The River Bargaining problem*

Jean-Christophe Pereau† and Remy Delille‡

April 24, 2014

Abstract

This article analyses the outcomes of different negotiation procedures between three agents located along a river in a Rubinstein alternating-offers model where agents bargain over transfers and water consumption levels. Unlike for simultaneous bargaining, agreements in sequential bargaining are not efficient for society, even if the period between stages becomes infinitely small. This inefficiency results from the player's inside options, which are given by their temporary disagreement payoffs. Results also show that depending on the sequence of moves, inside options and impasse points can strengthen or weaken the relative position of the players involved in the negotiation process.

JEL Classification: C78

Keywords: alternating-offers model, sequential bargaining, inside option, impasse point.

*This study has been carried out with financial support from the French National Research Agency (ANR) as part of the project "ADAPT'EAU" (ANR-11-CEPL-008) and in the frame of the Cluster of Excellence COTE (ANR-10-LABX-45).

†GREThA (UMR CNRS 5113) Université de Bordeaux. Av Leon Duguit, 33608 Pessac, France. E-mail: jean-christophe.pereau@u-bordeaux4.fr.

‡GREThA (UMR CNRS 5113) Université de Bordeaux. Av Leon Duguit, 33608 Pessac, France.

1 Introduction

Game theoretic analysis of the river sharing problem has been a very active research area over the past two decades (Dinar *et al.* 1992; Barret, 1994; Kilgour and Dinar, 1995; Ambec and Sprumont, 2002 and Beal *et al*, 2013 for a recent survey). The main motivation for this research relies on the conflicting nature of water uses between various agents or countries for residential, industrial or agricultural purposes. Analysing water allocation rules among agents or countries who are located along a river also raises some interesting questions in terms of efficiency and equity when property rights are not well defined. The sharing problem refers to a situation in which agents have unequal access to the resource depending their location on the river. The most upstream agent have a full access but as the river flow goes down downstream agents get the remaining water left by upstream agents who are located behind them. Game theory models and in particular bargaining models are relevant to deal which such a problem. The literature on water allocation can be classified into two broad approaches.

The first approach is based on cooperative game theory and, more precisely, on the core. The objective is to set up a burden-sharing rule able to favour the cooperation of all, ensuring that the rule prevents that any individual agent, but also any sub-group of agents, to leave the agreement. Based on a model taking explicitly into account directional flows, Ambec and Sprumont (2000, 2002) shows that the convexity of the cooperative game ensures a non-empty core. They analysed how a compromise solution between the two international law principles of water sharing in transboundary river bassins can be reached in a cooperative game with utility transfers. These two principles are the *absolute territorial sovereignty* (ATS) that prescribes that each agent is free to use all the water he controls on his territory and the *absolute territorial integrity* (ATI) that states that the amount of available water to an agent cannot be altered by all the agents who are located upstream from his location. Ansink and Weikard (2012) modelled the river sharing problem as a sequential bankruptcy game in which the sum of the claims of all the agents exceed the availability of the resource (Aumann and Maschler, 1985; and Thomson 2013 for a recent survey). They analysed several sharing rules specified in terms of amount of water given to each agent. When all the water originates at the head of the river, the sharing rule states that each agent gets the same proportion of his claim and the linear order of the agents does not matter. Otherwise both the distribution of claims and water endowments needed to be considered. Houba *et al.* (2013) applies the asymmetric Nash bargaining solution to the river problem. They show that the most downstream agent always prefers the ATI principle but at least one of the other agent prefers the ATS rule. Under the ATI rule, they assume that the most downstream agent can restrict all his upstream agents to zero extraction as long as no agreement is reached. Wang (2011) proposes a market-based approach using trading water rights to achieve a Pareto optimal welfare distribution but his approach restricts bilateral trading to neighboring agents.

The second approach is based on non-cooperative game theory, also with several categories. Ansink et al (2012) modelled the river sharing problem as a two-stage open-membership cartel game using the concept of internal and external stability (d'Aspremont *et al.*, 1983). Assuming that a river agreement is a group of agents (a coalition) who have merged and maximize their welfare jointly, their goal is to determine which coalitions are stable, in the sense that no agent wants to leave it or join it. Assuming that agents have identical benefit functions and only differ in their location along the river, results show that a coalition of at least four agents is not stable. Ansink and Weikard (2009) modelled the river sharing problem as a contested game over the property rights in which two agents can decide to bargain or not. In the latter case agents use their outside options by asking a third agent to implement an equitable solution. Results show that agents can end up in an inefficient equilibrium instead of bargaining an efficient outcome. Carraro *et al.* (2007) review several bargaining models to water issues in order to show how an agreement is reached among sectors or countries. They emphasize complex negotiation problems dealing with multilateral and multi-issues features that can only be solved by use of computer simulations. In an alternating-offer Rubinstein model, Houba (2008) interprets the model of Ambec and Sprumont in a bargaining perspective but only in a bilateral case with one upstream agent and one downstream agent.

