Environmental Policy and the Long-Run Management of a Growing Stock of Waste

(Preliminary version)

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Abstract

Most environmental policies consist of assigning a cost to pollution flows, even when the externality stems from the stock of accumulated pollution rather than from the flow itself. When the environmental damage also depends on other economic variables, notably the way the stock is managed, such policies cannot yield first-best outcomes. This is the case for resources such as nuclear fuels or rare earths, whose environmental impacts can be reduced through certain investments, but also for hydrocarbons when climate change adaptation is taken into account. We present an endogenous growth model with a polluting non-renewable resource and directed technical change, in which the environmental damage depends on the accumulated stock of waste as well as on a stock of green knowledge dedicated to its management. We show that taxing the flow of resource use or waste yields no activity in the green R&D sector; the social optimum can only be achieved by taxing the stock of environmental damage itself. We then study the properties of the first-best policy, and we analyze the effects of a second-best tax. Unlike standard environmental tools, the tax does not modify time profiles of resource use and waste production, but it does reallocate the R&D efforts, steering technical change in the green direction. Moreover, we show that the tax is detrimental to growth.

Keywords: directed technical change; endogenous growth; environmental policy; non-renewable resources; waste stock.

JEL classification: O32, O41, O44, Q38, Q32

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1 Introduction

Most of the environmental policies that are currently implemented, or that are debated, aim to assign a cost to the emissions of pollutants. In many cases however, the environmental externality arises from the accumulated stock of pollutant - e.g. atmospheric CO_2 - rather than from the flow of polluting emissions. Indeed, it is the atmospheric concentration of GHG that is regarded as the cause of the rise in the average temperature at the earth's surface. Economic theory tells us that one should tax the stock rather than just the flow of emissions. Benchekroun and van Long (1998), for instance, have studied this type of policy. They have proved that taxing the stock can implement the socially optimal outcome, and they have studied the properties of the optimal tax. However, taxing pollutant flows can have an impact equivalent to directly taxing the total stock. Indeed, correcting the way pollutant flows accumulate is equivalent to correcting the evolution of the stock of waste. Such is the case in an important part of the literature, from the early contributions of Plourde (1972), who considers a dynamic framework with accumulating polluting emissions associated with the use of a resource, and Schulze (1974), in which the resource is non-renewable, as well as in subsequent contributions like Forster (1980), van der Ploeg and Withagen (1991), Ulph and Ulph (1994), Farzin (1996), Tahvonen (1997) or Goulder and Mathai (2000), to cite a few. This equivalence principle continues to hold even when abatement is taken into account - as in Forster, Plourde, van der Ploeg and Withagen, Farzin, and Goulder and Mathai -, only here taxing resource use is not equivalent to taxing the polluting flow.

Things are different if the environmental damage does not depend solely on the stock of waste. Indeed, other variables - such as investments dedicated to the management of the stock - can alter the environmental impact of a given stock level. In this context, a tax on pollution flows cannot yield a first-best outcome, and the need to tax the environmental damage emerges. The aim of the present paper is to study such a policy. We analyze the main properties of a policy scheme that taxes the environmental damage, and we show how it affects a decentralized economy. In particular, we consider its impact on the time profiles of resource extraction, the waste stock, research and development (R&D) activity and on output growth.

In the context of climate change, the adaptation effort can be considered as the way a given accumulated stock of pollutant is handled: for a given level of atmospheric CO_2 concentration, and thereby a given amount of radiative forcing and subsequent warming, the physical consequences of the changing climate can be partially reduced through investments in, say, sea levees for flood-prone areas, or greater efficiency of water consumption in drought-stricken areas. One can consider that this is also the case for (long-term) radioactive waste[1], rare earths, heavy metals or asbestos for instance; in all of these cases, the extraction or the use of resources yields waste that has to be correctly handled so as to limit its environmental impact. Here, the environmental disutility depends not only on the accumulated stock of pollutants, but crucially also on the way the stocks are managed. For a given flow of waste, and thus a given increase in

[1] Of course, the nuclear issue is much broader. We greatly simplify the chain of production, in particular leaving aside questions linked to uncertainty (leaks, explosions, etc.) or to the investments required to plant commissioning and maintenance.

the existing stock, the rise in the environmental damage depends on the methods and technologies used to handle it and, in some cases, store it. Simply put, the production of an identical quantity of plutonium has a very different impact depending on whether it is illegally dumped in the ocean or geologically stored by the military. For this reason, contrary to van der Ploeg and Withagen (1991), Ulph and Ulph (1994), Farzin (1996) or Goulder and Mathai (2000) for instance, one generally cannot expect a first-best outcome from a tax on resource use, polluting emissions or even the waste stock. Indeed, such a policy scheme will not provide the right incentives to invest in waste management. One must tax the environmental damage itself.

This policy alternative can be difficult to implement. Firstly, even when the waste stock is measurable, it may be impossible to fully identify all of its emitters. Indeed, in the case of climate change, if one wants to tax the stock of CO_2, who will bear such tax? It seems difficult in practice to assign precisely this or that part of the stock of CO_2 to each of the agents who have produced it. However, Billette de Villemeur et Leroux (2011) show that such a policy could be considered if it is correctly based on past emissions data. Furthermore, according to them, it would have the virtue of being more politically acceptable, and would allow avoiding the thorny questions of the anticipations of economic behaviors - including emissions -, and of the psychological discount rate. In the case of the nuclear industry for instance, and more generally in the case of local pollution, such a policy scheme is more conceivable than for climate change. Indeed, it seems technically easier to observe radioactive waste at the time of its production and to identify the producer of each stock. The question that remains however is the assessment of how the stock is managed. One can think that the improvements in, say, radioactive waste storage, can be correctly evaluated by independent agencies. Thus, in some cases, public authorities can realistically measure the environmental damage. Of course, the damage will have a different nature depending on the type of waste. In the case of nuclear waste, it can be the danger felt by people to live close to radioactive waste deposits. In the case of heavy metals or asbestos, it is the impact of the remaining pollution on public health.

