The Optimal Regulation of a Risky Monopoly*

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Abstract

We study the potential conflict between cost minimization and investment in prevention for a risky venture. A natural monopoly is regulated i) first, for economic purpose; ii) second, because it can cause losses of substantial size to third parties (the environment or human beings). The regulator observes the cost without being able to distinguish the initial type (an adverse selection parameter) from the effort (a moral hazard variable). In addition, the investment in prevention is non observable (another moral hazard variable) and the monopoly is protected by limited liability. We fully characterize the optimal regulation in this context of asymmetric information plus limited liability. We show that incentives to reduce cost and to invest in safety care are always compatible. But, in some cases, higher rents have to be given up by the regulator.

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1 Introduction

We study the design of a suitable public policy for managing industrial and environmental accidents such as oil spills, nuclear accidents, fires, explosions or air/soil/water contaminations. The major wave of health, safety and environmental regulation that began in the 1970s, with the pioneering role of the United States, led to the enactment of new regulatory agencies with broad responsibilities for risk and environmental policy (see Viscusi, 2007). However, there are many drawbacks for any efficient public intervention. These limits include asymmetric information between public authorities and potential polluters regarding relevant parameters like the size of potential harm, the probability of an accident, the cost of prevention, the firm's assets level, for instance. In the case of regulated sectors, private information on firms' efficiency – the latter perturbating any economic regulation – can add to the private information on environmental parameters. Moreover, for the sake of realism, we take it for granted that firms are protected by limited liability. Different sources of inefficiency may thus be compounding. The objective of this paper is to better understand the optimal public policy toward a firm in such contexts. In particular, firms may face an internal conflict between cost minimization and prevention. How should public authorities tackle such a conflict? We provide a normative analysis to answer this question. We describe the optimal regulation of a risky venture that benefits from private information both on efficiency and safety care and, in addition, is protected by limited liability.

Consider a firm that undertakes some socially valuable activity but can potentially cause an accident affecting third parties, the environment or human beings in their health or private property. This firm can take actions to reduce the expected losses: it can invest to reduce the probability of accident, this investment in prevention being costly. We restrict our attention to a natural monopoly, which is already regulated for economic purpose.[1] This regulation takes place under asymmetric information regarding some relevant variables. More specifically, we assume that the regulator observes the production cost of the monopoly, but cannot distinguish the monopoly's type (an adverse selection parameter) from the monopoly's effort toward cost reduction (a moral hazard variable). As already said, the monopoly can also invest in prevention.[2] It is assumed that both types of investment or effort interact through a disutility function: the two levels of efforts

[1]Regulatory efforts induce a greater focus on cost minimization and can thus be considered as a shortcut for competition discipline.

[2]From an Incentive Theory viewpoint, we study a mixed model of adverse selection followed by moral hazard together with limited liability. See Ollier and Thomas (2013).

are not independent.

The externality between the natural monopoly and third parties calls for some public intervention. We assume that both the economic and the environmental regulations are designed by the same Agency.[3] Hence, the latter elaborates a regulatory contract with an eye on cost reduction and another on prevention. The Agency implements an incentive regulation that induces revelation on its type by the monopoly and safety care.

We first characterize the social optimum. We then proceed step by step in characterizing the optimal regulation. Let us start with the case where the monopoly has unlimited liability.

In a first benchmark, everything is observable except the prevention effort. In this setting where the monopoly's efficiency is common knowledge and there is no limited liability, moral hazard on safety care is easily solved by the regulator. Since the monopoly can be punished in case of accident – it is just asked to cover the losses – then incentives are given at no cost (the monopoly ends up with zero rent) to induce the first-best allocation.

In a second benchmark, the level of safety care remains unobservable. The novelty is that the monopoly is protected by limited liability: it cannot end up with a negative payoff, even when it is held responsible for an accident. Giving incentives for safety to the monopoly allows it to obtain an *ex ante* positive expected profit designated as a *limited liability rent* (Laffont-Martimort, 2002, chapter 4). Distortions on cost reducing effort are thus introduced to reduce this rent.

The most complex case comes when the regulator is unable to observe the monopoly's efficiency characteristic. It is shown that, in order to induce revelation of the cost, it is sometimes necessary to give up some informational rent to the monopoly. But, due to limited liability, the monopoly is also able to extract some limited liability rent. Then there is a subtle combination of both types of rents at the optimal regulation. We explore their respective roles in detail and show that two distinct cases arise depending on whether the limited liability rent increases or decreases with the level of cost reducing effort.

In the first case where the limited liability rent is decreasing with the cost reducing effort, the least efficient type of the monopoly only obtains a limited liability rent. All the other types receive an informational rent and a limited liability rent, both decreasing with the type. These rents being costly, distortions - in opposite directions - are introduced on cost reducing effort. We show that the priority for the regulator is to reduce the limited liability rent rather than the informational

[3]We abstract here from the multi-Principals setting: there is thus no conflict arising from the fact that different Principals with different objectives are regulating a same Agent.

rent. For all types, the third-best level of cost reducing effort is strictly larger than first-best and some pooling on cost over the whole distribution of types is optimal.

In the second case where the limited liability rent is increasing with the cost reducing effort, we show that no informational rent is given up to the monopoly, whatever its type. All types receive a limited liability rent: the latter is decreasing with the type and ensures revelation. Indeed, by its level and its slope, the limited liability rent plays all roles at the same time: allowing participation, giving incentives for prevention together with inducing revelation of the type. The third-best level of cost reducing effort is now strictly lower than first-best.

As a summary, we have shown that incentives to reduce cost and incentives to invest in safety care are compatible. But, in some circumstances, the optimal contract is more costly from a social viewpoint: in this case, higher rents have to be given up.

This article belongs to the Principal-Agent literature with a risky regulated firm. In this line of research,[4] Hiriart and Martimort (2006b) characterize the optimal regulation of a firm under adverse selection on efficiency and moral hazard on safety care. The main difference is that the production and prevention costs are independent in their setting. The fact that the two effort levels have an interaction through the disutility function in the present context renders the analysis much more complex. This interaction is the building block of Laffont (1995) who first investigated the potential conflict between cost minimization and prevention. His intuition is that regulatory efforts or simple competition, by inducing a greater focus on cost minimization, can tilt the Agent's tradeoff towards taking too much risk. His main conclusion is that high-powered incentives may conflict with safety care: less incentives should be given for cost minimization in order to induce prevention at a lesser cost for the regulator. In a discrete model with both the adverse selection parameter and the moral hazard variable being binary, the firm's optimal contract is characterized for two polar cases.[5] The main intuitions are provided on the mechanisms at work in the intermediary cases but there is no sketch of the optimal contract except for the two polar cases above-mentioned.

