effort allocations, i.e. quota or effort limits. McCay et al. (2014) examine a case study from the Pacific coast of Mexico, where a community-oriented fisheries management has been implemented based on fish species concessions. For instance, each cooperative has exclusive access and user rights to certain species like abalone, lobster or turban snail.

Thus, the literature has generally focused on the design of concessions without considering the issue of the duration of concessions or the ability to internalize spatial externalities. Still, such a contract may be awarded annually or for longer or smaller periods. Boscolo and Vincent (2000) provide a numerical analysis of forest regulations with some considerations of "lengthening concession agreements" and of the possibility to renew concessions based on performance. They outline that longer concessions provide loggers with small incentives to adopt a low-damage logging technology, or to comply with minimum policy requirements. Furthermore, in the proposal of the EU Commission, the fisheries concessions would be granted for a limited time. At the end of the period, the Commission can allocate concessions again.

Discussions on the importance of the tenure length may be found in papers focusing on the use of concessions for water utilities or Highways. The underlying questions are related to the impact of tenure length on possible network investments (Moretto et al., 2005) and to maintenance-related issues occurring at the end of the concession period (GonÃ§alves and Gomes, 2012), while in our contribution the focus is on the effect of tenure length on the concessionaires' incentives to cooperate.

We will take as given that resources can be mobile, and this model will raise important questions about the degree to which privatization can achieve economic efficiency. Precisely owing to the spatial connections, each owner's extraction imposes an externality on the other owners. On the flip side, an owner's conservation efforts spill over to other owners; no owner is a residual claimant of his conservation behavior. Thus, spatial connectivity weakens property rights and suggests that each owner will extract the resource at too-rapid a rate. To make matters worse, if production functions or spillovers are spatially heterogeneous, then the socially-optimal level of extraction will differ across space. Kaffine and Costello (2011) observed this market failure and derived a private "unitization" instrument that could theoretically induce all owners to cooperate over spatial extraction; following this instrument induces first-best harvesting behavior among all owners. While this instrument is theoretically attractive, and helps to frame the challenge of coordinating spatial property owners, its application to real-world resource conflict is limited because: (1) It requires *all* property owners to be aware of the full spatial and economic dynamics of the entire system, (2) It requires the resource manager to devolve all management responsibility, in perpetuity, to the spatial property rights holders, (3) it requires the sharing of profit (not revenue), which may be costly to credibly measure, and (4) It requires all harvesters to contribute 100% of profits to a common pool, which is later redistributed based on property-specific characteristics. In this paper we propose a limited duration property right instrument that achieves economically-efficient spatial resource use and overcomes the limitations noted above.

We begin by developing a model of spatial economic behavior among a set of spatially-distinct resource patches. The instrument we propose involves assigning limited-duration tenure to each patch to a private agent. Under this set of spatial concessions with limited tenure, each agent then faces an interesting, and to our knowledge unexplored, set of incentives. At first glance this instrument would seem to exacerbate the problem. First, the spatial externality has not been internalized, so each concessionaire will have a tendency to overexploit the resource. Second, the limited-duration tenure induces each concessionaire to extract the resource more rapidly than is socially optimal so as to extract the rents prior to the terminal date of his tenure. Both incentives appear to work in the wrong direction, and they lead to excessive harvest rates in all patches, which surely is socially inefficient.

To counteract this tendency toward overharvest, we implement two refinements. First, we allow the regulator to announce for each patch a "minimum stock," below which the concessionaire must never harvest. Second, the regulator commits to renewing the concession of any concessionaire who adheres to the minimum stock requirement every period. Indeed, this is a stylized version of how concessions are implemented in practice (CITE). Taken together, we have a system of N heterogeneous interconnected resource patches, where each is harvested by a single concessionaire for a fixed duration, T. If the concessionaire maintains the stock above the (pre-announced) minimum level for the duration of his tenure, his tenure will be renewed for another T years. If not, he loses his concession and it is allocated to another user. This process occurs in every resource patch, and in perpetuity. The two policies introduced to counteract overextraction (the possibility of renewal, and the requirement to stay above a minimum stock level) introduce yet another set of incentives to our concessionaires.

Under this setup, a concessionaire must decide whether to comply with the minimum stock requirement (in which case he cannot extract as much profit under the current tenure block, but he will retain ownership over multiple tenure blocks) or to defect (and harvest in way that maximizes his profit over the current tenure block only). Naturally, because the patches are interconnected, the payoffs under either strategy will depend on the strategy adopted by the other $N-1$ concessionaires. Thus, this system represents a spatial-temporal game. Our purpose is to explore the spatial and dynamic properties of this game in detail and to examine the welfare implications over time.

Despite the complexity of this setup, we are able to derive explicit analytically tractable results. First, we derive the optimal defection strategy for any single concessionaire, and use it to derive a set of conditions under which cooperation can emerge as an equilibrium outcome and whether this leads to fully efficient resource use. We then focus in on the properties of the system that ensure cooperation (or conversely, ensure defection). Importantly, the regulator in this setup has only loose control over the actual harvest achieved in each patch. His only instruments are the minimum stock announced for each patch and the tenure length. There we find an interesting, and somewhat counterintuitive result. We show that longer (not shorter) tenure is more likely to lead to defection from the efficient harvest rate. This odd result seems to contradict the economic intuition that more secure property rights (here, the longer the duration of tenure) give rise to more efficient resource use. In our setting, a long tenure period implies that the regulator essentially loses the ability to manipulate a concessionaire's harvest incentives via the promise of tenure renewal. In the extreme (with infinite-duration tenure), the concessionaire has *no* incentive to abide by the minimum stock requirement. Through this logic, we can show that for sufficiently long (but still finite) tenure length, defection will always be an equilibrium strategy.

Overall, we find that while strong (i.e. perpetual) property rights will not lead to socially efficient resource use in spatially-connected systems, it will often be possible to design limited-tenure property rights that will. The basic intuition is to induce private owners to adhere to socially-optimal resource extraction rates by promising renewed tenure if and only if they adhere to these guidelines. Under certain circumstances, even this will not be a sufficient incentive because the gap between what a private owner can capture through rapid overexploitation, and what he can gain through a low extraction rate (even into perpetuity), is just too great. In what follows we also consider a number of possible extensions, including whether one owner's defection will induce other owners to defect and the application of trigger strategies to induce cooperation.

2. Model & strategies

We begin by introducing a spatial model of natural resource exploitation with spatially-connected property owners. We then zero in on the incentives for different levels of harvest. Ultimately these harvest strategies will form the foundation of our analysis.

2.1. The model

We follow the basic setup of Kaffine and Costello (2011) and Costello et al. (2014) where a natural resource stock (denoted by x) is distributed heterogeneously across a discrete spatial domain consisting of N patches. Patches may be heterogeneous in size, shape, economic, and environmental characteristics, and resource extraction can occur in each patch. The resource is mobile and can migrate around this system. In particular, denote by $D_{ij} \geq 0$ the fraction of the resource stock in patch i that migrates to patch j in a single time period. The resource may grow, and the growth conditions may be patch-specific denoted by the parameter α_j. This patch-specific parameter reflects resource growth and has many possible interpretations including intrinsic rate of growth, carrying capacity, and the sheer size of the patch. Assimilating all of this information, the equation of motion, in the absence of harvest, is given as follows:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(x_{jt}, \alpha_j). \tag{1}$$

Here $g(x_{jt}, \alpha_j)$ is the period-t production in patch j. We follow the literature and require that $\frac{\partial g(x,\alpha)}{\partial x} > 0$, $\frac{\partial g(x,\alpha)}{\partial \alpha} > 0$, $\frac{\partial^2 g(x,\alpha)}{\partial x^2} < 0$, and $\frac{\partial^2 g(x,\alpha)}{\partial x \partial \alpha} > 0$. We also assume that extinction is absorbing ($g(0; \alpha_j) = 0$) and that the growth rate is finite ($\frac{\partial g(x,\alpha)}{\partial x}|_{x=0} < \infty$).[1] All standard biological production functions are special cases of $g(x, \alpha)$. All resource stock that is produced in patch j then disperses across the spatial domain: some fraction stays within patch j (D_{jj}) and some flows to other

[1]We will omit the growth-related parameter in most sections, except briefly before Section 3.2.1 and in Section 3.3.1, where the effect of this parameter will be analyzed. Thus, we will use notations $g_i'(x)$ and $g_i''(x)$ instead of (respectively) $\frac{\partial g(x,\alpha_i)}{\partial x}$ and $\frac{\partial^2 g(x,\alpha_i)}{\partial x^2}$ in most parts of the paper.

patches, e.g. to patch i, D_{ji}. Indeed, some may flow out of the system entirely, so the dispersal fractions need not sum to one: $\sum_i D_{ji} \leq 1$.

Harvest in patch i, period t is given by h_{it} and we follow the literature by defining the *residual stock*[2] left for reproduction which is given by $e_{it} \equiv x_{it} - h_{it}$. This gives rise to the patch-i equation of motion as follows:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g_j(e_{jt}). \tag{2}$$

We assume that both price and marginal harvest cost are constant in a patch, though they can differ across patches. The resulting *net price* is given by p_i. Profit in patch i, time t is given by:

$$\Pi_{it} = p_i \left(x_{it} - e_{it} \right). \tag{3}$$

We will employ this framework to compare the outcome and welfare implications of three different property right systems: (1) a benevolent social planner who seeks to maximize aggregate social welfare, (2) a set of decentralized and non-cooperative property rights owners, and (3) the same set of property right owners who operate in a limited tenure system we propose here.

2.1.1. Social Planner's Problem

We begin with the social planner who must solve the complicated problem of choosing the optimal spatial and temporal pattern of harvest to maximize the net present value of profit across the entire domain. This is a special case of the problem solved by Costello and Polasky (2008). The planner's objective is:

$$\max_{\{e_{1t},\ldots,e_{Nt}\}} \sum_{i=1}^{N} \sum_{t=0}^{\infty} \delta^t p_i \left(x_{it} - e_{it} \right), \tag{4}$$

subject to the equation of motion 2. Somewhat surprisingly, this complicated spatial optimization problem has a tractable solution. In any patch i, the planner should achieve a residual stock level as follows:

$$p_i \geq \sum_{j=1}^{N} \delta D_{ij} p_j g_i' \left(e_{it}^* \right), \tag{5}$$

where the equality holds if and only if e_{it}^* is positive. Furthermore, these optimal residual stock levels (which will vary across patches) are time and state independent. This implies that each patch has a single optimal residual stock level that

[2] In the fisheries literature, *residual stock* has been coined *escapement*.

should be achieved every period into perpetuity, and when that policy is followed, the net welfare of the system will be maximized. The optimal policy is:

$$e_{it}^* = e_i^*. \tag{6}$$

2.1.2. Decentralized Perpetual Property Right Holders

The second regime is the case in which each patch is owned in perpetuity by a single owner who seeks to maximize the net economic value of harvest from his patch. We assume that the owner of patch i makes her own decisions about harvest in patch i, with complete information about the stock, growth characteristics, and economic conditions present throughout the system. In that case owner i solves:

$$\max_{\{e_{it}\}} \sum_{t=0}^{\infty} \delta^t p_i \left(x_{it} - e_{it} \right). \tag{7}$$

Naturally, because owner i's stock (x_{it+1}) depends on owner j's residual stock (e_{jt}), this induces a game across the N players. This problem was solved by Kaffine and Costello (2011), who found that owner i will always harvest down to a residual stock level that satisfies:

$$1 = \delta D_{ii} g_i'(\bar{e}_i) \tag{8}$$

It is a straightforward matter to show that $\bar{e}_i \leq e_i^*$, and thus that achieving social efficiency in a spatially connected system will require some kind of cooperative instrument.

