Using a Tullock contest to approve/ban a new product/technology under asymmetric information?

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Abstract

We consider the regulatory problem to approve or to ban a new product/technology, in a context of scientific controversy about its environmental and/or health detrimental effects. We formalize the government decision-making as a Tullock contest (1980), the contestants being an industrial lobby, representing the economic agents who have developed the new product/technology, and an environmental lobby, representing the economic agents who will be harmed by its environmental and/or health detrimental effects. Assuming that the industrial lobby has private information about the environmental and/or health detrimental effects, but can be held liable for damage *ex post*, we derive the properties of the equilibrium of the contest. In particular, we give conditions under which it is socially better that the government decides according to the contest, rather than according to an *ex ante* cost-benefit analysis, using his prior beliefs. We find that the contest outperforms the *ex ante* cost-benefit analysis only if the risk of judgment-proofness is not too high.

Keywords : contest model, information asymmetry, law and economics, optimal regulation.

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1 Introduction

In recent years, several scientific and political controversies have arisen regarding the market introduction of new products and/or new technologies. Key

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examples are genetically modified organisms and hydraulic fracturing to extract shale gas. In both cases, a common feature is that the regulatory decision to approve, or to ban, the new product and/or new technology, faces the challenge of balancing large economic profits against uncertain (and possibly huge) environmental and/or health detrimental effects. The stylized fact is that this has lead to large differences in regulations among countries, which may reflect the preferences of various interest groups involved in the process ([1]).

This paper aims at representing the government decision-making process in this kind of situations, accounting for the intervention of interest groups in a context of scientific controversy about the environmental and/or health detrimental externalities. Formally, we analyse a Tullock contest (Tullock, 1980), with the two contestants being an industrial and an environmental lobby. A key assumption in our model is that the industrial lobby has private information about the detrimental externality, but can be held liable for damage *ex post*. In this setting, we determine the equilibrium of the contest and derive both its positive and normative properties.

The main result of the paper is to provide conditions under which it is socially better that the government decides to approve, or to ban, the new product and/or new technology, according to the contest, rather than according to an *ex ante* cost-benefit analysis, using his prior beliefs. The reason why this can be so, is because the *ex post* liability for harm induces the industrial lobby to reveal his private information about the the environmental and/or health detrimental externalities in the contest. The reason why this is not always so, is because the industrial lobby can be financially insolvent and the contest is a costly scheme. We find that the contest outperforms the *ex ante* cost-benefit analysis when the level of assets of the industrial lobby is not too small with respect to the harm done. In an extension, assuming the information about the damages becomes public, we also show that the contest is more efficient if the environmental lobby knows the harm prior to the contest.

The closest papers in the literature are Aidt (1998) and Graichen et al. (2001). Aidt (1998) derives the equilibrium environmental policy in a common agency model of politics (Grossman and Helpman, 1994), where the government determines policy so as to maximize the likelihood of being reelected. If all agents have their interests represented by an interest group, Aidt (1998) shows that the equilibrium environmental policy is the (socially optimal) Pigouvian tax. Graichen et al. (2001) analyse a Tullock contest, where a monopolistic utility and environmentalists confront for the contract for the supply of electricity. At the equilibrium, they show that this induces the monopolist to use a more environmentally friendly technology.

This paper pursues the same line of research, which is to understand whether the intervention of interest groups in the political decision-making process can

[1] See Vigani and Olper (2013), for genetically modified organisms, and Johnson and Boersma (2012), for hydraulic fracturing to extract shale gas.

help to design better environmental regulation, or not. Our principal contribution with respect to Aidt (1998) and Graichen et al. (2001) is to introduce the issue of scientific controversy about the environmental and/or health detrimental externalities, as represented by the private information of the industrial lobby, and to highlight the importance of the *ex post* liability system.

More specifically, we depart from Aidt (1998), by using a Tullock contest, instead of a common agency model of politics. Importantly, this means that the lobbying activity is viewed as costly in our setting, whereas it only involves transfers of numeraire in Aidt (1998). As a result, in the normative analysis, there is less chance that the intervention of the interest groups be optimal in our paper. Finally, we depart from Graichen et al. (2001) by going further into the analysis, to investigate the effect of the contest equilibrium on the social surplus. Graichen et al. (2001) did not consider this question.

The paper is organized as follows. Section 2 presents the model. Section 3 determines the equilibrium of the contest game. Section 4 and 5 respectively derive the positive and normative properties of the outcome of the contest. Section 6 amends the model, assuming that both lobbies know the damages. Section 7 concludes. Proofs of all propositions are contained in an appendix.

2 The model

Suppose that an industrial lobby, denoted by I, has developed a new product. If the new product is marketed, lobby I will obtain a benefit b from selling it. However, the new product can also have environmental and/or health detrimental effects on an environmental lobby, denoted by E. If the new product is marketed, it is assumed that lobby E will bear a damage δ. It is common knowledge that δ is uniformly distributed on $[0,1]$. Below, it is assumed that $b < 1$, which is the only case of interest ([2]). By assumption, only lobby I observes δ before the product is marketed ([3]) ([4]).

The new product needs to be approved by the regulator before it can be marketed by lobby I. Two ways of doing so will be considered and compared below. In the first one, the regulator simply decides according to his prior beliefs. As a result, he approves the new product if and only if $b > E[\delta] = 1/2$. In the second one, the regulator's decision depends on the two lobbyists' efforts, according to a simultaneous Tullock contest (Tullock, 1980). Formally, lobbies I and E simultaneously bid nonnegative values, respectively denoted by x and y,

[2] If $b \geq 1$, it is always socially optimal to market the new product.

[3] The situation where both lobbies observe δ is considered in Section 6.