Our model essentially elaborates on this paper. We extend this pioneering model by consid-

[4]See Pitchford (1995), Boyer and Laffont (1997), Balkenborg (2001), Newman and Wright (1990), or Hiriart and Martimort (2006a), though all these papers study essentially the case of *extended liability*.

[5]These two polar cases are the following: i) the probability of accident when not taking care approaches zero; ii) it approaches the probability of accident when taking care. Only asymptotic results are obtained in this complex setting.

ering a continuous case for the firm's adverse selection type - but safety care remains binary. Furthermore, the two types of efforts are not independent, though they are not perfect substitutes as assumed in Laffont (1995). Following his strategy, we restrict attention to the case where the highest level of prevention is implemented, which amounts to assuming that potential losses are large compared with prevention cost. We characterize the optimal regulation for all possible cases, thereby solving a problem left open until now. This provides more general results and allows to show precisely that no conflict arises between cost minimization and prevention.

This article is organized as follows. Section 2 presents the model. Section 3 gives some benchmarks of increasing complexity. Section 4 describes the optimal contract when all possible constraints are taken into account. Section 5 briefly concludes by pointing out alleys for further work.

2 The Model

We consider a natural monopoly that undertakes some risky activities and is subject to some incentive regulation. This activity is socially valuable and consumers derive a gross surplus S from it. With a probability $1-p$ an accident occurs and creates external losses of a given size D. We restrict our attention to unilateral accidents: victims have no means to reduce the expected losses, they play a passive role. In addition, there is no possible contractual or market relationship between the monopoly and potential victims, meaning that employees or consumers of the firm's products are excluded from our analysis.

The monopoly. Let us denote by U the monopoly's expected profit. The monopoly receives a transfer t from the rest of society for its activity. It bears a production cost $C = \beta - e$, where $\beta \in \mathcal{B} = [\underline{\beta}, \overline{\beta}]$ captures the firm's efficiency characteristic and $e \leq \underline{\beta}$ measures its cost reducing effort. The monopoly pays an amount f in case of accident, that can be interpreted as a regulatory fine or liability payment. The total transfer $t + C$ allows the monopoly to cover its total costs, meaning production costs, C, plus liability payment, f, in case of harm. By abuse of terminology, the probability $p \in \{p_0, p_1\}$ with $1 > p_1 > p_0 > 0$, of avoiding the accident is considered as a prevention effort.[6]

[6]Implicit is the assumption that the monopoly invests in prevention, and a level of safety care transforms into a probability of avoiding an accident. It is necessary to invest in safety to increase the probability p. Since investment is costly, then an increase in the probability p is costly. In addition, investment is assumed unobservable. Thus, p is a moral hazard variable.

Last, the monopoly incurs a non-monetary disutility $\psi(e, p_i)$.[7] Let $\psi_i(e) = \psi(e, p_i), i = 0, 1$. It is strictly increasing in e and p: i.e., $\psi_i'(e) > 0$ and $\psi_1(e) > \psi_0(e)$. It is also convex in e with $\psi_i''(e) > 0$. We assume that $\psi_1'(e) > \psi_0'(e)$, meaning that both types of efforts are not independent. There is some substitutability between both types of efforts in the disutility function.[8] In addition, we assume that $\psi_1''(e) > \psi_0''(e)$ and $\psi_i'''(e) > 0$. Finally, we consider that $p_1 \frac{\psi_1''(e)-\psi_0''(e)}{p_1-p_0} - \psi_1''(e) > 0$.

The monopoly's expected profit writes as:

$$U_i = t - (1 - p_i)f - \psi_i(e). \tag{1}$$

Regulator's objective. Following Laffont and Tirole (1993), we assume that there exists a positive marginal cost of public funds $\lambda > 0$. The rest of society's expected welfare V writes as:

$$V_i = S - (1 - p_i)D - (1 + \lambda)\left(t + C - (1 - p_i)f\right).$$

Social welfare simply aggregates the firm's expected profit and the rest of society's expected welfare: $W_i = U_i + V_i$. Replacing in W_i the expected transfers $t - (1 - p_i)f$ by their expression as a function of the firm's rent U_i, the regulator's objective function writes as:

$$W_i = S - (1 - p_i)D - (1 + \lambda)(C + \psi_i(e)) - \lambda U_i. \tag{2}$$

This expression shows that the monopoly's expected rent is costly for the regulator. The latter will try to reduce it as much as possible.

Information structure. The monopoly is privately informed about its efficiency parameter before the regulation contract is signed. Once the contract is signed, the monopoly exercises two levels of effort (cost reducing effort and safety care) that are non verifiable by the rest of society. The regulator observes the total production cost C but is unable to distinguish β and e. This regulator has beliefs on the distribution of the efficiency characteristic β with density function $g(.)$ and cumulative $G(.)$. We assume that this distribution satisfies the monotonicity of hazard rate property, i.e. $\left(\frac{G}{g}\right)' > 0$.[9]

[7] See multitask literature with Holmstrom and Milgrom (1991), or Laffont and Tirole (1993), chapter 4.

[8] Laffont (1995) had restrained attention to the case where both efforts were perfect substitutes with $\psi_i(e) = \psi(e + p_i)$. We obtain our results without this quite specific assumption.

[9] Most of the usual distributions satisfy this property, see Bagnoli and Bergstrom (2005).

3 Benchmarks

3.1 Social Optimum

Let us start by characterizing the first-best allocation. In this purpose, we assume that the efficiency characteristic β is observable and the prevention effort (p here) is verifiable. In addition, we assume that there is no limited liability: the monopoly can end up with a negative payoff.

The regulator's problem is to determine the triplet $\{U_i, e, p_i\}$ of rent and efforts that maximizes the expected social welfare W_i under the constraint that the monopoly participates. The participation constraint is such that:

$$U_i \geq 0, i = 0, 1. \tag{PC}$$

Let us denote the first-best solution by superscript FB.

No rent is left to the monopoly, since the objective function is decreasing with U_i. Hence, $U_i^{FB} = 0, i = 0, 1$.

As in Laffont (1995), we restrict our attention to the case where the highest level of prevention is socially optimal. This is true if and only if, for a given level of effort e, the expected social welfare at p_1 is larger than at p_0. The condition $W_1 \geq W_0$ rewrites as:

$$(1 + \lambda)\left(\psi_1(e) - \psi_0(e)\right) \leq (p_1 - p_0)D. \tag{3}$$

This condition says that the highest level of care is optimal if the increase in prevention cost from $\psi_0(e)$ to $\psi_1(e)$ (taking into account the marginal cost of public funds) is smaller than the reduction in expected harm due to the reduction in the probability of accident from $1 - p_0$ to $1 - p_1$. It is satisfied for large losses. In the following, we assume that the regulator implements this safety effort.