2.1.3. Decentralized, Limited-Tenure Property Rights

Here we introduce the third and final regime, and the one on which we focus for the rest of the paper. This regime is similar to the perpetual property rights case above, except that the property rights are of limited duration. This is a common practice in the real world, and has received some recent attention in the economics literature. Under this regime, ownership over patch i is granted to a private concessionaire for a duration of T periods, to which we will refer as the "tenure block." Following the existing literature, we will allow for the possibility of renewal for a second tenure block, a third tenure block, and so forth. Indeed, it is the possibility of renewal that will ultimately incentivize the concessionaire to deviate from her privately-optimal harvest rate and (ideally) follow the socially-optimal one.

The instrument is designed as follows. At $t = 0$ the regulator offers to concessionaire i a contract, which consists of two parameters: (1) A "target stock," $\mathcal{S}_i$, below which the concessionaire must never harvest and (2) a tenure period, $\mathcal{T}_i$. The regulator imposes only a single rule on the concessionaire: At the end of the tenure block (i.e. at time $\mathcal{T}_i$), the concession will be renewed if and only if the target stock condition has been met every period. Because $e_{it} \leq x_{it}$, this rule implies that concession i will be renewed if and only if:

$$e_{it} \geq \mathcal{S}_i \qquad \forall t < \mathcal{T}_i \tag{9}$$

This setup retains a great deal of autonomy for the concessionaire - in every period she is free to choose any harvest level that suits her. The regulator's challenge is to determine a set of target stocks $\{\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_N\}$ and tenure lengths $\{\mathcal{T}_1, \mathcal{T}_2, ...\mathcal{T}_N\}$ that will incentivize all concessionaires to simultaneously, and in every period, deliver the socially optimal level of harvest (which will, in general, differ across space; see Equation 5.

We begin by defining a candidate set of instrument parameters, and then evaluate the manner in which each concessionaire would respond to that set of incentives. Our proposed instrument is as follows:

Definition 1. *The* Limited-Tenure Spatial Concession Instrument *is defined by* $\mathcal{S}_i = e_i^*$ *and* $\mathcal{T}_i = T$ $\forall i$, *where* T *is a fixed positive integer which we will derive below.*

If all N concessionaires choose to comply with the target stocks in every period of every tenure block, we refer to this as *cooperation.* If they do not exceed the target stock (so the *just* comply), then concessionaire i's present value payoff would equal:

$$\Pi_i^c = p_i \left[x_{i0} - e_i^* + \frac{\delta}{1-\delta} \left(x_i^* - e_i^* \right) \right]. \tag{10}$$

where x_{i0} is the (given) starting stock and $x_i^* = \sum_j D_{ji} f(e_j^*, \alpha_j)$.

Instead, if a particular owner i fails to meet the target stock requirement (i.e., in some period she harvests the stock below $\mathcal{S}_i$), then, while she will retain ownership for the remainder of her tenure block (and thus be able to choose any harvest over that period), she will certainly not have her tenure renewed. In that case, owner i's payoff will be zero every period after her current tenure block runs out. If agent i defects during tenure block k, her present value payoff is given by:

$$\Pi_i^d = \sum_{t=0}^{kT-1} \delta^t p_i \left(x_{it} - e_{it} \right). \tag{11}$$

where $e_{it} \geq e_i^*$ for all $t < (k-1)T - 1$ (because she achieved the target stock prior to the k^{th} tenure block.

Because defection is a possibility and because an owner's payoff depends on the actions of the others, we must specify what happens in the event of a defection. We assume that if concessionaire i defects, then the concession is granted to a new concessionaire (though with identical characteristics) in the subsequent tenure block. If all initial owners decide to defect and are not renewed at the end of the current tenure, then the game ends.[3]

2.2. Defection Strategies

While our main purpose is to establish the conditions (and instrument parameters) under which a limited-tenure spatial concession instrument can deliver the socially optimal resource use, we must begin by characterizing the defection strategies pursued by concessionaires. Because this could happen during any tenure block, we consider the case where defection occurs during an arbitrary tenure block, k. We assume that all concessionaires, except i, follow simple cooperative strategies (that is, they are unconditional cooperators) and we compute the optimal defection strategy of concessionaire i, as follows:

Proposition 1. *The optimal defection strategy of concessionaire i in tenure block k is given by:*

$$\bar{e}_{ikT-1} = 0$$

and, for any period $(k-1)T \leq t \leq kT-2$, we have $\bar{e}_{it} = \bar{e}_i$ where:

$$1 = \delta D_{ii} g_i'(\bar{e}_i).$$

Proposition 1 states that a concessionaire who decides she will defect *sometime* during tenure block k, will decide to: (1) choose the non-cooperative level of harvest (see Section 2.1.2) up until the final period of the tenure block and then completely mine the resource, leaving nothing for the subsequent concessionaire.[4] An important implication of Proposition 1 is that this defection strategy does not depend on k. The finding that the defection strategy is independent of the tenure block greatly simplifies the characterization of equilibrium strategies. We will make extensive use of the defection strategy in what follows. We next turn to the conditions that give rise to cooperation.

3. Conditions for Cooperation

Here we derive the conditions under which all N concessionaires willingly choose to cooperate in perpetuity. We will proceed in three steps. First, we derive the target stocks that must be announced $(\mathcal{S}_1, ..., \mathcal{S}_N)$ by the regulator who

[3]This turns out to be irrelevant because, as we late show, if everyone defects, the natural resource is driven extinct.

[4]Note that if only one concessionaire defects, the entire population will not be driven extinct because patch i can be repopulated via dispersal from patches with owners who cooperated.

wishes to replicate the socially optimal level of extraction in every patch at every time. But whether all concessionaires willingly adhere to their targets will depend on patch-level characteristics such as price, cost, and dispersal. Thus, our second step is to derive necessary and sufficient conditions for cooperation to be sustained, as a function of these patch-level parameters. Finally, we will assess the influence of the tenure duration T on the emergence of cooperation, and provide comparative statics results.

3.1. The emergence of cooperation

Can the Limited Tenure Spatial Concessions Instrument *ever* lead to cooperation? Clearly, if the announced target stocks are sufficiently low (e.g. if $\mathcal{S}_i = 0 \; \forall i$), then "cooperation" is easily achieved.[5] But this is of little use because low target stocks result in low resource stocks and commensurately low social welfare. Our interest is in designing the instrument to replicate the socially-optimal harvest in each patch at every time. Given the goal of achieving the socially optimal spatial extraction, we first prove that the regulator *must* announce, as a patch-i target stock, the socially-optimal residual stock for that patch.

Lemma 1. *A necessary condition for social optimality is that the regulator announces as target stocks:* $\mathcal{S}_1 = e_1^*$, $\mathcal{S}_2 = e_2^*$,...,$\mathcal{S}_N = e_N^*$.

The proof for Lemma 1 makes use of two main results from above. First, because $\bar{e}_i \leq e_i^*$, if the regulator announces any $\mathcal{S}_i < e_i^*$, then the concessionaire will find it optimal to drive the stock below e_i^*, which is not socially optimal. Second, if the regulator sets a high target, so $\mathcal{S}_i > e_i^*$, then the concessionaire will either comply with the target (in which case the stock is inefficiently high) or will defect and reach an inefficiently low target stock. Either way, this is not socially optimal, so Lemma 1 provides the target stocks that must be announced.

Thus, we can restrict attention to the target stocks $\mathcal{S}_i = e_i^* \; \forall i$. In that case, compliance by concessionaire i requires that $e_{it} \geq e_i^* \; \forall t$, so she must never harvest below that level. Our next result establishes that, while concessionaire i is free to choose a residual stock that *exceeds* e_i^*, she will never do so.

Proposition 2. *If concessionaire i chooses to cooperate, she will do so by setting* $e_{it} = e_i^* \; \forall i, t$.

Proposition 2 establish that, if it can be achieved, cooperation will involve each concessionaire leaving precisely the socially-optimal residual stock in each period.

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[5]Recall, a firm is said to "cooperate" if its residual stock is at least as large as the announced target.

We proceed by assuming that concessionaires follow simple strategies, that is, they are unconditional cooperators. This allows us to assume compliance by $N-1$ agents and to explore the incentives of an arbitrary concessionaire i to cooperate or defect. In any given tenure block, the basic decision facing concessionaire i is whether or not to comply with the target stock requirement in each period. If she decides to defect, her optimal defection strategy is completely characterized by Proposition 1. If she chooses to cooperate, her optimal cooperation strategy is completely characterized by Proposition 2. Under the assumption of unconditional cooperation, she simply calculates her payoff from the optimal defection strategy (over a single tenure block, given by Proposition 1) and compares is to her payoff from the optimal cooperation strategy (over an infinite horizon, given by Proposition 2). Thus, each concessionaire must trade off between a *mining* effect, in which she achieves high short-run payoffs from defection during the current tenure block, and a *renewal* effect, in which she abides by the regulator's announced target stock, and thus receives lower short-run payoff, but ensures renewal in perpetuity. This comparison turns out to have a straightforward representation, given in the following result:

Proposition 3. *Cooperation emerges as an equilibrium outcome if and only if the following condition holds:*

$$\delta x_i^* - e_i^* > (1-\delta^{T-1})\left[\delta \bar{x}_i - \bar{e}_i\right],$$

where we assume that defection entails at least some harvest (i.e. that $\bar{x}_i = \sum_{j\neq i} D_{ji} g(e_j^*) + D_{ii} g(\bar{e}_i) \geq \bar{e}_i$). Proposition 3 shows that for concessionaire i, the socially optimal level of harvest $(x_i^* - e_i^*)$ must be sufficiently large compared to a discounted value of the defection harvest $(\bar{x}_i - \bar{e}_i)$. We will illustrate in the next subsection the applicability of this result by providing specific conditions on the spatial characteristics that ensure the optimality of cooperation.

3.2. Effects of Patch-Level Characteristics

Naturally, patch-level characteristics such as price, growth rates, and dispersal will affect a concessionaire's payoffs and may therefore play a role in the decision of whether to defect or cooperate. The fact that patch-level characteristics may also affect the announced target stocks further complicates the analysis. In this subsection we examine the effects of price, growth, and dispersal on the cooperation decision. Because the cooperation decision boils down to $\Pi_i^c \geq \Pi_i^d$, we define agent i's *willingness-to-cooperate* by:

$$W_i \equiv \Pi_i^c - \Pi_i^d \tag{12}$$

The variable W_i has an important implication, even when cooperation is strictly ensured by the model. For example, consider two concessionaires (A and B) who both cooperate but where $W_A >> W_B > 0$. Then we might have the intuitive idea that concessionaire A is more likely to continue cooperating under, for example, elevated transaction costs than is concessionaire B. We next explore how W_i depends on various parameters of the problem. Naturally, as a parameter changes, we must trace its effects through the entire system, including how it alters others' decisions. We summarize our findings in the following proposition:

Proposition 4. *Concessionaire i's willingness-to-cooperate (W_i) is:*

- *Increasing in p_i, α_j, and D_{ji},*
- *Decreasing in D_{ij},*
- *Can be Increasing or Decreasing in p_j and α_i, and*
- *Can be Increasing in D_{ii}.*

To help build intuition for these results, we will call out a few special cases. First, consider the effects of an increasing in productivity of connected patches (α_j). Since defection implies harvesting one's entire stock, there is little opportunity (under defection) to take advantage of one's neighbor's high productivity. But under cooperation, a larger α_j implies larger immigration, which translates into higher profit. A similar logic explains the result on D_{ji}. In contrast, consider the effect of a higher emigration rate (D_{ij}). It turns out that this reduces the incentive to cooperate. The intuition is that defection incentives are not altered much (since concessionaire i harvests the entire stock under defection), but cooperation incentives are reduced because the regulator will instruct concessionaire i to reduce her harvest under a larger D_{ij}.

3.2.1. Dispersal and cooperation

The results above suggest how the strength of the cooperation incentive for i depends on parameters of the problem. But whether this incentive is sufficiently strong to induce cooperation (i.e. whether $W_i > 0$) remains to be seen. We focus in this section on resource dispersal, which plays a pivotal role in our story. If the resource was immobile, the patches would not be interconnected, so no externality would exist and private property owners with secure property rights would harvest at a socially optimal level in perpetuity. It is dispersal that undermines this outcome and induces a spatial externality which leads to overexploitation and motivates the need for regulation. Naturally, then, the nature and degree of dispersal will play an important role in the cooperation decisions of each concessionaire.

