On the timing of political regime changes in Arab countries

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February 24, 2014

Abstract

We develop a dynamic game to provide with a comprehensive theory of Arab spring-type events. We consider two interacting groups, the elite *vs.* the citizens, two political regimes, dictatorship *vs.* a freer regime, the possibility to switch from the first to the second regime as a consequence of a revolution by the citizens and finally the opportunity, for the elite, to affect the citizens' decision through concession and/or repression strategies. In this framework, we provide a full characterization of the equilibrium of the political regime switching game. First, we emphasize the role of the direct switching cost of a revolution (for the citizens) and of the elite's self-preservation options. Under the concession strategy, when the switching cost is low, the elite can't avoid the political regime change. She optimally adapts to the overthrow of their political power by setting the rate of redistribution to the highest possible level, thereby extending the period during which she has full control on resources. This surprising result actually illustrates the role of the timing of events in these situations of interaction between the ruling elite and the people. When the direct switching is high, the elite can ultimately select the equilibrium outcome and adopts the opposite strategy, *i.e.* she chooses the lowest level of redistribution that allows her to stay in power forever. The same kind of results are obtained when the elite relies on repression to keep the citizens under control. Next, the equilibrium properties under a mix of repression and redistribution are analyzed. It is shown that in situations where neither repression (only) nor redistribution (only) protect the elite against the uprising of citizens, a subtle mixture of the two instruments is sufficient to make the dictatorship permanent. Based on our theoretical results, we finally examine the reason for such a large variety of decisions and outcomes during the Arab Spring events.

Keywords: political transitions, revolution, natural resources, optimal timing, regime switching, dynamic game.

JEL Classification: C61, D74, Q34.

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1 Introduction

The Arab spring, which started in Tunisia in late 2010, is arguably the most important event of the beginning of this century, certainly comparable to the fall of Berlin's Wall in 1989. While the political and geostrategic consequences of this event are not yet settled, it's enough to look a few months or years backward to realize how it is important and somehow striking: who could expect in early 2011 that fierce dictators and regimes as those of Benali or Mubarak will be ousted after quite a few weeks of heavy protests and demonstrations? Who could expect the cascade of events which have affected the whole Arab world after the Tunisian Jasmine revolution? After so many decades of brutal (and even insane) dictatorship, many analysts did lose hope in any institutional evolution (not speaking about democratization!) in the Arab world before the amazing sequence initialized in Tunisia. It's therefore fair to state that the Arab spring came as a surprise, not only to political leaders around the world, but also to the traditional media of the Arab World.[1]

i) A first puzzling aspect of the Arab spring is henceforth the **timing**. Why did Arab populations wait 4 or 5 decades (after independence) before revolting? All these countries have in common the fact that they have been ruled for decades by elites controlling fiercely the rents deriving either from the exploitation of natural resources (that's oil and gas) or from economic liberalization (like in Tunisia, celebrated as one of the most open Arab countries by the World Bank's *Doing Business* successive surveys prior to the Jasmine Revolution).[2] What are the mechanisms leading from such a regime with an omnipotent elite controlling the whole resource-dependent economy and deciding about everything to a regime change towards democracy?

ii) A second striking feature is **the large variety of outcomes** observed so far. In some cases, the ruling parties and dynasties were eventually ousted (as in Tunisia or Egypt) after weeks of political and social turmoil regularly punctuated by massive and sometimes deadly demonstrations. In others, the same massive demonstrations have not been so successful and the incumbents remained in office as in Bahrein: it is likely that the Shiite demonstrators didn't expect the bloody intervention of Saudi Arabia (for the Sunnite minority to keep power in Bahrein) when they started the uprising. Another case of external intervention is Libya, which resulted in the opposite outcome: the ruling dictator and associated "elite" were violently expelled by the opposition backed by NATO massive bombings. A much

[1] A funny example of this unawarness is this French minister spending her 2010 Christmas holidays in Tunisia, and showing little concern about the situation even after the first registered deaths. The whole French government led by Sarkozy was apparently sure that the firm ally, Benali, will be able to control the situation and stay in power at the end of the day!

[2] Egypt was ranked in the Top 10 of the most reforming countries in the world by *Doing Business* in 2008/2009, and Tunisia was ranked 55th out of 183 in 2011 by *Doing Business ranking.*

less bloody case is Algeria: with a mixture of soft repression and massive redistribution, the ruling elite has been able to retain the power (see Achy, 2011). Indeed, the January 2011 protests have led the Algerian government to markedly enlarge the scope of its staple food subsidy program and its youth employment support packages leading to a durable destructuring of the national budget. The Regulation Fund, created in 2000 for the stabilization of public expenditures thanks to the oils and gas exports, has been used to finance the ongoing budget deficits (about 50% of the 2011 budget deficit was financed by this Fund, according to Boucekkine and Bouklia-Hassane, 2011), which endangers the public finance system sustainability of this country. Morocco is another case where the elite (in other words, the King) has prevented a major uprising so far by conducting early institutional changes towards a constitutional monarchy (see Traub, 2012), signaling more redistribution of economic and political rights in the future. This variety of outcomes calls for a deep theoretical research on the use of redistribution/repression by ruling elites and the resulting economico-political equilibria. Two questions arise quite naturally: Couldn't the ruling elite, with all the powers in hands, systematically prevent the occurrence of such moves by taking the adequate decisions, either fierce/soft repression and/or massive/moderate redistribution of the rents, in the right time? For a given repression/redistribution policy of the elites, what does determine the decision of the opposition (that's the vast majority of the population) to go for a revolution and the inherent timing?

The two striking aspects described above suggest that the underlined phenomena cannot be properly addressed without two key ingredients: in first place, the dynamic aspects to be able to fully understand the timing of the events, and in second place, the strategic aspects with ruling elites acting as strategic leaders in the initial regime. Accordingly, we shall develop a full-fledged dynamic game framework to provide with a comprehensive theory of Arab spring-type phenomena as captured by the two features i) and ii). In this framework, we introduce hierarchy: elites do act as strategic leaders. Moreover, the considered games explicitly distinguish between the pre-revolution regime (dictatorship of the elite) and the post-revolution regime (say, common access to resources). Finally and more importantly, the determination of (Markov perfect) equilibria includes the timing of the transition (if it occurs) from the first to the second regime. This ultimately leads to a methodological innovation as the latter requires merging dynamic games with multi-stage optimal control (see Boucekkine et al., 2013a for the use of multi-stage optimal control in a non-strategic model of technology adoption).[3]

In the first regime, the elite has full control on the stock of resources (to be taken in a broad sense, it could either come from an – unmodelled – extraction

[3]Two related papers are Long et al., 2013, and Boucekkine et al., 2011. But, among other notable differences, the former does not have a strategic leader structure, and the latter is only concerned with open-loop equilibria.

sector or from economic resources derived from economic globalization, *e.g.* from FDIs or import licenses for example) and decides about: i) how much resources to transfer to population (redistribution or concession), ii) how much to use for own consumption, iii) how much resources to employ for repression (modelled as a flow) and iv) how much to invest. The production function is AK to have a chance to extract (partial) analytical results. Of course, redistribution and/or repression are the instruments the elites may use (simultaneously or not, we examine both cases) to retain the power (self-preservation) in exchange for own consumption and investment. From the point of view of the elites, neither redistribution nor repression make sense in the absence of a revolution threat. Given the elites' policy, the opposition has to decide whether she rebels (or not) against the elites and **when** (if she does so). A key aspect in the opposition tradeoff is the *direct (political regime) switching costs* (DSC hereafter) faced. These costs depend either on the repression exerted by the elites and/or their allies (possibly foreign countries) or on the coordination costs inherent in any collective action. Knowing the resulting reaction function of the opposition, the elites as strategic leaders will then choose their optimal redistribution and/or repression policies. We shall address all the research questions listed above within this far nontrivial framework.

In our analysis, we cannot explicitly model important aspects of the Arab spring in order to come with a comprehensive enough analytical characterization of the economico-political equilibria. As one can already guess from the description just above, our simplified setting is already heavily sophisticated from the algebraic point of view. In most of the cases, we are able however to identify shortcuts allowing to partially account for the missing pieces such like the education level or external military interventions. This is possible thanks to our broad interpretation of the DSC. For example, concerning the level of education, it's important to notice that the first country of the Arab spring, Tunisia, is the one which has invested the most in human capital in the recent decades in % of the national budgets (for example, about 23% of the Tunisian total public expenditures was devoted to education in 2008, according to the World Bank's Edstats).[4] If education increases the perception of grievance in the face of unequal distribution of rents, then of course it should play a role in the Arab spring story. Though we don't integrally consider such a channel in our model, we might view it partially through exogenous moves in the DSC. One might think that as the average education level of the countries becomes high enough, it is definitely easier to coordinate on a collective action,[5] thus pushing down the DSC. The contrary occurs in the case of an exogenous increase in repression due to external pro-elite intervention. A key aspect for the existence and nature of the economico-political equilibria is the controllability of the DSC by the elites. In reality, the DSC can be only partially controlled by the elites. Control obviously results from repression and is also linked to

[4]The Gulf monarchies and Jordan invest also a lot in public education, although slightly below 20% in the 2008 Edstats report.

[5]While this is a common idea in labour economics, see Becker and Murphy (1992), it seems relevant in any context involving collective actions.

education, given the importance of public education in these countries (see the Tunisian case above). A significant part of the DSC is however out of the control of the elites. Uncontrolled factors affecting the cost of revolting against the elites include opposite external military interventions (Libya or Syria), most of the elements linked to globalization (e.g. the role of worldwide communication networks), and demography. In this paper, we shall study the two polar cases: the case where the DSC are fully controllable *vs* fully exogenous. The mixed case is discussed in the last section before the conclusion.

Last but not least, we shall omit uncertainty. In the context of the Arab spring, an important source of uncertainty originates in the size of the rents (because of random resource prices in international markets for example). This uncertainty is likely to affect the elites' behaviour: in the Algerian case described by Boucekkine and Bouklia-Hassane (2011), it is shown that the ruling nomenklatura is much more prone to political and economic liberalization in periods where the oil prices are persistently low. Boucekkine et al. (2013b) have already provided with a stochastic game modelling of the latter aspect but they omitted the two fundamental components of our framework: elites as strategic leaders and a multi-regime setting. Indeed, incorporating the latter components in a proper stochastic dynamic game structure sounds as a daunting task.