We employ an endogenous growth model with Romer horizontal differentiation, in which the use of a non-renewable resource yields pollution flows that contribute to an existing stock. There are two kinds of knowledge. The first is "standard" knowledge, that is, knowledge that helps improving the productivity of output production. The second stock of knowledge is dedicated to reducing the negative impact of the accumulated stock of waste. Indeed, this pollution stock can be managed in order to limit its environmental impact: what (negatively) affects households' utility is a combination of this stock with the stock of dedicated knowledge, which we refer to as "green" knowledge. The environmental damage can be for example the inconvenience to live in the presence of radioactive waste stored in deep geological repositories[2]. It can also be the pollution of groundwater, which can be reduced by decontamination process. Here, for a given waste stock (or atmospheric concentration), the higher the level of green knowledge is, the lower the environmental damage, that is, we partially endogenize the environmental damage. Note that, contrary to Acemoglu-type models of polluting resources directed technical

[2]We do not take into account here issues like the necessary perpetual monitoring of the burried waste stock.

change like Grimaud and Rouge (2008) or Acemoglu et al. (2012), the second R&D sector does not improve the productivity of a backstop technology, but it is dedicated to limiting the environmental impact of the accumulated waste. For this reason, the two research sectors do not have symmetric effects on output and growth. We show that without environmental policy, or with a tax on resource use or polluting emissions, no green R&D is performed. We therefore consider a policy consisting of a tax on the environmental damage itself, and study its first-best properties. The implementation of such a policy implies that the government has the ability to measure the environmental damage through, for example, contingent valuation. Since it is likely that the policy maker will not be able to set the tax at its Pigovian level in the real world, we then analyze the economic effects of a second-best tax. We show that they are different from the ones of current environmental policies.

Formally, our framework is close to Grimaud and Rouge (2005), with three main differences. First, there are two types of knowledge. One is "standard knowledge", whose accumulation increases the productivity of the consumption good sector; the other one is the aforementioned green knowledge, which is dedicated to the management of the existing stock of waste. Secondly, it is not the flow of non-renewable resource that affects households' utility, but a joint function of the existing accumulated stock of pollution and the stock of green knowledge. Third, we study an economic policy consisting of a tax on the environmental damage, as opposed to a tax on the flow of pollution.

The decentralized economy asymptotically converges towards a steady-state in which the growth rate of resource extraction is negative and constant, and the stock of waste remains constant. Along the transition, the effort devoted to standard R&D increases, and the effort in green R&D decreases: the bias of technical progress is less and less green. If the tax grows fast enough, the environmental damage progressively decreases.

We then study the effects of a second-best - Pareto-improving - environmental policy consisting of a tax on the environmental damage growing at a rate inferior to its first-best level. We show that, contrary to standard environmental policies on resource extraction or polluting emissions, this policy does not affect the time profile of resource extraction and polluting emissions. Instead, it steers the bias of technical change in the green direction, and reduces the growth of the environmental damage. However, this policy is bad for output growth.

We present the model and the socially optimal outcome in section 2. Then, in section 3, we study the decentralized economy and the equilibrium conditions. We characterize the first-best environmental policy in section 4, and section 5 is devoted to the analysis of the effects of the environmental policy. The final section provides concluding remarks.

2 The model

At each date $t \in [0, +\infty)$, a quantity Y_t of consumption good is produced according to the following technology[3]:

$$Y_t = F(A_t, R_t). \tag{1}$$

A_t is a stock of "standard" knowledge, *à la Romer* - not to be confused with "green" knowledge, introduced below. R_t is a flow of non-renewable natural resource[4]. We will denote by $F_A(.)$ and $F_R(.)$ the marginal productivities; both are strictly positive.

Technology for the production of standard knowledge - *à la* Romer - is:

$$\dot{A}_t = \delta_A L_{At} A_t, \tag{2}$$

where δ_A is an exogenous parameter characterizing the efficiency of this research sector, and L_{At} is the amount of labour put into it.

The resource flow R_t is extracted from a finite stock S_t, according to the standard law of motion:

$$\dot{S}_t = -R_t. \tag{3}$$

As is often done in the literature on growth and non-renewable resources, extraction costs are omitted here.

The use of the natural resource yields a flow P_t of waste. We consider that P_t is a linear function of R_t: $P_t = hR_t$, where h is an exogenous, strictly positive parameter. This simple relationship means that the use of a given amount of resource to produce energy entails a given amount of waste production. This waste adds to the existing stock: $W_t = W_0 + \int_0^t hR_s ds$. We thus have the following law of motion:

$$\dot{W}_t = P_t = hR_t. \tag{4}$$

Like in Schulze (1974), we do not consider pollution decay. Tahvonen (1997) points out that this allows simplifying the relation between the stock of resource and the stock of waste: at each date t, we have $W_t = W_0 + h(S_0 - S_t)$. We assume here that the waste stock is nil at date 0, thus we simply have $W_t = h(S_0 - S_t)$.

Any positive amount of waste stock is stored and managed. In the real world, in the case of uranium for instance, waste management varies from illegal dumping to highly-managed and capital-intensive geologic disposal. Here, we consider that, at any time t, there is one unique technique for waste management, which depends on the current state of dedicated knowledge: B_t. Technology for the production of waste management knowledge, to which we will henceforth

[3] In the first part of the paper, the functional forms of the consumption good technology, the environmental damage and utility are not specified in order to get more general results; we provide specified functional forms in section 5.

[4] Note that we do not consider labour as an input of the consumption good's production function as in Grimaud and Rouge (2005). If we did so, this flow of labour would be constant in equilibrium, and it would not be affected by the environmental policy. Therefore, its presence would add no further insight to the main results of our analysis.

refer to as "green" knowledge, is:

$$\dot{B}_t = \delta_B L_{Bt} B_t, \tag{5}$$

where δ_B is an efficiency parameter, and L_{Bt} is the flow of labour dedicated to this specific research.