In this model, dispersal is completely characterized by the NxN matrix whose rows sum to something less than or equal to 1 ($\sum_j D_{ij} \leq 1$). Thus, in theory, there are N^2 free parameters that describe dispersal, so at first glance it seems difficult to get general traction on how dispersal affects cooperation. But Proposition 1 provides a useful insight: *If* concessionaire i decides to defect, she will optimally do so by considering only D_{ii}, thus totally ignoring all other $N^2 - 1$ elements of the dispersal matrix. This insight allows us first to show that a high degree of self-retention (D_{ii}) in all patches is a sufficient condition for cooperation.

Proposition 5. *Let patch i be the patch with smallest self-retention parameter. For sufficiently large D_{ii}, complete cooperation over all N concessions can be sustained as an equilibrium outcome.*

The basic intuition underlying Proposition 5 is that if all patches have sufficiently high self-retention, then the externality is relatively small, which implies that the *renewal* effect outweighs the *mining* effect in all patches. That is, when spatial externalities are not too large, the concession instrument overcomes the externality caused by strategic interaction. The inverse is also intuitive: If self-retention is very low, then a large externality exists, and it may be more difficult to sustain cooperation. Naturally, formalizing this intuition is not quite as straightforward because D_{ii} also plays a role in e_j^* for *all* patches j, and thus affect defection incentives in all patches. After accounting for all of these dynamics, we arrive at the following result:

Proposition 6. *Let patch i be the patch with the largest self-retention parameter. For sufficiently small D_{ii}, cooperation will not emerge as an equilibrium outcome provided the following condition is satisfied:*

$$\delta \sum_{j \neq i} D_{ji} g(e_j^*) < \sum_{j \neq i} D_{ij} \frac{p_j}{p_i} g'(e_i^*) e_i^*. \tag{13}$$

Proposition 6 establishes that if any patch has sufficiently low self-retention, then cooperation will be destroyed and the instrument will fail to return the socially optimal harvest. This result relies on a condition on the interplay between spatial characteristics and economic returns.

3.3. Effect of the time horizon

Thus far we have focused on inherent features of patches and the system as a whole that affect a concessionaire's incentives to cooperate or defect. We have derived sufficient conditions for full cooperation and for defection, and have assumed an exogenously given tenure period. But the time horizon is a policy choice that interacts with other model parameters to affect the cooperation decision. Recall that the regulator assigns to concessionaire i a target stock ($\mathcal{S}_i = e_i^*$ by Proposition 5) and a tenure duration ($\mathcal{T}_i = T$). This subsection focuses on the determination of T.

A basic tenet of property rights and resource exploitation is that more secure property rights lead to more efficient resource use. Apropos of this observation, Costello and Kaffine (2008) found that longer tenure duration indeed increased the likelihood of cooperation in limited-tenure fishery concessions. So at first glance, one might expect a similar finding here. In fact, we find the opposite, summarized as follows:

Proposition 7. *For sufficiently long tenure duration, T, cooperation cannot be sustained as an equilibrium outcome.*

Proposition 7 seems to contradict basic economic intuition; it states that if tenure duration is long, it is impossible for the regulator to induce socially-optimal extraction of a spatially-connected resource, at least using the instrument analyzed here. But upon deeper inspection this result accords with economic principles. Consider the case of very long tenure duration - in the extreme, when tenure is infinite, the promise of renewal has no effect on incentives, so each concessionaire acts in his own best interest, which involves the defection path identified in Proposition 1.[6] This result obtains precisly because the spatial externality of resource dispersal drives a wedge between the privately optimal decision and the socially optimal one.

Proposition 7 makes it clear that *long* tenure durations can never reproduce the socially optimal spatial and temporal pattern of harvest in a spatially-connected renewable resource. On the other hand, short tenure duration harbors two incentives for cooperation: First, when tenure is short, the payoff from defection is relatively small because the concessionaire has few periods in which to defect. Second, the renewal promise is significant because it involves a much longer future horizon that does the current tenure block. In fact, it can be shown that there exists a threshold tenure length for which cooperation is sustained if and only if T is smaller than the threshold value, which we summarize as follows

Proposition 8. *Assume the following holds for concessionaire i:*

$$\delta x_i^* - e_i^* > (1-\delta)\left(\delta \bar{x}_i - \bar{e}_i\right); \tag{14}$$

Then there exists a threshold value $\bar{T} > 1$ such that cooperation is sustained as an equilibrium outcome if and only if $T \leq \bar{T}$.

[6] The proof to Proposition 7 requires specifying the punishment strategies chosen by cooperators. Following our approach above, we focus on the incentives of a single concessionaire, and assume that the other concessionaires are unconditional cooperators. A more sophisticated set of strategies (e.g. under trigger or other punishment strategies), might weaken Proposition 7; we return to this issue in Section 4.1.

The condition in Proposition 8 is a restatement of the result of Proposition 3 for a tenure period of $T = 1$. Thus, by Proposition 3 we know that a tenure period of 1 will guarantee cooperation. Proposition 8 reveals that some longer tenure durations will also sustain cooperation, but if tenure is too long (i.e. if $T > \bar{T}$), defection will surely arise.

3.3.1. Dependence of $\bar{T}$ on system characteristics

We have shown that the regulator can reproduce the socially-optimal spatial and temporal pattern of harvest by announcing target stocks $\mathcal{S}_i = e_i^*$ and tenure lengths $\mathcal{T}_i < \bar{T}$. It turns out that the threshold tenure length ($\bar{T}$) depends on patch level characteristics, so we define by $\bar{T}_i$ the threshold value for concessionaire i. Then, by Proposition 8, cooperation can be achieved by assigning to all N concessionaires $T = \min_i\{\bar{T}_i\}$. Here, we briefly examine the dependence of $\bar{T}_i$ on patch, and system-level characteristics.

To do so, we consider a benchmark case in which patches are symmetric with respect to connectivity and economic returns, so $D_{ii} = D\ \forall i$, $D_{ji} = Q\ \forall i \neq j$, and $p_i = b\ \forall i$. In this setting patches are identical so $\bar{e}_i = \bar{e}$ and $e_i^* = e^*$ for all concessionaires, and all concessionaires have the same threshold tenure value $\bar{T}_i = \bar{T}$. Using this as a starting point, we then consider an increase in a single parameter in a single patch (e.g. self-retention increases in patch 1). This approach allows us to derive concrete results about the effects of parameters on the threshold tenure duration.

In what follows, we explore the dependence of $\bar{T}_i$ on various parameters of interest. The time-threshold for concessionaire i can be written as follows:

$$\bar{T}_i = \frac{\ln\left(\frac{\delta(\bar{x}_i - x_i^*) + e_i^* - \bar{e}_i}{\delta\bar{x}_i - \bar{e}_i}\right)}{\ln(\delta)} \tag{15}$$

But because the variables $\bar{e}_i$, $\bar{x}_i$, e_i^*, and x_i^* all depend on model parameters, deriving comparative statics is non-trivial. Recalling Proposition 4 (which addresses how concessionaire i's willingness to cooperate depends on parameters of the problem), it is intuitive to think that similar results will be obtained here. Indeed, we obtain qualitatively similar results (see Appendix).

4. Robustness

To maintain analytical tractability, and to sharpen the analysis, we have made a number of simplifying assumptions about the strategies pursued by cooperators. Here, we examine three relaxed assumptions. First, we ask whether defection begets defection, that is, whether one concessionaire's defection will strengthen, or weaken, others' incentives to defect. Second, we explore trigger strategies as a means by which cooperators might punish defectors. Finally, we examine whether a finite horizon (rather than infinite, as is assumed above) can still induce cooperation.

4.1. Does defection beget defection?

In this subsection we show that when one concessionaire defects, it makes other concessionaires more likely to defect. To do, we simplify the scope of the analysis by considering $N = 2$ concessionaires, and explore whether the defection of one concessionaire (say concessionaire 2) will increase, or decrease, concessionaire 1's payoff from cooperation. Moreover, we will highlight how the difference in payoffs depend on other features of the system.

We assume that concessionaire 2 defects and we consider the worst-case scenario in which he defects immediately (from period $t = 0$ to period $t = T-1$), is replaced at period T by an identical concessionaire who again defects immediately, and so on.[7]

We deduce from Proposition 1 that concessionaire 2's optimal strategy will be characterized, for a given tenure block k and any period t such that $(k-1)T \leq t \leq kT-2$, by:

$$e_{2,kT-1} = 0 \ ; \ \ e_{2,t} = \bar{e_2}.$$

This implies that concessionaire 1's payoff from cooperating (and thus being renewed) can be written:

$$\Pi_1^{uc} = p_1 \left[x_{10} - e_1^* + \frac{\delta^T}{1-\delta^T}(\underline{x_1} - e_1^*) + \frac{\delta}{1-\delta^T}\frac{(1-\delta^{T-1})}{1-\delta}(\bar{x_1} - e_1^*) \right],$$

where the stock level $\bar{x_1} = D_{11}g(e_1^*) + D_{21}g(\bar{e_2})$ (respectively, $\underline{x_1} = D_{11}g(e_1^*) + D_{21}g(0) = D_{11}g(e_1^*)$) corresponds to all but the *first* period of a given tenure, since the representative second concessionaire defects in all but the *last* period of a given tenure by choosing $\bar{e_2}$ (respectively, to the first period of a given tenure).

We can now compute the difference as follows:

$$\Pi_1^c - \Pi_1^{uc} = \frac{\delta D_{21} p_1}{(1-\delta)(1-\delta^T)} \left[(1-\delta^T)g(e_2^*) - (1-\delta^{T-1})g(\bar{e_2}) \right].$$

We can now assess whether defection begets defection, and how T and spatial characteristics influence this effect. We can state the conclusions as follows:

[7] A more general scenario is for concessionaire 2 to cooperate over k tenure blocks and then defect in the subsequent block. However, it is straightforward to show that this will never occur: If the concessionaire has the incentive to defect in the $k+1$ tenure block, then he has the incentive to defect at every other tenure block as well. Thus, if a concessionaire will *ever* find it in his best interest to defect, he will do so in the first tenure block.

Proposition 9. *Defection begets defection, that is $\Pi_1^c > \Pi_1^{uc}$. Moreover:*

1. *A longer tenure period T makes concessionaire 1 <u>less</u> likely to defect;*
2. *A higher value of D_{21} makes concessionaire 1 <u>more</u> likely to defect;*
3. *The effect of a higher value of D_{22} is <u>ambiguous</u>: for small initial values of the parameter, an increase makes concessionaire 1 more likely to defect, while the conclusion is reversed for high initial values.*

When one concessionaire defects, she does so by excessively harvesting in the short run and thus drives down the resource stock (see Proposition 1). Precisely because of the spatial connections between resource patches, this has important consequences for the other concessionaires. Now, receiving a smaller "input" of resource stock from the defecting concessionaire, a given owner will find it even more challenging to adhere to the target stock set by the regulator. This calculus gives rise to an unambiguous result: It enhances the incentives of other concessionaires to defect. The subsequent findings in Proposition 9 also inform how system-level characteristics influence a cooperating concessionaire's decision about whether to defect (given that another concessionaire has defected). While we have found that longer tenure periods reduce the defection incentive for concessionaire 1, this must be viewed with caution. Indeed, Proposition 7 states that if tenure is too *long*, concessionaire 1 will defect. Together Propositions 7 and 9 suggest that, for any renewal horizon shorter than the threshold, increasing the duration required for renewal (but still making sure that it is shorter than the threshold) will make the first concessionaire less likely to defect if the second concessionaire does so. Regarding the effect of dispersal on the defection incentives of concessionaire 1, we find ambiguous results. Higher migration from patch 2 to patch 1 increases the first concessionaire's stakes when defecting. By contrast, the effect of self-retention in the second patch flips depending on the initial value of this parameter.