[4] In this paper, we disregard the question of how pressure groups form. Implicitly, it is assumed that there is no free-riding and, consequently, the two lobbies truly represent their members' interests.

and the regulator he approves the new product/technology with the probability

$$\pi(x,y) = \begin{cases} \frac{x}{x+y}, & \text{if } x+y>0, \\ 0, & \text{otherwise.} \end{cases}$$

Finally, we assume that lobby I can be held liable for damage and a strict liability rule prevails. In other words, if lobby I sells the product and lobby E bears a damage δ, lobby I will be asked in court to pay δ to lobby E. However, the maximum amount that lobby I can pay is limited to k, which can represent either a statutory limit on damages or his level of assets ([5]). Hence, assuming that $k<1$, lobby I will sometimes pay k instead of δ, with $k<\delta$. Below, it is assumed that $k<b$, which is the only case of interest ([6]).

From this, *ex post*, given δ, the expected utilities of I and E are respectively equal to

$$\pi(x,y)\left(b-\min(\delta,k)\right)-x,$$
$$\pi(x,y)\left(\min(\delta,k)-\delta\right)-y.$$

3 Equilibrium of the contest

Suppose that the regulator relies on the lobbying' efforts to decide whether the product will be marketed or not. We seek for an equilibrium where lobby I with type δ plays $x^*(\delta)$ and lobby E plays y^*.

Consider first the equilibrium behavior of lobby I. Anticipating lobby E's effort y, he seeks x to maximize his expected utility

$$\left(\frac{x}{x+y}\right)\left(b-\min(\delta,k)\right)-x.$$

We can differentiate it with respect to x to obtain the first-order condition describing lobby I's optimal effort

$$\left(\frac{y}{(x+y)^2}\right)\left(b-\min(\delta,k)\right)-1\leq 0,$$

[5] Under the latter interpretation, the fact that lobby I can pay a judgment of only k reflects the implicit simplifying assumption that b cannot also be paid. Otherwise the available assets would be $b+k$. This is usually justified saying that b is a utility benefit (Shavell, 1984; Shavell, 2005). This justification does not apply in our application, where by definition b is a monetary benefit. Here, an alternative justification, in line with our application, could be that the damages will happen in such a distant future, that the benefit would no more be available at the time of the judgment.

[6] If $k\geq b$, it can be shown that, in equilibrium, $x+y$ is negligible, for all δ, and $\pi(x,y)=1$, if $b>\delta$, $\pi(x,y)=0$, otherwise. Hence, the contest implements the optimal decision at no cost. It is thus the best scheme to decide whether to market the new product/technology or not.

which holds with equality for an interior solution. It follows that the lobby I's reaction function is ([7]).

$$x=\begin{cases}\sqrt{(b-\min(\delta,k))\,y}-y, & \text{if } y<b-\min(\delta,k)\,,\\ 0, & \text{otherwise.}\end{cases}$$

Consider now the equilibrium behavior of lobby E. Anticipating lobby I's effort $x(\delta)$, for all δ, he seeks y to maximize his expected utility

$$\int_0^1\left(\frac{x(\delta)}{x(\delta)+y}\right)(\min(\delta,k)-\delta)\,d\delta-y.$$

Given that $\min(\delta,k)-\delta=0$, for all $\delta\leq k$, and $\min(\delta,k)-\delta=k-\delta$, for all $\delta>k$, lobby E's expected utility can also be written

$$\int_k^1\left(\frac{x(\delta)}{x(\delta)+y}\right)(k-\delta)\,d\delta-y.$$

We can differentiate it with respect to y to obtain the first-order condition describing lobby E's optimal effort

$$\int_k^1\left(\frac{x(\delta)}{(x(\delta)+y)^2}\right)(\delta-k)\,d\delta-1\leq 0,$$

which holds with equality for an interior solution.

Finally, let us determine an equilibrium $x^*(\delta)$, for all δ, and $y^*$of the contest. Assume that $x^*(\delta)>0$, for all δ, and $y^*>0$. Then, the equilibrium effort of lobby I satisfies

$$x^*(\delta)=\sqrt{(b-\min(\delta,k))\,y^*}-y^*,\text{ for all }\delta.$$

Substituting this into the first-order condition describing lobby E's optimal effort, we get the condition

$$\left(\frac{1}{\sqrt{(b-k)\,y^*}}-\frac{1}{b-k}\right)\int_k^1(\delta-k)\,d\delta=1,$$

implying that

$$y^*=\left(\frac{(1-k)^2}{2(b-k)+(1-k)^2}\right)^2(b-k)\,.$$

It is direct to verify that $x^*(\delta)>0$, for all δ, and $y^*>0$ ([8]) ([9]).

[7] In equilibrium, it will be shown that lobby E's effort y^* is smaller than $b-k$, implying an interior solution $x^*(\delta)>0$, for all δ.

[8] Remark that $y^*<b-k$. Hence, $y^*<b-\min(\delta,k)$ and $x^*(\delta)>0$, for all δ.

[9] It is worth noting that the result that lobby I's effort is always strictly positive, for all

4 Comparative statics

We investigate here some properties of the equilibrium of the contest.