Most of these approaches and models above mentioned have in common to partly abstract from the negotiating process. This paper uses a simplified Rubinstein model with three agents who can bargain sequentially with endogenous disagreement points. It is assumed that the ATS rule prevails and the most downstream agent is constraint in his water consumption. Hence negotiation can take place between agents but not limited to neighboring agents as in Wang (2011). In modelling terms, the contribution of this paper is to emphasize the role of inside options in a Rubinstein framework with several sequential bargaining procedures. Inside options refer to the payoffs that agents obtain when they temporarily disagree. By assuming in each case that delay between negotiation rounds vanishes, the goals of the paper are to explain the source of possible inefficiencies and the different negotiation outcomes that can result from sequential bargaining. We consider three different negotiation procedures. The first procedure refers to a *simultaneous* negotiation in which the most downstream agent bargains with each upstream agent over water extraction and transfers (side payments). The second procedure assumes that the most downstream agent bargains sequentially with both upstream agents. The last procedure considers that the most upstream agent bargains sequentially with his downstream agents. In each case, the outcome in terms of water extraction and transfers is analyzed and compared with the social optimum, showing that with both sequential procedures the outcome is inefficient from the point of view of society. The intuition for this inefficiency that we observe comes from the fact that in our game agents bargain over transfers *and* water consumption levels. Thus, not only the distribution of the net surplus is different under different protocols, the water extraction level and, hence, the net surplus itself, is also different. We also show that the inefficiency in the sequential procedures comes from the inside options. Finally, we show that, depending on the sequence of moves in the sequential negotiation, inside options can strengthen or weaken the relative position of the agents involved in the negotiation.

The rest of the paper is organized as follows. We start in section 2 by analyzing the model under cooperative, non cooperative cases and simultaneous negotiation. Section 3 is devoted to the analysis of sequential negotiations with two protocols. In section 4 a specific example shows the main results of the paper.

2 The model

Consider three agents that are located along a river in a lexicographic ordering. Agent 1 is the most upstream agent while agent 3 is located at the mouth of the river. Agent 2 is the middle agent. We denote by $i < j$ the fact that agent i is upstream of agent j. Following notation by Ansink and Weikard (2009) $U_i = \{j \in N : j < i\}$ stands for the set of agents located upstream of agent i and reciprocally $D_i = \{k \in N : i < k\}$. Each agent extracts an amount of water $x_i \geq 0$ from which he earns $B_i(x_i)$. The benefit function is assumed to be concave with $B' > 0$ and $B'' < 0$. The total water available is E. We consider this river flow at its source is not increased by inflows along the river that might enter after the location of every agent. Each agent can only consume water that enters into its territory.

Non cooperative outcome: In the non cooperative case, the most upstream agent chooses how much to extract, x_1, under the constraint that this level does not exceed the available amount E. Then agent 2 chooses x_2 from the remaining water $E - x_1$ (the water left by agent 1) and so on until the most downstream agent. The sub-game perfect equilibria (SPE) of this game shows that each agent i extracts the maximum between his non cooperative level $\arg\max_{x_i} B_i(x_i)$ and the amount of water available at his location $E - \sum\limits_{j \in U_i} x_j$ such that

$$x_i^{nc} = \begin{cases} \arg\max_{x_i} B_i(x_i) & if \quad \arg\max_{x_i} B_i(x_i) \leq E - \sum\limits_{j \in U_i} x_j \\ E - \sum\limits_{j \in U_i} x_j & if \quad \arg\max_{x_i} B_i(x_i) > E - \sum\limits_{j \in U_i} x_j \end{cases}$$

Assume that the scarcity constraint on the resource doesn't allow agent i to obtain his non cooperative extraction, it implies that $\forall j \in U_i$ and $\forall k \in D_i$

$$\begin{array}{rclcrcl} x_j^{nc} & = & \arg\max_{x_j} B_j(x_j) & \Leftrightarrow & B_j^{nc} & = & B_j(x_j^{nc}) \\ x_i^{nc} & = & E - \sum\limits_{j \in U_i} x_j^{nc} & \Leftrightarrow & B_i^{nc} & = & B_i(E - \sum\limits_{j \in U_i} x_j^{nc}) \\ x_k^{nc} & = & 0 & \Leftrightarrow & B_k^{nc} & = & 0 \end{array}$$

It is easy to show that $B_j^{nc} > B_j^c \ \forall j \in U_i$ and $B_j^c > B_j^{nc} \ \forall j \in D_i$. In the sequel of the paper, we assume that only the most downstream agent 3 is constrained. It implies the following configuration $\sum_{i=1}^{3} x_i^{nc} > E > \sum_{i=1}^{2} x_i^{nc}$.

Cooperative outcome: The cooperative solution is given by the program

$$\max_{x_1,x_2,x_3} \sum_{i}^{3} B_i(x_i) \quad \text{sc} \quad \sum_{i}^{3} x_i \leq E$$

Water extraction levels $x^c = \{x_1^c, x_2^c, x_3^c\}$ are solution of

$$B_1'(x_1) = B_2'(x_2) = B_3'(x_3)$$

with $x_3 = E - x_1 - x_2$

Efficiency requires that the river flow E is share so as to equalize marginal benefits among agents.

To move from the non cooperative outcome to the cooperative one, monetary compensation mechanisms are necessary to encourage upstream agents to refrain from consuming water and let it pass to downstream agents who derive a higher benefit from consumption. When the resource is scarce, there is an interest for the upstream agent to reduce its consumption to give up water from the downstream agent who was rationed. The strict concavity of the benefit function ensures that the loss incurred by the upstream agent will be lower than the gain obtained by the downstream agent. There exists a bargaining scheme that ensures that agents will find profitable to move from the non cooperative outcome to the cooperative one.