4.2. The scope of applicability of trigger strategies

In section 3, we focused on the use of simple strategies in order to highlight the potential of the concession instrument. By using trigger strategies in case one concessionaire defects, all other concessionaires will know for sure that they will not get renewed at the end of their current tenure block. Thus, trigger strategies imply a form of self-punishment, which can be seen as an additional incentive scheme. We summarize our findings on trigger strategies as follows:

Proposition 10. *Assume that concessionaires follow trigger strategies. Then cooperation will emerge as an equilibrium outcome if and only if the following condition holds (for any concessionaire i):*

$$\delta x_i^* - e_i^* - (1 - \delta^{T-1}) \left[\delta \bar{\bar{x}}_i - \bar{e}_i\right] > 0,$$

where it is assumed that $\bar{\bar{x}}_i = \sum_j D_{ji} g(\bar{e}_j) \geq \bar{e}_i$ and that $\bar{e}_i$ is positive.

The proof confirms one of our previous claims regarding the incentives to defect: it intuitive and straightforward to show that incentives to defect are the same at any given period, that is, they are not time dependent. Moreover, this proposition enables us to quickly deduce that the incentives to defect increase with a longer time horizon.[8] Moreover, it is easily checked that the inequality characterizing the scope of trigger strategies is less restrictive than the similar condition in Proposition 5. Thus, using trigger strategies in addition to the concession instrument enlarges the scope for full cooperation.

4.3. Robustness of the instrument: finite-horizon version

We have assumed an infinite horizon, so concessionaires must trade off a finite single tenure block against an infinite number of renewed tenure blocks. Even though this is not an unreasonable assumption per se, it begs the question of whether the instrument developed here is still effective at inducing cooperation when the horizon is finite. Here we prove that this is the case. Suppose time ends after K tenure blocks where $1 < K < \infty$ after which all agents' payoffs are zero. We prove here that provided cooperation was subgame perfect under an infinite horizon, it remains subgame perfect under the finite horizon problem described here. We formalize the result as follows:

Proposition 11. *Suppose time ends after the K^{th} tenure block. Then, if the instrument induces cooperation over the infinite horizon then it will do so for the first $K-1$ tenure blocks of the finite horizon problem*

5. Conclusion

We have studied a decentralized property rights system over a spatially-connected renewable natural resource, such as a fishery. To overcome the excessive harvest that is incentivized by decentralized spatial property rights, we propose a new instrument based on limited-tenure concessions with the possibility of renewal. Somewhat surprisingly, we find that this instrument can be designed to be extremely effective in overcoming the tragedy of the commons; indeed it is often the case that this instrument can induce the concessionaires to implement the socially optimal outcome. This is remarkable as it does not rely on any transfer or side-payment. Second, unlike an initial intuition, the effect of a longer time horizon is usually negative. This is in contradiction with the case without strategic interactions as depicted in Costello and Kaffine (2008). Overall, these results suggest that if implemented with care, the limited-tenure spatial concession can achieve near

[8]This conclusion follows if one differentiates the expression of the difference between payoffs as a function of the time horizon.

socially-optimal outcomes and yet still allow concessionaries to make decentralized decisions over harvest.

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Appendix

Proof of Proposition 1

We proceed by backward induction. At final period $kT-1$, concessionaire i's problem is to maximize

$$\max_{e_{i,kT-1}\geq 0} p_i \left(x_{i,kT-1} - e_{i,kT-1}\right)$$

Using the first order condition enables us to conclude immediately that $\bar{e}_{i,kT-1} = 0$, that is, concessionaire i extracts the entire stock at the final period. Now, moving backward, at period $T-2$, this concessionaire's problem becomes:

$$\max_{e_{i,kT-2}\geq 0} p_i \left[x_{i,kT-2} - e_{i,kT-2} + \delta \left(\sum_{j\neq i} D_{ji} g(\bar{e}_{j,kT-2}) + D_{ii} g(\bar{e}_{i,kT-2}) - \bar{e}_{i,kT-1} \right) \right].$$

Using the first order condition (with respect to $\bar{e}_{i,kT-2}$) and $\bar{e}_{i,kT-1} = 0$, we obtain that $\bar{e}_{i,kT-2}$ is characterized by the following condition:

$$\delta D_{ii} g'(\bar{e}_{i,kT-2}) = 1.$$

Repeating the same argument of backward induction it is easily checked that any equilibrium residual stock level $\bar{e}_{i,t}$ (where $(k-1)T \leq t \leq kT-3$) is characterized by the same condition. This concludes the proof.

Proof of Proposition 3

If concessionaire i deviates during tenure $k+1$ (while other concessionaires follow simple strategies) then this concessionaire's payoff is $\Pi_i^d = p_i A$, where :

$$A = \left[x_{i0} - e_i^* + \frac{\delta(1-\delta^{kT})}{1-\delta}(x_i^* - e_i^*) + \delta^{kT}(e_i^* - \bar{e}_i) + \frac{\delta^{kT+1}(1-\delta^{T-1})}{1-\delta}(\bar{x}_i - \bar{e}_i) + \delta^{(k+1)T-1}\bar{e}_i \right].$$

Now, computing the difference $\Pi_i^c - \Pi_i^d = p_i B$, we obtain:

$$B = \left[\frac{\delta^{kT+1}}{1-\delta}(x_i^* - e_i^*) - \delta^{kT}(e_i^* - \bar{e}_i) - \frac{\delta^{kT+1}(1-\delta^{T-1})}{1-\delta}(\bar{x}_i - \bar{e}_i) - \delta^{(k+1)T-1}(1-\delta)\bar{e}_i \right]$$

or, after simplification:

$$\begin{aligned}
\Pi_i^c - \Pi_i^d &= \frac{p_i}{1-\delta}\left[\delta^{kT+1}x_i^* - \delta^{kT}e_i^* + \delta^{kT}(1-\delta^{T-1})\bar{e}_i - \delta^{kT}(1-\delta^{T-1})\delta\bar{x}_i\right] \\
&= \delta^{kT}\frac{p_i}{1-\delta}\left\{\delta x_i^* - e_i^* - (1-\delta^{T-1})\left[\delta\bar{x}_i - \bar{e}_i\right]\right\}.
\end{aligned}$$

The conclusion follows from this equality.

Proof of Proposition 5

We will show that concessionaire i does not have incentives to deviate, which will be sufficient to prove the result. First, we proove that the concessionaire does not have incentive to deviate from the initial period until the end of the first tenure. From the proof of Proposition we know that:

$$\begin{aligned}
\Pi_i^c - \Pi_i^d &= p_i[\bar{e}_i - e_i^* + \frac{\delta}{1-\delta}(x_i^* - e_i^*) - \frac{\delta(1-\delta^{T-1})}{1-\delta}(\bar{x}_i - \bar{e}_i) - \delta^T\bar{x}_i] \\
&= \frac{p_i}{1-\delta}[(1-\delta)(\bar{e}_i - e_i^*) + \delta(x_i^* - e_i^*) - \delta(1-\delta^T)(\bar{x}_i - \bar{e}_i) - \delta^T(1-\delta)\bar{e}_i].
\end{aligned}$$

When D_{ii} gets arbitrarily close to one, the characterizations of $\bar{e}_i$ and e_i^* enable to conclude that $\bar{e}_i$ gets arbitrarily close to e_i^*. We can deduce that $\Pi_i^c - \Pi_i^d$ gets arbitrarily close to the following expression:

$$\frac{\delta^{T+1}}{1-\delta}(x_i^* - e_i^*) - \delta^T e_i^* = \frac{\delta^T}{1-\delta}[\delta x_i^* - e_i^*]. \tag{16}$$

Again, when D_{ii} converges to one, x_i^* gets arbitrarily close to $g(e_i^*)$. Then, for $D_{ii} = 1$ we know that $1 = \delta g'(e_i^*)$, and we can rewrite equation (16) as follows:

$$\frac{\delta^T}{1-\delta}[\delta x_i^* - e_i^*] = \frac{\delta^T}{1-\delta}[\delta g(e_i^*) - \delta g'(e_i^*)e_i^*] = \frac{\delta^{T+1}}{1-\delta}[g(e_i^*) - g'(e_i^*)e_i^*]. \tag{17}$$

The concavity of g (together with the fact that $g(0) = 0$) enables to quickly deduce that $g(e_i^*) - g'(e_i^*)e_i^*$ is positive. Thus, for $D_{ii} = 1$ we know that $\Pi_i^c - \Pi_i^d > 0$ which, by a continuity argument, enables to conclude that the above deviation is not profitable (for concessionaire i) for sufficiently large values of self retention of this concessionaire's patch.

Second, we conclude the proof of the result by showing that concessionaire i does not have incentives to deviate during any other tenure. We consider that defection might occur during tenure $k+1$. Using similar calculations than in the first part of the proof we obtain:

$$\begin{aligned}\Pi_i^c - \Pi_i^d &= p_i[\delta^{kT}(\bar{e_i} - e_i^*) + \sum_{t=KT+1}^{(k+1)T-1} \delta^t [x_i^* - e_i^* - \bar{x}_i + \bar{e_i}] \\ &+ \frac{\delta^{(k+1)T}}{1-\delta}(x_i^* - e_i^*) - \delta^{(k+1)T-1}\bar{e_i}].\end{aligned}$$

When D_{ii} gets arbitrarily close to one, the characterizations of $\bar{e_i}$ and e_i^* enable to conclude that $\bar{e_i}$ gets arbitrarily close to e_i^*, and $\bar{x}_i$ gets arbitrarily close to x_i^* (since g is continuous). We can deduce that $\Pi_i^c - \Pi_i^d$ gets arbitrarily close to $p_i \frac{\delta^{(k+1)T-1}}{1-\delta}(\delta x_i^* - e_i^*)$. We can then deduce that the deviation is not profitable for concessionaire i (for sufficiently large values of D_{ii}). This proves that concessionaire i does not have incentive to defect. The same reasoning holds for any other concessionaire, which concludes the proof.

Proof of Proposition 6

Using Proposition 5, we know that concessionaire i would defect if the following condition is satisfied:

$$\delta x_i^* - e_i^* - (1-\delta)(\delta \bar{x}_i - \bar{e_i}) \leq 0.$$

If D_{ii} is sufficiently small then $\bar{e_i} = 0$ and we focus on cases where e_i^* is still positive. Then, using $\bar{x}_i = \sum_{j \neq i} D_{ji} g(e_j^*)$ and the characterization of e_i^*, we can rewrite this inequality as follows:

$$\delta x_i^* - e_i^* - (1-\delta)\delta\bar{x}_i = \delta \left[\delta \sum_{j\neq i} D_{ji} g(e_j^*) - \sum_{j \neq i} D_{ij} \frac{p_j}{p_i} g'(e_i^*) e_i^* \right].$$

If the left hand side of this equality is negative (which is the case provided that condition (13) holds), then $\delta x_i^* - e_i^* - (1-\delta)\delta\bar{x}_i$ is negative, which concludes the proof.

Proof of Proposition 7

We claim that, as T gets arbitrarily large, any concessionaire i will defect from full cooperation. Let us assume that any concessionaire $j \neq i$ follows a full cooperation path; we now analyze concessionaire i's incentives to defect. One possible deviation is described in Proposition 1. Specifically, concessionaire i might deviate from the initial period until period T. Then this concessionaire will not be renewed. According to Proposition 1, this concessionaire's payoff from defecting will then be equal to Π_i^d.

