# Optimal Timing of CCS Policies under Decreasing Returns to Scale*

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**Abstract**

Carbon capture and sequestration (CCS) can help to mitigate the climate change transition. Usually, in models where the atmospheric carbon stock is constrained by an institutional stabilization cap and under constant average CCS cost, the use of CCS must be delayed up to the time at which the constraint begins to be effective. In this paper, we show that, when abatement activity are submitted to decreasing returns to scale, abatement must start earlier, before the climate constraint becomes to bind, but they must also be stopped strictly before the climate constraints ceases to be active. Depending on the solar energy costs, either there is a return toward dirty energy or either a progressive rise of solar energy at the expense of abatement activities.

**Keywords:** Energy resources; carbon stabilization cap; carbon capture and storage; decreasing returns to scale.

**JEL classifications:** Q32, Q42, Q54, Q58.

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# 1 Introduction

Basically, two kinds of devices mainly allow for reducing the anthropogenic $CO_2$ emissions generated by the consumption of non-renewable energy resources. The first one consists in improving the energy efficiency of the process at the disposal of the industries and the final consumers. Increasing the output/energy ratio can be obtained either by substituting capital for energy in a given state of the technological knowledge or by substituting new technologies for the older ones, the new technologies resulting from a technological progress or a mere learning effect which is nothing but that another form of the technological knowledge accumulation. The second one consists in capturing and sequestering the potential carbon emissions, or at least some part of the potential flow, to reduce the current flow of carbon release in the atmosphere, what we call CCS.[1]

In a model in which the atmospheric carbon stock is constrained to stay under some well defined limit, a ceiling, Chakravorty *et al.* (2006) have shown that the use of the CCS technology must be delayed up to the time at which this atmospheric limit stock is attained, assuming that its cost is sufficiently low to be ever brought into operation along an optimal path. In their model the instantaneous unitary CCS cost is constant. This model has been generalized by Lafforgue *et al.* (2008) to take into account the possibly limited capacity of the reservoirs in which the captured carbon is sequestered. In their model the instantaneous unitary CCS cost increases with the cumulative captured carbon stock the reservoirs being exploited by increasing order of economic accessibility. They show that the same property holds: Never capture before the date at which the atmospheric carbon stock constraint begins to be binding. One of the key explanations that drive this result is the nature and the shape of the instantaneous unitary cost function.[2] When the instantaneous unitary cost function is constant, as assumed by these two previous studies, then it is never optimal to capture before being constrained by the ceiling constraint. When this unit cost function exhibits more sophisticated properties, this result can be modified, but not necessarily. For instance, Amigues *et al.* (2014-b) show that introducing a learning-by-doing process in the CCS technology does not change the conclusion that society must wait to be constrained by the ceiling before undertake abatement.

The present paper still studies the question of the optimal timing of CCS policy, rel-

[1] See Hamilton *et al.* and Herzog (2011) for a technical and economical presentation of this technology. See also Kalkuhl *et al.* (2012) for a balanced account of the true competitiveness of this option.

[2] Another factor is the level of heterogeneity of energy consumers relatively to their access to the CCS technology, as shown in Amigues *et al.* (2014-a).

atively to the timing of emissions and to the time at which they are constrained by the carbon cap, but by assuming now a flow-dependent and increasing CCS cost function. An empirical justification of decreasing-returns-to-scale in CCS technology, at least in the long run, can be found in Bielicki (2008), Durmaz and Schroyen (2013) or Gerlagh and van der Zwaan (2006). Under this new assumption, we show that beginning to capture before being constrained by the ceiling constraint is the optimal policy now. In this case, since the first unit of sequestration is the cheaper one due to decreasing-returns-to-scale, discounting the flow of net surplus implies to deploy the CCS option as soon as possible. Moreover, the problem of the competitiveness of the CCS technology cannot be separated from the problem of the competitiveness of other clean technologies that is essentially the competitiveness of the clean energy sources. Both competitiveness problems are linked through the stringency of the ceiling constraint. Hence we systematically examine the both cases in which the clean renewable substitute is competitive and not when the economy is blockaded by the ceiling constraint.