Our research can be related to two distinct streams of the economic literature, one in the line of democratization games *à la* Acemoglu and Robinson (2006, 2008) and the other one (see for instance, Torvik, 2002) viewing natural resources as a source of conflicts. It's definitely more closely related to the former where a key question is the strategic behaviour of the elites as a central actor of institutional change. While the dynamic aspect is not always absent (see Acemoglu and Robinson, 2008), no dynamic games with explicit state variable and timing decisions are considered. We are not aware of any other paper along this line dealing with the fundamental features i) and ii) outlined above. An important aspect of the Arab spring and other democratization processes which is well accounted for in Acemoglu and Robinson (2008) and which is missing in our paper is the distinction between political power and economic power, and therefore between political and economic liberalization. As mentioned above about Tunisia and Egypt, what led to the disgrace of their respective dictators is the lack of political liberalization while these economies are close to fully liberalized. Non-simultaneity of both liberalizations has been more than problematic in these countries as analysed in Dunne and Revkin (2011). Concerning the second stream, several authors have recently pointed out that resource abundance induces political instability if competing factions try to obtain control over the associated rents. Resources cause rent seeking and fighting activities between rival groups, which weaken property rights and lead to a curse (see Torvik, 2002, Hodler, 2006, Mehlum et al., 2006, or Gonzalez, 2007). Cross-country evidence suggests a positive correlation between rents from natural resources and civil wars. Perhaps the major contribution along this line is the greed and grievance story told by Collier and Hoeffler (2004). In this seminal paper, the causes of civil war are studied using a new data set of wars during 1960-99. The authors outline that rebellion may be explained by atypically severe grievances, such as high inequality, a lack of political rights, or ethnic and religious divisions in society. Accordingly, most subsequent studies have considered conflicts as a result of grievance and greed between rival groups. Lower transfers to the opposition might induce grief as they sense injustice, and conflict over resource rents are further exacerbated when society is fractionalized by competing interest groups (Caselli and Coleman, 2006). Perhaps the most comprehensive theories on resource wars so far are due to Acemoglu et al. (2012) and van der Ploeg and Rohner (2012). In the latter for example, some mechanisms leading to this type of wars are singled out in a quite comprehensive setting where rebels fight a government, and fighting, armament, and extraction method, speed and investment, are all endogenous. Interestingly enough, rapacious resource exploitation can be preferred in this context to balanced depletion due to lowered incentives for future rebel attacks. Acemoglu et al. (2012) build their resource war model in the framework of international trade with unequal endowment of resources across countries. While in both cases the stories told are quite far from the Arab spring, one can hardly build on their respective conceptual settings to infer a theory for the latter. Indeed, a major ingredient of the Arab spring is lacking: a strong hierarchical relationship from the strategic point of view between the ruling elites and population. That's in the Arab spring story, the elites play the role of strategic leaders. As correctly pointed out by Caselli and Cunnigham (2009) in an excellent theoretical reflection on the "channels through which resource rents will alter the incentives of a political leader,...these mechanisms cannot be fully understood without simultaneously studying leader behaviour". We eagerly adopt this point of view and build dynamic games theories for the Arab spring with elites acting as strategic leaders. Pioneering works using dynamic games in contexts of strategic exploitation of resources by rival groups are due to Tornell (1996) and Lane and Tornell (1996). We depart from these contributions in two major ways. First, we do introduce hierarchy as mentioned just above, and elites act as strategic leaders. Second, the considered games explicitly distinguish between the pre-revolution regime (dictatorship of the elite) and the post-revolution regime (say, common access to resources). More importantly, the determination of (Markov perfect) equilibria includes the timing of the transition (if it occurs) from the first to the second regime.

On theoretical grounds, we provide a comprehensive analysis of the equilibrium of a political regime switching game when the elites adopt a redistribution strategy and/or a repression policy. First, we emphasize the role of the direct switching cost of the citizens and of the elite's self-preservation options by focusing on the particular cases where the elites use a single instrument. Concession made by the elites affects the citizens' uprising decision indirectly and may not be sufficient to avoid a revolution. The direct switching cost is crucial in understanding the equilibrium outcome. When this cost is low, a political regime switching always occurs. The crucial point is then to understand how the elites optimally adapt to the overthrow of their political power. Contrary to conventional wisdom, we show that the optimal strategy for the elites is to redistribute as many resources as possible to the people in the first dictatorial regime. This result clearly differs from the rapacious resource exploitation emphasized by van der Ploeg and Rohner (2012). It highlights the importance of the first striking aspect underlined above, *i.e.* the timing of events of our revolution game. Once this temporal dimension is taken into account, it comes at no surprise that making high redistribution levels is worthwhile since it allows the elites to lengthen the period of political and economic control. When the direct switching is high, the elites are able to provide sufficient transfers to the opposing citizens and choose the lowest level of redistribution compatible with a permanent dictatorship. The same kind of results are obtained when the elites decide to keep the people under control through repressive means. Repression makes it possible to directly change the switching cost and generally implies that there is more room for self-preservation. In situations where neither repression (only) nor redistribution (only) prevent the elites from being removed, a policy-mix between these two instruments succeeds in maintaining the non-democratic regime. Based on the criterion of retaining power, there is a temptation to conclude that this policy-mix is the best strategy for the elites. However, it is not always true that the payoffs associated with an equilibrium featuring a permanent dictatorship are higher than those corresponding to a solution with a regime change. Finally, we address the second important point raised in the beginning of the introduction, which is related to the large variety of outcomes that has characterized the Arab Spring. A discussion on the determinants of the cost of revolution (here, switching cost) is conducted. We emphasize how the results of our theoretical model may contribute to the analysis of the Arab Spring events by explaining why some countries have experienced a transition to (more) democratic political regimes while others are still stuck in dictatorships.

The article is organized as follows. Section 2 presents the details of our dynamic setup. Section 3 then explains the benchmark case when a revolutionary threat is absent and presents our approach. We formally discuss our political conflict game when the elite adopts a strategy based on concessions for **given** DSC in Section 4. Section 5 then analyzes the other polar case where the elite uses repression in order to keep the citizens under control, and where repression is the unique determinant of the DSC. Section 6 provides a synthesis of our analysis by considering the equilibrium under a policy-mix where. Section 7 discusses the implications of our analysis for the Arab spring. Finally, Section 8 concludes.

2 The setup

We consider a modified AK growth model where the representative agent is replaced by two infinitely lived groups. We note them as the following players: the incumbent elite (E) and the opposition (P, for the poor citizens). These groups comprise a fractionalized society with a resource-dependent economy. We abstract from any assumption regarding the size of the population and each rival group. Emulating the framework of Lane and Tornell (1996), let K be the stock of resources and A the rate of return on this asset. *Resources* are defined in a broad sense and may refer to economic resources, or wealth and windfalls from natural resources.

There are two political or institutional regimes that describe the ways in which these groups interact: dictatorship (regime 1) and a freer regime with common access to the resources (regime 2), the switch from regime 1 to regime 2 being the result of a successful revolution by the citizens. The economy initially belongs to regime 1. In this regime, the elite has the full control over the economy and the economic resources and has to divide them between three different uses: self-preservation, consumption, and investment. In order to lengthen the dictatorial system, the elite may choose either to make concessions to the citizens or to rely on repression. Self-preservation through concessionary spending takes the form a transfer of a share $1 - u_E$ of the output AK to the opposition. More formally, the concession strategy of the elite is modeled as a choice $u_E \in [\underline{u}_E, \overline{u}_E]$, where u_E, the share of resources accruing to the elite, also represents the inequality in the access to economic resources. Alternatively or jointly, the elite may decide to opt for a repression strategy. The repression strategy consists of a flow of military expenditures $r_E \in [\underline{r}_E, \overline{r}_E]$, that can be interpreted as bribes paid to those who repress (or a periodic fixed cost to pay to benefit from the protection to get some military support). The two last decisions of the elite involve the classical consumption-investment intertemporal tradeoff. Let C_E be the elite consumption at period t.[6] Then, the dynamics of the stock of economic resources are:

$$\dot{K} = u_E AK - C_E - r_E, \tag{1}$$

with $K(0) = K_0$, given.

Citizens' only source of wealth is from the elite's transfers, used for consumption: $C_P = (1 - u_E)AK$.[7] Concessions and repression affect the revolution strategy of the citizens in two different ways. From the expression of the citizens' consumption, one can easily see that concessions indirectly shape citizens' decisions by modifying the opportunity cost of the revolution. Other things equal, the higher the transfer (the lower u_E), the lower the incentive to revolt. By contrast, repression directly impinges upon the opposition. It is worth clarifying this point by describing the sequence of events leading to a regime change and its consequences. A switch from regime 1 to regime 2 results from a revolution by the opposition. A revolution (if any) succeeds with probability one. A revolt is associated with a global cost $\chi > 0$. This is the continuous time analog of Acemoglu et al. (2012) who assume that a fixed amount of resource is destroyed when violent uprisings occur:

$$K(T^+) = K(T) - \chi, \tag{2}$$

[6]When there is no risk of confusion, the time index is omitted.

[7]Leaving aside self-preservation, regime 1 is the one wherein the elite consumes and invests, while the opposition only consume. This structure shares similarities with the literature on the interaction between capitalists and workers (Lancaster, 1973, Hoel, 1978), except that in our framework citizens are completely passive and subject to the control of the elite.

which means that the state variable experiences a downward jump at the right of the switching time T, provided that T is finite. T represents the **switching strategy** of the citizens.

In addition, when conducting a revolt, the opposition incurs a direct switching cost (DSC), $\psi > 0$. This cost may be due to efforts from collective action, *i.e.* the opposing citizens need to coordinate when trying to instigate conflict. Nonetheless, the magnitude of the DSC can also be interpreted as a measure of regime contestability. The higher is this cost, the more difficult it is for the opposition to lead a popular uprising against the ruling elite. The important point to note is that repression by the elite makes the cost of revolting relatively higher.[8] As explained in the introduction section, the DSC are therefore partially controllable by the elites through r_E. In this paper, we will study the polar cases where the DSC are fully exogenous (Section 4) *vs* fully controllable (Sections 5 and 6) via r_E. Hereafter, we make some assumptions on the dependence of ψ on r_E when it is postulated:

Assumption 1 *$\psi(r_E)$ from $[\underline{r}_E, \overline{r}_E]$ to $[\underline{\psi}, \overline{\psi}]$, with $0 < \underline{\psi} < \overline{\psi} < \infty$, $\psi'(r_E) > 0$ and $\psi''(r_E) \leq 0$.*

After a revolution, the system switches to regime 2 where common access to economic resources prevails. The dynamics of the stock K simply becomes:

$$\dot{K} = AK - C_E - C_P. \tag{3}$$

Our view is that this second regime is characterized by more (political) freedom but may not be a well functioning democracy yet. Put differently, our groups engage in a political rent-seeking competition: Both groups try to extract transfers from resource wealth in a non-cooperative manner. The elite is no longer a leader but survives to the revolution, *i.e.* continues to take decisions even after she loses the control of the economy. This assumption differs from what is usually done in the literature (Acemoglu and Robinson, 2006). We do believe however that it is relevant to describe the situations of some Arab countries where the Arab spring events have been successful in overthrowing the ruling elite.

The preferences of the two groups are the same and invariant with the regime. A logarithmic function is utilized, e.g. $U(C_i^j) = \ln(C_i^j)$ with $i = E, P$ and $j = 1, 2$. The rate of pure time preference is δ and the time horizon is infinite.

In the next section, we first present the benchmark situation where the elite is not subject to the revolution threat. Then, we examine the differential game corresponding to the second regime. This preliminary discussion will serve as a basis for the original part of the paper, devoted to the analysis of the political regime change game under concessions and repression.

[8] Alternatively, we may assume that a revolution does not succeed with probability one and make the probability of success dependent on the level of repression. This would not change the general message delivered by the subsequent analysis.

3 Perpetual dictatorship, common pool game

3.1 Perpetual dictatorship of the elite

Our problem may generate two possible political modes: a perpetual dictatorial regime, and a regime switching from elite rule to common access. It is first worth studying the benchmark case with perpetual dictatorship, *i.e.* a regime in which one player (the incumbent elite) has permanent control of the political system. It boils down to considering that the ruling elite faces no threat of revolution. For the time being, assume that both u_E and r_E are given. Then, the elite solves the following optimization program:

$$\max_{\{C_E\}} \int_0^\infty \ln(C_E) e^{-\delta t} dt$$

subject to (1) and $K(0) = K_0$ given.