We now prove that $\Pi_i^c - \Pi_i^d \leq 0$ for sufficiently large values of T. We have:

$$\Pi_i^c - \Pi_i = p_i[\bar{e_i} - e_i^* + \frac{\delta}{1-\delta}(x_i^* - e_i^*) - \frac{\delta(1-\delta^{T-1})}{1-\delta}(\bar{x}_i - \bar{e_i}) - \delta^T \bar{x}_i].$$

When T gets arbitrarily large, $\Pi_i^c - \Pi_i^d$ gets close to

$$p_i[\bar{e_i} - e_i^* + \frac{\delta}{1-\delta}(x_i^* - e_i^* - \bar{x}_i + \bar{e_i})]. \tag{18}$$

Now, we know that $x_i^* - \bar{x}_i = D_{ii}(g(e_i^*) - g(\bar{e}_i))$ and we obtain the following inequality (by concavity of function g):

$$x_i^* - \bar{x}_i = D_{ii}(g(e_i^*) - g(\bar{e}_i)) < D_{ii}g'(\bar{e}_i)(e_i^* - \bar{e}_i).$$

This enables us to deduce the following inequality regarding equation (18):

$$\frac{p_i}{1-\delta}[\delta D_{ii}(g(e_i^*) - g(\bar{e}_i)) - (e_i^* - \bar{e}_i)] < \frac{p_i}{1-\delta}[\delta D_{ii}g'(\bar{e}_i) - 1](e_i^* - \bar{e}_i). \tag{19}$$

But we know (from the characterization of $\bar{e}_i$) that $\bar{e}_i$ satisfies $\delta D_{ii}g'(\bar{e}_i) = 1$, which implies that the right hand side of the above inequality is equal to zero. We conclude that the expression (18) is negative which, by a continuity argument, implies that $\Pi_i^c - \Pi_i^d \leq 0$ for sufficiently large values of T. This concludes the proof.

Proof of lemma 1

For a given concessionaire i, let us consider $\bar{T}_i$ defined implicitly by:

$$\bar{e}_i - e_i^* + \frac{\delta}{1-\delta}(x_i^* - e_i^*) - \frac{\delta(1-\delta^{\bar{T}_i - 1})}{1-\delta}(\bar{x}_i - \bar{e}_i) - \delta^{\bar{T}_i}\bar{x}_i = 0.$$

Since the characterization of $\bar{e}_i$ and e_i^* ensure that residual stock levels (and thus stock levels) do not depend on the value of the time horizon, we can differentiate the left hand side of the equality as a function of T, and we obtain the following expression:

$$\delta^T \frac{ln(\delta)}{1-\delta}\left\{\delta\bar{x}_i - \bar{e}_i\right\}$$

which is negative since $ln(\delta) < 0$ as $0 < \delta \leq 1$ and $\delta\bar{x}_i - \bar{e}_i$ is positive. This implies that the left hand side of the equality is a decreasing and continuous function of T (where T is assumed to take continuous values). Since the proof of Proposition 2 implies that this function takes on negative values as T becomes large, if one can prove that it has a positive value when $T = 1$ this would imply that $\bar{T}_i$ is uniquely defined and that $\bar{T}_i > 1$. Then, again using the proof of Proposition 7 enables us to conclude that concessionaire i will have incentives to defect as soon as the renewal time horizon is larger than $\bar{T}_i$.

For $T = 1$ the value of the function is given by the following expression:

$$\bar{e}_i - e_i^* + \frac{\delta}{1-\delta}\left(x_i^* - e_i^*\right) - \delta\bar{x}_i = \frac{1}{1-\delta}\left[\delta x_i^* - e_i^* - (1-\delta)\left(\delta\bar{x}_i - \bar{e}_i\right)\right].$$

Our assumption implies that the right hand side of this equality is positive, which enables us to conclude about the existence and uniqueness of

$$\bar{T}_i = \frac{ln\left[\frac{\delta\bar{x}_i - \bar{e}_i - (\delta x_i^* - e_i^*)}{\delta\bar{x}_i - \bar{e}_i}\right]}{ln(\delta)}.$$

This concludes the proof of the result since $\bar{T} = \min_i \bar{T}_i$ is easily checked to qualify as the appropriate threshold value.

Proof of Proposition 9

We know that $(1-\delta^T) > (1-\delta^{T-1})$ for any renewal horizon $T \geq 1$ and any discount factor $0 < \delta \leq 1$. Moreover, we know that $e_2^* > \bar{e_2}$ and thus $g(e_2^*) > g(\bar{e_2})$: together, this implies that

$$(1-\delta^T)g(e_2^*) - (1-\delta^{T-1})g(\bar{e_2}) > 0,$$

and we can conclude that $\Pi_1^c - \Pi_1^{uc}$ is positive. Regarding the two properties, we have:

1. Differentiating the difference between payoffs with respect to T, the sign of the derivative is given by that of the following expression:

$$\Delta_T = \frac{\left[-\delta^T ln(\delta) g(e_2^*) + \delta^{T-1} ln(\delta) g(\bar{e_2})\right](1-\delta^T) + \left[(1-\delta^T)g(e_2^*) - (1-\delta^{T-1})g(\bar{e_2})\right]\delta^T ln(\delta)}{(1-\delta)(1-\delta^T)^2},$$

   which can be simplified as:
$$\frac{\delta^{T-1} ln(\delta)(1-\delta)g(\bar{e_2})}{(1-\delta)(1-\delta^T)^2}.$$
   All terms are positive except $ln(\delta)$ since $0 < \delta \leq 1$ by assumption. We conclude that the sign of the derivative is negative.
2. Differentiating the difference between payoffs with respect to D_{21}, the sign of the derivative is given by that of the following expression:
$$\Delta_{D_{21}} = (1-\delta^T)g(e_2^*) - (1-\delta^{T-1})g(\bar{e_2}) + D_{21}(1-\delta^T)g'(e_2^*)\frac{\partial e_2^*}{\partial D_{21}}.$$
   Using the characterization of e_2^*, we deduce easily that:
$$\frac{\partial e_2^*}{\partial D_{21}} = \frac{-p_1 g'(e_2^*)}{\left[D_{22}p_2 + D_{21}p_1\right]g''(e_2^*)} > 0,$$
   which in turn enables us to conclude that Δ_D is positive as the sum of two positive terms. This concludes the proof.
3. Differentiating the difference between payoffs with respect to D_{22}, the sign of the derivative is given by that of the following expression:
$$\Delta_{D_{22}} = (1-\delta^T)g'(e_2^*)\frac{\partial e_2^*}{\partial D_{22}} - (1-\delta^{T-1})g'(\bar{e_2})\frac{\partial \bar{e_2}}{\partial D_{22}}.$$
   Using the characterizations of e_2^* and $\bar{e_2}$, we deduce easily that:
$$\frac{\partial e_2^*}{\partial D_{22}} = \frac{-p_2 g'(e_2^*)}{\left[D_{22}p_2 + D_{21}p_1\right]g''(e_2^*)} > 0, \quad \frac{\partial \bar{e_2}}{\partial D_{22}} = \frac{-g'(\bar{e_2})}{D_{22}g''(\bar{e_2})}$$
   which in turn enables us to rewrite $\Delta_{D_{22}}$ as
$$\frac{-p_2(1-\delta^T)(g'(e_2^*))^2 D_{22}g''(\bar{e_2}) + (1-\delta^{T-1})(g'(\bar{e_2}))^2[D_{22}p_2 + D_{21}p_1]g''(e_2^*)}{D_{22}\left[D_{22}p_2 + D_{21}p_1\right]g''(e_2^*)g''(\bar{e_2})}$$
   If D_{22} is sufficiently small then $\bar{e_2} = 0$ and the numerator of $\Delta_{D_{22}}$ gets close to
$$\left(1-\delta^{T-1}\right)\left(g'(0)\right)^2 D_{21}p_1 g''(0) < 0,$$

which enables us to conclude that $\Delta_{D_{22}}$ is negative for this case. If D_{22} is now sufficiently large then D_{21} is small and $\bar{e_2}$ gets close to e_2^*, which in turn implies that the numerator of $\Delta_{D_{22}}$ gets close to

$$\begin{aligned} & \left(1-\delta^{T-1}\right)\left(g'(e_2^*)\right)^2 D_{22} p_2 g''(e_2^*) - (p_2)\left(1-\delta^{T}\right)\left(g'(e_2^*)\right)^2 D_{22} g''(e_2^*) \\ = & \left(\delta^{T}-\delta^{T-1}\right) D_{22} p_2 \left(g'(e_2^*)\right)^2 g''(e_2^*). \end{aligned}$$

Since $\delta^T - \delta^{T-1} < 0$ we conclude that the numerator of $\Delta_{D_{22}}$ is positive, which in turn implies that $\Delta_{D_{22}}$ is positive as well. This concludes the proof.

Proof of Proposition 10

If concessionaire i deviates during tenure $k+1$ (while other concessionaires follow trigger strategies) then this concessionaire's payoff is Π_i^d, where :

$$p_i \left[x_{i0} - e_i^* + \frac{\delta(1-\delta^{kT})}{1-\delta}\left(x_i^* - e_i^*\right) + \delta^{kT}\left(e_i^* - \bar{e}_i\right) + \frac{\delta^{kT+1}(1-\delta^{T-1})}{1-\delta}\left(\bar{\bar{x}}_i - \bar{e}_i\right) + \delta^{(k+1)T-1}\bar{e}_i \right].$$

Now, computing the difference $\Pi_i^c - \Pi_i^d$, we obtain:

$$p_i \left[\frac{\delta^{kT+1}}{1-\delta}\left(x_i^* - e_i^*\right) - \delta^{kT}\left(e_i^* - \bar{e}_i\right) - \frac{\delta^{kT+1}(1-\delta^{T-1})}{1-\delta}\left(\bar{\bar{x}}_i - \bar{e}_i\right) - \delta^{(k+1)T-1}(1-\delta)\bar{e}_i \right]$$

or, after simplification:

$$\begin{aligned} \Pi_i^c - \Pi_i^d &= \frac{p_i}{1-\delta}\left[\delta^{kT+1}x_i^* - \delta^{kT}e_i^* + \delta^{kT}(1-\delta^{T-1})\bar{e}_i - \delta^{kT}(1-\delta^{T-1})\delta\bar{\bar{x}}_i\right] \\ &= \delta^{kT}\frac{p_i}{1-\delta}\left\{\delta x_i^* - e_i^* - (1-\delta^{T-1})\left[\delta\bar{\bar{x}}_i - \bar{e}_i\right]\right\}. \end{aligned}$$

The conclusion follows from this equality.

Computations of derivatives

We have the following stock when patch i defects:

$$\bar{x}_i = D_{ii} g\left(\bar{e}_i, \alpha_i\right) + \sum_{j \neq i} D_{ji} g\left(e_j^*, \alpha_j\right) \tag{20}$$

and when all patches cooperate:

$$x_i^* = \sum_j D_{ji} g\left(e_j^*, \alpha_j\right) \tag{21}$$

We assume that one parameter, $\theta_i = \{p_i, \alpha_i, D_{ii}, D_{ij}\}$ or $\theta_j = \{p_j, \alpha_j, D_{ji}\}$, is elevated. We obtain the general following forms for the stocks:

$$\frac{d\bar{x}_i}{d\theta_i} = \frac{\partial \bar{x}_i}{\partial \bar{e}_i} \cdot \frac{\partial \bar{e}_i}{\partial \theta_i} + \frac{\partial \bar{x}_i}{\partial \theta_i} + \sum_{j \neq i} \frac{\partial \bar{x}_i}{\partial e_j^*} \cdot \frac{\partial e_j^*}{\partial \theta_i} \tag{22}$$

$$\frac{d\bar{x}_i}{d\theta_j} = \frac{\partial \bar{x}_i}{\partial \theta_j} + \sum_{l \neq i} \frac{\partial \bar{x}_i}{\partial e_l^*} \cdot \frac{\partial e_l^*}{\partial \theta_j} \tag{23}$$

$$\frac{dx_i^*}{d\theta_i} = \frac{\partial x_i^*}{\partial \theta_i} + \sum_{j} \frac{\partial x_i^*}{\partial e_j^*} \cdot \frac{\partial e_j^*}{\partial \theta_i} \tag{24}$$