The paper is organized as follows. The model is laid down in Section 2. In Section 3 we express the social planner program and we derive the first-order conditions. In Section 4 we review the main qualitative properties of the optimal path. In section 5 we provide two examples of energy consumption and price trajectories, depending on whether solar energy cost is high or low. Last, we conclude in Section 6

## 2 Model and notations

We consider a dynamic model of energy use with two primary energy sources which are perfect substitutes: coal and solar energy. Solar is renewable and carbon-free whereas coal is non-renewable and carbon-emitting. However some part of the potential pollution flow can be abated using the CCS technology. We define *clean* coal as that part of coal consumption whose emissions are captured, and *dirty* coal as the other part whose emissions are directly released into the atmosphere. We denote by $x_c(t)$, $x_d(t)$ and $y(t)$ the consumption of clean coal, dirty coal and solar energy at time $t$, respectively. The total energy consumption is then given by $q(t) = x_c(t) + x_d(t) + y(t)$. This consumption generates an instantaneous gross surplus $u(q)$. The utility function $u(.)$ is assumed to satisfy the standard properties (strictly increasing and strictly concave) and to confirm the Inada condition: $\lim_{q \downarrow 0} u'(q) = +\infty$. We define $p(q) \equiv u'(q)$ as the marginal gross surplus, i.e. the energy consumer price, and $q^d(p) \equiv p^{-1}(p)$ as the energy demand function.

Taking $X(t)$ to denote the available stock of coal at time $t$, with $X(0) \equiv X^0$ being the initial reserves, the dynamics of extraction is given by:

$$\dot{X}(t) = -[x_c(t) + x_d(t)] \tag{1}$$

We assume that returns to scale in extraction are constant, but that extraction becomes more and more costly as coal reserves run low. The average extraction cost function $c(.)$, common to the two types of coal, is strictly decreasing and convex in $X$, with $\lim_{X \downarrow 0} c(X) = +\infty$. However, producing energy services from clean coal is costlier than from dirty coal since an additional CCS cost must be incurred. Assuming decreasing returns to scale in CCS technology, this additional cost, per unit of clean coal, is given by $a(x_c)$, with $a(.)$ strictly increasing and convex in $x_c$ and such that $a(0) \equiv \underline{a}$. We define $ma$ as the marginal CCS cost: $ma(x_c) = a(x_c) + a'(x_c)x_c > 0$, with $ma(0) = \underline{a}$. Since $a$ is convex, $ma$ is strictly increasing.

Let $Z(t)$ be the atmospheric carbon stock at time $t$, and $Z^0$ be the initial concentration inherited from the past, with $Z(0) \equiv Z^0$. The instantaneous natural regeneration of this pollution stock is given by $\alpha(Z)$, where $\alpha(.)$ is a strictly increasing and concave function.[3] Since only dirty coal feeds the atmospheric carbon stock, the dynamics of $Z$ is:

$$\dot{Z}(t) = \zeta x_d(t) - \alpha(Z(t)) \tag{2}$$

The pollution damage is negligible as long as $Z$ does not overshoot some critical level $\bar{Z}$. Beyond this threshold, the damage is supposed immeasurably high and irreversible.[4] Thus any optimal path must satisfy the following constraint:

$$\bar{Z} - Z(t) \geq 0 \tag{3}$$

For the problem to be meaningful, we assume $Z^0 < \bar{Z}$. When the ceiling $\bar{Z}$ is reached, i.e. when (3) is binding, dirty coal consumption is constrained and, from (2), its maximal level must be equal to $\bar{x}_d \equiv \alpha(\bar{Z})/\zeta$, i.e. the exact quantity whose emissions are balanced by the natural regeneration of the atmosphere.

The other primary energy source is solar energy, whose natural flow is supposed to be large enough to provide all the energy needs of the society, even in the absence of coal.

[3]See Toman and Whitagen (2000) for an exploration of alternative formulations giving rise to non convexity necessitating global comparisons for determining which path is the optimal one.

[4]See Chakravorty *et al.* (2006) for a justification of this specific damage function, which implies a marginal damage which is nil for $Z < \bar{Z}$ and infinite for $Z \geq \bar{Z}$. Amigues *et al* (2011) show that the main qualitative properties of the optimal paths do not change when *small* damages too are also taken into account.

It is processed at a constant average cost $b$.[5] We assume that $b$ is larger than $c(X^0)$ to justify the use of coal, at least during an initial period. At the end of the coal exploitation period, if only dirty coal is used as we shall show in the next section, then the grade $X_b$ of remaining coal reserves is such that $c(X_b) = b$. Hence, whatever the optimal path, the cumulative coal consumption must always be equal to $X^0 - X_b$. Last, once coal exploitation has ceased for good, the optimal solar energy consumption amounts to $\tilde{y} \equiv q^d(b)$, provided that the natural flow of solar energy be sufficiently large.