Simultaneous negotiation: Suppose that there is a negotiation about the pair of monetary transfers or side-payments (τ_{13}, τ_{23}) that agent 3 (the proposer) should grant to agents 1 and 2 to bring them on board of a deal over a water extraction reduction pair $x_i < x_i^{nc}$ for $i = 1, 2$. Hence 3 is assumed to bargain simultaneously in two bilateral rounds with 1 over $o_1 = (x_1, \tau_{13})$ and with 2 over $o_2 = (x_2, \tau_{23})$. We assume no transfers between 1 and 2 ($\tau_{12} = 0$). To simplify notation we write: $B_i = B_i(x_i)$. Thus, the net payoff function (after transfers) for the 3 agents of an agreement reached at period τ is:

$$\pi_1(o_1; t) = B_1^{nc} + \delta_1^t \left(B_1 - B_1^{nc} - \tau_{12} + \tau_{13}\right) \quad (1)$$

$$\pi_2(o_2, ; t) = B_2^{nc} + \delta_2^t \left(B_2 - B_2^{nc} + \tau_{12} + \tau_{23}\right) \quad (2)$$

$$\pi_3\left(o_1, o_2; t\right) = B_3^{nc} + \delta_3^t \left(B_3 - B_3^{nc} - \tau_{13} - \tau_{23}\right) \quad (3)$$

where δ_i^t $(i = 1, 2, 3)$ stands for the discount rate, and $t = \{0, 1, ..\}$ the periods in which the offers-counter-offers are made. The Rubinstein Bargaining Solution (RBS) that we are looking for is the unique[1] SPE given by the following conditions (Rubinstein, 1982; Muthoo, 1999) for the negotiation between 1 and 3

$$\pi_1(o_1^{(3)}; 0) = \pi_1(o_1^{(1)}; 1) \text{ and } \pi_3(o_1^{(1)}, o_2; 0) = \pi_3(o_1^{(3)}, o_2; 1).$$

and between 2 and 3

$$\pi_2(o_2^{(3)}; 0) = \pi_2(o_2^{(2)}; 1) \text{ and } \pi_3(o_1, o_2^{(2)}; 0) = \pi_3(o_1, o_2^{(3)}; 1).$$

It gives the proposition

[1] As shown by Shaked and reported by Sutton (1986), the use of Rubinstein model in a multilateral bargaining framework may yield multiple equilibria under the unanimity rule.

Proposition 1 *The Rubinstein bargaining solution shows that extraction levels are optimal* $\mathrm{x}^c = \{x_1^c, x_2^c, x_3^c\}$ *implying the highest created surplus* $\pi = \sum_{i=1}^{3}(B_i^c - B_i^{nc}) > 0$ *and the payoffs after transfers for* $i = 1, 2$. $j \neq i$

$$\begin{aligned} \pi_i^* &= B_i^{nc} + \mu_i \pi \\ \pi_3^* &= B_3^{nc} + (1 - \mu_i - \mu_j)\pi \end{aligned}$$

with $\mu_i = \frac{\delta_i(1-\delta_j)(1-\delta_3)}{(1-\delta_i\delta_3)(1-\delta_j\delta_3)-\delta_i\delta_j(1-\delta_3)^2}$.

Proof. see proof 1 ■

In the limit case $\delta \to 1$, each agent gets his impasse point (permanent disagreement payoff) plus a equal share (1/3) of the total created surplus. This simultaneous negotiation accounts as a benchmark case for the sequential bargaining procedures.

3 Sequential negotiation

Based on the same set of payoffs (1)-(3), two bargaining procedures are analysed. In the first procedure, 3 is assumed to bargain twice with 2 over (x_2, τ_{23}) in the first round and with 1 over (x_1, τ_{13}) in the second round (while $\tau_{12} = 0$). The second procedure assumed that 3 bargains only once with 1 in the second round over (x_1, τ_{13}) while 1 also bargains with 2 over (x_2, τ_{12}) in the first round (and $\tau_{23} = 0$). The negotiation is solved by backward induction in both cases.

3.1 The downstream-sequential protocol

We assumed that 3 bargains twice as a proposer. In round two, 3 bargains with 1 knowing that an agreement has been reached in round one with 2 and this agreement will be effective even in the case of a disagreement in this current round. This first round agreement refers to the inside option for agent 3. It determines his impasse point.

Proposition 2 *The Rubinstein bargaining solution shows that when 3 bargains with 2 in round 1 and with 1 in round 2*

1. Water extractions (x_1^*, x_2^*) are solution of

$$\begin{aligned} B_1'(x_1) &= B_3'(x_3) \\ B_2'(x_2) &= B_3'(x_3) + \frac{\delta_1(1-\delta_3)}{(1-\delta_1\delta_3)}\left(B_3'\left(x_3^d\right) - B_3'(x_3)\right) \end{aligned}$$

   with $x_3^d = E - x_1^{nc} - x_2$

2. Equilibrium payoffs after transfers are

$$\begin{aligned} \pi_1^* &= B_1^{nc} + \mu_1\pi - (1 - \psi_2 - \psi_3)\left[\left(B_3^d - B_3^{nc}\right) - \left(B_2^{nc} - B_2^*\right)\right] \\ \pi_2^* &= B_2^{nc} + \mu_2\pi + \psi_2\left[\left(B_3^d - B_3^{nc}\right) - \left(B_2^{nc} - B_2^*\right)\right] \\ \pi_3^* &= B_3^{nc} + (1 - \mu_1 - \mu_2)\pi + \psi_3\left[\left(B_3^d - B_3^{nc}\right) - \left(B_2^{nc} - B_2^*\right)\right] \end{aligned}$$