We distinguish between the actual stock of waste and its environmental impact. The environmental damage consists of the local or global effects caused by waste disposal of long duration. This damage can include the risk of leaks or accidental or purposeful unearthing of these dangerous stockpiles, but also the discomfort that people have to live in the presence of such hazard. In the case of climate change, this is the remaining negative impact of global warming for a given level of adaptation (e.g. sea level increases overcoming levees and floodwalls). Obviously, the more advanced the technique of maintenance of the stock - represented here by the level of green knowledge, B_t-, the lower the damage. The latter is denoted by Ω_t. This environmental damage thus depends on the stock of waste, W_t, and on the technology used to manage it, that is, the stock of green knowledge. We have the following functional relation:

$$\Omega_t = \Omega(W_t, B_t). \tag{6}$$

Ω_W is the marginal environmental damage caused by waste, and Ω_B is the marginal environmental benefit from green knowledge. Thus, we assume $\Omega_W(.) > 0$ and $\Omega_B(.) < 0$.

The representative household is endowed with a constant flow L of labour, which we normalize to one. Labour has two competing uses: research in the general purpose sector (L_{At}), and research in the waste management sector (L_{Bt}):

$$1 = L_{At} + L_{Bt}. \tag{7}$$

The representative household's instantaneous utility depends positively on the current level of consumption C_t, which is equal to the entire production of good Y_t ($C_t = Y_t$), and negatively on the environmental impact of the stock of waste, Ω_t. We denote by u_C and u_Ω the marginal utility of consumption and the marginal disutility of the environmental damage; thus $u_C(.) > 0$ and $u_\Omega(.) < 0$. The intertemporal utility function is

$$U_0 = \int_0^{+\infty} u(C_t, \Omega_t) e^{-\rho t} dt, \tag{8}$$

where ρ is the psychological discount rate.

3 Decentralized economy and environmental policy tools

The price of good Y is normalized to one, and w_t, p_t^R and r_t are, respectively, the wage, the resource price and the interest rate on a perfect financial market.

3.1 Environmental policy tools

In the economy studied here, a standard environmental policy cannot yield first-best results. For the firm producing the consumption good, knowledge can have two uses: either as an input to production of this good or as an input to the management of the stock of waste generated by the production activity. Consequently, knowledge is also going to be used within the $R\&D$ sector, since it is an input of its own production (see equations (2) and (5)). Therefore, if the consumption good sector does not value knowledge, it will not be produced, since markets produce no incentive to $R\&D$. This is what happens for green knowledge if no environmental policy is implemented, or if the environmental tax is levied on the flow of resource or the flow of waste: in these cases, it has no value. Consider the case of a unit tax θ_t on polluting emissions, like a carbon tax. At each date t, the profit of the firm producing the consumption good is given by $\pi_t^Y = F(A_t, R_t) - p_t^R R_t - \theta_t h R_t$, since polluting emissions are equal to hR_t (see formula (4)). For all $\theta_t \geq 0$, the marginal profitabilities of the two inputs that are standard knowledge and the resource flow are respectively $F_A(.)$ and $F_R(.) - (p_t^R + \theta_t h)$. Both are independent from the level of green knowledge (B_t). It is then straightforward that this policy instrument does not yield any incentive to produce green knowledge, despite the fact that it is necessary to attain the socially optimal level of environmental damage[5]. The same applies to an ad valorem tax θ'_t on resource use: the profit is then $\pi_t^Y = F(A_t, R_t) - p_t^R R_t - \theta'_t h R_t$, and here also the marginal profitabilities are independent from green knowledge.

We thus consider here a unit tax τ_t levied at each date t on the environmental damage Ω_t itself[6]. As previously mentioned, we consider here that the government has perfect information on the environmental damage. For given levels of tax and stock of waste W_t, the firm will pay more or less depending on whether the state of knowledge in the field of waste management, B_t, is low or high.

At each time t, the firm's instantaneous profit is

$$\pi_t^Y = F(A_t, R_t) - p_t^R R_t - \tau_t \Omega(W_t, B_t).$$

The resource demand function is given by the differentiation of π_t^Y with respect to R_t. We have $F_R = p_t^R$: the tax does not appear, contrary to what we have with more usual environmental policies such as a tax on resource use.

3.2 Agents' behaviour

3.2.1 Consumption good sector

Because of the particular type of tax we use, an intertemporal dimension is added to the standard maximization program of the firm. Given that using a flow of resource R_t at date $\underline{t}$ means increasing the stock of waste, for a given time profile of the stock of green knowledge, the total sum paid at $\underline{t}$ and at any date $t > \underline{t}$ is also increasing: in other words, costs rise ad infinitum. At each date t, the chosen resource use generates a pollution flow, which determines

[5] We prove this point later in section 4.

[6] We show later that this tool, when set at a certain level -presented in section 4.2-, yields a first-best solution.

an associated specific time path for the stock of waste, and consequently a specific temporal profile of environmental tax payments from date t onwards. Formally, profit function π_t^Y features the control variable R_t and, because of the tax τ_t, the state variable W_t which depends on R_s for all $s \in (0, t)$, as stated in (4). The program of the firm is thus:

$$\max_R \int_0^{+\infty} [F(A_t, R_t) - p_t^R R_t - \tau_t \Omega(W_t, B_t)] e^{-\int_0^t r_u du} dt \qquad (9)$$
$$\text{subject to } \dot{W}_t = hR_t \text{ for all } t.$$

From the maximum principle, we obtain two first order conditions with respect to R_t and W_t which, after elimination of the costate variable, yield the following condition:

$$\tau_t h \Omega_W = (F_R - p_{Rt}) r_t - (F_{RA} \dot{A}_t + F_{RR} \dot{R}_t - \dot{p}_{Rt}). \qquad (10)$$

This condition describes how the profit-maximizing firm uses the resource and thereby manages the accumulation of waste. At each date t, a marginal increase in resource use yields an additional profit equal to $F_R - p_{Rt}$. Investing it in the financial market generates an instantaneous income equal to $(F_R - p_{Rt})r_t$. Not keeping this resource in situ yields a potential loss due to the evolution of the resource's marginal productivity and its price: $\dot{F}_R - \dot{p}_{Rt}$. Hence, the right-hand side (hereafter RHS) of (10) stands for the net profitability of this marginal increase in resource use, without taking into account the environmental policy. This additional resource use leads to an increase in the stock of waste by h units, and consequently in the environmental damage of $h\Omega_W(W_t, B_t)$. Hence, at each date t, because of the environmental policy, the cost for the firm is $\tau_t h \Omega_W(W_s, B_s)$, that is, the left-hand side (hereafter LHS) of condition (10). This condition thus tells us that the cost of extracting more resource must be equal to its benefit.