## 3 Program of the social planner

The problem of the social planner consists in determining the path $\{(x_c(t), x_d(t), y(t)), t \geq 0\}$ that maximizes the intertemporel net surplus, given the ceiling constraint on the stock of pollution. Denoting by $\rho$ the social discount rate, the optimal program is then:[6]

$$\max_{\{x_d, x_c, y\}} \int_0^\infty [u(x_c + x_d + y) - c(X)(x_c + x_d) - a(x_c)x_c - by]\, e^{-\rho t} dt$$

subject to (1), (2), (3) and to the non-negativity constraints on $x_c$, $x_d$ and $y$. Let $\lambda_X$ and $-\lambda_Z$ be the co-state variables of $X$ and $Z$ respectively.[7] Let $\nu_Z$ be the Lagrange multiplier associated with the ceiling constraint on $Z$ and $\gamma$ be those corresponding to the non-negativity constraints on the control variables. The current valued Lagrangian of the program is:

$$\begin{aligned}\mathcal{L} &= u(x_c + x_d + y) - c(X)(x_c + x_d) - a(x_c)x_c - by - \lambda_X[x_c + x_d] - \lambda_Z[\zeta x_d - \alpha(Z)] \\ &\quad + \nu_Z[\bar{Z} - Z] + \gamma_c x_c + \gamma_d x_d + \gamma_y y\end{aligned}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial x_c} = 0 \quad \Rightarrow \quad p = c(X) + \lambda_X + ma(x_c) - \gamma_c \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial x_d} = 0 \quad \Rightarrow \quad p = c(X) + \lambda_X + \zeta\lambda_Z - \gamma_d \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \quad \Rightarrow \quad p = b - \gamma_y \tag{6}$$

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{\partial X} \quad \Rightarrow \quad \dot{\lambda}_X = \rho\lambda_X + c'(X)(x_c + x_d) \tag{7}$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \quad \Rightarrow \quad \dot{\lambda}_Z = [\rho + \alpha'(Z)]\lambda_Z - \nu_Z \tag{8}$$

[5] The case of increasing average solar costs is developed in Chakravorty *et al* (2012).

[6] We have dropped the time index for the sake of convenient notation as far as possible.

[7] Using $-\lambda_Z$ as the co-state variable of $Z$, we can directly interpret $\lambda_Z \geq 0$ as the social marginal cost of the pollution stock. Note that, in a decentralized economy without any other externality than the environmental one, the optimal carbon tax per unit of dirty coal would be $\zeta\lambda_Z$.

together with the usual complementary slackness conditions and the following transversality conditions:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = \lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{9}$$

Before reviewing the main qualitative properties of the optimal paths, we provide here some direct implications of the optimality conditions. First, let us consider the part $p^F$ ($F$ for free of pollution tax and cleaning cost) of the full marginal cost of coal which is common to the two types of coal: $p^F \equiv c(X) + \lambda_X$. Denoting by $t_X$ the time at which coal exploitation ceases, time differentiating $p^F$ and using (7), we get:

$$t < t_X \quad \Rightarrow \quad \dot{p}^F(t) = \rho\lambda_X(t) > 0 \tag{10}$$

Then, $p^F$ always increases during any coal exploitation period.

Next, let $\underline{t}_Z$ and $\bar{t}_Z$ be the dates at which the ceiling constraint begins and ceases to be active, respectively, i.e. $Z(t) = \bar{Z}\ \forall t \in [\underline{t}_Z, \bar{t}_Z]$. For $t < t_Z$, since $\nu_Z = 0$, (8) implies $\dot{\lambda}_Z = [\rho + \alpha'(Z)]\lambda_Z > 0$, hence:

$$t < \underline{t}_Z \quad \Rightarrow \quad \lambda_Z(t) = \lambda_{Z0}e^{A(t)} \tag{11}$$

where $\lambda_{Z0} \equiv \lambda_Z(0)$ and $A(t) \equiv \int_0^t [\rho + \alpha'(Z(\tau))]d\tau$. Clearly once the ceiling constraint is no longer active, i.e. after $\bar{t}_Z$, $\lambda_Z$ must be nil:[8]

$$t > \bar{t}_Z \quad \Rightarrow \quad \lambda_Z(t) = 0 \tag{12}$$

Finally, we define as $\underline{t}_c$ and $\bar{t}_c$ the times at which the clean coal begins and ceases to be exploited, respectively, and as $t_y$ the date at which the exploitation of solar energy begins.