with $\pi = \sum_{i=1}^{3}(B_i^* - B_i^{nc}) > 0$ the surplus generated, $B_3^d = B_3(x_3^d)$ and the coefficients $\mu_1 = \frac{\delta_1(1-\delta_3)}{(1-\delta_1\delta_3)}$, $\mu_2 = \frac{\delta_2(1-\delta_3)(1-\delta_1)}{(1-\delta_2\delta_3)(1-\delta_1\delta_3)}$, $\psi_2 = \frac{\delta_1\delta_2(1-\delta_3)^2}{(1-\delta_1\delta_3)(1-\delta_2\delta_3)}$ and $\psi_3 = \frac{\delta_1(1-\delta_2)(1-\delta_3)}{(1-\delta_2\delta_3)(1-\delta_1\delta_3)}$.

Proof. see proof 2. ■

Water extractions given by proposition 2 do not maximize social welfare ($x_i^* \neq x_i^c \; \forall i$). It implies that the net surplus is lower in the sequential case than in the cooperative case. The intuition for this inefficiency comes from the fact that changing the bargaining protocol does not only change the distribution of the net surplus, but also the net surplus itself, as agents are bargaining over transfers and water extractions. Optimality conditions suggest that for agent 1 $x_1^{nc} > x_1^* > x_1^c$ and for agent 2 $x_2^* < x_2^c < x_2^{nc}$ while 3 obtains $x_3^* > x_3^c > x_3^{nc}$.

In the first negotiation round, 3 bargains with 2 over a low water extraction level or equivantently a high reduction with respect to his initial situation $(x_2^{nc} - x_2^*)$ in exchange of a high transfer. Based on this agreement, 3 negotiates in the second round with 1 over a lower reduction effort and a lower transfer. A failure of the negotiation with 1 will have lesser consequences for 3 since he has already secured an agreement with 2 in the first round.

The agreement reached in round one increases the impasse point of 3 in round two and gives him advantage when bargaining with 1. Thus, both 2 and 3 are better off at the expense of 1, which is only involved in the second round of the negotiation. In terms of payoffs each agent gets a share of the created surplus but even if 1 gets a higher share (1/2 in the limit $\delta \to 1$ and 1/4 for 2 and 3), his payoff is reduced by an amount measured by the positive term into brackets $(\left(B_3^d - B_3^{nc}\right) > (B_2^{nc} - B_2^*))$. This loss of payoff for 1 is a gain for 2 and 3. Since the problem is symmetric, a first round between 3 and 1 (instead of 2) would imply the term $\left(\left(B_3^d - B_3^{nc}\right) > (B_1^{nc} - B_1^*)\right)$. The relative importance of these terms can be used by 3 to decide whether to bargain first with 1 or with 2, showing that the order of the partners in the sequential process becomes a strategic variable. This downstream-sequential bargaining protocol shows that it is better off for an agent to be involved in the first round rather than in the second round of a negotiation.

3.2 The upstream-sequential protocol

Assuming now that 1 bargains twice as a proposer. In round two 1 bargains with 3 knowing that an agreement has been reached in round one with 2.

Proposition 3 *The Rubinstein bargaining solution shows that when 1 bargains with 2 in round 1 and with 3 in round 2*

1. Water extractions (x_1^*, x_2^*) are solution of

$$\begin{aligned} B_1'(x_1) &= B_3'(x_3) \\ x_2 &= x_2^{nc} \end{aligned}$$

2. Equilibrium payoffs after transfers are

$$\begin{aligned}\pi_1^* &= B_1^{nc} + \mu_1 \pi \\ \pi_2^* &= B_2^{nc} + \mu_2 \pi \\ \pi_3^* &= B_3^{nc} + (1 - \mu_1 - \mu_2)\,\pi\end{aligned}$$

with $\pi = B_1^* - B_1^{nc} + B_3^* - B_3^{nc} > 0$ the created surplus and $\mu_1 = \frac{(1-\delta_2)(1-\delta_3)}{(1-\delta_2\delta_1)(1-\delta_1\delta_3)}$ and $\mu_2 = \frac{\delta_2(1-\delta_1)(1-\delta_3)}{(1-\delta_2\delta_1)(1-\delta_1\delta_3)}$ the sharing parameters.

Proof. see proof 3. ■

The upstream-sequential protocol also implies that water extractions do not maximize social welfare. As 2 behaves as in the non-cooperative case, his water consumption is the one that maximizes his payoff $x_2^* = x_2^{nc}$. It turns out that $x_1^* < x_1^c$. As usual, each agent gets its impasse point and a share of the net surplus, but to be only involved in the second round is now advantageous. The reason is that the impasse point of 1 in the second round weakens his relative position with respect to 3. In case of perpetual disagreement with 3, agent 1 will consume x_1^{nc}, but will bear the cost of the transfer to 2 ($\tau_{12} > 0$). For that reason, 1 is urged to sign an agreement with 3 over a low level of water extraction ($x_1^c > x_1^*$) or a high reduction ($x_1^{nc} - x_1^*$), implying a high value $\left(B_1^{nc} - B_1^*\right)$, which increases the transfer τ_1 paid by 3 and decreases the transfer τ_{12} that 1 pays to 2.