$$\frac{dx_i^*}{d\theta_j} = \frac{\partial x_i^*}{\partial \theta_j} + \sum_{l} \frac{\partial x_i^*}{\partial e_l^*} \cdot \frac{\partial e_l^*}{\partial \theta_j} \tag{25}$$

and the residual stock levels

θ	$\frac{\partial e_i^*}{\partial\theta}$	$\frac{\partial\bar{e}_i}{\partial\theta}$	$\frac{\partial e_i^*}{\partial\theta}-\frac{\partial\bar{e}_i}{\partial\theta}$
p_i	$\frac{1-\delta D_{ii}g_{e_i}}{\sum_{j=1}^N \delta D_{ij}p_j g_{e_ie_i}}<0$	0	$+$
p_j	$-\frac{D_{ij}g_{e_i}}{\sum_{l=1}^N D_{il}p_l g_{e_ie_i}}>0$	0	$+$
α_i	$-\frac{g_{e_i\alpha_i}}{g_{e_ie_i}}>0$	$-\frac{g_{e_i\alpha_i}}{g_{e_ie_i}}>0$	?
α_j	0	0	0
D_{ii}	$-\frac{p_i g_{e_i}}{\sum_{j=1}^N D_{ij}p_j g_{e_ie_i}}>0$	$-\frac{g_{e_i}}{g_{e_ie_i}}>0$	$-\frac{g_{e_i}}{g_{e_ie_i}}\left(\frac{p_i}{\sum_{j=1}^N D_{ij}p_j}-1\right)$ $=-\frac{g_{e_i}}{g_{e_ie_i}}\left(\frac{p_i(1-D_{ii})-\sum_{j\neq i}D_{ij}p_j}{\sum_{j=1}^N D_{ij}p_j}\right)$ $=-\frac{g_{e_i}}{g_{e_ie_i}}\left(\frac{\sum_{j\neq i}D_{ij}\left(p_i-\sum_{j\neq i}p_j\right)}{\sum_{j=1}^N D_{ij}p_j}\right)$ it depends on $p_i-\sum_{j\neq i}p_j$
D_{ij}	$-\frac{p_j g_{e_i}}{\sum_{j=1}^N D_{ij}p_j g_{e_ie_i}}>0$	0	$+$
D_{ji}	0	0	0

Impact on the emergence of cooperation

Impact of net price, p

Impact of p_i

We first analyze the impact of p_i on agent i's willingness to cooperate by using expressions (22) to (25) and the table in order to compute the following expression:

$$
\begin{aligned}
\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dp_i} &= \frac{\delta^{kT}}{1-\delta}\left[\delta x_i^*-e_i^*-(1-\delta^{T-1})(\delta\bar{x}_i-\bar{e}_i)\right] \\
&+ \frac{\delta^{kT}p_i}{1-\delta}\left[\delta\sum_j\frac{\partial x_i^*}{\partial e_j^*}\frac{\partial e_j^*}{\partial p_i}-\frac{\partial e_i^*}{\partial p_i}-\delta(1-\delta^{T-1})\sum_{j\neq i}\frac{\partial\bar{x}_i}{\partial e_j^*}\frac{\partial e_j^*}{\partial p_i}\right]
\end{aligned}
$$

Let us focus on the second term between brackets and rewrite it as follows:

$$\frac{\partial e_i^*}{\partial p_i}\left(\delta\frac{\partial x_i^*}{\partial e_i^*}-1\right)+\sum_{j\neq i}\frac{\partial e_j^*}{\partial p_i}\left[\delta\frac{\partial x_i^*}{\partial e_j^*}-\delta(1-\delta^{T-1})\frac{\partial \bar{x}_i}{\partial e_j^*}\right] \tag{26}$$

$$\Leftrightarrow\quad \frac{\partial e_i^*}{\partial p_i}\left(\delta D_{ii}g_{e_i^*}-1\right)+\sum_{j\neq i}\frac{\partial e_j^*}{\partial p_i}\left[\delta D_{ji}g_{e_j^*}-\delta(1-\delta^{T-1})D_{ji}g_{e_j^*}\right] \tag{27}$$

$$\Leftrightarrow\quad -\frac{\partial e_i^*}{\partial p_i}\left(1-\delta D_{ii}g_{e_i^*}\right)+\sum_{j\neq i}\frac{\partial e_j^*}{\partial p_i}\delta^T D_{ji}g_{e_j^*}>0 \tag{28}$$

because we have $\frac{\partial e_i^*}{\partial p_i}<0$, $1-\delta D_{ii}g_{e_i^*}>0$ and $\frac{\partial e_j^*}{\partial p_i}>0$. Thus, we can conclude that $\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dp_i}>0$ if the condition regarding agent i's *willingness-to-cooperate* is satisfied. This means that an increase in p_i results in an increase in the value of $\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dp_i}$, thus an increase in the *willingness-to-cooperate*.

If initially this agent was not willing to cooperate (that is, if initially $\Pi_i^c-\Pi_i^d<0$) then the overall expression of $\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dp_i}$ is the sum of two terms, one which is negative, the other which is positive, and the net effect is ambiguous (as it depends on the initial value of net price p_i and on other factors).

Effect of p_j, $j\neq i$

In this case we have

$$\begin{aligned}
\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dp_j} &= \frac{\delta^{kT}p_i}{1-\delta}\left[\delta\sum_l\frac{\partial x_i^*}{\partial e_l^*}\frac{\partial e_l^*}{\partial p_j}-\frac{\partial e_i^*}{\partial p_j}-\delta(1-\delta^{T-1})\sum_{l\neq i}\frac{\partial \bar{x}_i}{\partial e_l^*}\frac{\partial e_l^*}{\partial p_j}\right]\\
&= \frac{\delta^{kT}p_i}{1-\delta}\left[\frac{\partial e_i^*}{\partial p_j}\left(\delta\frac{\partial x_i^*}{\partial e_i^*}-1\right)+\sum_{l\neq i}\left(\delta\frac{\partial x_i^*}{\partial e_l^*}\frac{\partial e_l^*}{\partial p_j}-\delta(1-\delta^{T-1})\frac{\partial \bar{x}_i}{\partial e_l^*}\frac{\partial e_l^*}{\partial p_j}\right)\right]\\
&= \frac{\delta^{kT}p_i}{1-\delta}\left[-\frac{\partial e_i^*}{\partial p_j}\left(1-\delta D_{ii}g_{e_i^*}\right)+\delta^T\sum_{l\neq i}\frac{\partial e_l^*}{\partial p_j}D_{li}g_{e_l^*}\right]\\
&= \frac{\delta^{kT}p_i}{1-\delta}\left[\underbrace{-\frac{\partial e_i^*}{\partial p_j}\left(1-\delta D_{ii}g_{e_i^*}\right)}_{<0}+\delta^T\left(\underbrace{\frac{\partial e_j^*}{\partial p_j}D_{ji}g_{e_j^*}}_{<0}+\underbrace{\sum_{l\neq i,j}\frac{\partial e_l^*}{\partial p_j}D_{li}g_{e_l^*}}_{>0}\right)\right]
\end{aligned}$$

Using the expressions provided in the table and **focusing on the spatial relationship between the patch of interest and the patch where the parameter is elevated, (i and j)**, we deduce the following conclusions:

- **First, if both dispersal rate between these two patches,** D_{ij} and D_{ji} are sufficiently small, then the first and second term between brackets on the RHS of the equality are small, which implies that $\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dp_j}$ is positive;

Explanation: When D_{ij} and D_{ji} small then $\frac{\partial e_i^*}{\partial p_j}$ and $\frac{\partial e_j^*}{\partial p_j} D_{ji} g_{e_j^*}$ are small. And the sign of the term in brackets (and thus of $\frac{d\left(\Pi_i^c - \Pi_i^d\right)}{dp_j}$) is similar to the sign of $\sum_{l \neq i,j} \frac{\partial e_l^*}{\partial p_j} D_{li} g_{e_l^*}$, which is positive.

- Second, if the spatial relationship of the two patches and their own self-retention rate are sufficiently large (or if both patches i and j are weakly spatially-connected with other patch), respectively $D_{ii} + D_{ij}$ and $D_{jj} + D_{ji}$ are sufficiently large, then the term $\sum_{l \neq i,j} \frac{\partial e_l^*}{\partial p_j} D_{li} g_{e_l^*}$ is small, which implies that $\frac{d\left(\Pi_i^c - \Pi_i^d\right)}{dp_j}$ is negative.

Impact of growth, α

Effect of α_i

We analyze the effect of α_i on agent i's willingness to cooperate. We have:

$$
\begin{aligned}
\frac{d\left(\Pi_i^c - \Pi_i^d\right)}{d\alpha_i} &= \frac{\delta^{kT} p_i}{1-\delta} \left\{ \delta \left(\frac{\partial x_i^*}{\partial \alpha_i} + \frac{\partial x_i^*}{\partial e_i^*} \frac{\partial e_i^*}{\partial \alpha_i} \right) - \frac{\partial e_i^*}{\partial \alpha_i} - (1-\delta^{T-1}) \left[\delta \left(\frac{\partial \bar{x}_i}{\partial \alpha_i} + \frac{\partial \bar{x}_i}{\partial \bar{e}_i} \frac{\partial \bar{e}_i}{\partial \alpha_i} \right) - \frac{\partial \bar{e}_i}{\partial \alpha_i} \right] \right\} \\
&= \frac{\delta^{kT} p_i}{1-\delta} \left[\frac{\partial e_i^*}{\partial \alpha_i} \left(\delta D_{ii} g_{e_i^*} - 1 \right) + \delta D_{ii} g_{\alpha_i^*} - (1-\delta^{T-1}) \left(\frac{\partial \bar{e}_i}{\partial \alpha_i} (\delta D_{ii} g_{\bar{e}_i} - 1) + \delta D_{ii} g_{\bar{\alpha}_i} \right) \right] \\
&= \frac{\delta^{kT} p_i}{1-\delta} \left[\frac{\partial e_i^*}{\partial \alpha_i} \left(\delta D_{ii} g_{e_i^*} - 1 \right) + \delta D_{ii} \left(g_{\alpha_i^*} - (1-\delta^{T-1}) g_{\bar{\alpha}_i} \right) \right]
\end{aligned}
$$

If D_{ii} is small while $\bar{e}_i > 0$, then $\frac{d\left(\Pi_i^c - \Pi_i^d\right)}{d\alpha_i} < 0$ and an increase in α_i decreases agent i's incentives to cooperate.

If $D_{ii} = 1$, then $1 - \delta D_{ii} g_{e_i^*} = 0$ and $\frac{d\left(\Pi_i^c - \Pi_i^d\right)}{d\alpha_i} > 0$ since $g_{\alpha_i^*} - (1-\delta^{T-1}) g_{\bar{\alpha}_i}$ is positive. By a continuity argument, this conclusion remains valid when D_{ii} is sufficiently large.

Effect of α_j, $j \neq$

We analyze the effect of α_j on agent i'd willingness to cooperate. We have:

$$
\begin{aligned}
\frac{d\left(\Pi_i^c - \Pi_i^d\right)}{d\alpha_j} &= \frac{\delta^{kT} p_i}{1-\delta} \left[\delta \left(\frac{\partial x_i^*}{\partial \alpha_j} + \frac{\partial x_i^*}{\partial e_j^*} \frac{\partial e_j^*}{\partial \alpha_j} \right) - \delta(1-\delta^{T-1}) \left(\frac{\partial \bar{x}_i}{\partial \alpha_j} + \frac{\partial \bar{x}_i}{\partial e_j^*} \frac{\partial e_j^*}{\partial \alpha_j} \right) \right] \\
&= \frac{\delta^{kT+1} p_i}{1-\delta} \left[\delta^{T-1} D_{ji} g_{\alpha_j^*} + \frac{\partial e_j^*}{\partial \alpha_j} \delta^{T-1} D_{ji} g_{e_j^*} \right] \\
&= \frac{\delta^{(k+1)T} p_i}{1-\delta} D_{ji} \left(g_{\alpha_j^*} + g_{e_j^*} \right) > 0
\end{aligned}
$$

An increase in α_j increases the willingness-to-cooperate of agent i.