3.2.2 Resource sector

On the competitive natural resource market, the maximization of the profit function $\int_t^{+\infty} p_s^R R_s e^{-\int_t^s r_u du} ds$ subject to $\dot{S}_s = -R_s$, $S_s \geq 0, R_s \geq 0, s \geq t$, yields the standard Hotelling rule in the decentralized equilibrium:

$$\dot{p}_t^R / p_t^R = r_t, \text{ for all } t. \qquad (11)$$

3.2.3 Representative household

At each date t, the representative household maximizes the utility function (8) subject to the following budget constraint: $\dot{b}_t = r_t b_t + w_t + p_t^R R_t - T_t - C_t$, where b_t is the stock of bonds at date t, and T_t is the lump-sum tax levied by the government to finance research. This maximization leads to the following Ramsey-Keynes condition:

$$r_t = \rho - \frac{u_{CC}\dot{C} + u_{C\Omega}\dot{\Omega}}{u_C}. \qquad (12)$$

3.2.4 R&D sectors

The basic structure of each *R&D* sector is identical to the one in Grimaud and Rouge (2005). As in their model, knowledge is directly financed and we assume that once an innovation has occurred, the government pays to the innovator a sum equal to the willingnesses to pay of the sectors using it[7].

For both R&D sectors, the profit on innovations produced at date t, is $\pi_t^{RD(i)} = \delta_i L_{it} i_t V_{it} - w_t L_{it}$, with $i = A, B$. Here, V_{it} is the value of one innovation in sector i, and $V_{it} \equiv \int_0^{+\infty} v_{is} e^{-\int_t^s r_u du} ds$, where v_{is} is the price of this innovation at date s. The maximization of this profit function with respect to L_{it} leads to the following first-order condition: $\delta_i i_t V_{it} = w_t$, with $i = A, B$. Then, log-differentiating this condition with respect to time yields the condition:

$$\frac{\dot{w}_t}{w_t} - r_t = -\frac{v_{it}}{V_{it}} + \delta_i L_{it}, \text{ for all } t, \text{ with } i = A, B. \tag{13}$$

The value for one unit of knowledge in sector i is $v_{it} = \frac{\partial \pi_t^Y}{\partial i_t} + \frac{\partial \pi_t^{RD}}{\partial i_t}$ with $i = A, B$. Hence, in the standard R&D sector, condition (13) becomes

$$\frac{\dot{w}_t}{w_t} - r_t = -F_A \delta_A A_t / w_t, \tag{14}$$

and, in the green R&D sector, it becomes

$$\frac{\dot{w}_t}{w_t} - r_t = \delta_B \tau_t \Omega_B B_t / w_t. \tag{15}$$

Conditions (14) and (15) express the marginal return of labour in the standard and the green R&D sectors, respectively. Note that the value of green research is only induced by the environmental policy. In other words, there would be no green research without the tax[8].

3.3 General equilibrium conditions

Equations (14) and (15) together yield

$$F_A \delta_A A_t = -\tau_t \Omega_B \delta_B B_t. \tag{16}$$

This is the equilibrium non-arbitrage condition between the R&D sectors. This condition states that the rate of return must be the same in both R&D sectors.

Equation (11) allows us replacing $\dot{p}_{Rt}$ by $p_{Rt} r_t$ in (10): one obtains $h \tau_t \Omega_W = F_R r_t - (F_{RA} \dot{A}_t +$

[7] By doing this, we avoid distortions (monopoly rents, intertemporal spillovers...) in the R&D sectors that would greatly increase the complexity of the model. This allows us to focus on the distortion caused by the use of the non-renewable resource. As a consequence, when the tax is set at its first-best level, the decentralized economy reaches the social optimum.

[8] In section 5.3.1, we show that the socially optimal effort in green research is positive. Hence, to avoid the corner solution where only the standard R&D sector is active, we focus on the case where the tax is high enough to trigger a non zero - if not optimal - activity in this R&D sector.

$F_{RR}\dot{R}_t$). Then, eliminating r_t in this equation and (12), we get

$$\rho - \frac{u_{CC}\dot{C} + u_{C\Omega}\dot{\Omega}}{u_C} = \frac{F_{RA}\dot{A}_t + F_{RR}\dot{R}_t}{F_R} + \frac{h\tau_t\Omega_W}{F_R}. \tag{17}$$

Equilibrium condition (17) states that, if the firm marginally decreases production at date t, then the benefit at date $t + \Delta t$ is equal to the quantity of good that compensates consumers at date $t+\Delta t$ from the marginal loss of consumption at t. To illustrate this, consider a given growth path of the economy, and suppose that the firm producing the consumption good marginally reduces production at date t. The LHS of condition (17) corresponds to the value of the amount of consumption good that compensates households at date $t + \Delta t$ for the marginal loss of consumption good at date t. Not producing this amount of good allows the firm to save a quantity of resource equal to $1/F_{Rt}$. The RHS of equation (17) represents the firm's advantage in leaving this resource quantity unextracted, which contains two separate components. The first benefit is a higher productivity of the resource - represented by the first term in the RHS. The second benefit stems from the fact that, for a given path of the stock of green knowledge, B_t, forgoing this flow of resource, and hence not increasing the stock of environmental damage, means smaller payments of environmental taxes - this is represented by the second term in the RHS. The equilibrium condition (17) equates the sum of these two benefits to the firm to the amount of good that allows keeping households' intertemporal utility unchanged.