The necessary condition for maximizing W_1 with respect to the cost reducing effort gives:

$$1 = \psi_1'(e^{FB}). \tag{4}$$

The socially optimal level of effort equates the marginal social (and private) cost of effort with its marginal social benefit, equal to one. Indeed, one unit of effort directly reduces the total cost of production by one.

In this restricted setting, the optimal regulation is:

$$\begin{cases} 1 = \psi_1'(e^{FB}), \\ p^{FB} = p_1, \\ U_1^{FB} = 0. \end{cases} \tag{FB}$$

Before diving into the full-fledged model developed in Section 4, let us first consider intermediate settings where one-dimensional asymmetric information, then limited liability, put constraints on the definition of the optimal regulation. We proceed step by step through benchmarks of increasing complexity.

3.2 Benchmark 1. Moral hazard on safety care

In this first benchmark, we keep the same information structure as in the first-best, except that the prevention effort p is no longer observable. There is still no limited liability constraint.

Choice of safety care by the firm. The safety care p_1 is implemented if the expected profit U_1 is larger than U_0:

$$p_1 = \arg\max_{p_i \in \{p_0, p_1\}} U_i \Leftrightarrow t - (1-p_1)f - \psi_1(e) \geq t - (1-p_0)f - \psi_0(e), \tag{5}$$

$$\Leftrightarrow f \geq \frac{\psi_1(e) - \psi_0(e)}{p_1 - p_0}. \tag{ICp}$$

This condition says that the monopoly chooses the highest prevention effort if the increase in prevention cost from $\psi_0(e)$ to $\psi_1(e)$ is smaller than the reduction in expected fine due to the reduction in the probability of accident from $1 - p_0$ to $1 - p_1$. Since unobservable, the prevention effort cannot be imposed by the regulator to the firm. The moral hazard incentive constraint (ICp) should thus be taken into account whenever trying to define an optimal incentive regulation. The participation constraint remains (PC).

The regulator's problem. The regulator needs to determine the monopoly's rent and the level of cost reducing effort that maximize the expected social welfare (2) under the participation constraint (PC), given the moral hazard incentive constraint (ICp):

$$\begin{cases} \max\limits_{\{U_1, e\}} \; W_1 = S - (1-p_1)D - (1+\lambda)\left(\beta - e + \psi_1(e)\right) - \lambda U_1, \\ s.t. \quad U_1 \geq 0, \\ \text{with } \; f \geq \dfrac{\psi_1(e) - \psi_0(e)}{p_1 - p_0}. \end{cases}$$

The solution is exactly the socially optimal one given by (FB). As long as the firm's expected profit can be reduced to zero, moral hazard on safety care can be solved at no cost for the regulator: the optimal regulation remains the first-best. Comparing (3) and (ICp), it can be implemented by a policy $f = \frac{D}{1+\lambda}$ which aligns private and social incentives to exercise care. The monopoly may end up with negative payoff in case of accident (recall that it is not protected by limited liability in this benchmark). Participation is ensured as long as the expected profit remains positive. This constraint is satisfied since $U_1^{FB} = 0$.

Let us now investigate how the optimal regulation is modified when the monopoly is protected by limited liability.

3.3 Benchmark 2. Moral hazard on safety care and limited liability

The problem faced by the regulator is essentially the same as Benchmark 1, except that the monopoly is protected by limited liability. It means that, in case of accident, the monopoly's payoff cannot be negative. This constraint writes as:

$$t - f \geq 0. \tag{LLC}$$

Using (ICp) and (1), it can be rewritten as:

$$U_1 \geq \mathcal{R}_1(e) = p_1 f - \psi_1(e), \tag{6}$$

where $\mathcal{R}_1(e)$ is the limited liability rent of the monopoly. Since U_1 is socially costly and increasing with the fine f, the latter will be reduced to its minimum while satisfying (ICp). Hence we have $f = \frac{\psi_1(e)-\psi_0(e)}{p_1-p_0}$, and $\mathcal{R}_1(e) = p_1 \frac{\psi_1(e)-\psi_0(e)}{p_1-p_0} - \psi_1(e)$. We restrict our attention to the relevant cases where $\mathcal{R}_1(e)$ is positive.

Since a gap between the payoffs obtained in the two states of nature (accident/no accident) must be created in order to give incentives for safety care provision,[10] and given that the monopoly cannot be punished in case of accident (due to limited liability), then the monopoly is able to extract a positive rent. As a consequence, (6) implies (PC), ensuring participation of the monopoly.

At this point, it is interesting to see how the rent evolves with respect to the cost reducing effort e. The expression of $\mathcal{R}_1'(e) = p_1 \frac{\psi_1'(e)-\psi_0'(e)}{p_1-p_0} - \psi_1'(e)$ does not allow to have a definite conclusion on its sign. Hence we have two cases. First, when $\mathcal{R}_1'(e) < 0$, the limited liability rent is strictly

[10]The gap is here created by inflicting the fine f in case of accident.

decreasing with the effort e. It implies that the higher the effort one wants to induce, the lower the level of the limited liability rent that must be given up to the monopoly. The reverse happens when $\mathcal{R}_1'(e) > 0$.

The regulator's problem. The regulator needs to determine the monopoly's rent and the level of cost reducing effort that maximize the expected social welfare under the limited liability constraint (6):

$$\begin{cases} \max\limits_{\{U_1,e\}} \;\; W_1 = S - (1-p_1)D - (1+\lambda)\left(\beta - e + \psi_1(e)\right) - \lambda U_1, \\ \\ s.t. \quad U_1 \geq \mathcal{R}_1(e). \end{cases}$$

Let us denote the second-best optimal solution by superscript SB. The limited liability constraint is binding: $U_1^{SB} = \mathcal{R}_1(e^{SB})$. The optimal cost reducing effort is such that:

$$1 = \psi_1'(e^{SB}) + \frac{\lambda}{1+\lambda}\mathcal{R}_1'\big(e^{SB}\big). \tag{7}$$

Let us now compare the second-best cost reducing effort level obtained in (7) with the first-best level characterized by (FB). Due to limited liability, rents cannot anymore be reduced to zero. Two cases arise.