Straightforward manipulation of the necessary optimality conditions yield the time path of K, C_E and C_P:

$$\begin{cases} K(t) = (K_0 - \frac{r_E}{u_E A}) e^{(u_E A - \delta)t} + \frac{r_E}{u_E A} \\ C_E(t) = \delta \left(K_0 - \frac{r_E}{u_E A} \right) e^{(u_E A - \delta)t} \\ C_P(t) = (1 - u_E) A \left[\left(K_0 - \frac{r_E}{u_E A} \right) e^{(u_E A - \delta)t} + \frac{r_E}{u_E A} \right] \end{cases} \tag{4}$$

The resulting present value, for the elite, is given by:

$$V_E(u_E, r_E) = \frac{1}{\delta} \left[\ln(\delta) + \ln \left(K_0 - \frac{r_E}{u_E A} \right) + \frac{u_E A}{\delta} - 1 \right]. \tag{5}$$

Several remarks can be made. First, the growth rate of the elite's consumption is equal to $g = u_E A - \delta$. Therefore, modifying the sharing rule is accompanied by a positive *growth effect*. By increasing u_E, the elite has more resources available for consumption and investment. This stimulates growth. In addition, there is also a positive scale effect. Given the level of repression, an increase in u_E also implies that the elite has more resources left for consumption. In the absence of threat of conflict, the optimal choice of the elite is to set the sharing rule to the largest possible value, *i.e.* to $\overline{u}_E$. Second, the repression decision only involves a negative scale effect. So, it is also clear that when the elite does not have any political challenger, there is no incentive to repress and the optimal repression level is $\underline{r}_E$. For simplicity, hereafter, we will take $\underline{r}_E = 0$. Third, the present value earned by the citizens, in this case, is:

$$V_P(\overline{u}_E, 0) = \frac{1}{\delta} [\ln((1 - \overline{u}_E)A) + \ln(K_0) + \frac{\overline{u}_E A}{\delta} - 1], \tag{6}$$

while they also benefit from the growth effect associated with an increase in u_E, they incur a negative *rent capturing effect*. Increasing u_E means that the elite grabs more resources at the expense of the opposition.

Fourth, when a revolutionary threat is present, one logically expects that the elite will no longer be able to set her instruments to the levels $\overline{u}_E$ and $\underline{r}_E = 0$. In the subsequent analysis, we will envision two extreme cases:

• **Case 1.** Concessions, no repression: In this first case, the elite will respond to the threat of revolution by making concessions only, which allow her to modify the opportunity cost of revolting for the citizens. Therefore, the strategy will be to choose $u_E \in [\underline{u}_E, \overline{u}_E]$, taking $r_E = 0$.

• **Case 2.** Repression, no concessions: In contrast, here we consider that the elite sets the sharing rule to her most preferred level $\overline{u}_E$ and deals with the threat of revolution through military expenditures. Thus, the choice is to pick up a $r_E \in [0, \overline{r}_E]$.

Let's now discuss the critical boundaries for the domain of definition of our strategies, in the two cases. The basic idea we want to convey is that whenever the elite can maintain the citizens under control by appropriately choosing her instrument, she should benefit from her resulting leadership. Put differently, it seems reasonable to disregard situations where player E controls the economy and can't take advantage of this power. This would mean that she obtains a value that is less than the value of player P, who is subject to her domination. In a permanent dictatorial regime, we rather want to have: $V_E(u_E, 0) \geq V_P(u_E, 0)$ for all $u_E \in [\underline{u}_E, \overline{u}_E]$ (case 1) and $V_E(\overline{u}_E, r_E) \geq V_P(\overline{u}_E, r_E)$ for all $r_E \in [0, \overline{r}_E]$ (case 2).[9] This consideration leads us to impose the following restrictions, summarized in two different assumptions:

Assumption 2 *Under the concession strategy, the lower bound of the domain of definition of u_E is defined such that $V_P(\underline{u}_E) = V_E(\underline{u}_E) \Leftrightarrow \underline{u}_E = 1 - \frac{\delta}{A}$.*

Assumption 3 *Under the repression strategy, the upper bound of the domain is defined by: $\overline{r}_E = \frac{\overline{u}_E A K_0(\delta - (1-\overline{u}_E)A)}{\delta}$, which implies that $V_E(r_E) > V_P(r_E)$ for all $r_E \leq \overline{r}_E$.*

All the proofs are relegated in the appendices (see Appendix A.1).

In order to simplify the analysis even further we will consider simple once-and-for-all strategies for concessions and repression. In addition, note that in both cases, the model exhibits a leader-follower structure regarding decisions u_E and/or r_E and T. It thus encompasses a two-stage optimal control/differential game problem that has to be solved backward. In the next section, we briefly examine the game played by our two players following a revolution.

3.2 After the regime switch: common access

After the revolution, the two groups interact in a common-pool resource differential game. Indeed, regime 2 is characterized by the lack of strong institutions. It can be seen as reduced form model where each group has its own resource

[9] To save notations, we will not make the dependance of the value function with respect to the second fixed instrument explicit. For instance, in the first case, we will denote the value function of player i as $V_i(u_E)$, $i = E, P$.

stock, with ability to appropriate part of the other's wealth (Lane and Tornell, 1996). Given the structure of the model described above, this game is symmetric. Using Markov-perfect equilibrium (MPE) as the solution concept and guessing linear (proportional) feedback strategy for players, which implies that $C_j = a_j + b_j K$ with a_j, b_j two constants, each player solves:

$$\max_{\{C_i\}} \int_T^\infty \ln(C_i) e^{-\delta t} dt$$

subject to (3), given $K(T^+) = \tilde{K} - \chi$, where $\tilde{K}$ is the level of the capital stock at the instant of the revolution, and the guess formulated above. Direct manipulations of the necessary optimality conditions yield the MPE (the superscript 2 refers to the second regime)

$$\begin{cases} K^2(t) = (\tilde{K} - \chi) e^{(A-2\delta)(t-T)}, \\ C_i^2(t) = \delta(\tilde{K} - \chi) e^{(A-2\delta)(t-T)}, \end{cases} \tag{7}$$

and the value corresponding to this problem is, for each player $i = E, P$:

$$V_i^2(\tilde{K} - \chi) = \frac{1}{\delta} \left[\ln(\delta) + \ln(\tilde{K} - \chi) + \frac{A}{\delta} - 2 \right].$$

This value yield the continuation payoff as seen from regime 1.

We conclude this section by discussing growth prospects in the different regimes. A restriction on the parameters is imposed, which is necessary and sufficient to ensure that the regime following a revolution is compatible with non-negative growth:

Assumption 4 *The productivity parameter is high enough compared to the discount rate:* $A - 2\delta \geq 0$.

This assumption is the most relevant in our AK model where the analysis is conducted in terms of balanced growth path.[10] Given the lower bound on the sharing rule (see Assumption 2), this condition also ensures that elite's consumption grows at a positive rate in – the non permanent dictatorial – regime 1, whatever the case under scrutiny.

We are now equipped to analyze the political regime change problem. The next two sections alternatively conduct the equilibrium analysis in the first and the second case considered. Mixed strategies, involving the simultaneous use of redistribution and repression, are examined in section 6.

4 Equilibrium under the concession strategy

The purpose of the analysis below is twofold. First, we want to discuss the conditions under which a revolution might occur or, on the contrary the elite might

[10] Allowing for negative post-revolution growth, as it is observed in Tunisia or Lybia following the Arab Spring revolutions, would only make sense in the transitional dynamics, *i.e.* in the short run, that are absent here.

stay in power forever. Second in case of unavoidable uprising, we wonder what is the best concession strategy for the elite. If a revolution takes place in finite time, then the optimization problem can be decomposed into two subproblems, one for each regime. This is solved backward starting from the differential game studied above.

4.1 Before the regime switch: Elite rule

Let us proceed with the analysis of the problem faced by the elite given the potential occurrence of a revolution at some date T. Under the once and for all choice of the resource sharing by the incumbent elite,[11] first we solve the optimal control problem for any u_E. Then, we examine the elite's choice of the sharing rule. The optimization program of the elite is given by:

$$\max_{C_E} \int_0^T \ln(C_E) e^{-\delta t} dt + e^{-\delta T} V_E^2(\tilde{K} - \chi)$$

subject to (1), $K(0) = K_0$, and given that $T \leqq \infty$. The level $K(T) = \tilde{K}$ is free if $T < \infty$.

Simple computations give the general solution, valid in the dictatorial regime, indexed by 1, for any u_E:

$$\begin{cases} K^1(t) = \phi e^{u_E A t} + (K_0 - \phi) e^{(u_E A - \delta)t} \\ C_E^1(t) = \delta (K_0 - \phi) e^{(u_E A - \delta)t} \\ C_P^1(t) = (1 - u_E) A [\phi e^{u_E A t} + (K_0 - \phi) e^{(u_E A - \delta)t}] \end{cases} \tag{8}$$

with ϕ an unknown. If $T < \infty$, then $K^1(T) = \tilde{K}$ is free and the corresponding transversality condition is:[12]

$$e^{-\delta T} \lambda_E^1(T) = e^{-\delta T} \frac{\partial V_E^2(.)}{\partial K}. \tag{10}$$

The crucial point is that the elite does not directly choose whether $T \leqq \infty$. However, in some circumstances, he will be able to influence the choice of the citizens to revolt or not. The purpose of the next section is to discuss the condition(s) under which either a solution with $T < \infty$, or $T = \infty$ exists.

[11] To keep the forthcoming solutions tractable, we assume that there is no hold-up problem. The commitment mechanism of the elite is credible.

[12] Otherwise ($T = \infty$), we have the usual standard transversality condition (and we get back to the permanent dictatorship studied in section 3.1):

$$\lim_{t \to \infty} e^{-\delta t} \lambda_E^1(t) K^1(t) = 0. \tag{9}$$

4.2 Timing of the revolt

If the opposition finds it optimal to challenge political control by the elite, then he earns the following present value:[13]

$$V_P(K_0, T) = \int_0^T \ln(C_P^1) e^{-\delta t} dt + e^{-\delta T}[V_P^2(\tilde{K} - \chi) - \underline{\psi}]. \tag{11}$$

The optimal condition for switching results from the maximization of (11) w.r.t T, which yields:

$$\ln(C_P^1(T)) - V_P^2(\tilde{K} - \chi) = -\delta \underline{\psi}.$$

If there exists an optimal T for switching then the marginal benefit from delaying the switch (LHS) must be equal to the marginal direct switching cost at this instant (RHS). Using (2) and (8), this condition can be rewritten as:

$$\ln\left(\frac{\tilde{K}}{\tilde{K} - \chi}\right) = \frac{A}{\delta} - 2 + \ln\left(\frac{\delta}{[1 - u_E]A}\right) - \delta \underline{\psi}. \tag{12}$$

The LHS is a measure of the extent of the switching cost. The first term in the RHS represents the growth rate achieved under the second regime. The second term is a ratio of the proportion of resource consumed by the citizens in the second and first regimes. The last entity represents the discounted private switching cost. Denote the RHS of (12) as $\omega(u_E)$ and define the critical threshold for the DSC, $\tilde{\psi}_u$, as follows:

$$\tilde{\psi}_u = \frac{1}{\delta}\left(\frac{A}{\delta} - 2\right) \tag{13}$$

Then, it can be established that:

Lemma 1 *A necessary and sufficient condition for the existence of a solution to (12) is:* $\omega(u_E) > 0$.