4 First-best environmental policy

4.1 First-best social optimum

We now analyze the socially optimal outcome for the economy. The social planner's solves the following maximization problem: max (8) subject to (2)-(7). The Hamiltonian associated with this program is:

$$\mathcal{H} = u\left[F(A_t, R_t), \Omega(h(S_0 - S_t), B_t)\right] + \mu_{At}\delta_A L_{At} A_t + \mu_{Bt}\delta_B(1 - L_{At})B_t - \mu_{St}R_t, \tag{18}$$

where μ_{At}, μ_{Bt} and μ_{St} are the costate variables associated to constraints (2), (5) and (3) respectively. The first order conditions for L_{At}, A_t and B_t together yield the following condition

$$u_C F_A \delta_A A_t = u_\Omega \Omega_B \delta_B B_t. \tag{19}$$

This condition, which is the counterpart of condition (16) in the first-best context, establishes the equality between the marginal utilities of labor in the two research sectors. Suppose a marginal decrease in the flow of labor dedicated to standard research, L_{At}, at date t. This reduction has an impact on the accumulation of knowledge, in turn on output production, and subsequently on consumption and utility. This decline in the instantaneous utility is described by the LHS of the equation. Suppose now that the amount of labor that has been accordingly saved is transferred to the green research sector: L_{Bt} marginally increases. Then, the stock of green knowledge, B_t, increases, which, for a given non-renewable resource extraction path and, in turn, a given polluting emissions path, diminishes the environmental damage, and increases instantaneous utility. This rise in utility is given by the RHS of the equation. Condition (19), which equalizes these two variations of utility, describes the socially optimal arbitrage between the two research sectors.

By differentiating the first order condition for R_t with respect to time, and using the expression of the growth rate of μ_{St} obtained in the first order condition for S_t, one gets the counterpart of condition (17):

$$\rho - \frac{u_{CC}\dot{C} + u_{C\Omega}\dot{\Omega}}{u_C} + \frac{h}{F_R}\left(\frac{u_\Omega}{u_C}\right)\Omega_W = \frac{F_{RA}\dot{A}_t + F_{RR}\dot{R}_t}{F_R}. \tag{20}$$

This corresponds to the Ramsey-Keynes condition in the particular context of this model. As in simple growth models, that is, without resources or pollution, this condition equates two marginal rates of substitution, one for consumer utility, the other for production. A marginal decrease in consumption at date t yields a decrease in instantaneous utility that can be compensated in terms of discounted utility by a rise in consumption at date $t + \Delta t$. In the LHS of condition (20), the term $\frac{h}{F_R}\left(\frac{u_\Omega}{u_C}\right)\Omega_W$, negative by definition, shows that this increase is lower than in the standard case. Indeed, the reduction in consumption at t is due to a decrease in R_t, which, for a given level of knowledge B_t, corresponds to a decrease in the environmental damage, and thus an increase in utility. The RHS represents the growth rate of the marginal productivity of the resource: a marginal decrease of output at date t allows saving a quantity of resource $1/F_{Rt}$, which, used at date $t + \Delta t$, allows an increase in output Y_t by $\dot{F}_{Rt}/F_{Rt}$.

4.2 First-best environmental policy

In order to characterize the first-best environmental policy, we need to compare the preceding decentralized equilibrium conditions (16) and (17), with their socially optimal counterparts, (19) and (20). It is straightforward that the optimal level of the environmental tax is (henceforth, the upper-script ^{o} is used to denote socially optimal values):

$$\tau_t^o = -\frac{u_\Omega}{u_C}. \tag{21}$$

Proposition 1 *A tax τ_t levied on the level of environmental damage Ω_t at each date t, and equal to $-u_\Omega/u_C$, allows achieving the economy's first-best outcome.*

The socially optimal level of the environmental tax is equal to the marginal disutility of the environmental damage divided by the marginal utility of consumption; in other words, it is a simple measure of the social cost of the stock of environmental damage. This contrasts with the standard result obtained when the environmental policy consists of a tax on resource use or on the flow of pollution itself, such as a standard carbon tax. In such a case, it is not the pollution disutility at date t that appears in the expression of the optimal tax, but the discounted sum of its instantaneous disutilities from t to infinity - see for instance Goulder and Mathai (2000) or Grimaud et al. (2011). In contrast, the present model directly taxes the environmental externality. Therefore, the numerator of (21) features the disutility of the stock of environmental damage at date t. This point may not be negligible when the question of policy implementation arises. Indeed, such a policy design does not require the accurate forecast and actualization of future emissions, which is undoubtedly a difficult task, and with controversial results. The same type of argument is also put forward by Billette de Villemeur and Leroux (2011), when they argue in favor of a carbon tax on the stock of accumulated CO_2 instead of the flow.

5 Impact of a second-best environmental policy

5.1 Specification and characterization of the decentralized economy

For various reasons, including a lack of international political consensus, the environmental policy implemented by the policy maker will not be socially optimal at each date t in the real world. We thus study here the effects of such a second-best environmental policy. To do so, we apply to (1), (6) and (8) the following functional forms:

$$Y_t = A_t^{\nu} R_t, \text{ with } \nu > 0, \tag{22}$$

$$\Omega(W_t, B_t) = W_t B_t^{-\eta}, \text{ with } \eta > 0, \tag{23}$$

$$\text{and } u(C_t, \Omega_t) = \ln C_t - \omega \Omega_t, \text{ with } \omega > 0. \tag{24}$$

From now on, we denote by g_{Xt} the growth rate $\dot{X}_t / X_t$ of any variable X_t.