When the limited liability rent is decreasing with the cost reducing effort level, an upward distortion on e^{SB} is introduced with respect to the first-best level. The second-best level of effort still equalizes the marginal social benefit with the marginal social cost. But, the cost reducing effort reduces the limited liability rent: there is a marginal social benefit associated to an increase in effort e. Hence, the marginal social cost of increasing e is $\psi_1'(e)$ and the marginal social benefit is $1 - \frac{\lambda}{1+\lambda}\mathcal{R}_1'(e)$. Indeed, there is a kind of win-win situation here for the regulator: a higher level of effort e allows both to reduce the production cost of the monopoly and the limited liability rent that must be given up to it.

On the reverse, when the limited liability rent is increasing with the cost reducing effort level and since this rent is socially costly, a downward distortion on e^{SB} is introduced with respect to the first-best level. The second-best level of effort equalizes the marginal social benefit, equal to 1, with the marginal social cost: the marginal social cost of the rent, namely $\frac{\lambda}{1+\lambda}\mathcal{R}_1'(e)$, now adds to $\psi_1'(e)$.

The existence of these two cases is highly structuring for the resolution of the full-fledged model developed in the following section.

4 Optimal Regulation under Limited liability

Let us now move to the case where the monopoly's efficiency characteristic β is not observable by the regulator. The latter only observes the production cost C. The unobservability of the type β leads to a loss of control over the cost reducing effort e. This private information on the cost side adds to the non-verifiability of the prevention effort level. In addition, the monopoly is protected by limited liability.

We focus our attention to direct revelation mechanisms where the monopoly reports $\hat{\beta}$ on its type and this report is truthful: $\hat{\beta} = \beta$. From the Revelation Principle,[11] there is no loss of generality in considering such direct and truthful mechanisms that define transfers, production costs and fines $\left\{t(\hat{\beta}), C(\hat{\beta}), f(\hat{\beta})\right\}$ contingent on the report $\hat{\beta}$ made by the monopoly on its type. The expected profit of the monopoly when claiming $\hat{\beta}$ when its true type is indeed β writes as:

$$t(\hat{\beta}) - (1 - p_1) f(\hat{\beta}) - \psi_1 \left(\beta - C(\hat{\beta})\right).$$

Choice of prevention effort by the monopoly. By analogy with Benchmark 1, if the regulator wants to implement the highest level of safety care p_1, then the following moral hazard incentive constraint must be taken into account:

$$f(\hat{\beta}) \geq \frac{\psi_1(e(\hat{\beta})) - \psi_0(e(\hat{\beta}))}{p_1 - p_0}. \tag{8}$$

Choice of the report $\hat{\beta}$ by the monopoly. The expected profit $U_1(\hat{\beta}, \beta)$ of the monopoly claiming $\hat{\beta}$ when its true type is indeed β writes as:

$$U_1(\hat{\beta}, \beta) = t(\hat{\beta}) - (1 - p_1) f(\hat{\beta}) - \psi_1(\beta - C(\hat{\beta})). \tag{9}$$

Incentive constraints require that, $\forall \beta, \hat{\beta} \in \mathcal{B}$:

$$\mathcal{U}_1(\beta) = U_1(\beta, \beta) \geq U_1(\hat{\beta}, \beta). \tag{10}$$

That is, the report $\hat{\beta} = \beta$ maximizes the monopoly's expected profit $U_1(\hat{\beta}, \beta)$.

Lemma 1 *Necessary and sufficient conditions for truth-telling on its type by the monopoly write as:*[12]

$$\mathcal{U}_1'(\beta) = -\psi_1'(e(\beta)) < 0, \tag{11}$$

$$e'(\beta) \leq 1 \ \Leftrightarrow C'(\beta) \geq 0. \tag{12}$$

[11]See Myerson (1982).

[12]See the proof in Appendix A.

Except for the least efficient type, all types have an incentive to overstate their cost, but none has any incentives to understate them. The condition (11) says that, to induce revelation, more efficient types should receive higher rents. In addition, the production cost should be weakly increasing with the type.

Limited liability. We finally get to the problem at stake where all possible constraints are taken into account. Since it is protected by limited liability, the monopoly's payoff in case of accident cannot be negative. The constraint is:

$$t(\beta) - f(\beta) \geq 0, \;\; \forall \beta \in \mathcal{B}. \tag{13}$$

From Benchmark 2, we know that the compounding of limited liability and moral hazard allows the monopoly to extract a limited liability rent $\mathcal{R}_1(e(\beta))$. To implement the pair of effort levels $(e(\beta), p_1)$, the regulator must ensure that the monopoly's expected profit satisfies the condition:

$$\mathcal{U}_1(\beta) \geq \mathcal{R}_1(e(\beta)) = p_1 f(\beta) - \psi_1(e(\beta)), \forall \beta \in \mathcal{B}.$$

By analogy with Benchmark 2, the moral hazard incentive constraint (8) is binding, and thus

$$\mathcal{U}_1(\beta) \geq \mathcal{R}_1(e(\beta)) = p_1 \frac{\psi_1(e(\beta)) - \psi_0(e(\beta))}{p_1 - p_0} - \psi_1(e(\beta)), \forall \beta \in \mathcal{B}.$$

Optimal Regulation. We can decompose the overall expected rent, $\mathcal{U}_1(\beta)$, received by the monopoly into a limited liability component, $\mathcal{R}_1(e(\beta))$, and an informational component $\mathcal{I}_1(\beta)$ such that:

$$\mathcal{I}_1(\beta) = \mathcal{U}_1(\beta) - \mathcal{R}_1(e(\beta)), \forall \beta \in \mathcal{B}. \tag{14}$$

The slope of $\mathcal{I}_1(\beta)$ is crucial to determine the optimal regulation. Indeed, differentiating (14), we get:

$$\mathcal{I}_1'(\beta) = -\psi_1'(e(\beta)) - \left(p_1 \frac{\psi_1'(e(\beta)) - \psi_0'(e(\beta))}{p_1 - p_0} - \psi_1'(e(\beta)) \right) e'(\beta). \tag{15}$$

The sufficient condition (12), $e'(\beta) \leq 1$, implies that $\mathcal{I}_1'(\beta) < 0$ when $\mathcal{R}_1'(e(\beta)) < 0$.[13] By contrast, if $\mathcal{R}_1'(e(\beta)) > 0$, then $\mathcal{I}_1'(\beta)$ can have, *a priori*, any sign. As indicated in Benchmark 2, two cases arise depending on the sign of $\mathcal{R}_1'(e(\beta))$.

Let us denote the third best solutions by superscript TB. We first analyze the case where the limited liability rent is decreasing with the cost reducing effort.