1. *If* $\underline{\psi} < \tilde{\psi}_u$ ***(low DSC)*** *then,* $\omega(u_E) > 0$ *for all* $u_E \in [\underline{u}_E, \overline{u}_E]$*: There always exists a unique* $\tilde{K}(u_E)$ *for switching with,*

$$\tilde{K}(u_E) = \frac{\chi e^{\omega(u_E)}}{e^{\omega(u_E)} - 1}. \tag{14}$$

2. *Else,* $\underline{\psi} \geq \tilde{\psi}_u$ ***(high DSC)****, there exists a critical threshold* $\tilde{u}_E$ *such that* $\omega(u_E) > 0 \Leftrightarrow u_E > \tilde{u}_E$ *with,*

$$\tilde{u}_E = 1 - \frac{\delta}{A} e^{\delta(\tilde{\psi}_u - \underline{\psi})}, \tag{15}$$

this threshold is admissible i.e. $\tilde{u}_E > \underline{u}_E$.

[13] Under the concession strategy, the DSC is equal to the lowest possible value $\underline{\psi}$, *i.e.* the elite can't change the DSC directly.

Irrespective of the size of the global switching cost (GSC), χ, the occurrence of a political regime change is more likely when the DSC is low enough. For a high enough DSC, the decision to undertake a revolution will be bound to the sharing of resource fixed by the elite. So, the actual DSC and its position compared to the threshold $\tilde{\psi}_u$ is of crucial importance to understand the options available to the elite. Clearly, when the DSC is low, the elite can't avoid the revolution through concessions and the question is what is her best strategy given that the regime change is inevitable. By contrast, when the cost of switching regime is high enough, the elite seems to have the choice between making sizeable concessions in order to stay in power forever or grabbing a lot of economic resources in the first regime till a switch to regime 2 occurs.

Based on this discussion, hereafter a distinction will be made between two scenarios, depending on the magnitude of the opposing citizens' switching cost.

4.3 Magnitude of the switching costs and nature of the equilibrium

4.3.1 Low DSC scenario

In the case where the DSC of the citizens is low, $\psi < \tilde{\psi}_u$, a series of questions naturally arise: Does a solution with a revolution always exist? What does the incumbent do in its anticipation of a future regime change? Despite the existence of an interior solution, is the alternative (permanent elite dictatorship) still possible?

Part of the answer to the first question is provided by the proposition below where we characterize, for u_E given, the solution to the switching problem.

Proposition 1 *The optimal switching time is implicitly given by*

$$K_0 e^{(u_{Ea}A-\delta)T} = \chi \left(\frac{1}{e^{\omega(u_{Ea})} - 1} + e^{-\delta T} \right). \tag{16}$$

This equation has a unique solution, denoted by $T(u_E)$, if and only if

$$\tilde{K}(u_E) > K_0 \Leftrightarrow \frac{\chi e^{\omega(u_E)}}{e^{\omega(u_E)} - 1} > K_0. \tag{17}$$

where $\tilde{K}(u_E)$ is defined in (14).

If $\tilde{K}(\bar{u}_E) > K_0$, and furthermore $\chi > K_0$, then this solution exists for all $u_E \in [\underline{u}_E, \overline{u}_E]$.

Proof. See the Appendix A.2. ■

Proposition 1 states that the switching level must be higher than the initial stock. This implies that the opposition allows the resource to accumulate first. This ensures that the remaining "cake" after the switch is high enough to compensate for the loss incurred during the regime change. This also guarantees that what is competed for in the common access regime is abundant. More importantly, Proposition 1 indicates that the existence condition is dependent on the GSC. The existence of a solution, for a given u_E, surprisingly requires the GSC be high compared to the initial stock, K_0. The cost of switching for society must be high enough for the citizens to revolt against the elite. Indeed, the explanation relies on a remaining *cake size effect*. In order for the second regime to be valuable to the citizens, the amount of resource available at the beginning of the common pool game should be sufficiently high. In other words, the remaining cake $\tilde{K}(u_E) - \chi$ should be large enough for the revolution to occur. Now, this critical level is increasing in χ.

Assuming that a regime change will occur in finite time, we now tackle the choice of the sharing rule by the elite. Before proceeding to the analysis, it is useful to assess the features of the solution in regime 1:

$$\begin{cases} K^1(t) = \chi e^{u_E A(t-T)} \left(1 + \frac{e^{-\delta(t-T)}}{e^{\omega(u_E)}-1}\right), \\ C_E^1(t) = \frac{\delta\chi}{e^{\omega(u_E)}-1} e^{(u_E A-\delta)(t-T)}, \\ C_P^1(t) = (1-u_E)AK^1(t). \end{cases} \tag{18}$$

Let us start the discussion with a comparative statics exercise. From (14) and (16), it appears that:

$$\begin{cases} \frac{\partial \tilde{K}}{\partial u_E} < 0; \ \frac{\partial \tilde{K}}{\partial \chi} > 0; \ \frac{\partial \tilde{K}}{\partial \psi} > 0; \\ \frac{\partial T}{\partial u_E} < 0; \ \frac{\partial T}{\partial K_0} < 0; \ \frac{\partial T}{\partial \chi} > 0; \ \frac{\partial T}{\partial \underline{\psi}} > 0. \end{cases}$$

As expected the desired switching level, for the citizens, is decreasing in u_E: The higher u_E, the lower the opportunity cost of switching. A high u_E implies that the first regime is painful for the citizens. In contrast, the higher the GSC or the DSC, the higher the desired switching level of resource. This is due to the cake size effect. Regarding the switching date, the revolution will occur more rapidly if the elite chooses an unequal sharing rule during the first regime. The switching date decreases with the initial endowment too. The more abundant is the initial stock of natural resource in the economy, the more rapidly will conflict occur. With a higher initial stock, the resource level that triggers the revolt is achieved earlier. This observation is consistent with resource curse literature related to civil wars (see Hodler, 2006; Ploeg and Rohner, 2012 for detailed examples). Finally, the impact of a change in the switching costs on T is positive. To make the revolt valuable, the citizens must accept a longer phase of resource accumulation during the first regime in order to reach a larger $\tilde{K}(u_E)$ that will compensate for the cost.

Also central to the elite decision is the impact of a change in u_E on her consumption level during the first regime.[14] The elite's consumption can be

[14] Another interesting feature of the first regime concerns the evolution of the ratio between the consumption levels of the two groups, which can be seen as a measure of the level of inequalities in the society. From (18), it appears that the ratio $\frac{C_P^1(t)}{C_E^1(t)} = \frac{(1-u_E)A}{\delta}[1+(e^{\omega(u_E)}-1)e^{\delta(t-T)}]$ increases over time. As time passes by, the difference between consumption levels

rewritten as:

$$C_E^1(t; u_E) = \delta\hat{K}_0(u_E)e^{(u_E A-\delta)t} \text{ with } \hat{K}_0(u_E) = K_0 - \chi e^{-u_E AT(u_E)} \tag{19}$$

Comparing this expression with the one in (4), with $r_E = 0$, it turns out that the growth rate of consumption is similar. The striking difference is the existence of a scale effect. Elite consumption is lower at the equilibrium with a regime change, u_E being given. The derivative of elite consumption with respect to u_E is

$$\frac{\partial C_E^1(t; u_E)}{\partial u_E} = \delta\left[\hat{K}_0'(u_E) + At\hat{K}_0(u_E)\right]e^{(u_E A-\delta)t}, \tag{20}$$

with,

$$\hat{K}_0'(u_E) = \chi AT(u_E)e^{-u_E AT(u_E)}[1 + \sigma(T)],$$

where $\sigma(T) = \frac{u_E T'(u_E)}{T(u_E)}$ is the elasticity of the switching date with respect to the sharing rule u_E.

The RHS of (20) can be decomposed into two terms. The second term is a *positive growth effect*. A larger u_E implies greater investment. This eventually translates into a higher consumption growth rate. The first term is a combination of two opposing forces: *the wealth effect due to rent-capturing, and the regime instability effect.* When u_E increases, indeed, the elite is able to consume more. However, increasing u_E always decreases the waiting time $T(u_E)$ when a revolution will occur. A higher u_E makes elite rule less cohesive.

If $|\sigma(T)| \leq 1$, the wealth effect dominates the instability effect. Consumption in the first regime increases with u_E. On the other hand, if the impact of u_E on T is strong enough ($|\sigma(T)| > 1$), then the instability effect dominates. In this case, when investment u_E increases, elite consumption takes the opposite direction. In anticipation of conflict, the elite is willing to sacrifice her own consumption. From now on, we assume $|\sigma(T)| \leq 1$, i.e. the switching time chosen by the citizens is not too sensitive to the sharing rule. This is a relevant characteristic of resource-dependent economies with mediocre levels of social capital, e.g. awareness towards collective action.

Let us now look at the incumbent elite's choice of u_E, under the constraint that $u_E \in [\underline{u}_E, \overline{u}_E]$. The value obtained by the elite in the equilibrium with an interior regime switching is:

$$V_E(u_E) = \int_0^{T(u_E)} \ln[C_E^1(t; u_E)]e^{-\delta t}dt + e^{-\delta T(u_E)}V_E^2[\tilde{K}(u_E) - \chi]. \tag{21}$$

decreases. However, the consumption level of the elite at any date up to T is larger than the one of the citizens because $C_E^1(T) > C_P^1(T)$. This feature may appear to be at odd compared to what is observed in reality. It however reflects the optimal reaction of the elite to the future occurrence of a political regime change. In anticipation to the coming events, the elite optimally decides to smooth her consumption during the transition between the two regimes. This is ultimately accompanied by a decrease of the consumption ratio. Note however that what may seem to be a drawback of our stylized model is not robust to the removal of the normalization of the lower bound on the repression strategy to nought. With a strictly positive $\underline{r}_E$, this by-product of the analysis will not be observed.

The derivative of V_E w.r.t. u_E is:

$$\frac{\partial V_E(u_E)}{\partial u_E} = \begin{cases} \int_0^{T(u_E)} \frac{\frac{\partial C_E^1(t;u_E)}{\partial u_E}}{C_E^1(t;u_E)} e^{-\delta t} dt \\ +T'(u_E)e^{-\delta T(u_E)}\{\ln[C_E^1(T(u_E))] - \delta V_E^2[\tilde{K}(u_E) - \chi]\} \\ +e^{-\delta T(u_E)} \frac{\partial V_E^2[\tilde{K}(u_E)-\chi]}{\partial K} \tilde{K}'(u_E) \end{cases} \quad (22)$$

The first term represents the cumulative impact of a change in the redistribution rate on consumption. As shown above, this is positive as long as $|\sigma(T)| \leq 1$. The second positive term states a comparison between the cost and benefit at the switching time $T(u_E)$. The term between the brace brackets is the marginal gain of delaying the switch. It is weighted by the derivate of the switching date w.r.t u_E, which is negative. It compares the discounted value of what the elite gains in the second regime, and her loss in utility (from consumption) during the switch. The third term exhibits the remaining cake size effect. This is always negative as $\tilde{K}'(u_E)$. Simply put, this term indicates the loss due to resource destruction at the start of the second regime.[15] Careful analysis of this derivative leads to:

Proposition 2 *If $|\sigma(T)| \leq 1$ for all $u_E \in [\underline{u}_E, \overline{u}_E]$, then under the conditions of Proposition 1, there exists an equilibrium with political regime switching in finite time in which the elite sets the sharing of resource to the level $u_E^* = \underline{u}_E$.*

Proof. See the Appendix A.3. ■

Given that a revolt will occur in the future, one may expect that the elite's best response is to set the sharing rule to the highest possible value $\overline{u}_E$ as she takes advantage of her period of control. This is the kind of message delivered by the related literature (see for instance Ploeg and Rohner, 2012). It turns out that the optimal choice of the elite is the exact opposite, *i.e.* she chooses an u_E equal to the lower bound $\underline{u}_E$. This quite surprising result emphasizes the role of the **timing**, which is typically ignored by the literature. However, once we account for the timing issue, it comes at no surprise that setting $\underline{u}_E$ is good to the elite. It allows her to delay the regime switching and to lengthen the duration in office.