## 4 Qualitative properties of the optimal paths

Three energy sources are under competition: dirty coal and the two clean options (clean coal and solar). The composition of the optimal energy-mix and its dynamics thus result from the comparison of their respective full marginal costs, as given by (4)-(6). Under constant average CCS cost, Lafforgue *et al.* (2008) conclude that when it is optimal to use it, the clean coal exploitation must take place at the beginning of the ceiling period and its consumption rate must be decreasing. Moreover, clean coal and solar energy are never simultaneously exploited. As we shall show now, those results are no longer valid

[8]This point can be easily proved by integrating (2) and (8), and by replacing the resulting expressions of $Z$ and $\lambda_Z$ in the transversality condition (9).

under decreasing returns in CCS technology. To get this key result, we need to establish the three following lemmas beforehand.

**Lemma 1** *Along any optimal path, the exploitation of solar energy cannot begin before the ceiling constraint is binding, neither before the beginning of the clean coal exploitation:* $t_y \geq \underline{t}_Z$ *and* $t_y \geq \underline{t}_c$.

**Proof:** i) Assume that, at some time $t'$, with $t' < \underline{t}_Z < t_X$, solar energy is competitive: $b \leq \min\left\{p^F(t') + \zeta\lambda_{Z0}e^{A(t')}, p^F(t') + ma(x_c(t'))\right\}$. Since, from (10), $p^F(t)$ increases for any $t < t_X$ and since $A(t)$ also increases for any $t < \underline{t}_Z$, then we must have $b < p^F(t)+\zeta\lambda_{Z0}e^{A(t)}$, $\forall t \in (t', \underline{t}_Z)$. Dirty coal is thus not competitive relatively to solar energy although it could be competitive, or not, relatively to clean coal. Whatever the case, only carbon-free energy (solar or clean coal) is used between $t'$ and $\underline{t}_Z$ implying that $Z$ decreases. This results in $Z(\underline{t}_Z) < \bar{Z}$, which is a contradiction.

ii) Assume now that there exists a time interval $(t', \underline{t}_c)$ during which solar energy is more competitive than clean coal: $b < p^F(t) + \underline{a}$, $\forall t \in (t', \underline{t}_c)$. Since both $p^F$ and $ma(x_c)$ are increasing then the inequality holds even after $\underline{t}_c$, meaning that once solar energy is competitive relatively to clean coal, it is competitive forever. Hence, if clean coal has ever to be exploited, its exploitation cannot take place after the beginning of the solar energy exploitation. ■

These first results are proved to be an immediate implication of the constant average solar cost assumption. The next lemma shows that, if clean coal is exploited during the ceiling period, it is not optimal to delay its exploitation after $\underline{t}_Z$, and that the exploitation must cease strictly before the end of this period, i.e. before $\bar{t}_Z$.

**Lemma 2** *It is never optimal to delay clean coal exploitation once the ceiling constraint is binding and its exploitation must cease before the end of the ceiling period.*

**Proof:** i) Assume that clean coal exploitation begins strictly after $\underline{t}_Z$. We can thus consider two time intervals within the ceiling period, $(\underline{t}_Z, t')$ and $(t', t'')$, with $\underline{t}_Z < t' < t'' \leq \bar{t}_Z$, during which, respectively, first $x_c = 0$ and then $x_c > 0$. During the first time interval, since, from Lemma 1, solar energy cannot be exploited before clean coal, only dirty coal is used and we must have: $p^F(t) + \zeta\lambda_Z(t) = u'(\bar{x}_d) \leq p^F(t) + \underline{a}$, $\forall t \in (\underline{t}_Z, t')$. Since $p^F(\underline{t}_Z) + \zeta\lambda_Z(\underline{t}_Z) = u'(\bar{x}_d) \leq p^F(\underline{t}_Z) + \underline{a}$ and $p^F$ is increasing, then:

$$\lim_{t \uparrow t'} \left\{p^F(t) + \zeta\lambda_Z(t)\right\} = u'(\bar{x}_d) < \lim_{t \uparrow t'} \left\{p^F(t) + \underline{a}\right\} \tag{13}$$