Using these specifications, equations (16) and (17) can be respectively rewritten as

$$\nu \delta_A A_t^{\nu} R_t = \eta \delta_B \tau_t W_t B_t^{-\eta} \tag{25}$$

and

$$\rho + g_{Yt} = \nu g_{At} + h \tau_t B_t^{-\eta} / A_t^{\nu}. \tag{26}$$

Time differentiating (22) yields

$$g_{Yt} = \nu g_{At} + g_{Rt}. \tag{27}$$

Plugging (27) into condition (17), we get $\rho + g_{Rt} = h \tau_t B_t^{-\eta} / A_t^{\nu}$. Then, using (25), we have: $\frac{hR_t}{W_t} = \frac{\eta \delta_B}{\nu \delta_A}(g_{Rt} + \rho)$. Together with formula (4), this gives a linear relation between g_{Rt} and g_{Wt}:

$$g_{Rt} = \frac{\nu \delta_A}{\eta \delta_B} g_{Wt} - \rho. \tag{28}$$

Differentiating (4) with respect to time, we obtain $\frac{\dot{g}_{Wt}}{g_{Wt}} = g_{Rt} - g_{Wt}$. Substituting g_{Rt} with its value given in (28), we get the following differential equation: $\dot{g}_{Wt} = \frac{\nu \delta_A - \eta \delta_B}{\eta \delta_B} {g_{Wt}}^2 - \rho g_{Wt}$. This differential equation is solved through a change of variable: changing g_{Wt} we introduce $x_t \equiv 1/g_{Wt}$ and replace it in the differential equation, which becomes: $\dot{x}_t = \rho x_t - \frac{\nu \delta_A - \eta \delta_B}{\eta \delta_B}$. Solving this equation and then replacing x_t by $1/g_{Wt}$ yields the equilibrium value of the growth rate of the stock of waste:

$$g_{Wt} = \frac{1}{e^{\rho t}\left(\frac{1}{g_{W0}} - \frac{\nu\delta_A - \eta\delta_B}{\eta\delta_B\rho}\right) + \frac{\nu\delta_A - \eta\delta_B}{\eta\delta_B\rho}}. \tag{29}$$

Since we consider that the stock of waste is nil at date 0 (see the comments below formula (4)), then $1/g_{W0}$ is nil. This means

$$g_{Wt} = \frac{\eta\delta_B\rho}{(1 - e^{\rho t})(\nu\delta_A - \eta\delta_B)}. \tag{30}$$

By definition, this growth rate is positive at any time. This implies $\eta\delta_B > \nu\delta_A$; in other words, the green sector - by green sector, we mean management of the stock of waste and its associated R&D - has to be sufficiently efficient with respect to the standard sector.

Differentiating (23) and (25) with respect to time and using (27), we can see that the equilibrium growth rate of the environmental damage equals output growth minus the growth rate of the environmental tax:

$$g_{\Omega t} = g_{Wt} - \eta g_{Bt} = g_{Yt} - g_{\tau t}. \tag{31}$$

Then, using (27), (2) and (5), equation (31) can be rewritten as $g_{Wt} - \eta\delta_B L_{Bt} = \nu\delta_A L_{At} + g_{Rt}$. Since $L_{At} = 1 - L_{Bt}$ (see equation (7)) and since we already have an expression of g_{Rt} in terms of g_{Wt} given by (28), we get the equilibrium level of effort dedicated to the two types of R&D:

$$L_{At} = \left[(1 - \nu\delta_A/\eta\delta_B)g_{Wt} + g_{\tau t} + \rho - \eta\delta_B\right]\frac{1}{\nu\delta_A - \eta\delta_B}, \tag{32}$$

$$L_{Bt} = \left[(\nu\delta_A/\eta\delta_B - 1)g_{Wt} - g_{\tau t} - \rho + \nu\delta_A\right]\frac{1}{\nu\delta_A - \eta\delta_B}. \tag{33}$$

Then, inserting (2), (32) and (28) into (27), we obtain:

$$g_{Yt} = \left[\nu\delta_A g_{\tau t} - \eta\delta_B(\nu\delta_A - \rho)\right]\frac{1}{\nu\delta_A - \eta\delta_B}. \tag{34}$$

Replacing this expression of g_{Yt} into (31) gives the equilibrium growth rate of the environmental damage:

$$g_{\Omega t} = \left[\eta\delta_B g_{\tau t} - \eta\delta_B(\nu\delta_A - \rho)\right]\frac{1}{\nu\delta_A - \eta\delta_B}. \tag{35}$$

Finally, the bias of technical change, $g_{Bt} - g_{At}$, is computed from equations (2), (5), (32) and (33):

$$g_{Bt} - g_{At} = \left[(\nu\delta_A/\eta\delta_B - 1)g_{Wt} - g_{\tau t} - \rho + \frac{\delta_B\nu\delta_A + \delta_A\eta\delta_B}{\delta_A + \delta_B}\right]\frac{\delta_A + \delta_B}{\nu\delta_A - \eta\delta_B}. \tag{36}$$

5.2 Transitional dynamics

We now study the transitional dynamics of this decentralized economy. The relationship between the growth rates of the environmental damage and resource extraction is a linear function, as stated by (28). Moreover, as mentioned above, the time differentiation of (4) yields $\frac{\dot{g}_{Wt}}{g_{Wt}} = g_{Rt} - g_{Wt}$. This enables us to construct the phase diagram depicted in Figure 1.