The last question is whether there exist multiple equilibria in the low DSC scenario. The other possible solution corresponds to the situation where the opposing citizens never find it optimal to undertake a revolt. This means that we have a corner solution for the regime switching problem. Assuming that the initial regime is dictatorship and immediate switching is not allowed, the corner solution of interest is of the never switching type. If there is no $T < \infty$ for

[15] An alternative reading of the expression in (22) is that the impact of a change in u_E can be divided between three different effects, depending on the period during! which the change is felt: The marginal impact of choosing u_E before the regime change (first term), the marginal impact at this instant of the regime change (second term) and the marginal impact of varying u_E after the political switch (last term).

switching, then it must hold that:

$$\lim_{T\to\infty} \frac{\partial V_P(.)}{\partial T} \geq 0 \Leftrightarrow \lim_{T\to\infty} e^{-\delta T} \left[\ln\left(\frac{\tilde{K}}{\tilde{K}-\chi} \right) - \omega(u_E) \right] \geq 0 \tag{23}$$

with $V_P(.)$ defined in (11). This condition implies that postponing the switch is associated with a marginal gain that is not lower than the marginal loss of foregoing for an instant the benefit from switching to the second regime.

Given that a general solution with perpetual elite dictatorship is characterized by (4), we have:

Proposition 3 *For a given u_E, a necessary condition for the citizens' problem to have a never switching solution is $\omega(u_E) \leq 0$. A sufficient condition for the converse to be true is $\omega(u_E) > 0$.*

Proof. See the Appendix A.4. ■

To sum up, the analysis conducted in the first scenario can be summarized in the following corollary.

Corollary 1 *In the low DSC scenario, there exists a unique equilibrium where the citizens challenge political control by the elite in finite time and the elite sets the sharing rule to the lower bound $\underline{u}_E$.*

According to Lemma 1, in the low DSC scenario, $\omega(u_E) > 0$ for all admissible u_E. Despite her leadership and ability to manipulate the sharing rule, the incumbent elite is unable to avoid the revolution by the opposition.

4.3.2 High DSC scenario

We now pay attention to the second scenario characterized by a high DSC, $\psi \geq \tilde{\psi}_u$. This is an interesting case because the elite now has the capacity to indirectly influence the choice of the citizens. When the elite is willing to make sufficient concessions by sharing the resource in such a way that citizens are not too harmed, then the latter may not instigate conflict. Hence, there might be conditions wherein the opposition prefer staying under permanent elite rule. In what follows, we conduct the analysis of the high DSC scenario using the material presented in the previous subsection.

As stated in Lemma 1, when the DSC is high (and such that $\omega(\underline{u}_E) < 0$), the elite has two options available. Either, she chooses a sharing rule $u_E \in [\underline{u}_E, \tilde{u}_E]$, which implies that $\omega(u_E) \leq 0$. In this case, from Lemma 1 and Proposition 3, the unique equilibrium must be of the never switching type. Or, the elite can fix the sharing rule to a level $u_E \in (\tilde{u}_E, \overline{u}_E]$. In this case, the equilibrium candidate exhibits a regime change, characterized in Lemma 1 Proposition 1.

Nonetheless, even if there are two possible solutions, we cannot have multiple equilibria, i.e. one equilibrium of each type for the same u_E. In the end, the elite seems to have the power to select the particular outcome that is best for her. The reason is that in this second scenario, he can strategically affect the switching decision of the opposition. Consequently, the elite has a choice to make between these two options. Obviously, she will select the option yielding the highest value.

To better understand this decision, we further examine the optimal sharing rule corresponding to each of these options. In the first - never switching - case, based on the findings for a perpetual dictatorship with no revolutionary option (see Section 3.1), the elite should choose $u_E^* = \tilde{u}_E$: Her value, defined in (5), with r_E put equal to zero, is increasing in u_E, which implies that she sets the sharing rule to the highest level compatible with a permanent dictatorship. The conditions are rather different under the second option as it now considers a revolt by the citizens. The elite must decide on the sharing rule in $(\tilde{u}_E, \overline{u}_E]$ with the aim of maximizing her value, which is now given by (21). If we want to be consistent with the condition imposed in Proposition 2 (regarding the elasticity of the switching date with respect to u_E), then the answer is trivial. Given that the value of the elite is decreasing in u_E, her optimal decision would be to set u_E to the lowest possible level. However, this critical redistribution rate, $\tilde{u}_E$, is not achievable because the interval corresponding to a solution with a regime change is open on the left. Since there is no solution to the elite's problem, there is no equilibrium featuring a transition between the two political regimes. This analysis is summarized in:

Corollary 2 *In the high DSC scenario, there exists a unique equilibrium where the elite stays in power forever and sets the sharing rule to the intermediate level $\tilde{u}_E$, defined in (15).*

5 Equilibrium under the repression strategy

The analysis of the second case where the elite adopts the repression strategy for given redistribution is similar to the one conducted in Section 4. We shall therefore briefly summarize our main findings. Suppose that the share u_E is fixed at the largest possible value, $\overline{u}_E$.

From the resolution of the timing problem of the citizens and the definition of:

$$\omega(r_E) = \frac{A}{\delta} - 2 + \ln\left(\frac{\delta}{[1-\overline{u}_E]A}\right) - \delta\psi(r_E),$$

and,

$$\tilde{\psi}_r = \frac{1}{\delta}\left(\ln(\delta) - \ln[(1-\overline{u}_E)A] + \frac{A}{\delta} - 2\right),$$

the second critical threshold for the DSC in this case,[16] we obtain

Lemma 2 *A necessary and sufficient condition for the existence of a solution to the switching problem of the citizens is: $\omega(r_E) > 0$.*

[16]Quite naturally we have $\tilde{\psi}_r > \tilde{\psi}_u$: Other things equal, when the elite doesn't provide any transfers to the citizens, the latter group is willing to revolt for a relatively lower switching cost. The main difference with the first case is that now the elite is able to modify directly this switching cost.

1. *If $\underline{\psi} \geq \tilde{\psi}_r$ then dictatorship is necessarily permanent.*

2. *If $\overline{\psi} < \tilde{\psi}_r$ (**low returns to repression, RR**) then the revolution is unavoidable and occurs for a switching level $\tilde{K}(r_E) = \frac{\chi e^{\omega(r_E)}}{e^{\omega(r_E)}-1}$.*

3. *If $\overline{\psi} \geq \tilde{\psi}_r > \underline{\psi}$ (**high RR**) then there exists a critical repression level $\tilde{r}_E = \psi^{-1}(\tilde{\psi}_r)$ such that: Any $r_E < \tilde{r}_E$ will trigger a revolution in finite time whereas choosing $r_E \geq \tilde{r}_E$ is a means to avoid the revolution.*

Proof. See the appendix A.5. ■

The solution to the first case is trivial. From the discussion conducted in Section 3.1, the optimal solution is $r_E^* = 0$. By analogy with Section 4, two interesting cases remain. On the one hand, when the RR are low, the elite is not able to keep the citizens under control and the political regime change is inevitable. On the other, the elite may avoid the revolution provided that the repression technology is efficient enough and by investing at least $\tilde{r}_E$ in military expenditures. Put differently, a permanent policy that consists in devoting a constant level of resources $\tilde{r}_E$ to the military budget protects the elite from an uprising of the citizens.

Next, we can establish that:

Proposition 4 *Suppose r_E is given. If $\chi > K_0$ then the switching time $T(r_E)$ is uniquely and implicitly defined by:*

$$\left(K_0 - \frac{r_E}{\overline{u}_E A}\right) e^{(\overline{u}_E A - \delta)T} = \tilde{K}(r_E) - \chi + \left(\chi - \frac{r_E}{\overline{u}_E A}\right) e^{-\delta T},$$

with $\tilde{K}(r_E)$ defined in Lemma 2. The comparative statics are:

$$\frac{\partial T}{\partial r_E} > 0, \frac{\partial \tilde{K}}{\partial r_E} > 0; \frac{\partial T}{\partial \overline{u}_E} < 0, \frac{\partial \tilde{K}}{\partial \overline{u}_E} < 0; \frac{\partial T}{\partial \chi} > 0, \frac{\partial \tilde{K}}{\partial \chi} > 0 \text{ and } \frac{\partial T}{\partial K_0} < 0.$$

Proof. See the Appendix A.5. ■

The same condition as in Proposition 1, for the concession strategy, is also sufficient for the existence of a solution to the switching problem. As far as the impact of repression on this solution is concerned, intuitively we obtain that increasing repression expenditures is a means to delay the revolution, which will occur for a larger stock of resource. Indeed, the larger r_E, the higher the cost of switching and the larger the compensation must be for the citizens. This compensation takes the form of the achievement of the second regime, whose profitability is determined by the difference $\tilde{K}(r_E) - \chi$ (the remaining cake size). Regarding the other parameters, the larger $\overline{u}_E$, *i.e.* the more unequal the country is in the absence of concessions by the elite, the lower the opportunity cost of switching and the sooner the political regime change. In addition, with a large $\overline{u}_E$, the citizens accept to start the second regime with a lower amount of economic resources. Finally, a larger initial wealth of stock of resources tends to expedite the decision to revolt.

Suppose that $T(r_E)$ exists. Then, the solution corresponding to regime 1 can be written as:

$$\begin{cases} K^1(t;r_E) = \left(\chi - \frac{r_E}{\overline{u}_E A}\right) e^{\overline{u}_E A(t-T(r_E))} + (\tilde{K}(r_E) - \chi)e^{(\overline{u}_E A-\delta)(t-T(r_E))} + \frac{r_E}{\overline{u}_E A} \\ C_E^1(t;r_E) = \delta(\tilde{K}(r_E) - \chi)e^{(\overline{u}_E A-\delta)(t-T(r_E))} \\ C_P^1(t;r_E) = (1-\overline{u}_E)AK^1(t;r_E). \end{cases} \tag{24}$$

The last proposition summarizes our results in the two possible scenarios, low *vs.* high RR. Let us first define $\sigma(\tilde{K}) = \frac{r_E \tilde{K}'(r_E)}{\tilde{K}(r_E)} > 0$. Then,

Proposition 5 *If* $\sigma(\tilde{K}) < -\frac{r_E \psi''(r_E)}{\psi'(r_E)}$ *for all* $r_E \in [0, \overline{r}_E]$ *and* $\frac{\partial C_E^1(t;r_E)}{\partial r_E}|_{r_E = \overline{r}_E} > 0$ *then,* $\frac{\partial V_E(r_E)}{\partial r_E} > 0$ *for all admissible* r_E*.*

- *In the low RR scenario, there is a unique equilibrium where the revolution occurs in finite time and the elite sets* $r_E^* = \overline{r}_E$*.*
- *In the high RR scenario, the unique equilibrium features permanent dictatorship and* $r_E^* = \tilde{r}_E$*.*

Proof. See the Appendix A.5. ■

Under the repression strategy, the trick consists once again in determining the sign of the derivative of the value function of the elite with respect to the r_E. This boils down to determining how the consumption of the elite, in the first regime, responds to a change in the repression level. It appears that an increase in r_E has two opposing (scale) effects on the elite consumption. The effects are more easily seen when rewriting consumption as:

$$C_E^1(t;r_E) = \delta\left[K_0 - \frac{r_E}{\overline{u}_E A} - \left(\chi - \frac{r_E}{\overline{u}_E A}\right) e^{-\overline{u}_E A T(r_E)}\right] e^{(\overline{u}_E A - \delta)t}$$

Both the initial condition and the GSC are reduced by an amount $\frac{r_E}{\overline{u}_E A}$. The resulting differences define the *true* values, or the values that matter to the elite when deciding how much to repress. Then, the analysis runs as follows. As in the first case (concession strategy, see (19)). The constant term in the expression above is a rescaling of the true initial condition obtained by subtracting the true GSC incurred at the date of the revolution, $T(r_E)$, discounted from the initial period at the autonomous growth rate of the stock of economic resources. Other things equal, more repression means that revolution will occur later and consequently this loss is felt less acutely, which stimulates consumption. However, at the same time, an increase in r_E implies that less resources are left for consumption and investment at every date in regime 1. This in turn tends to lower the elite consumption. Now, it turns out that if the elasticity of the desired switching level, $\tilde{K}(r_E)$, with respect to repression is low enough and lower than the sensitivity of the marginal switching with respect to this strategy, then the overall effect is positive: regime 1 consumption is higher the larger r_E.[17] Under

[17] The second sufficient condition is Proposition 5 is a technical boundary condition, see the Appendix A.5.

this condition we finally obtain that the value function of the elite is strictly increasing in r_E when the political regime switch is inevitable. Therefore, in the low RR scenario, the elite will choose the highest level of repression in order to lengthen the first dictatorial regime. By contrast, when the RR are high, a permanent dictatorship is the only possible equilibrium. In this case, and given that her present value is now decreasing with the level of repression, the elite sets the repression level to the lowest possible value, $\tilde{r}_E$, allowing for the absence of regime switch.