Consider now the second interval $(t', t'')$. Since both dirty and clean coal are used, we must have: $p^F + \zeta\lambda_Z = p^F + am(x_c)$. Hence:

$$\lim_{t\downarrow t'} \left\{p^F(t) + \zeta\lambda_Z(t)\right\} = \lim_{t\downarrow t'} \left\{p^F(t) + am(x_c(t))\right\} \geq \lim_{t\downarrow t'} \left\{p^F(t) + \underline{a}\right\} \tag{14}$$

(13) and (14) imply that $\lambda_Z$ is not continuous at time $t'$, which is not possible given the assumptions of the model. Then, clean coal exploitation cannot begin strictly after $\underline{t}_Z$.

ii) Next, since $\lambda_Z(\bar{t}_Z) = 0$, the full marginal cost of dirty coal must be such that $\lim_{t\uparrow\bar{t}_Z} \left\{p^F(t) + \zeta\lambda_Z(t)\right\} = p^F(\bar{t}_Z)$. Hence there must exist a time interval $(\bar{t}_Z - \Delta, \bar{t}_Z)$, with $0 < \Delta < \bar{t}_Z - \underline{t}_Z$, during which $p^F + \underline{a} > p^F + \zeta\lambda_Z$. During this time interval, clean coal is necessarily less competitive than dirty coal and it is then no longer used, which concludes the second part of the proof. ■

**Lemma 3** *When clean coal is exploited during the ceiling period, its consumption rate must decrease. Assuming that solar energy is exploited simultaneously, then its production rate has to increase.*

**Proof:** Assume first that only coal is exploited during the ceiling period. Then from (4), we must have: $u'(x_c + \bar{x}_d) = p^F + ma(x_c)$. Time differentiating this equation, substituting for $\dot{p}^F$ and rearranging, we get: $\dot{x}_c = \frac{\rho\lambda_X}{u''(x_c+\bar{x}_d) - ma'(x_c)} < 0$.

Assume now that solar energy is also exploited during the same period, then from (4) and (6) we have $b = p^F + ma(x_c)$, which implies: $\dot{x}_c = -\frac{\rho\lambda_X}{ma'(x_c)} < 0$.

From (6), $x_c + \bar{x}_d + y = \tilde{y}$, hence the above inequality implies $\dot{y} > 0$. ■

During the ceiling period, the production rate of clean coal must thus decrease. Contrary to the constant CCS cost case, when the solar cost is low, both clean and dirty coals can now be exploited together with solar energy. Furthermore, since the average CCS cost increases, assuming that clean coal is not exploited before the ceiling period would imply that the shadow cost of the pollution stock is discontinuous at the time $\underline{t}_Z$ at which the constraint begins to be active, which is clearly not possible (or, at least, not optimal) given the assumptions of the model.

Last, Proposition 1 below shows that, when the average CCS cost is increasing, it is optimal to deploy the CCS option before being constrained by the ceiling.

**Proposition 1** *Under decreasing returns in CCS technology, the clean coal exploitation must begin before the ceiling constraint binds, i.e.* $\underline{t}_c \leq \underline{t}_Z$. *During this pre-ceiling period, the clean coal consumption rate must increase.*

**Proof:** i) If clean coal is used during the ceiling period then, from Lemma 2, its exploitation must begin at time $\underline{t}_Z$ at the latest. Hence there exists some time interval $(\underline{t}_Z, \underline{t}_Z + \Delta)$, $\Delta > 0$, during which $p^F + \zeta\lambda_Z = p^F + ma(x_c)$. Since $x_c$ decreases within this interval from Lemma 3 and $ma$ is an increasing function of $x_c$, then:

$$\lim_{t \downarrow \underline{t}_Z} \left\{p^F(t) + \zeta\lambda_Z(t)\right\} = \lim_{t \downarrow \underline{t}_Z} \left\{p^F(t) + ma(x_c(t))\right\} > p^F(\underline{t}_Z) + \underline{a} \tag{15}$$

Assume now that clean coal is not competitive yet before $\underline{t}_Z$, then: $p^F(t) + \zeta\lambda_Z(t) \leq p^F(t) + \underline{a}$, $\forall t < \underline{t}_Z$. Hence:

$$\lim_{t \uparrow \underline{t}_Z} \left\{p^F(t) + \zeta\lambda_Z(t)\right\} \leq p^F(\underline{t}_Z) + \underline{a} \tag{16}$$

(15) and (16) imply that $\lambda_Z$ is discontinuous at time $\underline{t}_Z$, which is not possible in the present context. This concludes the first part of the proof.

ii) When clean and dirty coals are exploited before the ceiling, then, from (4) and (5), we must have $p^F + \zeta\lambda_{Z0}e^A = p^F + ma(x_c)$. Time differentiating, we get: $\dot{x}_c = \frac{\zeta\lambda_{Z0}e^A \dot{A}}{ma'(x_c)} > 0$. ■

# 5 Example of optimal paths

In the last section, we have shown that decreasing return in CCS technology implies that it can be optimal to use clean coal and solar energy simultaneously during the ceiling period. Then, two main scenarios have to be considered, depending on whether the cost of solar energy is high or low.