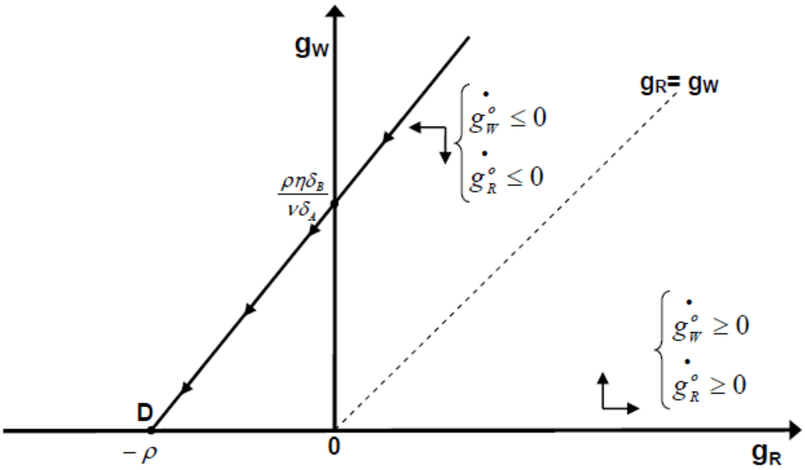

Figure 1: Phase diagram

We consider the dynamics of the economy for a given environmental policy design, that is, here, a given constant g_τ. We observe one unique and stable steady-state, D, towards which the economy asymptotically converges. In this steady-state, the growth rate of resource extraction is equal to $-\rho$, and and g_{Wt} is nil. In other words, the economy asymptotically tends to a regime where no resource is used and the stock of waste remains constant. Henceforth, the environmental damage unambiguously decreases over time if green knowledge is progressively improved, that is, when $L_{Bt} > 0$. Equation (33) shows that this occurs if the tax grows sufficiently fast: $g_\tau > \nu\delta_A - \rho$[9].

Along the transition towards D, the growth rate of resource extraction, g_{Rt}, decreases, and asymptotically tends to $-\rho$, its lower limit. At the same time, g_{Wt}, the rate of growth of the stock of waste, decreases down to zero. Thus the stock of waste always increases over time, but this accumulation is slower and slower.

The dynamics of the effort levels dedicated to standard and green research depend on the relative efficiencies of both sectors. Since $\eta\delta_B > \nu\delta_A$, $\partial L_{At}/\partial g_{Wt}$ and $\partial L_{Bt}/\partial g_{Wt}$ are respectively negative and positive (see equations (32) and (33)). This means that, along the transition path, the effort put in the standard research sector progressively increases, and asymptotically tends to its upper limit, $(g_\tau + \rho - \eta\delta_B)/(\nu\delta_A - \eta\delta_B)$ (see equation (32)). The growth rate of standard knowledge, g_{At} follows the same dynamics (see (2)). Simultaneously, the effort in green research, L_{Bt}, decreases during the transition to the steady-state (see equation (33)) - as does the growth rate of green knowledge, g_{Bt} (see (5)) - towards its lower limit: $(g_{\tau t} - \rho + \nu\delta_A)/(\nu\delta_A - \eta\delta_B)$. Note that the difference between the growth rates of green and standard knowledge, $g_{Bt} - g_{At}$, which we refer to as the bias of technical change, in reference to Acemoglu (2002), decreases

[9]Recall that $\eta\delta_B > \nu\delta_A$ (see the comments below equation (30)), that is, the parameters characterizing the efficiency of the green sector are higher than those of the standard sector, is an existence condition for the equilibrium.

over time: technical progress is less and less green-oriented.

The dynamics of the output growth rate are simple: production growth is constant and positive if $g_\tau < (\nu\delta_A - \rho)\eta\delta_B/\nu\delta_A$ - this can be observed from equation (34). In other words, growth is positive if and only if the environmental tax does not increase too rapidly.

The growth rate of the environmental damage, $g_{\Omega t}$, follows the same dynamics as output growth, since $g_{\Omega t} = g_{Yt} - g_\tau$ (see (31)). This growth rate is also constant. Since $\eta\delta_B > \nu\delta_A$, the environmental damage only decreases if the growth rate of the environmental tax is sufficiently high: $g_\tau > \nu\delta_A - \rho$. We have seen (equation (31)) that $g_{\Omega t} = g_{Wt} - \eta g_{Bt}$; this means that in this case, the long term growth of green knowledge overcomes the growth of the stock of waste.

Following these last two points, one can see that in order to have a positive long-term growth coupled with a declining environmental damage, we must have $\nu\delta_A - \rho < g_\tau < (\nu\delta_A - \rho)\eta\delta_B/\nu\delta_A$.

We sum up the main features of the decentralized economy's dynamics in the following proposition.

Proposition 2 *The economy asymptotically converges towards a unique and stable steady-state characterized by a (negative) constant growth rate of resource extraction and a constant stock of waste. Along the transition path, the bias of technical change is less and less green oriented, and output growth is constant and positive if $g_\tau < (\nu\delta_A - \rho)\eta\delta_B/\nu\delta_A$. Simultaneously, the environmental damage decreases if $g_\tau > \nu\delta_A - \rho$.*

5.3 Impact of the environmental policies

Now we study the impact of Pareto-improving environmental policies on the main relevant variables of the model.

5.3.1 First-best environmental policy in the specified case

We first characterize the first-best policy using the chosen specific functional forms. Applying specifications (22)-(24) to equation (21), we obtain the socially optimal design for the environmental tax: $\tau_t^o = \omega Y_t^o$. Log-differentiating this equality with respect to time yields the optimal growth rate of the tax:

$$g_{\tau t}^o = g_{Yt}^o. \tag{37}$$

In other words, the socially optimal environmental tax grows at the same rate as output.

We now compute g_{Yt}^o. The expression of this rate of growth is derived from the general conditions of the socially optimal economy, (19) and (17). Using specifications (22)-(24), condition (19) yields $\Omega_t^o = W_t^o (B_t^o)^{-\eta} = \frac{\nu\delta_A}{\omega\eta\delta_B}$. Thus, the socially optimal environmental damage remains constant over time; in other words, the improvement in green knowledge totally offsets the ongoing accumulation of waste. Log-differentiating this equality and using (5), we obtain $L_{Bt}^o = \frac{g_{Wt}^o}{\eta\delta_B}$, which implies $L_{At}^o = \frac{\eta\delta_B - g_{Wt}^o}{\eta\delta_B}$. Then, using condition (20) and the fact that $g_{Yt}^o = \nu g_{At}^o + g_{Rt}^o$, we obtain $g_{Rt}^o = \omega h R_t^o B_t^{o-\eta} - \rho$. Together with (4), this yields $g_{Rt}^o = g_{Wt}^o \frac{\nu\delta_A}{\eta\delta_B} - \rho$. From (22) and (2), we have $g_{Yt}^o = \nu\delta_A L_{At}^o + g_{Rt}^o$. Thus, we finally get $g_Y^o = \nu\delta_A - \rho$. We assume here that $\nu\delta_A > \rho$, that is, the standard sector is efficient enough relative to the psychological discount rate. In this case, the socially optimal output's growth rate is positive, which means, by (37), that $g^o_{\tau t}$ is positive: the optimal environmental tax is increasing at constant rate.