The aggregate lobbying effort equals

$$x^*(\delta) + y^* = \begin{cases} \frac{(1-k)^2(b-k)}{2(b-k)+(1-k)^2}\sqrt{\frac{b-\delta}{b-k}}, & \text{if } \delta < k, \\ \frac{(1-k)^2(b-k)}{2(b-k)+(1-k)^2}, & \text{otherwise.} \end{cases}$$

The probability that the product will be marketed is

$$\pi(x^*(\delta), y^*) = \begin{cases} 1 - \frac{(1-k)^2}{2(b-k)+(1-k)^2}\sqrt{\frac{b-k}{b-\delta}}, & \text{if } \delta < k, \\ 1 - \frac{(1-k)^2}{2(b-k)+(1-k)^2}, & \text{otherwise.} \end{cases}$$

The expected social surplus equals

$$\begin{pmatrix} \pi(x^*(\delta), y^*)(b-\delta) \\ -x^*(\delta) - y^* \end{pmatrix} = \begin{cases} (b-\delta) - 2\frac{(1-k)^2}{2(b-k)+(1-k)^2}\sqrt{(b-\delta)(b-k)}, & \text{if } \delta < k, \\ (b-\delta) - \frac{(1-k)^2}{2(b-k)+(1-k)^2}(2b-\delta-k), & \text{otherwise.} \end{cases}$$

The following proposition gives sufficient conditions to characterize the relevant comparative statics whenever possible. Note that the proposed conditions can be much stricter than necessary. The proof is given in the appendix.

Property 1. (Comparative statics).

- Consider a state such that $\delta < k$: Assume $2b - k^2 \leq 1$. The aggregate lobbying effort $x^*(\delta) + y^*$ is increasing in b and decreasing in k. The probability of approving the new product/technology $\pi(x^*(\delta), y^*)$ is increasing in k. The expected social surplus is $\pi(x^*(\delta), y^*)(b-\delta) - x^*(\delta) - y^*$ is increasing in k.

- Consider a state such that $\delta \geq k$: The aggregate lobbying effort $x^*(\delta) + y^*$ is increasing in b and decreasing in k. The probability of approving the new product/technology $\pi(x^*(\delta), y^*)$ is increasing in b and is decreasing in k if $2b - k \leq 1$. The expected social surplus is $\pi(x^*(\delta), y^*)(b-\delta) - x^*(\delta) - y^*$ is increasing in k if $2b - k^2 \leq 1$.

5 Optimal regulation

The purpose here is to determine conditions under which it can be socially worthwhile for the regulator to pay attention to the lobbying efforts to decide whether the product should be marketed or not.

δ, is not specific to the uniform distribution of damage. In fact, it would hold true for any probability distribution. Indeed, assume we get a corner solution for some types of lobby I. Then there exists $\bar{\delta} \leq k$ such that $x^*(\delta) = 0$, for all $\delta \geq \bar{\delta}$. However, this implies that lobby E has no incentive to exert effort, as $\min(\delta, k) - \delta = 0$, for all $\delta < \bar{\delta}$, and $x^*(\delta) = 0$, for all $\delta \geq \bar{\delta}$. This contradics our assumption of a corner solution for lobby I's effort. We thank David Malueg for pointing us this extension.

Suppose the regulator decides whether the product should be marketed or not on the basis of his prior beliefs. Then, he anticipates that the expected social surplus will equal $b - 1/2$, if the product is marketed, and 0, otherwise. Accordingly, he will approve the product if and only if $b > 1/2$. It follows that, if the regulator decides on the basis of his prior beliefs only, the expected social surplus will equal $\max(0, b - 1/2)$.

Suppose now that the regulator's decision is the outcome of the lobbying efforts of I and E, according to a Tullock contest (1980), as analyzed above. Anticipating the equilibrium of the contest, the expected social surplus will equal

$$\begin{aligned} s^*(b,k) &= \int_0^1 \left[\pi\left(x^*(\delta), y^*\right)(b-\delta) - x^*(\delta) - y^*\right] d\delta, \\ &= \left(b - \frac{1}{2}\right) + \frac{(1-k)^2}{2(b-k) + (1-k)^2}\left(\frac{1}{2} - 2b + k + \frac{1}{3}b^2 f\left(\frac{k}{b}\right)\right), \end{aligned}$$

where we define $f(x) = 4\left(1 - \sqrt{1-x}\right) - 2x - x^2/2$, for all $0 \leq x \leq 1$.

Clearly, from the social viewpoint, it is optimal to pay attention to the lobbyists' activity if and only if

$$s^*(b,k) > \max(0, b - 1/2).$$

The purpose of the remaining of this section is to clarify the circumstances under which this condition holds true. Proposition 1 and Figure 1 below summarize our results.

Proposition 1. For all b, let $\kappa(b) = \max\left(2\sqrt{1-b}\left(1 - \sqrt{1-b}\right), 2\sqrt{b} - 1\right) \in (0, b)$. For all b, there exists $a(b) \in (\kappa(b)/b, 1)$ such that it is socially better to use a Tullock contest, to decide whether the new product/technology can be marketed or not, if and only if $k/b > a(b)$.

In other words, Proposition 1 shows that if the lobby I is rich enough, in the sense that the ratio of his level of assets with respect to his benefit, k/b, is larger than a given threshold, $a(b)$, then it is socially preferable, *ex ante*, that the government decides to approve, or to ban, the new product/technology, according to the contest, rather than according to a cost-benefit analysis, using his prior beliefs. Figure 1 plots the frontier $a(b)$, showing that it lies between 0.9 and 1.

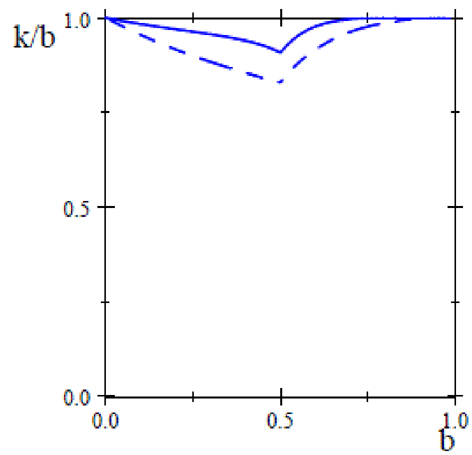

Figure 1: $a(b)$(solid) and $\kappa(b)/b$ (dash)

Intuitively, the result stated in Proposition 1 can be explained as follows. On the one hand, as the lobbying activity diverts resources from productive activities, the contest is a costly decision mechanism. This favors a decision according to a cost-benefit analysis with the prior beliefs. On the other hand, as lobby I is held liable for damage *ex post*, the larger the damage is, as long as lobby I can afford to pay for it (i.e., $\delta < k$), the less effort lobby I has incentives to expend in the contest. In equilibrium, the probability of approval of the new product/technology is decreasing in the harm, as long as $\delta < k$. This favors a decision according to the contest. The result stated in Proposition 1 holds true, simultaneously because increasing k reduces the lobbying efforts and enlarges the region where the probability of approval of the new product/technology is negatively correlated to the harm. The picture is complete remarking that the contest performs very badly when $k = 0$ and implements the optimal decision rule when $k = 1$.