## 5.1 The high solar cost scenario

In this first scenario, solar energy is not exploited during the ceiling period. This is the kind of path resulting from high solar cost, $b > u'(\bar{x}_d)$, as illustrated in Figure 1.

Figure 1 here

Initially only dirty coal is exploited. Its consumption decreases and the energy price increases, due both to the resource scarcity rent and to the carbon tax arguments, up to the time $\underline{t}_c < \underline{t}_Z$ at which $p^F(t) + \zeta\lambda_Z(t) = p^F(t) + \underline{a}$. The next phase is still a pre-ceiling phase, but during which the two types of coal are simultaneously used.[9] Since both

[9]Note that the first phase $[0, \underline{t}_c)$ during which only dirty coal is used may disappear. For $Z^0$ sufficiently high, it may happen that $\underline{t}_c = 0$ and that the clean coal exploitation must begin immediately. Since dirty coal is also exploited and its exploitation rate is higher than $\bar{x}_d$, and $Z(t) < \bar{Z}$, then $Z(t)$ increases. This case would not be possible under constant returns to scale in CCS technology. Because clean coal would never be exploited before the ceiling period, the first phase would necessarily be a phase of exclusive dirty coal exploitation.

are competitive, we must have $\zeta\lambda_Z = ma(x_c)$ which is increasing due to the increasing returns in CCS technology. Then, during this phase, dirty coal exploitation decreases and, simultaneously, the clean coal exploitation increases. This consumption attains its maximum at time $\underline{t}_Z$ when the atmospheric carbon stock reaches the ceiling. The next phase is the first one within the ceiling period, during which both types of coal are still used. However, clean coal consumption decreases down to 0 at $t = \bar{t}_c < \bar{t}_Z$ and the price of the energy services steadily increases up to $u'(\bar{x}_d)$ at time $t = \bar{t}_c < \bar{t}_Z$. The next phase is the last phase at the ceiling during which only dirty coal is exploited: $x_d(t) = \bar{x}_d$. At the end of the phase $\lambda_Z = 0$ forever. Next comes a phase of increasing price and dirty oil exploitation, up to the time $t_y$ at which $p^F(t) = b$. Then the coal exploitation is closed, the last grade of coal which is exploited being this grade $X_b = X(t_y)$ for which $c(X_b) = b$, opening the way for the solar energy exploitation: $y(t) = \tilde{y},\ t > t_y$.

### 5.2 The low solar cost scenario

When the cost of solar energy is low enough, i.e. for $b < u'(\bar{x}_d)$, it can be optimal to deploy this energy during the ceiling period and simultaneously to the clean coal exploitation. This scenario is illustrated in Figure 2.

Figure 2 here

Now the period at the ceiling includes three phases. During the first one, both clean and the dirty coal are used, the production of clean coal being decreasing up to the time at which the solar energy becomes competitive. The next phase is a phase during which the three types of energy are exploited, the clean coal production decreases again, down to zero, and the solar energy production increases. The third phase is a phase of simultaneous use of dirty coal and solar energy, up to the time at which the coal grade $X_b$ is attained and the coal exploitation comes to an end.

## 6 Conclusion

Operation scale is a main challenge for emissions mitigation technologies. This paper has explored this issue in the time-to-build context of a transition between fossil fuel based energy generation and carbon-free energy generation techniques and by assessing the consequences of long run decreasing to scale abatement technologies over the optimal management of climate change in a second best context. Within this framework, we have concluded that the abatement process must start strictly before the climate constraint begins to be binding. This stands in sharp contrast with the usual conclusions derived from constant average abatement cost models. Until the climate constraint binds, the abatement rate must also increase over time.