6 Policy mix

In the two preceding sections, the analysis has revealed under which conditions a revolution might occur in an economy ruled by an elite having two options to control the citizens. Let us now go back to the other important question raised by the paper: What is the best strategy for the elite? The answer to this question is more complicated. At first glance, one may reply that the best strategy is the one that allows the elite to stay in power forever, if possible. A quick inspection of the ordering between the critical switching costs gives some insights into this crucial point. Assume that $\underline{\psi} < \tilde{\psi}_u (< \tilde{\psi}_r)$. In this situation, adopting the concession strategy doesn't succeed in avoiding the revolution whereas repression may be sufficient to keep the citizens under control provided that it renders the switching cost prohibitive, which requires $\tilde{\psi}_r \leq \overline{\psi}$. Thus based on this criterion, the elite should implement a repression policy. The conclusion is reversed however, when the ordering is $\tilde{\psi}_u \leq \underline{\psi} < \overline{\psi} < \tilde{\psi}_r$. Here repression is not efficient enough to prevent political challenge by the citizens. In this case, an elite mostly interested in holding power should rather provide citizens with large transfers so that regime 1 is good enough to them and the opportunity cost of revolting is too high compared to the resulting benefits associated with a freer political system.

A last question arises ultimately: could a strategy mix perform better than the single-instrument strategies studied so far? The answer to this question is far nontrivial. Let us see under which conditions the strategy mix is better for the elite in the sense that it allows him to stay in power when a single strategy fails to do so.

From lemmas 1 and 2, we know that the revolution is unavoidable under concession (resp. repression) when $\underline{\psi} < \tilde{\psi}_u$ (resp. $\overline{\psi} < \tilde{\psi}_r$). Let's assume that these two conditions hold and the overall ordering is: $\underline{\psi} < \tilde{\psi}_u < \overline{\psi} < \tilde{\psi}_r$.[18] In that case, using a strategy mix can be a means to maintain the dictatorship.

The proof is as follows: From now on, we allow r_E and u_E to vary in their respective domains of definition, *i.e.* $[0, \overline{r}_E]$ and $[\underline{u}_E, \overline{u}_E]$. Adopting the same methodology as before, we can show that the switching problem of the citizens

[18]The following reasoning can easily be extended to the other possible cases.

defines a unique $\tilde{K}(r_E, u_E)$ if and only if $\omega(r_E, u_E) > 0$ with:

$$\omega(r_E, u_E) = \frac{A}{\delta} - 2 + \ln\left(\frac{\delta}{[1-u_E]A}\right) - \delta\psi(r_E).$$

Solving the equation $\omega(r_E, u_E) = 0$ boils down to solving $\psi(r_E) = \varphi(u_E)$. The function $\varphi(u_E) = \tilde{\psi}_r - \frac{1}{\delta}[\ln(1-u_E) - \ln(1-\overline{u}_E)](> 0$ for $u_E \geq \underline{u}_E)$ satisfies $\varphi'(u_E) > 0$, $\varphi(\underline{u}_E) = \tilde{\psi}_u$ and $\varphi(\overline{u}_E) = \tilde{\psi}_r$. Given our ordering, it's clear that $\psi(r_E) = \varphi(u_E)$ has a solution because we have $\underline{\psi} < \varphi(\underline{u}_E) < \overline{\psi} < \varphi(\overline{u}_E)$. More precisely, we can define $\hat{u}_E \in (\underline{u}_E, \overline{u}_E)$ such that $\varphi(\hat{u}_E) = \overline{\psi}$ and $\hat{r}_E \in (0, \overline{r}_E)$ such that $\psi(\hat{r}_E) = \tilde{\psi}_u$. Given that ψ is invertible, for any $u_E \in [\underline{u}_E, \hat{u}_E]$, the equation $\psi(r_E) = \varphi(u_E)$ can rewritten as a relationship $r_E = \kappa(u_E)(= \psi^{-1\prime}(\varphi(u_E)))$ with $\kappa'(u_E) > 0$, $\kappa(\underline{u}_E) = \hat{r}_E$ and $\kappa(\hat{u}_E) = \overline{r}_E$. By definition, any pair (r_E, u_E) belonging to this locus is such that $\omega(r_E, u_E) = 0$. Then, let ζ be the set such that for any pair taken in this set a revolution cannot occur (dictatorship is permanent) because $\omega(r_E, u_E) \leq 0$:

$$\zeta = \left\{(r_E, u_E) \in [\underline{u}_E, \hat{u}_E] \times [\hat{r}_E, \overline{r}_E] / r_E \geq \kappa(u_E)\right\},$$

the complementary set being denoted by $\overline{\zeta}$. For any pair in $\overline{\zeta}$, there might exist an equilibrium with a revolution in finite time.

The second step consists in determining the optimal combination $(r_E, u_E) \in \zeta$. Under permanent dictatorship, the value of the elite is given by (5), which is decreasing in r_E and increasing in u_E. Thus, for any, u_E, the elite chooses the lowest level of repression compatible with dictatorship. This means that the optimal combination necessarily lies in the frontier given by $r_E = \kappa(u_E)$. Then, the value can be rewritten as:

$$V_E(u_E, \kappa(u_E)) = \frac{1}{\delta}\left[\ln(\delta) + \ln\left(K_0 - \frac{\kappa(u_E)}{u_E A}\right) + \frac{u_E A}{\delta} - 1\right].$$

Taking the derivative w.r.t u_E, one obtains:

$$\frac{\partial V_E}{\partial u_E} = \frac{\kappa(u_E)}{u_E^2 A(K_0 - \frac{\kappa(u_E)}{u_E A})}(1 - \sigma(\kappa)) + \frac{A}{\delta^2} \text{ with } \sigma(\kappa) = \frac{u_E \kappa'(u_E)}{\kappa(u_E)},$$

and the condition $\sigma(\kappa) < 1$, together with $K_0 - \frac{\kappa(u_E)}{u_E A} > 0$, is sufficient to have $\frac{\partial V_E}{\partial u_E} > 0$. The conclusion is that the optimal combination is $(\hat{u}_E, \overline{r}_E)$, which yields a present value:

$$V_E^P(\hat{u}_E, \overline{r}_E) = \frac{1}{\delta}\left[\ln(\delta) + \ln\left(K_0 - \frac{\overline{r}_E}{\hat{u}_E A}\right) + \frac{\hat{u}_E A}{\delta} - 1\right]. \tag{25}$$

The third step shows that permanent dictatorship is indeed the unique equilibrium when the ordering is $\underline{\psi} < \tilde{\psi}_u < \overline{\psi} < \tilde{\psi}_r$. Here our aim is to generalize the analyses of the previous sections and to prove that under the conditions used in propositions 1 and 5, very few is needed to obtain the result.

Let's assume that the elite picks up a policy mix from $\overline{\zeta}$. This implies that the switching problem has a solution $(\tilde{K}(r_E, u_E), T(r_E, u_E))$, which is similar to the one characterized in lemma 2, proposition 4 and (24) when one replaces $\overline{u}_E$ with any u_E provided that $(r_E, u_E) \in \overline{\zeta}$.[19] To determine under which conditions the general properties derived in the two preceding sections are still valid, let's see how the first period elite's consumption, $C_E^1(t; r_E, u_E)$ responds to changes in r_E and u_E given that:

$$C_E^1(t; r_E, u_E) = \delta(\tilde{K}(r_E, u_E) - \chi)e^{(u_E A - \delta)(t - T(r_E, u_E))},$$

with $\tilde{K}(r_E) = \frac{\chi e^{\omega(r_E, u_E)}}{e^{\omega(r_E, u_E)} - 1}$.

Partial derivative w.r.t r_E for any $u_E \in [\underline{u}_E, \hat{u}_E]$:[20]

$$\frac{\partial C_E^1(t; r_E, u_E)}{\partial r_E} = C_E^1(t; r_E, u_E)\left(\frac{\tilde{K}_{r_E}}{\tilde{K}(r_E, u_E) - \chi} - (u_E A - \delta)T_{r_E}\right),$$

We know that under the conditions of Proposition 5, this derivative is positive at $u_E = \overline{u}_E$. Given that $\frac{\tilde{K}_{r_E}}{\tilde{K}(r_E, u_E) - \chi} = \frac{\delta\psi'(r_E)\tilde{K}(r_E, u_E)}{\chi}$ is decreasing in u_E (because $\tilde{K}_{u_E} < 0$), we have $\frac{\tilde{K}_{r_E}}{\tilde{K}(r_E, u_E) - \chi}|_{u_E \leq \hat{u}_E} > \frac{\tilde{K}_{r_E}}{\tilde{K}(r_E, u_E) - \chi}|_{u_E = \overline{u}_E}$. Note also that $u_E A - \delta < \overline{u}_E A - \delta$ for all $u_E \in [\underline{u}_E, \hat{u}_E]$. Then the condition $T_{r_E u_E} > 0$ for all $(r_E, u_E) \in \overline{\zeta}$ is sufficient to conclude that $\frac{\partial C_E^1(t; r_E, u_E)}{\partial r_E} > 0$ provided that the conditions of Proposition 5, adapted to the case studied, hold. The first condition is $\sigma(\tilde{K}) < -\frac{r_E \psi''(r_E)}{\psi'(r_E)}$ where the RHS is independent of u_E and one can easily check that the LHS is decreasing in u_E. Thus, this condition must be strengthened to require that $\sigma(\tilde{K})|_{u_E = \underline{u}_E} < -\frac{r_E \psi''(r_E)}{\psi'(r_E)}$. The second technical condition is satisfied as well. The new condition, that involves the cross derivative $T_{r_E u_E} > 0$, basically means that when the elite makes less concessions (u_E increases), the switching time chosen by the citizens becomes more sensitive to the level of repression.