6 Extension

Up to here, we have dealt with the situation where only the industrial lobby knows the damage before the product is marketed. This assumption is appealing if the industrial lobby has registered a patent protecting his new product/technology, implying that he is in a privileged position to conduct laboratory experiments to analyse the potential detrimental effects of using his new product/technology. However, the alternative assumption, where both the industrial lobby and the environmental lobby are informed, is also relevant and needs to be investigated. This is the purpose of this section.

Formally, given that both lobbies observe δ before playing the contest, we seek for an equilibrium where lobby I plays $X^*(\delta)$ and lobby E plays $Y^*(\delta)$, for all δ. It is standard (Nti, 1999) to show that

$$X^*(\delta) = \begin{cases} 0^+ & \text{if } \delta < k, \\ \frac{(b-k)^2(\delta-k)}{(b+\delta-2k)^2}, & \text{otherwise.} \end{cases}$$

$$Y^*(\delta) = \begin{cases} 0 & \text{if } \delta < k, \\ \frac{(b-k)(\delta-k)^2}{(b+\delta-2k)^2}, & \text{otherwise.} \end{cases}$$

From this, we can calculate the aggregate lobbying effort, the probability of approving the new product/technology and the *ex post* expected social surplus, respectively equal to

$$X^*(\delta) + Y^*(\delta) = \begin{cases} 0^+, & \text{if } \delta < k, \\ \frac{(b-k)(\delta-k)}{b+\delta-2k}, & \text{otherwise.} \end{cases}$$

$$\pi(X^*(\delta), Y^*(\delta)) = \begin{cases} 1, & \text{if } \delta < k, \\ \frac{b-k}{b+\delta-2k}, & \text{otherwise.} \end{cases}$$

$$\begin{pmatrix} \pi(X^*(\delta), Y^*(\delta))(b-\delta) \\ -X^*(\delta) - Y^*(\delta) \end{pmatrix} = \begin{cases} (b-\delta), & \text{if } \delta < k, \\ \frac{(b-k)(b-2\delta+k)}{b+\delta-2k}, & \text{otherwise.} \end{cases}$$

The following proposition characterizes the relevant comparative statics whenever possible. The proof is given in the appendix.

Property 2. (Comparative statics).

- Consider a state such that $\delta < k$: The aggregate lobbying effort $X^*(\delta) + Y^*(\delta)$ and the probability of approving the new product/technology $\pi(X^*(\delta), Y^*(\delta))$ are invariant in b and k. The expected social surplus is $\pi(X^*(\delta), Y^*(\delta))(b-\delta) - X^*(\delta) - Y^*(\delta)$ is increasing in b and is invariant in k.
- Consider a state such that $\delta \geq k$: The aggregate lobbying effort $X^*(\delta) + Y^*(\delta)$ is increasing in b and decreasing in k. The probability of approving the new product/technology $\pi(X^*(\delta), Y^*(\delta))$ is increasing in b and is increasing (resp., decreasing) in k if $\delta < b$ (resp., if $\delta > b$). The expected social surplus is $\pi(X^*(\delta), Y^*(\delta))(b-\delta) - X^*(\delta) - Y^*(\delta)$ is increasing in k.

Like in the benchmark case with private information, we now wish to identify the circumstances under which it is socially worthwhile to organize a Tullock contest (1980). We then compare the two situations, to see whether a public information about the damage favours the use of a Tullock contest, or not.

Anticipating the equilibrium of the contest, the expected social surplus will equal

$$\begin{aligned} S^*(b,k) &= \int_0^1 \left[\pi(X^*(\delta), Y^*(\delta))(b-\delta) - X^*(\delta) - Y^*(\delta)\right] d\delta, \\ &= -2b + 2k + 3bk - \frac{5}{2}k^2 + 3(b-k)^2 \, ln\left(1 + \frac{1-k}{b-k}\right). \end{aligned}$$

From the social viewpoint, it is optimal to pay attention to the lobbyists' activity if and only if

$$S^*(b,k) > \max(0, b - 1/2).$$

Proposition 2 and Figure 2 below expounds the conditions such that the contest is socially preferable.

Proposition 2. For all b, let $K(b) = \max\left(2 - b - \sqrt{4(1-b)^2 + b^2}, 2 - b - \sqrt{5}(1-b)\right) \in (0, b)$. There exists $A(b) \in (1/2, K(b)/b)$ such that it is socially better to decide whether the new product can be marketed or not, according to the Tullock contest, if and only if $k/b > A(b)$.

In other words, Proposition 2 shows that if the lobby I is rich enough, in the sense that the ratio of his level of assets with respect to his benefit, k/b, is larger than a given threshold, $A(b)$, then it is socially preferable, *ex ante*, that the government decides to approve, or to ban, the new product and/or new technology, according to the contest, rather than according to a cost-benefit analysis, using his prior beliefs. Figure 2 plots in black the frontier $A(b)$, showing that it lies between 0.5 and 1.