Impact of dispersal rate, D

Effect of D_{ii}

We first analyze the effect of the self-retention rate on an agent's willingness to cooperate. We have:

$$
\begin{aligned}
\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dD_{ii}} &= \frac{\delta^{kT}p_i}{1-\delta}\left\{\delta\left(\frac{\partial x_i^*}{\partial D_{ii}}+\frac{\partial x_i^*}{\partial e_i^*}\frac{\partial e_i^*}{\partial D_{ii}}\right)-\frac{\partial e_i^*}{\partial D_{ii}}-(1-\delta^{T-1})\left[\delta\left(\frac{\partial \bar{x}_i}{\partial D_{ii}}+\frac{\partial \bar{x}_i}{\partial \bar{e}_i}\frac{\partial \bar{e}_i}{\partial D_{ii}}\right)-\frac{\partial \bar{e}_i}{\partial D_{ii}}\right]\right\} \\
&= \frac{\delta^{kT}p_i}{1-\delta}\left\{\frac{\partial e_i^*}{\partial D_{ii}}\left(\delta D_{ii}g_{e_i^*}-1\right)+\delta g(e_i^*,\alpha_i)-(1-\delta^{T-1})\left[\frac{\partial \bar{e}_i}{\partial D_{ii}}\left(\delta D_{ii}g_{\bar{e}_i}-1\right)+\delta g(\bar{e}_i,\alpha_i)\right]\right\} \\
&= \frac{\delta^{kT}p_i}{1-\delta}\left(\delta[g(e_i^*,\alpha_i)-g(\bar{e}_i,\alpha_i)]+\delta^T g(\bar{e}_i,\alpha_i)-\left(1-\delta D_{ii}g_{e_i^*}\right)\frac{\partial e_i^*}{\partial D_{ii}}\right).
\end{aligned}
$$

The overall effect of D_{ii} on $\Pi_i^c-\Pi_i^d$ is given by the sum of two terms of opposite signs, and is thus ambiguous (due to the expression of $\frac{\partial e_i^*}{\partial D_{ii}}$ provided in the table, when p_i is small one might expect $\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dD_{ii}}$ to be positive).

Effect of D_{ij}

We now analyze the effect of dispersal from patch i on agent i's willingness to cooperate. We have:

$$
\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dD_{ij}} = \frac{\delta^{kT}p_i}{1-\delta}\left(\delta\frac{\partial x_i^*}{\partial e_i^*}\frac{\partial e_i^*}{\partial D_{ij}}-\frac{\partial e_i^*}{\partial D_{ij}}\right) = -\frac{\delta^{kT}p_i}{1-\delta}\cdot\frac{\partial e_i^*}{\partial D_{ij}}\left(1-\delta D_{ii}g_{e_i^*}\right)<0
$$

An increase in dispersal from patch i decreases agent i's incentives to cooperate.

Effect of D_{ji}

We finally analyze the effect of dispersal from a given patch *to* patch i on agent i's willingness to cooperate. We have:

$$
\begin{aligned}
\frac{d\left(\Pi_i^c-\Pi_i^d\right)}{dD_{ji}} &= \frac{\delta^{kT}p_i}{1-\delta}\left[\delta\left(\frac{\partial x_i^*}{\partial D_{ji}}+\frac{\partial x_i^*}{\partial e_j^*}\frac{\partial e_j^*}{\partial D_{ji}}\right)-\delta(1-\delta^{T-1})\left(\frac{\partial \bar{x}_i}{\partial D_{ji}}+\frac{\partial \bar{x}_i}{\partial e_j^*}\frac{\partial e_j^*}{\partial D_{ji}}\right)\right] \\
&= \frac{\delta^{(k+1)T}p_i}{1-\delta}\left[\frac{\partial e_j^*}{\partial D_{ji}}D_{ji}g_{e_j^*}+g(e_j^*,\alpha_j)\right]>0
\end{aligned}
$$

An increase in dispersal from patch j *to* patch i increases agent i's incentives to cooperate.

Impact on the time threshold, $\bar{T}_i$

Let us remind the expression of the time threshold:

$$
\bar{T}_i = \frac{\ln\left(\frac{\delta(\bar{x}_i-x_i^*)+e_i^*-\bar{e}_i}{\delta\bar{x}_i-\bar{e}_i}\right)}{\ln(\delta)} \tag{29}
$$

Differentiating with respect to parameter θ, we have:

$$
\begin{aligned}
\frac{d\bar{T}_i}{d\theta} &= \frac{\partial \bar{T}_i}{\partial \theta} + \frac{\partial \bar{T}_i}{\partial \bar{x}_i}\frac{d\bar{x}_i}{d\theta} + \frac{\partial \bar{T}_i}{\partial \bar{e}_i}\frac{\partial \bar{e}_i}{\partial \theta} + \frac{\partial \bar{T}_i}{\partial x_i^*}\frac{dx_i^*}{d\theta} + \frac{\partial \bar{T}_i}{\partial e_i^*}\frac{\partial e_i^*}{\partial \theta} \\
&= \frac{1}{\ln(\delta)\left[\delta(\bar{x}_i - x_i^*) + e_i^* - \bar{e}_i\right]}\left[\frac{\partial e_i^*}{\partial \theta} - \delta\frac{dx_i^*}{d\theta} + \left(\frac{\delta x_i^* - e_i^*}{\delta \bar{x}_i - \bar{e}_i}\right)\left(\delta\frac{d\bar{x}_i}{d\theta} - \frac{\partial \bar{e}_i}{\partial \theta}\right)\right] \qquad (30)
\end{aligned}
$$

Since $\delta \in (0,1)$ and $\delta(\bar{x}_i - x_i^*) + e_i^* - \bar{e}_i > 0$, we know that the first term of equation (30) is always negative. Thus, in order to sign the effect of parameter θ on $\bar{T}_i$ we just need to study the variation of the term between brackets with respect to this parameter.

Impact of net price, b

Effect of p_i

We first assess the impact of the net price in patch i on its own time threshold. We use expressions (22)-(25) and those in the table. We can rewrite the relevant term as follows:

$$
\frac{\partial e_i^*}{\partial p_i}\left(1 - \delta\frac{\partial x_i^*}{\partial e_i^*}\right) - \delta\sum_{j\neq i}\frac{\partial x_i^*}{\partial e_j^*}\frac{\partial e_j^*}{\partial p_i} + \delta\left(\frac{\delta x_i^* - e_i^*}{\delta \bar{x}_i - \bar{e}_i}\right)\sum_{j\neq i}\frac{\partial \bar{x}_i}{\partial e_j^*}\frac{\partial e_j^*}{\partial p_i} \qquad (31)
$$

$$
\Leftrightarrow \quad \frac{\partial e_i^*}{\partial p_i}(1 - \delta D_{ii} g_{e_i}) + \delta\sum_{j\neq i} D_{ji} g_{e_j}\frac{\partial e_j^*}{\partial p_i}\left(\frac{\delta(x_i^* - \bar{x}_i) - e_i^* + \bar{e}_i}{\delta \bar{x}_i - \bar{e}_i}\right) \qquad (32)
$$

We know that $1 - \delta D_{ii} g_{e_i} \geqslant 0$ and $\frac{\delta(x_i^* - \bar{x}_i) - e_i^* + \bar{e}_i}{\delta \bar{x}_i - \bar{e}_i} < 0$, and these inequalities imply that the above term is negative, which in turn yields $\frac{d\bar{T}_i}{dp_i} > 0$. An increase in the net price in patch i increases the threshold value $\bar{T}_i$.

Effect of p_j, $j \neq i$

Now we analyze the effect of the net price in patch $j \neq i$ on the value of $\bar{T}_i$. We have:

$$
\frac{\partial e_i^*}{\partial p_j}\left(1 - \delta\frac{\partial x_i^*}{\partial e_i^*}\right) - \delta\sum_{l\neq i}\frac{\partial x_i^*}{\partial e_l^*}\frac{\partial e_l^*}{\partial p_j} + \delta\left(\frac{\delta x_i^* - e_i^*}{\delta \bar{x}_i - \bar{e}_i}\right)\sum_{l\neq i}\frac{\partial \bar{x}_i}{\partial e_l^*}\frac{\partial e_l^*}{\partial p_j} \qquad (33)
$$

$$
\Leftrightarrow \quad \frac{\partial e_i^*}{\partial p_j}(1 - \delta D_{ii} g_{e_i}) + \delta\sum_{l\neq i} D_{li} g_{e_l}\frac{\partial e_l^*}{\partial p_j}\left(\frac{\delta(x_i^* - \bar{x}_i) - e_i^* + \bar{e}_i}{\delta \bar{x}_i - \bar{e}_i}\right) \qquad (34)
$$

$$
\Leftrightarrow \quad \frac{\partial e_i^*}{\partial p_j}(1 - \delta D_{ii} g_{e_i}) + \delta\left(\frac{\delta(x_i^* - \bar{x}_i) - e_i^* + \bar{e}_i}{\delta \bar{x}_i - \bar{e}_i}\right)\left(D_{ji} g_{e_j}\frac{\partial e_j^*}{\partial p_j} + \sum_{l\neq i,j} D_{li} g_{e_l}\frac{\partial e_l^*}{\partial p_j}\right) \qquad (35)
$$

Using the expressions provided in the table, we can obtain conclusions that highlight that the effect on $\bar{T}_i$ depends on the dispersal process. Specifically, we have:

- First, if D_{ij} is small enough, then expression (18) is negative, which implies that the value of $\bar{T}_i$ increases when p_j increases;

- Second, if D_{ji} and $\sum_{l\neq i,j} D_{li}D_{lj}$ are small enough, then expression (18) is positive, which implies that the value of $\bar{T}_i$ decreases when p_j increases.
  Explanation: this leads to a small expression of the last term in brackets, $D_{ji}g_{e_j}\frac{\partial e_j^*}{\partial p_j} + \sum_{l\neq i,j} D_{li}g_{e_l}\frac{\partial e_l^*}{\partial p_j}$. Thus, the sign of of $\frac{d\bar{T}_i}{dp_j}$ depends only on $\frac{\partial e_i^*}{\partial p_j}(1-\delta D_{ii}g_{e_i})$ which is positive. We thus conclude that $\frac{\partial \bar{T}_i}{\partial p_j}$ is negative.

Impact of growth, α

Effect of α_i

We analyze the impact of the growth rate in patch i, α_i on the value of $\bar{T}_i$. Again, all we need to do is to focus on the term between brackets in expression (30). We have:

$$\begin{aligned}
&\frac{\partial e_i^*}{\partial \alpha_i}\left(1-\delta\frac{\partial x_i^*}{\partial e_i^*}\right)-\delta\frac{\partial x_i^*}{\partial \alpha_i}+\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)\left[\delta\left(\frac{\partial \bar{x}_i}{\partial \alpha_i}+\frac{\partial \bar{x}_i}{\partial \bar{e}_i}\frac{\partial \bar{e}_i}{\partial \alpha_i}\right)-\frac{\partial \bar{e}_i}{\partial \alpha_i}\right] \qquad (36)\\
\Leftrightarrow\quad &\frac{\partial e_i^*}{\partial \alpha_i}\left(1-\delta D_{ii}g_{e_i^*}\right)-\delta D_{ii}g_{\alpha_i^*}-\frac{\partial \bar{e}_i}{\partial \alpha_i}\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)(1-D_{ii}g_{\bar{e}_i})+\delta D_{ii}g_{\bar{\alpha}_i}\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)
\end{aligned}$$

If D_{ii} is sufficiently small while $\bar{e}_i$ remains positive, then the sign of expression (18) is positive, which implies that $\bar{T}_i$ would decrease when the growth-related parameter increases in patch i.