In the limit $\delta \to 1$, agents 1 and 2 get the same share of the created surplus $\pi_i = B_i^{nc} + \frac{1}{4}\pi$, $i = 1, 2$ but 3 is better off $\pi_3 = B_3^{nc} + \frac{1}{2}\pi$. This upstream-sequential protocol favors the most downstream agent but ensures an equal treatment for the upstream agents since 1 and 2 get the same share of the created surplus.

To be able to compare all the protocols, an example is needed.

4 Example

Let us illustrate the outcomes of the different bargaining protocols discussed above with an example. Assume the following benefit function as in Ambec and Sprumont (2000)

$$B_i = a x_i - \frac{b_i}{2} x_i^2$$

In the non cooperative case, the optimality condition $B_i' = 0$ implies $x_i^{nc} = a/b_i$.

Players are ranked such that $x_2^{nc} = \lambda x_1^{nc}$ and $x_3^{nc} = \lambda^2 x_1^{nc}$ with $\lambda > 0$. For $\lambda < 1$ $x_1^{nc} > x_2^{nc} > x_3^{nc}$ and for $\lambda > 1$ $x_1^{nc} < x_2^{nc} < x_3^{nc}$. They are identical when $\lambda = 1$. It implies $\lambda b_2 = b_1$ and $\lambda^2 b_3 = b_1$.

To ensure that negotiation takes place due to the scarcity constraint, we set $\left(1 + \lambda + \lambda^2\right) x_1^{nc} > E > (1 + \lambda)\, x_1^{nc}$.

Table 1. Outcomes under different bargaining protocols

	Protocol		
	SIM	Downstream$^{(d)}$	Upstream$^{(u)}$
x_1	$\frac{1}{\lambda^2+\lambda+1}E$	$\frac{x_1^{nc}+2\lambda E}{2\lambda^3+2\lambda^2+2\lambda+1}$	$\frac{E-\lambda x_1^{nc}}{\lambda^2+1}$
x_2	λx_1	$\frac{\left(1+2\lambda^2\right)E-\left(1+\lambda^2\right)x_1^{nc}}{2\lambda^3+2\lambda^2+2\lambda+1}$	λx_1^{nc}
x_3	$\lambda^2 x_1$	$\lambda^2 x_1$	$\lambda^2 x_1$
$\pi_1 - B_1^{nc}$	$\frac{1}{3}\frac{(\lambda+1)}{(\lambda^2+\lambda+1)}\Pi$	$\varphi\Pi$	$\frac{1}{4}\frac{1}{(\lambda^2+\lambda+1)}\Pi$
$\pi_2 - B_2^{nc}$	$\pi_1 - B_1^{nc}$	$\psi\Pi$	$\pi_1 - B_1^{nc}$
$\pi_3 - B_3^{nc}$	$\pi_1 - B_1^{nc}$	$\pi_2 - B_2^{nc}$	$2\left(\pi_1 - B_1^{nc}\right)$

with $\Pi = \frac{b_1}{2\lambda^2}\left(\left(1+\lambda+\lambda^2\right)x_1^{nc} - E\right)^2 > 0$, $\varphi = \frac{4\lambda^2\left(4\lambda^8+24\lambda^7+40\lambda^6+44\lambda^5+40\lambda^4+24\lambda^3+13\lambda^2+4\lambda+1\right)}{(4\lambda^6+12\lambda^5+16\lambda^4+16\lambda^3+10\lambda^2+4\lambda+1)^2}$, $\psi = \frac{\left(16\lambda^{11}+88\lambda^{10}+240\lambda^9+400\lambda^8+484\lambda^7+452\lambda^6+332\lambda^5+194\lambda^4+88\lambda^3+30\lambda^2+7\lambda+1\right)}{2(4\lambda^6+12\lambda^5+16\lambda^4+16\lambda^3+10\lambda^2+4\lambda+1)^2}$ with $\psi > \varphi$.

It can be show that in terms of water extraction that $x_1^d > x_1^{sim} > x_1^u$, $x_2^u > x_2^{sim} > x_2^d$ and $x_3^d > x_3^{sim} > x_3^u$ and in terms of payoffs $\pi_1^{sim} > \pi_1^u > \pi_1^d$, $\pi_2^d > \pi_2^{sim} > \pi_2^u$ and $\pi_3^u > \pi_3^d > \pi_3^{sim}$. This example shows that the most upstream agent prefers the simultaneous negotiation while the most downstream agent prefers the upstream-sequential negotiation and the middle agent the downstream-sequential negotiation.

5 Conclusion

This article has analyzed a bargaining game over two variables, water extractions and monetary compensations, between three agents located along a river. Simultaneous and two sequential negotiation procedures have been considered. Results show that simultaneous bargaining leads to a global agreement which is optimal as social welfare is maximized. On the contrary, the two sequential procedures considered lead to agreements that are not efficient for society. A first contribution of the paper has been to explain the role of inside options in explaining this inefficiency by focusing on the case where the delay between offers and negotiation rounds vanishes. In sequential protocols, the outcome of the agreement signed in the first round determines the temporary disagreement payoff for the second round.