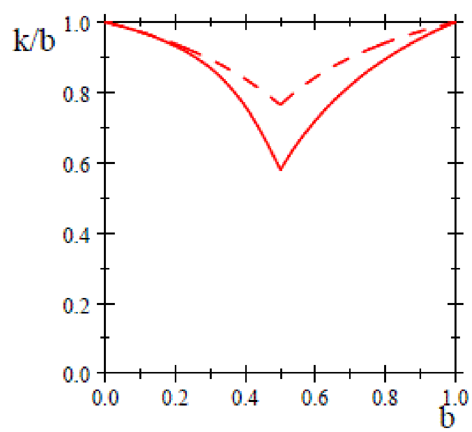

Figure 2: $A(b)$(solid) and $K(b)/b$ (dash)

Now, one would like to know whether the setting with complete information favours the use of a contest, or not. Proposition 3 below shows that this is indeed the case. The strategy for proving it is to rank the lower and upper boundaries of the two frontiers $a(b)$ and $A(b)$, respectively $\kappa(b)/b$ and $K(b)/b$ (see propositions 1 and 2), corresponding to the dashed lines in Figures 1 and 2.

Proposition 3. For given parameters, if it is socially better to use a contest in the setting with asymmetric information, then it is also socially better to use a contest in the setting with complete information. However, the converse may not be true. Formally, $A(b) < K(b)/b < \kappa(b)/b < a(b)$, for all b.

7 Conclusion

In this paper, we consider the regulatory problem to approve or to ban a new product/technology. We formalize the government decision-making as a Tullock contest in which two lobbies are competing against one another. The originality of our paper is to assume that the industrial lobby has private information about the environmental and/or health detrimental effects, but can be held liable for damage *ex post*. We determine the equilibrium of the contest and derive both its positive and normative properties.

Our main result is to give conditions under which it is socially better that the government decides according to the contest, rather than according to an *ex ante* cost-benefit analysis, using his prior beliefs. The reason why this can be so, is because the *ex post* liability for harm induces the industrial lobby to reveal his private information about the environmental and/or health detrimental externalities in the contest. The reason why this is not always so, is because the liability system can be imperfect, when the maximum assets of the industrial lobby is less than the harm, and the contest is a costly scheme.

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Appendix

Proof of property 1.

- Consider a state such that $\delta < k$:
Assume that $1-2b+k^2 \geq 0$. By differenciation, we show that

$$\begin{aligned}
\frac{\partial}{\partial b}\left(x^*(\delta)+y^*(\delta)\right) &= \frac{(1-k)^2}{2(b-k)+(1-k)^2}\sqrt{\frac{b-\delta}{b-k}}\left(\frac{(1-k)^2}{2(b-k)+(1-k)^2}-\frac{1}{2}\frac{k-\delta}{b-\delta}\right) > 0,\\
\frac{\partial}{\partial k}\left(x^*(\delta)+y^*(\delta)\right) &= -\frac{1}{2}\frac{(1-k)}{\left(2(b-k)+(1-k)^2\right)^2}\sqrt{\frac{b-\delta}{b-k}}\left(8(b-k)^2+(1-k)\left(1-2b+k^2\right)\right) < 0,\\
\frac{\partial}{\partial b}\pi\left(x^*(\delta),y^*(\delta)\right) &= \frac{(1-k)^2}{2(b-k)+(1-k)^2}\sqrt{\frac{b-k}{b-\delta}}\left(\frac{2}{2(b-k)+(1-k)^2}-\frac{1}{2}\frac{k-\delta}{(b-\delta)(b-k)}\right),\\
\frac{\partial}{\partial k}\pi\left(x^*(\delta),y^*(\delta)\right) &= \frac{1}{2}\frac{(1-k)}{\left(2(b-k)+(1-k)^2\right)^2}\sqrt{\frac{b-\delta}{b-k}}\left(\frac{8(b-k)^2+(1-k)\left(1-2b+k^2\right)}{b-k}\right) > 0.
\end{aligned}$$

$$\frac{\partial}{\partial b}\begin{pmatrix}\pi\left(x^*(\delta),y^*(\delta)\right)(b-\delta)\\ -x^*(\delta)-y^*(\delta)\end{pmatrix} =,$$

$$\frac{\partial}{\partial k}\begin{pmatrix}\pi\left(x^*(\delta),y^*(\delta)\right)(b-\delta)\\ -x^*(\delta)-y^*(\delta)\end{pmatrix} = \frac{(1-k)}{\left(2(b-k)+(1-k)^2\right)^2}\sqrt{\frac{b-\delta}{b-k}}\left(8(b-k)^2+(1-k)\left(1-2b+k^2\right)\right) > 0.$$

The signs of all derivatives except the first one directly follow from our assumption that $1-2b+k^2 \geq 0$. To verify that the first derivative is positive, first note that $(k-\delta)/(b-\delta) < 1$. A sufficient condition is thus $(1-k)^2/\left(2(b-k)+(1-k)^2\right) \geq 1/2$. It is easy to see that this condition is equivalent to $1-2b+k^2 \geq 0$.