[13]Indeed, $\mathcal{I}_1'(\beta) < 0$ is equivalent to $-\frac{\psi_1'}{p_1 \frac{\psi_1' - \psi_0'}{p_1 - p_0} - \psi_1'} > e'$ when $\mathcal{R}_1'(e(\beta)) < 0$. By assumption, $\frac{\psi_1' - \psi_0'}{p_1 - p_0} > 0$. Thus, the left-hand side is greater than 1. Since, from (12), the right-hand side is lower than 1, we are sure that $\mathcal{I}_1'(\beta) < 0$ when $\mathcal{R}_1'(e(\beta)) < 0$.

Proposition 2 *When $\mathcal{R}_1'(e(\beta)) < 0$, the optimal contract is such that:*[14]

i) The informational rent is decreasing and $\mathcal{I}_1^{TB}(\overline{\beta}) = 0$. All types receive a limited liability rent, which is also decreasing.

ii) There is full pooling on production cost C. The cost reducing effort is $e^{TB}(\beta) = \beta + k^{TB}$ with k^{TB} satisfying:

$$1 = \int_{\mathcal{B}} \left(\psi_1'(\beta + k^{TB}) + \frac{\lambda}{1+\lambda} \psi_1''(\beta + k^{TB}) \frac{G(\beta)}{g(\beta)} \right) g(\beta) d\beta + \frac{\lambda}{1+\lambda} \mathcal{R}_1'(\overline{\beta} + k^{TB}). \tag{16}$$

iii) The third best level of cost reducing effort is strictly larger than first best ($\forall \beta \in \mathcal{B}, e^{TB}(\beta) > e^{FB}$) and thus, the third-best level of fine is also strictly larger than first best ($\forall \beta \in \mathcal{B}, f^{TB}(\beta) > f^{FB}$).

Interpretation. We need to keep in mind that rents are socially costly, hence they will be reduced to their minimum except if some positive level is required for incentive reasons.

First, as Lemma 1 shows, notice that no type of the monopoly has incentives to mimic a more efficient one. Obviously, there exist incentives to mimic less efficient types. Let us start by the least efficient type of the monopoly, $\bar{\beta}$. There is no need to give any informational rent to it, since it cannot mimic anybody. Type $\bar{\beta}$ just receives the limited liability rent. All the other types of the monopoly are able to extract some positive informational rent in addition to the limited liability rent. Since both rents are costly, the regulator seeks to limit them. This is done by introducing two types of distortions on the third-best level of cost reducing effort, reflecting two different trade-offs between efficiency and rent extraction.

The trade-off between efficiency and extraction of the informational rent is reflected in the second term between brackets on the right-hand side of (16). Since the informational rent is increasing in the effort e, the cost of the former pushes toward a downward distortion on effort.

The trade-off between efficiency and extraction of the limited liability rent is reflected by the existence of the last term on the right-hand side of (16). Since the limited liability rent is decreasing in the effort e, this pushes toward an upward distortion on effort.

Hence, summarizing, there are two forces at play. The first one pushes the third-best level of effort to be lower than first-best except for $\underline{\beta}$ since $G(\underline{\beta}) = 0$; the second one pushes the third-best level of effort to be larger for $\beta = \bar{\beta}$. Equivalently, $C^{TB}(\beta) > C^{FB}(\beta), \forall \beta < \bar{\beta}$ and $C^{TB}(\bar{\beta}) < C^{FB}(\bar{\beta})$. This suggests a reduction in production cost at $\bar{\beta}$. But such a reduction is constrained by

[14] See the proof in Appendix B.

(12). The result is some pooling on the upper part of the interval of types $\mathcal{B}$. Appendix B shows that the second effect dominates: pooling is optimally extended to all types of the monopoly. In other words, the priority for the regulator is to reduce the limited liability rent rather than the informational rent.

For the same argument, the third-best level of effort is strictly larger than first-best. Since $C'(\beta) = 0$, $e'^{TB}(\beta) = 1$ and the distortion is strictly increasing with the type. The moral hazard incentive constraint (8) indicates a one-to-one mapping between fine and effort. Thus, (16) also defines the third-best level of fine $f^{TB}(\beta) = \frac{\psi_1(e^{TB}(\beta)) - \psi_0(e^{TB}(\beta))}{p_1 - p_0}$.

Let us now move to the case where the limited liability rent is increasing with the cost reducing effort.

Proposition 3 *When $\mathcal{R}_1'(e(\beta)) > 0$, the optimal contract is such that:*[15]

i) No informational rent is given up to the monopoly: $\mathcal{I}_1(\beta) = 0, \forall \beta \in \mathcal{B}$. A limited liability rent is given up to all types.

ii) Let $e(\beta) = \Gamma(\beta) + k$ be the solution in e of $\mathcal{I}_1'(\beta) = 0$. The cost reducing effort $e^{TB}(\beta)$ is such that k^{TB} satisfies:

$$1 = \int_{\mathcal{B}} \left(\psi_1'(\Gamma(\beta) + k^{TB}) + \frac{\lambda}{1+\lambda} \mathcal{R}_1'(\Gamma(\beta) + k^{TB}) \right) g(\beta) d\beta. \tag{17}$$

iii) The third best level of effort is strictly lower than first best ($\forall \beta \in \mathcal{B}, e^{TB}(\beta) < e^{FB}$) and thus, the third best level of fine is strictly lower than first best ($\forall \beta \in \mathcal{B}, f^{TB}(\beta) < f^{FB}$).

Interpretation. As indicated earlier in the comments following (15), when $\mathcal{R}_1'(e) > 0$, the slope of the informational rent can have, *a priori*, any sign. In particular, this slope can be zero. Since rents are costly, a solution where none of the monopoly's types receives an informational rent is socially optimal. However, all types of the monopoly receive a limited liability rent. Hence, (17) describes a trade-off between efficiency in cost reducing effort and extraction of the limited liability rent. This trade-off is however modified with respect to the standard case of pure moral hazard since it is constrained by setting the informational rent $\mathcal{I}_1(\beta)$ at zero, implying that $e^{TB}(\beta) = \Gamma(\beta) + k^{TB} \neq e^{SB}(\beta)$. Contrary to Proposition 2, the limited liability rent is now increasing with the cost reducing effort e. This now leads to a downward distortion of effort and fine for the whole distribution of types.

[15]See the proof in Appendix C.

Coming back to (14), since $\mathcal{I}_1'(\beta) = 0$ then the limited liability rent has the same slope as $\mathcal{U}(\beta)$. The latter is negative, hence the limited liability rent is strictly decreasing with the type.

Discussion. We will now comment a little the optimal third-best policy described in this section.