A second contribution has been to analyze how the disagreement payoffs which determine the impasse points can strengthen or weaken the relative position of agents, depending on the sequence of moves. With the downstream-sequential procedure, the most downstream agent bargains first with one upstream agent and then with the other. The impasse point of the most downstream agent in the second round increases his relative bargaining position when facing the other upstream agent in the second round. In the upstream-sequential case, the two upstream agents bargain in the first round and one of them bargains with the most downstream agent in the second round. Results show that the impasse point of the upstream agent in the second round weakens his relative bargaining position when negotiating with the most downstream agent.

6 Proofs

6.1 Proof of Proposition 1

The two indifference conditions give respectively

$$B_1\left(x_1^{(3)}\right)+\tau_{13}^{(3)} = (1-\delta_1)\,B_1^{nc}+\delta_1\left(B_1\left(x_1^{(1)}\right)+\tau_{13}^{(1)}\right), \tag{4}$$

$$B_3\left(x_1^{(1)},x_2\right)-\tau_{13}^{(1)}-\tau_{23} = (1-\delta_3)\,B_3^{nc}+\delta_3\left(B_3\left(x_1^{(3)},x_2\right)-\tau_{13}^{(3)}-\tau_{23}\right), \tag{5}$$

and

$$B_2\left(x_2^{(3)}\right)+\tau_{23}^{(3)} = (1-\delta_2)\,B_2^{nc}+\delta_2\left(B_2\left(x_2^{(2)}\right)+\tau_{23}^{(2)}\right), \tag{6}$$

$$B_3\left(x_1,x_2^{(2)}\right)-\tau_{13}-\tau_{23}^{(2)} = (1-\delta_3)\,B_3^{nc}+\delta_3\left(B_3\left(x_1,x_2^{(3)}\right)-\tau_{13}-\tau_{23}^{(3)}\right), \tag{7}$$

To ensure optimality, the offer made by 1 $o_1^{(1)}=\left(x_1^{(1)},\tau_{13}^{(1)}\right)$ has to maximise his payoff $\pi_1 = B_1(x_1)+\tau_{13}$ under the constraint given by (5). Maximising returns the optimality condition

$$B_1'(x_1)=B_3'(x_3). \tag{8}$$

(8) gives the optimal offer made by 1 $x_1^{(1)}=x_1$. By symmetry the optimal offer of 3 is $x_1^{(3)}=x_1$.

We proceed in the same way for the second negotiation between 2 and 3. The optimal offer made by 2 $o_2^{(2)}=\left(x_2^{(2)},\tau_{23}^{(2)}\right)$ maximises his payoff under (7). It yields the first order condition

$$B_2'(x_2)=B_3'(x_3) \tag{9}$$

that gives the optimal offer of 2 $x_2^{(2)}=x_2$. By symmetry, 3 offers $x_2^{(3)}=x_2$. Optimality conditions (8) and (9) show that the abatement levels agreed are the cooperative abatement $x_i^*=x_i^c$ implying $B_i^*=B_i^c$.

Substitution in (4)-(5) and (6)-(7) gives the transfers when 3 is assumed to be the proposer $\tau_{13}^a=\tau_{13}^{(3)}$ and $\tau_{23}^a=\tau_{23}^{(3)}$

$$\tau_{13}^a = -\frac{(1-\delta_1)}{(1-\delta_1\delta_3)}\left(B_1^c-B_1^{nc}\right)+\frac{\delta_1(1-\delta_3)}{(1-\delta_1\delta_3)}\left(B_3^c-B_3^{nc}\right)-\frac{\delta_1(1-\delta_3)}{(1-\delta_1\delta_3)}\tau_{23}^a$$

$$\tau_{23}^a = -\frac{(1-\delta_2)}{(1-\delta_2\delta_3)}\left(B_2^c-B_2^{nc}\right)+\frac{\delta_2(1-\delta_3)}{(1-\delta_2\delta_3)}\left(B_3^c-B_3^{nc}\right)-\frac{\delta_2(1-\delta_3)}{(1-\delta_2\delta_3)}\tau_{13}^a$$

It gives the equilibrium transfers with $\eta = (1-\delta_1\delta_3)(1-\delta_2\delta_3) - \delta_1\delta_2(1-\delta_3)^2 > 0$

$$\tau_{13}^* = -\frac{(1-\delta_1)(1-\delta_2\delta_3)}{\eta}(B_1^c - B_1^{nc}) + \frac{\delta_1(1-\delta_2)(1-\delta_3)}{\eta}(B_2^c - B_2^{nc} + B_3^c - B_3^{nc})$$
$$\tau_{23}^* = -\frac{(1-\delta_2)(1-\delta_1\delta_3)}{\eta}(B_2^c - B_2^{nc}) + \frac{\delta_2(1-\delta_1)(1-\delta_3)}{\eta}(B_1^c - B_1^{nc} + B_3^c - B_3^{nc})$$

6.2 Proof of Proposition 2

In round two, the two indifference conditions are given by

$$B_1(x_1^{(3)}) + \tau_{13}^{(3)} = (1-\delta_1) B_1^{nc} + \delta_1 \left(B_1(x_1^{(1)}) + \tau_{13}^{(1)}\right) \quad (10)$$
$$B_3(x_1^{(1)}, x_2^a) - \tau_{13}^{(1)} - \tau_{23}^a = (1-\delta_3)\left(B_3^d - \tau_2^a\right) + \delta_3 \left(B_3(x_1^{(3)}, x_2^a) - \tau_{13}^{(3)} - \tau_{23}^a\right) \quad (11)$$