- Consider a state such that $\delta \geq k$:
By differenciation, we show that

$$\begin{aligned}
\frac{\partial}{\partial b}\left(x^*(\delta)+y^*(\delta)\right) &= \frac{(1-k)^4}{\left(2(b-k)+(1-k)^2\right)^2} > 0,\\
\frac{\partial}{\partial k}\left(x^*(\delta)+y^*(\delta)\right) &= -\frac{(1-k)}{\left(2(b-k)+(1-k)^2\right)^2}\left(4(b-k)^2+(1-k)^3\right) < 0,\\
\frac{\partial}{\partial b}\pi\left(x^*(\delta),y^*(\delta)\right) &= 2\frac{(1-k)^2}{\left(2(b-k)+(1-k)^2\right)^2} > 0,\\
\frac{\partial}{\partial k}\pi\left(x^*(\delta),y^*(\delta)\right) &= -2\frac{(1-k)}{\left(2(b-k)+(1-k)^2\right)^2}(1-2b+k),
\end{aligned}$$

$$\frac{\partial}{\partial b}\begin{pmatrix}\pi\left(x^*(\delta),y^*(\delta)\right)(b-\delta)\\ -x^*(\delta)-y^*(\delta)\end{pmatrix} =,$$

$$\frac{\partial}{\partial k}\begin{pmatrix}\pi\left(x^*(\delta),y^*(\delta)\right)(b-\delta)\\ -x^*(\delta)-y^*(\delta)\end{pmatrix} = \frac{(1-k)}{\left(2(b-k)+(1-k)^2\right)^2}\begin{pmatrix}8(b-k)^2+(1-k)\left(1-2b+k^2\right)\\ +(\delta-k)(1-2b+k)\end{pmatrix}.$$

The fourth derivative is positive under the assumption that $1-2b+k \geq 0$. The last derivative is positive under the assumption that $1-2b+k^2 \geq 0$ ([10]).
■

[10] As $k < 1$, $1-2b+k^2 \geq 0$ implies that $1-2b+k \geq 0$.

Calculus of the ex ante expected social surplus (private information)

The ex ante expected social surplus is

$$s^*(b,k) = \int_0^1 \left[\pi(x^*(\delta), y^*)(b-\delta) - x^*(\delta) - y^*\right] d\delta.$$

Using our results in section 4, we get

$$\begin{aligned} s^*(b,k) &= \int_0^1 (b-\delta)\, d\delta - \frac{(1-k)^2}{2(b-k)+(1-k)^2} \begin{pmatrix} 2\sqrt{b-k}\int_0^k \sqrt{b-\delta}d\delta \\ \int_k^1 (2b-\delta-k)\, d\delta \end{pmatrix}, \\ &= \left(b-\frac{1}{2}\right) - \frac{(1-k)^2}{2(b-k)+(1-k)^2} \begin{pmatrix} \frac{4}{3}\sqrt{b-k}\left(b^{\frac{3}{2}} - (b-k)^{\frac{3}{2}}\right) \\ -\frac{1}{2}(1-k)(1-4b+3k) \end{pmatrix}. \end{aligned}$$

Rearranging, this yields

$$s^*(b,k) = \left(b-\frac{1}{2}\right) + \frac{(1-k)^2}{2(b-k)+(1-k)^2}\left(\frac{1}{2} - 2b + k + \frac{1}{3}b^2 f\left(\frac{k}{b}\right)\right),$$

where we define $f(x) = 4\left(1-\sqrt{1-x}\right) - 2x - x^2/2$, for all $0 \leq x \leq 1$.■

Proof of proposition 1.
We first show the following lemmas.

Lemma 1. $f(x)$ is increasing in x and $0 < f(x) < 3x^2/2$, for all $0 < x < 1$.

Proof. Differenciation yields $f'(x) = 2(1-x)^{-1/2} - 2 - x$ and $f''(x) = (1-x)^{-3/2} - 1$. For all $0 < x < 1$, $f''(x) > 0$ and $f'(x)$ is strictly increasing. As $f'(0) = 0$, it follows that $f(x)$ is stricly increasing.

To show that $0 < f(x) < 3x^2/2$, simply observe that we can write

$$f(x) = \frac{1}{2}\left(1-\sqrt{1-x}\right)^2\left(\left(1-\sqrt{1-x}\right)^2 + 2x\right) > 0,$$

$$f(x) - \frac{3}{2}x^2 = 2\left(\left(1-\sqrt{1-x}\right)^2 - x^2\right) < 0 \text{ (using } 1-\sqrt{1-x} < x\text{)},$$

for all $0 < x < 1$. □

Lemma 2. If $1-2b+k^2 \geq 0$, the expected social surplus $s^*(b,k)$ is increasing in k. If $1-2b+k^2 < 0$, $s^*(b,k) < b - 1/2$.

Proof. The first part follows from property 1. To show the second part, first remark that $1-2b+k^2 < 0$ implies that $b > 1/2$. Then use lemma 1 to show that

$$s^*(b,k) < \left(b-\frac{1}{2}\right) + \frac{(1-k)^2}{2(b-k)+(1-k)^2}\left(\frac{1}{2} - 2b + k + \frac{1}{2}k^2\right).$$

Rearrange the term under brackets to write

$$s^*(b,k) < \left(b - \frac{1}{2}\right) + \frac{(1-k)^2}{2(b-k) + (1-k)^2} \left(\frac{1}{2}\left(1 - 2b + k^2\right) - (b-k)\right),$$

which proves the lemma given that $k < b$. □

The rest of the proof distinguishes two subcases:

- For all $b \leq 1/2$, we seek $\kappa(b)$ such that $s^*(b, \kappa(b)) = 0$.

We first show that $s^*\left(b, 2\sqrt{1-b}\left(1 - \sqrt{1-b}\right)\right) < 0$ ([11]). Indeed, lemma 1 implies that

$$s^*(b,k) < \left(b - \frac{1}{2}\right) + \frac{(1-k)^2}{2(b-k) + (1-k)^2} \left(\frac{1}{2} - 2b + k + \frac{1}{2}k^2\right).$$

For all k, remark that

$$\frac{(1-k)^2}{2(b-k) + (1-k)^2} < \frac{1}{1 + 2b - 2k}.$$

In particular, if $k = 2\sqrt{1-b}\left(1 - \sqrt{1-b}\right)$, we obtain (after rearrangement)

$$\frac{(1-k)^2}{2(b-k) + (1-k)^2} < \frac{1}{1 - 2b + 4\left(1 - \sqrt{1-b}\right)},$$

and

$$\frac{1}{2} - 2b + k + \frac{1}{2}k^2 = \left(\frac{1}{2} - b\right)\left(1 - 2b + 4\left(1 - \sqrt{1-b}\right)\right) \geq 0,$$

implying that

$$s^*\left(b, 2\sqrt{1-b}\left(1 - \sqrt{1-b}\right)\right) < \left(b - \frac{1}{2}\right) + \frac{1 - 2b + 4\left(1 - \sqrt{1-b}\right)}{1 - 2b + 4\left(1 - \sqrt{1-b}\right)}\left(\frac{1}{2} - b\right) = 0.$$

We then show that $s^*(b,b) = b^2/2 > 0$.