Comparison with a Pigovian policy. One may wonder how this third-best optimal policy is linked to the level of harm. A quick look at the benchmarks allows to better understand this relationship. It should be noticed that in Benchmark 1, where safety care is not observable but the firm can be punished since not protected by limited liability, the optimal policy remains first-best: private and social objectives are aligned by imposing a fine equal to the level of harm - taking into account the cost of public funds - i.e. $f = \frac{D}{1+\lambda}$. Moral hazard on prevention solely does not impede a Pigovian policy. As soon as the firm cannot be punished anymore - it cannot end up with a negative payoff - then the optimal fine departs from the level of harm because of limited liability rent. This can be seen in Benchmark 2. The results would be different if the firm had some assets to start with. If it were rich enough, then there would be no distortion on the policy and it would pay $\frac{D}{1+\lambda}$ in case of accident. The distortion is introduced by the fact that the firm is not able to cover the losses in case of accident. Then, in order to provide incentives for prevention while ensuring participation, the limited liability rent $\mathcal{R}$ must be given up to the monopoly. The fine f is still equal to $\frac{\psi_1(e)-\psi_0(e)}{p_1-p_0}$. However, the distortion introduced on e^{SB} because of the limited liability rent creates a distortion on f: the optimal second-best fine is necessarily linked to the level of the limited liability rent. Hence, when frictions are introduced in the setting, the fine gets away from the harm level and is affected by the rents that are necessary for incentive and participation purpose. This is even more the case when, in addition to moral hazard on prevention and limited liability, we also consider the unobservability of the cost β, as in Section 4. Then, for the same arguments, the fine f in this third-best setting is still affected by the rents that must be given up to the firm, but an informational rent $\mathcal{I}$ can now add to the limited liability rent $\mathcal{R}$ (see Proposition 2).

Comparison with Laffont (1995). The optimal contracts described in Propositions 1 and 2 are not present in Laffont (1995) since this author restricted his attention to two polar cases. We have characterized the optimal policy in all possible cases. It is however possible to locate some of our results with respect to his own.

Our Proposition 2 can be related to the Laffont's Proposition 3 where the probability of avoiding the accident when not exercising any care is small (the first polar case). This author underlines the necessity of giving up informational rent, except for the least efficient type of the monopoly, $\overline{\beta}$. But we show that even this type is able to obtain a rent, namely a limited liability rent. The latter influences the optimal regulation. Hence, the optimal contract departs from the one described by Laffont.

Our Proposition 3 depicts a situation where no informational rent is necessary to elicit private information. The level and the slope of the limited liability rent are sufficient for this purpose. This striking result is also completely new.

Overall, we have shown that there is no incompatibility between cost minimization and prevention, as long as the necessary rents are provided to the monopoly. However, there is a cost for this regulatory policy, due to these rents, since regulatory transfers have to be left to the firm. Then, if the rents necessary to implement the third-best policies described earlier were too costly, the regulator would give up implementing efforts for cost minimization. We would be back to $e = 0$, the corner solution obtained in Laffont (1995), corresponding to a situation where the probability of accident when exercising no effort approaches the probability of accident when exercising effort (the second polar case).

5 Conclusion

This paper is a follow-up of Laffont (1995). Generalizing the model to a continuous case for the adverse selection parameter on efficiency and the moral hazard variables on cost reducing effort, we have characterized the optimal incentive regulation for a whole range of cases that had not been explored by this author. Investigating the possible conflict between incentives for cost minimization and for prevention in the case of a natural monopoly, we have obtained many new results. In particular, we have shown that the optimal regulatory contract is strongly driven by the limited liability rent obtained in this setting by the monopoly.

Following Laffont (1995)'s pioneering work, we have restricted attention to regulations implementing the highest level of safety care, a policy which is socially optimal as long as we deal with high levels of environmental losses, let us say disasters. Revelation of the cost by the monopoly is always possible, and the third-best level of cost reducing effort can be higher or lower than first-best. However, once again, there is a cost for this regulatory policy, a dimension which has been left aside in the present analysis.

A first natural extension of our work would be to go a step further and explore the optimal regulation for a lower range of possible environmental damages, in which case it is not necessarily optimal to induce the highest level of prevention. This is kept for further research.

In our paper, the monopoly is regulated both for economic and environmental purposes. To better understand the possible conflict between cost minimization and prevention, we have provided a normative analysis where a unique regulator is in charge of both types of regulations. In the real life, two different entities are usually in charge of these different tasks. Then the potential conflict could be affected and eventually aggravated. A second natural extension of this work is to move to a multi-Principals setting in which an economic regulator and a distinct environmental one both control a same risky natural monopoly. This is also kept for further research.

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Appendix

Appendix A. First and second-order conditions are respectively:

$$\frac{\partial U_1}{\partial \hat{\beta}}(\beta, \beta) = 0, \tag{18}$$

$$\frac{\partial^2 U_1}{\partial \hat{\beta}^2}(\beta, \beta) \leq 0. \tag{19}$$

Using (8) and (9), first-order condition (18) is equivalent to:

$$t'(\beta) - (1 - p_1)f'(\beta) + \psi_1'(e(\beta))C'(\beta) = 0. \tag{20}$$

Since $\mathcal{U}_1(\beta) = U_1(\beta, \beta)$, we use the Envelop Theorem to understand how it evolves with the efficiency characteristic β. We obtain:

$$\mathcal{U}_1'(\beta) = \frac{\partial U_1}{\partial \hat{\beta}}(\beta, \beta) + \frac{\partial U_1}{\partial \beta}(\beta, \beta) = \frac{\partial U_1}{\partial \beta}(\beta, \beta) = -\psi_1'(e(\beta)) < 0.$$

Differentiating (18), we obtain $\frac{\partial^2 U_1}{\partial \hat{\beta}^2}(\beta,\beta) + \frac{\partial^2 U_1}{\partial \beta \hat{\beta}}(\beta,\beta) = 0$. So, from (20), the second-order condition (19) can be rewritten as:

$$\psi_1''(e(\beta))C'(\beta) \geq 0. \tag{21}$$

But, with $\psi_1'' > 0$ and $C'(\beta) = 1 - e'(\beta)$, we get $e'(\beta) \leq 1$. This local condition is also globally sufficient.

Appendix B.

□ *Proof of Proposition 2 i).* Since $\mathcal{I}_1'(\beta) < 0$ when $\mathcal{R}_1'(e(\beta)) < 0$ and rents are costly from the regulator's viewpoint, then the type $\bar{\beta}$ will not receive any informational rent at the optimal solution, i.e. $\mathcal{I}_1(\bar{\beta}) = 0 \Leftrightarrow \mathcal{U}_1(\bar{\beta}) = \mathcal{R}_1(e(\bar{\beta}))$.