5.3.2 Second-best environmental policies

Here, we assume that the policy maker cannot set $g_{\tau t}$ at its first-best level, and that it can only set it at a lower level at each date t. We consider a second-best environmental policy consisting in setting an increasing tax on the stock of environmental damage in an economy where such a tool does not exist, or increasing the growth rate of this tax if there is already one; but in neither case $g^o_{\tau t}$ is reached. By observing the expressions of the equilibrium variables given in section 5.1, we can easily deduce the impact of such policy.

The literature has established that a unit tax on resource use that grows at a rate lower (resp. higher) than the interest rate of the economy postpones (resp. accelerates) resource extraction (see for instance Dasgupta et al. 1981). Here, the time profile of resource extraction, and hence the growth of waste, g_{Wt}, are not affected by the environmental policy - see equations (28) and (30). To understand this, one has to consider that an economic policy can modify the time profile of resource extraction through two channels: by changing the time profile of the resource price, and/or by changing the time profile of the resource's demand function itself. Condition (11) states that in this economy, as in standard Hotelling models, the resource price's growth rate equals the interest rate. To understand how the interest rate r_t is affected by the environmental policy, apply specification (24) to the Ramsey-Keynes condition (12) to obtain $r_t = g_{Yt} + \rho$. Using (34) and differentiating g_{Yt} with respect to $g_{\tau t}$, we finally get: $\partial r_t / \partial g_{\tau t} = \nu\delta_A/(\nu\delta_A - \eta\delta_B)$. As mentioned earlier, an existence condition for the decentralized equilibrium is that the green sector is more efficient than the standard one: $\eta\delta_B > \nu\delta_A$. Thus, $\partial r_t/\partial g_{\tau t}$ is negative; in other words, the environmental policy holds back the growth of the resource price.

The resource demand function is given by the differentiation of the profit function presented in (9) with respect to R_t, and it is equal to F_R. Using specification (22), we have $F_R = A_t^{\nu}$. The growth rate of this demand is thus $\nu\delta_A L_{At}$ (see equation (2)). Using (32), the differentiation of this expression with respect to $g_{\tau t}$ yields $\partial \nu\delta_A L_{At}/\partial g_{\tau t} = \nu\delta_A/(\nu\delta_A - \eta\delta_B)$. Thus, one can see that the environmental policy affects the dynamics of both the resource price and the resource demand in the same way. Both effects compensate each other, and the time profile of resource extraction - and thus waste production - are not modified. Therefore, the effect of the tax entirely falls on the allocation of efforts into both R&D sectors - the green and the standard - and thus on the bias of technical change.

Differentiating equations (32) and (33) with respect to $g_{\tau t}$, we get the following result: an increase in $g_{\tau t}$ fosters green research. Indeed, the effort put into this specific R&D sector, L_{Bt}, as well as the growth of the associated knowledge stock, g_{Bt}, increase with $g_{\tau t}$ - see (5). However, this comes at the expense of the standard R&D sector: L_{At} and g_{At} are both reduced - see (2). In other words, implementing an environmental policy, or making it more stringent favors green research and holds back the standard one. Overall, this policy steers technical change in the green direction: $g_{Bt} - g_{At}$ increases (see equation (36)).

At the same time, since the growth of waste is unchanged, and green knowledge grows faster, by (35) we can see that the rate of growth of the environmental damage, $g_{\Omega t}$, is reduced. This means that the environmental policy slows down the increase in the environmental damage. As exposed in the previous subsection, the limit case occurs when the first best environmental policy is implemented: equations (31) and (37) show that in this case the damage remains constant over time.

By equation (22), we know that $g_{Yt} = \nu g_{At} + g_{Rt}$. Since the growth rate of resource extraction is unaffected by the environmental policy, and the growth of standard knowledge decreases, the growth of output declines. Therefore, as it redirects part of the global R&D effort towards the green sector, this policy is detrimental to economic growth.

Proposition 3 *A Pareto-improving climate policy consisting in an increasing tax on the environmental damage does not affect the time profiles of resource extraction and waste production. Instead, it steers the bias of technical change in the green direction, so that the environmental damage grows less quickly. Such a policy is detrimental to economic growth.*

6 Concluding remarks

We have used a simple endogenous growth model with directed technical change in which an environmental damage stems from the accumulation of waste through the use of a non-renewable resource. This damage can be reduced by improving the technology used to manage the stock of waste. However, no green R&D is carried out without the right environmental policy, such as a tax on resource use or polluting emissions. The decentralized economy converges towards a unique and stable steady-state in which the stock of waste remains constant and the environmental damage decreases. Along the transition, the effort in green R&D as well as the growth rate of the associated knowledge progressively decrease, while the standard sector's importance grows: the bias of technical change is less and less green-oriented. If the tax level is set within an appropriate range, output's growth can be positive while the environmental damage decreases.

We have shown that a first-best policy consists in taxing the damage itself, that is, a function of two stocks: the stock of waste and the stock of green knowledge. We have studied the effects of such a tool in the specified version of the model. It does not modify the time profiles of resource extraction nor of associated polluting flow. It just reallocates the R&D efforts towards the green sector, which allows reducing - offsetting, in the first-best case - the growth of the environmental damage.

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