From the mean-value theorem, there exists $\kappa(b)$ such that $2\sqrt{1-b}\left(1 - \sqrt{1-b}\right) < \kappa(b) < b$ and $s^*(b, \kappa(b)) = 0$. From lemma 2, $\kappa(b)$ is unique and $s^*(b, \kappa(b)) > 0$, for all $k > \kappa(b)$.

- For all $b > 1/2$, we seek $\kappa(b)$ such that $s^*(b, \kappa(b)) = b - 1/2$.

We first show that $s^*\left(b, 2\sqrt{b} - 1\right) < b - 1/2$ ([12]). Indeed, lemma 1 implies that

$$s^*(b,k) < \left(b - \frac{1}{2}\right) + \frac{(1-k)^2}{2(b-k) + (1-k)^2} \left(\frac{1}{2} - 2b + k + \frac{1}{2}k^2\right).$$

[11] It is immediate to verify that $k = 2\sqrt{1-b}\left(1 - \sqrt{1-b}\right) \in (0,b)$.

[12] It is immediate to verify that $k = 2\sqrt{b} - 1 \in (0,b)$.

In particular, if $k = 2\sqrt{b} - 1$, we get

$$s^*(b,k) < b - \frac{1}{2}.$$

We then show that $s^*(b,b) = b^2/2 > 0$.

From the mean-value theorem, there exists $\kappa(b)$ such that $2\sqrt{b}-1 < \kappa(b) < b$ and $s^*(b,\kappa(b)) = 0$. From lemma 2, $\kappa(b)$ is unique and $s^*(b,\kappa(b)) > 0$, for all $k > \kappa(b)$. ■

Proof of property 2.

Only states such that $\delta \geq k$ need to be considered.

By differenciation, we show that

$$\frac{\partial}{\partial b}\left(X^*(\delta) + Y^*(\delta)\right) = \frac{(\delta - k)^2}{(b + \delta - 2k)^2} > 0,$$

$$\frac{\partial}{\partial k}\left(X^*(\delta) + Y^*(\delta)\right) = -\frac{(b-k)^2 + (\delta - k)^2}{(b + \delta - 2k)^2} < 0,$$

$$\frac{\partial}{\partial b}\pi\left(X^*(\delta), Y^*(\delta)\right) = \frac{(\delta - k)}{(b + \delta - 2k)^2} \geq 0,$$

$$\frac{\partial}{\partial k}\pi\left(x^*(\delta), y^*(\delta)\right) = \frac{(b - \delta)}{(b + \delta - 2k)^2},$$

$$\frac{\partial}{\partial b}\left(\begin{array}{c}\pi\left(X^*(\delta), Y^*(\delta)\right)(b-\delta) \\ -X^*(\delta) - Y^*(\delta)\end{array}\right) = \frac{1}{(b + \delta - 2k)^2}\left((b-k)^2 + 2(b-\delta)(\delta-k)\right),$$

$$\frac{\partial}{\partial k}\left(\begin{array}{c}\pi\left(X^*(\delta), Y^*(\delta)\right)(b-\delta) \\ -X^*(\delta) - Y^*(\delta)\end{array}\right) = \frac{1}{(b + \delta - 2k)^2}\left((b-\delta)^2 + (b-\delta)(\delta-k) + (\delta-k)^2\right) > 0.$$

The signs of all derivatives are immediate, except for the last one, which can be obtained by showing that the quadratic form $q(x,y) = x^2 + xy + y^2$ is positive definite (i.e., $\partial^2 q(x,y)/\partial x^2 > 0$ and $\left(\partial^2 q(x,y)/\partial x^2\right)\left(\partial^2 q(x,y)/\partial y^2\right) - \left(\partial^2 q(x,y)/\partial x \partial y\right)^2 > 0$). ■

Calculus of the ex ante expected social surplus (perfect information)

The ex ante expected social surplus is

$$S^*(b,k) = \int_0^1 \left[\pi\left(X^*(\delta), Y^*(\delta)\right)(b-\delta) - X^*(\delta) - Y^*(\delta)\right] d\delta.$$

Using our results in section 4, we get

$$\begin{aligned} S^*(b,k) &= \int_0^k (b-\delta)\,d\delta - (b-k)\int_k^1 \left(\frac{b - 2\delta + k}{b + \delta - 2k}\right) d\delta, \\ &= -2b + 2k + 3bk - \frac{5}{2}k^2 + 3(b-k)^2\, ln\left(1 + \frac{1-k}{b-k}\right). \end{aligned}$$

observing that $(b-2\delta+k)/(b+\delta-2k)$ admits $3(b-k)\,ln\,(b+\delta-2k)-2\delta$ is an antiderivative. ■

Proof of proposition 2.

We first show the following lemmas.

Lemma 3. For all $0<k<b<1$, $S^*(b,k)>-2b+2k-bk+2b^2-\frac{1}{2}k^2$.