□ *Proof of Proposition 2 ii).* The monopoly's expected profit is strictly decreasing with the type β. We can then rewrite the expected profit of any type of monopoly as:

$$\mathcal{U}_1(\beta) = \int_{\beta}^{\bar{\beta}} \psi_1'(e(\tilde{\beta}))d\tilde{\beta} + \mathcal{U}_1(\bar{\beta}).$$

From the last expression, we can write the expected profit on the whole distribution of types $\int_{\mathcal{B}} \mathcal{U}_1(\beta)g(\beta)d\beta$ as:

$$\int_{\mathcal{B}} \left[\int_{\beta}^{\bar{\beta}} \psi_1'(e(\tilde{\beta}))d\tilde{\beta}\right] g(\beta)d\beta + \mathcal{U}_1(\bar{\beta}).$$

Proceeding to an integration by parts, we obtain:

$$\int_{\mathcal{B}} \mathcal{U}_1(\beta)g(\beta)d\beta = \int_{\mathcal{B}} \psi_1'(e(\beta))G(\beta)d\beta + \mathcal{U}_1(\bar{\beta}). \tag{22}$$

Using *i)*, $\mathcal{U}_1(\bar{\beta}) = \mathcal{R}_1(e(\bar{\beta}))$. The regulator's objective writes as:

$$\begin{aligned}\int_{\mathcal{B}} \Big(S - (1-p_1)D - (1+\lambda)(\beta - e(\beta) + \psi_1(e(\beta)))\\ - \lambda\psi_1'(e(\beta))\frac{G(\beta)}{g(\beta)}\Big)g(\beta)d\beta - \lambda\mathcal{R}_1(e(\bar{\beta})).\end{aligned} \tag{23}$$

This takes into account the informational rent received by all types of the monopoly except type $\bar{\beta}$, and the limited liability rent $\mathcal{R}_1(e(\bar{\beta})) = p_1\frac{\psi_1(e(\bar{\beta})) - \psi_0(e(\bar{\beta}))}{p_1 - p_0} - \psi_1(e(\bar{\beta}))$ received by the worst type $\beta = \bar{\beta}$. The constraint faced by the regulator is $e'(\beta) \leq 1$.

From now on, argument of functions will be omitted unless necessary. Using optimal control techniques with scrap value, we let $C' = y$, where C is a state variable and y is a control variable.

We denote by μ the costate variable associated to this evolution equation. Then the Hamiltonian writes as:

$$H = \left(-(1+\lambda)(C + \psi_1(\beta - C)) - \lambda \psi_1'(\beta - C)\frac{G}{g} \right) g + \mu y.$$

Denoting by η the multiplier of constraint $y \geq 0$, the Lagrangian writes:

$$L = H + \eta y.$$

There are two necessary conditions for maximization:

$$L_y = \mu + \eta = 0. \tag{24}$$

$$\mu' = -H_C = -\left(-(1+\lambda)(1 - \psi_1') + \lambda \psi_1'' \frac{G}{g} \right) g. \tag{25}$$

There are also two transversality conditions. The first at $\underline{\beta}$ writes as:[16]

$$\mu(\underline{\beta}) = 0. \tag{26}$$

The second one at $\bar{\beta}$ writes as:

$$\mu(\bar{\beta}) = \lambda \mathcal{R}_1'(\bar{\beta} - C(\bar{\beta})). \tag{27}$$

There is a slackness condition:

$$\eta \geq 0, \; \eta y = 0. \tag{28}$$

Since the Lagrangian is linear in y, necessary and transversality conditions are also sufficient if H and $-\lambda \mathcal{R}_1$ are concave in C. The first sufficient condition is $H_{CC} < 0$, which amounts to $\left(-(1+\lambda)\psi_1'' - \lambda \psi_1''' \frac{G}{g} \right) g < 0$. The assumptions $\psi_1'' > 0$ and $\psi_1''' > 0$ ensure that the last condition is satisfied. The second sufficient condition is $-\lambda \mathcal{R}_1'' = -\lambda \left(p_1 \frac{\psi_1'' - \psi_0''}{p_1 - p_0} - \psi_1'' \right) < 0$. It is satisfied using assumptions on ψ.

Now, let us show the optimality of a pooling contract.

Combining (24) and (28), we both have:

$$\eta = -\mu \geq 0. \tag{29}$$

Our demonstration proceeds in four steps. Each step is a statement which proof is given straight below.

[16] Transversality conditions in similar optimization problems with scrap value function can be found in Seierstad and Sydsæter (1987), Theorem 3 page 182.

a) *There exists $\beta^\circ < \overline{\beta}$ such that $C' = 0$ for $\beta \geq \beta^\circ$.*

Since $\mathcal{R}_1'(e(\bar{\beta})) < 0$, from (27), we have $\mu(\bar{\beta}) < 0$. By continuity of the costate variable μ and (26), there exists β° such that:

$$\begin{cases} \mu = 0 \text{ if } \beta \leq \beta^\circ, \\ \mu < 0 \text{ if } \beta > \beta^\circ. \end{cases} \tag{30}$$

Using (29), this implies that:

$$\begin{cases} \eta = 0 \text{ if } \beta \leq \beta^\circ, \\ \eta > 0 \text{ if } \beta > \beta^\circ. \end{cases} \tag{31}$$

Hence, using (28), we obtain $y = 0 \Rightarrow C' = 0$ for $\beta > \beta^\circ$. Moreover, by continuity of C, we must have, using (12):

$$C(\beta) = C(\beta^\circ) \text{ for any } \beta \geq \beta^\circ. \tag{32}$$

b) *We have: $C' > 0$ for $\beta < \beta^\circ$.*

From (30), we have $\mu = 0$ on $[\underline{\beta}, \beta^\circ]$. Therefore, $\mu' = 0$ on $[\underline{\beta}, \beta^\circ[$. From (25) and (29), it follows that C is such that:

$$1 - \psi_1' - \frac{\lambda}{1+\lambda}\psi_1''\frac{G}{g} = 0. \tag{33}$$

Differentiating with respect to β, yields:

$$e' = -\frac{\frac{\lambda}{1+\lambda}\psi_1''\left(\frac{G}{g}\right)'}{\psi_1'' + \frac{\lambda}{1+\lambda}\psi_1'''\frac{G}{g}} < 0. \tag{34}$$

Therefore, $C' = 1 - e' > 0$ on $[\underline{\beta}, \beta^\circ[$.

c) *We have: $\eta' > 0$ if $\beta > \beta^\circ$.*

Using (29), $\eta' > 0$ is equivalent to $\mu' < 0$. Assume that the statement c) does not hold: hence, there exists some $\tilde{\beta} > \beta^\circ$ such that $\mu'(\tilde{\beta}) = 0$. By continuity of C, $C(\beta^\circ)$ is given by (33). Moreover, if $\tilde{\beta} > \beta^\circ$, we know, from (32), that $C(\tilde{\beta}) = C(\beta^\circ)$. But if, by assumption, $\mu'(\tilde{\beta}) = 0$, then $C(\tilde{\beta})$ is the solution of (33). Then using the same argument developed for statement b), we must have $C(\tilde{\beta}) > C(\beta^\circ)$. This is a contradiction. Therefore, $\eta' > 0$ on $]\beta^\circ, \bar{\beta}]$.

d) *We have:* $\beta^\circ = \underline{\beta}$.