Assuming that 1 and 3 maximises their payoffs (respectively under constraints (10) or (11)), it turns out that their optimal offers $x_1^{(1)} = x_1^{(3)} = x_1$ will be solution of

$$B_1'(x_1) = B_3'(x_3) \quad (12)$$

Assuming that 3 is the proposer, the transfer paid by 3 to 1 is equal to

$$\tau_{13}^{(3)} = -\frac{(1-\delta_1)}{(1-\delta_1\delta_3)}(B_1 - B_1^{nc}) + \frac{\delta_1(1-\delta_3)}{(1-\delta_1\delta_3)}(B_3 - B_3^{nc}) + \frac{\delta_1(1-\delta_3)}{(1-\delta_1\delta_3)}\left(B_3^{nc} - B_3^d\right) \quad (13)$$

In round one, the two indifference conditions associated to the negotiation between 3 and 2 are

$$B_2(x_2^{(3)}) + \tau_{23}^{(3)} = (1-\delta_2) B_2^{nc} + \delta_2 \left(B_2(x_2^{(2)}) + \tau_{23}^{(2)}\right) \quad (14)$$
$$B_3(x_1, x_2^{(2)}) - \tau_{13} - \tau_{23}^{(2)} = (1-\delta_3) B_3^{nc} + \delta_3 \left(B_3(x_1, x_2^{(3)}) - \tau_{13} - \tau_{23}^{(3)}\right) \quad (15)$$

The disagreement payoff of 3 differs from the second round. We assumed that in case of a failure, the negotiation is over and all agents get their non cooperative payoffs.

Using the implicit relations in round two $x_1 = x_1(x_2)$ and $\tau_{13} = \tau_{13}(x_2)$, the optimal water extraction level x_2 is solution of the optimal first order condition

$$\left(1 + \frac{\partial x_1}{\partial x_2}\right) B_3'(x_3) + \frac{\partial \tau_{13}}{\partial x_2} = B_2'(x_2) \quad (16)$$

Optimality conditions (12) and (16) implies the equilibrium pair (x_1^*, x_2^*). The transfer associated to that negotiation when 3 is the proposer is given by

$$\tau_2^{(3)} = -\frac{(1-\delta_2)}{(1-\delta_2\delta_3)}(B_2 - B_2^{nc}) + \frac{\delta_2(1-\delta_3)}{(1-\delta_2\delta_3)}(B_3 - B_3^{nc}) - \frac{\delta_2(1-\delta_3)}{(1-\delta_2\delta_3)}\tau_{13} \quad (17)$$

Solving the system (13)-(17) gives $\tau_{13}^{(3)} = \tau_{13}^{*}$ and

$$\tau_{23}^{*} = \frac{\delta_2 (1-\delta_3)(1-\delta_1)}{(1-\delta_2\delta_3)(1-\delta_1\delta_3)} (B_1 - B_1^{nc} + B_3 - B_3^{nc}) - \frac{(1-\delta_2)}{(1-\delta_2\delta_3)} (B_2 - B_2^{nc}) + \frac{\delta_1\delta_2 (1-\delta_3)^2}{(1-\delta_1\delta_3)(1-\delta_2\delta_3)} \left(B_3^{d} - B_3^{nc}\right) \tag{18}$$

Substitute the expression $\frac{\partial \tau_{13}}{\partial x_2}$ derived from (18) in (13) gives

$$\begin{aligned} B_1'(x_1) &= B_3'(x_3) \\ B_2'(x_2) &= B_3'(x_3) + \frac{\delta_1 (1-\delta_3)}{(1-\delta_1\delta_3)} \left(B_3'\left(x_3^{d}\right) - B_3'(x_3)\right) \end{aligned}$$

Let us note that $x_3^* = E - x_1^* - x_2^* > x_3^d = E - x_1^{nc} - x_2^*$ since $x_1^* < x_1^{nc}$, it implies that $B_3'\left(x_3^d\right) > B_3'\left(x_3^a\right)$ since B_3' is decreasing ($B'' < 0$). Simple manipulations yield the payoffs for every agent.

6.3 Proof of Proposition 3

In round 2, 1 bargains with 3 based on an agreement in round 1. The two indifference conditions are

$$\begin{aligned} B_1(x_1^{(3)}) + \tau_{13}^{(3)} - \tau_{12} &= (1-\delta_1)(B_1^{nc} - \tau_{12}) + \delta_1 \left(B_1(x_1^{(1)}) + \tau_{13}^{(1)} - \tau_{12}\right) \\ B_3\left(x_1^{(1)}, x_2\right) - \tau_{13}^{(1)} &= (1-\delta_3) B_3^{d} + \delta_3 \left(B_3(x_1^{(3)}, x_2) - \tau_{13}^{(3)}\right) \end{aligned}$$

with $B_3^d = B_3(x_1^{nc}, x_2)$.