If solar energy is so expensive that it should not be used before the complete depletion of fossil fuels, then the production of clean coal energy decreases permanently during the climate constrained phase and disappears strictly before the end of this phase. If solar energy is sufficiently cheap, it will be used before the depletion of fossil fuels. A typical scenario is the following. The abatement process, or equivalently the clean coal production, starts before the climate constraint begins to be binding while solar energy is not exploited. Once the climate constraint begins to be active, the abatement activity begins to decrease over time while the production of dirty coal is constrained by the carbon cap. The solar energy production starts during this time phase. A substitution from clean coal production toward solar energy generation occurs until the end of abatement efforts. The next phase combines solar energy production and dirty coal production until the complete depletion of fossil fuels. To understand this complex pattern remark that clean coal energy generation is submitted to the increasing scarcity of the non-renewable resource. This explains why solar energy, while more costly to produce than coal energy, replaces progressively the mitigation of carbon emissions to deal with the climate problem.

Last, as usual in this type of model the optimal policy may decentralized by taxing the pollution emissions. The unitary tax rate is increasing up the time at which the cap constraint is effective and next decreasing down to zero during the phase of constrained emission flow at the ceiling.

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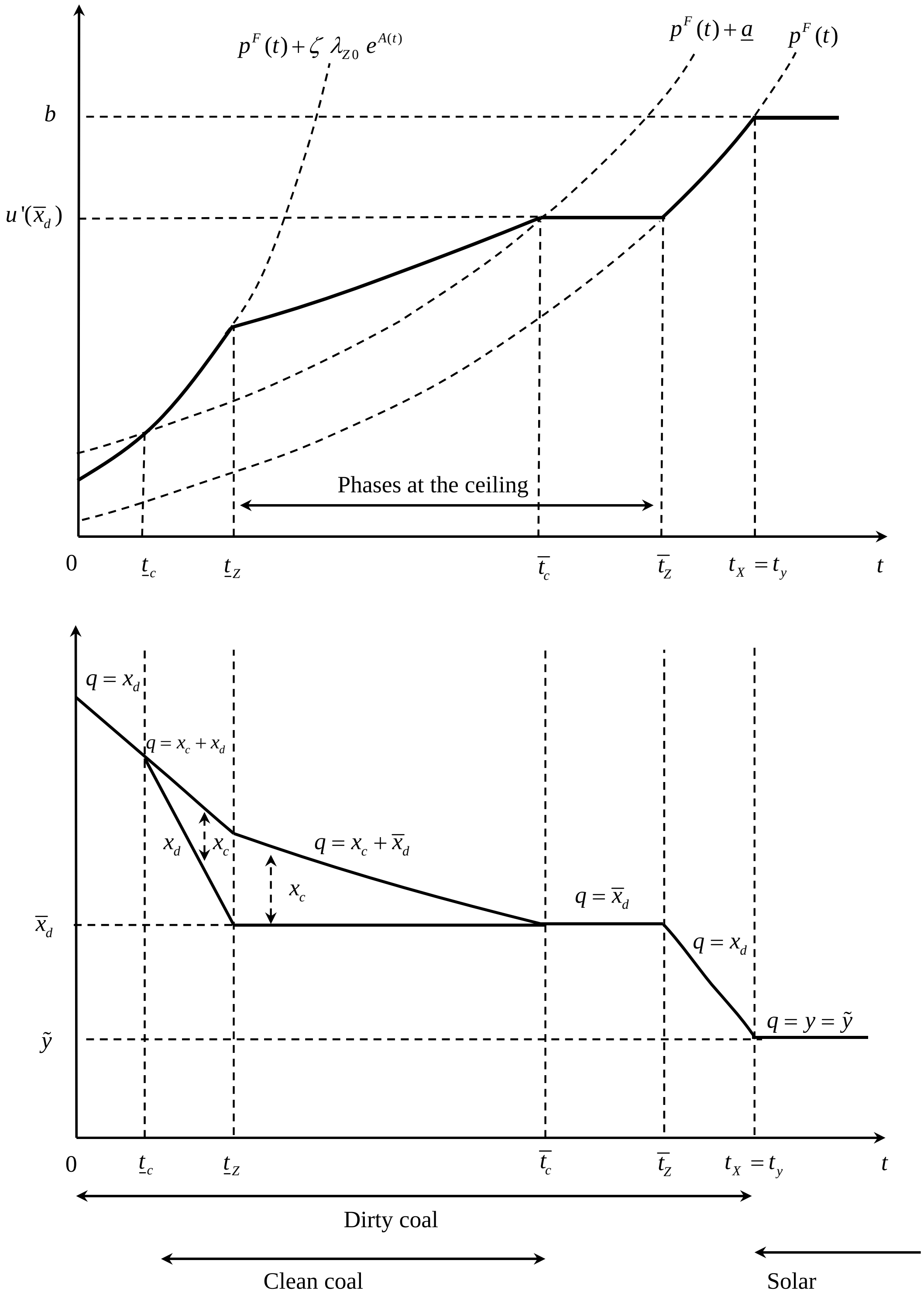