The general idea of the proof is the following. It cannot exist $\beta^\circ > \underline{\beta}$ satisfying the continuity of μ, and, using (29), the continuity of η. Using (27) and (29), we have $\eta(\overline{\beta})\big|_{C=C(\beta^\circ)} = -\lambda\mathcal{R}_1'(\bar{\beta} - C(\beta^\circ))$. So, from the convexity of $\mathcal{R}_1$, we must have:

$$\frac{\partial\eta(\overline{\beta})\big|_{C=C(\beta^\circ)}}{\partial C(\beta^\circ)} = \lambda\mathcal{R}_1''(\bar{\beta} - C(\beta^\circ)) > 0. \tag{35}$$

Moreover, using (25) and (29):

$$\eta'|_{C=C(\beta^\circ)} = H_C|_{C=C(\beta^\circ)}\,. \tag{36}$$

By concavity of the Hamiltonian, this yields:

$$\frac{\partial\ \eta'|_{C=C(\beta^\circ)}}{\partial C(\beta^\circ)} = H_{CC}|_{C=C(\beta^\circ)} < 0. \tag{37}$$

Assume the existence of a $\beta^\circ > \underline{\beta}$. From b), we know that $C(\beta^\circ) > C(\underline{\beta})$. This implies $0 < \eta'|_{C=C(\beta^\circ)} < \eta'|_{C=C(\underline{\beta})}$ from c) and (37). Moreover, the interval over which η is strictly positive moves on the right. Since the slope is lower, then by continuity of η, the path must end at a lower level: $\eta(\overline{\beta})|_{C=C(\beta^\circ)} < \eta(\overline{\beta})|_{C=C(\underline{\beta})}$. A contradiction with (35). Hence, $\beta^\circ = \underline{\beta}$. Using a), the cost C is constant over the whole interval: a pooling contract is optimal.

□ *Proof of Proposition 2 iii)*. From the proof of (c) above, we know that $\eta' > 0,\ \forall\beta \in \mathcal{B}$. In particular, we have $\eta'(\underline{\beta}) > 0$. Using (25), (29) and $G(\underline{\beta}) = 0$, we get $\eta'(\underline{\beta}) > 0 \Leftrightarrow \psi_1' > 1$ at $\underline{\beta}$. We also know that $e'^{\,TB} = 1$ since $C'^{\,TB} = 0$ (from the proof of ii)). So, from equation (FB), we obtain $e^{TB} > e^{FB}$, for all β in $\mathcal{B}$, since $1 = e'^{\,TB} > e'^{\,FB} = 0$.

Appendix C.

□ *Proof of Proposition 3 i).*

a) Assume that $\mathcal{I}_1(\beta)$ is increasing on $[\underline{\beta}, \underline{\beta} + \epsilon[$ with $\bar{\beta} - \underline{\beta} > \epsilon > 0$. The best type only gets $\mathcal{R}_1$. The transversality condition (27) becomes $\mu(\underline{\beta}) = -\lambda\mathcal{R}_1' < 0$. Using (30), this implies $\eta > 0$. From (29), this yields $y = C' = 0$. But in this case, (15) shows that $\mathcal{I}_1' = -\psi_1'(1-e') - p_1\frac{\psi_1'-\psi_0'}{p_1-p_0} = \psi_1'C' - p_1\frac{\psi_1'-\psi_0'}{p_1-p_0} = -p_1\frac{\psi_1'-\psi_0'}{p_1-p_0} < 0$. A contradiction with the assumption that $\mathcal{I}_1(\beta)$ is increasing.

b) Assume that $\mathcal{I}_1(\beta)$ is decreasing on $]\overline{\beta} - \epsilon, \overline{\beta}]$ with $\bar{\beta} - \underline{\beta} > \epsilon > 0$. The worst type only gets $\mathcal{R}_1$. The transversality condition (28) implies that $\mu(\overline{\beta}) > 0$. This contradicts (30). As a consequence, the informational rent cannot be decreasing on this interval.

c) From the two preceding points, we conclude that: if the informational rent is positive on some intervals of $\mathcal{B}$, then the only possible shape for $\mathcal{I}_1(\beta)$ is that it is first decreasing for lower values of β, then it is flat for intermediary values and finally increasing for higher values of β. Let us assume that $\mathcal{I}_1(\beta)$ follows this pattern. Let us define the thresholds (β_1, β_2) such that $\underline{\beta} \leq \beta_1 \leq \beta_2 \leq \bar{\beta}$[17] and, i) $\mathcal{I}_1(\beta)$ is strictly decreasing on $[\underline{\beta}, \beta_1]$; ii) $\mathcal{I}_1(\beta)$ is constant on $[\beta_1, \beta_2]$; iii) $\mathcal{I}_1(\beta)$ is strictly increasing on $[\beta_2, \overline{\beta}]$.

In this case, the types belonging to $[\beta_1, \beta_2]$ do not receive any informational rent, since socially costly: they will just receive the limited liability rent $\mathcal{R}_1$.

On $[\underline{\beta}, \beta_1]$, we find the same contradiction as the one shown in b): the informational rent cannot be decreasing on $[\underline{\beta}, \beta_1]$. This interval thus boils down to $\beta_1 = \underline{\beta}$. A similar reasoning applies for the upper interval. On $[\beta_2, \overline{\beta}]$, we find the same contradiction as the one shown in a): the informational rent cannot be increasing on this interval. The latter degenerates and $\beta_2 = \bar{\beta}$. As a consequence, $\mathcal{I}_1(\beta) = 0$ on $\mathcal{B}$: all types receive the limited liability rent $\mathcal{R}_1$.

[17]Obviously, the degenerated case $\underline{\beta} = \beta_1 = \beta_2 = \bar{\beta}$ will not be considered.