In round 1, the two indifference conditions of the negotiation between 1 and 2 are

$$\begin{aligned} B_1\left(x_1(x_2^{(2)})\right) + \tau_{13} - \tau_{12}^{(2)} &= (1-\delta_1) B_1^{nc} + \delta_1 \left(B_1 x_1(x_2^{(1)}) + \tau_{13} - \tau_{12}^{(1)}\right) \\ B_2(x_2^{(1)}) + \tau_{12}^{(1)} &= (1-\delta_2) B_2^{nc} + \delta_2 \left(B_2(x_2^{(2)}) + \tau_{12}^{(2)}\right) \end{aligned}$$

The first negotiation implies

$$B_1'(x_1) = B_3'(x_3)$$

and

$$\tau_{13}^{(1)} = -\frac{\delta_3 (1-\delta_1)}{(1-\delta_1\delta_3)} (B_1 - B_1^{nc}) + \frac{(1-\delta_3)}{(1-\delta_1\delta_3)} \left(B_3 - B_3^{d}\right)$$

The second negotiation gives

$$\left(\frac{\partial x_1}{\partial x_2}\right) B_1'(x_1(x_2)) + \frac{\partial \tau_{13}}{\partial x_2} + B_2'(x_2) = 0$$

and the transfer

$$\tau_{12}^{(1)} = \frac{\delta_2 (1-\delta_1)}{(1-\delta_2\delta_1)} (B_1 - B_1^{nc}) - \frac{(1-\delta_2)}{(1-\delta_2\delta_1)} (B_2 - B_2^{nc}) + \frac{\delta_2 (1-\delta_1)}{(1-\delta_2\delta_1)} \tau_{13}$$

Substitute the derivative $\frac{\partial\tau_{13}}{\partial x_2}$ in the optimality condition gives

$$B_2'(x_2) = -\frac{(1-\delta_3)}{(1-\delta_1\delta_3)}\left(B_3'\left(x_3^d\right) - B_3'(x_3)\right)$$

Let us note that $B_3'\left(x_3^d\right) > B_3'(x_3)$ since $x_3 > x_3^d$ and B_3' is decreasing ($B'' < 0$), it implies $B_2'(x_2) < 0$. This case is rule out since $B_2'(x_2)$ is defined on $[0, +\infty[$, it means that $B_2'(x_2) = 0$ giving the maximum value of $x_2 = x_2^{nc}$ and as a consequence $B_2^* = B_2^{nc}$ and $B_3^{nc} = B_3^d$

Equilibrium transfers are $\tau_{13}^* = \tau_{13}^{(1)}$ and $\tau_{23}^* = \tau_{23}^{(1)}$

$$\begin{aligned}
\tau_{13}^* &= -\frac{\delta_3(1-\delta_1)}{(1-\delta_1\delta_3)}\left(B_1^* - B_1^{nc}\right) + \frac{(1-\delta_3)}{(1-\delta_1\delta_3)}\left(B_3^* - B_3^{nc}\right) \\
\tau_{12}^* &= \frac{\delta_2(1-\delta_1)(1-\delta_3)}{(1-\delta_2\delta_1)(1-\delta_1\delta_3)}\left(B_1^* - B_1^{nc} + B_3^* - B_3^{nc}\right) > 0
\end{aligned}$$

Simple manipulations give the proposition.

References

Ambec, S. and Sprumont, Y. (2000) Sharing a river, Paper No 06, University of Laval, Recherche en Energie, Québec.

Ambec, S. and Sprumont, Y. (2002) Sharing a river, *Journal of Economic Theory*, 107:453–462.

Ansink, E. and Weikard, H.-P. (2012) Sequential sharing rules for river sharing problems, *Social Choice and Welfare*, 38:187–210.

Ansink, E., Gengenbach M. and Weikard, H.-P. (2012) River Sharing and Water Trade, FEEM Nota Di Lavoro 17.2012.

Aumann, R. and Maschler, M. (1985) Game theoretic analysis of a bankruptcy problem from the Talmud, *Journal of Economic Theory,* 36:195–213.

Barrett S (1994) Conflict and cooperation in managing international water resources. World Bank Policy, Research Working Paper 1303.

Béal, S., Ghintran, A., Remila, E. and Solal. P. (2013) The river sharing problem: a survey, *International Game Theory Review*, 15(3):

Carraro, C., Marchiori, C. and Sgobbi, A. (2007) Negotiating on water: Insights from non-cooperative bargaining theory, *Environment and Development. Economics*, 12:329–349.

d'Aspremont, C., A. Jacquemin, J. J. Gabszewicz and J. A. Weymark, 1983. On the stability of collusive price leadership. *Canadian Journal of Economics* 16(1): 17–25.

Houba, H. (2008). Computing alternating offers and water prices in bilateral river basin management. *International Game Theory Review*, 10:257-78.

Houba, H, van der Laan G. and Zeng Y. (2013) Asymmetric Nash Solutions in the River Sharing Problem, Tinbergen Institute Discussion Pare TI 2013-051.

Kilgour, D. M. and Dinar, D. (1995) Are stable agreements for sharing international river water now possible? Policy Research Working Paper 1474, World Bank,Washington.

Muthoo, A., 1999. *Bargaining theory with applications.* Cambridge University Press, Cambridge.

Rubinstein, A. (1982). Perfect Equilibrium in a Bargaining Model. *Econometrica* 50:97-109.

Wang, Y. (2011) Trading water along a river, *Mathematical Social Science*, 61:124–130.

Thomson, W (2013) Game-theoretic analysis of bankrupcy and taxation problems: some recent advances, *International Game Theory Review*, 15(3):