Effect of α_j

We now analyze the effect of α_j ($j\neq i$) on the value of $\bar{T}_i$. We have:

$$\begin{aligned}
&-\delta\left(\frac{\partial x_i^*}{\partial \alpha_j}+\frac{\partial x_i^*}{\partial e_j^*}\frac{\partial e_j^*}{\partial \alpha_j}\right)+\delta\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)\left(\frac{\partial \bar{x}_i}{\partial \alpha_j}+\frac{\partial \bar{x}_i}{\partial e_j^*}\frac{\partial e_j^*}{\partial \alpha_j}\right)\\
\Leftrightarrow\quad &-\delta D_{ji}\left[g_{\alpha_j^*}+g_{e_j^*}\frac{\partial e_j^*}{\partial \alpha_j}+\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)\left(g_{\alpha_j^*}+g_{e_j^*}\frac{\partial e_j^*}{\partial \alpha_j}\right)\right]\\
\Leftrightarrow\quad &-\delta D_{ji}\left(g_{\alpha_j^*}+g_{e_j^*}\frac{\partial e_j^*}{\partial \alpha_j}\right)\left(1+\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)<0
\end{aligned}$$

We thus conclude that $\frac{d\bar{T}_i}{d\alpha_j}>0$, an increase in the value of the growth-related parameter in another patch results in an increase in the value of $\bar{T}_i$.

Impact of dispersal rate, D

Effect of D_{ii}

We first assess the effect of the self-retention rate in patch i on the value of $\bar{T}_i$. We have:

$$\begin{aligned}
&\frac{\partial e_i^*}{\partial D_{ii}}\left(1-\delta\frac{\partial x_i^*}{\partial e_i^*}\right)-\delta\frac{\partial x_i^*}{\partial D_{ii}}+\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)\left[\delta\left(\frac{\partial \bar{x}_i}{\partial D_{ii}}+\frac{\partial \bar{x}_i}{\partial \bar{e}_i}\frac{\partial \bar{e}_i}{\partial D_{ii}}\right)-\frac{\partial \bar{e}_i}{\partial D_{ii}}\right]\\
\Leftrightarrow\quad &\frac{\partial e_i^*}{\partial D_{ii}}\left(1-\delta D_{ii}g_{e_i^*}\right)-\delta g(e_i^*,\alpha_i)+\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)\left[\delta g(\bar{e}_i,\alpha_i)+\frac{\partial \bar{e}_i}{\partial D_{ii}}\left(\delta D_{ii}g_{\bar{e}_i}-1\right)\right]\\
\Leftrightarrow\quad &\frac{\partial e_i^*}{\partial D_{ii}}\left(1-\delta D_{ii}g_{e_i^*}\right)-\delta\underbrace{\left[g(e_i^*,\alpha_i)-\left(\frac{\delta x_i^*-e_i^*}{\delta \bar{x}_i-\bar{e}_i}\right)g(\bar{e}_i,\alpha_i)\right]}_{>0}
\end{aligned}$$

If $D_{ii} = 1$, then $1 - \delta D_{ii} g_{e_i^*} = 0$. Therefore, $\frac{d\bar{T}_i}{dD_{ii}} > 0$.

No evident conclusion.

Effect of D_{ji}

We now assess the effect of dispersal from another patch $j \neq i$ *to* patch i on the value of $\bar{T}_i$. In this case we know that $\frac{\partial e_i^*}{\partial D_{ji}} = \frac{\partial \bar{e}_i}{\partial D_{ji}} = 0$, which implies that we have:

$$
\begin{aligned}
& -\delta \left[\frac{\partial x_i^*}{\partial D_{ji}} + \frac{\partial x_i^*}{\partial e_j^*} \frac{\partial e_j^*}{\partial D_{ji}} \right] + \delta \left(\frac{\delta x_i^* - e_i^*}{\delta \bar{x}_i - \bar{e}_i} \right) \left[\frac{\partial \bar{x}_i}{\partial D_{ji}} + \frac{\partial \bar{x}_i}{\partial e_j^*} \frac{\partial e_j^*}{\partial D_{ji}} \right] \\
\Leftrightarrow \quad & -\delta \left[g(e_j^*, \alpha_j) + D_{ji} g_{e_j^*} \frac{\partial e_j^*}{\partial D_{ji}} \right] + \delta \left(\frac{\delta x_i^* - e_i^*}{\delta \bar{x}_i - \bar{e}_i} \right) \left[g(e_j^*, \alpha_j) + D_{ji} g_{e_j^*} \frac{\partial e_j^*}{\partial D_{ji}} \right] \\
\Leftrightarrow \quad & -\delta \left(g(e_j^*, \alpha_j) + D_{ji} g_{e_j^*} \frac{\partial e_j^*}{\partial D_{ji}} \right) \left(1 - \frac{\delta x_i^* - e_i^*}{\delta \bar{x}_i - \bar{e}_i} \right) < 0
\end{aligned}
$$

We thus conclude that $\frac{d\bar{T}_i}{dD_{ji}} > 0$, an increase in dispersal from patch $j \neq i$ *to* patch i results in an increase in the value of $\bar{T}_i$.

Effect of D_{ij}

We finally study the effect of dispersal from patch i on the value of $\bar{T}_i$. In this case we know that $\frac{\partial \bar{e}_i}{\partial D_{ij}} = \frac{\partial \bar{x}_i}{\partial D_{ij}} = 0$, which implies that we have:

$$
\frac{\partial e_i^*}{\partial D_{ij}} \left(1 - \delta \frac{\partial x_i^*}{\partial e_i^*} \right) = \frac{\partial e_i^*}{\partial D_{ij}} \left(1 - \delta D_{ii} g_{e_i^*} \right) > 0
$$

Thus, we conclude that $\frac{d\bar{T}_i}{dD_{ij}} < 0$: the value of $\bar{T}_i$ decreases as the dispersal from patch i increases.

All results are summarized in the following table:

θ	p_i	p_j	α_i	α_j	D_{ii}	D_{ij}	D_{ji}
$\frac{dT_i}{d\theta}$	$+$	If D_{ij} small, then $+$ If D_{ji} and $\sum_{l \neq i,j} D_{li} D_{lj}$ small, then $-$	If D_{ii} small then $-$	$+$	?	$-$	$+$

Proof of Proposition 11

We know that the infinite-horizon instrument will induce all agents to cooperate if and only if, for any agent i, the following condition holds:

$$
\delta x_i^* - e_i^* - \left(1 - \delta^{T-1} \right) \left(\delta \bar{x}_i - \bar{e}_i \right) > 0, \tag{37}
$$

where $\bar{x}_i = D_{ii} g(\bar{e}_i) + \sum_{j \neq i} D_{ji} g(e_j^*)$ as before. We are going to prove that the finite version of the instrument induces all agents to cooperate during all but the last tenure block if condition (37) is satisfied for all agents. This will conclude the proof of the proposition.

First, consider what happens during the final tenure block K: then using backward induction reveals that any agent i's strategy during that block is characterized as follows:

$$e_{i,KT-1} = 0$$

and, for any other period $(K-1)T \le t \le KT-2$ we have $e_{i,t} = \bar{e}_i$ where:

$$1 = \delta D_{ii} g'(\bar{e}_i).$$

In other words, anticipating that he will not get renewed for sure at the end of the final tenure block, any agent i will defect. But in order to reach the final tenure block all agents will have managed the resource cooperatively (for the first $K-1$ tenure blocks). Thus, cooperative agents will play as follows (the first periode of the first tenure block being $t = 0$):

- during the first $K-1$ tenure blocks (thus from $t = 0$ to $t = (K-1)T - 1$) agent i chooses $e_i = e_i^*$: from $t = 1$ to $t = (K-1)T - 1$ the stock level is $x_i = x_i^*$, at period $t = 0$ one has $x_i = x_{i,0}$;
- then, at period $t = (K-1)T$, agent i chooses $e_i = \bar{e}_i$, and stock level at this same period $(K-1)T$ is still $x_i = x_i^*$;
- In all other periods of the final block but the last one, agent i chooses $e_i = \bar{e}_i$ and stock level is $\hat{x}_i = \sum_j D_{ji} g(\bar{e}_j)$;
- Finally, at $t = KT - 1$ we have $e_i = 0$ and $x_i = \hat{x}_i$.

This implies that the payoffs from cooperation are this time given by:

$$\Pi_i^c = p_i \left[x_{i,0} - e_i^* + \sum_{t=1}^{(K-1)T-1} \delta^t (x_i^* - e_i^*) + \delta^{(K-1)T} (x_i^* - \bar{e}_i) + \sum_{t=(K-1)T+1}^{KT-2} \delta^t (\hat{x}_i - \bar{e}_i) + \delta^{KT-1} \hat{x}_i \right].$$

Now, we have to consider agent i's potential unilateral deviation strategy. Assuming that this agent defects during tenure block $1 \le k < K$ (thus knowing that he will not be renewed following tenure block k) the timing of his strategy then becomes:

- From $t = 0$ to $t = (k-1)T - 1)$ agent i chooses $e_i = e_i^*$: from $t = 1$ to $t = (k-1)T - 1$ the stock level is $x_i = x_i^*$, at period $t = 0$ one has $x_i = x_{i,0}$;
- Then, at period $t = (k-1)T$, agent defects by choosing $e_i = \bar{e}_i$, and stock level at this same period $(k-1)T$ is still $x_i = x_i^*$;
- In all other periods of tenure block k but the last one, agent i chooses $e_i = \bar{e}_i$ and stock level is $x_i = \bar{x}_i$;
- Finally, at $t = kT - 1$ we have $e_i = 0$ and $x_i = \bar{x}_i$.

This implies that the payoffs from unilaterally deviating during tenure block $k < K$ are this time given by:

$$\Pi_i^d = p_i \left[x_{i,0} - e_i^* + \sum_{t=1}^{(k-1)T-1} \delta^t (x_i^* - e_i^*) + \delta^{(k-1)T} (x_i^* - \bar{e}_i) + \sum_{t=(k-1)T+1}^{kT-2} \delta^t (\bar{x}_i - \bar{e}_i) + \delta^{kT-1} \bar{x}_i \right].$$

Using the expressions of Π_i^c and Π_i^d (and computing the sums used in these expressions) we obtain:

$$\Pi_i^c - \Pi_i^d = p_i \frac{\delta^{(k-1)T}}{1-\delta} \{ \left(1 - \delta^{(K-k)T}\right) [\delta x_i^* - e_i^* + (1-\delta)\bar{e}_i]$$

$$+\delta(1-\delta^{T-2})\left[\delta^{(K-k)T}(\hat{x_i}-\bar{e_i})-(\bar{x_i}-\bar{e_i})\right]+\delta^{T-1}(1-\delta)\left[\delta^{(K-k)T}\hat{x_i}-\bar{x_i}\right]\}.$$

Now we prove our claim: let us assume that condition (37) is satisfied for any agent i. Then we obtain the following inequality:

$$\Pi_i^c-\Pi_i^d>p_i\frac{\delta^{(k-1)T}}{1-\delta}\{\left(1-\delta^{(K-k)T}\right)\left[(1-\delta^{T-1})\left(\delta\bar{x_i}-\bar{e_i}\right)+(1-\delta)\bar{e_i}\right]$$

$$+\delta(1-\delta^{T-2})\left[\delta^{(K-k)T}(\hat{x_i}-\bar{e_i})-(\bar{x_i}-\bar{e_i})\right]+\delta^{T-1}(1-\delta)\left[\delta^{(K-k)T}\hat{x_i}-\bar{x_i}\right]\}.$$

Rewriting the above inequality and simplifying, we obtain:

$$\Pi_i^c-\Pi_i^d>p_i\frac{\delta^{(k-1)T}}{1-\delta}\left[\delta^{T-1}(1-\delta+\delta^{(K-k)T})\bar{x_i}+\delta^{(K-k)T+1}(1-\delta^{T-2})\hat{x_i}+\delta^{(K-k)T+T-1}(1-\delta)\hat{x_i}\right]>0,$$

which implies that agent i does not have incentives to use a unilateral deviation: thus the finite version of the instrument induces agents to cooperate during all but the final tenure block.