By analogy with section 4, consumption can be rewritten as: $C_E^1(t; u_E, r_E) = \delta\hat{K}_0(u_E, r_E)e^{(u_E A - \delta)t}$ with

$$\hat{K}_0(u_E, r_E) = K_0 - \frac{r_E}{A u_E} - (\chi - \frac{r_E}{A u_E})e^{-u_E A T(u_E, r_E)}.$$

The partial derivative w.r.t u_E for any $r_E \in [\hat{r}_E, \overline{r}_E]$:

$$\frac{\partial C_E^1(t; u_E)}{\partial u_E} = \delta\left[\hat{K}_{0,u_E} + At\hat{K}_0(u_E)\right]e^{(u_E A - \delta)t},$$

with,

$$\hat{K}_{0,u_E} = \frac{r_E}{A u_E^2}\left(1 - e^{-u_E A T(r_E, u_E)}\right) + \left(\chi - \frac{r_E}{u_E A}\right)AT(r_E, u_E)(1 + \sigma(T)),$$

[19] Of course, the argument u_E becomes apparent in the functions $\tilde{K}(.)$ and $T(.)$.

[20] With a slight abuse of notation, we will denote the derivate w.r.t r_E for $\tilde{K}$ and T as $\tilde{K}_{r_E}$ and T_{r_E}.

where $\sigma(T) = \frac{u_E T_{u_E}}{T(r_E, u_E)}$ is increasing in r_E. Then, extending the condition of Proposition 2 to $|\sigma(T)|_{r_E=\overline{r}_E} \leq 1$ is sufficient to conclude that $\frac{\partial C_E^1(t;u_E)}{\partial u_E} > 0$.

Finally, under the conditions stated above, it's straightforward to show that the present value of the elite at a solution featuring a regime change is decreasing in u_E and increasing in r_E. Thus, there is no equilibrium with a political regime switching because one cannot find a pair (r_E, u_E) that maximizes the elite value. For instance, the elite would like to choose the highest concession level $\underline{u}_E$ but then it doesn't exist a repression level such that the pair of instruments belongs to $\overline{\zeta}$. In sum, when the ordering is $\underline{\psi} < \tilde{\psi}_u < \overline{\psi} < \tilde{\psi}_r$, there exists a unique equilibrium with permanent dictatorship, which produces a payoff equal to (25).

In concluding this discussion, we want to emphasize a crucial point. At the beginning of this section we argued that a success in keeping the political control is a good criterion to evaluate the best strategy. However, this criterion may not be so clear and forceful. Indeed, it may well be that the elite, by using an instrument only and accepting the regime change, is better off than by implementing a policy mix of the type discussed above. Put differently, the present value associated with the former strategy may likely be higher than the one yielded by the latter. Therefore, according to the criterion of (maximizing) the present value, accepting the revolution and adapting to this event may constitute the optimal strategy of the elite. Unfortunately, any attempt to go deeper into this discussion is vain. The comparison between the values provided by the different strategies appears to be a difficult exercise and doesn't allow us to identify the set of conditions under which this provocative result holds.[21] As an illustration of the various conflicting (growth and scale) effects at stake, let's have a quick look at the comparison between the concession strategy and the policy mix. In this particular case, it can easily be shown that growth prospects are higher at the policy mix solution. But, how the scale effect exactly play is unclear. If the elite redistributes much more resources at the solution with concession, she doesn't spend a penny to repress the citizens whereas, with the policy mix, she devotes a lot of resources to her military budget. In general, it is possible neither to know which solution the scale effect benefit to, nor to conclude which effect prevails (when the scale effect pushes in the opposite direction as the growth effect).

[21] From the results of sections 4 and 5, we know that the degenerated combination of instruments corresponding to these two cases are respectively given by $(0, \underline{u}_E)$ (concessions) and $(\overline{r}_E, \overline{u}_E)$ (repression). The values produced by these extreme strategies are:

$$V_E^C(0, \underline{u}_E) = \frac{1}{\delta}\left(\ln(\delta) + \ln(\tilde{K}(0, \underline{u}_E) - \chi) - (\underline{u}_E A - \delta)T(0, \underline{u}_E) + \frac{\underline{u}_E A - \delta}{\delta}\right),$$
$$V_E^R(\overline{r}_E, \overline{u}_E) = \frac{1}{\delta}\left[e^{-\delta T(\overline{r}_E, \overline{u}_E)}\left(\frac{A(1-\overline{u}_E)-\delta}{\delta}\right) + \ln(\delta) + \ln(\tilde{K}(\overline{r}_E, \overline{u}_E) - \chi) - (\overline{u}_E A - \delta)T(\overline{r}_E, \overline{u}_E) + \frac{\overline{u}_E A - \delta}{\delta}\right],$$

and one cannot find simple conditions to find the ordering between these values and the one produced by the policy mix (25).

7 Implications for the Arab spring

As mentioned in the introduction, the DSC are neither fully controllable as in Sections 5 and 6 nor totally exogenous as in Section 4. Recall that the DSC reflect either the cost of a collective action (or the cost of coordination) or the extent to which the regime set by the elites is contestable, that's in particular the extent of their repressive forces. Because the DSC are affected in real life by such diverse factors as national demography, international geopolitics and connectability of the countries to global markets and media, our theoretical analysis (of the polar cases) need to be qualified. A few points are worth to make in the Arab spring context.

a) A major finding of the theoretical analysis is that the equilibrium essentially depends on the level of the DSC. An interesting example is Bahrein. Since his arrival in office in 1999, Sheikh Hamad has started a sequence of democratization and political liberalization steps leading many international organizations to believe that the country has definitely improved regarding protection of human rights. This low level of repression was however associated with what the religious majority of the country (the Shias) felt as an unfair redistribution of resources. In this context, our theory would predict a revolution and a regime change at finite time (Section 4). Indeed, the Shias revolt took place in 2011 but the massive external intervention of Saudi Arabia to back the Khalifa dynasty changed the outcome. A large enough exogenous increase in the DSC through external intervention as in Bahrein would change the equilibrium to permanent dictatorship.[22] We might also interpret the Libyan case symmetrically: Initially repression was strong in this country (and so was the military threat of the former dictator) and redistribution quite sizeable, leading to permanent dictatorship according to our theory if no external intervention occurs. The NATO bombings implied a rapid drop in the DSC faced by the opposition, opening the doors wide open for an alternative equilibrium with a revolution and regime change at finite time.

b) As referred to in the introduction, as the DSC can also be interpreted as coordinations costs, it might help explaining the role of human capital level in the genesis of the Arab spring. In particular, if one admits that the larger the education level, the lower the coordination costs in collective actions (so the lower the DSC), the Tunisian lead sounds more natural given the weight of public education in the budget of this country and the impressive literacy performances registered in the relevant population age classes.[23] While this single criterion cannot explain the whole picture, it is a relevant part of the story. The institutional stability of the Gulf

[22] In a straightforward extension of our model with random revolution success such as the probability of success is a decreasing function of total repression forces, we would get that the Shias would have not revolted if they would have anticipated the Saudi intervention.

[23] According to the World Bank Edstats, the literacy rate in the 15-24 years old population is Tunisia was about 97% in 2008, among the very best in the MENA region.

monarchies, which are also known to have high education performances, does not go at odds with this view: Just like Algeria, these countries have spent a substantial part of their petrodollars to buy time. In none of the latter cases, the status-quo seems a stable economico-political equilibria. In particular, the Gulf monarchies' stability is undermined by deep contradictions, the discord between religious and secular schooling, and between women's education and their actual role in public life being only two examples of these contradictions among others (see the illuminating analysis by Bahgat, 1998, many years before the Arab spring).[24] The repression force of the *Big Brother* of this region, Saudi Arabia, has been so far, if not persuasive, certainly implacable (as it transpires from the Bahrein case). Much of the future of this region depends on how this monarchy will tackle its contradictions. It is far unclear whether the recent steps taken (for example in favor of women's rights) by the King are necessary and sufficient for that.

c) The interpretation of the DSC as a coordination cost is also interesting to understand the cascade of events following the Jasmine revolution. One can perfectly attribute to the striking success of this early revolution a significant psychological effect on the Arab populations, a kind of awareness shock easing their rallying towards a common objective. In this sense, the Jasmine revolution has supposed a significant drop in the DSC faced by these populations, and therefore a larger propensity to revolt. It is because the rulers of the Arab world are precisely aware of this driving force that they have launched so many initiatives after the Tunisian revolution towards more concessionary policies both with respect to economic and political rights. This concerns countries of the Maghreb (Algeria and Marocco as mentioned in the introduction) but also Gulf monarchies starting with Saudi Arabia.

d) Last but not least, the demographic aspects, and in particular the relative demographic size of the elites, are worth accounting for (see the modelling in Acemoglu and Robinson, 2008). Just like human capital accumulation, demographic change involves slow dynamics and might be a good candidate for explaining the timing of the revolts in the Arab countries. But the Arab world demographics are tricky to model for several reasons (see for example Rashad, 2000, about the demographic transition in Arab countries), and a minimally realistic (endogenous) specification of these demographics would have made our analytical work quite uncomfortable. In our theoretical setting, population growth has a priori an ambiguous effect on the DSC: on one hand, it lowers them because repression is potentially less efficient when the size of the opposition increases; but on ther other hand, for a given fixed education level, coordination costs across a bigger population are larger leading to a DSC increment.

[24] See also the excellent recent book by Christopher Davidson (2013) on the sustainability of Gulf monarchies.

Moreover, demographic growth has also an impact on the redistribution as it leads to a mechanical increase of the redistribution bill. Accordingly, a straightforward extension of our model including exogenous growth of the opposition size would deliver a threshold value for this growth rate above which a regime change occurs.

8 Conclusion

In this paper, we develop a dynamic game to provide with a comprehensive theory of Arab spring-type events with the following main ingredients. We have two interacting groups, the elite *vs.* the citizens, two political regimes, dictatorship *vs.* a freer regime, the possibility to switch from the first to the second regime as a consequence of a revolution by the citizens and finally the opportunity, for the elite, to affect the citizens' decision through concession and/or repression strategies. In this framework, we provide a full characterization of the equilibrium of our political regime switching game. First, we emphasize the role of the direct switching cost of the citizens and of the elite's self-preservation options. Under the concession strategy, when the switching cost is low, the elite can't avoid the political regime change. She optimally adapts to the overthrow of their political power by setting the rate of redistribution to the highest possible level, thereby extending the period during which she has full control on resources. This surprising result actually illustrates the role of the timing of events in these situations of interaction between the ruling elite and the people. When the direct switching is high, the elite can ultimately select the equilibrium outcome and adopt the opposite strategy, that consists in choosing the lowest level of redistribution that allows her to stay in power forever. The same kind of results are obtained when the elites rely on repression to keep the citizens under control. Next, the equilibrium properties under a mix of repression and redistribution are analyzed. It is shown that in situations where neither repression (only) nor redistribution (only) protect the elite against the uprising of citizens, a subtle mixture of the two instruments is sufficient to make the dictatorship permanent. Based on our theoretical results, we finally examine the reason for such a large variety of decisions (taken by the elites) and outcomes during the Arab Spring events.

Our analysis can be extended in several ways. First, we may allow the elite to revise her repression and/or redistribution strategy in face of the threat of revolution. This can be done by formulating the choice of u_E and r_E as a regime switching problem (and applying the methodology developed by Long et al., 2013). Second, we should account for additional aspects of the mechanics of the Arab Spring. In particular the role of human capital and education (through their impact on citizens' claims for freedom) or the importance of the demographic structure seem to be crucial for understanding the timing and success of the revolts in Arab countries. Among the other promising developments of our work are the introduction of (endogenous) probabilities of (successful) revolution, capital evasion (when the elite can divert resources to its own benefit) and secure property rights in the second regime.

A Appendix

A.1 Assumption 2 & 3

The elite in the first case (resp. the second case) is obtained by replacing $r_E = 0$ (resp. $u_E = \overline{u}_E$) in (5). The value of the citizens in the first case is derived from (6) by substituting $\overline{u}_E$ with any u_E.