Figure 1: Optimal paths – The high solar cost case

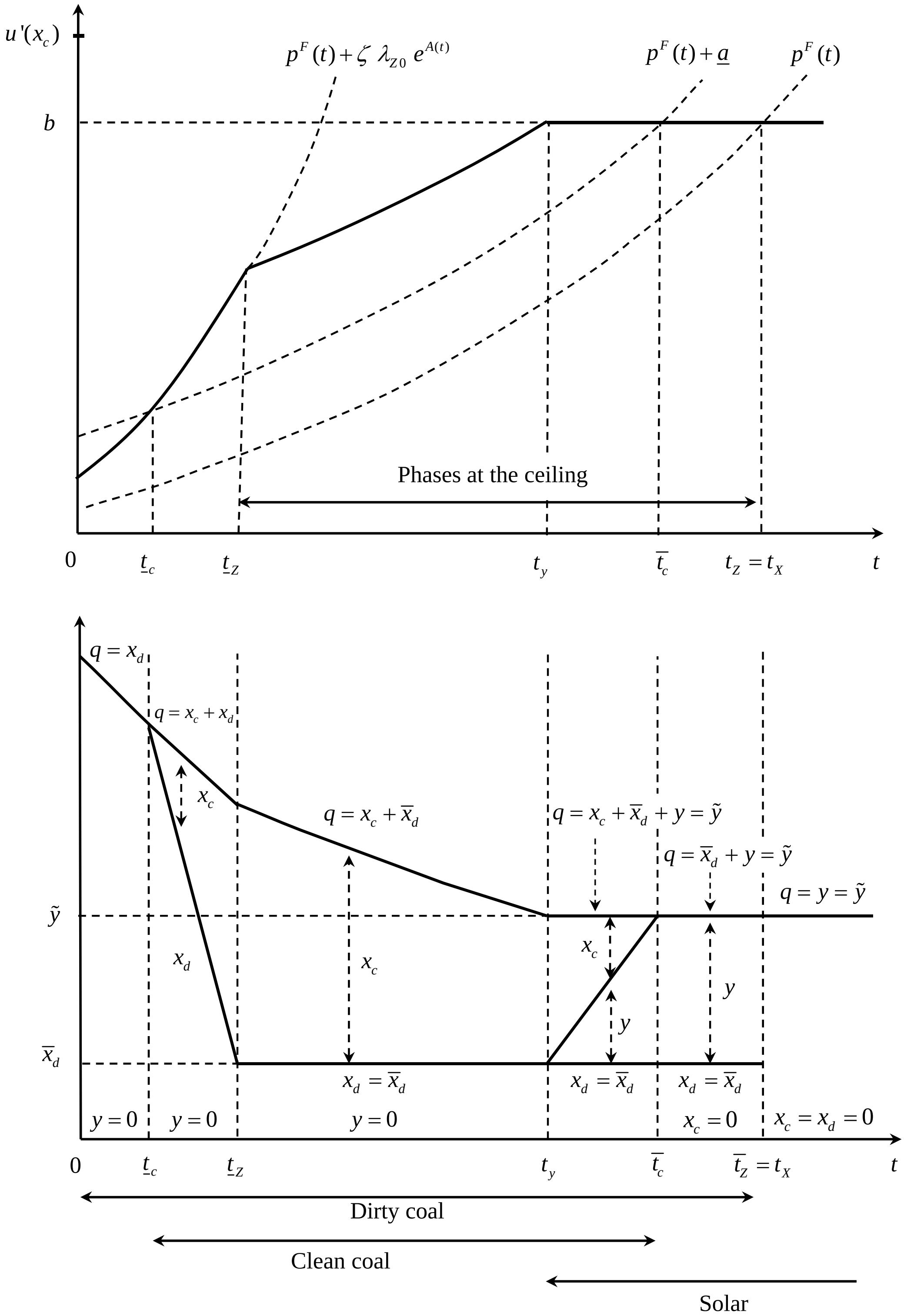


Figure 2: Optimal paths – The low solar cost case