Proof. Given that $0<k<b<1$,$(1-k)/(b-k)>1$ and $ln\,(1+(1-k)/(b-k))>ln\,(2)>2/3$. This directly implies that $S^*(b,k)>-2b+2k+3bk-\frac{5}{2}k^2+2(b-k)^2$, which yields lemma 3 after rearrangement. □

The rest of the proof distinguishes two subcases:

- For all $b\leq 1/2$, we seek $K(b)$ such that $S^*(b,K(b))=0$.

We first show that $S^*(b,b/2)<0$. Indeed, we can write

$$S^*(b,b/2)=-\frac{b}{8}\left(8-\left(6\ln\left(\frac{2}{b}\right)+7\right)b\right).$$

The term under brackets is decreasing in b ([13]). Thus, it attains a minimum $9/2-3\ln(4)$, when $b=1/2$. As this minimum is positive, this proves our assertion.

We then show that $S^*\left(b,2-b-\sqrt{4(1-b)^2+b^2}\right)>0$ ([14]). Indeed, lemma 3 implies that

$$S^*(b,k)>-2b+2k-bk+2b^2-\frac{1}{2}k^2.$$

In particular, if $k=2-b-\sqrt{4(1-b)^2+b^2}$, we get $S^*\left(b,2-b-\sqrt{4(1-b)^2+b^2}\right)>0$.

From the mean-value theorem, there exists $K(b)$ such that $b/2<K(b)<2-b-\sqrt{4(1-b)^2+b^2}$ and $S^*(b,K(b))=0$. From property 2, $K(b)$ is unique and $S^*(b,K(b))>0$, for all $k>K(b)$.

- For all $b>1/2$, we seek $K(b)$ such that $S^*(b,K(b))=b-1/2$.

We first show that $S^*(b,b/2)<b-1/2$. Indeed, we can write

$$S^*(b,b/2)-(b-1/2)=-\frac{b}{8}\left(16-\left(6\ln\left(\frac{2}{b}\right)+7\right)b-\frac{4}{b}\right).$$

Remarking that the term under brackets is concave ([15]) and is positive for $b=1/2$ and when $b=1$ ([16]), we can show that the term under brackets is

[13] First-order differenciation yields $-6\ln(2/b)-1<0$, for all $0<b<1$.

[14] It is immediate to verify that $k=2-b-\sqrt{4(1-b)^2+b^2}\in(0,b)$.

[15] Second-order differenciation yields $-\left(8-6b^2\right)/b^3<0$, for all $0<b<1$.

[16] It is equal to $9/2-3\ln(4)$, when $b=1/2$, and to $5-6\ln(2)$, when $b=1$.

positive, since the graph of a concave function lies above any chord between two points on the graph. This proves our assertion.

We then show that $S^* \left(b, 2-b-\sqrt{5}(1-b)\right) > b - 1/2$ ([17]). Indeed, lemma 1 implies that

$$S^*(b,k) - \left(b - \frac{1}{2}\right) > \frac{1}{2} - 3b + 2k - bk + 2b^2 - \frac{1}{2}k^2.$$

In particular, if $k = 2 - b - \sqrt{5}(1-b)$, we get $S^*(b,k) > 0$.

From the mean-value theorem, there exists $K(b)$ such that $0 < K(b) < 2 - b - \sqrt{5}(1-b)$ and $S^*(b, K(b)) = b - 1/2$. From property 2, $K(b)$ is unique and $S^*(b, K(b)) > b - 1/2$, for all $k > K(b)$. ■

Proof of proposition 3.

Let $\triangle(b) = \kappa(b) - K(b)$, for all $0 < b < 1$. We need to show that $\Delta(b) > 0$. The proof distinguishes two subcases:

- First consider $b \leq 1/2$. From propositions 1 and 2,

$$\begin{aligned}\triangle(b) &= \left(2\sqrt{1-b}\left(1-\sqrt{1-b}\right)\right) - \left(2 - b - \sqrt{4(1-b)^2 + b^2}\right), \\ &= -4 + 2\sqrt{1-b} + \sqrt{4(1-b)^2 + b^2} + 3b.\end{aligned}$$

To show that $\triangle(b) > 0$, first differenciate it to get

$$\triangle'(b) = -\frac{1}{\sqrt{1-b}} - \frac{4-5b}{\sqrt{4(1-b)^2 + b^2}} + 3.$$

From this, we can show that $\triangle(0) = \triangle'(0) = 0$, implying that x-axis is tangent to the graph of $\triangle(b)$at the origin. Now, a second-order differenciation yields

$$\triangle''(b) = \frac{1}{2}\frac{8(1-b)^{3/2} - \left(4(1-b)^2 + b^2\right)^{3/2}}{(1-b)^{3/2}\left(4(1-b)^2 + b^2\right)^{3/2}},$$

proving that $\triangle(b)$ is strictly convex for all $b \leq 1/2$ ([18]). Finally, use the fact that the graph of a convex function lies lies above its tangents.

- Now consider $b > 1/2$. From propositions 1 and 2,

$$\begin{aligned}\triangle(b) &= \left(2\sqrt{b} - 1\right) - \left(2 - b - \sqrt{5}(1-b)\right), \\ &= -\left(3 - \sqrt{5}\right) + 2\sqrt{b} - \left(\sqrt{5} - 1\right)b,\end{aligned}$$

To see that $\triangle(b) > 0$, simply remark that $\triangle(b)$ is strictly concave and is positive when $b = 1/2$ and when $b = 1$ ([19]). Then use the fact that the graph of a concave function lies above any chord between two points on the graph. ■

[17] It is immediate to verify that $k = 2 - b - \sqrt{5}(1-b) \in (0, b)$.

[18] Precisely, $\triangle''(b) > 0$ if and only if $0 < b < 4/5$.

[19] Precisely, $\triangle(1/2) = \sqrt{2} - \sqrt{5}\left(\sqrt{5} - 1\right)/2 > 0$ and $\triangle(1) = 0$.