First case: One can directly check that $V_E(u_E) \geq V_P(u_E) \Leftrightarrow B \geq \ln[\frac{(1-u_E)A}{\delta}]$ or $u_E \geq 1 - \frac{\delta}{A}$. This leads to the Assumption 2.

Second case: The citizens' value cannot be computed so easily. So we resort to the condition that the elite's consumption is always higher than the citizens' consumption under permanent dictatorship. A sufficient condition for this to hold is $C_E(0) > C_P(0) \Leftrightarrow r_E \leq \frac{\overline{u}_E A K_0(\delta-(1-\overline{u}_E)A)}{\delta}$. This in turn ensures that $V_E(r_E) > V_P(r_E)$. All this information is summarized in Assumption 3.

A.2 Proof of proposition 1

First, K is continuous everywhere except at T, where it's only left-continuous. The left-continuity of the capital stock at the switching date T, the general expression of $K^1(t)$ for all $t \leq T$ being given in (8), implies

$$K^1(T) = \tilde{K}(u_E) \Leftrightarrow \phi e^{u_E A T} + (K_0 - \phi)e^{(u_E A-\delta)T} = \tilde{K}(u_E). \qquad (26)$$

Second, we use the fact that the switching decision is taken by citizens but citizens don't have any influence on the dynamics of the first regime i.e. the value of the state at which the switch occurs is controlled by the elite. So, condition (10) must hold, which together with (8) yields:

$$e^{-\delta T}\lambda_E^1(T) = e^{-\delta T}\frac{\partial V_E^2(.)}{\partial K} \Leftrightarrow (K_0 - \phi)e^{(u_E A-\delta)T} = \tilde{K}(u_E) - \chi. \qquad (27)$$

In sum, we are left with a system of two conditions (26)-(27) in the two remaining unknowns ϕ and T. Substituting the value of ϕ given by (26) in (27), one obtains

$$K_0 e^{(u_E A-\delta)T} = \chi\left(\frac{1}{e^{\omega(u_E)}-1} + e^{-\delta T}\right),$$

which must be studied to show the existence of a strictly positive and finite switching date. Noticing that under Assumption 1., necessarily we have $u_E A - \delta > 0$ for all u_E, then we obtain that there exists a unique $0 < T(u_E) < \infty$ iff $\tilde{K}(u_E) > K_0$. Finally, given that $\omega'(u_E) = \frac{1}{1-u_E}$ and $\tilde{K}'(u_E) =$

$-\frac{-\chi\omega'(u_E)e^{\omega(u_E)}}{(e^{\omega(u_E)}-1)^2} < 0$, $\tilde{K}(\bar{u}_E) > K_0$ is sufficient to ensure existence for all $u_E \in [\underline{u}_E, \bar{u}_E]$. [25]

A.3 Proof of proposition 2

After direct manipulations, the derivative in (22) can be rewritten as

$$\frac{\partial V_E(u_E)}{\partial u_E} = \frac{1}{\delta}\left\{ \begin{array}{l} \frac{1}{1-u_E}\left(\frac{(1-u_E)A}{\delta} - \frac{e^{\omega(u_E)}}{e^{\omega(u_E)}-1}\right) - AT(u_E)[1+\sigma(u_E)] + \delta T'(u_E) \\ -e^{-\delta T(u_E)}\left\{T'(u_E)\left[(1-u_E)A-\delta\right] + \frac{A}{\delta}\right\}. \end{array}\right. \tag{28}$$

Under Assumption 2, it's easy to check that if $u_E \geq \underline{u}_E = 1-\frac{\delta}{A}$ and $|\sigma(u_E)| \leq 1$ then $\frac{\partial V_E(u_E)}{\partial u_E} < 0$ for all $u_E \in [\underline{u}_E, \bar{u}_E]$. Hence, the optimal choice is $u_E = \underline{u}_E$ (if the second order optimality condition is satisfied).

A.4 Proof of proposition 3

Under Assumptions 2 and 4, we have $u_E > \frac{\delta}{A}$ for all admissible u_E. This implies that $\lim_{t\to\infty} K(t) = \infty$ under a permanent regime 1 and we know that $\lim_{K\to\infty} \ln\left[\frac{K}{K-\chi}\right] = 0$. Then, (23) is equivalent to $\ln\left[\frac{(1-u_E)A}{\delta}\right] - \frac{A-2\delta}{\delta} \geq 0 \Leftrightarrow \omega(u_E) \leq 0$.

A.5 Proof of lemma 2 and Propositions 4 & 5

- Citizens' switching problem (Lemma 2): The opposition solves

$$\max_T V_P(K_0, T) = \int_0^T \ln(C_P^1)e^{-\delta t}dt + e^{-\delta T}[V_P^2(\tilde{K}-\chi) - \psi(r_E)],$$

The necessary optimal condition for switching is (the sufficient optimality condition is satisfied):

$$\ln\left(\frac{\tilde{K}}{\tilde{K}-\chi}\right) = \frac{A}{\delta} - 2 + \ln\left(\frac{\delta}{[1-\overline{u}_E]A}\right) - \delta\psi(r_E). \tag{29}$$

Denote the RHS of (29) as $\omega(r_E)$. As before, there exists a unique $\tilde{K}(r_E) = \frac{\chi e^{\omega(r_E)}}{e^{\omega(r_E)}-1}$ that solves (29) iff $\omega(r_E) > 0$. In addition, we can define a second critical threshold for the DSC:

$$\tilde{\psi}_r = \frac{1}{\delta}\left(\ln(\delta) - \ln[(1-\overline{u}_E)A] + \frac{A}{\delta} - 2\right).$$

This is sufficient to state Lemma 2.

[25] One may be concerned with the meaning of this sufficient condition because till now we haven't discussed the level of the upper bound $\bar{u}_E$. For the sake of simplicity, given that the share $\bar{u}_E$ is bounded from above by 1, we may simply replace $\tilde{K}(\bar{u}_E) > K_0$ with the stronger requirement: $\tilde{K}(1) > K_0$.

• Switching date (Proposition 4): From the transversality condition of regime 1 and the left continuity of the resource stock we obtain

$$\phi = \left(\tilde{K}(r_E) - \frac{r_E}{\overline{u}_E A} - \left(K_0 - \frac{r_E}{\overline{u}_E A} \right) \right) e^{-\overline{u}_E A T} \left(1 - e^{-\delta T} \right)^{-1}$$

And T is implicitly given by:

$$\left(K_0 - \frac{r_E}{\overline{u}_E A} \right) e^{(\overline{u}_E A - \delta) T} = \tilde{K}(r_E) - \chi + \left(\chi - \frac{r_E}{\overline{u}_E A} \right) e^{-\delta T} \tag{30}$$

The condition $\chi > K_0$ is sufficient for the existence of a unique solution $T(r_E)$ to this equation (for a given r_E). This is the same condition as the one derived in the draft for the concession case.

Suppose that $T(r_E)$ exists. Then, the solution corresponding to regime 1 reads as follows:

$$\begin{cases} K^1(t; r_E) = \left(\chi - \frac{r_E}{\overline{u}_E A} \right) e^{\overline{u}_E A (t - T(r_E))} + (\tilde{K}(r_E) - \chi) e^{(\overline{u}_E A - \delta)(t - T(r_E))} + \frac{r_E}{\overline{u}_E A} \\ C_E^1(t; r_E) = \delta (\tilde{K}(r_E) - \chi) e^{(\overline{u}_E A - \delta)(t - T(r_E))} \end{cases}$$

• Proof of Proposition 5:

Impact of a change in r_E on consumption (other comparative statics can easily be derived from the definition of $\tilde{K}(r_E)$ and (**??**)):

$$\frac{\partial C_E^1(t; r_E)}{\partial r_E} = C_E^1(t; r_E) \left(\frac{\tilde{K}'(r_E)}{\tilde{K}(r_E) - \chi} - (\overline{u}_E A - \delta) T'(r_E) \right),$$

From (30),

$$T'(r_E) = \frac{\tilde{K}'(r_E) + \frac{1}{\overline{u}_E A} \left(e^{(\overline{u}_E A - \delta) T(r_E)} - e^{-\delta T(r_E)} \right)}{\tilde{K}(r_E) - \chi + \frac{\overline{u}_E A}{\overline{u}_E A - \delta} \left(\chi - \frac{r_E}{\overline{u}_E A} \right) e^{-\delta T(r_E)}}$$

Then, $\frac{\partial C_E^1(t; r_E)}{\partial r_E} > 0 \Leftrightarrow \frac{\tilde{K}'(r_E)}{\tilde{K}(r_E) - \chi} > (\overline{u}_E A - \delta) T'(r_E)$, which after straightforward manipulations, is equivalent to:

$$\frac{\overline{u}_E A}{\overline{u}_E A - \delta} \frac{\tilde{K}'(r_E)}{\tilde{K}(r_E) - \chi} \left(\chi - \frac{r_E}{\overline{u}_E A} \right) > \frac{1}{\overline{u}_E A} \left(e^{\overline{u}_E A T(r_E)} - 1 \right). \tag{31}$$

Denote the RHS of the inequality by $G(r_E)$. This function is increasing in r_E. Let's define

$$\sigma(\tilde{K}) = \frac{r_E \tilde{K}'(r_E)}{\tilde{K}(r_E)} > 0 \text{ and } \sigma(\psi') = -\frac{r_E \psi''(r_E)}{\psi'(r_E)} > 0$$

If $\sigma(\psi') > \sigma(\tilde{K})$ for all $r_E \in [0, \overline{r}_E]$ then, the LHS of the inequality above is decreasing in r_E. Let $F(r_E)$ be this LHS. Now, imposing $F(\overline{r}_E) > G(\overline{r}_E)$ is sufficient to conclude that $C_E^1(t; r_E)$ is increasing in r_E for all $t \geq 0$.

Optimal choice of the repression level $r_E \in [0, \overline{r}_E]$: The present value of the elite when she uses repression but the revolution occurs in finite time is:

$$V_E(r_E) = \int_0^{T(r_E)} \ln(C_E^1(t; r_E))e^{-\delta t}dt + e^{-\delta T(r_E)}V_E^2(\tilde{K}(r_E) - \chi),$$

Taking the derivative w.r.t r_E, one obtains:

$$\frac{\partial V_E(r_E)}{\partial r_E} = \begin{cases} \int_0^{T(r_E)} \frac{\frac{\partial C_E^1(t;r_E)}{\partial r_E}}{C_E^1(t;r_E)} e^{-\delta t}dt \\ + \left(\ln(C_E^1(T(r_E); r_E) - \delta V_E^2(\tilde{K}(r_E) - \chi)\right) T'(r_E)e^{-\delta T(r_E)} \\ + \frac{\partial V_E^2}{\partial K}\tilde{K}'(r_E)e^{-\delta T(r_E)}. \end{cases}$$

This can be decomposed into 3 different effects: the marginal impact of choosing r_E before the regime change (first term); the marginal impact at this instant of the regime change (second term) and the marginal impact of varying r_E after the switch (last term). Using the expression of C_E^1, this derivative can be rewritten as:

$$\frac{\partial V_E(r_E)}{\partial r_E} = \frac{1}{\delta}\left(\frac{\tilde{K}'(r_E)}{\tilde{K}(r_E) - \chi} - \left(\overline{u}_E A - \delta + (A(1 - \overline{u}_E) - \delta)e^{-\delta T(r_E)}\right) T'(r_E)\right),$$

which is positive under the same two conditions that yield a consumption increasing in r_E.

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